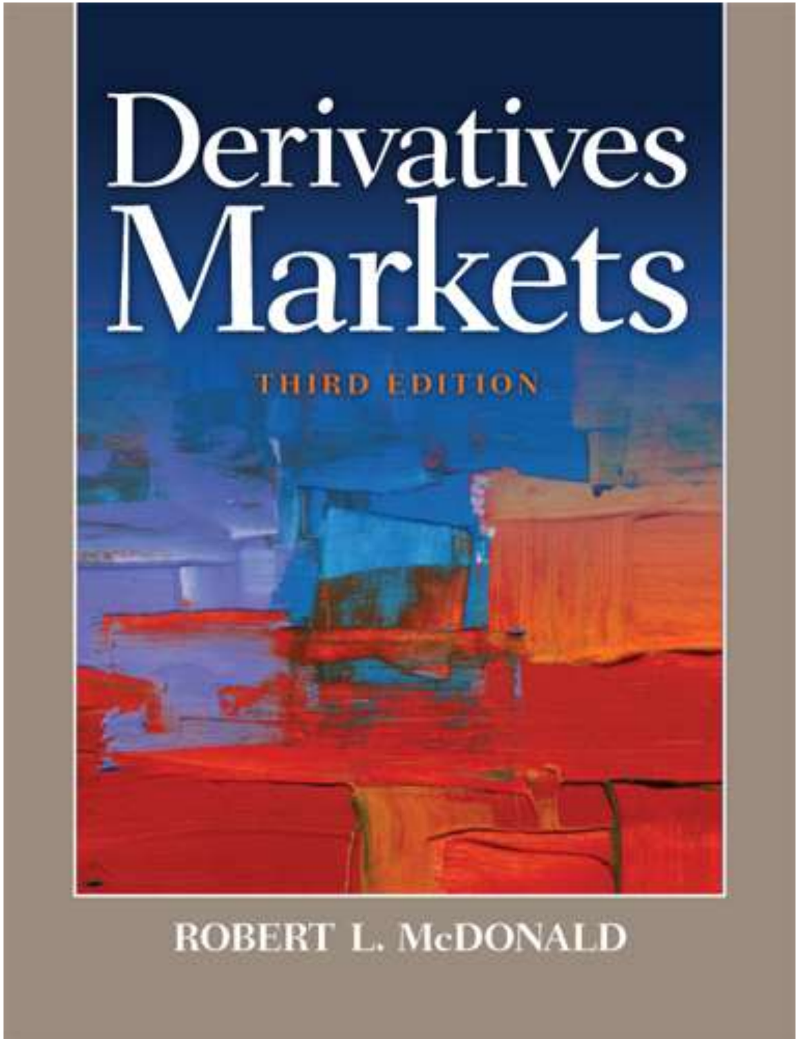


Chapter 6

(Chapter 7 in the textbook)

Bond Basics

Points to Note

1. Definition of $r_t(t_1,t_2)$, see P.4.
2. What is the relationship between the bond price $P(0, n)$ and $r(0,n)$? See P.5.

   yield to maturity
3. How to find YTM from the zero coupon price? See P.7.
4. How to find the implied forward rate? See P.8 – 9.
5. How to find the implied forward zero-coupon price? See P.10.
6. Coupon bonds, see P.12.
7. Bootstrapping zero-coupon price from coupon bonds, see P.14 – 15.
8. Definition of continuously compounded yields $r^{cc}(0, t)$.

Bond Basics (cont'd)

- Zero-coupon bonds make a single payment at maturity

TABLE 7.1 Five ways to present equivalent information about default-free interest rates. All rates but those in the last column are effective annual rates.

Years to Maturity	(1) Zero-Coupon Bond Yield	(2) Zero-Coupon Bond Price		(3) ?? One-Year Implied Forward Rate	(4) Par Coupon	(5) Continuously Compounded Zero Yield
1	6.00%	0.943396	①	6.00000%	6.00000%	5.82689%
2	6.50	0.881659	②	7.00236	6.48423	6.29748
3	7.00	0.816298	③	8.00705	6.95485	6.76586

$① = \frac{1}{(1+r(0,1))} = \frac{1}{1+6\%}$

$② = \frac{1}{(1+6.5\%)^2}$

$③ = \frac{1}{(1+7\%)^3}$

Coupon bond = portfolio of zero - coupon bonds

Bond Basics (cont'd)

- Zeros from Coupons
  - **Bootstrapping**: the procedure in which zero coupon bond prices are deduced from a set of coupon bond prices.
  - From Column (4) in Table 7.1, we have

bond price =

$$1 = (1 + 0.06)P(0,1)$$

$$P(0,1) = 0.943396$$

The second par coupon bond gives us

$$1 = 0.0648423P(0,1) + 1.0648423P(0,2)$$

$$P(0,2) = 0.881659$$

Bond Basics (cont'd)

– Similarly, we find

$$1 = 0.0695485P(0,1) + 0.0695485P(0,2) + 1.0695485P(0,3)$$

$$P(0,3) = 0.816298$$

Bond Basics (cont'd)

- Continuously Compounded Yields
  - In general, if we have a zero-coupon bond paying $1 at maturity, we can write its price in terms of an annualized continuously compounded yield, $r^{cc}(0,t)$, as

