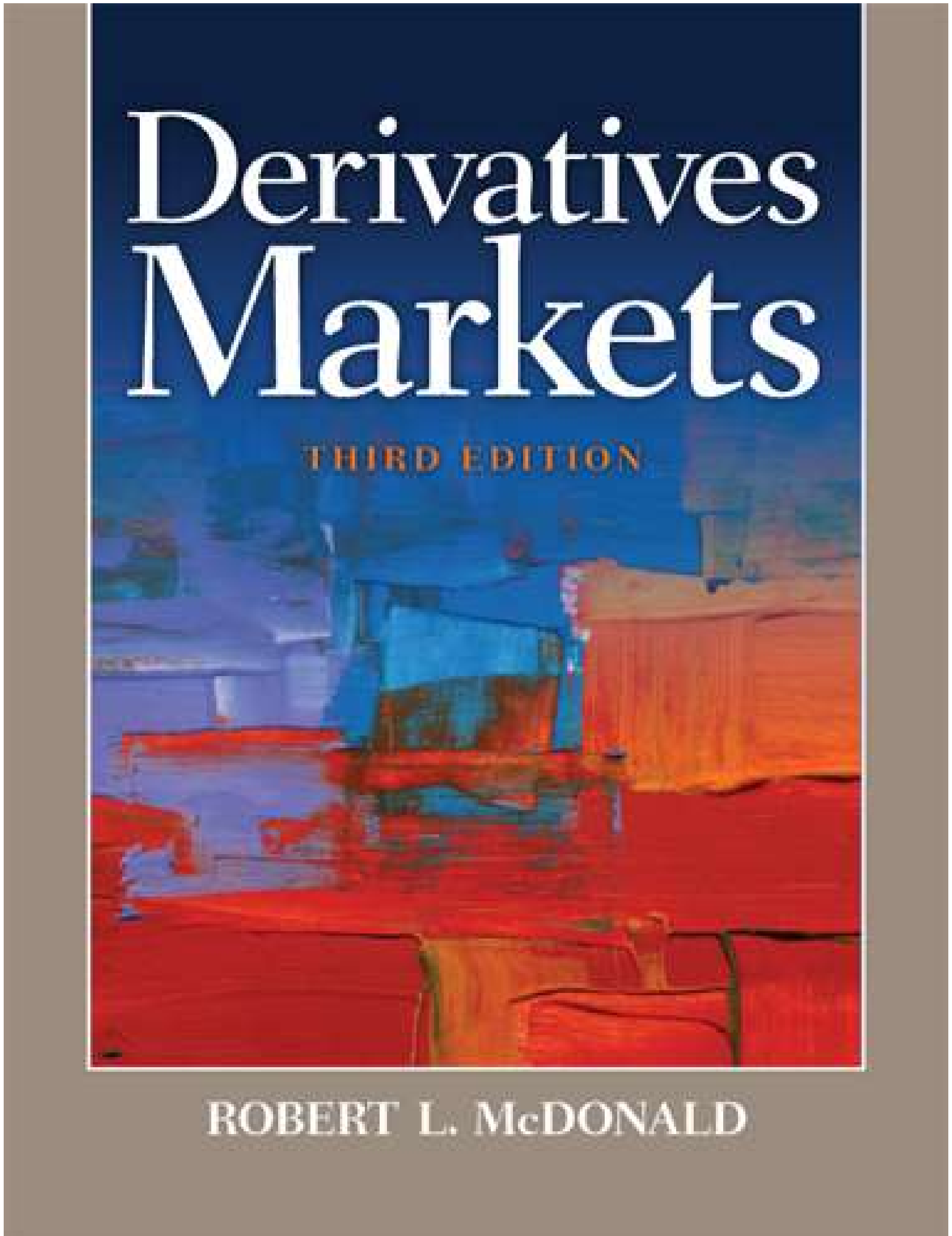


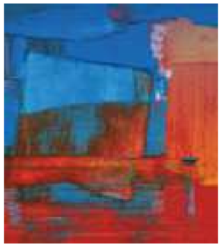
Chapter 6

(Chapter 7 in the textbook)

Bond Basics

Points to Note

1. Definition of $r_t(t_1,t_2)$, see P.4.
2. What is the relationship between the bond price $P(0, n)$ and $r(0,n)$? See P.5.
3. How to find YTM from the zero coupon price? See P.7.
4. How to find the implied forward rate? See P.8 – 9.
5. How to find the implied forward zero-coupon price? See P.10.
6. Coupon bonds, see P.12.
7. Bootstrapping zero-coupon price from coupon bonds, see P.14 – 15.
8. Definition of continuously compounded yields $r^{cc}(0, t)$.

