Revision

11-oct-2018

Forward on commodity

$$F_{0,T} = S_0 \, e^{(r \underbrace{-y}_{\text{convenience yield}} \overbrace{+u}^{\text{storage cost}})T}$$

$$F_{0,T} = (S_0 + \underbrace{u - y}_{PV(\quad)}) \, e^{rT}$$

Expenses of Asset owner $\Rightarrow \uparrow F_{0,T}$

Income of " " $\Rightarrow \downarrow F_{0,T}$

Loan market exist

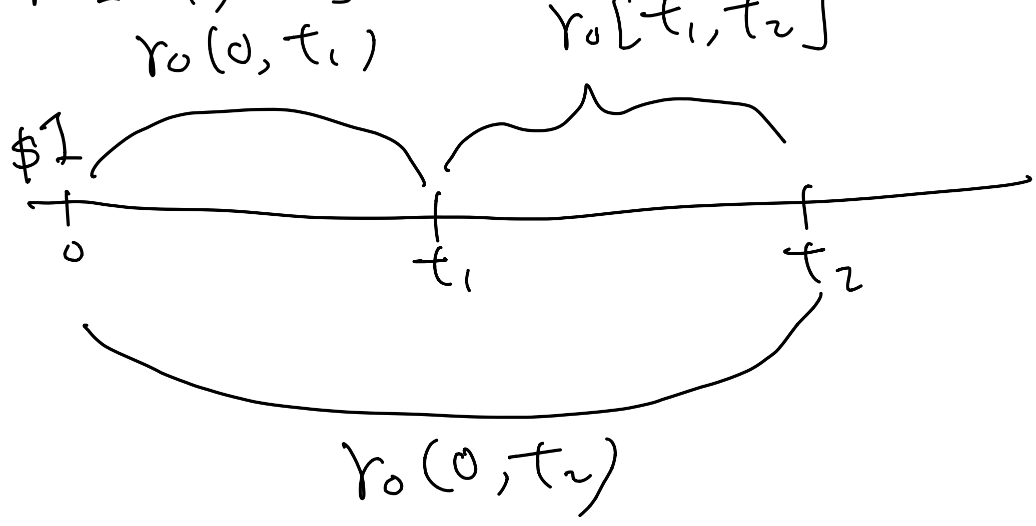
leased rate = Net benefit of asset owner

$$\delta_l = y - u$$

$$F_{0,T} = S_0 e^{(r - \delta_l) T}$$

Implied forward rate

$r_0[t_1, t_2]$



$$(1 + r_0(0,t_1))^{t_1} (1 + r_0[t_1,t_2])^{t_2 - t_1}$$

$$= (1 + r_0(0,t_2))^{t_2}$$

$$(1 + r_0[t_1,t_2])^{t_2-t_1} = \frac{(1 + r_0(0,t_2))^{t_2}}{(1 + r_0(0,t_1))^{t_1}}$$

$$= \boxed{\frac{P(0,t_1)}{P(0,t_2)}}$$

$$F_{0,T} = S_0 e^{rT} = \frac{S_0}{P(0,T)}$$

t_2-year Forward price of t_1-year zero-coupon bond

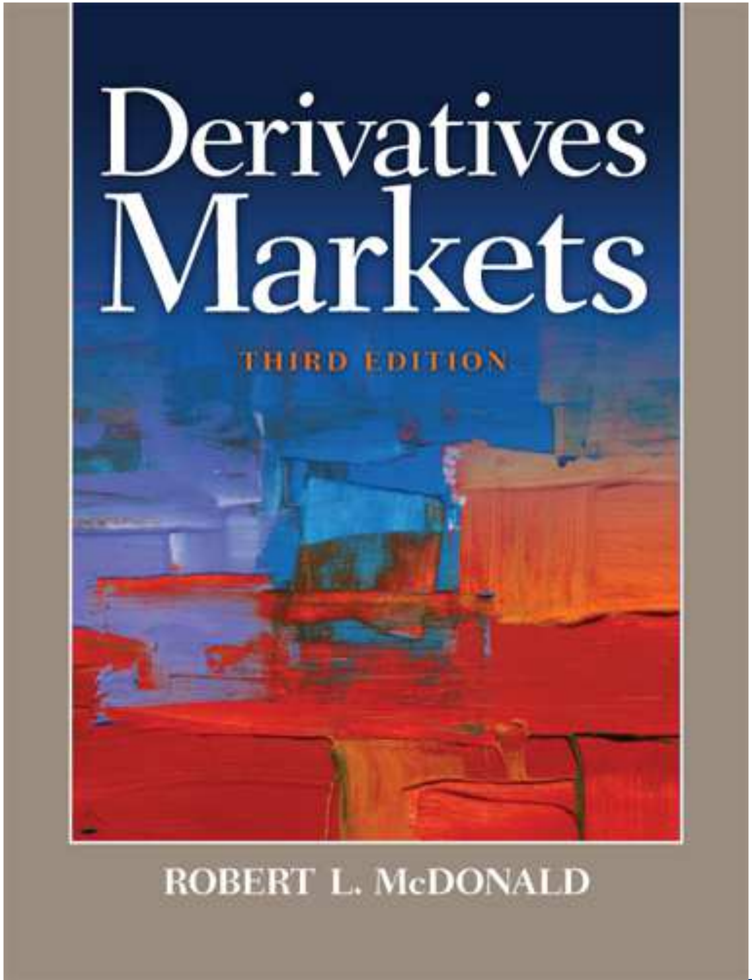
$$F_{0,T} = FV(S_0) = (S_0)\boxed{\left(\frac{1}{P(0,T)}\right)}$$