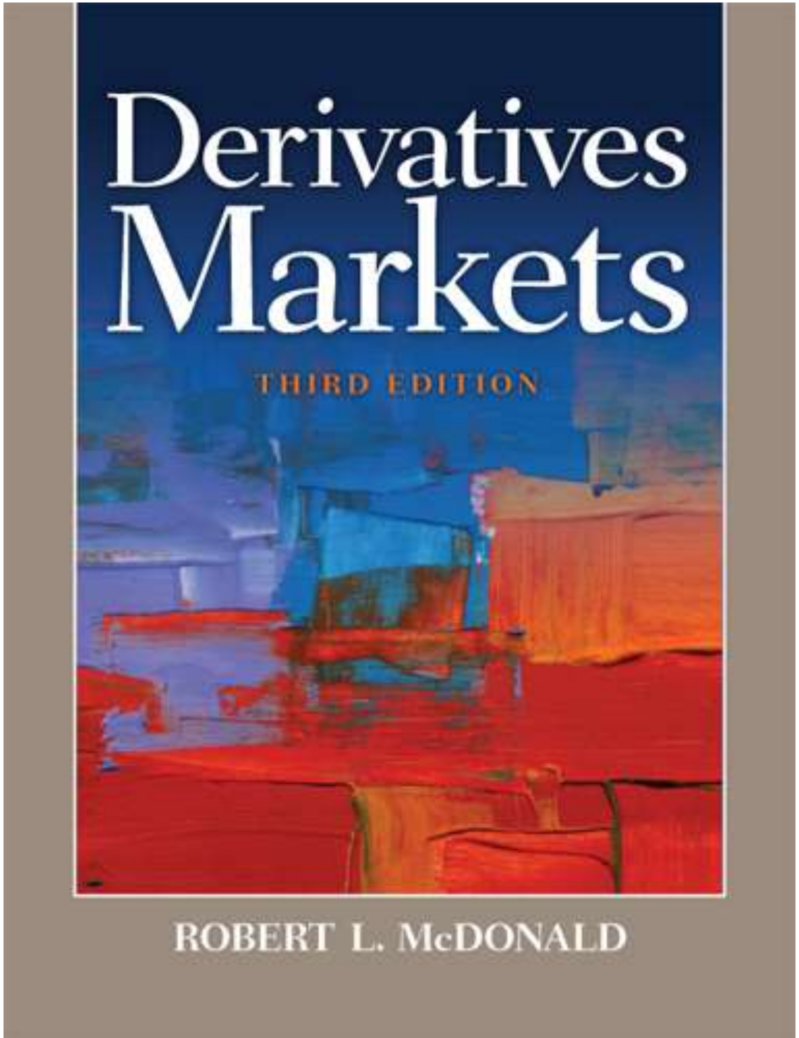
$$P(0,t) = e^{-r^{CC}(0,t)t}$$

$$P(0,t) = \frac{1}{(1+r(0,t))^t}$$

discrete

$r(0,t)$

zero-coupon yield



Chapter 7
(Chapter 8 in the textbook)

Swaps

a portfolio of forward contracts

Points to Note

1. How does the swap compare with the forward contracts? See P.3 – 6.
2. What are the meaning of the long and short positions? See P. 7.
3. What is the difference between the physical settlement and financial settlement? See P. 7 – 13.
4. Understand TWO different ways in which the dealer uses to hedge his swap positon. See P.14 – 16.
5. The market value of the a swap. See P.17 – 19.
6. Computing the swap rate. See P.20 – 25.

Introduction to Swaps

- A **swap** is a contract calling for an exchange of payments, on one or more dates, determined by the difference in two prices.
- A swap provides a means to hedge a *stream* of risky payments.
- A single-payment swap is the same thing as a cash-settled forward contract.

An Example of a Commodity Swap

- An industrial producer, IP Inc., needs to buy 100,000 barrels of oil 1 year from today and 2 years from today.

- The forward prices for deliver in 1 year and 2 years are \$110 and \$111/barrel.

  $F_{0,1}$ $F_{0,2}$

- The 1- and 2-year zero-coupon bond yields are 6% and 6.5% (annual effective interest rate).

IP Inc

t=0	1yr	2yr
	100,000 barrels	100,000 barrels

① Forward contract: 1yr: \$110 → oil; 2yr: \$111 → oil

② \$Y at t=0; oil at 1yr; oil at 2yr

$$\$Y = PV(110) + PV(111)$$

$$= \frac{110}{1+6\%} + \frac{111}{(1+6.5\%)^2}$$

pre-paid swap

③ 1yr: \$x → oil; 2yr: \$x → oil

$$\frac{x}{1+6\%} + \frac{x}{(1+6.5\%)^2} = \frac{110}{1+6\%} + \frac{111}{(1+6.5\%)^2}$$

$$x = \$110.483$$

$\boxed{\text{SWAP}}$

An Example of a Commodity Swap (cont'd)

- IP can guarantee the cost of buying oil for the next 2 years by entering into long forward contracts for 100,000 barrels in each of the next 2 years. The PV of this cost per barrel is

$$\frac{\$110}{1.06}+\frac{\$111}{1.065^2}=\$201.638$$

- Thus, IP could pay an oil supplier \$201.638, and the supplier would commit to delivering one barrel in each of the next two years.
- A **prepaid swap** is a single payment today to obtain *multiple* deliveries in the future.

