MFE5130 – Financial Derivatives

Class Activity (12-November-2019) (Solution)

Important Notes:

1. This class activity is counted toward to your class participation score. **Fail** to hand in this class activity worksheet in the class will receive **0 score** for that class.
2. **0 mark** will be received if you leave the solution blank.

Name:	Student No.:

Problem 1

The current price of a stock is \$130. The volatility of the stock is 35%. The dividend yield of the stock is 2%.

The continuously compounded risk-free interest rate is 7%.

An 8-month European call option on the stock has a strike price of \$247.

The option is priced using the forward tree with 8 periods.

Calculate the value of the European call option.

Solution

The value of *h* is 1/12 since the intervals are monthly periods. The values of *u* and *d* are:

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.07-0.02)(1/12)+0.35\sqrt{1/12}} = 1.1109,$$
$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(0.07-0.02)(1/12)-0.35\sqrt{1/12}} = 0.9077.$$

The risk-neutral probability of an upward movement is :

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{(0.07-0.02)(1/12)} - 0.9077}{1.1109 - 0.9077} = 0.4748.$$

Let k be the smallest integer such that $u^k d^{n-k} S_0 \geq K$, that is

$$k \geq \frac{\ln\left(\frac{K}{S_0 d^n}\right)}{\ln\left(\frac{u}{d}\right)} = \frac{\ln\left(\frac{247}{130(0.9077)^8}\right)}{\ln\left(\frac{1.1109}{0.9077}\right)} = 7.0124.$$

So, $k = 8$.

The option value is then given by

$$
\begin{aligned}
C &= S_0 C_8^8 \left(p^*\right)^8 \left(1-p^*\right)^0 \frac{u^8 d^0}{e^{rT}} - Ke^{-rT} C_8^8 \left(p^*\right)^8 \left(1-p^*\right)^0 \\
&= 130 e^{-0.07(8/12)} \left(0.4748\right)^8 \left(1.1109\right)^8 - 247 e^{-0.07(8/12)} \left(0.4748\right)^8 \\
&= 0.1344.
\end{aligned}
$$