observed date t_0 — t_1 — t_2

$r(t_1, t_2)$ interest rate

$r_t(t_1, t_2)$

zero-coupon bond price pay 1\$ at T=n

$P(0,n) = \frac{1}{[1+r(0,n)]^n}$ ← discount factor $= e^{-r(0,t_i)t_i}$

↑ annualized zero-coupond bond yield

discrete / continous

pay C_t at T=t $C_t \times P(0,t) = \frac{C_t}{[1+r(0,t)]^t}$

zero-coupon Bond yield (Yield curve).

$r(0,n)$

one-year implied forward rate

$(r_0(0,1) \quad r_0(1,2) \quad r_0(2,3) \ldots)$

$[1+r_0(t_1,t_2)]^{t_2-t_1} = \frac{[1+r_0(0,t_2)]^{t_2}}{[1+r_0(0,t_1)]^{t_1}} = \frac{P(0,t_1)}{P(0,t_2)}$

t_1 t_2 $[1+r(0,t_2)]^{t_2}$

$t \quad t_1 \quad t_2 \quad t_i \quad T$

par coupon. (the coupon rate at which the bond will be priced at par).

$C = \frac{1-P_t(t,T)}{\sum_{i=1}^n P_t(t,t_i)}$ $B_t = 1$

Coupon bonds price (face value).

- issue t=t. maturing T
  pay n coupon of size C at time t_i
  maturity payment of \$1

$B_t(t,T,c,n) = \sum_{i=1}^{n} c P_t(t,t_i) + 1 P_t(t,T)$

$= \sum_{i=1}^{n} \frac{c}{(1+y_m)^i} + \frac{1}{(1+y_m)^n}$

make m payment/year.
per period YTM $= y_m$.
annualized YTM $= y = m \times y_m$

Yield to maturity (YTM) $(n) = \frac{1}{P(0,n)} - 1 = (1+r(0,n))^n - 1$

Annualized compounded yields $r^{cc}(0,t)$ $P(0,t) = e^{-r^{cc}(0,t) \cdot t}$

pay \$1 at maturity

pre-paid swap: single payment today
obtain multiple delieveries in futures

swap (equal payment).

110 111

$\frac{110}{1.06} + \frac{111}{1.065^2} = 201.638$

$\frac{x}{1.06} + \frac{x}{1.065^2} = 201.638$

Buyer. long position $\xrightarrow{\text{swap price}}$ Counterparty short position

∘ spot price − swap price > 0 counterparty pay buyer

$\boxed{\text{Swap}}$ → forward contract coupled with borrowing & lending

implied forward yield $= r_0(t_1,t_2)$

dealer: back-to-back transaction
buyer $\xleftarrow{\text{short swap}}$ dealer $\xrightarrow{\text{long swap}}$ seller
√ credit risk
√ bid-ask spread
× expose to prize risk

market value of swap (or long → forward)
√ interest rate exposure
→ according to new forward price

long position $= PV$ (New swap rate − original swap rate) × Notional

implied forward zero-coupon price

$P_0(t_1,t_2) = \frac{1}{[1+r_0(t_1,t_2)]^{t_2-t_1}} = \frac{P(0,t_2)}{P(0,t_1)}$

pre-paid swap $= \sum_{i=1}^{n} F_{0,t_i} P(0,t_i)$

fixed swap price $R = \frac{\sum_{i=1}^n P(0,t_i) F_{0,t_i}}{\sum_{i=1}^n P(0,t_i)}$ (PV of payment implied by strip of F_{0,t_i}) / (PV of \$1 annuity)

$\underline{PV(\text{forward})} = \underline{PV(\text{swap})}$
prices / fix payments

$= \sum_{i=1}^n \left[\frac{P(0,t_i)}{\sum_{i=1}^n P(0,t_i)}\right] F_{0,t_i}$. weighted average of F_{0,t_i}

variable quantity (RQ) & price (R)

