Tutorial - Class Activity (Solution)

24 September, 2019

For the following problems assume the effective 6-month interest rate is 2%, the S&R 6-month forward price is \$1020, and use these premiums for S&R options with 6 months to expiration:

Strike	Call	Put
\$950	\$120.405	\$51.777
1000	93.809	74.201

3.9 Construct payoff and profit diagrams for the purchase of a 950-strike S&R call and sale of a 1000-strike S&R call. Verify that you obtain exactly the same *profit* diagram for the purchase of a 950-strike S&R put and sale of a 1000-strike S&R put. What is the difference in the payoff diagrams for the call and put spreads? Why is there a difference?

Solution

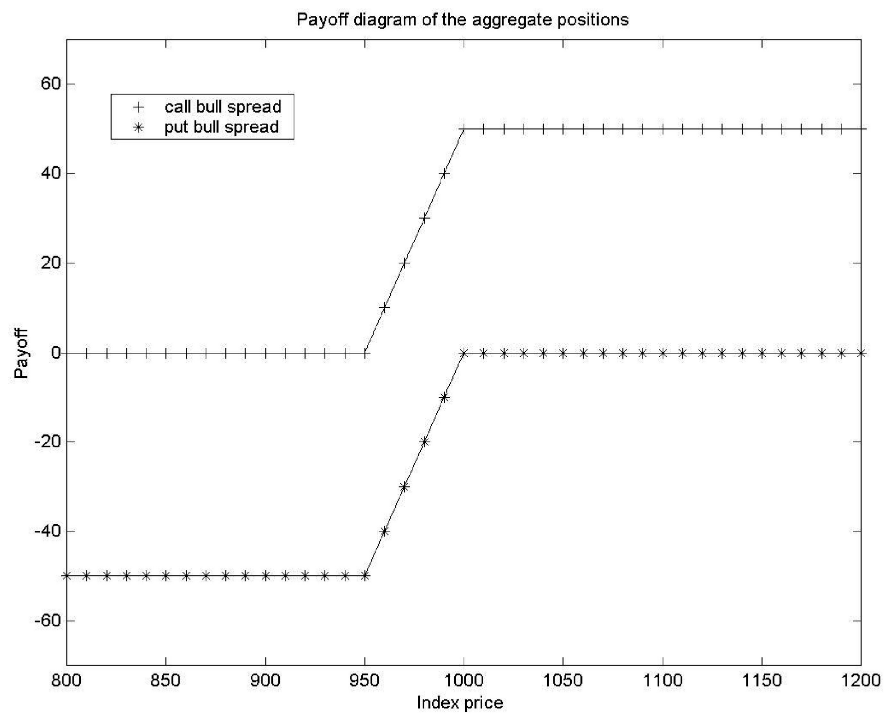
The strategy of buying a call (or put) and selling a call (or put) at a higher strike is called call (put) bull spread. In order to draw the profit diagrams, we need to find the future value of the cost of entering in the bull spread positions. We have:

Cost of call bull spread: (\$120.405 − \$93.809) × 1.02 = \$27.13
Cost of put bull spread: (\$51.777 − \$74.201) × 1.02 = −\$22.87

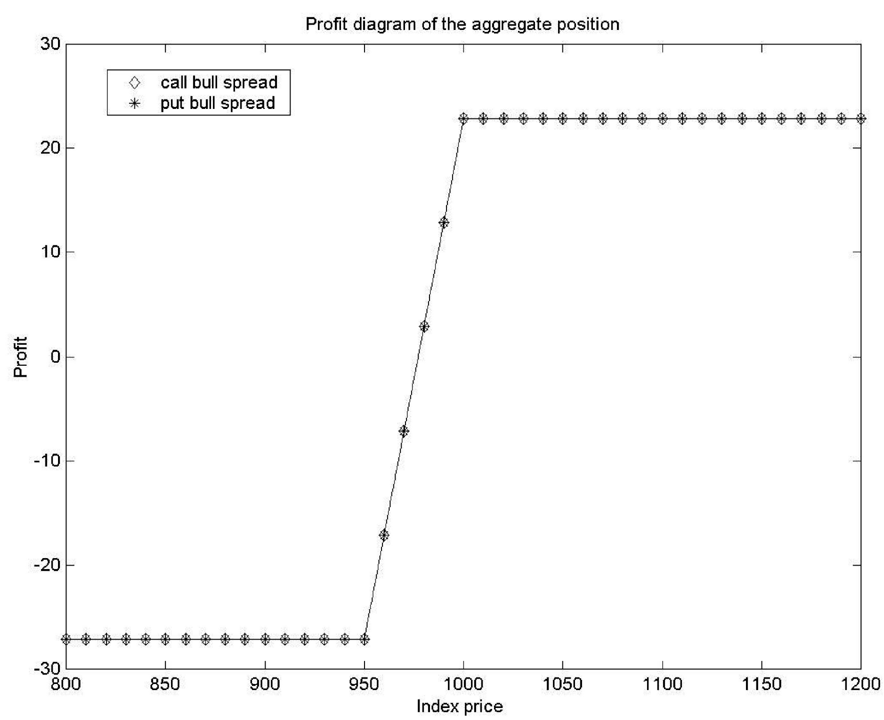
The payoff diagram shows that the payoffs to the put bull spread are uniformly less than the payoffs to the call bull spread. There is a difference, because the put bull spread has a negative initial cost (i.e., we are receiving money if we enter into it). The difference is exactly \$50, the value of the difference between the two strike prices. It is equivalent as well to the value of the difference of the future values of the initial premia.

