数 学 作 业 纸

科目 Derivatives

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1. Chapter 2: Exercise 4

a. Payoff to long position = Spot Price − Forward Price

For prices of \$40, \$45, \$50, \$55 and \$60, the payoff is −\$10, −\$5, 0, \$5, \$10 respectively.

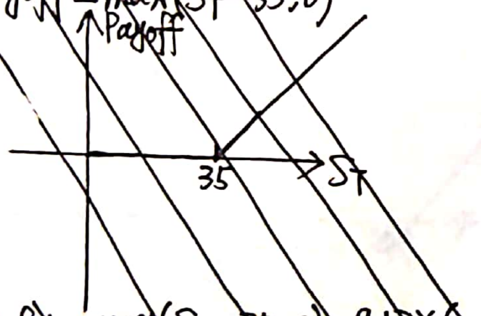
b. Payoff = $\max(S_T - K, 0)$

For prices of \$40, \$45, \$50, \$55 and \$60, the payoff is 0, 0, 0, \$5, \$10 respectively.

c. The 6 month \$50-strike call option should be more expensive because its expected payoff is larger than that of the forward contract. (less likely to have negative payoffs)

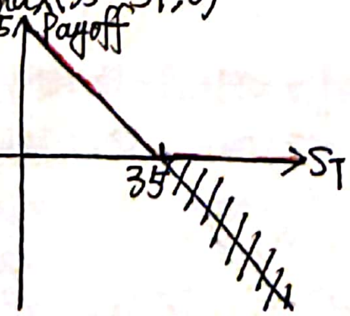
2. Chapter 2: Exercise 14

~~a. Payoff = $\max(S_T - 35, 0)$~~

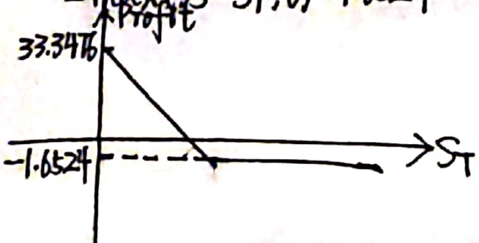


~~Profit = $\max(S_T - 35, 0) - 9.12 \times ($~~

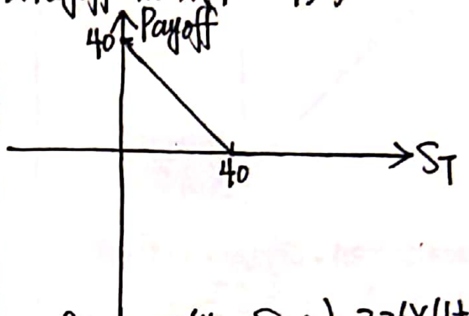
a. Payoff = $\max(35 - S_T, 0)$



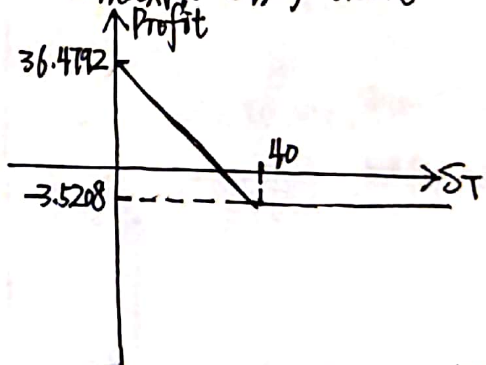
Profit = $\max(35 - S_T, 0) - 1.53 \times (1 + 8\%)$
$= \max(35 - S_T, 0) - 1.6524$



