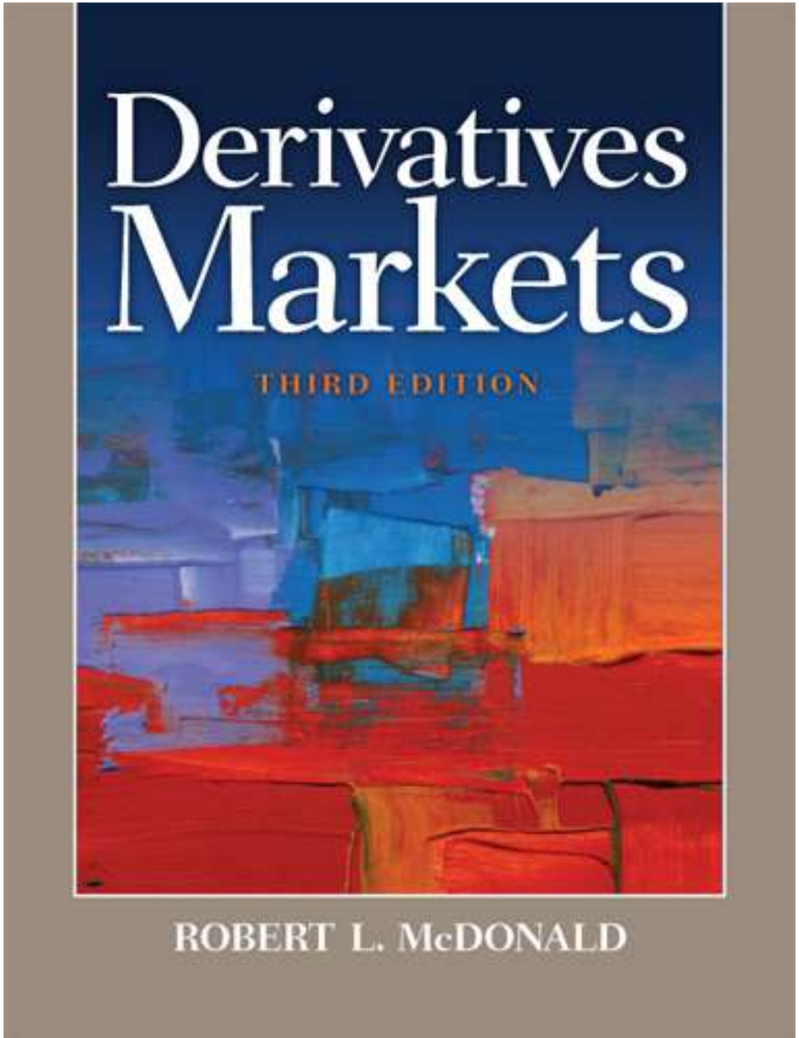


Chapter 7
(Chapter 8 in the textbook)

Swaps

a portfolio of forward contracts

Points to Note

1. How does the swap compare with the forward contracts? See P.3 – 6. Swap : portfolio of forwards
2. What are the meaning of the long and short positions? See P. 7.
3. What is the difference between the physical settlement and financial settlement? See P. 7 – 13.
4. Understand TWO different ways in which the dealer uses to hedge his swap positon. See P.14 – 16.
5. The market value of the a swap. See P.17 – 19.
6. Computing the swap rate. See P.20 – 25.

1) PV(swap fixed payment) = PV(forward prices)

$$\underset{\text{Swaprate}}{R} = \frac{\sum_{i=1}^{n} F_{0,t_i} P(0,t_i)}{\sum_{i=1}^{n} P(0,t_i)}$$

