MFE5130 – Financial Derivatives

Class Activity (23-October-2019) (Solution)

Important Notes:

1. This class activity is counted toward to your class participation score. **Fail** to hand in this class activity worksheet in the class will receive **0 score** for that class.
2. **0 mark** will be received if you leave the solution blank.

Name:	Student No.:

Problem 1

Suppose that (spot) exchange rate is \$0.009/¥, the yen-denominated continuously compounded interest rate is 1%, the dollar-denominated continuously compounded interest rate is 5%, and the price of a 1-year \$0.009-strike dollar-denominated European yen call is \$0.0006. What is the price of a 1-year ¥ $\frac{1}{0.009}$ -strike yen-denominated dollar call?

Solution

The dollar-denominated yen call is related to the yen-denominated dollar put by the equation

$$C_{\$}(x_0, K, T) = x_0 K P_{\text{Yen}}\left(\frac{1}{x_0}, \frac{1}{K}, T\right).$$

Thus,

$$P_{\text{Yen}}\left(\frac{1}{0.009}, \frac{1}{0.009}, 1\right) = \frac{0.0006}{0.009} \times \frac{1}{0.009} = 7.4074\,\text{Yens}.$$

Using the put-call parity by treating the dollar-denominated continuously compounded interest rate 5% as the dividend yield of the underlying asset (USD), we have

$$\begin{aligned}
C_{\text{Yen}}\left(\frac{1}{0.009}, \frac{1}{0.009}, 1\right) - P_{\text{Yen}}\left(\frac{1}{0.009}, \frac{1}{0.009}, 1\right) &= \frac{1}{x_0} e^{-r_{US}T} - \frac{1}{K} e^{-r_{Yen}T} \\
&= \frac{1}{0.009} e^{-0.05} - \frac{1}{0.009} e^{-0.01} \\
C_{\text{Yen}}\left(\frac{1}{0.009}, \frac{1}{0.009}, 1\right) &= P_{\text{Yen}}\left(\frac{1}{0.009}, \frac{1}{0.009}, 1\right) + \\
&\quad \frac{1}{0.009} e^{-0.05} - \frac{1}{0.009} e^{-0.01} \\
&= 3.0939\,\text{Yens}.
\end{aligned}$$

Remark:

In general, the forward price on currency with the current spot exchange rate x_0 is given by

$$F_{0,T} = x_0 e^{(r-r_f)T}$$

where T is the maturity date and r_f is the annual continuously compounded foreign interest rate.

Problem 2

Two European put options expire in 1 year. The put options have the same underlying asset, but they have different strike prices and premiums.

Put Option	A	B
Strike	40.00	45.00
Premium	4.00	8.78

The continuously compounded risk-free rate of return is 8%.
A profit-maximizing arbitrageur constructs an arbitrage strategy.
Arbitrage profits are accumulated at the risk-free rate of return.
If the stock price is \$35 at the end of the year, then the accumulated arbitrage profits are \$*X*.
If the stock price is \$43 at the end of the year, then the accumulated arbitrage profits are \$*Y*.
Find *X* and *Y*.

Solution

The prices of the European put options violate

$$P(K_2,1) - P(K_1,1) \le e^{-r}(K_2 - K_1)$$

where $P(K, 1)$ is the price of the European put option with strike price K and 1 year until expiration, because when $K_1 = 40$ and $K_2 = 45$, we have:

$$8.78 - 4 > (45 - 40)e^{-0.08}$$
$$4.78 > 4.6156.$$

Arbitrage is available by buying a put bull spread:

Buy 40-strike put and sell 45-strike put.

The cost of the put bull spread = 4 – 8.78 = –4.78.

The strategy produces the following payoff table:

		Time 1		
Transaction	**Time 0**	$S_1 < 40$	$40 \leq S_1 \leq 45$	$45 < S_1$
Buy 40-strike put	–4.00	$40 - S_1$	0	0
Sell 45-strike put	8.78	$-(45 - S_1)$	$-(45 - S_1)$	0
Total	4.78	–5.00	$-(45 - S_1)$	0

The accumulated profit of the strategy at $t = 1$ is

	Time 1		
Transaction	$S_1 < 40$	$40 \leq S_1 \leq 45$	$45 < S_1$
Accumulated profit	$-5.00 + 4.78\, e^{0.08} = 0.1781$	$-(45 - S_1) + 4.78\, e^{0.08} = S_1 - 39.8219 > 0$	$4.78\, e^{0.08} = 5.1781$

It can be observed that the accumulated profits at $t = 1$ for all scenarios of S_1 are positive. So, this strategy creates the arbitrage profit.

If the final stock price is \$35, then the accumulated arbitrage profits are:

$X = -5 + 4.78\, e^{0.08} = 0.1781.$

If the final stock price is \$43, then the accumulated arbitrage profits are:

$Y = -(45 - 43) + 4.78\, e^{0.08} = 3.1781.$