Yet, the higher initial cost for the call bull spread is exactly offset by the higher payoff so that the profits of both strategies are the same. It is easy to show this with the put-call-parity.

Payoff diagram:



Profit diagram:



Proof by using the put-call parity

Let Call(K, t) and Put(K, t) denote the premiums of options with strike price K and <u>time t until expiration</u>, and $F_{s,t}$ be the forward price at time s with time to expiration of t.

At time 0, for the call, put and forwards with time to expiration of t, the put-call parity states that

$$\text{Call}(K,t) - \text{Put}(K,t) = PV\left(F_{0,t} - K\right).$$

So, we have

$$\text{Call}(950,t) = \text{Put}(950,t) + PV\left(F_{0,t} - 950\right);$$
$$\text{Call}(1000,t) = \text{Put}(1000,t) + PV\left(F_{0,t} - 1000\right).$$

Now, we take T be the maturity of the call, put and forwards.

At time T (maturity of the options and forwards), the time until expiration is 0.

At time 0, the time until expiration is T.

The payoff of the call bull spread is then given by

$$\begin{aligned}
&\text{Call}(950,0) - \text{Call}(1000,0) \\
&= \text{Put}(950,0) - \text{Put}(1000,0) + PV\left(F_{T,0} - 950\right) - PV\left(F_{T,0} - 1000\right) \\
&= \text{Put}(950,0) - \text{Put}(1000,0) + 50 \\
&= \text{Payoff of the put bull spread} + 50.
\end{aligned}$$

The cost of the call bull spread at time 0 is then given by

$$\begin{aligned}
&\text{Call}(950,T) - \text{Call}(1000,T) \\
&= \text{Put}(950,T) - \text{Put}(1000,T) + PV\left(F_{0,T} - 950\right) - PV\left(F_{0,T} - 1000\right) \\
&= \text{Put}(950,T) - \text{Put}(1000,T) + PV(50).
\end{aligned}$$

The profit of the call bull spread at T is then given by

$$\begin{aligned}
&\text{Call}(950,0) - \text{Call}(1000,0) - FV\left(\text{Call}(950,T) - \text{Call}(1000,T)\right) \\
&= \text{Put}(950,0) - \text{Put}(1000,0) + 50 - FV\left[\text{Put}(950,T) - \text{Put}(1000,T) + PV(50)\right] \\
&= \text{Put}(950,0) - \text{Put}(1000,0) + 50 - FV\left[\text{Put}(950,T) - \text{Put}(1000,T)\right] - FV\left(PV(50)\right) \\
&= \text{Put}(950,0) - \text{Put}(1000,0) - FV\left[\text{Put}(950,T) - \text{Put}(1000,T)\right] \\
&= \text{Profit of the put bull spread at } T.
\end{aligned}$$

2. Which statement about zero-cost purchased collars on a non-dividend paying stock is FALSE?

   (A) A zero-width, zero-cost collar can be created by setting both the put and call strike prices at the forward price.

   (B) There are an infinite number of zero-cost collars.

   (C) The put option can be at-the-money.

   (D) The call option can be at-the-money.

Solution

Answer (D)

Cost for taking the long position of a collar = $Put(K_1, t) - Call(K_2, t)$, where $K_1 \le K_2$.

(A)

From the put-call parity, we have

$$Call\left(K, t\right) - Put(K, t) = PV\left(F_{0,t} - K\right)$$

If $F_{0,t} = K$, then we have $Put(K, t) - Call(K, t) = 0$. So, a zero-width, zero-cost collar will be created by setting both the put and call strike prices at the forward price.

(B)

There are infinite number of pairs of K_1 and K_2 which make

$$Put(K_1, t) - Call(K_2, t) = 0.$$

(C)

Under the non-dividend paying stock, $F_{0,t} = FV_{0,t}(S_0)$.

Use the put-call parity, the cost of the purchased collar can be rewritten as

$$\begin{aligned}
&Put(K_1,t) - Call\left(K_2,t\right) \\
&= Call(K_1,t) - S_0 + PV(K_1) - Call\left(K_2,t\right) \\
&= Call(K_1,t) - Call\left(K_2,t\right) - \left(S_0 - PV(K_1)\right).
\end{aligned}$$

When $K_1 = S_0$, we have

$$\begin{aligned}
&Call(S_0,t) - Call\left(K_2,t\right) - \left(S_0 - PV(S_0)\right) = 0 \\
&Call\left(K_2,t\right) = Call(S_0,t) - \left(S_0 - PV(S_0)\right) < Call(S_0,t).
\end{aligned}$$

Since call option premium is a decreasing function of the strike price, it is possible to find K_2 to create a zero-cost collar.

