数 学 作 业 纸

科目

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Chapter 5: Exercise 2

a. $F_{0,T}^P = \$50 - \sum_{i=1}^{4} \$1 \cdot e^{-0.06 \times \frac{3}{12} i}$

$= \$50 - \$e^{-0.015} - \$e^{-0.03} - \$e^{-0.045} - \$e^{-0.06}$

$= \$46.1467$

b. $F_{0,T} = F_{0,T}^P e^{0.06 \times 1} = \$49.00$

2. Chapter 5: Exercise 4

a. $F_{0,T} = S_0 e^{rT} = \$35 e^{5\% \times \frac{1}{2}} \approx \$35.8860$

b. annualized forward premium

$= \frac{1}{T} \ln\left(\frac{F_{0,T}}{S_0}\right) = \frac{1}{0.5} \ln\left(\frac{\$35.50}{\$35}\right) \approx 0.0284$

~~3. Chapter 5: Exercise 5~~

c. annualized forward premium

$= \frac{1}{T} \ln\left(\frac{F_{0,T}}{S_0}\right) = \frac{1}{T} \ln\left(\frac{S_0 e^{T(r-\delta)}}{S_0}\right)$

$= r - \delta$

$5\% - \delta = 0.0284$

$\delta = 0.0216$

3. Chapter 5: Exercise 5

a. $F_{0,T} = S_0 e^{rT} = \$1100 e^{5\% \times \frac{9}{12}} \approx \$1142.033$

b.

	Today	9 Month
Long the index forward	0	$S_T - F_{0,T}$
Sell short the index	$+S_0$	$-S_T$
Lend S_0	$-S_0$	$+S_0 e^{rT}$
Total	0	$S_0 e^{rT} - F_{0,T}$

So we engage in a reverse cash and carry strategy.

	Today	9 Month
Long the index forward	0	$S_T - \$1142.033$
Sell short the index	$+\$1100$	$-S_T$
Lend $\$1100$	$-\$1100$	$+\$1100 e^{0.05 \times \frac{3}{4}}$
Total	0	0

Therefore, we fully hedge the resulting ~~short~~ long position.

~~4. Chapter 5: Exercise 8~~

c.

	Today	9 Month
Short the index forward	0	$F_{0,T} - S_T$
Long the index	$-S_0$	$+S_T$
~~Len~~ Borrow S_0	$+S_0$	$-S_0 e^{rT}$
Total	0	$F_{0,T} - S_0 e^{rT}$

We engage in a cash and carry strategy

	Today	9 Month
Short the index forward	0	$\$1142.033 - S_T$
Buy the index	$-\$1100$	S_T
Borrow $\$1100$	$+\$1100$	$-\$1100 e^{0.05 \times \frac{3}{4}}$
Total	0	0

Therefore, we fully hedge the resulting short position.

4. Chapter 5: Exercise 8

a. If there is no arbitrage, then the price of the forward price:

$$F_{0,T} = S_0 e^{(r-\delta)T} = \$1100 \times e^{(5\%-2\%)\times\frac{1}{2}} = \$1116.62$$

If the forward price is $1120, then the forward is too expensive. So we can short the forward at $1120 and create a synthetic long forward at $1116.62.

	Today	9 Months
Short the forward	0	$\$1120 - S_T$
Long the tailed position in index	$-\$1100e^{-0.02\times\frac{1}{2}} = -\$1089.055$	S_T
Borrow $1089.055	$+\$1089.055$	$-\$1116.62$
Total	0	$3.38

b. If the forward price is $1110, then the forward is too cheap. Therefore, we can long the forward at $1110 and create a synthetic short forward for $1116.62 and the profit is $6.62 without investment

	Today	9 Months
Long the forward	0	$S_T - \$1110$
Short the tailed position in index	$+\$1100e^{-0.02\times\frac{1}{2}} = +\$1089.055$	$-S_T$
Lend $1089.055	$-\$1089.055$	$+\$1116.62$
Total	0	$6.62

Transaction	Dec Year0	March Year1	June Year1	Sept Year1
Short the forward	~~$3.00~~ 0			$\$2.750 - S_T$
Buy the widget	$-\$3.000$			S_T
Storage Cost		$-\$0.03$	$-\$0.03$	$-\$3.00$

$$3e^{r\times\frac{9}{12}} + 0.03e^{r\times\frac{6}{12}} + 0.03e^{r\times\frac{3}{12}} + 0.03 = 2.45$$

$r = -0.16108 < 0$ The answer is not sensible

5. Chapter 5: Exercise 12

a. The notional value of my position

$\$10 \times 950 \times \$250 = \$2375000$

b. After one week, our initial margin grows to:

$\$237500e^{0.06\times\frac{1}{52}} \approx \$237774.20$

$\$237774.20 + (F_1 - 950) \times \$2500 \leq \$237500 \times 0.8$

$F_1 \leq \$930.89$

Therefore, the greatest S&P 500 index future price at which we will receive a margin call is $930.89.