② Long Swap: pay fixed and recieve underlying

Short swap: reieve fixed and pay underlying

③

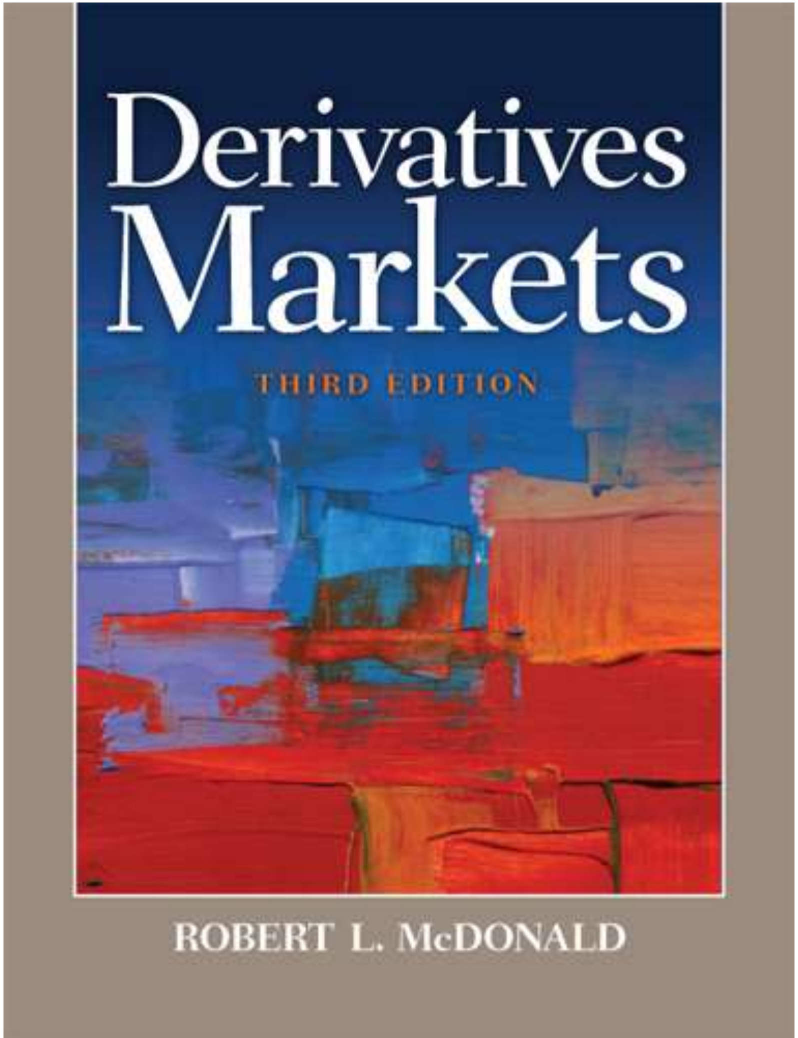
④ hedging $\begin{cases} \text{back to back hedging} \\ \text{use forwards} \end{cases}$

Long swap + $\underbrace{\boxed{\text{Short a porfolio of forwards}}}$ Short a synthetic swap

⑤ market value of the swap $\begin{cases} \text{Long} \\ \text{Short} \end{cases}$

= Long swap @ t=0 — Short the swap at t=1

= market value of the swap at t=1



Chapter 8
(Chapter 9 in the textbook)

Parity and Other Option Relationships

Points to Note

1. Important relation: Put-call parity. See P.5
2. Generalized put-call parity on exchange options. See P.9
3. The relationship between call and put options on exchange rate. See P.15
4. Compare the prices of European and American options. See P.16
5. The upper and lower bounds of the option price. See P.17 – 18
6. Early exercises of American call and put options. See P.19 – 24
7. Relationship between time to expiration and option price. See P. 25
8. Relationship between strike prices and option prices. See P.26 - 30
9. Convexity property of option prices with respect to strike prices. See P.31

① put-call parity

$$C(K,T) - P(K,T) = PV(F_{0,T}) - PV(K)$$

non-dividend
→ discrete divid
→ continuous divid

② Generalized option A: S_t B: Q_t

$\max(S_T - K, 0)$
$\max(K - S_T, 0)$

payoff : $\max(\overset{A}{S_T} - \overset{B}{Q_T}, 0)$

① Call option on A, strike asset is B

At T: right to give up B in exchange for A.

② put option on B, strike asset is A

① ⟺ ②

$C(S_t, Q_t, T-t)$ — underlying, strike, time to expiration

put-call parity

$$C(S_t, Q_t, T-t) - P(S_t, Q_t, T-t)$$

$$= F^P_{t,T}(S) - F^P_{t,T}(Q)$$ where $F^P_{t,T}(\cdot)$ pre-paid forward price

Special case : $Q_t = K$ ⇒ put-call parity in Chapter 3.

$$C(S_t, Q_t, T-t) = P(Q_t, S_t, T-t)$$

Tutorial - Class Activity

22 October, 2019 (Solution)

Question 1 ① position of swap ② Cashflow of the swap / forward

1. Suppose that 1-year, 2-year and 3-year oil forward prices are \$20 per barrel, \$21 per barrel, and \$22 per barrel respectively. The annualized continuously compounded yield of the zero-coupon bonds for different maturities are given as follows:

Maturity (in year)	Annualized continuously compounded yield
1	5.83%
2	6.30%
3	6.77%

$P(0,t) = e^{-r_{cc}(0,t)\,t}$

Consider a 3-year swap on oil with the swap price of \$20.9519 per barrel of oil and the notional amount of 1 barrel of oil. Suppose a dealer is paying the fixed price and receiving floating price in the swap contract.