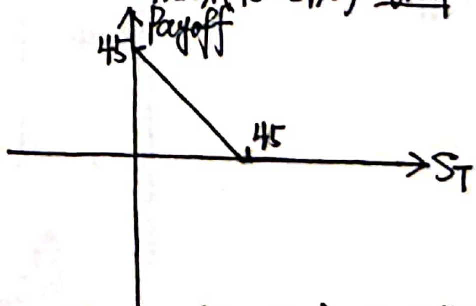
b. Payoff = $\max(40 - S_T, 0)$



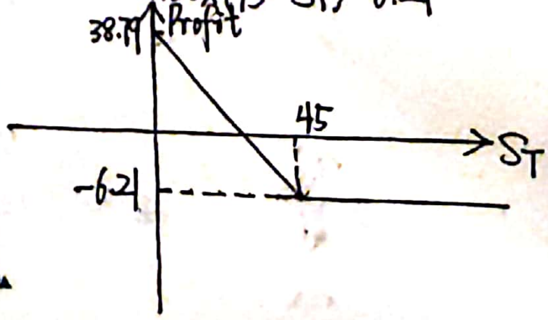
Profit = $\max(40 - S_T, 0) - 3.26 \times (1 + 8\%)$
$= \max(40 - S_T, 0) - 3.5208$



c. Payoff = $\max(45 - S_T, 0)$ ~~$- 5.75 \times (1+8\%)$~~
$= \max(45 - S_T, 0)$ ~~$- 6.21$~~



Profit = $\max(45 - S_T) - 5.75 \times (1 + 8\%)$
$= \max(45 - S_T) - 6.21$



b. Because as the strike price increases, the payoff for ~~an option is~~ a put option is expected to be higher and more risky.

3. Chapter 3: Exercise 13

a. $Cost = [Call(1050,t) + Put(1050,t)] \times 1.04$

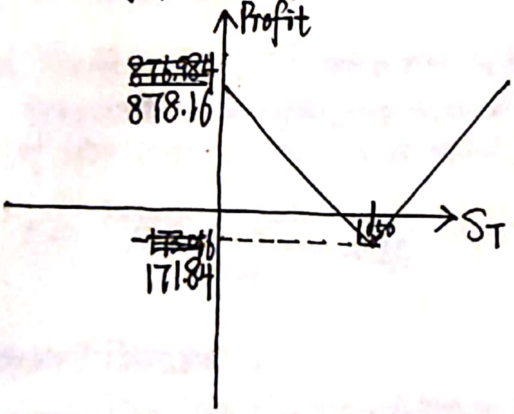
$Payoff = \max(S_T - K, 0) + \max(K - S_T, 0)$

$\cancel{Profit} = \max(S_T - 1050, 0) + \max(1050 - S_T, 0)$

$Profit = \max(S_T - 1050, 0) + \max(1050 - S_T, 0) - [Call(1050,t) - Put(1050,t)] \times 1.04$

$\cancel{= \begin{cases} -173.016, & S_T < 1050 \\ & S_T > 1050 \end{cases}}$

$= \begin{cases} \cancel{876.984}\ 878.16 - S_T, & S_T < 1050 \\ S_T - \cancel{1223.016}\ 1221.84, & S_T \geq 1050 \end{cases}$

