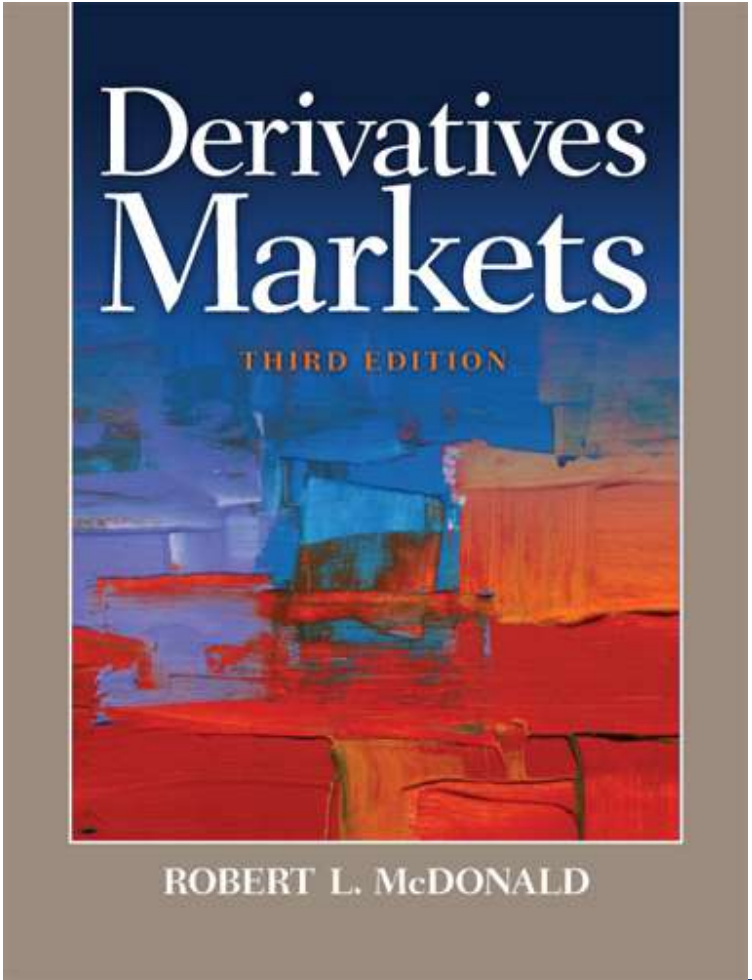


Chapter 4

(Chapter 5 in the Textbook)

Financial Forwards and Futures

Points to note

1. Alternative ways to buy a stock, see P. 4 to 5.
2. Pricing pre-paid forwards, see P. 6 to 17.
3. Pricing forwards on stock, see P. 18 to 20.
4. Creating a synthetic forward, see P. 21 to 25.
5. Synthetic forwards in market-making and arbitrage, see P. 26 – 28.
6. An interpretation of the forward pricing formula, see P. 29 – 30.
7. Future contracts, see P. 31 – 34.
8. Future contracts – marking to market, see P. 35 – 41.
9. Comparing futures and forward prices, see P. 42 – 43.
10. Quanto Index Contracts, see P. 44 – 46.

Pre-paid forward vs forward

Difference between Forward and futures

Issues

1) Pricing $F_{0,T} = ??$

2) hedging of forward

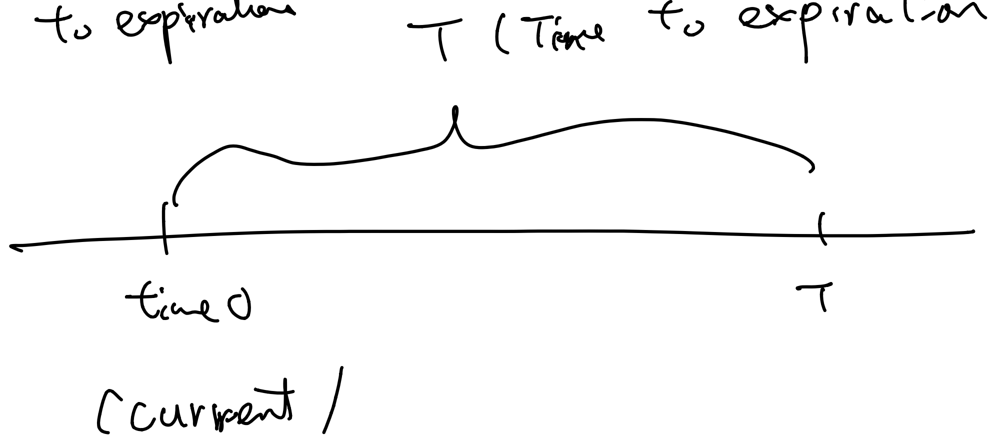
Non-dividend paying stock

$F^{P}_{0,T}$: pre-paid forward price

pre-paid forward with time to expiration of T.

current time

time to expiration



$F_{0,T}$: forward price

$$F_{0,T} = FV(F^{P}_{0,T})$$

$$F^{P}_{0,T} = S_0 \quad \text{(non-dividend paying stock)}$$

$$\Rightarrow F_{0,T} = FV(S_0) = \begin{cases} S_0 e^{rT} & \text{continuous comp} \\ S_0(1+r)^T & \text{discrete} \end{cases}$$

Introduction

- Financial futures and forwards
  - On stocks and indexes
  - On currencies
  - On interest rates
- How are they used?
- How are they priced?
- How are they hedged?

