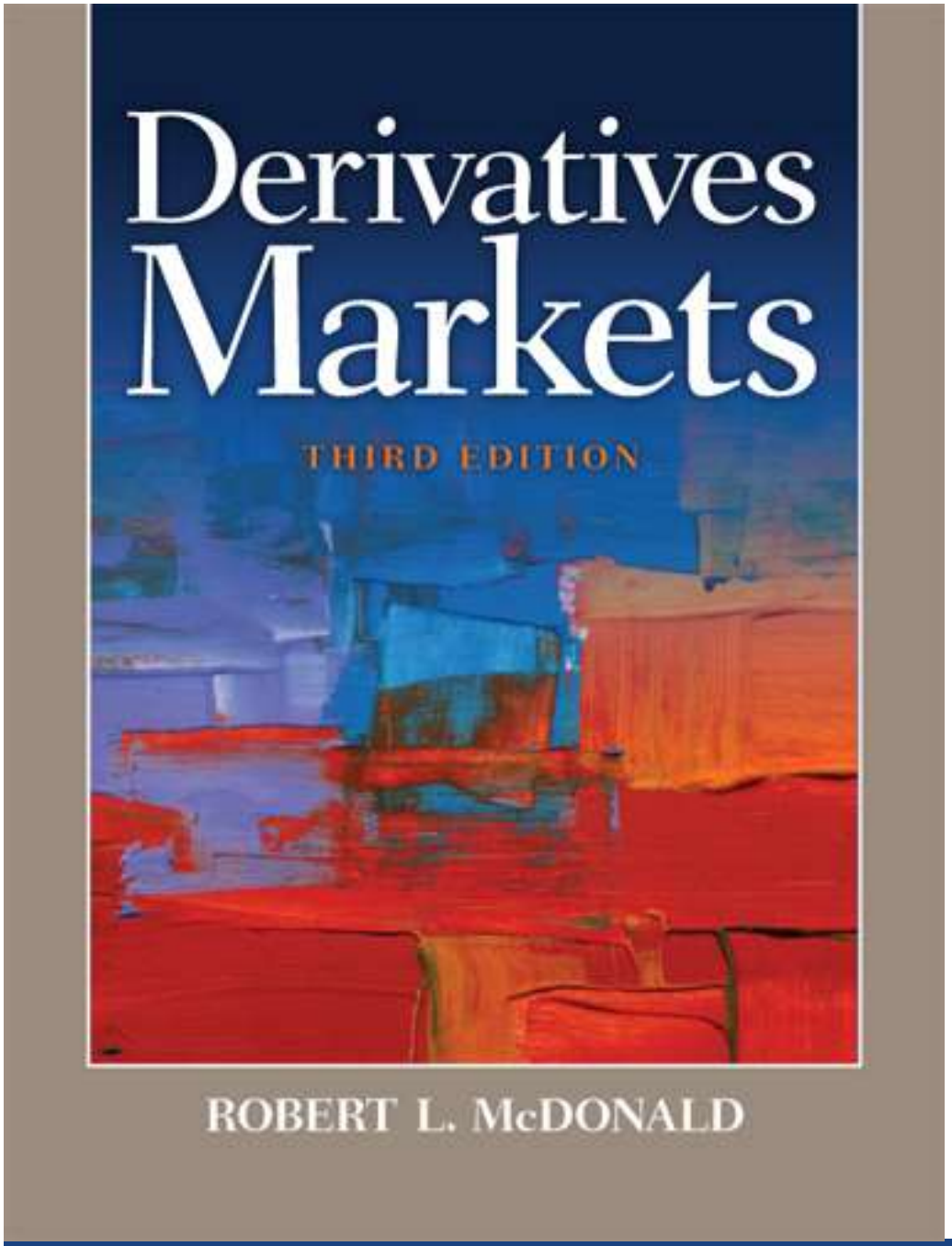


Chapter 7
(Chapter 8 in the textbook)

Swaps

Points to Note

1. How does the swap compare with the forward contracts? See P.3 – 6.
2. What are the meaning of the long and short positions? See P. 7.
3. What is the difference between the physical settlement and financial settlement? See P. 7 – 13.
4. Understand TWO different ways in which the dealer uses to hedge his swap positon. See P.14 – 16.
5. The market value of the a swap. See P.17 – 19.
6. Computing the swap rate. See P.20 – 25.

