MFE5130 – Financial Derivatives
First Term, 2017-18
Final Examination

Exam Duration: 2 hours

Instruction

1. Total Marks: 100 points.
2. Answer **ALL** questions.
3. You must show all the steps in order to get full mark for each question.
4. When using the standard normal distribution table attached on P.5, do not interpolate.
   - Use the nearest z-value in the table to find the probability. Example: Suppose that you are to find $\Pr(Z < 0.759)$, where Z denotes a standard normal random variable. Because the z-value in the table nearest to 0.759 is 0.76, your answer is $\Pr(Z < 0.76) = 0.7764$.
   - Use the nearest probability value in the table to find the z-value. Example: Suppose that you are to find z such that $\Pr(Z < z) = 0.7$. Because the probability value in the table nearest to 0.7 is 0.6985, your answer is 0.52.

1. (20 points) Assume the Black-Scholes framework. Consider a six-month financial derivative on a stock with the following payoff:

$$\text{Payoff} = \begin{cases} S(0.5) & \text{if } 0 \le S(0.5) \le 80, \\ 80 & \text{if } S(0.5) > 80. \end{cases}$$

where $S(0.5)$ is the stock price at $t = 0.5$.

You are given that

(i) The time-0 stock price is \$90.

(ii) The stock's volatility is 30%.

(iii) The stock pays dividends at a continuously compounded rate of 3%.

(iv) The continuously compounded risk-free interest rate is 6%.

Calculate the elasticity of this financial derivative at time 0.

2. The current price of a stock is \$90. The stock pays dividends at a continuously compounded rate of 3%. The volatility of the stock is 30%. The continuously compounded risk-free interest rate is 4%.

A 1-year American put option on the stock has a strike price of \$86. The option is priced using a binomial (forward) tree with $n = 3$ (i.e., a three-period binomial tree).

a. (20 points) Calculatue the price of the American put option at time 0.

b. (5 points) Calculate the value of delta of the American put option at time 0.

3. (15 points) The price of a stock is \$59. The stock does not pay dividends. The four call options described below are written on the stock.

Type	Strike	Maturity	Price	Delta	Gamma
Call	90	91 days	8.81	0.7015	0.0350
Call	95	91 days	6.03	0.5646	0.0381
Call	100	91 days	3.94	0.4388	0.0297
Call	95	180 days	9.18	0.5958	0.0208

A market-maker buys 200 of \$90-strike 91-day call, sells 150 of \$95-strike 91-day call and sells 90 of \$100-strike 91-day call.

The market-maker uses the stock and the \$95-strike 180-day call option to delta-gamma hedge the position.

Determine the number of shares of the stock and the number of \$95-strike 180-day call options that the market-maker has to use to delta-gamma hedge his portfolio.

4. (25 points) Let $X(t)$ and $Y(t)$ be the price processes of the two stocks.

Suppose that $X(0) = 12$ and $Y(0) = 15$. Neither stock pays dividends. Under the risk-neutral measure Q, $X(t)$ and $Y(t)$ are governed by

$$\frac{dX(t)}{X(t)} = 0.04dt + 0.28dZ_1(t) + 0.12dZ_2(t)$$

$$\frac{dY(t)}{Y(t)} = 0.04dt + 0.31dZ_2(t)$$

where $Z_1(t)$ and $Z_2(t)$ are two independent standard Brownian motions under Q.

Consider a European type financial derivative with 3 years to expiration. The payoff at the maturiy is given by

$$\max\left[X(3)Y^2(3) - 235X(3), 0\right].$$

Find the price of the financial derivative at time 0.

5. (15 points) Let $S_1(t)$ and $S_2(t)$ be two stochastic processes. The dynamics of $S_1(t)$ and $S_2(t)$ under the probability measure P are governed by

$$\frac{dS_1(t)}{S_1(t)} = 0.06dt + 0.42dZ_1(t),$$
$$\frac{dS_2(t)}{S_2(t)} = 0.03dt + 0.19dZ_2(t),$$

where $Z_1(t)$ and $Z_2(t)$ are two correlated standard Brownian motions under the probability measure P and $dZ_1(t)dZ_2(t) = 0.2dt$.

At time 0, we have $S_1(0) = 4$ and $S_2(0) = 2$.

Under the probability measure P, find $\Pr(S_1(3) > 3S_2(3))$.

End

NORMAL DISTRIBUTION TABLE

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z<z)
The value of z to the first decimal is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Values of z for selected values of Pr(Z<z)							
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995