Alternative Ways to Buy a Stock

- Four different payment and receipt timing combinations:
  - Outright purchase: ordinary transaction.
  - Fully leveraged purchase: A purchase in which you borrow the entire purchase price of the security and repay the borrowed amount later.
  - Prepaid forward contract: pay today (not necessarily the stock price), receive the share later.
  - Forward contract: agree on price now, pay/receive later.

Alternative Ways to Buy a Stock (cont'd)

Suppose we want to have a stock at T

TABLE 5.1 Four different ways to buy a share of stock that has price S_0 at time 0. At time 0 you agree to a price, which is paid either today or at time T. The shares are received either at 0 or T. The interest rate is r.

	Description	Pay at Time	Receive Security at Time	Payment	
1	Outright purchase	0	0	S_0 at time 0	
2	Fully leveraged purchase	T	0	S_0e^{rT} at time T	
3	Prepaid forward contract	0	T	?	??
4	Forward contract	T	T	$? \times e^{rT}$	

Pricing Prepaid Forwards

- If we can find the *prepaid* forward price for an asset bought at time 0 and delivered at time T which is denoted as $F^P_{0,T}$, then we can calculate the price for a forward contract

Forward price $F = \boxed{\text{Future value of } F^P_{0,T}}$

- Three possible methods to price prepaid forwards:
  - Pricing by analogy.
  - Pricing by discounted present value.
  - Pricing by arbitrage.

- For now, assume that there are no dividends.

Compare ① and ③

Payoff ① at T $\qquad$ Payoff ③ at T

$$S_T = S_T$$

$\Rightarrow$ Cost ① = Cost ③

$$S_0 = F_{0,T}^{P}$$ (pre-paid forward price)

$$F_{0,T} \text{ (forward price)} = FV(F_{0,T}^{P})$$
$$= FV(S_0)$$
$$= \begin{cases} S_0 e^{rT} & \text{continuous compounding} \\ S_0 (1+r)^T & \text{discrete compound.} \end{cases}$$

key assumption: stock pays no dividend

Pricing Prepaid Forwards (cont'd)

- Pricing by analogy
  - In the absence of dividends, the timing of delivery is irrelevant.
  - Price of the prepaid forward contract same as current stock price.
  - $F^{P}_{0,T} = S_0$ (where the asset is bought at t = 0, delivered at t = T).

Pricing Prepaid Forwards (cont'd)

- Pricing by discounted present value

  – If expected stock price at time T based on information we have at time 0 is $E_0(S_T)$, then

$$F^{P}{}_{0,T} = E_0(S_T)e^{-\alpha T}$$

  where α is the risk-adjusted discount rate.

  – By the definition of the expected return, we have

$$E_0(S_T) = S_0 e^{\alpha T}$$

  – Combining them,

$$F^{P}{}_{0,T} = S_0$$

Pricing Prepaid Forwards (cont'd)

Want to prove: $F^P_{0,T} = S_0$

Suppose $F^P_{0,T} \neq S_0$

- Pricing by arbitrage
  - Arbitrage: a situation in which one can generate positive cash flow by simultaneously buying and selling related assets, with no net investment of funds and with no risk ➔ free money!!!
  - An *extremely important pricing principle* is that the price of a derivative should be such that no arbitrage is possible.
    - Suppose that $F^P_{0,T} > S_0$. The arbitrage can be created as in Table 5.2

Pricing Prepaid Forwards (cont'd)

TABLE 5.2 Cash flows and transactions to undertake arbitrage when the prepaid forward price, $F^P_{0,T}$, exceeds the stock price, S_0.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Buy stock @ S_0	$-S_0$	$+S_T$
Sell prepaid forward @ $F^P_{0,T}$	$+F^P_{0,T}$	$-S_T$
Total	$F^P_{0,T} - S_0$	0

Buy Low

Sell high

> 0

Pricing Prepaid Forwards (cont'd)

- Now suppose on the other hand that $F^P_{0,T} < S_0$. The arbitrage can be formed by
  - buying the prepaid forward and shorting the stock, earning

$$S_0 - F^P_{0,T} > 0$$

  - One year from now, we acquire the stock via the prepaid forward and we use the stock to close the short position. So, there is an arbitrage profit.

