MFE5130 – Financial Derivatives

Class Activity (9-October-2019) (Solution)

Important Notes:

1. This class activity is counted toward to your class participation score. **Fail** to hand in this class activity worksheet in the class will receive **0 score** for that class.
2. **0 mark** will be received if you leave the solution blank.

Name:	Student No.:

Problem 1

6.5 Suppose the gold spot price is \$300/oz, the 1-year forward price is 310.686, and the continuously compounded risk-free rate is 5%.

a. What is the lease rate?

b. What is the return on a cash-and-carry in which gold is not loaned?

c. What is the return on a cash-and-carry in which gold is loaned, earning the lease rate?

Solution

a) The spot price of gold is \$300.00 per ounce. With a continuously compounded annual risk-free rate of 5 percent, and at a one-year forward price of 310.686, we can calculate the lease rate according to the formula:

$$\delta_l = r - \frac{1}{T}\ln\left(\frac{F_{0,T}}{S_0}\right) = 0.05 - \ln\left(\frac{310.686}{300}\right) = 0.015$$

b) Suppose gold cannot be loaned. Then our cash and carry trade is:

Transaction	Time 0	Time $T = 1$
Short forward	0	$310.686 - S_T$
Buy gold	−300	S_T
Total	−300	310.686

Let *y* be the rate of return of the cash and carry trade.

$$310.686 = 300e^{y}$$
$$y = 3.5\%.$$

So, the return of the cash and carry trade is less than the prevailing risk-free interest rate of 5%.

The forward price bears an implicit lease rate. Therefore, if we try to engage in a cash and carry arbitrage but we do not have access to the gold loan market and thus do not have access to the additional revenue on our long gold position, it is not possible for us to replicate the forward price. We incur a loss.

c) If gold can be loaned, we engage in the following cash and carry arbitrage:

Transaction	Time 0	Time $T = 1$
Short forward	0	$310.686 - S_T$
Buy tailed gold position, lease rate @ 1.5%	$300e^{-1.5\%} = -295.5336$	S_T
Total	-295.5336	310.686

$$310.686 = 295.5336e^{y}$$
$$y = 5\%.$$

Now the forward was fairly priced by taking the implicit lease rate into account.