An Example of a Commodity Swap (cont'd)

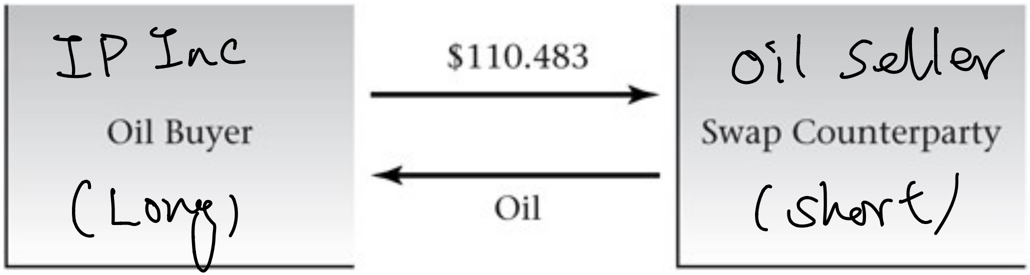
- With a prepaid swap, the buyer might worry about the resulting credit risk. Therefore, a more attractive solution is to defer payment until the oil is delivered, while still fixing the total price.
- Any payments that have a present value of $201.638 are acceptable. Typically, a swap will call for equal payments in each year.
  - For example, the payment per year per barrel, x, will have to be $110.483 to satisfy the following equation

$$\frac{x}{1.06} + \frac{x}{1.065^2} = \$201.638$$

- We then say that the 2-year swap price is $110.483.

Physical Versus Financial Settlement

- **Physical settlement** of the swap



Fixed swap rate payer (oil buyer) is said to take a **long position** of the swap while the (fixed rate receiver) swap counterparty is in the **short position** of the swap.

Physical Versus Financial Settlement (cont'd)

- **Financial settlement** of the swap
  - The oil buyer, IP, pays the swap counterparty the difference between $110.483 and the spot price, and the oil buyer then buys oil at the spot price.
  - If the difference between $110.483 and the spot price is negative, then the swap counterparty pays the buyer.

Physical Versus Financial Settlement (cont'd)

- Whatever the market price of oil, the net cost to the buyer is the swap price, \$110.483.

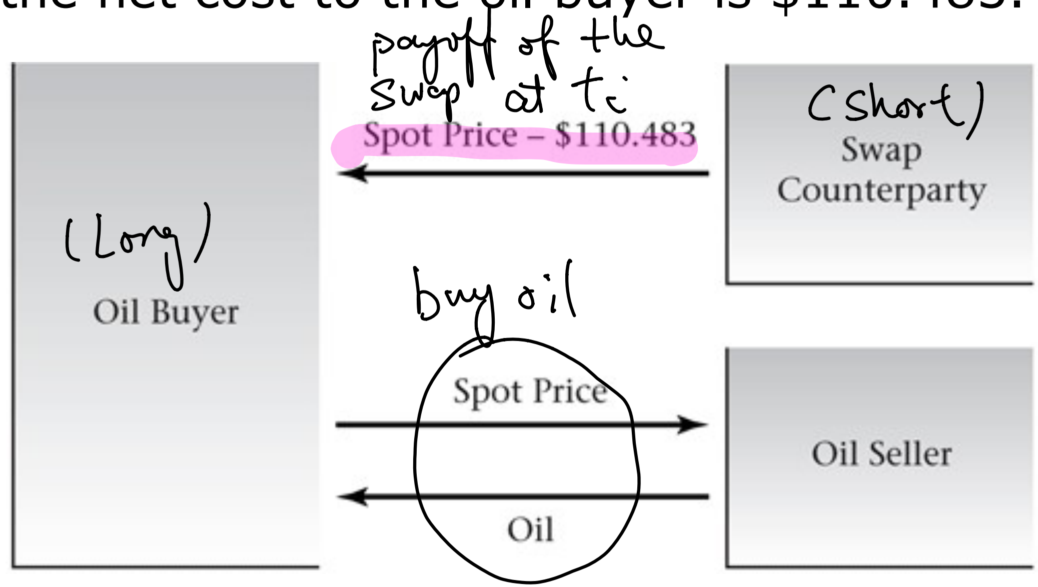
$$\underbrace{\text{Spot price} - \text{swap price}}_{\text{Swap payment}} - \underbrace{\text{Spot price}}_{\text{Spot purchase of oil}} = -\text{Swap price}$$

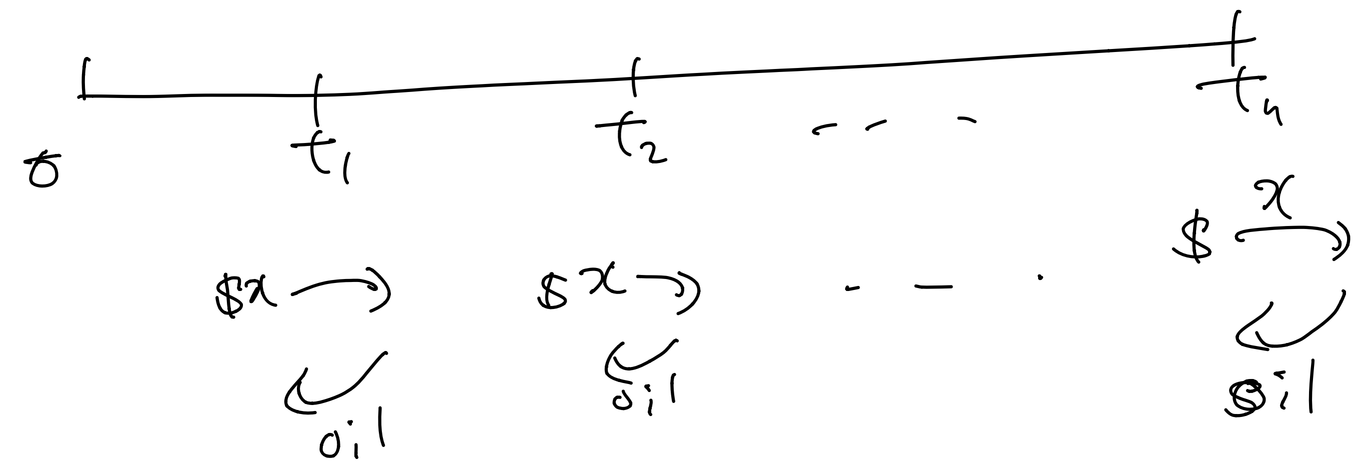
Physical Versus Financial Settlement (cont'd)

- The results for the buyer are the same whether the swap is settled physically or financially. In both cases, the net cost to the oil buyer is $110.483.

