Tutorial - Class Activity

15 October, 2019 (Solution)

Question 1

Suppose that copper costs \$3.00 per pound today and the lease rate for copper is 5%. The continuously compounded interest rate is 10%. The copper price in 1 year is uncertain and copper can be stored costlessly.

a. If you short-sell a pound of copper for 1 year, what payment do you have to make to the copper lender?
b. Find the equilibrium forward price.

Solution

(a)

As we need to borrow a pound of copper to sell it short, we must pay the lender the lease rate for the time we borrow the asset, i.e., until expiration of the contract in one year.

Including the leasing cost, the short seller needs to pay back $e^{0.05}$ pounds of copper to the lender at the end of one year.

At the end of one year, the amount that the short seller has to pay to the copper lender

$$= S_T \times e^{0.05} = 1.05127 \times S_T .$$

(b)

The equilibrium forward price is calculated according to our pricing formula:

$$F_{0,T} = S_0 \times e^{(r-\delta_l)\times T} = \$3.00 \times e^{(0.10-0.05)\times 1} = \$3.00 \times 1.05127 = \$3.1538.$$

Question 2

Suppose that the November price of corn is \$2.50/bushel, the effective monthly interest rate is 1%, and the storage cost per bushel are \$0.05/month. Assuming that corn is stored from November to February (i.e. the storage cost is paid on Dec, Jan and Feb).

(a) Find the theoretical forward price of this February forward.
(b) Suppose the February forward price had been \$2.80. What would the arbitrage be?

Solution

(a)

The future value of the storage cost

$\$0.05 + (\$0.05 \times 1.01) + (\$0.05 \times 1.01^2) = \$0.1515$

Thus, the February forward price will be

$2.5 \times (1.01)^3 + 0.1515 = 2.7273.$

(b)

If the February corn forward price is $2.80, the observed forward price is too expensive relative to our theoretical price of $2.7273. We will therefore sell the February contract short, and create a synthetic long position, engaging in cash and carry arbitrage:

Transaction	Nov	Dec	Jan	Feb
Short Feb forward	0			$2.80 - S_T$
Buy spot	−2.50			S_T
Borrow purchasing cost	+2.50			−2.57575
Pay storage cost Dec,		−0.05		−0.051005
borrow storage cost		+0.05		
Pay storage cost Jan,			−0.05	−0.0505
borrow storage cost			+0.05	
Pay storage cost Feb				−0.05
Total	0	0	0	0.072745

We made an arbitrage profit of 0.07 dollar.

Question 3

7.4 Suppose you observe the following 1-year implied forward rates: 0.050000 (1-year), 0.034061 (2-year), 0.036012 (3-year), 0.024092 (4-year), 0.001470 (5-year). For each maturity year compute the zero-coupon bond prices, effective annual and continuously compounded zero-coupon bond yields, and the par coupon rate.

Solution

Maturity	Zero-Coupon Bond Yield	Zero-Coupon Bond Price	One-Year Implied Forward Rate	Par Coupon	Cont. Comp. Zero Yield
1	0.05000	0.95238	0.05000	0.05000	0.04879

2	0.04200	0.92101	0.03406	0.04216	0.04114
3	0.04000	0.88900	0.03601	0.04018	0.03922
4	0.03600	0.86808	0.02409	0.03634	0.03537
5	0.02900	0.86681	0.00147	0.02962	0.02859

Notes

1. From the one-year implied forward rate, we can find $P(0, t_j)$.
   For example,
$$\begin{aligned}
1+r_0(0,1) &= \frac{P(0,0)}{P(0,1)} \\
&= \frac{1}{P(0,1)} \\
P(0,1) &= \frac{1}{1+r_0(0,1)} \\
&= \frac{1}{1+0.05} \\
&= 0.95238. \\
1+r_0(1,2) &= \frac{P(0,1)}{P(0,2)} \\
&= \frac{0.95238}{P(0,2)} \\
P(0,2) &= \frac{0.95238}{1+r_0(1,2)} \\
&= \frac{0.95238}{1+0.034061} \\
&= 0.92101.
\end{aligned}$$
2. The **zero-coupon bond yield** and **continuous compounding zero-coupon yield** can be calculated by using the following formula:
$$P(0,t) = \frac{1}{\left(1+r(0,t)\right)^t},$$
$$P(0,t) = e^{-r_{CC}(0,t)t}.$$

   It is worth to note that the 1-year implied forward rate which matures at the end of the first year is our $r(0, 1)$.

3. The **par coupon** for the t_n maturity bond ($c(t_n)$) can be calculated by the following formula:

$$c(t_n) = \frac{1 - P(0, t_n)}{\sum_{i=1}^{n} P(0, t_i)}.$$