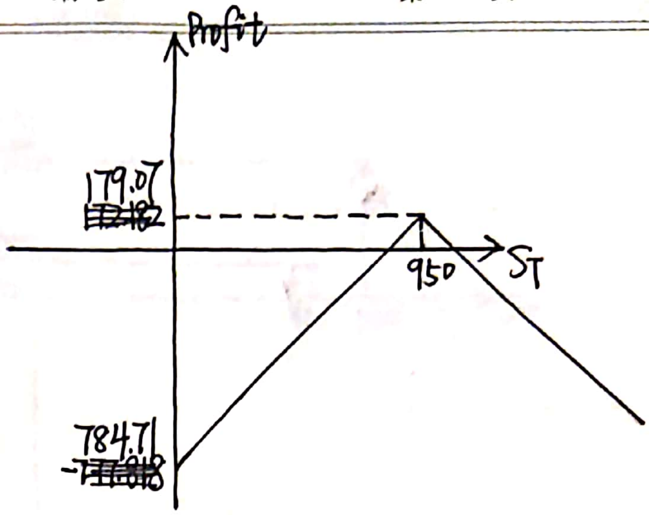


b. $Cost = -[Call(950,t) - Put(950,t)] \times 1.04$

$Payoff = -\max(S_T - 950) - \max(950 - S_T)$

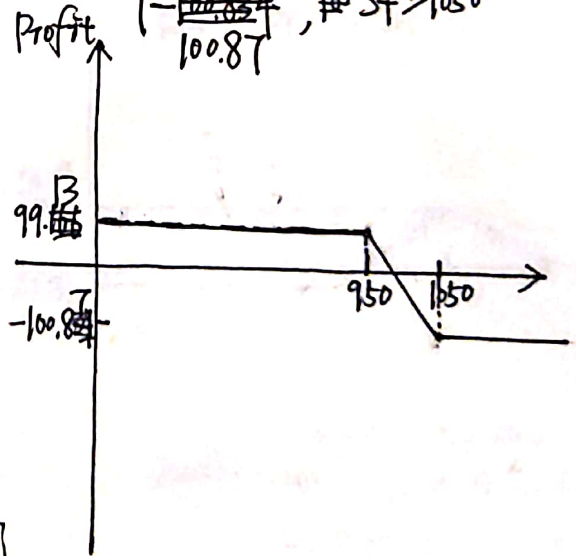
$Profit = -\max(S_T - 950) - \max(950 - S_T) + [Call(950,t) + \cancel{\max(950 - S_T}\ Put(950,t)] \times 1.04$

$= \begin{cases} S_T - \cancel{777.818}\ 784.71, & S_T < 950 \\ \cancel{1122.182}\ 1129.07 - S_T, & S_T \geq 950 \end{cases}$



c. $Profit = \max(S_T - 1050, 0) + \max(1050 - S_T, 0) + [-Call(1050,t) - Put(1050)] \times 1.04 - \max(S_T - 950) - \max(950 - S_T) + [Call(950,t) + Put(950,t)] \times 1.04$

$= \begin{cases} 99.13, & S_T \leq 950 \\ 1999.13 - 2S_T, & 950 < S_T \leq 1050 \\ -100.87, & S_T > 1050 \end{cases}$



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4. Chapter ~~18~~3: Exercise 14

a. $Payoff = \max(S_T-950,0)-\max(S_T-1000,0)$
$\quad -\max(950-S_T,0)+\max(1000-S_T,0)$

$$= \begin{cases} 50, & S_T \le 950 \\ 50, & 950 < S_T \le 1000 \\ 50, & S_T > 1000 \end{cases}$$

$= 50$

Therefore, there is no S&R price risk in this transaction.

b. $Cost = Call(950,t) - Call(1000,t)$
$\quad - Put(950,t) + Put(1000,t)$
$= 120.405 - 93.809 - 51.777 + 74.201$
$= 49.027$

c. The value of the position after 6 months is ~~49.027~~ 50

d. There is no S&R price risk in this transaction. Therefore, the rate of return of this transaction should equal to r.

$$r = \frac{Payoff - Cost}{Cost} \approx 2\%$$

5. Chapter 3: Exercise 15

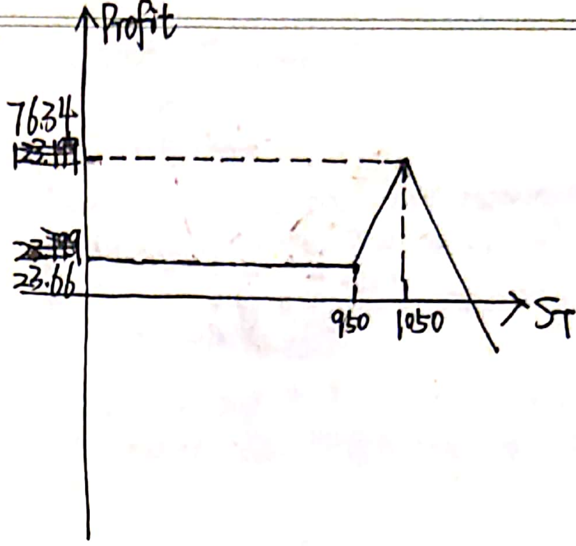
a. $Cost = [Call(950,t) - 2Call(1050,t)] \times 1.02$
$= \cancel{23.199}\ 23.66$