At t_i

$i = 1, \ldots, n$





($100)

Oil price at the end of 1 yr = $120 per barrel.

From the swap: Long = $120 − 110.483

= $9.517

$100 − 110.483 = −$10.483

Buy the oil from the oil seller

pay $120 ($100) to oil seller

Total cost = $120 + (−$9.517)

= $110.483

Total cost = $100 + (−(−10.483))

= $110.483

Physical Versus Financial Settlement (cont'd)

- If the swap has the **notional amount** of 100,000, meaning that 100,000 barrels is used to determine the magnitude of the payments when the swap is settled financially.
- Fig. 8.3 shows a **term sheet** for an oil swap. Term sheets are commonly used by broker-dealers to succinctly convey the important terms of a financial transaction.

FIGURE 8.3

Illustrative example of the terms for an oil swap based on West Texas Intermediate (WTI) crude oil.

Fixed-Price Payer:	Broker-dealer
Floating-Price Payer:	Counterparty
Notional Amount:	100,000 barrels
Trade Date:	April 18, 2011
Effective Date:	July 1, 2011
Termination Date:	September 31, 2011
Period End Date:	Final Pricing Date of each Calculation Period as defined in the description of the Floating Price.
Fixed Price:	110.89 USD per barrel
Commodity Reference Price:	OIL-WTI-NYMEX
Floating Price:	The average of the first nearby NYMEX WTI Crude Oil Futures settlement prices for each successive day of the Calculation Period during which such prices are quoted
Calculation Period:	Each calendar month during the transaction
Method of Averaging:	Unweighted
Settlement and Payment:	If the Fixed Amount exceeds the Floating Amount for such Calculation Period, the Fixed Price Payer shall pay the Floating Price Payer an amount equal to such excess. If the Floating Amount exceeds the Fixed Amount for such Calculation Period, the Floating Price Payer shall pay the Fixed Price Payer an amount equal to such excess.
Payment Date:	5 business days following each Period End Date

Physical Versus Financial Settlement (cont'd)

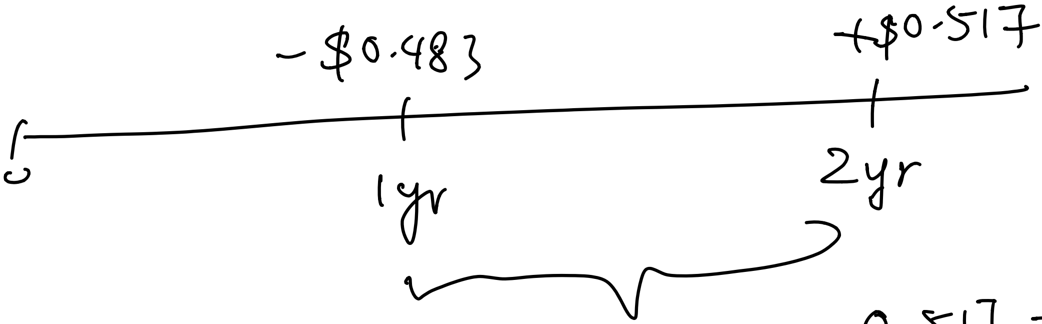
Swap is a combination of forward and zeros

- Swaps are nothing more than forward contracts coupled with borrowing and lending money
  - Consider the swap price of \$110.483/barrel. Relative to the forward curve price of \$110 in 1 year and \$111 in 2 years, we are overpaying by \$0.483 in the first year, and we are underpaying by \$0.517 in the second year.
  - Thus, by entering into the swap, we are lending the counterparty money for 1 year. The interest rate on this loan is

$$0.517 / 0.483 - 1 = 7\%$$

  - Given 1- and 2-year zero-coupon bond yields of 6% and 6.5%, 7% is the 1-year implied forward yield from year 1 to year 2.
- If the deal is priced fairly, the interest rate on this loan should be the implied forward interest rate.

$r_0[1,2]$



$$\text{return\%} = \frac{0.517 - 0.483}{0.483}$$

$$= 7\%$$

$$r_0[1,2] = \frac{P(0,1)}{P(0,2)} - 1 = \frac{\left[\frac{1}{(1+6\%)}\right]}{\left[\frac{1}{(1+6.5\%)^2}\right]} - 1$$