FV

$$F^P_{0,T} = PV(F_{0,T}) = e^{-rT} F_{0,T}$$
$$= \boxed{P(0,T)}\, F_{0,T}$$

"PV"

Corp



Chapter 7
(Chapter 8 in the textbook)

Swaps

An Example of a Commodity Swap

- An industrial producer, IP Inc., needs to buy 100,000 barrels of oil 1 year from today and 2 years from today.
- The forward prices for deliver in 1 year and 2 years are $110 and $111/barrel.
- The 1- and 2-year zero-coupon bond yields are 6% and 6.5% (annual effective interest rate).

	0	1	2
		Long 1-yr Forward	Long 2-yr Forward
①		\$110	\$111
②	\$P	0	0

$$\$P = PV(110) + PV(111)$$

$$= \frac{110}{(1+6\%)} + \frac{111}{(1+6.5\%)^2}$$

	0	1	2
③		\$x	\$x

$$\left.\frac{x}{1+6\%} + \frac{x}{(1+6.5\%)^2} = P \right\} \text{swap}$$

$$\Rightarrow x = \$110.483$$

An Example of a Commodity Swap (cont'd)

- IP can guarantee the cost of buying oil for the next 2 years by entering into long forward contracts for 100,000 barrels in each of the next 2 years. The PV of this cost per barrel is

$$\frac{\$110}{1.06} + \frac{\$111}{1.065^2} = \$201.638$$

- Thus, IP could pay an oil supplier \$201.638, and the supplier would commit to delivering one barrel in each of the next two years.
- A **prepaid swap** is a single payment today to obtain *multiple* deliveries in the future.

An Example of a Commodity Swap (cont'd)

- With a prepaid swap, the buyer might worry about the resulting credit risk. Therefore, a more attractive solution is to defer payment until the oil is delivered, while still fixing the total price.
- Any payments that have a present value of \$201.638 are acceptable. Typically, a swap will call for equal payments in each year.
  - For example, the payment per year per barrel, x, will have to be \$110.483 to satisfy the following equation

$$\frac{x}{1.06} + \frac{x}{1.065^2} = \$201.638$$

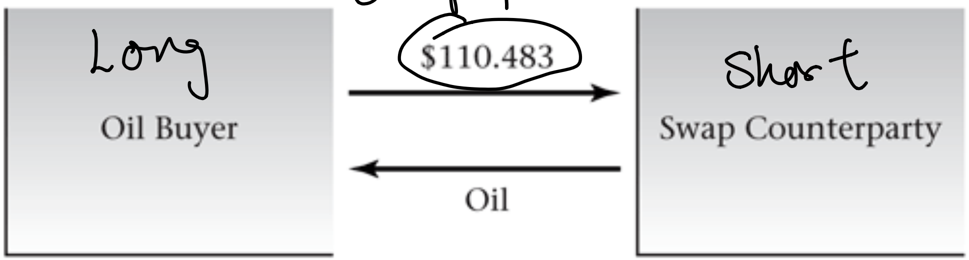
- We then say that the 2-year swap price is \$110.483.