(D)

Use the put-call parity, the cost of the purchased collar can be rewritten as

$$
\begin{aligned}
&Put(K_1,t)-Call\left(K_2,t\right)\\
&=Put(K_1,t)-\left[Put(K_2,t)+S_0-PV(K_2)\right]
\end{aligned}
$$

When $K_2 = S_0$, we have

$$
\begin{aligned}
&Put(K_1,t)-\left[Put\left(S_0,t\right)+S_0-PV\left(S_0\right)\right]=0\\
&Put(K_1,t)=Put\left(S_0,t\right)+S_0-PV\left(S_0\right)>Put\left(S_0,t\right).
\end{aligned}
$$

Since put option premium is an increasing function of the strike price, it is impossible to find K_1 to create the zero-cost collar.

3. The current price of a non-dividend paying stock is \$40 and the continuously compounded annual risk-free rate of return is 8%. The following table shows call and put option premiums for three-month European options of various exercise prices:

Exercise Price	Call Premium	Put Premium
\$35	\$6.13	\$0.44
\$40	\$2.78	\$1.99
\$45	\$0.97	\$5.08

A trader interested in speculating on volatility in the stock price is considering two investment strategies. The first is a 40-strike straddle – i.e., purchase of one 40-strike call option and one 40-strike put option. The second is a strangle consisting of purchasing of one 35-strike put and one 45-strike call.

Determine the range of stock prices in 3 months for which the profit of the strangle is greater than the profit of the straddle in 3 months.

Solution

The straddle consists of buying a 40-strike call and buying a 40-strike put.

The cost of the straddle = 2.78 + 1.99 = 4.77.

The future value of the cost of the straddle at three months = $4.77e^{8\%\times 0.25} = 4.87$.

The strangle consists of buying a 35-strike put and buying a 45-strike call.

The cost of the strangle = 0.44 + 0.97 = 1.41.

The future value of the cost of the strangle at three months = $1.41e^{8\%\times 0.25} = 1.44$.

Let $S_{0.25}$ be the stock price in three months.

The **payoff table** for the straddle and strangle at three months can be obtained as follows:

	$S_{0.25} < 35$	$35 \le S_{0.25} < 40$	$40 \le S_{0.25} < 45$	$S_{0.25} \ge 45$
Buy a 40-strike call	0	0	$S_{0.25} - 40$	$S_{0.25} - 40$
Buy a 40-strike put	$40 - S_{0.25}$	$40 - S_{0.25}$	0	0
The payoff of the straddle	$40 - S_{0.25}$	$40 - S_{0.25}$	$S_{0.25} - 40$	$S_{0.25} - 40$
Buy a 35-strike put	$35 - S_{0.25}$	0	0	0
Buy a 45-strike call	0	0	0	$S_{0.25} - 45$
The payoff of the strangle	$35 - S_{0.25}$	0	0	$S_{0.25} - 45$

Hence, the **profit table** for the straddle and strangle at three months is

	$S_{0.25} < 35$	$35 \le S_{0.25} < 40$	$40 \le S_{0.25} < 45$	$S_{0.25} \ge 45$
The profit of the straddle (A)	$40 - S_{0.25} - 4.87$ $= 35.13 - S_{0.25}$	$40 - S_{0.25} - 4.87$ $= 35.13 - S_{0.25}$	$S_{0.25} - 40 - 4.87$ $= S_{0.25} - 44.87$	$S_{0.25} - 40 - 4.87$ $= S_{0.25} - 44.87$
The profit of the strangle (B)	$35 - S_{0.25} - 1.44$ $= 33.56 - S_{0.25}$	-1.44	-1.44	$S_{0.25} - 45 - 1.44$ $= S_{0.25} - 46.44$
(A) – (B)	1.57	$36.57 - S_{0.25}$	$S_{0.25} - 43.43$	1.57

The profit of the strangle is higher than the profit of the straddle in 3 months if (A) – (B) < 0. Hence, the strangle **CANNOT** outperform straddle in 3 months when $S_{0.25} < 35$ and $S_{0.25} \ge 45$.

When $35 \le S_{0.25} < 40$, the strangle will outperform the straddle if

$$36.57 - S_{0.25} < 0 \text{ or } S_{0.25} > 36.57.$$

Combining with $35 \le S_{0.25} < 40$, we have $36.57 < S_{0.25} < 40$. (I)

When $40 \le S_{0.25} < 45$, the strangle will outperform the straddle if

$$S_{0.25} - 43.43 < 0 \text{ or } S_{0.25} < 43.43.$$

Combining with $40 \le S_{0.25} < 45$, we have $40 \le S_{0.25} < 43.43$. (II)

From (I) and (II), the strangle will outperform the straddle when

$$36.57 < S_{0.25} < 43.43.$$