MFE5130 – Financial Derivatives
Class Activity (29-November-2018) (Solution)

Important Notes:

1. This class activity is counted toward to your class participation score. **Fail** to hand in this class activity worksheet in the class will receive **0 score** for that class.

2. **0 mark** will be received if you leave the solution blank.

Name:	Student No.:

Problem 1

You are given:

$$\frac{dS(t)}{S(t)} = 0.5dt + 0.4dZ(t) \qquad G(t) = t^2 S(t)$$

$\{Z(t): t \geq 0\}$ exhibits standard Brownian motion.

Find an expression for $dG(t)$.

Solution

The expression for $G(t)$ is $t^2 S(t)$.

The partial derivatives are:

$$G_S = t^2, \ \ G_{SS} = 0, \ \ G_t = 2tS.$$

From Itô's lemma, we have:

$$dG(t) = G_S dS(t) + \frac{1}{2} G_{SS} [dS(t)]^2 + G_t dt$$

$$= t^2 dS(t) + 2tS(t)dt$$

$$= t^2 [0.5S(t)dt + 0.4S(t)dZ(t)] + 2tS(t)dt$$

$$= t^2 S(t)[0.5dt + 0.4dZ(t)] + t^2 S(t)\frac{2}{t}dt.$$

$$= G(t)\left[\left(\frac{0.5t + 2}{t}\right)dt + 0.4dZ(t)\right].$$