**MFE5130 – Financial Derivatives**

**Class Activity (6-December-2018) (Solution)**

**Important Notes:**

1. This class activity is counted toward to your class participation score. **Fail** to hand in this class activity worksheet in the class will receive **0 score** for that class.
2. **0 mark** will be received if you leave the solution blank.

| Name: | Student No.: |
|---|---|

**Problem 1**

Given that $S(t)$ is a GBM (Geometric Brownian motion) which follows

$$\frac{dS(t)}{S(t)} = 0.06dt + 0.3dZ(t)$$

where $Z(t)$ is a standard Brownian motion under measure $P$.
Find another measure $Q$ by specifying the Radon-Nikodym derivative of $Q$ with respect to $P$, $\frac{dQ}{dP}$, such that $S(t)$ is governed by

$$\frac{dS(t)}{S(t)} = 0.02dt + 0.3d\tilde{Z}(t)$$

under the measure $Q$, where $\tilde{Z}(t)$ is a standard Brownian motion under $Q$.

**Solution**

Let $\eta = \frac{0.06-0.02}{0.3} = \frac{2}{15}$. and consider the Radon Nikodym derivative of $Q$ with respect to $P$ based on the information up to time $t$:

$$\frac{dQ}{dP} = \exp\left(-\frac{2}{15}Z(t) - \frac{1}{2}\left(\frac{2}{15}\right)^2 t\right) = \exp\left(-\frac{2}{15}Z(t) - \frac{2}{225}t\right).$$

Under the measure $Q$, the stochastic process

$$\tilde{Z}(t) = Z(t) + \frac{2}{15}t$$

is a standard Brownian motion under $Q$ by the Girsanov Theorem.

It is seen that when we set $\eta = \frac{2}{15}$, then

$$0.06dt + 0.3dZ(t) = 0.06dt + 0.3\left( d\tilde{Z}(t) - \frac{2}{15}dt \right) = 0.02dt + 0.3d\tilde{Z}(t).$$

Therefore, $S(t)$ is governed by

$$\frac{dS(t)}{S(t)} = 0.02dt + 0.3d\tilde{Z}(t)$$

under measure $Q$.