Introduction to Swaps

- A **swap** is a contract calling for an exchange of payments, on one or more dates, determined by the difference in two prices.
- A swap provides a means to hedge a *stream* of risky payments.
- A single-payment swap is the same thing as a cash-settled forward contract.

An Example of a Commodity Swap

- An industrial producer, IP Inc., needs to buy 100,000 barrels of oil 1 year from today and 2 years from today.
- The forward prices for deliver in 1 year and 2 years are $110 and $111/barrel.
- The 1- and 2-year zero-coupon bond yields are 6% and 6.5% (annual effective interest rate).

An Example of a Commodity Swap (cont'd)

- IP can guarantee the cost of buying oil for the next 2 years by entering into long forward contracts for 100,000 barrels in each of the next 2 years. The PV of this cost per barrel is

$$\frac{\$110}{1.06}+\frac{\$111}{1.065^2}=\$201.638$$

- Thus, IP could pay an oil supplier $201.638, and the supplier would commit to delivering one barrel in each of the next two years.
- A **prepaid swap** is a single payment today to obtain *multiple* deliveries in the future.

An Example of a Commodity Swap (cont'd)

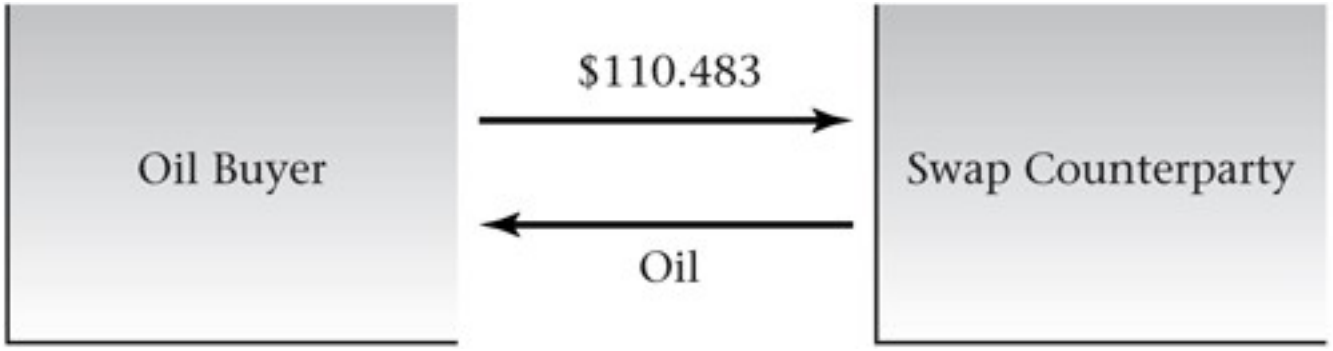
- With a prepaid swap, the buyer might worry about the resulting credit risk. Therefore, a more attractive solution is to defer payment until the oil is delivered, while still fixing the total price.
- Any payments that have a present value of $201.638 are acceptable. Typically, a swap will call for equal payments in each year.
  - For example, the payment per year per barrel, x, will have to be $110.483 to satisfy the following equation

$$\frac{x}{1.06}+\frac{x}{1.065^2}=\$201.638$$

- We then say that the 2-year swap price is $110.483.

Physical Versus Financial Settlement

- **Physical settlement** of the swap



Fixed swap rate payer (oil buyer) is said to take a **long position** of the swap while the (fixed rate receiver) swap counterparty is in the **short position** of the swap.

