## Tutorial - Class Activity

## 10 October, 2018 (Solution)

### Question 1

Suppose that the November price of corn is $2.50/bushel, the effective monthly interest rate is 1%, and the storage cost per bushel are $0.05/month. Assuming that corn is stored from November to February (i.e. the storage cost is paid on Dec, Jan and Feb).

(a) Find the theoretical forward price of this February forward.

(b) Suppose the February forward price had been $2.80. What would the arbitrage be?

### Solution

(a)

The future value of the storage cost

$$\$0.05 + (\$0.05 \times 1.01) + (\$0.05 \times 1.01^2) = (\$0.05/.01) \times \left[(1+0.01)^3 - 1\right] = \$0.1515$$

Thus, the February forward price will be

$$2.50 \times (1.01)^3 + 0.1515 = 2.7273$$

(b)

If the February corn forward price is $2.80, the observed forward price is too expensive relative to our theoretical price of $2.7273. We will therefore sell the February contract short, and create a synthetic long position, engaging in cash and carry arbitrage:

| Transaction | Nov | Dec | Jan | Feb |
|---|---|---|---|---|
| Short Feb forward | 0 | | | $2.80 - S_T$ |
| Buy spot | −2.50 | | | $S_T$ |
| Borrow purchasing cost | +2.50 | | | −2.57575 |
| Pay storage cost Dec, | | −0.05 | | |
| borrow storage cost | | +0.05 | | −0.051005 |
| Pay storage cost Jan, | | | −0.05 | |

| | | | | |
|---|---|---|---|---|
| borrow storage cost | | | +0.05 | −0.0505 |
| Pay storage cost Feb | | | | −0.05 |
| Total | 0 | 0 | 0 | 0.072745 |

We made an arbitrage profit of 0.07 dollar on Feb.

## Question 2

**7.4** Suppose you observe the following 1-year implied forward rates: 0.050000 (1-year), 0.034061 (2-year), 0.036012 (3-year), 0.024092 (4-year), 0.001470 (5-year). For each maturity year compute the zero-coupon bond prices, effective annual and continuously compounded zero-coupon bond yields, and the par coupon rate.

### Solution

| Maturity | Zero-Coupon Bond Yield | Zero-Coupon Bond Price | One-Year Implied Forward Rate | Par Coupon | Cont. Comp. Zero Yield |
|---|---|---|---|---|---|
| 1 | 0.05000 | 0.95238 | 0.05000 | 0.05000 | 0.04879 |
| 2 | 0.04200 | 0.92101 | 0.03406 | 0.04216 | 0.04114 |
| 3 | 0.04000 | 0.88900 | 0.03601 | 0.04018 | 0.03922 |
| 4 | 0.03600 | 0.86808 | 0.02409 | 0.03634 | 0.03537 |
| 5 | 0.02900 | 0.86681 | 0.00147 | 0.02962 | 0.02859 |

Notes

1. From the one-year implied forward rate, we can compute $P(0, t_i)$.
   For example,

$$
\begin{aligned}
\left(1+r_0\left(0,1\right)\right) &= \frac{P\left(0,0\right)}{P(0,1)} \\
&= \frac{1}{P(0,1)} \\
P\left(0,1\right) &= \frac{1}{\left(1+r_0\left(0,1\right)\right)} \\
&= \frac{1}{\left(1+0.05\right)} \\
&= 0.95238.
\end{aligned}
$$

$$
\begin{aligned}
\left(1+r_0\left(1,2\right)\right) &= \frac{P\left(0,1\right)}{P(0,2)} \\
&= \frac{0.95238}{P(0,2)} \\
P\left(0,2\right) &= \frac{0.95238}{\left(1+0.034061\right)} \\
&= 0.92101.
\end{aligned}
$$

2. The **zero-coupon bond yield** and **continuous compounding zero-coupon yield** can be calculated by using the following formula:

$$
\begin{aligned}
P\left(0,t\right) &= \frac{1}{\left(1+r\left(0,t\right)\right)^t} \\
P\left(0,t\right) &= e^{-r^{CC}(0,t)t}.
\end{aligned}
$$

It is worth to note that the 1-year implied forward rate which matures at the end of the first year is our $r(0,1)$.

3. The **par coupon** for the $t_n$ maturity coupon bond ($c(t_n)$) can be calculated by the following formula:

$$
c(t_n) = \frac{1-P(0,t_n)}{\sum_{i=1}^{n} P\left(0,t_n\right)}.
$$

假设债券平价发行，票面价值为1