Tutorial - Class Activity

29 October, 2019 (Solution)

Problem 1

The current exchange rate is 0.42 British pounds per Australian dollar.

A pound-denominated European Australian dollar put has a strike price of 0.4 pounds and a premium of 0.0133 pounds. The put expires in 1 year.

A continuously compounded interest rate available on British pounds is 8%. The continuously compounded interest rate available on Australian dollars is 7%.

Calculate the value of an Australian dollar-denominated European British pound put that has a strike price of 2.5 Australian dollars and expires in 1 year.

Solution

The price of Australian dollar-denominated European British pound call is given by

$$\begin{aligned} C_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) &= \left(\frac{1}{0.42}\right)(2.5)\, P_{Pound}\left(0.42, \frac{1}{2.5}, 1\right) \\ &= \left(\frac{1}{0.42}\right)(2.5)(0.0133) \\ &= \text{AUD } 0.07917. \end{aligned}$$

By the put-call parity,

$$\begin{aligned} C_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) - P_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) &= \frac{1}{0.42}e^{-0.08} - 2.5e^{-0.07} \\ 0.07917 - P_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) &= \frac{1}{0.42}e^{-0.08} - 2.5e^{-0.07} \\ P_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) &= \text{AUD } 0.2123. \end{aligned}$$

Problem 2

An American call option on a stock has a strike price of 85 and expires in 5 months. You are given

i. The continuously compounded risk-free interest rate is 4%.
ii. The dividend of 1.5 payable at the end of today, and another dividend of 1.5 is payable in 3 months.
iii. The current price of the stock is 100.
iv. A European put option with a strike price of 85 which expires in 5 months costs 0.82.

Could it be rational to exercise the option immediately, before the dividend is paid?

Solution

By the put-call parity, we have

$$
\begin{aligned}
& C_{Eur}\left(100, 85, \frac{5}{12}\right) \\
& = P_{Eur}\left(100, 85, \frac{5}{12}\right) + 100 - \left(1.5 + 1.5e^{-(0.04)(3/12)}\right) - 85e^{-(0.04)(5/12)} \\
& = P_{Eur}\left(100, 85, \frac{5}{12}\right) + 100 - \left(1.5 + 1.5e^{-(0.04)(3/12)}\right) - 85e^{-(0.04)(5/12)} \\
& = 14.2399 \\
& < 15 = 100 - 85 \quad (\text{or } S - K).
\end{aligned}
$$

So, it may be rational to exercise the option early.

Problem 3

Three European put options expire in 1 year. The put options have the same underlying asset, but they have different strike prices and premiums.

Put Option	A	B	C
Strike	\$50.00	\$55.00	\$61.00
Premium	\$3.00	\$7.00	\$11.00

The continuously compounded risk-free interest rate is 11%.

a. What no-arbitrage property is violated?

b. What spread position would you use to effect arbitrage?

c. Demonstrate that the spread position is an arbitrage.

Solution

(a)

The prices of the options violate the following inequality

$$\frac{P(K_2)-P(K_1)}{K_2-K_1} \leq \frac{P(K_3)-P(K_2)}{K_3-K_2}$$

Because:

$$\frac{7-3}{55-50} > \frac{11-7}{61-55}$$
$$\frac{4}{5} > \frac{4}{6}$$

(b)

The above violated inequality can be rewritten as

$$\frac{P(55)-P(50)}{55-50} > \frac{P(61)-P(55)}{61-55}$$
$$6\left(P(55)-P(50)\right) > 5\left(P(61)-P(55)\right)$$
$$0 > 6P(50)-11P(55)+5P(61).$$

The arbitrage profit can be obtained by using the *asymmetric butterfly spread* with the following transactions:

Buy 6 of the 50-strike put options

Sell 11 of the 55-strike put options

Buy 5 of the 61-strike put options

(c)

		$t = 1$ year			
Transaction	$t = 0$	$S_1 < 50$	$50 \leq S_1 \leq 55$	$55 < S_1 \leq 61$	$61 < S_1$
Buy 6 of $P(50)$	–6(3.00)	$6(50 - S_1)$	0.00	0.00	0.00
Sell 11 of $P(55)$	11(7.00)	$-11(55 - S_1)$	$-11(55 - S_1)$	0.00	0.00
Buy 5 of $P(61)$	–5(11.00)	$5(61 - S_1)$	$5(61 - S_1)$	$5(61 - S_1)$	0.00
Total	4.00	0.00	$6S_1 - 300$	$305 - 5S_1$	0.00

where $P(K)$ is the price of the K-strike put option.

This strategy has strictly positive cash inflow at $t = 0$, and has a nonnegative payoff for all possible values S_1 of at $t = 1$ year. Therefore, this is an arbitrage strategy.