Tutorial - Class Activity

24 October, 2018 (Solution)

Problem 1

The current exchange rate is 0.42 British pounds per Australian dollar.

A pound-denominated European Australian dollar put has a strike price of 0.4 pounds and a premium of 0.0133 pounds. The put expires in 1 year.

A continuously compounded interest rate available on British pounds is 8%. The continuously compounded interest rate available on Australian dollars is 7%.

Calculate the value of an Australian dollar-denominated European British pound put that has a strike price of 2.5 Australian dollars and expires in 1 year.

Solution

The price of Australian dollar-denominated European British pound call is given by

$$\begin{aligned} C_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) &= \left(\frac{1}{0.42}\right)(2.5)\, P_{Pound}\left(0.42, \frac{1}{2.5}, 1\right) \\ &= \left(\frac{1}{0.42}\right)(2.5)(0.0133) \\ &= \text{AUD } 0.07917. \end{aligned}$$

By the put-call parity,

$$\begin{aligned} C_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) - P_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) &= \frac{1}{0.42}e^{-0.08} - 2.5e^{-0.07} \\ 0.07917 - P_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) &= \frac{1}{0.42}e^{-0.08} - 2.5e^{-0.07} \\ P_{AUD}\left(\frac{1}{0.42}, 2.5, 1\right) &= \text{AUD } 0.2123. \end{aligned}$$

Problem 2

The cum-dividend price of a stock is \$58 just before a dividend of \$3 is to be paid. The stock will also pay a dividend of \$2 in 9 months. The continuously compounded risk-free interest rate is 10% per annum.

The table below describes the strike prices and time unit maturity (T) in years for 5 different American call options on the stock.

Option	Strike Price	T (in years)
A	40	1.5
B	50	1.5
C	50	1
D	52	1
E	59	0.75

Determine which of the options **might be optimal** to be exercised now and which of the options should not be optimal to be exercised now.

Solution

Early exercise should not occur if the interest on the strike price exceeds the value of the dividends obtained through early exercise:

No early exercise if: $K - Ke^{-r(T-t)} > PV_{t,T}$(dividends).

The present value of the dividends is:

$PV_{t,T}$(dividends) = $3 + 2\,e^{-10\%(0.75)} = 4.86$.

The interest cost of paying the strike price early is shown in the rightmost column below:

Option	Strike Price	T (in years)	$K - Ke^{-rT}$
A	40	1.5	5.57
B	50	1.5	6.96
C	50	1	4.76
D	52	1	4.95
E	59	0.75	4.26

Only Option C and Option E have interest on the strike price that is less than the present value of the dividends of \$4.86. Option E is not in the money though, because its strike price exceeds the stock price of \$58, so it is not optimal to exercise option E. Therefore, Option C is the only option for which early exercise **might be** optimal.

Problem 3

Three European put options expire in 1 year. The put options have the same underlying asset, but they have different strike prices and premiums.

Put Option	A	B	C
Strike	\$50.00	\$55.00	\$61.00
Premium	\$3.00	\$7.00	\$11.00

The continuously compounded annual risk-free interest rate is 11%.

a. What no-arbitrage property is violated?

b. What spread position would you use to effect arbitrage?

c. Demonstrate that the spread position is an arbitrage.

Solution

(a)

The prices of the options violate the following inequality

$$\frac{P(K_2)-P(K_1)}{K_2-K_1} \le \frac{P(K_3)-P(K_2)}{K_3-K_2}$$

Because:

$$\frac{7-3}{55-50} > \frac{11-7}{61-55}$$
$$\frac{4}{5} > \frac{4}{6}$$

(b)

The above violated inequality can be rewritten as

$$\frac{P(55)-P(50)}{55-50} > \frac{P(61)-P(55)}{61-55}$$
$$6\big(P(55)-P(50)\big) > 5\big(P(61)-P(55)\big)$$
$$0 > 6P(50)-11P(55)+5P(61).$$

The arbitrage profit can be obtained by using the *asymmetric butterfly spread* with the following transactions:

Buy 6 of the 50-strike put options

Sell 11 of the 55-strike put options

Buy 5 of the 61-strike put options

(c)

		$t = 1$ year			
Transaction	**$t = 0$**	$S_1 < 50$	$50 \le S_1 \le 55$	$55 < S_1 \le 61$	$61 < S_1$
Buy 6 of $P(50)$	$-6(3.00)$	$6(50 - S_1)$	0.00	0.00	0.00
Sell 11 of $P(55)$	$11(7.00)$	$-11(55 - S_1)$	$-11(55 - S_1)$	0.00	0.00
Buy 5 of $P(61)$	$-5(11.00)$	$5(61 - S_1)$	$5(61 - S_1)$	$5(61 - S_1)$	0.00
Total	4.00	0.00	$6S_1 - 300$	$305 - 5S_1$	0.00

where $P(K)$ is the price of the K-strike put option.

This strategy has strictly positive cash inflow at $t = 0$, and has a nonnegative payoff for all possible values S_1 of at $t = 1$ year. Therefore, this is an arbitrage strategy.