Tutorial - Class Activity (Solution)

12 September, 2018

Problem 1

2.9 An *off-market* forward contract is a forward where either you have to pay a premium or you receive a premium for entering into the contract. (With a standard forward contract, the premium is zero.) Suppose the effective annual interest rate is 10% and the S&R index is 1000. Consider 1-year forward contracts.

a. Verify that if the forward price is \$1100, the profit diagrams for the index and the 1-year forward are the same.

b. Suppose you are offered a long forward contract at a forward price of \$1200. How much would you need to be paid to enter into this contract?

c. Suppose you are offered a long forward contract at \$1000. What would you be willing to pay to enter into this forward contract?

In this problem, suppose that you can borrow money to buy the S&R index today.

Note:
The effective annual interest rate r_{eff} is related to the annual continuously compounded interest rate r by the following formula:

$$1 + r_{eff} = e^{r}.$$

Solution

a) <u>Long the forward contract</u>

If the forward price is \$1,100, then the buyer of the one-year forward contract receives at expiration after one year a profit of: $\$S_T - \$1{,}100$, where S_T is the (unknown) value of the S&R index at expiration of the forward contract in one year. Remember that it costs nothing to enter the forward contract.

<u>Purchasing the index</u>

We borrow money to finance the purchase of the index today, so that we do not need any initial cash. If we borrow \$1,000 today to buy the S&R index (that costs \$1,000), we have to repay in one year: $\$1{,}000 \times (1 + 0.10) = \$1{,}100$. Our total profit in one year from borrowing to buy the S&R index is therefore: $\$S_T - \$1{,}100$.

The profits from the two strategies (long the forward contract and purchasing the index) are identical.

b) Let P_1 be the premium of the forward contract which you need to pay today to enter into this contract.

So, the profit of the forward contract at expiration is given by

$$\$S_T - \$1{,}200 - \$\,P_1(1 + 10\%) = \$S_T - \$1{,}200 - \$1.1\,P_1.$$

To make there is no advantage in buying either the stock or forward at a price of \$1,200, we have

$$\begin{aligned} S_T - 1200 - 1.1P_1 &= S_T - 1100 \\ P_1 &= 100/1.1 \\ &= -90.91. \end{aligned}$$

The negative sign means that you are to be paid with the amount of $90.91 from the forward seller.

c) Let P_2 be the premium of the forward contract which you need to pay today to enter into this contract. Similar to (b), we have

$$\begin{aligned} S_T - 1000 - 1.1P_2 &= S_T - 1100 \\ P_2 &= 100/1.1 \\ &= 90.91. \end{aligned}$$

So, you need to pay $90.91 to the forward seller in order to enter into this forward contract.

Problem 2

You are given that the effective 6-month interest rate is 2% and the S&R 6-month forward price is $1020. Also, the premium for 6-month 950-strike S&R call and put options are $120.405 and $51.777 respectively.

Suppose you buy the S&R index for $1000 and buy a 950-strike put. Construct payoff and profit diagrams for this position. Verify that you obtain the same payoff and profit diagram by investing $931.37 in zero-coupon bonds and buying a 950-strike call.

Note:
The effective 6-month interest rate r_{eff} is related to the annual continuously compounded interest rate r by the following formula:

$$\left(1 + r_{eff}\right) = e^{r \times 0.5}.$$

Solution

In order to be able to draw profit diagrams, we need to find the future value of the put premium, the call premium, and the investment in zero-coupon bonds. We have for:

the put premium: $51.777 × (1 + 0.02) = $52.81,
the call premium: $120.405 × (1 + 0.02) = $122.81, and
the zero-coupon bond: $931.37 × (1 + 0.02) = $950.00

Now, we can construct the payoff and profit diagrams of the aggregate position:

