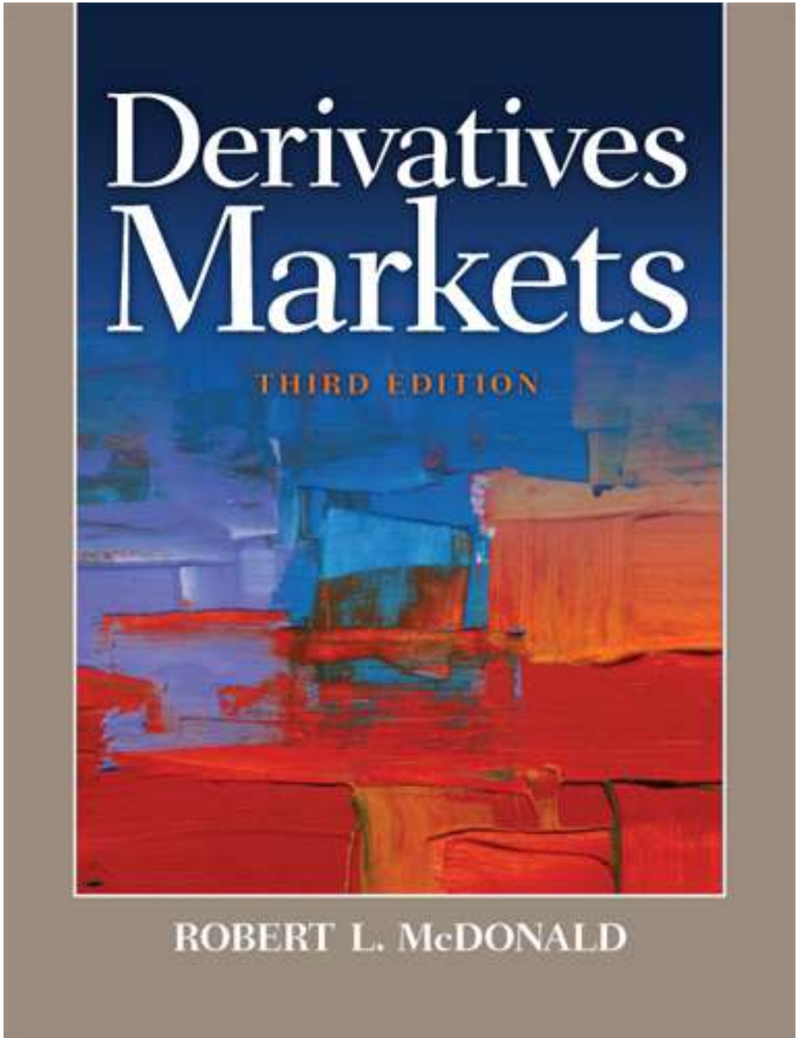


Chapter 8
(Chapter 9 in the textbook)

Parity and Other Option Relationships

Points to Note

1. Important relation: Put-call parity. See P.5
2. Generalized put-call parity on exchange options. See P.9
3. The relationship between call and put options on exchange rate. See P.15
4. Compare the prices of European and American options. See P.16
5. The upper and lower bounds of the option price. See P.17 – 18
6. Early exercises of American call and put options. See P.19 – 24
7. Relationship between time to expiration and option price. See P. 25
8. Relationship between strike prices and option prices. See P.26 - 30
9. Convexity property of option prices with respect to strike prices. See P.31

Revision

① Currency option

currency ① currency ②

x_0 : exchange rate (price of ② in ①)

eg. x_0 : \$1.2 / € (price of 1 € in \$)

denominated ①

$$C_{①}(\underset{\Phi}{x_0}, \underset{\Phi}{K}, T) = K x_0 P_{②}\left(\underset{\Phi}{\frac{1}{x_0}}, \frac{1}{K}, T\right)$$

① - denominated

call option

underlying : ②

② - denominated

put option

underlying : ①

put - call parity

$$C - P = F^{P}_{0,T}(S) - PV(K)$$

$S_0 e^{-\delta T}$ Ke^{-rT}

$$\underbrace{C_{①}(x_0, K, T) - P_{①}(x_0, K, T)}_{\text{underlying } ②}$$

$$= x_0 e^{-\delta T} - K e^{-rT}$$

δ : $r_{②}$

r : $r_{①}$

depends on currency

$C_{Amer} \geq C_{EUR}$

$P_{Amer} \geq P_{EUR}$

Early Ex of American options

	Call	Put
non-dividend asset	NE	E
dividend paying asset	E	E

NE : no early Ex, E : Early Ex is possible

$C_{Amer} = C_{EUR}$ if the asset does not have dividend.

