MFE5130 – Financial Derivatives

First Term, 2018-19

Midterm Examination (Solution)

Question 1

(a)

Let $F_{0,T}$ be the T-year oil forward price at time 0.

Let $r(0,T)$ be the annualized continuously compounded yield of the T-year zero-coupon bond.

Let R be the fixed swap price for one barrel of oil.

$$R = \frac{\sum_{k=1}^{3} F_{0,k} e^{-r(0,k)k}}{\sum_{k=1}^{3} e^{-r(0,k)k}}$$
$$= \frac{74.5e^{-(4.5\%)1} + 78.9e^{-(6\%)2} + 81.5e^{-(7.3\%)3}}{e^{-(4.5\%)1} + e^{-(6\%)2} + e^{-(7.3\%)3}}$$
$$=78.0997.$$

Hence, the fixed swap price for one barrel of oil is \$78.0997.

(b)

Since the dealer is paying the floating price and receiving the fixed swap price, she generates the cash-flows (cash inflow) depicted in column 2. To hedge the oil price risk in his position, the dealer enters into three long forward positions, one contract for each year of the active swap. Her payoffs on the forward contracts are depicted in column 3, and the aggregate net cash inflow position is in column 4.

Year	Net Cash Inflow from Swap	Cash Inflow from Long Forwards	Net Cash Inflow Position
1	\$78.0997 − S_1	S_1 − \$74.5	3.5997
2	\$78.0997 – S_2	S_2 − \$78.9	–0.8003
3	\$78.0997 – S_3	S_3 − \$81.5	–3.4003

$$PV(\text{Net Cash Inflow}) = 3.5997e^{-(4.5\%)(1)} - 0.8003e^{-(6\%)(2)} - 3.4003e^{-(7.3\%)(3)} = 0$$

Hence, the present value of the locked-in net cash inflows of the dealer at the inception of the swap contract is zero.

(c)

The new fixed swap price is given by

$$
\begin{aligned}
R_{new} &= \frac{\sum_{k=1}^{3} F_{0,k} e^{-r(0,k)k}}{\sum_{k=1}^{3} e^{-r(0,k)k}} \\
&= \frac{75.8e^{-(4.5\%)1} + 79.9e^{-(6\%)2} + 80.2e^{-(7.3\%)3}}{e^{-(4.5\%)1} + e^{-(6\%)2} + e^{-(7.3\%)3}} \\
&= 78.5099.
\end{aligned}
$$

The market value of the swap from the dealer's perspective (perspective of the short position) is given by

$$
\begin{aligned}
&\text{Notional Amount} \times \left(R - R_{new}\right)\left(e^{-(4.5\%)1} + e^{-(6\%)2} + e^{-(7.3\%)3}\right) \\
&= 1 \times \left(78.0997 - 78.5099\right)\left(e^{-(4.5\%)1} + e^{-(6\%)2} + e^{-(7.3\%)3}\right) \\
&= -1.0855.
\end{aligned}
$$

Question 2

We are told that the price of a European call option with a strike price of \$54.08 has a value of \$10.16. The payoff of the call option at time 2 is:

$$\max[S(2) - 54.08, 0].$$

The payoff of the financial product is given by

$$\begin{aligned}
\text{Payoff} &= P(1-y\%)\times\max\left[\frac{S(2)}{50}, 1.04^2\right] \\
&= P(1-y\%)\times\frac{1}{50}\times\max\left[S(2), 50\times 1.04^2\right] \\
&= P(1-y\%)\times\frac{1}{50}\times\max\left[S(2), 54.08\right] \\
&= P(1-y\%)\times\frac{1}{50}\times\left\{\max\left[S(2)-54.08, 0\right]+54.08\right\}.
\end{aligned}$$