Physical Versus Financial Settlement

- **Physical settlement** of the swap

Swap price

IP



Oil Supplier

Fixed swap rate payer (oil buyer) is said to take a **long position** of the swap while the (fixed rate receiver) swap counterparty is in the **short position** of the swap.

Physical Versus Financial Settlement (cont'd)

- **Financial settlement** of the swap
  - The oil buyer, IP, pays the swap counterparty the difference between \$110.483 and the spot price, and the oil buyer then buys oil at the spot price.
  - If the difference between \$110.483 and the spot price is negative, then the swap counterparty pays the buyer.

Physical Versus Financial Settlement (cont'd)

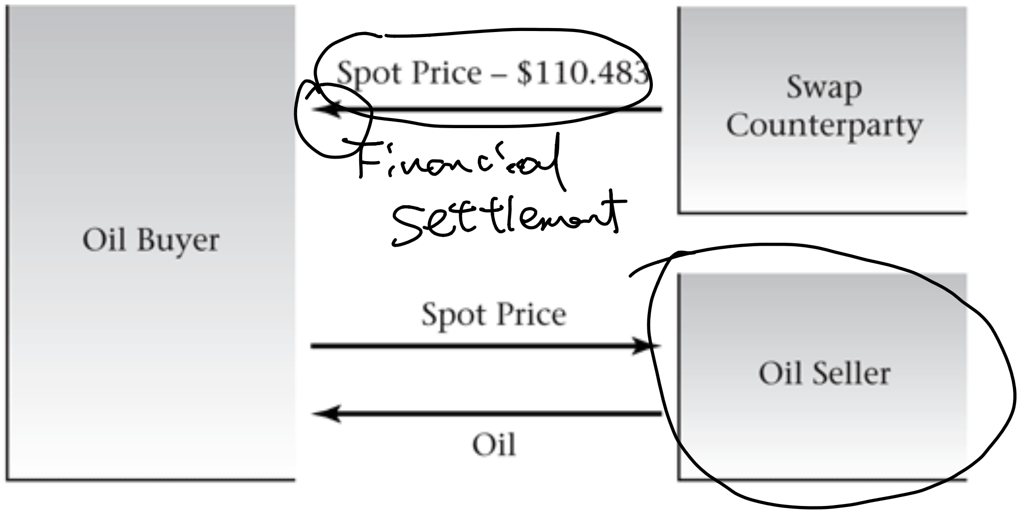
- Whatever the market price of oil, the net cost to the buyer is the swap price, \$110.483.

$$\underbrace{\text{Spot price} - \text{swap price}}_{\text{Swap payment}} - \underbrace{\text{Spot price}}_{\text{Spot purchase of oil}} = \boxed{-\text{Swap price}}$$

$\boxed{\text{Cost of the swap}}$ $\boxed{\text{market oil price}}$ || cost of the oil

Physical Versus Financial Settlement (cont'd)

- The results for the buyer are the same whether the swap is settled physically or financially. In both cases, the net cost to the oil buyer is $110.483.



IP has a swap : swap price

$= 110.483$

spot
oil price = \$ 120

1 Under the swap :

IP has to pay $\boxed{\underbrace{110.483 - 120}_{-\$9.517}}$ to

swap counterparty

2 IP buys the oil from the market

= \$120

Net cost of IP = 2 + 1

$= 120 + (-9.517) = \boxed{\$110.483}$ swap price

Spot oil price = \$105 , Net cost of IP = ?

1 = IP pays $110.483 - 105 = 5.483$

2 = \$105

$1 + 2 = 110.483$

FIGURE 8.3

Illustrative example of the terms for an oil swap based on West Texas Intermediate (WTI) crude oil.

Fixed-Price Payer:	Broker-dealer
Floating-Price Payer:	Counterparty
Notional Amount:	100,000 barrels
Trade Date:	April 18, 2011
Effective Date:	July 1, 2011
Termination Date:	September 31, 2011
Period End Date:	Final Pricing Date of each Calculation Period as defined in the description of the Floating Price.
Fixed Price:	110.89 USD per barrel
Commodity Reference Price:	OIL-WTI-NYMEX
Floating Price:	The average of the first nearby NYMEX WTI Crude Oil Futures settlement prices for each successive day of the Calculation Period during which such prices are quoted
Calculation Period:	Each calendar month during the transaction
Method of Averaging:	Unweighted
Settlement and Payment:	If the Fixed Amount exceeds the Floating Amount for such Calculation Period, the Fixed Price Payer shall pay the Floating Price Payer an amount equal to such excess. If the Floating Amount exceeds the Fixed Amount for such Calculation Period, the Floating Price Payer shall pay the Fixed Price Payer an amount equal to such excess.
Payment Date:	5 business days following each Period End Date

