MFE5130 – Financial Derivatives
First Term, 2019 – 20

Assignment 2 (Solution)

Question 5.2

a) The owner of the stock is entitled to receive dividends. As we will get the stock only in one year, the value of the prepaid forward contract is today's stock price, less the present value of the four dividend payments:

$$F_{0,T}^{P} = \$50 - \sum_{i=1}^{4} \$1e^{-0.06\times\frac{3}{12}i} = \$50 - \$0.985 - \$0.970 - \$0.956 - \$0.942$$
$$= \$50 - \$3.853 = \$46.147$$

b) The forward price is equivalent to the future value of the prepaid forward. With an interest rate of 6 percent and an expiration of the forward in one year, we thus have:

$$F_{0,T} = F_{0,T}^{P} \times e^{0.06\times 1} = \$46.147 \times e^{0.06\times 1} = \$46.147 \times 1.0618 = \$49.00$$

Question 5.4

a) We plug the continuously compounded interest rate and the time to expiration in years into the valuation formula and notice that the time to expiration is six months, or 0.5 years. We have:

$$F_{0,T} = S_0 \times e^{r\times T} = \$35 \times e^{0.05\times 0.5} = \$35 \times 1.0253 = \$35.886$$

b) The annualized forward premium is calculated as:

$$\text{annualized forward premium} = \frac{1}{T}\ln\left(\frac{F_{0,T}}{S_0}\right) = \frac{1}{0.5}\ln\left(\frac{\$35.50}{\$35}\right) = 0.0284$$

c) For the case of continuous dividends, the forward premium is simply the difference between the risk-free rate and the dividend yield:

$$\text{annualized forward premium} = \frac{1}{T}\ln\left(\frac{F_{0,T}}{S_0}\right) = \frac{1}{T}\ln\left(\frac{S_0 \times e^{(r-\delta)T}}{S_0}\right)$$
$$= \frac{1}{T}\ln\left(e^{(r-\delta)T}\right) = \frac{1}{T}(r-\delta)T$$
$$= r - \delta$$

Therefore, we can solve:

$$0.0284 = 0.05 - \delta$$
$$\Leftrightarrow \delta = 0.0216$$

The annualized dividend yield is 2.16%.

Question 5.5

a) We plug the continuously compounded interest rate and the time to expiration in years into the valuation formula and notice that the time to expiration is nine months, or 0.75 years. We have:

$$F_{0,T} = S_0 \times e^{r \times T} = \$1{,}100 \times e^{0.05 \times 0.75} = \$1{,}100 \times 1.0382 = \$1{,}142.02$$

b) We engage in a reverse cash and carry strategy. In particular, we do the following:

Description	Today	In nine months
Long forward, resulting from customer purchase	0	$S_T - \$1{,}142.02$
Sell short the index	\$1,100	$-S_T$
Lend \$ 1,100	−\$1,100	$\$1{,}100 \times e^{0.05\times 0.75}$
		$= \$1{,}142.02$
TOTAL	0	0

Therefore, the market maker is perfectly hedged. She does not have any risk in the future because she has successfully created a synthetic short position in the forward contract.

c) Now, we will engage in cash and carry arbitrage:

Description	Today	In nine months
Short forward, resulting from customer purchase	0	$\$1{,}\ 142.02 - S_T$
Buy the index	−\$1,100	S_T
Borrow \$1,100	\$1,100	$-\$1{,}100 \times e^{0.05\times 0.75}$
		$= -\$1{,}142.02$
TOTAL	0	0

Again, the market maker is perfectly hedged. He does not have any index price risk in the future, because he has successfully created a synthetic long position in the forward contract that perfectly offsets his obligation from the sold forward contract.