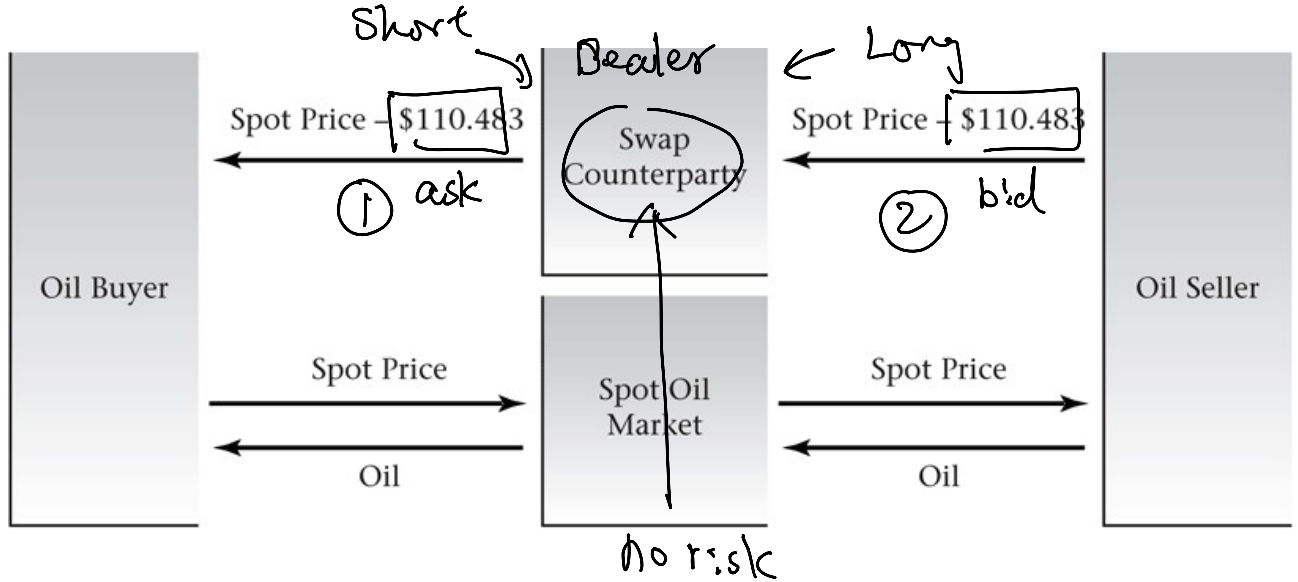
$$= 7\%$$

The Swap Counterparty

- The swap counterparty is a dealer, who is, in effect, a broker between buyer and seller.
- The dealer can hedge the oil price risk resulting from the swap in several ways.
- The fixed price paid by the buyer, usually, exceeds the fixed price received by the seller. This price difference is a bid-ask spread, and is the dealer’s fee.
- The dealer bears the credit risk of both parties, but is not exposed to price risk.

The Swap Counterparty (cont'd)

- The situation where the dealer matches the buyer and seller is called a **back-to-back transaction** or "matched book" transaction.



The Swap Counterparty (cont'd)

(Short the swap + $\boxed{\text{Long forwards}}$)

Synthetic swap

- Alternatively, the dealer can serve as counterparty and hedge the transaction by entering into long forward or futures contracts.

TABLE 8.1	Positions and cash flows for a dealer who has an obligation to receive the fixed price in an oil swap and who hedges the exposure by going long year 1 and year 2 oil forwards.

portfolio of forward

Year	Payment from Oil Buyer	Long Forward	Net
1	\$110.483 − year 1 spot price	Year 1 spot price − \$110	\$0.483
2	\$110.483 − year 2 spot price	Year 2 spot price − \$111	−\$0.517

Swap

$$\frac{0.483}{1+6\%} + \frac{(-0.517)}{(1+6.5\%)^2} = 0$$

- Note that the net cash flow for the hedged dealer is a loan, where the dealer receives cash in year 1 and repays it in year 2.
- Thus, the dealer also has interest rate exposure (which can be hedged by using Eurodollar contracts or forward rate agreements).

The Market Value of a Swap

- The market value of a swap is zero at inception.
- Once the swap is struck, however, its market value will generally no longer be zero because
  - The forward prices for oil and interest rates will change over time.
  - Even if prices do not change, the market value of swaps can change over time due to the implicit borrowing and lending.
- A buyer wishing to exit the swap could negotiate terms with the original counterparty to eliminate the swap obligation or enter into an offsetting swap with the counterparty offering the best price.

The Market Value of a Swap (cont'd)

- The original swap called for the oil buyer to pay the fixed price and receive floating; the offsetting swap has the buyer receive the fixed price and pay floating. The original obligation would be cancelled except to the extent that the fixed prices are different.
- The market value of the swap in the perspective of the **long position** is

  PV of the payments with the amount of (New swap rates – Original swap rates).

The Market Value of a Swap (cont'd)

Example

After 1 day

- Suppose the forward curve for oil rises by $2 in years 1 and 2. Thus, the year 1 forward price becomes $112 and the year-2 forward price becomes $113. $F_{0,1} = 112$, $F_{0,2} = 113$.
- Assuming interest rates are unchanged, the new swap price is $112.483 (*Verify!*). new swap rate = ??
- The market value of the swap in the perspective of the **long position** is

$$\frac{(112.483-110.483)}{1.06}+\frac{(112.483-110.483)}{(1.065)^2}=\$3.650$$

Market value of short position = − $3.650

Original swap: swap rate $= \$110.483$

$F_{0,1} = 110, \quad F_{0,2} = 111.$

After 1 day: $F_{0,1} = 112, \quad F_{0,2} = 113$

Swap rate $= \$112.483$

Cash inflow

	0	1yr	2yr
Long original swap		$S_1 - 110.483$	$S_2 - 110.483$
Short new swap		$\$112.483 - S_1$	$\$112.483 - S_2$
Total		$\$112.483 - 110.483$	$\$112.483 - 110.483$

Value of swap $=$ PV(all the net cash inflows)

Computing the Swap Rate

- Notation
  - Suppose there are n swap settlements, occurring on dates t_i, $i = 1,\ldots, n$.
  - The forward prices on these dates are given by F_{0,t_i}
  - The price of a zero-coupon bond maturing on date t_i is $P(0, t_i)$.
  - The fixed swap rate is R.
- If the buyer at time zero were to enter into forward contracts to purchase one unit on each of the n dates, the present value of payments would be the present value of the forward prices, which equals the price of the prepaid swap:

$$\text{Prepaid swap} = \sum_{i=1}^{n} F_{0,t_i} P(0, t_i)$$

Computing the Swap Rate (cont'd)