$R_S \sum_{i \in summer} P(0,t_i) Q_{t_i} + R_W \sum_{i \in winter} P(0,t_i) Q_{t_i} = \sum_{i=1}^{n} P(0,t_i) Q_{t_i} F_{0,t_i}$

$\boxed{\text{Put-call parity}}$

Call − put = PV(forward price − strike price)

$C(K,T) - P(K,T) = PV_{0,T}(F_{0,T} - K) = e^{-rT}(F_{0,T} - K)$

$C(K,T) = P(K,T) + [S_0 - PV_{0,T}(Div)] - e^{-rT}(K) = P(K,T) + S_0 e^{-\delta T} - PV_{0,T}(K)$

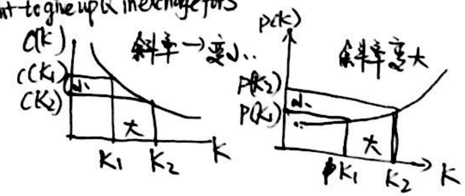
$\boxed{\text{Generalized parity}}$

$C(S_t, Q_t, T-t) - P(S_t, Q_t, T-t) = F^P_{t,T}(S) - F^P_{t,T}(Q)$

$C(S_t, Q_t, T-t) = P(Q_t, S_t, T-t)$

$C(S_T, Q_T, 0) = \max(0, S_T - Q_T)$. S_T − underlying
$P(S_T, Q_T, 0) = \max(0, Q_T - S_T)$. Q_T − striking

Right to give up Q in exchage for S



$X_0 = \frac{\$1.2}{€1}$ measured in dollar
initial price of foreign
K = strike price. \$1.2
right to buy €1 for \$K

$C_\$(X_0, K, T)$
↑
dollar-denominated

$\frac{1}{X_0} = \frac{€x}{\$1}$ K = € ... right to give up \$1 for K€

$P_€(\frac{1}{X_0}, \frac{1}{K}, T)$
↑
euro-denominated

汇率 数量

$C_\$(X_0, K, T) = X_0 K P_f(\frac{1}{X_0}, \frac{1}{K}, T)$

$C(K,T)$ $P(K,T)$ ← time to maturity F_0, T = maturity date

American option → exercise at any time

$C_{Amer}(S,K,T) \geq C_{EUr}(S,K,T)$

$P_{Amer}(S,K,T) \geq C_{EUr}(S,K,T)$

Maximum & minimum option price

$S > C_{Amer}(S,K,T) \geq C_{EUr}(S,K,T) \geq \max(0, PV_{0,T}(F_{0,T}) - PV_{0,T}(K))$

stock price put-call parity zer put

$K > P_{Amer}(S,K,T) \geq P_{EUr}(S,K,T) \geq \max[0, PV_{0,T}(K) - PV_{0,T}(F_{0,T})]$

Early exercise?

non-dividend stock:

$C_{EUr}(S_t,K,T-t) = S_t - K + P_{EUr}(S_t,K,T-t) + K(1-e^{-r(T-t)})$

exercise value / insurance against $S_t < K$ / positive / interest rate time value for K

⇓

$C_{Amer} \geq Eur > S_t - K$.

never early ~~exer~~ exercise a non-dividend pay call

you can sell it.

Dividend paid:

$C(S_t,K,T-t) = P(S_t,K,T-t) + S_t - PV_{t,T}(Div) - PV_{t,T}(K)$

$= \underline{S_t - K} + P(S_t,K,T-t) + K - PV_{t,T}(K) - PV_{t,T}(Div)$

early exercise

early exercise possible if < 0.

summary		No Div
call	↑Div ⇒ ↑early Ex	never
Put	↑Div ⇒ ↓early Ex	Early ex smaller value of "C"