- Therefore, to preclude the arbitrage, we have

$$F^P_{0,T} = S_0$$

Pricing Prepaid Forwards (cont'd)

discrete dividend
continuous dividend.

- When a stock pays dividend, the owner of stock receives dividends but the owner of the prepaid forward contact does not.
- It is necessary to adjust the prepaid forward price to reflect dividends that are received by the shareholder but not by the holder of prepaid forward contact.

Pricing Prepaid Forwards (cont'd)

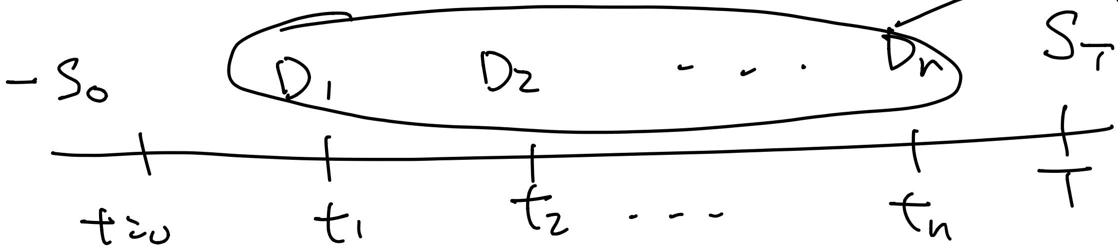
Discrete Dividends

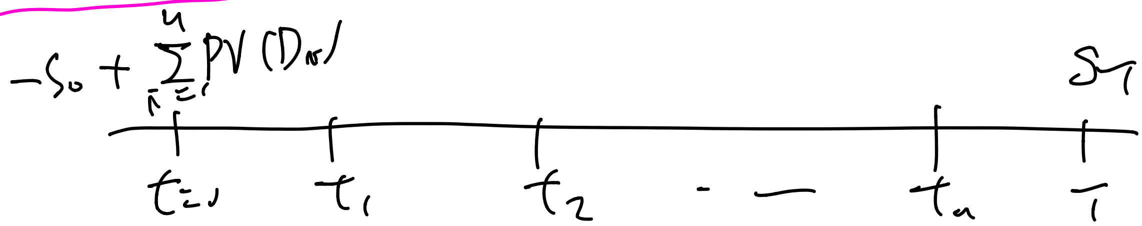
- Suppose a stock is expected to make dividend payments of D_{t_i} at times t_i, $i = 1, 2, \ldots, n$.
- A prepaid forward contract will entitle you to receive the stock at time T but without receiving the interim dividends.
- Thus, the price of a prepaid forward contract will be the stock price less the present value of dividends to be paid over the life of the contract

$$F_{0,T}^{P} = S_0 - \sum_{i=1}^{n} PV_{0,t_i}\left(D_{t_i}\right)$$

where PV_{0,t_i} denotes the time 0 present value of a time t_i payment.

Discrete Dividend ($D_1, D_2, \ldots D_n$ dividends) deterministic





Pre-paid

$$F^P_{0,T} = S_0 - \sum_{i=1}^{n} PV_{0,t_i}(D_i)$$

$$= S_0 - \sum_{i=1}^{n} D_i e^{-rt_i} \qquad PV_{0,t_i}(1) = e^{-rt_i}$$

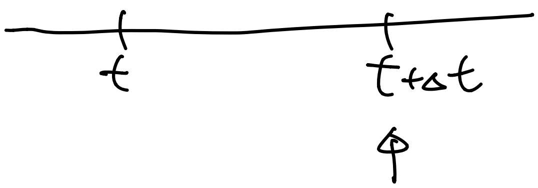
Pricing Prepaid Forwards (cont'd)

Continuous Dividends

- Suppose the dividend is being paid continuously at a rate proportional to the level of the stock price; i.e., the dividend yield, is constant.
- **Dividend yield**: the annualized dividend payment divided by the stock price.
- Let δ be the dividend yield.
- Now suppose we wish to invest today in order to have one share at time *T*. We can buy $e^{-\delta T}$ shares today. Because of **dividend reinvestment**, at time *T*, we will end up with *exactly one share*.