- We determine the fixed swap price, *R*, by requiring that the present value of the swap payments equal the value of the prepaid swap

PV(fixed Swap payment)
$$\sum_{i=1}^{n} RP(0,t_i) = \sum_{i=1}^{n} F_{0,t_i} P(0,t_i) \qquad (8.2)$$
PV(forward prices)

- Equation (8.2) can be rewritten as

Swap rate
$$R = \frac{\sum_{i=1}^{n} P(0,t_i) F_{0,t_i}}{\sum_{i=1}^{n} P(0,t_i)} \qquad (8.3)$$

where $\sum_{i=1}^{n} P(0,t_i) F_{0,t_i}$ is the present value of payments implied by the strip of forward rates, and $\sum_{i=1}^{n} P(0,t_i)$ is the present value of a \$1 annuity.

Computing the Swap Rate (cont'd)

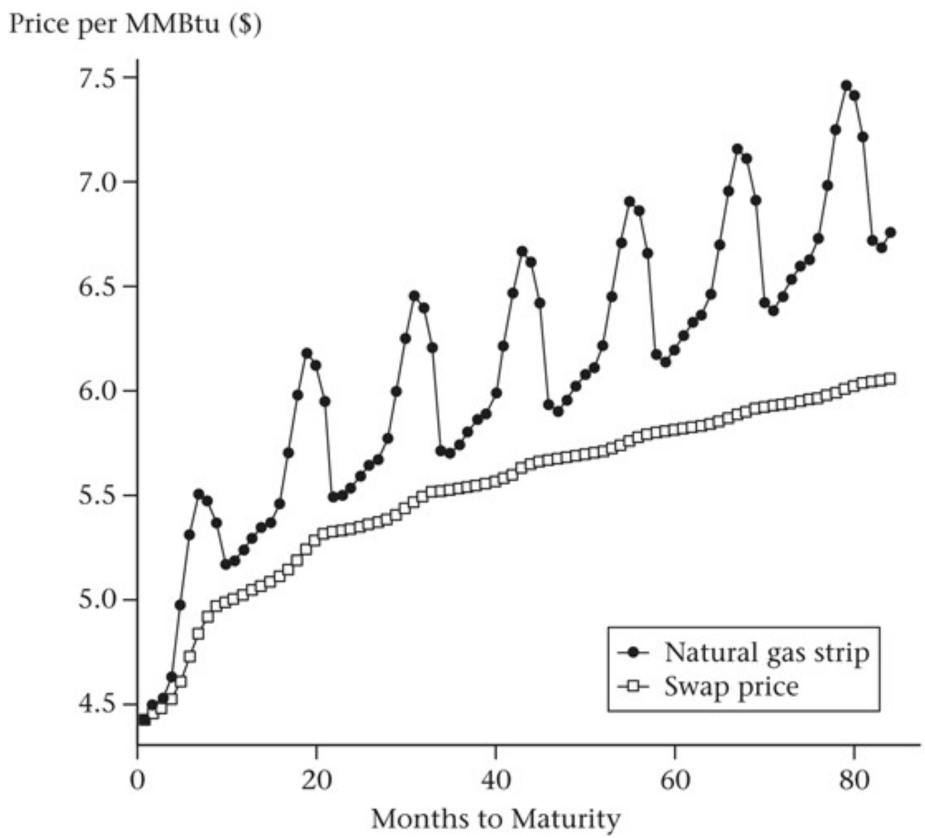
- We can rewrite equation (8.3) to make it easier to interpret

$$R = \sum_{i=1}^{n} \left[\frac{P(0,t_i)}{\sum_{j=1}^{n} P(0,t_j)} \right] F_{0,t_i}$$

- Thus, the fixed swap rate is as a weighted average of the forward prices, where zero-coupon bond prices are used to determine the weights.

FIGURE 8.6

Natural gas swap curve, June 2, 2010. The swap curve displays the fixed price for a natural gas swap beginning June 2010 and continuing, with monthly settlement, for the number of months specified on the x-axis.



Swaps With Variable Quantity and Prices

- A buyer with seasonally varying demand (e.g., someone buying gas for heating) might enter into a swap, in which *quantities* vary over time.
- Consider a swap in which the buyer pays RQ_{t_i}, for Q_{t_i} units of the commodity. The present value of these fixed payments (fixed per unit of the commodity) must equal the prepaid swap price

