

MFE5130 – Financial Derivatives
Class Activity (8-November-2018) (Solution)

Important Notes:

1. This class activity is counted toward to your class participation score. **Fail** to hand in this class activity worksheet in the class will receive **0 score** for that class.
2. **0 mark** will be received if you leave the solution blank.

Name:	Student No.:
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Problem 1

The current price of a stock is \$130. The volatility of the stock is 35%. The dividend yield of the stock is 2%.

The continuously compounded risk-free interest rate is 7%.

An 8-month European call option on the stock has a strike price of \$247.

The option is priced using the forward tree with 8 periods.

Calculate the value of the European call option.

Solution

The value of h is $1/12$ since the intervals are monthly periods. The values of u and d are:

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{(0.07-0.02)(1/12) + 0.35\sqrt{1/12}} = 1.1109,$$
$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{(0.07-0.02)(1/12) - 0.35\sqrt{1/12}} = 0.9077.$$

The risk-neutral probability of an upward movement is :

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{(0.07-0.02)(1/12)} - 0.9077}{1.1109 - 0.9077} = 0.4748.$$

Let k be the smallest integer such that $u^k d^{n-k} S_0 \geq K$, that is

$$k \geq \frac{\ln\left(\frac{K}{S_0 d^n}\right)}{\ln\left(\frac{u}{d}\right)} = \frac{\ln\left(\frac{247}{130(0.9077)^8}\right)}{\ln\left(\frac{1.1109}{0.9077}\right)} = 7.0124.$$

So, $k = 8$.

The option value is then given by

$$\begin{aligned}
C &= S_0 C_8^8(p^*)^8 (1-p^*)^0 \frac{u^8 d^0}{e^{rT}} - K e^{-rT} C_8^8(p^*)^8 (1-p^*)^0 \\
&= 130 e^{-0.07(8/12)} (0.4748)^8 (1.1109)^8 - 247 e^{-0.07(8/12)} (0.4748)^8 \\
&= 0.1344.
\end{aligned}$$