Question 5.8

First, we need to find the fair value of the forward price. We plug the continuously compounded interest rate, the dividend yield and the time to expiration in years into the valuation formula and notice that the time to expiration is six months, or 0.5 years. We have:

$$F_{0,T} = S_0 \times e^{(r-\delta)\times T} = \$1{,}100 \times e^{(0.05-0.02)\times 0.5} = \$1{,}100 \times 1.01511 = \$1{,}116.62$$

a) If we observe a forward price of \$1,120, we know that the forward is too expensive, relative to the fair value we have determined. Therefore, we will sell the forward at \$1,120, and create a synthetic forward for \$1,116.82, making a sure profit of \$3.38. As we sell the real forward, we engage in cash and carry arbitrage:

Description	Today	In six months
Short forward	0	$\$1{,}120.00 - S_T$
Buy tailed position in index	$-\$1{,}100 \times e^{(-0.02)\times 0.5}$ $= -\$1{,}089.055$	S_T
Borrow \$1,089.055	\$1,089.055	−\$1,116.62
TOTAL	0	\$3.38

This position requires no initial investment, has no index price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy.

b) If we observe a forward price of \$1,110, we know that the forward is too cheap, relative to the fair value we have determined. Therefore, we will buy the forward at \$1,110, and create a synthetic short forward for \$1,116.62, thus making a sure profit of \$6.62. As we buy the real forward, we engage in a reverse cash and carry arbitrage:

Description	Today	In six months
Long forward	0	$S_T - \$1{,}110.00$
Sell short tailed position in index	\$1,100 × .99 = \$1,089.055	$-S_T$
Lend \$1,089.055	−\$1,089.055	\$1,116.62
TOTAL	0	\$6.62

This position requires no initial investment, has no index price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy.

<u>Question 5.12</u>

a) The notional value of 10 contracts is 10 × \$250 × 950 = \$2,375,000, because each index point is worth \$250, we buy 10 contracts and the S&P 500 index level is 950.

With an initial margin of 10 percent of the notional value, this results in an initial dollar margin of:

$$\$2{,}375{,}000 \times 0.10 = \$237{,}500.$$

b) We first obtain an approximation. Because we have a 10 percent initial margin, a 2 percent decline in the futures price will result in a 20 percent decline in margin. As we will receive a margin call after a 20 percent decline in the initial margin, the smallest futures price that avoids the maintenance margin call is 950 × 0.98 = 931. However, this calculation ignores the interest that we are able to earn in our margin account.

Let us now calculate the details. We have the right to earn interest on our initial margin position. As the continuously compounded interest rate is currently 6 percent, after one week, our initial margin has grown to:

$$\$237,500e^{0.06\times\frac{1}{52}} = \$237,774.20$$

We will get a margin call if the initial margin falls by 20 percent. We calculate 80 percent of the initial margin as:

$$\$237,500 \times 0.8 = \$190,000$$

Ten long S&P 500 futures contracts obligate us to pay $2,500 times the forward price at expiration of the futures contract.

Therefore, we have to solve the following equation:

$$\begin{aligned}
& \$237,774.20 + (F_{1W} - 950) \times \$2,500 \geq \$190,000 \\
\Leftrightarrow\quad & \$47774.20 \geq -(F_{1W} - 950) \times \$2,500 \\
\Leftrightarrow\quad & 19.10968 - 950 \geq -F_{1W} \\
\Leftrightarrow\quad & F_{1W} \geq 930.89
\end{aligned}$$

Therefore, the greatest S&P 500 index futures price at which we will receive a margin call is 930.88.

Question 6.6

Both the present value and future value methods can be used to calculate the storage cost.

b) Let us take the December Year 0 forward price as a proxy for the spot price in December Year 0. We can then calculate with our cash and carry arbitrage tableau:

Transaction	Time 0	Time $T = 3/12$
Short March forward	0	$3.075 - S_T$
Buy December Forward (= Buy spot)	−3.00	S_T
Pay storage cost		−0.03
Total	−3.00	3.045

Present Value Method

We can calculate the annualized rate of return as:

$$\frac{3.075}{3.00 + 0.03e^{-6\% \times 0.25}} = e^{rT}$$

$$1.015 = e^{r0.25} \qquad = e^{(r) \times T}$$

$$r = 0.0596.$$

which is the prevailing risk-free interest rate of 0.06. This result seems to make sense.

