



# Useful Results for GBM

Given

$$dX(t) = X(t) [\mu dt + \sigma dZ(t)],$$

where  $\mu$  and  $\sigma$  are constants,  $Z(t)$  is a Brownian motion.

The following results can be deduced:

- (i)  $E(\max(X(T) - K, 0)) = X(0)e^{\mu T}N(\tilde{d}_1) - KN(\tilde{d}_2),$
- (ii)  $E(\max(K - X(T), 0)) = KN(-\tilde{d}_2) - X(0)e^{\mu T}N(-\tilde{d}_1),$



## Exchange Options (cont'd)

where

$$\tilde{d}_1 = \frac{\ln\left(\frac{X(0)}{K}\right) + (\mu + 0.5\sigma^2)T}{\sigma\sqrt{T}}, \quad \tilde{d}_2 = \tilde{d}_1 - \sigma\sqrt{T}.$$