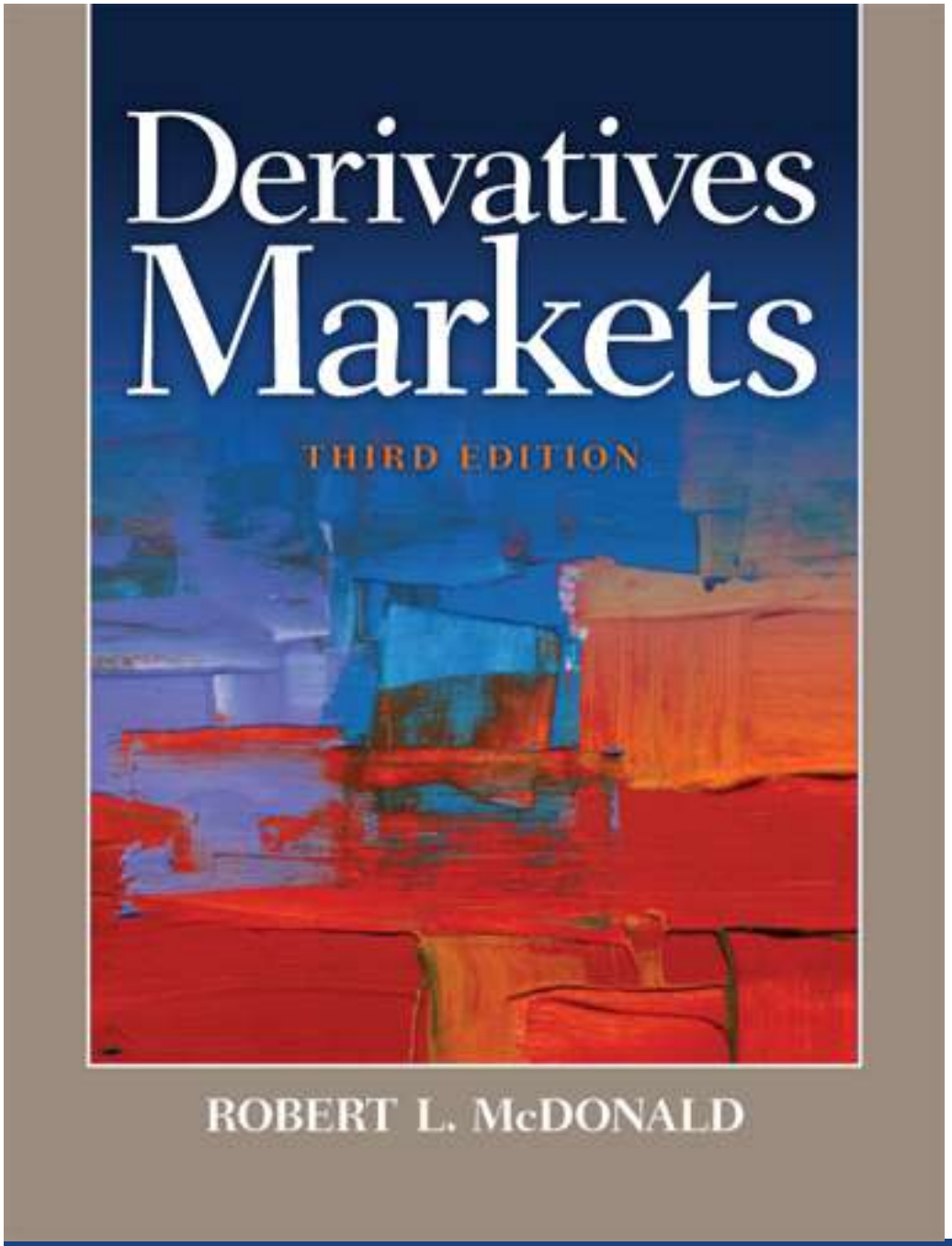



# Chapter 15

## (Chapter 23 in the textbook)

Exotic Options

# Points to Notes

1. What are the all-or-nothing options? See P. 3 – 4.
2. How are the all-or-nothing options related to the BS call and put options? See P. 5 – 9.
3. What are the Asian options? See P. 10 - 11
4. What are the differences between the arithmetic and geometric average? See P. 12 – 14.
5. What are the barrier options? See P. 15.
6. How are the barrier options related to the ordinary call and put options? See P. 16 – 18.