Physical Versus Financial Settlement (cont'd)

- **Financial settlement** of the swap
  - The oil buyer, IP, pays the swap counterparty the difference between $110.483 and the spot price, and the oil buyer then buys oil at the spot price.
  - If the difference between $110.483 and the spot price is negative, then the swap counterparty pays the buyer.

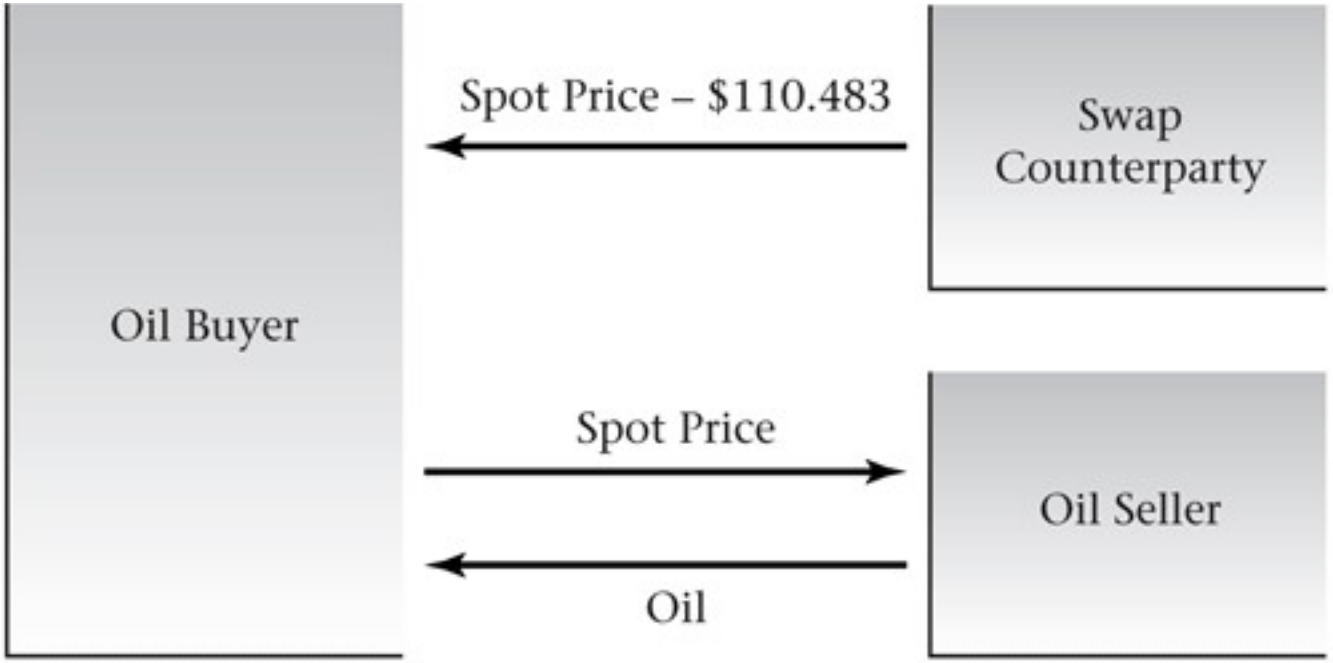
Physical Versus Financial Settlement (cont'd)

- Whatever the market price of oil, the net cost to the buyer is the swap price, $110.483.

$$\underbrace{\text{Spot price} - \text{swap price}}_{\text{Swap payment}} - \underbrace{\text{Spot price}}_{\text{Spot purchase of oil}} = -\text{Swap price}$$

Physical Versus Financial Settlement (cont'd)

- The results for the buyer are the same whether the swap is settled physically or financially. In both cases, the net cost to the oil buyer is $110.483.