$$\sum_{i=1}^{n} Q_{t_i} F_{0,t_i} P(0,t_i) = \sum_{i=1}^{n} Q_{t_i} RP(0,t_i)$$

Notional = variable Q_{t_i}

- Solving for R gives

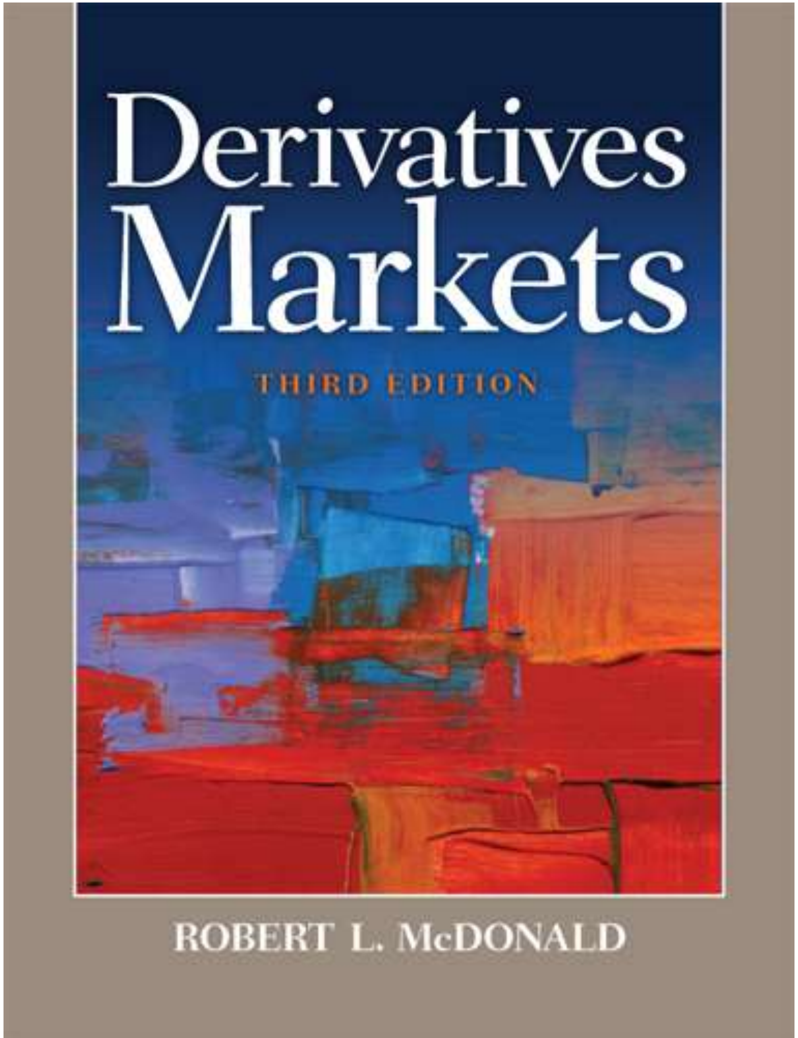
$$R = \frac{\sum_{i=1}^{n} Q_{t_i} P(0,t_i) F_{0,t_i}}{\sum_{i=1}^{n} Q_{t_i} P(0,t_i)}$$

Swaps With Variable Quantity and Prices (cont'd)

- It is also possible for *prices* to be time-varying
  - For example, we let the summer swap price be denoted by R_s and the winter price by R_w, then the summer and winter swap prices can be any prices for which the value of the prepaid swap equals the present value of the fixed swap payment:

$$R_s \sum_{i \in \text{summer}}^{n} P(0,t_i) Q_{t_i} + R_w \sum_{i \in \text{winter}}^{n} P(0,t_i) Q_{t_i} = \sum_{i=1}^{n} P(0,t_i) Q_{t_i} F_{0,t_i}$$

Once we fix one of R_s and R_w, the equation will give us the other.



Chapter 8
(Chapter 9 in the textbook)

Parity and Other Option Relationships

Points to Note

1. Important relation: Put-call parity. See P.5
2. Generalized put-call parity on exchange options. See P.9
3. The relationship between call and put options on exchange rate. See P.15
4. Compare the prices of European and American options. See P.16
5. The upper and lower bounds of the option price. See P.17 – 18
6. Early exercises of American call and put options. See P.19 – 24
7. Relationship between time to expiration and option price. See P. 25
8. Relationship between strike prices and option prices. See P.26 - 30
9. Convexity property of option prices with respect to strike prices. See P.31

IBM Option Quotes

TABLE 9.1 IBM option prices, dollars per share, May 6, 2011. The closing price of IBM on that day was $168.89.

		Calls		Puts	
Strike	Expiration	Bid ($)	Ask ($)	Bid ($)	Ask ($)
160	June	10.05	10.15	1.16	1.20
165	June	6.15	6.25	2.26	2.31
170	June	3.20	3.30	4.25	4.35
175	June	1.38	1.43	7.40	7.55
160	October	14.10	14.20	5.70	5.80
165	October	10.85	11.00	7.45	7.60
170	October	8.10	8.20	9.70	9.85
175	October	5.80	5.90	12.40	12.55

Source: Chicago Board Options Exchange.

Put-Call Parity

Important Note: Starting from here, the meaning of T in $F_{0,T}$ is changed to mean the maturity date of the forward contract.

Notations:

- $C(K, T)$ and $P(K, T)$ are the prices of a European call and put with the strike price K and the **time to expiration *T*** respectively;
- $F_{0,T}$ be the time 0 price of the forward contract with the **maturity date at time *T***.

Put-Call Parity (cont'd)

For European options with the same strike price and time to expiration the parity relationship is

Call – put = *PV* (forward price – strike price)

Or

$$C(K,T) - P(K,T) = PV_{0,T}(F_{0,T} - K) = e^{-rT}(F_{0,T} - K)$$

Bid and ask $\Rightarrow$ put - call parity NOT an identity

- Intuition
  - Buying a call and selling a put with the strike equal to the forward price ($F_{0,T} = K$) creates a synthetic forward contract and hence must have a zero price.
- In general, put-call parity *fails* for American style options.