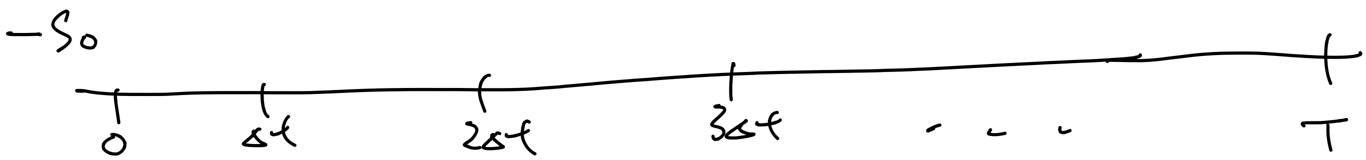
$$F_{0,T}^{P} = S_0 e^{-\delta T}$$

δ: dividend yield



Annualized dividend

$$\underset{[t,\, t+\Delta t]}{\text{dividend}} = \boxed{\delta \times S_{t+\Delta t}} \times \Delta t$$



dividend $\quad S_{\Delta t} \times \Delta t \times \delta \quad S_{2\Delta t} \times \Delta t \times \delta \quad \ldots$

stochastic

Re-investment of dividend

Current stock price $= S_0$, $N = \dfrac{T}{\Delta t}$

dividend yield $= \delta$, $a = e^{-\delta T}$

t	0	Δt
Stock price @ t	S_0	$S_{\Delta t}$ — use dividend to buy stock
No. of share of stock	a	$a + \dfrac{a\delta \times \cancel{S_{\Delta t}} \times \Delta t}{\cancel{S_{\Delta t}}} = a + a\delta\Delta t = a(1+\delta\Delta t)$
dividend @ t	0	$a\delta \times S_{\Delta t}\,\Delta t$
Value of stock	aS_0	$a(1+\delta\Delta t)S_{\Delta t}$

	$2\Delta t$
stock price	$S_{2\Delta t}$
No. of share	$a(1+\delta\Delta t) + \dfrac{\delta a(1+\delta\Delta t)\cancel{S_{2\Delta t}} \times \Delta t}{\cancel{S_{2\Delta t}}} = a(1+\delta\Delta t)^2$
dividend	$\delta \times a(1+\delta\Delta t) \times S_{2\Delta t}\,\Delta t$
Value	$a(1+\delta\Delta t)^2 S_{2\Delta t}$

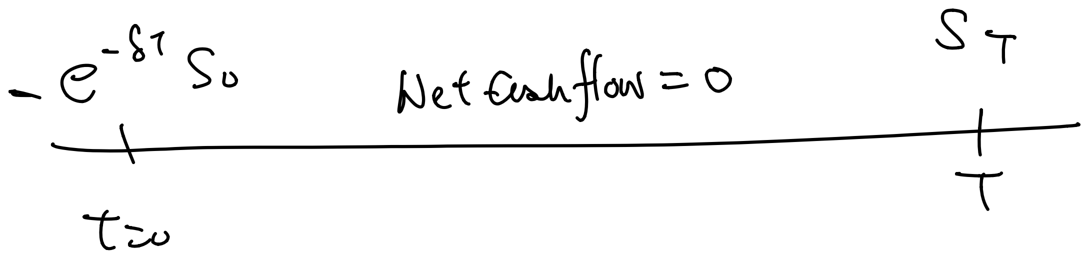
	$N\Delta t = T$
stock price	$S_{N\Delta t}$
No. of share	$a(1+\delta\Delta t)^N$
dividend	
Value	

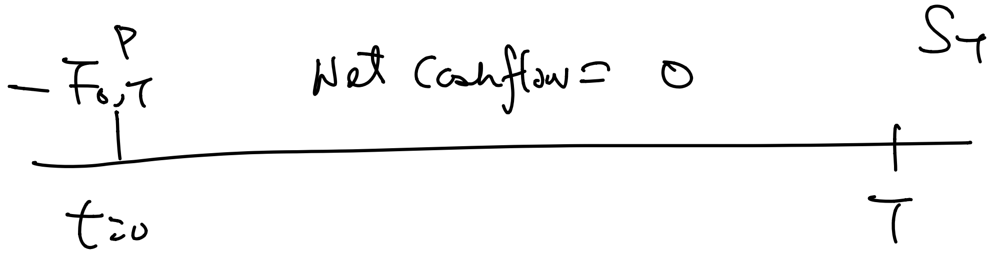
No. of share at $T = a(1+\delta\Delta t)^N$

$$\lim_{\Delta t \to 0} a\boxed{(1+\delta\Delta t)^N}$$

$$= a e^{\delta T} = e^{-\delta T} \cdot e^{\delta T} = 1$$

Continuous Dividend





$$\Rightarrow \quad F_{0,T}^{P} = e^{-\delta T} S_0$$

$$F_{0,T} = FV(F_{0,T}^{P}) = e^{(r-\delta)T} S_0$$

Let $a = e^{-\delta T}$ and $N = T/\Delta t$.

Let S_t be the stock price at time t.

Time	**0**	$\boldsymbol{\Delta t}$	$\boldsymbol{2\Delta t}$	**…**	$\boldsymbol{N\Delta t = T}$
No. of shares of stock	a	$a+a\delta\Delta t$ $=a(1+\delta\Delta t)$	$a(1+\delta\Delta t)+a(1+\delta\Delta t)\delta\Delta t$ $=a(1+\delta\Delta t)^2$		$a(1+\delta\Delta t)^N$
Value of the stock	aS_0	$aS_{\Delta t}$ + $a\delta S_{\Delta t}\Delta t$	$a(1+\delta\Delta t)\,S_{2\Delta t}+a(1+\delta\Delta t)\delta$ $\Delta t\,S_{2\Delta t}$		$a(1+\delta\Delta t)^N S_T$

By taking $\Delta t \rightarrow 0$, we have $(1+\delta\Delta t)^N \rightarrow e^{\delta T}$. So, we end up with 1 unit of stock at time T.