Non-dividend paying asset

$$C_{EUR} = \underbrace{P_{EUR}}_{>0} + S_t - K + \underbrace{K(1 - e^{-r(T-t)})}_{>0}$$

$$> S_t - K$$

$C_{Amer} > S_t - K$ (No Early Ex)

Dividend paying asset

① $\begin{cases} >0 \\ <0 \end{cases}$ depends on PV(Div)

$$C_{EUR} = S_t - K + \boxed{P_{EUR} + (K - PV_{t,T}(K)) - PV(\text{Divid})}$$

Early Ex ← C_{Amer}

C_{Amer}

C_{EUR} $\quad S_t - K$

if ① < 0

C_{Amer} if 1 < 0

C_{EUR}

if ① 0

Inequalities

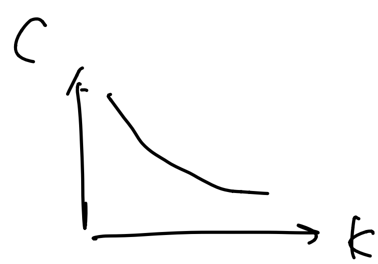
$$C(K_1) - C(K_2) \le K_2 - K_1$$
$$P(K_2) - P(K_1) \le K_2 - K_1$$

both Eur & Amer

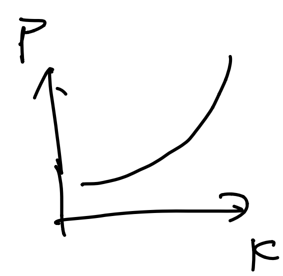
$$C(K_1) - C(K_2) \le PV(K_2 - K_1)$$
$$P(K_2) - P(K_1) \le PV(K_2 - K_1)$$

European

$$\frac{C(K_1) - C(K_2)}{K_2 - K_1} \ge \frac{C(K_2) - C(K_3)}{K_3 - K_2}$$



$$\frac{P(K_2) - P(K_1)}{K_2 - K_1} \le \frac{P(K_3) - P(K_2)}{K_3 - K_2}$$



Tutorial - Class Activity

29 October, 2019 (Solution)

Problem 1

underlying

£0.42 price of 1 AUD

The current exchange rate is 0.42 British pounds per Australian dollar.

A pound-denominated European Australian dollar put has a strike price of 0.4 pounds and a premium of 0.0133 pounds. The put expires in 1 year.

$P_{£}(0.42, 0.4, 1)$

A continuously compounded interest rate available on British pounds is 8%. The continuously compounded interest rate available on Australian dollars is 7%. $r_{£} = 8\%$ $r_{AUD} = 7\%$

Calculate the value of an Australian dollar-denominated European British pound put that has a strike price of 2.5 Australian dollars and expires in 1 year.

underlying

$P_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right)$

Solution

The price of Australian dollar-denominated European British pound call is given by

$$
\begin{aligned}
C_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) &= \left(\frac{1}{0.42}\right)(2.5)\, P_{Pound}\left(0.42, \frac{1}{2.5}, 1\right) \\
&= \left(\frac{1}{0.42}\right)(2.5)(0.0133) \\
&= \text{AUD } 0.07917.
\end{aligned}
$$

0.4

$P_{£} \xrightarrow{??} P_{AUD}$

① Find C AUD from the $P_{£}$

By the put-call parity,

r pound

$$
C_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) - P_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) = \frac{1}{0.42} e^{-0.08} - 2.5 e^{-0.07}
$$

r_{AUD}

$$
0.07917 - P_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) = \frac{1}{0.42} e^{-0.08} - 2.5 e^{-0.07}
$$

$$
P_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) = \text{AUD } 0.2123.
$$

② Use the put-call parity to find PAUD from CAUD

2nd method

① use the put-call parity to find $C_{£}$ from $P_{£}$

② find PAUD from $C_{£}$ use the identity.

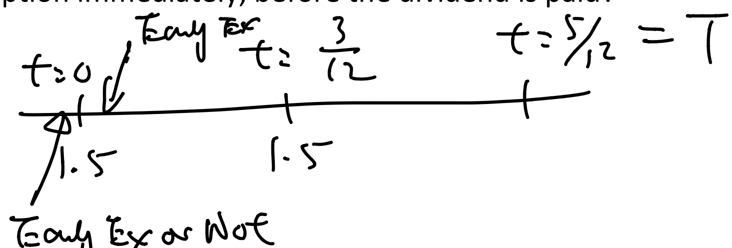
Problem 2 $K=85$ $T=\frac{5}{12}$

An American call option on a stock has a strike price of 85 and expires in 5 months. You are given

i. The continuously compounded risk-free interest rate is 4%. $r = 4\%$
ii. The dividend of 1.5 payable at the end of today, and another dividend of 1.5 is payable in 3 months.
iii. The current price of the stock is 100. $P = 0.82$
iv. A European put option with a strike price of 85 which expires in 5 months costs 0.82.

