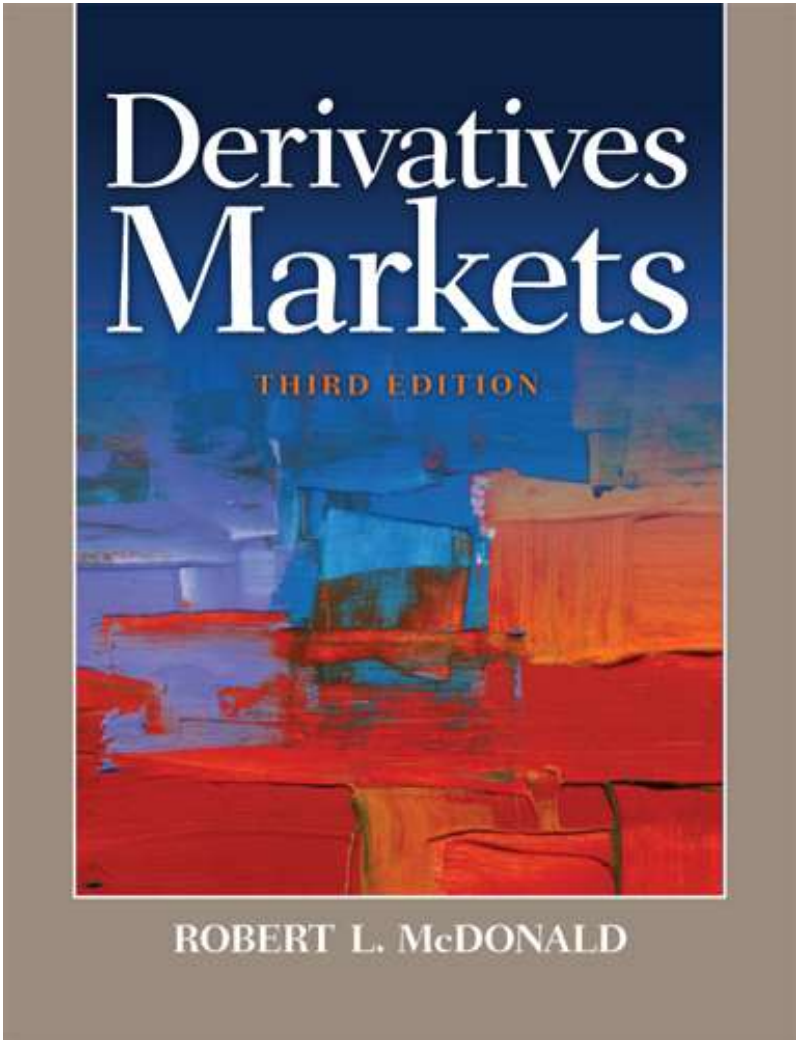


Chapter 15

(Chapter 23 in the textbook)

Exotic Options

Points to Notes

1. What are the all-or-nothing options? See P. 3 – 4.
2. How are the all-or-nothing options related to the BS call and put options? See P. 5 – 9.
3. What are the Asian options? See P. 10 - 11
4. What are the differences between the arithmetic and geometric average? See P. 12 – 14.
5. What are the barrier options? See P. 15.
6. How are the barrier options related to the ordinary call and put options? See P. 16 – 18.

Barrier Options (path dependent option)

- The payoff depends on whether over the option life the underlying price reaches a specified level, called the *barrier*.
  - Path-dependent.
  - Since barrier puts and calls never pay more than standard puts and calls, they are no more expensive than standard puts and calls.
  - Widely used in practice.

knock-in option (Barrier : B)

Call

$$\text{payoff} = \begin{cases} \max(S_T - K, 0), & \text{asset price touch } B \text{ over } [0,T] \\ 0 \ (R) & \text{otherwise} \end{cases}$$

Φ rebate option

put

$$\text{payoff} = \begin{cases} \max(K - S_T, 0) & \text{asset price touch } B \text{ over } [0,T] \\ 0 \ (R) & \text{otherwise} \end{cases}$$