$C - P = S_0 - PV(K)$ non-dividend paying stock

$C - P - S_0 + PV(K) = 0$ (No bid-ask spread)

When there is a bid-ask spread,

$C_{ask} - P_{bid} - S_{bid} + K P_{ask}(0,T)$

Payoff of the portfolio = Long call, short put, short underlying + Long Bond = 0

No arbitrage $\Rightarrow$

$C_{ask} - P_{bid} - S_{bid} + K P_{ask}(0,T) \geq 0$

$-C + P + S - K P(0,T) = 0$

$-C_{bid} + P_{ask} + S_{ask} - K P_{bid}(0,T) \geq 0$

Parity for Options on Stocks

- If underlying asset is a stock and $PV_{0,T}(\text{Div})$ is the present value of the dividends payable over the life of the option, then $e^{-rT} F_{0,T} = S_0 - PV_{0,T}(\text{Div})$, therefore

$$C(K,T) = P(K,T) + [S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})] - e^{-rT}(K)$$

- For index options, $S_0 - \mathrm{PV}_{0,T}(\mathrm{Div}) = S_0 e^{-\delta T}$, therefore

$$C(K,T) = P(K,T) + S_0 e^{-\delta T} - \mathrm{PV}_{0,T}(K)$$

Parity for Options on Stocks (cont'd)

- Examples 9.1 & 9.2
  - Price of a non-dividend-paying stock: \$40, r=8%, option strike price: \$40, time to expiration: 3 months, European call: \$2.78, European put: \$1.99. ➡ $\$2.78=\$1.99+\$40 - \$40e^{-0.08 \times 0.25}$.
  - Additionally, if the stock pays \$5 just before expiration, call: \$0.74, and put: \$4.85. ➡ $\$0.74-\$4.85=(\$40 - \$5e^{-0.08 \times 0.25}) - \$40e^{-0.08 \times 0.25}$.
- Synthetic security creation using parity
  - Synthetic stock: buy call, sell put, lend PV of strike and dividends.
  - Synthetic T-bill: buy stock, sell call, buy put.
  - Synthetic call: buy stock, buy put, borrow PV of strike and dividends.
  - Synthetic put: sell stock, buy call, lend PV of strike and dividends.

Call option : $\max(S_T - K, 0)$ $\max(S_T - USD(0, 0)$

pay \$K to get 1 unit of the underlying

give up \$K in exchange 1 unit of the underlying

Asset A : price at t = S_t

Asset B : price at t = Q_t

payoff : $\max(S_T - Q_T, 0)$

Call option on A :

At expiration T : right to give up 1 unit of B in exchange for 1 unit of A

Put option on B :

At expiration : right to give up 1 unit of asset B in exchange for 1 unit of A

Generalized Parity and Exchange Options

- Suppose we have an option to exchange one asset for another.
- Let the underlying asset, asset A, have price S_t, and the strike asset, asset B, have the price Q_t.
- Let $F_{t,T}^{P}(S)$ denote the time t price of a prepaid forward on the underlying asset, paying S_T at time T.
- Let $F_{t,T}^{P}(Q)$ denote the time t price of a prepaid forward on the underlying asset, paying Q_T at time T.

time to expiration

- Let $C(S_t, Q_t, T - t)$ denote the time t price of an option with $T-t$ periods of expiration, which gives us the right to give up asset B in exchange for asset A.
- Let $P(S_t, Q_t, T - t)$ denote the time t price of an option with $T-t$ periods of expiration, which gives us the right to give up asset A in exchange for asset B.

underlying give up

Generalized Parity and Exchange Options (cont'd)

- At time T, we have $C(S_t, Q_t, 0) = P(Q_t, S_t, 0)$

  $C(S_T, Q_T, 0) = \max(0, S_T - Q_T)$ and $=$

  $P(S_T, Q_T, 0) = \max(0, Q_T - S_T)$
- Then for European options we have this form of the parity equation:

$$C(S_t, Q_t, T-t) - P(S_t, Q_t, T-t) = F_{t,T}^P(S) - F_{t,T}^P(Q)$$

Generalized Parity Relationship

TABLE 9.2 Payoff table demonstrating that there is an arbitrage opportunity unless $-C(S_t, Q_t, T-t) + P(S_t, Q_t, T-t) + F^P_{t,T}(S) - F^P_{t,T}(Q) = 0$.

		Expiration	
Transaction	Time 0	$S_T \leq Q_T$	$S_T > Q_T$
Buy call	$-C(S_t, Q_t, T-t)$	0	$S_T - Q_T$
Sell put	$P(S_t, Q_t, T-t)$	$S_T - Q_T$	0
Sell prepaid forward on A	$F^P_{t,T}(S)$	$-S_T$	$-S_T$
Buy prepaid forward on B	$-F^P_{t,T}(Q)$	Q_T	Q_T
Total cost	$\boxed{-C(S_t, Q_t, T-t) + P(S_t, Q_t, T-t) + F^P_{t,T}(S) - F^P_{t,T}(Q)}$	0	0

So,

$$-C(S_t, Q_t, T-t) + P(S_t, Q_t, T-t) + F^P_{t,T}(S) - F^P_{t,T}(Q)$$

$$\geq 0 \qquad (1)$$

Transaction	Time 0	Expiration $S_T \leq Q_T$	Expiration $S_T > Q_T$
Sell Call	$C(S_t, Q_t, T-t)$		$-(S_T - Q_T)$
Buy Put	$-P(S_t, Q_t, T-t)$	$-(S_T - Q_T)$	
Buy Pre-paid forward on A	$-F^P_{t,T}(S)$	S_T	S_T
Sell Pre-paid forward on B	$F^P_{t,T}(Q)$	$-Q_T$	$-Q_T$
	$\boxed{C(S_t, Q_t, T-t) - P(S_t, Q_t, T-t) - F^P_{t,T}(S) + F^P_{t,T}(Q)}$ (2)	0	0

≥ 0

Combining (1) and (2), we have

$$C(S_t, Q_t, T-t) - P(S_t, Q_t, T-t) - F^P_{t,T}(S) + F^P_{t,T}(Q) = 0$$