The current value of a payoff of $\max[S(2) - 54.08, 0]$ is \$10.16. The current value of \$54.08 at time 2 is:

$$54.08e^{-0.06\times 2} = 47.9647.$$

Therefore, the price of this financial product at time 0 is given by:

$$P(1-y\%)\times\frac{1}{50}\times(10.16+47.9647). \qquad (1)$$

$y\%$ is chosen to make (1) equal to P. So,

$$\begin{aligned}
P(1-y\%)\times\frac{1}{50}\times(10.16+47.9647) &= P \\
(1-y\%)\times 1.1625 &= 1 \\
y\% &= 13.9785\%.
\end{aligned}$$

Question 3

The put option that allows its owner to give up Stock B in exchange for Stock A has Stock B as its underlying asset:

$$P_{Eur}\left(B_t, A_t, 1\right) = 11.49\,.$$

Using put-call parity, we can find the value of the call option have Stock B as its underlying asset:

$$C_{Eur}\left(B_t, A_t, 1\right) - P_{Eur}\left(B_t, A_t, 1\right) = F_{t,T}^{P}\left(B_t\right) - F_{t,T}^{P}\left(A_t\right)$$
$$C_{Eur}\left(B_t, A_t, 1\right) - 11.49 = 67e^{-0.053(1)} - 70$$
$$C_{Eur}\left(B_t, A_t, 1\right) = 5.0315.$$

We can describe the call option as a put option by switching the underlying asset and the strike asset:

$$C_{Eur}\left(B_t, A_t, 1\right) = P_{Eur}\left(A_t, B_t, 1\right)$$
$$P_{Eur}\left(A_t, B_t, 1\right) = 5.0315.$$

Therefore, the value of a put option giving its owner the right to give up a share of Stock A in exchange for a share of Stock B is $5.0315.

Question 4

The prices of Option A and Option B violate the following inequality

$$C_{Eur}\left(K_1\right)-C_{Eur}\left(K_2\right)\leq\left(K_2-K_1\right)e^{-rt}, \quad \text{where } K_1 = 50 \text{ and } K_2 = 55,$$

because

$$\begin{aligned}14-9.25 &> \left(55-50\right)e^{-0.07(1)}\\ 4.75 &> 4.6620.\end{aligned}$$

Arbitrage is available using the following strategy:

Buy 55-strike call

Sell 50-strike call

The strategy produces the following payoff table:

		***t* = 1**		
Transaction	**Time 0**	$S_1 < 50$	$50 \leq S_1 \leq 55$	$S_1 > 55$
Buy 55-strike call	–9.25	0	0	$S_1 - 55$
Sell 50-strike call	14	0	$-(S_1 - 50)$	$-(S_1 - 50)$
Total	4.75	0	$-(S_1 - 50)$	–5

The accumulated profit of the strategy at $t = 1$ is

	***t* = 1**		
	$S_1 < 50$	$50 \leq S_1 \leq 55$	$S_1 > 55$
Accumulated Profit	$4.75e^{0.07}$	$4.75e^{0.07}-(S_1 - 50)$ $= 55.0944 - S_1 > 0$	$4.75e^{0.07}-5$ $= 0.0944$

It can be observed that the strategy has positive accumulated profit for all scenarios of S_1. So, we have the arbitrage opportunity.

If the final stock price is \$52, then the accumulated arbitrage profits are:

$X = 55.0944 - 52 = 3.0944.$

If the final stock price is \$60, then the accumulated arbitrage profits are:

$Y = 0.0944.$

Question 5

The theoretical forward price = $350\ e^{(0.09-0.055)(0.75)} = 359.3091$.

Now, the observed forward price is lower than the theoretical one. To have the arbitrage profit, we long the forward contract in the market and short the synthetic forward as follows:

Transactions	**Cash Flows**	
	$t = 0$	$t = 0.75$
Long one forward	0	$S_1 - 353.22$
Short $e^{-(0.055)(0.75)}$ oz. of gold	$e^{-(0.055)(0.75)} \times 350 = 335.8562$	$-S_1$
Lend \$335.8562 at $t = 0$	-335.8562	$335.8562e^{(0.09)(0.75)} = 359.3091$
Total	0	6.0891

This position requires no initial investment, has no gold price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy. The accumulated profit at the end of 9 months is \$6.0891.