6. Chapter 6: Exercise 6

b. Storage cost = 0.03

The forward price $= \$3.000\, e^{6\%\times\frac{1}{4}} + 0.03 = 3.075$

Our cash and carry trade is

Transaction	Dec Year0	March Year1
Short the forward	0	$\$3.075 - S_T$
Buy the widget	$-\$3.000$	$S_T - \$0.03$
Total	$-\$3.000$	$3.045

the annualized rate of return

$= \cancel{\frac{3.075 - 3.000}{3.000} \times 4 = 10\%}$

$\ln\left(\frac{3.045}{3.000}\right) \times 4 \approx 0.05955$

The answer is sensible

c. The forward price $= \$3.000\, e^{6\%\times\frac{3}{4}} + \$0.03\, e^{6\%\times\frac{1}{2}} + \$0.03\, e^{6\%\times\frac{1}{4}} + \$0.03$

$= \$3.229$

Our cash and carry trade is

Transaction	Dec Year0	Sept Year1
Short the forward	0	$\$3.229 - S_T$
Buy the widget	$-\$3.000$	S_T
Total	$-\$3.000$	$3.229

~~The annualized~~ rate of return $= \ln\left(\frac{3.229}{3.000}\right) \times 4$

~~Transaction~~	~~Dec Year 0~~	~~Sept Year 1~~

7. Chapter 7: Exercise 3

Maturity	Zero-Coupon Bond Yield	Zero-Coupon Bond Price	Continuously Compounded Zero-Coupon Bond ~~Year~~ Yield	Par Coupon Rate	1-Year Implied Forward Rate
1	0.030	~~0.97087~~ 0.97087	~~0.02956~~ 0.02956	~~0.030~~ 0.03	~~0.030~~ 0.030
2	0.035 ~~0.03250~~	~~0.93804~~ 0.93351	~~0.03198~~ 0.03440	~~0.03246~~ 0.03491	~~0.035~~ 0.04002
3	0.0400 ~~0.03499~~	~~0.90197~~ 0.88900	~~0.03439~~ 0.03922	~~0.03488~~ 0.03974	~~0.040~~ 0.05007
4	0.045 ~~0.03748~~	~~0.86314~~ 0.83856	~~0.03679~~ 0.04402	~~0.03725~~ 0.04445	~~0.045~~ 0.06014
5	0.050 ~~0.03997~~	~~0.82204~~ 0.78353	~~0.03919~~ 0.04879	~~0.03958~~ 0.04903	~~0.050~~ 0.07024

8. Chapter 7: Exercise 7

a. $r_1 = -\ln(0.96154) = 0.03922$

$r_2 = \frac{-\ln(0.91573)}{2} = 0.04402$

$r_3 = \frac{-\ln(0.87630)}{3} = 0.04402$

$r = \frac{\ln(e^{3r_3}/e^{r_1})}{2} \approx 0.04642$

$$R = \sqrt{\frac{[1+r_0(0,3)]^3}{1+r_0(0,1)}} - 1 = \sqrt{\frac{P_0(0,1)}{P_0(0,3)}} - 1 = \sqrt{\frac{0.943396}{0.816298}} - 1 \approx 0.07504$$

9. Additional problem 1

The present value of the storage costs are

$\cancel{0.24}\,0.06 + \cancel{0.24}\,0.06\,e^{-10\% \times \frac{1}{4}} + \cancel{0.24}\,0.06\,e^{-10\% \times \frac{1}{2}} \approx \cancel{\$0.70237}\ \$0.17559$

The futures price $= (9 + \cancel{0.70237}\,0.17559)\,e^{10\% \times \frac{3}{4}} \approx \cancel{\$10.45803}\ 9.8902$

10. Additional Problem 2

$r_1 = -\frac{\ln(0.9912)}{1} \approx 0.00884$ $\qquad F^1_{0,T} = \$4.8e^{(0.00884+1.8\%-4.2\%)} \approx \$4.72778$

$r_2 = -\frac{\ln(0.9701)}{2} \approx 0.01518$ $\qquad F^2_{0,T} = \$4.8e^{\cancel{0.00884}\,0.01518 \times 2 + (1.8\% - 4.2\%) \times 2} \approx \$4.71607$

$r_3 = -\frac{\ln(0.9518)}{3} \approx 0.01647$ $\qquad F^3_{0,T} = \$4.8e^{0.01647 \times 3 + (1.8\% - 4.2\%) \times 3} \approx \$4.69278$