Pricing Prepaid Forwards (cont'd)

- Example 5.1
  - XYZ stock costs \$100 today and is expected to pay a quarterly dividend of \$1.25. If the risk-free rate is 10% compounded continuously, how much does a 1-year prepaid forward cost?

$$F^{p}_{0,1} = \$100 - \Sigma^{4}_{i=1} \$1.25e^{-0.025i} = \$95.30$$

within [0, 1yr]

Pricing Prepaid Forwards (cont'd)

- Example 5.2
  - The index is \$125 and the dividend yield is 3% continuously compounded. How much does a 1-year prepaid forward cost?
  - $F^P_{0,1} = \$125e^{-0.03} = \$121.31$

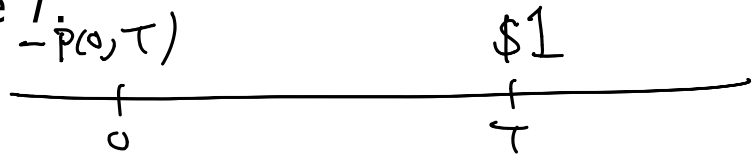
$S_0 e^{-\delta T}$

Pricing Forwards on Stock

- Forward price is the future value of the *prepaid* forward price

$$F_{0,T} = FV(F^{P}_{0,T}) = F^{P}_{0,T}e^{rT} = \frac{F^{P}_{0,T}}{P(0,T)}$$

where *r* is the yield to maturity for a default-free zero coupon bond with maturity at time *T* and *P*(*t*,*T*) is the time *t* price of a zero-coupon bond maturing at time *T*.



$$P(0,T) = e^{-rT}$$

Pricing Forwards on Stock (cont'd)

- Forward premium
  - Definition
    - Forward premium = $F_{0,\,T} \,/\, S_0$.
    - Annualized forward premium = $(1/T)\ \ln(F_{0,\,T} \,/\, S_0)$.
  - For the case of continuous dividends, the annualized forward premium is $r - \delta$.
  - The forward price can be used to infer the price of the underlying asset when it is unobserved. For example, the future contract of the S&P 500 index trades at times when the NYSE is not open. The asset price implied by the forward pricing formulas is called the *fair value* of the underlying stock.

Does the Forward Price Predict the Future Spot Price?

- The forward price is the expected future spot price discounted at the risk premium

$$F_{0,T} = e^{rT} F_{0,T}^{P} = E_0(S_T) e^{-(\alpha - r)T}$$

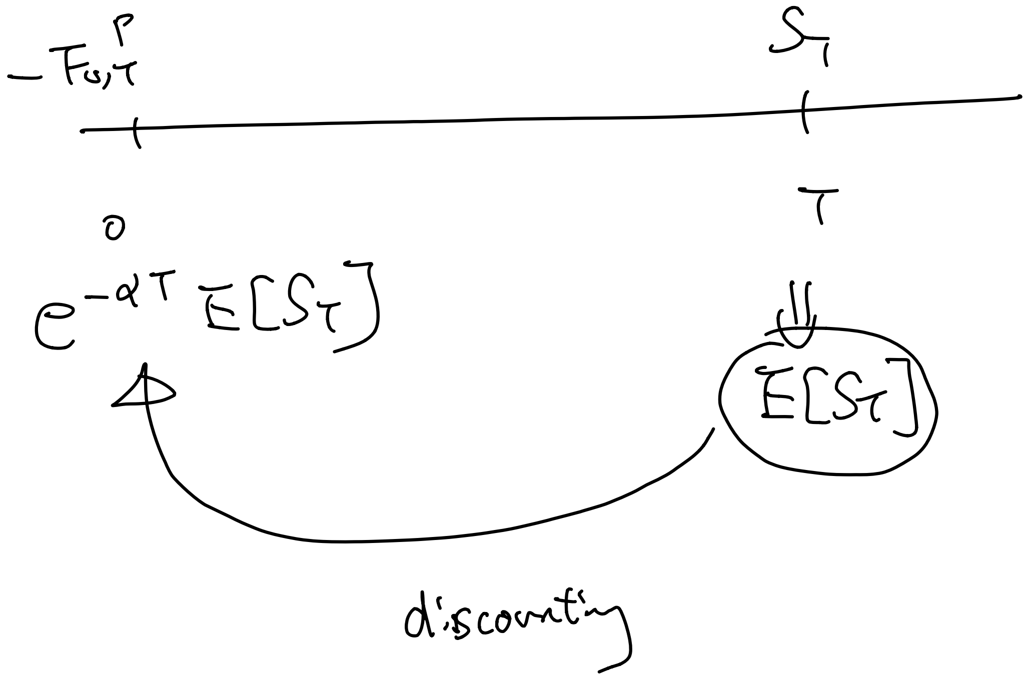
α (risk return)

r

where α is the expected return on a stock and r is the risk-free interest rate.