	0	1	2
Long Forward		pay $110	pay $111
Long Swap		$110.483	$110.483
Swap − Forward		$0.483	$−0.517
		Lending	repayment of Loan

$\Rightarrow$ Implied forward Lending rate $r_0[1,2]$

$$r_0[1,2] = \frac{0.517}{0.483} - 1$$

Physical Versus Financial Settlement (cont'd)

- Swaps are nothing more than forward contracts coupled with borrowing and lending money
  - Consider the swap price of \$110.483/barrel. Relative to the forward curve price of \$110 in 1 year and \$111 in 2 years, we are overpaying by \$0.483 in the first year, and we are underpaying by \$0.517 in the second year.
  - Thus, by entering into the swap, we are lending the counterparty money for 1 year. The interest rate on this loan is

$$0.517 / 0.483 - 1 = 7\%$$

  - Given 1- and 2-year zero-coupon bond yields of 6% and 6.5%, 7% is the 1-year implied forward yield from year 1 to year 2.
- If the deal is priced fairly, the interest rate on this loan should be the implied forward interest rate.

The Swap Counterparty

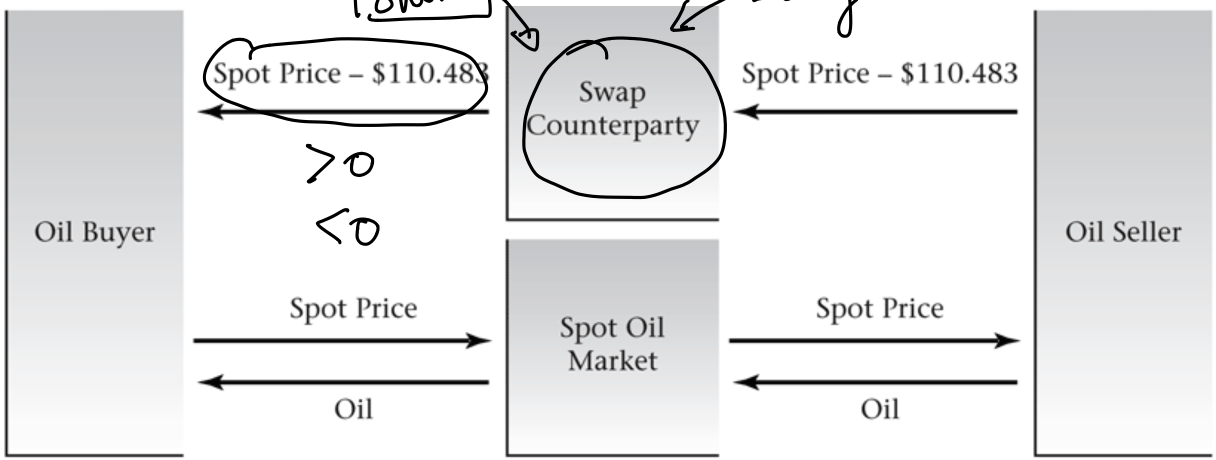
- The swap counterparty is a dealer, who is, in effect, a broker between buyer and seller.
- The dealer can hedge the oil price risk resulting from the swap in several ways.
- The fixed price paid by the buyer, usually, exceeds the fixed price received by the seller. This price difference is a bid-ask spread, and is the dealer’s fee.
- The dealer bears the credit risk of both parties, but is not exposed to price risk.

The Swap Counterparty (cont'd)

- The situation where the dealer matches the buyer and seller is called a **back-to-back transaction** or "matched book" transaction.

Short

Long



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The Swap Counterparty (cont'd)

- Alternatively, the dealer can serve as counterparty and hedge the transaction by entering into long forward or futures contracts.

Short swap

TABLE 8.1 Positions and cash flows for a dealer who has an obligation to receive the fixed price in an oil swap and who hedges the exposure by going long year 1 and year 2 oil forwards.

PV(net) = 0

Year	Payment from Oil Buyer	Long Forward	Net
1	\$110.483 − year 1 spot price	Year 1 spot price − \$110	$\boxed{\$0.483}$
2	\$110.483 − year 2 spot price	Year 2 spot price − \$111	$\boxed{-\$0.517}$

  - Note that the net cash flow for the hedged dealer is a loan, where the dealer receives cash in year 1 and repays it in year 2.
  - Thus, the dealer also has interest rate exposure (which can be hedged by using Eurodollar contracts or forward rate agreements).