# All-or-Nothing Options

## Terminology

| Notation | Meaning |
|---|---|
| Asset | Payment at expiration is one unit of the asset |
| Cash | Payment at expiration is \$1 |
| Call | Payment received if $S(T) > K$. |
| Put | Payment received if $S(T) < K$. |

## Definition

$$d_1 = \frac{\left[\ln\left(S(t)/K\right) + \left(r - \delta + 0.5\sigma^2\right)\left(T - t\right)\right]}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

# All-or-Nothing Options

- Simple all-or-nothing options pay the holder a discrete amount of cash or a share if some particular event occurs.
- Cash-or-nothing
  - Call: pays \$1 if $S_T > K$ and zero otherwise

$$\text{CashCall}(S,K,\sigma,r,T-t,\delta) = e^{-r(T-t)}N(d_2)$$

  - Put: pays \$1 if $S_T < K$ and zero otherwise

$$\text{CashPut}(S,K,\sigma,r,T-t,\delta) = e^{-r(T-t)}N(-d_2)$$

- Asset-or-nothing
  - Call: pays $S_T$ (one unit share) if $S_T > K$ and zero otherwise

$$\text{AssetCall}(S,K,\sigma,r,T-t,\delta) = Se^{-\delta(T-t)}N(d_1)$$

  - Put: pays $S_T$ (one unit share) if $S_T < K$ and zero otherwise

$$\text{AssetPut}(S,K,\sigma,r,T-t,\delta) = Se^{-\delta(T-t)}N(-d_1)$$

# All-or-Nothing Options (cont'd)

- + 1 asset-or-nothing call option with strike price $K$
  – $K$ cash-or-nothing call option with strike price $K$
  = 1 ordinary call option with strike price $K$

$$\begin{aligned}
&\mathrm{BSCall}(S,K,\sigma,r,T-t,\delta)\\
&=\mathrm{AssetCall}(S,K,\sigma,r,T-t,\delta)-K\times\mathrm{CashCall}(S,K,\sigma,r,T-t,\delta)\\
&=Se^{-\delta(T-t)}N(d_1)-Ke^{-r(T-t)}N(d_2)
\end{aligned}$$

# All-or-Nothing Options (cont'd)

- Similarly, a put option can be created by buying $K$ cash-or-nothing puts, and selling 1 asset-or-nothing put

$$\begin{aligned}&\text{BSPut}\left(S,K,\sigma,r,T-t,\delta\right)\\&=K\times\text{CashPut}\left(S,K,\sigma,r,T-t,\delta\right)-\text{AssetPut}\left(S,K,\sigma,r,T-t,\delta\right)\end{aligned}$$

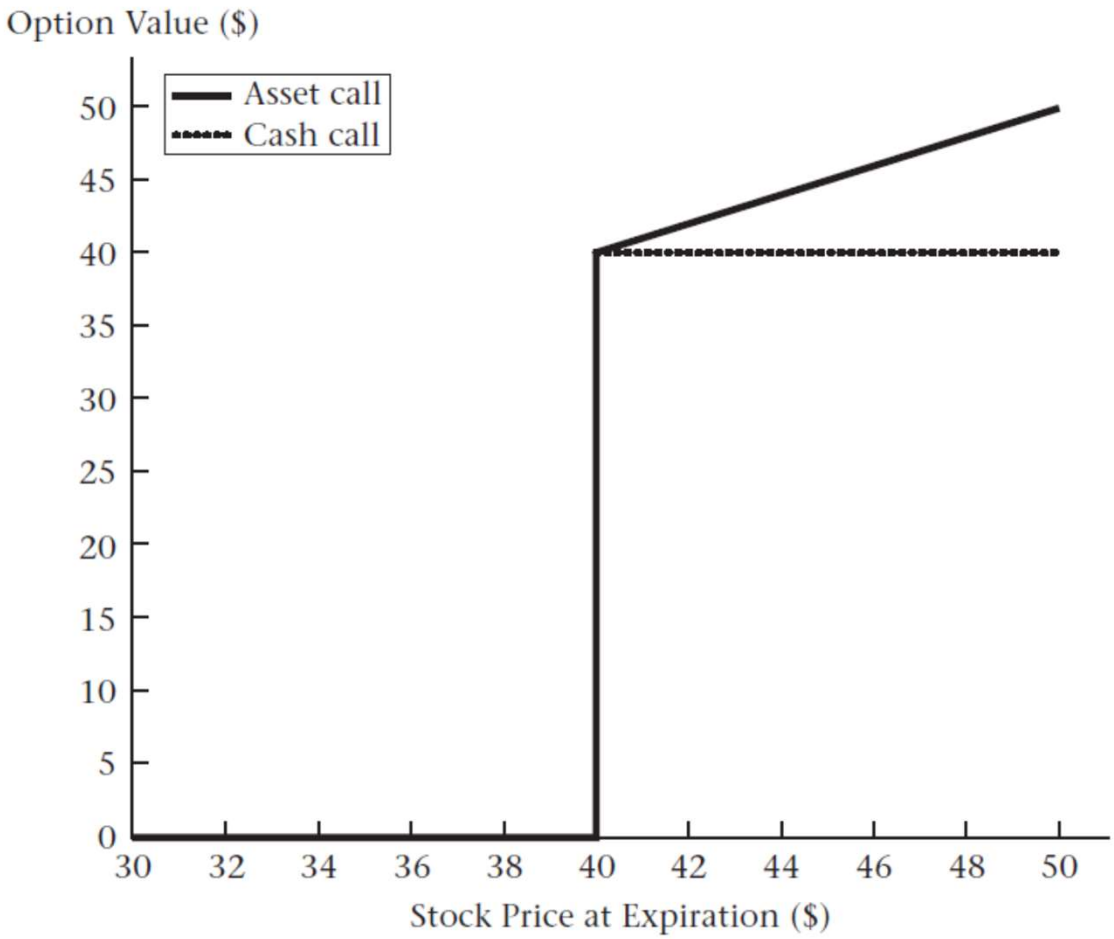
# All-or-Nothing Options (cont'd)