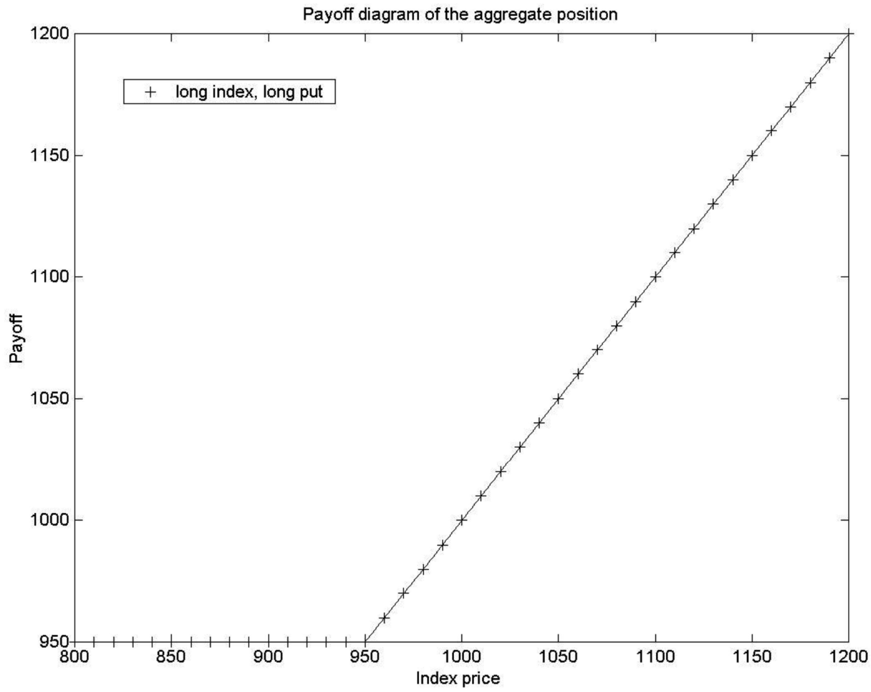
The cash flows for the portfolio of buying S&R index and 950-strike put are given as follows

		t = 6-month	
Transactions	*t* = 0	$S_{0.5} \leq 950$	$S_{0.5} > 950$
Buy S&R index	–1000	$S_{0.5}$	$S_{0.5}$
Long 950-strike put	–51.777	$950 - S_{0.5}$	0
Total	**–1051.777**	**950**	$\boldsymbol{S_{0.5}}$

The cash flows for the portfolio of buying a zero-coupon bond and 950-strike call are given as follows

		t = 6-month	
Transactions	$t = 0$	$S_{0.5} \leq 950$	$S_{0.5} > 950$
Buy zero-coupon bond	–931.37	\$931.37 × (1 + 0.02) = \$950.00	\$931.37 × (1 + 0.02) = \$950.00
Long 950-strike call	–120.405	0	$S_{0.5} - 950$
Total	**–1051.775**	**950**	$\boldsymbol{S_{0.5}}$

Payoff diagram:



From this figure, we can already see that the combination of a long put and the long index looks exactly like a certain payoff of \$950, plus a call with a strike price of 950. But this is the alternative given to us in the question. We have thus confirmed the equivalence of the two combined positions for the payoff diagrams. Also, the two positions have the same FV of the cost:

Position 1 (Buy the S&R index and buy the put) = 1051.777(1.02) = 1,072.81.

Position 2 (Buy the zero-coupon bond and the call) = 1051.775(1.02) = 1,072.81.

Therefore, the two positions have the same profit diagram.

Problem 3

Suppose the S&P index is 1,300 initially and an investor invests \$10,000. If the index is below 1,300 after 5 years, the CD returns to the investor the original \$10,000 investment. If the index is above 1,300 after 5 years, the investor receives \$10,000 plus 70% of the percentage gain on the index. However, the gain is capped at 100%.

Write down the final payoff of this equity linked CD in form of the payoff of call options.

Solution

$$\text{Final payoff} = 10{,}000\left[1 + 70\% \times \min\left(\max\left(\frac{S_5 - 1300}{1300}, 0\right), 1\right)\right]$$

$$= 10{,}000 + \frac{7{,}000}{1{,}300} \times \min\left(\max\left(S_5 - 1300, 0\right), 1300\right).$$

	$0 < S_5 < 1{,}300$	$1{,}300 \le S_5 < 2{,}600$	$S_5 \ge 2{,}600$
min(max(S_5 – 1,300, 0), 1,300)	0	S_5 – 1300	1,300

The final payoff can then be written as

$$10{,}000 + \frac{7{,}000}{1{,}300} \times \left[\max\left(S_5 - 1{,}300, 0\right) - \max\left(S_5 - 2{,}600, 0\right)\right].$$

Therefore, the equity linked CD is made of

i. Long a 5-year zero coupon bond with face value of 10,000;
ii. Long 7,000/1,300 units of 1,300-strike call option with 5 years to expiration;
iii. Short 7,000/1,300 units of 2,600-strike call option with 5 years to expiration.