Physical Versus Financial Settlement (cont'd)

- If the swap has the **notional amount** of 100,000, meaning that 100,000 barrels is used to determine the magnitude of the payments when the swap is settled financially.
- Fig. 8.3 shows a **term sheet** for an oil swap. Term sheets are commonly used by broker-dealers to succinctly convey the important terms of a financial transaction.

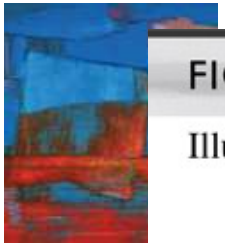


FIGURE 8.3

Illustrative example of the terms for an oil swap based on West Texas Intermediate (WTI) crude oil.

Fixed-Price Payer:	Broker-dealer
Floating-Price Payer:	Counterparty
Notional Amount:	100,000 barrels
Trade Date:	April 18, 2011
Effective Date:	July 1, 2011
Termination Date:	September 31, 2011
Period End Date:	Final Pricing Date of each Calculation Period as defined in the description of the Floating Price.
Fixed Price:	110.89 USD per barrel
Commodity Reference Price:	OIL-WTI-NYMEX
Floating Price:	The average of the first nearby NYMEX WTI Crude Oil Futures settlement prices for each successive day of the Calculation Period during which such prices are quoted
Calculation Period:	Each calendar month during the transaction
Method of Averaging:	Unweighted
Settlement and Payment:	If the Fixed Amount exceeds the Floating Amount for such Calculation Period, the Fixed Price Payer shall pay the Floating Price Payer an amount equal to such excess. If the Floating Amount exceeds the Fixed Amount for such Calculation Period, the Floating Price Payer shall pay the Fixed Price Payer an amount equal to such excess.
Payment Date:	5 business days following each Period End Date

Physical Versus Financial Settlement (cont'd)

- Swaps are nothing more than forward contracts coupled with borrowing and lending money
  - Consider the swap price of $110.483/barrel. Relative to the forward curve price of $110 in 1 year and $111 in 2 years, we are overpaying by $0.483 in the first year, and we are underpaying by $0.517 in the second year.
  - Thus, by entering into the swap, we are lending the counterparty money for 1 year. The interest rate on this loan is

$$0.517 / 0.483 - 1 = 7\%$$

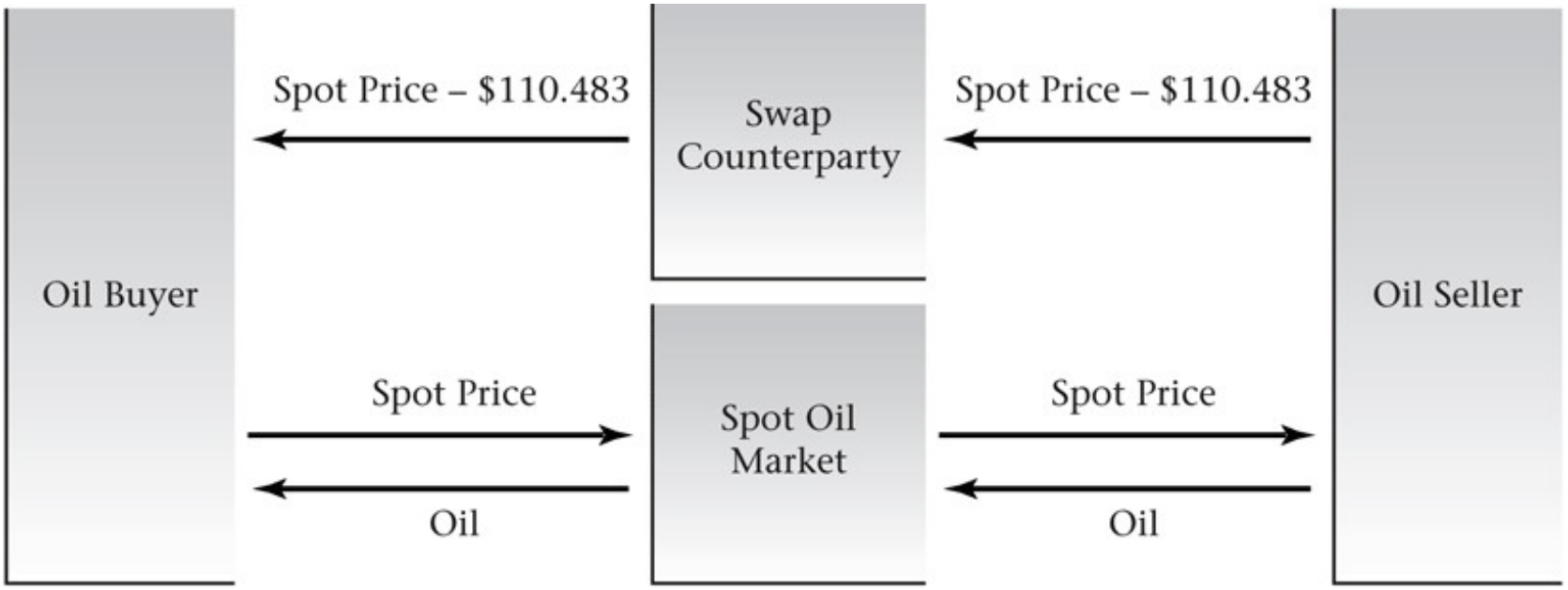
  - Given 1- and 2-year zero-coupon bond yields of 6% and 6.5%, 7% is the 1-year implied forward yield from year 1 to year 2.
- If the deal is priced fairly, the interest rate on this loan should be the implied forward interest rate.