- All-or-nothing options are easy to price but hard to hedge.
- Fig. 1 shows that a small swing in the stock price can determine whether the option is in- or out-of-the money, with the payoff changing discretely.
- Fig. 2 shows that hedging is straightforward and delta is well behaved when 3 months to expiration. However, with 2 minutes to expiration, the cash call delta at $40 is 15. *For the at-the-money option, delta and gamma approach infinity at expiration because an arbitrarily small change in the price can result in a $1 change in the option's value.*

# All-or-Nothing Options (cont'd)

FIGURE 23.1

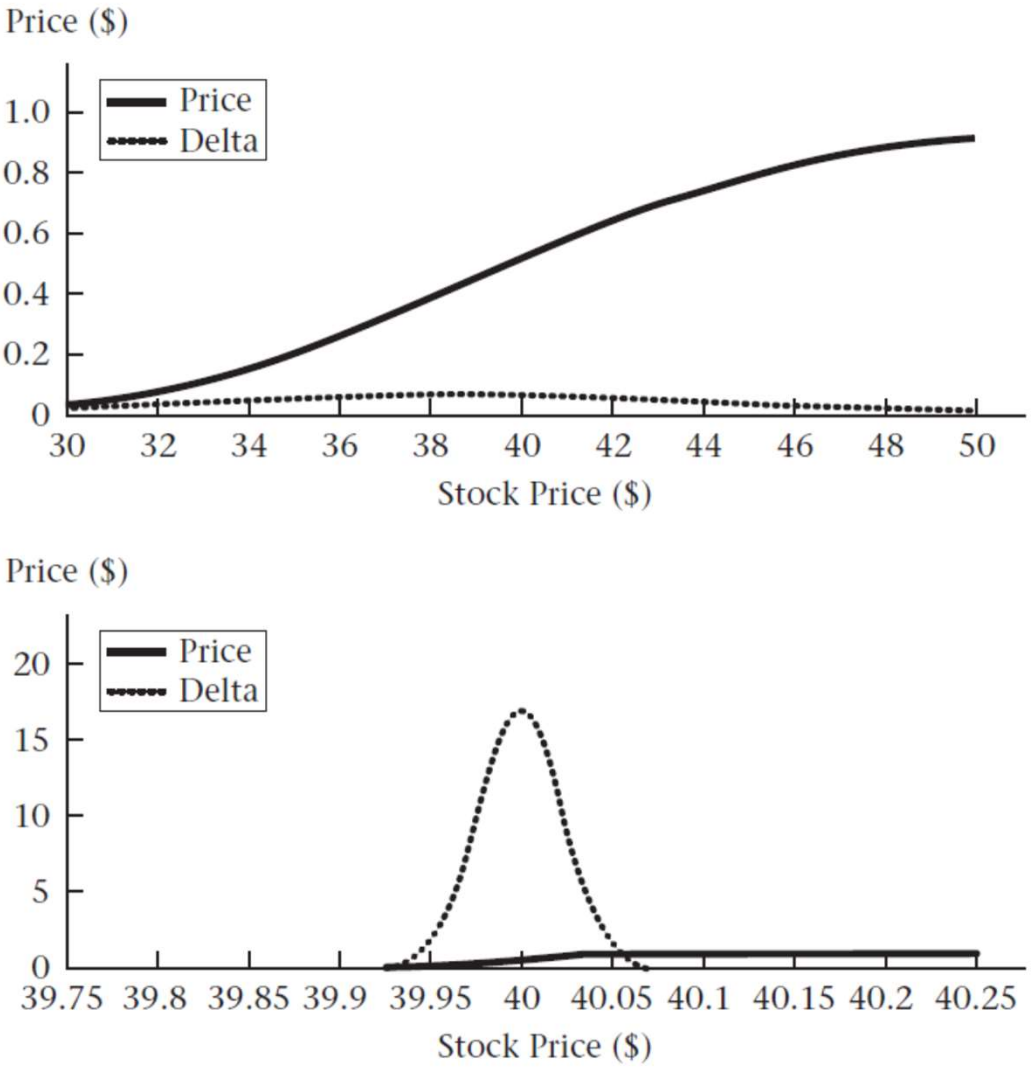
Payoff at maturity to one asset call and 40 cash calls. Assumes $K = \$40$, $\sigma = 0.30$, $r = 0.08$, and $\delta = 0$. The payoff to both is zero for $S < \$40$.