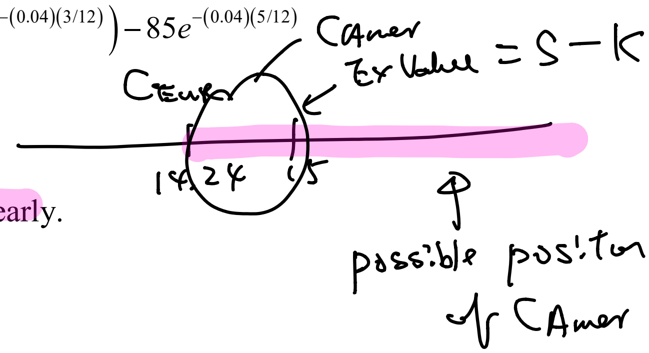
Could it be rational to exercise the option immediately, before the dividend is paid?

Solution



By the put-call parity, we have

$$
\begin{aligned}
&C_{Eur}\left(100,85,\frac{5}{12}\right)\\
&= P_{Eur}\left(100,85,\frac{5}{12}\right)+100-\left(1.5+1.5e^{-(0.04)(3/12)}\right)-85e^{-(0.04)(5/12)}\\
&= P_{Eur}\left(100,85,\frac{5}{12}\right)+100-\left(1.5+1.5e^{-(0.04)(3/12)}\right)-85e^{-(0.04)(5/12)}\\
&=14.2399\\
&<15 = \underbrace{100-85}_{\text{Ex Value}} \text{ (or } S-K\text{)}.
\end{aligned}
$$



So, it may be rational to exercise the option early.

Problem 3

Three European put options expire in 1 year. The put options have the same underlying asset, but they have different strike prices and premiums.

Put Option	A	B	C
Strike	\$50.00	\$55.00	\$61.00
Premium	\$3.00	\$7.00	\$11.00

The continuously compounded risk-free interest rate is 11%.

a. What no-arbitrage property is violated?

b. What spread position would you use to effect arbitrage?

$P(K_2) - P(K_1) \le K_2 - K_1$

$P(K_2) - P(K_1) \le PV(K_2 - K_1)$

$\frac{P(K_2) - P(K_1)}{K_2 - K_1} \le \cdots$

c. Demonstrate that the spread position is an arbitrage.

Solution

(a)

The prices of the options violate the following inequality

$$\frac{P(K_2)-P(K_1)}{K_2-K_1} \leq \frac{P(K_3)-P(K_2)}{K_3-K_2}$$

Because:

$$\frac{7-3}{55-50} > \frac{11-7}{61-55}$$

$$\frac{4}{5} > \frac{4}{6}$$

LHS > RHS

(b)

The above violated inequality can be rewritten as

LHS high

$$\frac{P(55)-P(50)}{55-50} > \frac{P(61)-P(55)}{61-55}$$

RHS RHS Low

$$6\big(P(55)-P(50)\big) > 5\big(P(61)-P(55)\big)$$

$$0 > 6P(50)-11P(55)+5P(61).$$

The arbitrage profit can be obtained by using the <u>*asymmetric butterfly spread*</u> with the following transactions:

Buy 6 of the 50-strike put options
Sell 11 of the 55-strike put options
Buy 5 of the 61-strike put options

put

(c)

		***t* = 1 year**			
Transaction	***t* = 0**	$S_1 < 50$	$50 \leq S_1 \leq 55$	$55 < S_1 \leq 61$	$61 < S_1$
Buy 6 of $P(50)$	–6(3.00)	$6(50 - S_1)$	0.00	0.00	0.00
Sell 11 of $P(55)$	11(7.00)	$-11(55 - S_1)$	$-11(55 - S_1)$	0.00	0.00
Buy 5 of $P(61)$	–5(11.00)	$5(61 - S_1)$	$5(61 - S_1)$	$5(61 - S_1)$	0.00
Total	4.00	0.00	$6S_1 - 300$	$305 - 5S_1$	0.00

>0

≥ 0

where $P(K)$ is the price of the K-strike put option.

This strategy has strictly positive cash inflow at $t = 0$, and has a nonnegative payoff for all possible values S_1 of at $t = 1$ year. Therefore, this is an arbitrage strategy.

Buy (RHS + Sell (LHS

(Long 5 61-put
Short 5 55-put)

(Long 6 55-put)
Short 6 50-put)

⇒ Long 5 61-put + Short 5 55-put + Short 6 55-put + Long 6 50-put

⇒ Long 5 61-put + Short 11 55-put + Long 5 50-put