	Forward contract (futures) Forward price = K		Call option Strike price = K Premium = C		Put option Strike Price = K Premium = P	
	Long	Short	Long Right at T	Short	Long Right at T	Short
Obligation at T	To buy	To sell	To buy	To sell if asked.	To sell	To buy if asked.
	the underlying asset at the price K					
Cost @($t = 0$)	0	0	C	$-C$	P	$-P$
Payoff	$S_T - K$	$K - S_T$	$\max(S_T - K, 0)$	$-\max(S_T - K, 0)$	$\max(K - S_T, 0)$	$-\max(K - S_T, 0)$
Profit at T	$S_T - K$	$K - S_T$	$\max(S_T - K, 0) - FV(C)$	$-\max(S_T - K, 0) + FV(C)$	$\max(K - S_T, 0) - FV(P)$	$-\max(K - S_T, 0) + FV(P)$

Horizontal calendar spread	Sell T1-K1-call+Buy T2-K2-call
Diagonal spread	Different strike+different maturity
Box spread	Long Synthetic K1 forward(long k1 call+short K1 put) +short synthetic K2 forward(short K2 call+long K2 put)
	Lend at t0 c(k1)-c(k2)+p(k1)-p(k2) Return at T k2-k1 (compare to bank interest) determistic
Ratio spread	Long m k1 cal(put)+short n k2 call(put)
Asymmetric butterfly spread	λ=(k2-k)/(k2-k1) k=λk1+(1-λ)k2

	Protection(insure)				speculation (vertical)	
	Floors	Caps	Covered Call	Covered Put	Bull Spread	Bear Spread
At $t = 0$	Long a K-strike put + Long the underlying asset	Long a K-strike call + Short the underlying asset	Sell a K-strike call + Long the underlying asset	Sell a K-strike put + Short the underlying asset	Long a K_1-strike call + Sell a K_2-strike call safe. low initial cost	Sell a K_1-strike call + Long a K_2-strike call
Cost $t = 0$	$P(K, T) + S_0$	$C(K, T) - S_0$	$-C(K, T) + S_0$	$-P(K, T) - S_0$	$C(K_1, T) - C(K_2, T)$	$-C(K_1, T) + C(K_2, T)$
Payoff @ T	$\max(K - S_T, 0) + S_T$	$\max(S_T - K, 0) - S_T$	$-\max(S_T - K, 0) + S_T$	$-\max(K - S_T, 0) - S_T$	$\max(S_T - K_1, 0) - \max(S_T - K_2, 0)$	$-\max(S_T - K_1, 0) + \max(S_T - K_2, 0)$
Profit @ T	payoff − FV($P(K, T)$ + S_0) insure long position	Payoff − FV(C(K, T) − S0) insure short position	Payoff − FV(− C(K, T) +S0)	Payoff − FV(− $P(K, T)$ − S_0)	Payoff − FV($C(K_1, T) - C(K_2, T)$)	payoff − FV($-C(K_1, T) + C(K_2, T)$)
Remark	Profit S	Profit S_T	P S	P S	same maturity K1 < K2. Can be constructed using puts. market ↑ alittle P S	P ↓ alittle S

Let $C(K,T)$ and $P(K,T)$ be the time 0 premium (price) of the K-strike call option and K-strike put option with time T until expiration respectively. Assume the underlying asset pays no dividends.