The Swap Counterparty

- The swap counterparty is a dealer, who is, in effect, a broker between buyer and seller.
- The dealer can hedge the oil price risk resulting from the swap in several ways.
- The fixed price paid by the buyer, usually, exceeds the fixed price received by the seller. This price difference is a bid-ask spread, and is the dealer's fee.
- The dealer bears the credit risk of both parties, but is not exposed to price risk.

The Swap Counterparty (cont'd)

- The situation where the dealer matches the buyer and seller is called a **back-to-back transaction** or "matched book" transaction.



The Swap Counterparty (cont'd)

- Alternatively, the dealer can serve as counterparty and hedge the transaction by entering into long forward or futures contracts.

TABLE 8.1 Positions and cash flows for a dealer who has an obligation to receive the fixed price in an oil swap and who hedges the exposure by going long year 1 and year 2 oil forwards.

Year	Payment from Oil Buyer	Long Forward	Net
1	\$110.483 – year 1 spot price	Year 1 spot price – \$110	\$0.483
2	\$110.483 – year 2 spot price	Year 2 spot price – \$111	–\$0.517

  - Note that the net cash flow for the hedged dealer is a loan, where the dealer receives cash in year 1 and repays it in year 2.
  - Thus, the dealer also has interest rate exposure (which can be hedged by using Eurodollar contracts or forward rate agreements).

The Market Value of a Swap

at inception开始时

- The market value of a swap is zero at inception.
- Once the swap is struck, however, its market value will generally no longer be zero because
  - The forward prices for oil and interest rates will change over time.
  - Even if prices do not change, the market value of swaps can change over time due to the implicit borrowing and lending.
- A buyer wishing to exit the swap could negotiate terms with the original counterparty to eliminate the swap obligation or enter into an offsetting swap with the counterparty offering the best price.

The Market Value of a Swap (cont'd)

- The original swap called for the oil buyer to pay the fixed price and receive floating; the offsetting swap has the buyer receive the fixed price and pay floating. The original obligation would be cancelled except to the extent that the fixed prices are different.
- The market value of the swap in the perspective of the **long position** is

  PV of the payments with the amount of (New swap rates – Original swap rates).