The Market Value of a Swap

- The market value of a swap is zero at inception.
- Once the swap is struck, however, its market value will generally no longer be zero because
  - The forward prices for oil and interest rates will change over time.
  - Even if prices do not change, the market value of swaps can change over time due to the implicit borrowing and lending.
- A buyer wishing to exit the swap could negotiate terms with the original counterparty to eliminate the swap obligation or enter into an offsetting swap with the counterparty offering the best price.

The Market Value of a Swap (cont'd)

- The original swap called for the oil buyer to pay the fixed price and receive floating; the offsetting swap has the buyer receive the fixed price and pay floating. The original obligation would be cancelled except to the extent that the fixed prices are different.

- The ~~market value~~ of the swap in the perspective of the **long position** is

  PV of the payments with the amount of (New swap rates – Original swap rates).

X

Notional

Short position

Market value of swap (short)

= PV (Original swap rate − New swap rate) × Notional

= − market value of swap (Long)

The Market Value of a Swap (cont'd)

Example

- Suppose the forward curve for oil rises by $2 in years 1 and 2. Thus, the year 1 forward price becomes $112 and the year-2 forward price becomes $113.
- Assuming interest rates are unchanged, the new swap price is $112.483 (*Verify!*).
- The market value of the swap in the perspective of the **long position** is

$$\frac{(112.483-110.483)}{1.06}+\frac{(112.483-110.483)}{(1.065)^2}=\$3.650$$

Computing the Swap Rate

- Notation
  - Suppose there are n swap settlements, occurring on dates t_i, $i = 1, \ldots, n$.
  - The forward prices on these dates are given by F_{0,t_i}
  - The price of a zero-coupon bond maturing on date t_i is $P(0, t_i)$.
  - The fixed swap rate is R.
- If the buyer at time zero were to enter into forward contracts to purchase one unit on each of the n dates, the present value of payments would be the present value of the forward prices, which equals the price of the prepaid swap:

$$\text{Prepaid swap} = \sum_{i=1}^{n} F_{0,t_i} P(0, t_i)$$

Computing the Swap Rate (cont'd)

- We determine the fixed swap price, *R*, by requiring that the present value of the swap payments equal the value of the prepaid swap

$$\sum_{i=1}^{n} RP(0,t_i) = \sum_{i=1}^{n} F_{0,t_i} P(0,t_i) \qquad (8.2)$$

- Equation (8.2) can be rewritten as

$$R = \frac{\sum_{i=1}^{n} P(0,t_i) F_{0,t_i}}{\sum_{i=1}^{n} P(0,t_i)} \qquad (8.3)$$

where $\sum_{i=1}^{n} P(0,t_i) F_{0,t_i}$ is the present value of payments implied by the strip of forward rates, and $\sum_{i=1}^{n} P(0,t_i)$ is the present value of a $1 annuity.

Computing the Swap Rate (cont'd)

- We can rewrite equation (8.3) to make it easier to interpret

Weight

$$R = \sum_{i=1}^{n} \left[\frac{P(0,t_i)}{\sum_{j=1}^{n} P(0,t_j)} \right] F_{0,t_i}$$

- Thus, the fixed swap rate is as a weighted average of the forward prices, where zero-coupon bond prices are used to determine the weights.

Swaps With Variable Quantity and Prices

- A buyer with seasonally varying demand (e.g., someone buying gas for heating) might enter into a swap, in which *quantities* vary over time.
- Consider a swap in which the buyer pays RQ_{t_i}, for Q_{t_i} units of the commodity. The present value of these fixed payments (fixed per unit of the commodity) must equal the prepaid swap price

PV(forward)

$$\sum_{i=1}^{n} Q_{t_i} F_{0,t_i} P(0,t_i) = \boxed{\sum_{i=1}^{n} Q_{t_i} R P(0,t_i)}$$

PV(swap payment)

- Solving for R gives

$$R = \frac{\sum_{i=1}^{n} Q_{t_i} P(0,t_i) F_{0,t_i}}{\sum_{i=1}^{n} Q_{t_i} P(0,t_i)}$$