Φ rebate

knock-out option

Call

$$\text{payoff} = \begin{cases} \max(S_T - K, 0), & \text{otherwise} \\ 0 & \text{asset price touch } B \text{ over } [0,T] \end{cases}$$

put

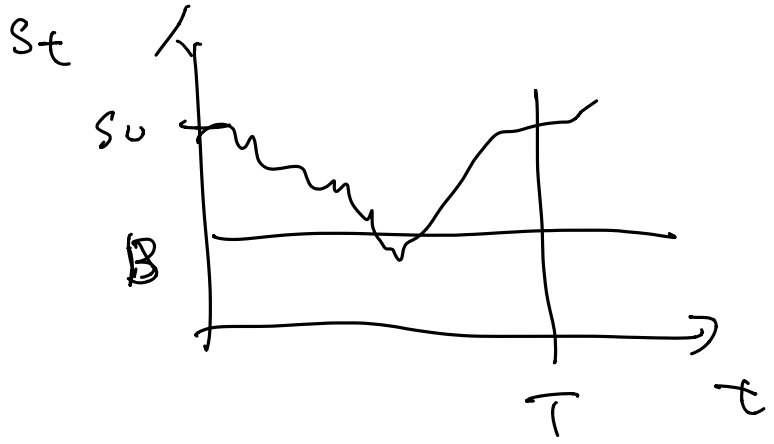
$$\text{payoff} = \begin{cases} \max(K - S_T, 0) & \text{otherwise} \\ 0 & \text{asset price touch } B \text{ over } [0,T] \end{cases}$$

knock-in call

payoff $= \max(S_T - K, 0)\, 1\{\text{asset price over } [0,T] \text{ touch } B\}$

Suppose $S_0 > B$

payoff $= \max(S_T - K, 0)\, 1\{\min_{0 \le t \le T} S_t \le B\}$



$$V(0) = e^{-rT} E^Q\left[\max(S_T - K, 0)\, 1\{\min_{0 \le t \le T} S_t \le B\}\right]$$

joint pdf S_T, $\min_{0 \le t \le T} S_t$

function of 1st passage time of standard BM.

Barrier Options (cont'd)

- Barrier puts and calls
  - Knock-out options: *go out of* existence (are "knocked-out")
    - down-and-out: if the asset price *falls* to reach the barrier.
    - up-and-out: if the asset price *rises* to reach the barrier.
  - Knock-in options: *come into* existence (are "knocked-in")
    - down-and-in: if the asset price *falls* to reach the barrier.
    - up-and-in: if the asset price *rises* to reach the barrier.
  - The important parity relation for barrier options is

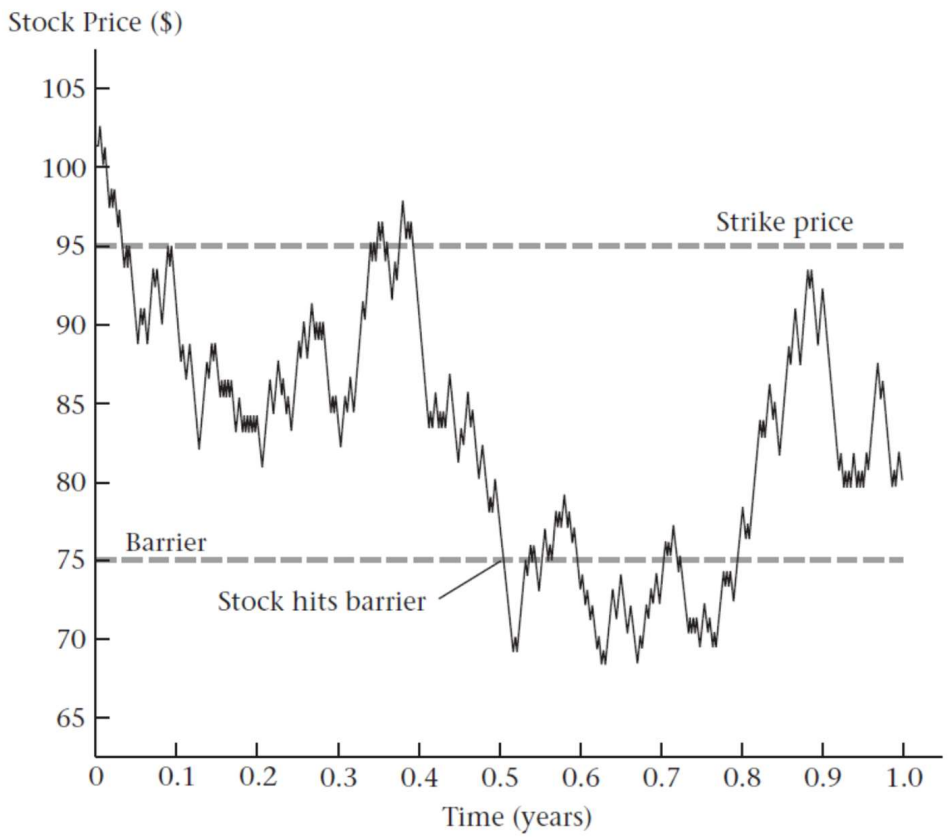
$$\underset{B,K,T}{\text{"Knock-in" option}} + \underset{B,K,T}{\text{"Knock-out" option}} = \text{Ordinary option}$$