⇓ LONG

a. What position in oil forward contracts will hedge oil price risk in the dealer's position?
b. What is the present value of the locked-in net cash inflows of the dealer at the inception of the swap contract?
c. Just immediately after the inception of the swap contract all interest rates fall 50 basis points, what is the new present value of the locked-in cash inflows of the dealer?

1 bp = 0.01%
50 bp = 0.5%

Solution

inflow

a) Since the dealer is paying fixed and receiving floating, she generates the cash-flows depicted in column 2. Suppose that the dealer enters into three short forward positions, one contract for each year of the active swap. Her payoffs are depicted in column 3, and the aggregate net cash inflow position is in column 4.

synthetic swap

Year	Net Cash Inflow from Swap	Cash Inflow from Shorting Forwards	Net Cash Inflow Position
1	$S_1 - \$20.9519$	$\$20 - S_1$	−0.9519
2	$S_2 - \$20.9519$	$\$21 - S_2$	+0.0481
3	$S_3 - \$20.9519$	$\$22 - S_3$	+1.0481

> 0 or < 0

Constant

b) PV (net cash inflows) = $-0.9516e^{-(5.83\%)(1)} + 0.0481e^{-(6.30\%)(2)} + 1.0481e^{-(6.77\%)(3)} = -0.0001.$

Hence, the present value of the locked-in net cash inflows of the dealer at the inception of the swap contract is nearly zero.

(own practice : ↓ 50 bps)

c) If all interest rates fall 50 basis points just after the inception of the swap contract, we have

$$\text{PV (net cash inflows)} = -0.9519e^{-(5.33\%)(1)} + 0.0481e^{-(5.8\%)(2)} + 1.0481e^{-(6.27\%)(3)} = 0.0087.$$

$\neq 0$

Question 2

The table below describes 4 stocks:

Stock	Price	Dividend yield (δ)
A	40	4%
B	50	0%
C	60	0%
D	75	2%

X: 1A + 2B

Y: 1C + 1D

A 1-year European call option gives its owner the right to give up 1 share of Stock C and 1 share of Stock D in exchange for 1 share of Stock A and 2 shares of Stock B. The value of this call option is \$10.

Give up Y in exchange for X

Find the value of a 1-year European call option that gives its owners the right to give up 1 share of Stock A and 2 shares of Stock B in exchange for 1 share of Stock C and 1 share of Stock D.

give up X in exchange for Y

Solution

Let's establish two portfolios. Portfolio X consists of 1 share of Stock A and 2 shares of Stock B. Portfolio Y consists of 1 share of Stock C and 1 share of Stock D.

The first call option described in the question has Portfolio X as its underlying asset:

$$C_{Eur}(X_0, Y_0, 1) = 10.$$

We can find the prepaid forward prices for both portfolios:

$$F_{0,1}^P(X_0) = 40e^{-0.04(1)} + 2(50) = 138.4316;$$
$$F_{0,1}^P(Y_0) = 60 + 75e^{-0.02(1)} = 133.5149.$$

We can now use put-call parity to find the value of the corresponding put option:

$$C_{Eur}(X_0, Y_0, 1) - P_{Eur}(X_0, Y_0, 1) = F_{0,1}^P(X_0) - F_{0,1}^P(Y_0)$$
$$10 - P_{Eur}(X_0, Y_0, 1) = 138.4316 - 133.5149$$
$$P_{Eur}(X_0, Y_0, 1) = 5.0833.$$

We can describe a put option as a call option by switching the underlying asset with the strike asset:

D	75	2%

A 1-year European call option gives its owner the right to give up 1 share of Stock C and 1 share of Stock D in exchange for 1 share of Stock A and 2 shares of Stock B. The value of this call option is $10.