$\alpha \neq r \Rightarrow F_{0,T} \neq E[S_T]$

- Therefore, the forward price *systematically errs* in predicting the future stock price. If the asset has a positive risk premium, the future spot price will on average be greater than the forward price.



$$F_{0,T} = FV(F_{0,T}^P) = e^{rT}(F_{0,T}^P) = e^{(r-\alpha)T} E[S_T]$$

$$\Rightarrow F_{0,T} \neq E[S_T]$$

Creating a *Synthetic* Forward

- One can offset the risk of a forward by creating a *synthetic* forward to offset a position in the actual forward contract.
- How can one do this? (assume continuous dividends at rate δ)
  - Recall the long forward payoff at expiration: $= S_T - F_{0,T}$.
  - Borrow and purchase shares as follows.

Long forward

How to use

Stock + cash (bond) $\Longrightarrow$ Forward

$S_T - F_{0,T}$ $\Leftarrow$ Payoff

$= S_T - S_0 e^{(r-\delta)T}$

$= \underbrace{S_T}_{\text{one unit of stock}} - \underbrace{\boxed{(S_0 e^{-\delta T}) e^{rT}}}_{\text{face value of zero-coupon bond}}$

	$t=0$	$t=T$
Stock (Buy)	$-e^{-\delta T} S_0$	S_T
Bond (Short)	$+S_0 e^{-\delta T}$	$-(S_0 e^{-\delta T}) e^{rT}$
total	0	$\underbrace{S_T - S_0 e^{(r-\delta)T}}_{\text{payoff of forward}}$

Synthetic forward = Buy $e^{-\delta T}$ units of stock

+ Short a zero-coupon bond with face value $= S_0 e^{(r-\delta)T}$

Synthetic stock

$\boxed{\text{Forward} = S_T - F_{0,T}}$

$S_T = \text{Forward} + F_{0,T}$ $\leftarrow$ cash

Synthetic Bond $= -\text{Forward} + S_T = F_{0,T}$

Creating a *Synthetic* Forward (cont'd)

TABLE 5.3 Demonstration that borrowing $S_0 e^{-\delta T}$ to buy $e^{-\delta T}$ shares of the index replicates the payoff to a forward contract, $S_T - F_{0,T}$.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Buy $e^{-\delta T}$ units of the index	$-S_0 e^{-\delta T}$	$+ S_T$
Borrow $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$- S_0 e^{(r-\delta)T}$
Total	0	$S_T - S_0 e^{(r-\delta)T}$

Note that the total payoff at expiration is same as forward payoff.

Creating a *Synthetic* Forward (cont'd)

- The idea of creating synthetic forward leads to following:
  - (Synthetic) Forward = Stock – zero-coupon bond.
  - (Synthetic) Stock = Forward + zero-coupon bond.
  - (Synthetic) Zero-coupon bond = Stock – forward.

	$t=0$	$t=T$
Long forward	0	$S_T - S_0 e^{(r-\delta)T}$
Long zero coupon bond	$-e^{-\delta T} S_0$	$S_0 e^{(r-\delta)T}$
	$-e^{-\delta T} S_0$	S_T

Creating a *Synthetic* Forward (cont'd)

TABLE 5.4 Demonstration that going long a forward contract at the price $F_{0,T} = S_0 e^{(r-\delta)T}$ and lending the present value of the forward price creates a synthetic share of the index at time T.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Long one forward	0	$S_T - F_{0,T}$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_0 e^{(r-\delta)T}$
Total	$-S_0 e^{-\delta T}$	S_T

Note that the total payoff at expiration is same as stock payoff.

Creating a *Synthetic* Forward (cont'd)

TABLE 5.5 Demonstration that buying $e^{-\delta T}$ shares of the index and shorting a forward creates a synthetic bond.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Buy $e^{-\delta T}$ units of the index	$-S_0 e^{-\delta T}$	$+S_T$
Short one forward	0	$F_{0,T} - S_T$
Total	$-S_0 e^{-\delta T}$	$F_{0,T}$

synthetic bond

The payoff at T is

$$F_{0,T} = S_0 e^{(r-\delta)T} = \left(S_0 e^{-\delta T}\right) e^{rT}$$

It is the payoff of $S_0 e^{-\delta T}$ unit of T-year zero coupon bond.