Future Value Method

We can calculate the annualized rate of return as:

$$\frac{3.045}{3.00} = e^{(r) \times T}$$

$$\Leftrightarrow \ln\left(\frac{3.045}{3.00}\right) = r \times 3/12$$

$$r = 0.05955$$

which is the prevailing risk-free interest rate of 0.06. This result seems to make sense.

c) Let us again take the December Year 0 forward price as a proxy for the spot price in December Year 0.

Present Value Method

Transaction	Time 0	Time T= 9/12
Short Sep forward	0	$2.75 - S_T$
Buy spot	−3.00	S_T
PV(Storage Mar)	−0.0296	
PV(Storage Jun)	−0.0291	
PV(Storage Sep)	−0.0287	
Total	−3.0874	2.75

We can calculate the annualized rate of return as:

$$\frac{2.75}{3.0874} = e^{(r) \times T} \Leftrightarrow \ln\left(\frac{2.75}{3.0874}\right) = r \times 9/12$$

$$r = -0.1543$$

Future Value Method

Transaction	Time 0	Time T= 9/12
Short Sep forward	0	$2.75 - S_T$
Buy spot	−3.00	S_T

Pay storage cost Sep		−0.03
FV(Storage Jun)		−0.0305
FV(Storage Mar)		−0.0309
Total	−3.00	2.6586

We can calculate the annualized rate of return as:

$$\frac{2.6586}{3.00} = e^{(r)\times T} \Leftrightarrow \ln\left(\frac{2.6586}{3.00}\right) = r \times 9/12$$

$$r = -0.16108$$

For both present value and future value methods, the result does not seem to make sense. We would earn a negative annualized return on such a cash and carry arbitrage. Therefore, it is likely that our naive calculations do not capture an important fact about the widget market. In particular, we will buy and hold the widget through a time where the forward curve indicates that there is a significant convenience yield attached to widgets.

Question 7.3

Maturity	Zero-Coupon Bond Yield	Zero-Coupon Bond Price	One-Year Implied Forward Rate	Par Coupon	Cont. Comp. Zero Yield
1	0.03000	0.97087	0.03000	0.03000	0.02956
2	0.03500	0.93351	0.04002	0.03491	0.03440
3	0.04000	0.88900	0.05007	0.03974	0.03922
4	0.04500	0.83856	0.06014	0.04445	0.04402
5	0.05000	0.78353	0.07024	0.04903	0.04879

Question 7.7

a) We are looking for $r_0(1, 3)$. We will use equation (7.3) of the main text, and the known one-year and three-year zero-coupon bond prices. We have to solve the following equation:

$$\left[1 + r_0(1,3)\right]^{3-1} = \frac{P(0,1)}{P(0,3)}$$

$$\Leftrightarrow r_0(1, 3) = \sqrt{\frac{P(0,1)}{P(0,3)}} - 1 = \sqrt{\frac{0.943396}{0.816298}} - 1 = 0.07504$$

Additional Problem 1

The present value of the storage costs for 9 months are

$$0.06+0.06e^{-0.25\times 0.1}+0.06e^{-0.5\times 0.1}=0.176.$$

The futures price is given by

$$F_{0,0.75}=(9+0.176)e^{0.1\times 0.75}=9.89.$$

That is, it is \$9.89 per ounce.

Additional Problem 2

It is known that

$$F_{0,T}=\frac{S_0 e^{(u-y)T}}{P(0,T)},$$

where S_0 is the current spot price of corn, u is the storage cost of corn and y is the convenience yield of corn.

So,

$$F_{0,T}=\frac{4.8e^{(1.8\%-4.2\%)T}}{P(0,T)}=\frac{4.8e^{-(2.4\%)T}}{P(0,T)}.$$

T (in Years)	1	2	3
$F_{0,T}$	4.7278	4.7161	4.6927