Volatility	Collar	Straddles	Written Straddles	Strangles out of money	Butterfly Spreads
At $t = 0$	Long a K_1-strike put + sell a K_2-strike call	Long a K-strike call + Long a K-strike put	Sell a K-strike call + sell a K-strike put	Long a K_1-strike put + Long a K_2-strike call	Sell a K-strike call + sell a K-strike put + Long a K_2-strike call + Long a K_1-strike put
Cost $t = 0$	$P(K_1, T) - C(K_2, T)$	$C(K, T) + P(K, T)$	$-C(K, T) - P(K, T)$	$P(K_1, T) + C(K_2, T)$	$-C(K, T) - P(K, T) + C(K_2, T) + P(K_1, T)$
Payoff @ T	$\max(K_1 - S_T, 0) - \max(S_T - K_2, 0)$	$\max(S_T - K, 0) + \max(K - S_T, 0)$	$-\max(S_T - K, 0) - \max(K - S_T, 0)$	$\max(K_1 - S_T, 0) + \max(S_T - K_2, 0)$	$-\max(S_T - K, 0) - \max(K - S_T, 0) + \max(S_T - K_2, 0) + \max(K_1 - S_T, 0)$
Profit @ T	Payoff − FV($P(K_1, T) - C(K_2, T)$)	Payoff − FV($C(K, T) + P(K, T)$)	Payoff+ FV($C(K, T) + P(K, T)$)	Payoff − FV($P(K_1, T) + C(K_2, T)$)	Payoff− FV($-C(K, T) - P(K, T) + C(K_1, T) + P(K_2, T)$)
Remark	Both options have the same maturity $K_1 < K_2$. P S	P K S high premiumn	P S	$K_1 < K_2$. k k_1 k_2	$K_1 < K < K_2$. Written K-strike Straddle + Long a Strangle with strike prices of K_1 and K_2. insure against large loss on straddle.

market ↓
resemble short forward
collated stock
pay initial cost ← buy put · sell call → collar
floor buy stock
zero-cost collar-
notice cost of interest of holding asset

$F^P_{0,T} = S_0$

$= S_0 - \sum_{i=1}^{n} PV(D_i) = S_0 - \sum_{i=1}^{n} e^{-rt_i} D_i$

(prepaid forward) $= S_0 e^{-\delta t}$

$= P(0,T) F_{0,T}$ ← deterministic "PV"

$= e^{-rt} F_{0,T}$ ← stochastic

$\boxed{\begin{array}{l} \times D_i \\ \text{Discrete } D_i \\ \text{Continous } D_i \end{array}}$ $F_{0,T}$

$= S_0 e^{rT}$

$= FV[S_0 - \sum_{i=1}^{n} PV_{0,t}(D_i)]$

Financial asset → $= S_0 e^{(r-\delta)T}$ $= \frac{S_0 e^{-\delta T}}{P(0,T)}$

$= \cancel{S_0} \cdot e^{rT} F^P_{0,T} = E_0(S_T) e^{-(\alpha - r)T}$ risk premium

Commodity → $= S_0 e^{(r+u-y)T}$

$= [S_0 + \sum_{i=1}^{n} PV_{0,T}(u_i)] e^{(r-y)T}$

y = convenience yield (income of commodity owner − $F_{0,T}$ ⇓) [compensation]
u = storage cost (expense − $F_{0,T}$ ⇑)
δ_l = leasing rate = y − u (like dividend).
$F_{0,T} = S_0 e^{(r-\delta_l)T}$
~~expense of~~.

- credit risk of counter party.
  OTC → major credit check collateral
  bank letter of credit
  traded contracts → exchange guarantees transaction
  require collateral
- in the money − positive payoff if exercise immediately
  at the money − zero-
  out of the money − negtive
  K_1 S_0 K_2
  long call in at out
  long put out at in
- bid price < ask price
  (sell to market maker) (buy from market maker)
  bid-ask spread = ask price − bid price
  (liquidity)
- risk sharing diversifiable risk vanish
  use of derivative ① risk mgt. hedging
  ② speculation. ③ ↓ transact cost
  ④ Regulatory abribtrage
- 无论 Buy 还是 sell
  都付 commission
- open interest: total # of buy/sell pair
  volumn.
- moneyness
  put: $\frac{\text{strike price} - S_0}{S_0}$ call: $\frac{S_0 - k}{S_0}$.
- cash and carry
  buy the underlying asset + short the offsetting forward contract
- cost of carry: $r - \delta$ (δ = dividend) r = interest rate
  (commodity) $r + u - y$
- Future contract = ① daily settlement.
  ② liquid. highly standarized ② different credit risk ③ price limit ④ interest rate vs forward