$Payoff = \max(S_T-950,0) - 2\max(S_T-1050,0)$

$$= \begin{cases} 0, & S_T \le 950 \\ S_T - 950, & 950 < S_T \le 1050 \\ 1150 - S_T, & S_T > 1050 \end{cases}$$

~~Profit = {23.199 66, S_T ≤ 950; S_T − 926.801 950 < S_T ≤ 1050; 1173.199 − S_T, S_T > 1050}~~

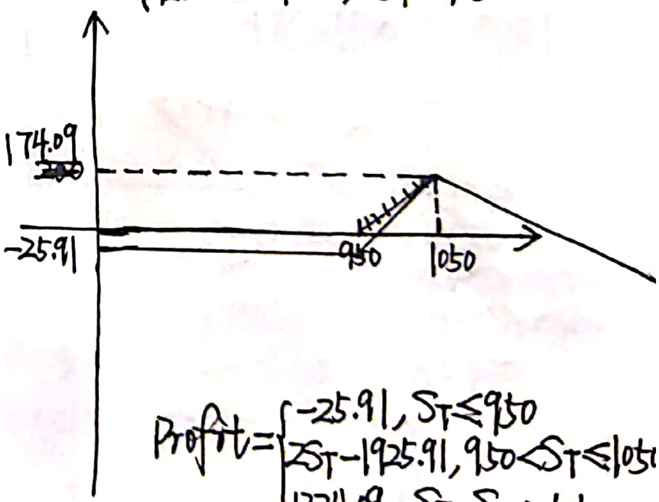
$$Profit = \begin{cases} -23.66, & S_T \le 950 \\ S_T - 973.66, & 950 < S_T \le 1050 \\ 1126.34 - S_T, & S_T > 1050 \end{cases}$$



b. $Cost = [2Call(950,t) - 3Call(1050,t)] \times 1.02$
$= \cancel{62.832}\ 25.91$

$Payoff = 2\max(S_T-950,0) - 3\max(S_T-1050,0)$

$$= \begin{cases} 0, & S_T \le 950 \\ 2S_T - 1900, & 950 < S_T \le 1050 \\ 1250 - S_T, & S_T > 1050 \end{cases}$$



$$Profit = \begin{cases} -25.91, & S_T \le 950 \\ 2S_T - 1925.91, & 950 < S_T \le 1050 \\ 1224.09 - S_T, & S_T > 1050 \end{cases}$$

5. Chapter3: Exercise 15

The ratio can be calculated by

$$120.405n - 71.802m = 0$$

$$\frac{n}{m} = \frac{71.802}{120.405} \approx 0.596 = 0.6$$

$\therefore n:m = 3:5$

6. Chapter3: Exercise 16

(1) bull spread:

$$Payoff = \max(S_T - K_1, 0) - \max(S_T - K_2, 0)$$

$\because K_1 < K_2$

$\therefore Payoff \geq 0$ ($Payoff = 0$ only when $S_T \leq K_1$)

Therefore, the payoff of the bull spread cannot be negative and there is no bull spread with zero initial premium.

(2) bear spread:

$$Payoff = -\max(S_T - K_1, 0) + \max(S_T - K_2, 0)$$

$K_1 < K_2$

$Payoff \leq 0$ ($Payoff = 0$ only when $S_T \leq K_1$)

$\therefore$ There is no bear spread with zero initial premium.

(3) butterfly spread:

$$Payoff = -\max(S_T - K, 0) - \max(K - S_T, 0) + \max(S_T - K_2, 0) + \max(K_1 - S_T, 0)$$

$\because K_1 < K < K_2$

$\therefore \max(S_T - K_2, 0) - \max(S_T - K, 0) \leq 0$

(equals to zero only when $S_T \leq K$)

$\max(K_1 - S_T, 0) - \max(K - S_T, 0) \leq 0$

(equals to zero only when $S_T \geq K$)

Therefore, payoff ≤ 0 and there is no butterfly spread with zero initial premium.

In a word, if the payoff of transactions is always greater or equal to zero or less or equal to zero (equals to zero only in some cases), the premium of transactions cannot be zero.