Give up Y in exchange for X

$$C(X_0, Y_0, 1) = 10 = P(Y_0, X_0, 1)$$

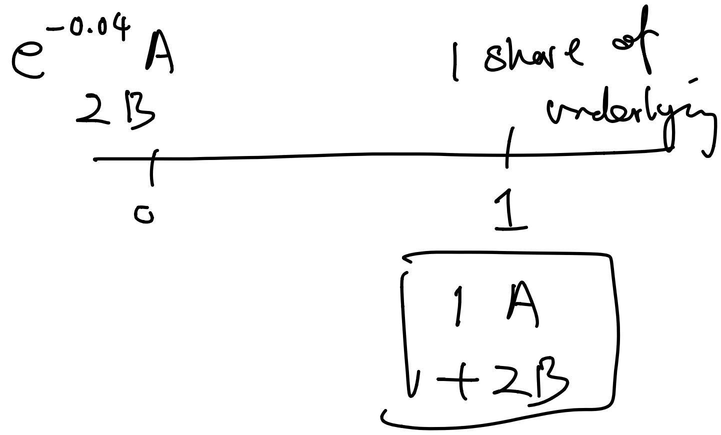
Find the value of a 1-year European call option that gives its owners the right to give up 1 share of Stock A and 2 shares of Stock B in exchange for 1 share of Stock C and 1 share of Stock D.

give up X in exchange for Y

$$C(Y_0, X_0, 1) = ?? = P(X_0, Y_0, 1)$$

$$\overset{=10}{C(X_0, Y_0, 1)} - \overset{??}{P(X_0, Y_0, 1)} = \overset{?}{F^P_{0,1}(X)} - \overset{?}{F^P_{0,1}(Y)}$$

X : 1A + 2B



$$F^P_{0,1}(X)$$

$$= e^{-0.04}(40) + 2(50)$$

$$F^P_{0,1}(Y)$$

$$= 60 + 75e^{-0.02}$$

$$P_{Eur}\left(X_0, Y_0, 1\right) = C_{Eur}\left(Y_0, X_0, 1\right)$$
$$C_{Eur}\left(Y_0, X_0, 1\right) = 5.0833.$$

Therefore, the call option giving its owner the right to give up Portfolio X in exchange for Portfolio Y has a value of $5.0833.

Question 3

Consider a European type financial claim with 1 year to expiration. The payoff of the claim depends on the prices of Stock 1 and Stock 2 and the payoff is given by

min[$S_1(1)$, $S_2(1)$, 17] price = 15.5

where $S_j(t)$ is the price of one share of Stock j, j = 1, 2, at time t.

The current price of this claim is $15.50.

You are also given that the current price of a 1-year European option with the payoff of

max[17 – min($S_1(1)$, $S_2(1)$), 0] price = 0.67

is $0.67.

Calculate the continuously compounded risk-free rate of return. r = ??

Solution

Let $X(t)$ = min($S_1(1)$, $S_2(1)$).

The payoff of the European option can be rewritten as

max[17 – min($S_1(1)$, $S_2(1)$), 0] = max[17 – $X(1)$, 0].

Hence, the European option can be regarded as a put option on asset X with strike price of 17.

Since

min[$S_1(1)$, $S_2(1)$, 17] = min[min($S_1(1)$, $S_2(1)$), 17],

the payoff of the given financial claim can be rewritten as

$$\begin{aligned} \min\left(S_1(1), S_2(1), 17\right) &= \min\left(X(1), 17\right) \\ &= \min\left(X(1) - 17, 0\right) + 17 \\ &= -\max\left(17 - X(1), 0\right) + 17. \end{aligned}$$

Hence, the given claim is made of

i. short one unit of 17-strike 1-year put option on asset *X* and
ii. long a 1-year zero-coupon bond with face value of 17.

Let $P(17, 1)$ be the current price of the 17-strike 1-year put option on asset *X*.

Let r be the continuously compounded risk-free rate of return.

By no-arbitrage, we have

put option

$$15.50 = -P(17,1) + 17e^{-r}$$

price of zero

$$15.50 = -0.67 + 17e^{-r}$$
$$16.17 = 17e^{-r}$$
$$r = 0.0501 \text{ or } 5.01\%.$$

$$\min(a, b, c) = \min(\min(a, b), c)$$

$$\min(S_1(1), S_2(1), 17) = \min(\underbrace{\min(S_1(1), S_2(1))}_{X(1)}, 17)$$

$$= \min(X(1), 17)$$

$$= \min(X(1) - 17, 0) + 17$$

$$= -\max(17 - X(1), 0) + 17$$

$$= \boxed{-\max(17 - \min(S_1(1), S_2(1)))} + 17$$

Short

Long