The rate of return (r) on this synthetic bond is called *implied repo rate.*

Synthetic Forwards in Market-Making and Arbitrage

- The market-maker can *offset* his risk on the short forward position by creating a synthetic long forward position.
- A transaction in which you buy the underlying asset and short the offsetting forward contract is called a *cash-and-carry*.

Cash & Carry = 1 + 3

TABLE 5.6 Transactions and cash flows for a cash-and-carry: A market-maker is short a forward contract and long a synthetic forward contract.

Long Synthetic forward

Transaction	Cash Flows Time 0	Cash Flows Time T (expiration)
Buy tailed position in stock, paying $S_0e^{-\delta T}$ 1	$-S_0e^{-\delta T}$	$+S_T$
Borrow $S_0e^{-\delta T}$ 2	$+S_0e^{-\delta T}$	$-S_0e^{(r-\delta)T}$
Short forward 3	0	$F_{0,T} - S_T$
Total	0	$F_{0,T} - S_0e^{(r-\delta)T}$

Synthetic Forwards in Market-Making and Arbitrage (cont'd)

TABLE 5.7 Transactions and cash flows for a reverse cash-and-carry: A market-maker is long a forward contract and short a synthetic forward contract. $= ① + ③$

Short synthetic forward

Transaction	Cash Flows: Time 0	Cash Flows: Time T (expiration)
Short tailed position in stock, receiving $S_0e^{-\delta T}$ ①	$+S_0e^{-\delta T}$	$-S_T$
Lend $S_0e^{-\delta T}$ ②	$-S_0e^{-\delta T}$	$+S_0e^{(r-\delta)T}$
Long forward ③	0	$S_T - F_{0,T}$
Total	0	$S_0e^{(r-\delta)T} - F_{0,T}$

theoretical price

$= 0 \quad \therefore \quad F_{0,T} = S_0e^{(r-\delta)T}$

??

Synthetic Forwards in Market-Making and Arbitrage (cont'd)

theoretical price

- If $F_{0,T} \neq S_0 e^{(r-\delta)T}$, then an arbitrageur can make a costless profit.

  market price

  – If $F_{0,T} > S_0 e^{(r-\delta)T}$, then an arbitrageur or market-maker can use the strategy in Table 5.6 to make a risk-free profit.

  Buy synthetic + sell market forward

  – On the other hand, if $F_{0,T} < S_0 e^{(r-\delta)T}$, then an arbitrageur or market-maker can use the strategy in Table 5.7 to make a risk-free profit.

An Interpretation of the Forward Pricing Formula

- Suppose you buy a unit of index that costs S and fund the position by borrowing at the risk-free rate. Over the short time interval Δt, you will
  - pay $rS\Delta t$ on the borrowed amount. outflow ①
  - Receive the dividend $\delta S\Delta t$. inflow ②
- Net cashflow for you to carry a long position in the asset = $(r - \delta)S\Delta t$. (net cash outflow) = ① − ②
- So, $(r - \delta)$ is the called the <u>*cost of carry*</u>.

per unit time per unit price

An Interpretation of the Forward Pricing Formula (cont'd)

- The forward contract, unlike the stock, requires no investment and makes no payouts and therefore has a zero cost of carry. Therefore, the forward contract saves our having to pay the cost of carry, we are willing to pay a higher price.

$$\text{Forward price} = \text{Spot price} + \underbrace{\text{Interest to carry the asset} - \text{Asset lease rate}}_{\text{Cost of carry} \times \text{spot price} \times \Delta t}$$

where Δt is the maturity of the forward contract.

$$F_{0,T} = S_0 e^{(r-\delta)T} \approx S_0 (1 + (r-\delta)T)$$

$= S_0 + S_0 (r - \delta) T$

Cost of carry

Futures Contracts

- Exchange-traded "forward contracts"
- Typical features of futures contracts
  - Standardized, with specified delivery dates, locations, procedures.
  - A clearinghouse
    - Matches buy and sell orders.
    - Keeps track of members' obligations and payments.
    - After matching the trades, becomes counterparty.
- Open interest: total number of buy/sell pairs.

Futures Contracts (cont'd)

- Differences from forward contracts
  - Futures contract is settled daily. The determination of who owes what to whom is called *marking-to-market*.
  - Because of daily settlement, futures contract is liquid – it is possible to offset an obligation on a given day by entering into the opposite position.
  - Highly standardized structure ➔ harder to customize.
  - Because of daily settlement, the nature of credit risk is different with the futures contract.
  - There are typically daily *price limits* in futures market. The price limit is a move in the futures price that triggers a temporary halt in trading.

The S&P 500 Futures Contract

- *Underlying asset:* S&P 500 stock index.
- *Notational value (the dollar value of the assets underlying one contract):*

  $250 x S&P 500 index

- *Cash-settled contract (not physical settlement)*:

On the expiration day, the S&P 500 futures contract is marked-to-market against the actual cash index. This final settlement against the cash index guarantees that the futures price equals the index value at contract expiration.