  - Rebate options: make a fixed payment if the asset price reaches the barrier
    - down rebates: if the asset price *falls* to reach the barrier.
    - up rebates: if the asset price *rises* to reach the barrier.

Barrier Options (cont'd)

FIGURE 14.1

Illustration of a price path where the initial stock price is \$100 and the barrier is \$75. At $t = 0.5$, the stock hits the barrier.



Barrier Options (cont'd)

TABLE 14.3 Premiums of standard, down-and-in, and up-and-out currency put options with strikes K. The column headed "standard" contains prices of ordinary put options. Assumes $x_0 = 0.9$, $\sigma = 0.1$, $r_\$ = 0.06$, $r_€ = 0.03$, and $t = 0.5$.

	Standard	Down-and-In Barrier ($)		Up-and-Out Barrier ($)		
Strike ($)	($)	0.8000	0.8500	0.9500	1.0000	1.0500
$K = 0.8$	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
$K = 0.9$	0.0188	0.0066	0.0167	0.0174	0.0188	0.0188
$K = 1.0$	0.0870	0.0134	0.0501	0.0633	0.0847	0.0869

MFE5130 – Financial Derivatives

Class Activity (11-December-2019) (Solution)

Important Notes:

1. This class activity is counted toward to your class participation score. **Fail** to hand in this class activity worksheet in the class will receive **0 score** for that class.
2. **0 mark** will be received if you leave the solution blank.

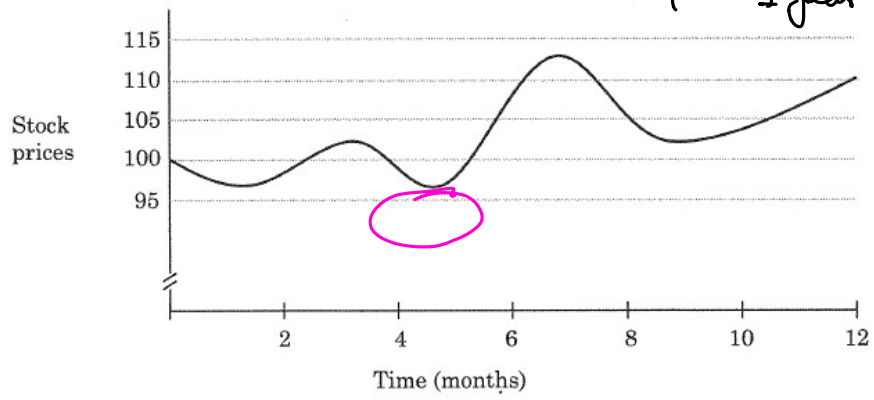
Name:	Student No.:

Problem 1

The price of Stock Y over the course of a year are shown below. The initial stock price is \$100, and the final stock price is \$110.

$S_0 = 100$, $S_T = 110$

$T = 1$ year



Five different exotic options were issued at the beginning of the year. Which of the five options has a payoff of exactly \$5 at the end of the year?

A. A knock-in call with a barrier of \$105 and a strike of \$100. $S_1 - 100 = 10$

B. A knock-in call with a barrier of \$95 and a strike of \$105. 0

C. A knock-out call with a barrier of \$95 and a strike of \$105. $S_1 - 105 = 5$

D. A knock-in put with a barrier of \$105 and a strike of \$105.

E. A knock-out put with a barrier of \$95 and a strike of \$120.

Solution

Answer: C

The payoffs of each of the options are:

	Payoff
A	10
B	0
C	5
D	0
E	10