7. Chapter 3: Exercise 17

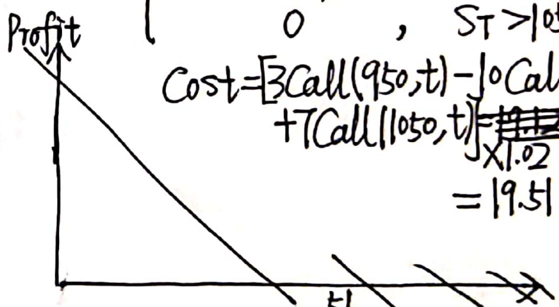
$$\lambda = \frac{1050 - 1020}{1050 - 950} = 0.3$$

Therefore, to construct the asymmetric butterfly, for every ~~950~~ 1020-strike call we write, we buy 0.3 950-strike call and 0.7 1050-strike call.

So we can buy 3 950-strike call, sell ten 1020-strike call, buy 7 1050-strike call.

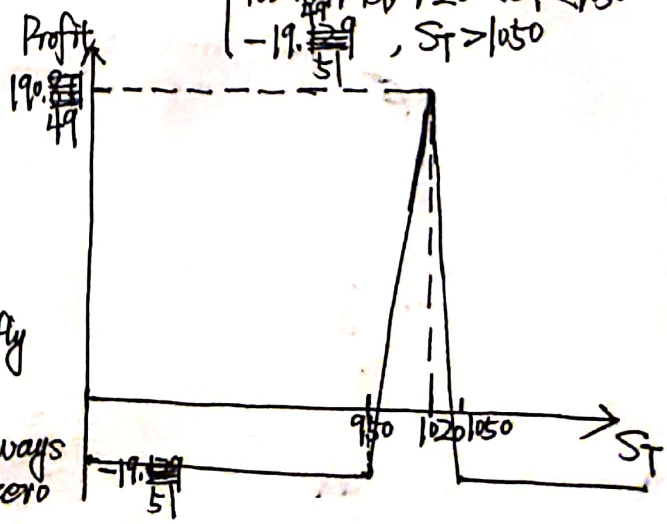
$$Payoff = 3\max(S_T - 950, 0) - 10\max(S_T - 1020, 0) + 7\max(S_T - 1050, 0)$$

$$= \begin{cases} 0, & S_T \leq 950 \\ 3S_T - 2850, & 950 < S_T \leq 1020 \\ 7350 - 7S_T, & 1020 < S_T \leq 1050 \\ 0, & S_T > 1050 \end{cases}$$



$$Cost = [3Call(950,t) - 10Call(1020,t) + 7Call(1050,t)] \times 1.02 = 19.51$$

$$Profit = \begin{cases} -19.51, & S_T \leq 950 \\ 3S_T - 2869.51, & 950 < S_T \leq 1020 \\ 7330.49 - 7S_T, & 1020 < S_T \leq 1050 \\ -19.51, & S_T > 1050 \end{cases}$$



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Additional Problem1

accumulated trading volume = 9+3+3+2=17

open interest = 8

Additional Problem2

~~Cost = C(K2,T)+P(K2,T)+P~~

~~Payoff~~

$$Cost = -C(K_2,T) - P(K_2,T) + P(K_1,T) + C(K_3,T)$$

$$Payoff = -max(S_T - K_2, 0) - max(K_2 - S_T, 0) + max(\cancel{S_T - K_1}\ K_1 - S_T, 0) + max(S_T - K_3, 0)$$

Put-Call Parity:

$$Call(K,t) - Put(K,t) = PV(F_{0,t} - K)$$

$$Payoff = -2Call(K_2,0) + Call(K_1,0) + Call(K_3,0) + K_1 - K_2$$

$$Cost = -2Call(K_2,T) + Call(K_3,T) + Call(K_1,T) + PV(K_1 - K_2)$$

$$\begin{aligned} Profit &= Payoff - FV(cost) \\ &= -2Call(K_2,0) + Call(K_1,0) + Call(K_3,0) \\ &\quad - FV[-2Call(K_2,T) + Call(K_3,T) + Call(K_1,T) \end{aligned}$$

= Profit of the butterfly spread.