Example: S&P 500 Futures

FIGURE 5.1

Specifications for the S&P 500 index futures contract.

Underlying	S&P 500 index
Where traded	Chicago Mercantile Exchange
Size	\$250 × S&P 500 index
Months	March, June, September, December
Trading ends	Business day prior to determination of settlement price
Settlement	Cash-settled, based upon opening price of S&P 500 on third Friday of expiration month

Margins and Marking to Market

- Suppose you would enter 8 long S&P 500 futures contracts with the futures price of 1,100.
- Total notional value

  $= 8 \text{x} \$250 \text{x} 1100 = \$2.2$ million.
- A broker executes your buy order by matching with another sell order. The total number of open positions (buy/sell pairs) is called the *open interest* of the contract.

Margins and Marking to Market (cont'd)

- Both buyers and sellers need to make a deposit, which can earn interest, with the broker. The deposit is called *margin* which is intended to protect the counterparty against your failure to meet your obligations.
- Here, we suppose the margin is *10%* and *weekly settlement*.
- Margin deposit = 10% x $2.2 million

  = $220,000

Margins and Marking to Market (cont'd)

Interest rate for the margin deposit: 6% p.a. (compounded continuously).

Suppose that over the first week, the futures price drops 72.01 points to 1,027.99, our margin balance after 1 week is

$$\$220{,}000e^{0.06 \times 1/52} + 8 \times 250 \times (1{,}027.99 - 1{,}100)$$

$$= \$76{,}233.99$$

TABLE 5.8

Mark-to-market proceeds and margin balance over 10 weeks from long position in 8 S&P 500 futures contracts. The last column does not include additional margin payments. The final row represents expiration of the contract.

Week	Multiplier ($)	Futures Price	Price Change	Margin Balance($)
0	8x250= 2000.00	1100.00	—	220,000.00
1	2000.00	1027.99	−72.01	76,233.99
2	2000.00	1037.88	9.89	96,102.01
3	2000.00	1073.23	35.35	166,912.96
4	2000.00	1048.78	−24.45	118,205.66
5	2000.00	1090.32	41.54	201,422.13
6	2000.00	1106.94	16.62	234,894.67
7	2000.00	1110.98	4.04	243,245.86
8	2000.00	1024.74	−86.24	71,046.69
9	2000.00	1007.30	−17.44	36,248.72
10	2000.00	1011.65	4.35	44,990.57

220,000
= − 2000x72.01

Margins and Marking to Market (cont'd)

The 10-week profit on the futures position

$$\$44{,}990.57 - \$220{,}000e^{0.06\times 10/52} = -\$177{,}562.6$$

In the case of forward contract, the 10-week profit is

$$(1{,}011.65 - 1{,}100) \times \$2{,}000 = -\$176{,}700$$

The difference is because the interest is earned on the mark-to-market proceeds in the case of futures contract.

Margins and Marking to Market (cont'd)

- The decline in the margin balance means the broker has significantly less protection should we default. For this reason, participants are required to maintain the margin at a minimum level, called the *maintenance margin*.
- If the margin balance falls below the maintenance margin, the broker would make a *margin call*, requesting additional margin to bring the margin balance to the level of initial margin.

Margins and Marking to Market (cont'd)

- If we failed to post additional margin, the broker would close the futures position (long/short position) and return the remaining margin to the corresponding party.

Comparing Futures and Forward Prices

- *If the interest rate were not random*, then forward and future prices would be the same. (see supplementary "forwardandfutures.pdf")
- If the interest rate is random and positively (negatively) correlated with the future price, the future price will be higher (less) than the price on an otherwise identical forward contract.

Comparing Futures and Forward Prices (cont'd)

- In general, the difference between the price of the short-lived forward and futures contract is small. However, for long-lived contracts, the difference can be significant.

Arbitrage in Practice: S&P 500 Index Arbitrage

- See P.143 to understand how the arbitrage can be achieved.

Quanto Index Contracts

- Contract specifications of Nikkei 225 index futures contract

FIGURE 5.2

Specifications for the Nikkei 225 index futures contract.

Underlying	Nikkei 225 Stock Index
Where traded	Chicago Mercantile Exchange
Size	$5 × Nikkei 225 Index
Months	March, June, September, December
Trading ends	Business day prior to determination of settlement price
Settlement	Cash-settled, based upon opening Osaka quotation of the Nikkei 225 index on the second Friday of expiration month

Quanto Index Contracts (cont'd)

- The Nikkei 225 futures contact can eliminate the exchange rate risk for the US investors by using dollars to settle instead of yen.
- The contract insulates investors from currency risk, permitting them to speculate solely on whether the underlying asset rises or falls. This kind of contract is called *quanto*.