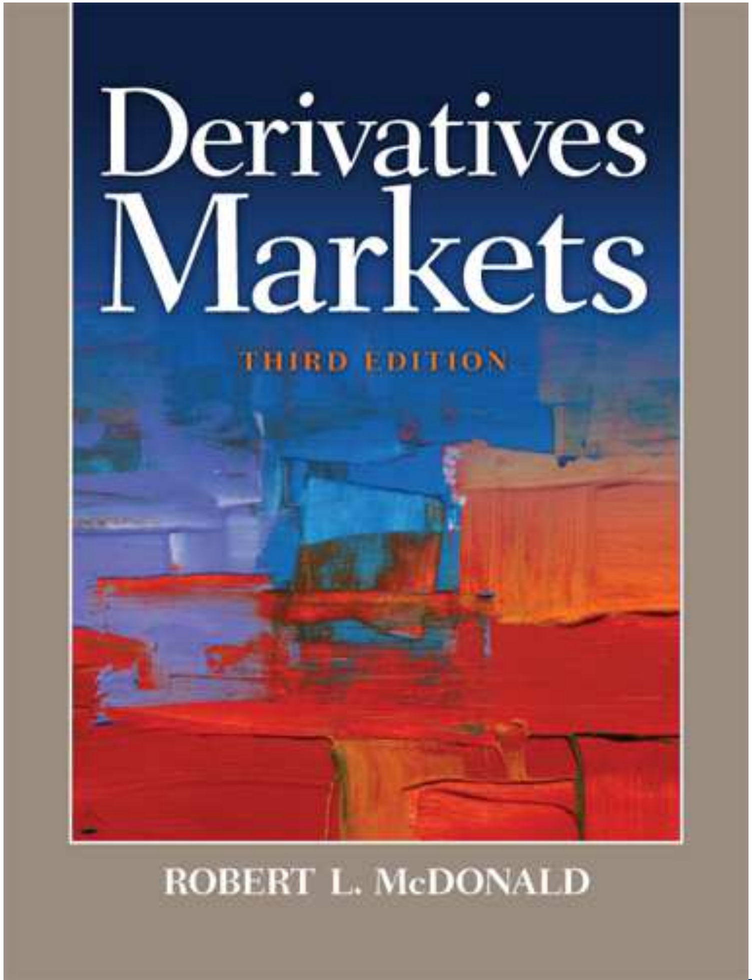
Swaps With Variable Quantity and Prices (cont'd)

- It is also possible for *prices* to be time-varying
  - For example, we let the summer swap price be denoted by R_s and the winter price by R_w, then the summer and winter swap prices can be any prices for which the value of the prepaid swap equals the present value of the fixed swap payment:

$$R_s \sum_{i \in \text{summer}}^{n} P(0,t_i) Q_{t_i} + R_w \sum_{i \in \text{winter}}^{n} P(0,t_i) Q_{t_i} = \sum_{i=1}^{n} P(0,t_i) Q_{t_i} F_{0,t_i}$$

Once we fix one of R_s and R_w, the equation will give us the other.

eg. $\boxed{R_s = 1.1\, R_w}$



Chapter 8
(Chapter 9 in the textbook)

Parity and Other Option Relationships

Put-Call Parity

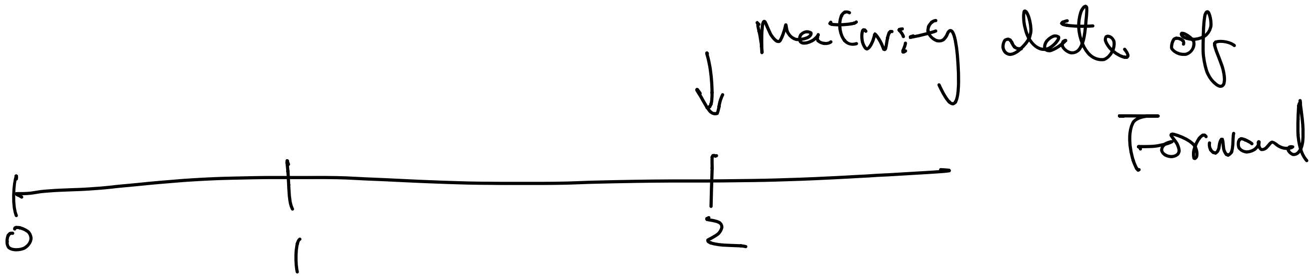
$F_{0,T}$ — maturity date

Important Note: Starting from here, the meaning of T in $F_{0,T}$ is changed to mean the maturity date of the forward contract.

time to expiration

Notations:

- $C(K, T)$ and $P(K, T)$ are the prices of a European call and put with the strike price K and the **time to expiration *T*** respectively;
- $F_{0,T}$ be the time 0 price of the forward contract with the **maturity date at time *T***.