# All-or-Nothing Options (cont'd)

FIGURE 23.2

Price and delta of a cash call at two different times to expiration: 3 months (top panel) and 2 minutes (bottom panel). Assumes $K = \$40$, $\sigma = 0.30$, $r = 0.08$, and $\delta = 0$.



# Asian Options

- The payoff of an Asian option is based on the average price over some period of time. An Asian options is an example of a path-dependent option.
- Situations when Asian options are useful:
  - When a business cares about the average exchange rate over time.
  - When a single price at a point in time might be subject to manipulation.
  - When price swings are frequent due to thin markets.

# Asian Options (cont'd)

- Asian options are less valuable than otherwise equivalent ordinary options, since the averaged price of the underlying asset is less volatile than the asset price itself, and an option on a lower volatility asset is worth less.

# Asian Options (cont'd)

- There are eight ($2^3$) basic kinds of Asian options:
  - Put or call.
  - Geometric or arithmetic average.
  - Average asset price is used in place of underlying. price or the strike price.
- Arithmetic versus geometric average:
  - Suppose we record the stock price every $h$ periods from $t = 0$ to $t = T$.
  - Arithmetic average: Geometric average:

$$A(T) = \frac{1}{N}\sum_{i=1}^{N} S_{ih} \qquad G(T) = (S_h \times S_{2h} \times \cdots \times S_{Nh})^{1/N}$$

# Asian Options (cont'd)

- Average used as the asset price: Average price option
  - Geometric average price call = max [0, $G(T) - K$].
  - Geometric average price put = max [0, $K - G(T)$].
- Average used as the strike price: Average strike option
  - Geometric average strike call = max [0, $S_T - G(T)$].
  - Geometric average strike put = max [0, $G(T) - S_T$].

# Asian Options (cont'd)

- All four options above could also be computed using arithmetic average instead of geometric average.
- Relatively simple pricing formulas exist for pricing European options on the geometric average but not for arithmetic average options.

# Barrier Options

- The payoff depends on whether over the option life the underlying price reaches a specified level, called the *barrier*.
  - Path-dependent.
  - Since barrier puts and calls never pay more than standard puts and calls, they are no more expensive than standard puts and calls.
  - Widely used in practice.

# Barrier Options (cont'd)

- Barrier puts and calls
  - Knock-out options: *go out of* existence (are "knocked-out")
    - down-and-out: if the asset price *falls* to reach the barrier.
    - up-and-out: if the asset price *rises* to reach the barrier.
  - Knock-in options: *come into* existence (are "knocked-in")
    - down-and-in: if the asset price *falls* to reach the barrier.
    - up-and-in: if the asset price *rises* to reach the barrier.
  - The important parity relation for barrier options is