The Market Value of a Swap (cont'd)

Example

- Suppose the forward curve for oil rises by $2 in years 1 and 2. Thus, the year 1 forward price becomes $112 and the year-2 forward price becomes $113.
- Assuming interest rates are unchanged, the new swap price is $112.483 (*Verify!*).
- The market value of the swap in the perspective of the **long position** is

$$\frac{(112.483-110.483)}{1.06}+\frac{(112.483-110.483)}{(1.065)^2}=\$3.650$$

Computing the Swap Rate

- Notation
  - Suppose there are n swap settlements, occurring on dates t_i, $i = 1,\ldots, n$.
  - The forward prices on these dates are given by F_{0,t_i}
  - The price of a zero-coupon bond maturing on date t_i is $P(0, t_i)$.
  - The fixed swap rate is R.
- If the buyer at time zero were to enter into forward contracts to purchase one unit on each of the n dates, the present value of payments would be the present value of the forward prices, which equals the price of the prepaid swap:

$$\text{Prepaid swap} = \sum_{i=1}^{n} F_{0,t_i} P(0,t_i)$$

Computing the Swap Rate (cont'd)

- We determine the fixed swap price, *R*, by requiring that the present value of the swap payments equal the value of the prepaid swap

$$\sum_{i=1}^{n} RP(0,t_i) = \sum_{i=1}^{n} F_{0,t_i} P(0,t_i) \tag{8.2}$$

- Equation (8.2) can be rewritten as

$$R = \frac{\sum_{i=1}^{n} P(0,t_i) F_{0,t_i}}{\sum_{i=1}^{n} P(0,t_i)} \tag{8.3}$$

where $\sum_{i=1}^{n} P(0,t_i) F_{0,t_i}$ is the present value of payments implied by the strip of forward rates, and $\sum_{i=1}^{n} P(0,t_i)$ is the present value of a $1 annuity.

Computing the Swap Rate (cont'd)

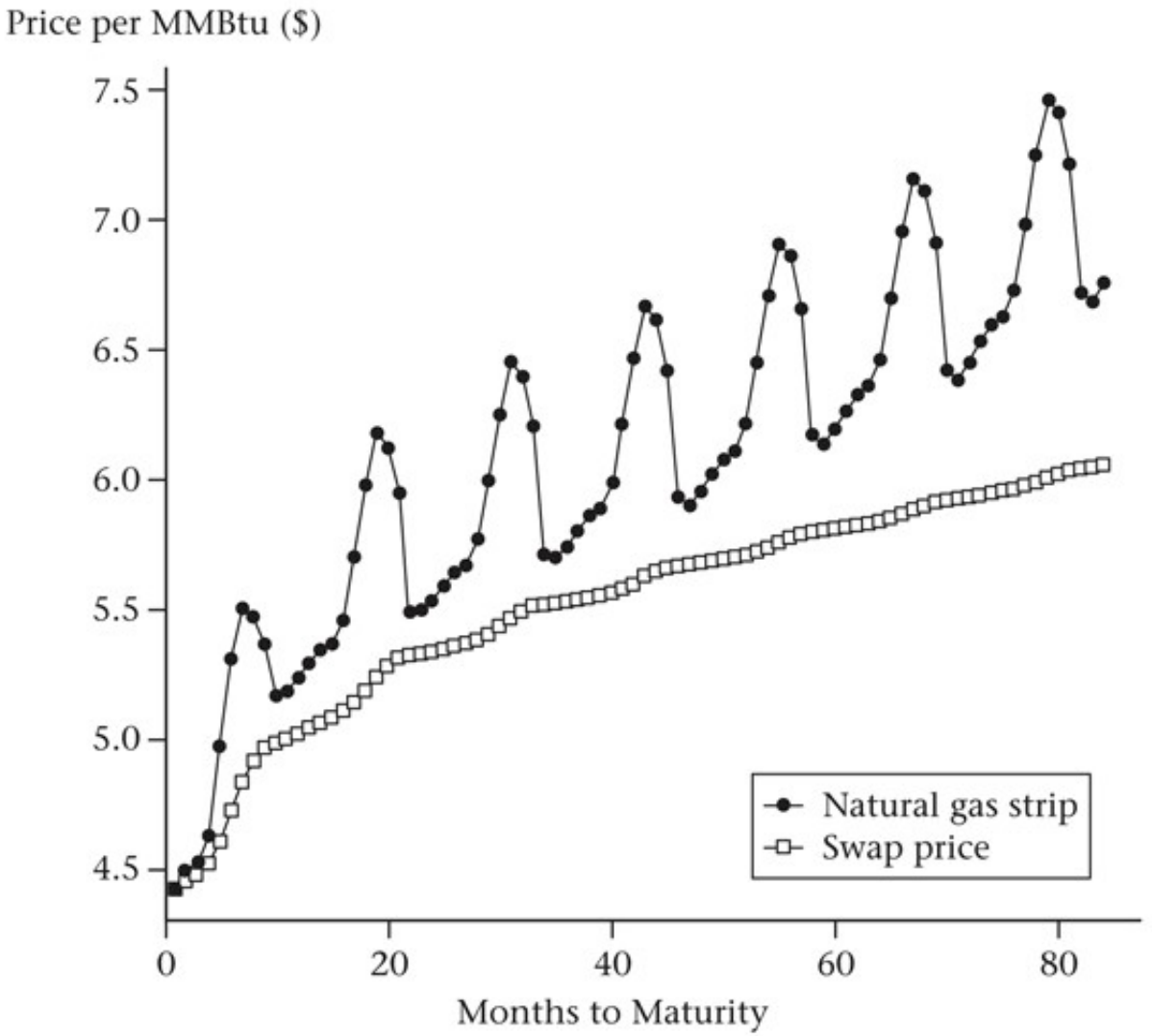
- We can rewrite equation (8.3) to make it easier to interpret