$F_{0,2}$ $\quad$ $F_{1,2}$ $\quad$ $F_{2,2}$

$C(K,2)$ $\quad$ $C(K,1)$ $\quad$ $C(K,0)$

$P(K,2)$ $\quad$ $P(K,1)$ $\quad$ $P(K,0)$

$C(K,2) - P(K,2) = PV(F_{0,2} - K)$

$C(K,1) - P(K,1) = PV(F_{1,2} - K)$

$C(K,0) - P(K,0) = PV(F_{2,2} - K)$

Put-Call Parity (cont'd)

For European options with the same strike price and time to expiration the parity relationship is

Call – put = *PV* (forward price – strike price)

Or

$$C(K,T) - P(K,T) = PV_{0,T}(F_{0,T} - K) = e^{-rT}(F_{0,T} - K)$$

- Intuition
  - Buying a call and selling a put with the strike equal to the forward price ($F_{0,T} = K$) creates a synthetic forward contract and hence must have a zero price.
- In general, put-call parity *fails* for American style options.

What is call option?

> At T, Right to use \$K
>
> to buy the underlying asset.

Right to give up \$K in exchange

for the underlying.

another asset.

Exchange option

underlying

$$C(S_t, Q_t, T-t)$$

give up asset.

Asset A : S_t

Asset B : Q_t

time to expiration

$$C(S_T, Q_T, 0) = \max(S_T - Q_T, 0)$$

> give a right to give up B in exchange
>
> for A

$$P(S_T, Q_T, 0) = \max(Q_T - S_T, 0)$$

give a right to give up A in exchange

for B.

Generalized Parity and Exchange Options (cont'd)

- At time T, we have

  $C(S_T, Q_T, 0) = \max(0, S_T - Q_T)$ and

  $P(S_T, Q_T, 0) = \max(0, Q_T - S_T)$

- Then for European options we have this form of the parity equation:

?

$$C(S_t, Q_t, T-t) - P(S_t, Q_t, T-t) = F^P_{t,T}(S) - F^P_{t,T}(Q)$$

≈K ≈K ≈K

cost of portfolio

$$\boxed{C - P - F^P_{t,T}(S) + F^P_{t,T}(Q)} = 0$$

Exchange rate

HKD 7.8 / USD $\Rightarrow$ use HKD 7.8 to buy USD 1

USD $\frac{1}{7.8}$ / HKD $\Rightarrow$ use USD $\frac{1}{7.8}$ to buy 1 HKD

Currency Options

- A currency transaction involves the exchange of one kind of currency for another.
- The idea that calls can be relabeled as puts is commonplace in currency markets.
- A term sheet for a currency option might specify

  "EUR Call USD Put, AMT: EUR 100 million, USD 120 million"

  It says explicitly that the option can be viewed either as a call on the euro or a put on the dollar. Exercise of the option will entail an exchange of €100 million for $120 million.
- A call in one currency can be converted into a put in the other.

$C(S_t, Q_t, T-t)$ Call on A

$P(Q_t, S_t, T-t)$ Put on B

$$C(S_t, Q_t, T-t) = P(Q_t, S_t, T-t)$$

Currency Options (cont'd)

Example

Suppose the current exchange rate is x_0 = \$1.25/€. Consider the following two options:

1. A *1-year dollar-denominated call option* on euros with a strike price of \$1.20 and premium of \$0.06545. In 1 year, the owner of the option has the right to buy €1 for \$1.20. the payoff on this option, in dollars, is therefore

$$\boxed{\max(0, x_1 - 1.20)}$$

payoff

Currency Options (cont'd)

2. A *1-year euro-denominated put option* on dollars with a strike price of 1/1.20 = €0.833. The premium of this option is €0.04363. In 1 year the owner of this put has the right to give up $1 and receive €0.833; the owner will exercise the put when $1 is worth less than €0.833. The euro value of $1 in 1 year will be $1/x_1$. Hence the payoff of this option is

$$\boxed{\max\left(0, \frac{1}{1.2} - \frac{1}{x_1}\right)}$$

1 unit of Dollar payoff.

BOTH the call and put options are exercised when $x_1 > 1.20$.