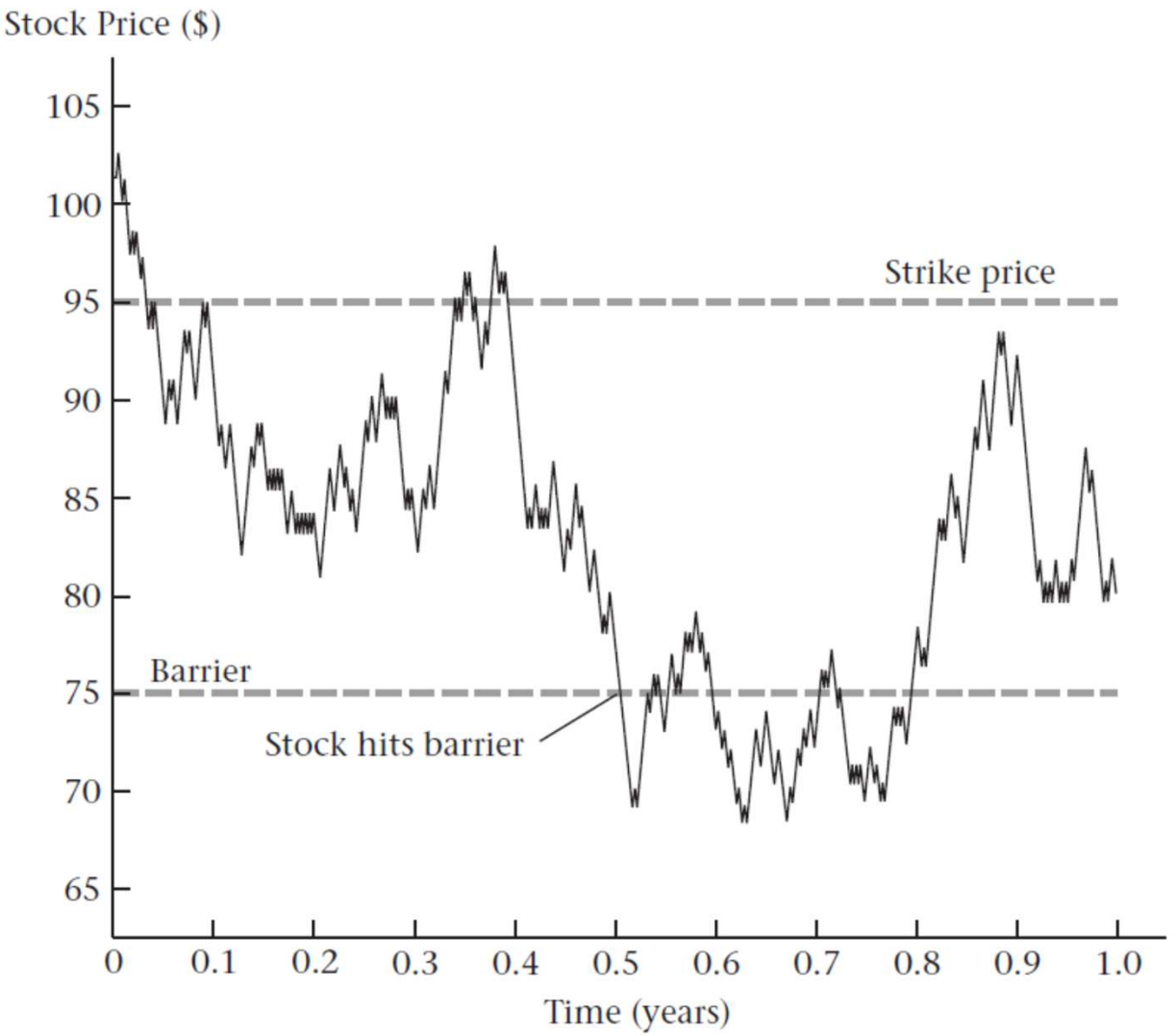
$$\text{"Knock}-\text{in"}\ \text{option} + \text{"Knock}-\text{out"}\ \text{option} = \text{Ordinary option}$$

  - Rebate options: make a fixed payment if the asset price reaches the barrier
    - down rebates: if the asset price *falls* to reach the barrier.
    - up rebates: if the asset price *rises* to reach the barrier.

# Barrier Options (cont'd)

FIGURE 14.1

Illustration of a price path where the initial stock price is \$100 and the barrier is \$75. At $t = 0.5$, the stock hits the barrier.



# Barrier Options (cont'd)

TABLE 14.3 Premiums of standard, down-and-in, and up-and-out currency put options with strikes $K$. The column headed "standard" contains prices of ordinary put options. Assumes $x_0 = 0.9$, $\sigma = 0.1$, $r_\$ = 0.06$, $r_€ = 0.03$, and $t = 0.5$.

| | Standard | Down-and-In Barrier ($) | | Up-and-Out Barrier ($) | | |
|---|---|---|---|---|---|---|
| Strike ($) | ($) | 0.8000 | 0.8500 | 0.9500 | 1.0000 | 1.0500 |
| $K = 0.8$ | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 |
| $K = 0.9$ | 0.0188 | 0.0066 | 0.0167 | 0.0174 | 0.0188 | 0.0188 |
| $K = 1.0$ | 0.0870 | 0.0134 | 0.0501 | 0.0633 | 0.0847 | 0.0869 |