$$R = \sum_{i=1}^{n} \left[\frac{P(0,t_i)}{\sum_{j=1}^{n} P(0,t_j)} \right] F_{0,t_i}$$

- Thus, the fixed swap rate is as a weighted average of the forward prices, where zero-coupon bond prices are used to determine the weights.

FIGURE 8.6

Natural gas swap curve, June 2, 2010. The swap curve displays the fixed price for a natural gas swap beginning June 2010 and continuing, with monthly settlement, for the number of months specified on the x-axis.



Swaps With Variable Quantity and Prices

- A buyer with seasonally varying demand (e.g., someone buying gas for heating) might enter into a swap, in which *quantities* vary over time.
- Consider a swap in which the buyer pays RQ_{t_i}, for Q_{t_i} units of the commodity. The present value of these fixed payments (fixed per unit of the commodity) must equal the prepaid swap price

$$\sum_{i=1}^{n} Q_{t_i} F_{0,t_i} P(0,t_i) = \sum_{i=1}^{n} Q_{t_i} RP(0,t_i)$$

- Solving for R gives

$$R = \frac{\sum_{i=1}^{n} Q_{t_i} P(0,t_i) F_{0,t_i}}{\sum_{i=1}^{n} Q_{t_i} P(0,t_i)}$$

Swaps With Variable Quantity and Prices (cont'd)

- It is also possible for *prices* to be time-varying
  - For example, we let the summer swap price be denoted by R_s and the winter price by R_w, then the summer and winter swap prices can be any prices for which the value of the prepaid swap equals the present value of the fixed swap payment:

$$R_s \sum_{i \in \text{summer}}^{n} P(0,t_i) Q_{t_i} + R_w \sum_{i \in \text{winter}}^{n} P(0,t_i) Q_{t_i} = \sum_{i=1}^{n} P(0,t_i) Q_{t_i} F_{0,t_i}$$

Once we fix one of R_s and R_w, the equation will give us the other.