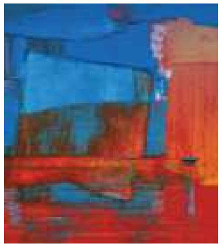


Bond Basics

- U.S. Treasury
  - Bills (<1 year), no coupons, sell at discount
  - Notes (1–10 years), Bonds (>10–30 years), with coupons
  - STRIPS (Separate Trading of Registered Interest and Principal Securities): claim to a single coupon or principal portion of a government bond, i.e., *zero-coupon bond*

Bond Basics (cont'd)

- Notation
  - $r_t\,(t_1,t_2)$: annual effective interest rate from time t_1 to t_2 prevailing at time t. If the interest rate is current –i.e., if $t = t_1$ - and if there is no risk of confusion, we will drop the subscript.
  - $P_{to}(t_1,t_2)$: price of a bond quoted at $t = t_0$ to be purchased at $t = t_1$ maturing at $t = t_2$
  - Yield to maturity: percentage increase in dollars earned from the bond



Bond Basics (cont'd)

- In general, the zero-coupon bond price that pays \$1 at year *n* is given by

$$P(0,n)=\frac{1}{\left[1+r(0,n)\right]^{n}}$$

where *r*(0, *n*) is called the annualized zero-coupon yield of the *n*-year zero-coupon bond.

- Zero-coupon bond price that pays C_t at year *t*:

$$C_t \times P(0,t)=\frac{C_t}{\left[1+r(0,t)\right]^{t}}$$

so, *P*(0, *t*) is a discount factor.

Bond Basics (cont'd)

- Zero-coupon bonds make a single payment at maturity

TABLE 7.1 Five ways to present equivalent information about default-free interest rates. All rates but those in the last column are effective annual rates.

Years to Maturity	(1) Zero-Coupon Bond Yield	(2) Zero-Coupon Bond Price	(3) One-Year Implied Forward Rate	(4) Par Coupon	(5) Continuously Compounded Zero Yield
1	6.00%	0.943396	6.00000%	6.00000%	5.82689%
2	6.50	0.881659	7.00236	6.48423	6.29748
3	7.00	0.816298	8.00705	6.95485	6.76586

Bond Basics (cont'd)

- One year zero-coupon bond: $P(0,1)=0.943396$
  - Pay \$0.943396 today to receive \$1 at t=1
  - Yield to maturity (YTM) = 1/0.943396 - 1 = 6% = $r(0,1)$

- Two year zero-coupon bond: $P(0,2)=0.881659$
  - YTM=1/0.881659 – 1

$$=0.134225 \quad =(1+r(0,2))^2 - 1$$

$$=> r(0,2)=0.065=6.5\%$$

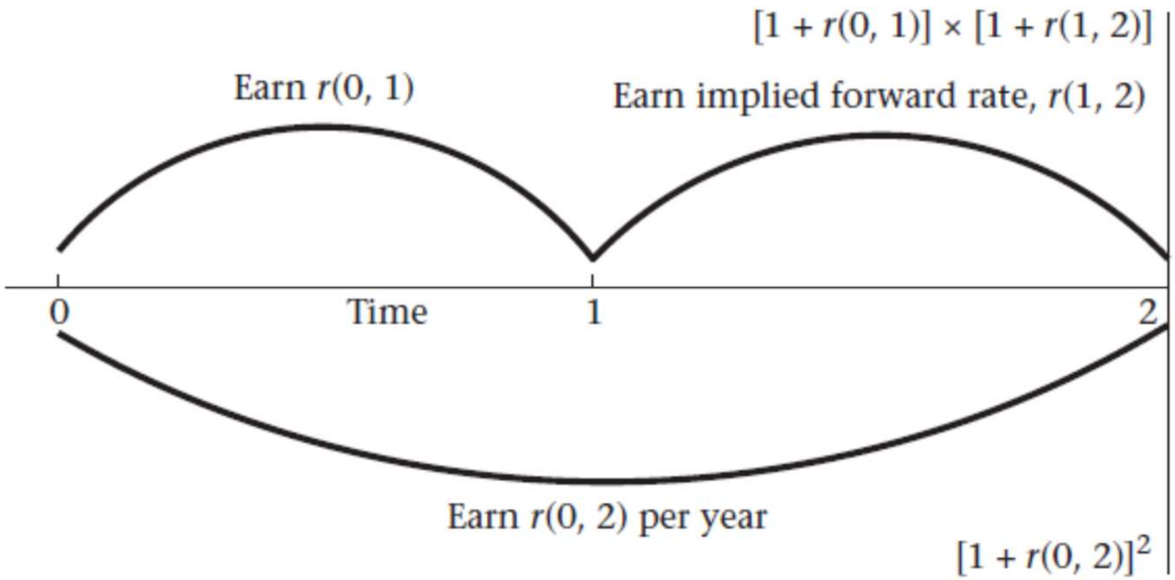
Bond Basics (cont'd)

- Yield curve: graph of annualized zero-coupon bond yields against time
- Implied forward rates
  - Implied interest rate for the future period
  - Suppose current one-year rate $r(0,1)$ and two-year rate $r(0,2)$
  - Current forward rate from year 1 to year 2, $r_0(1,2)$, must satisfy

$$[1 + r_0(0,1)][1 + r_0(1,2)] = [1 + r_0(0,2)]^2$$

FIGURE 7.1

An investor investing for 2 years has a choice of buying a 2-year zero-coupon bond paying $[1 + r_0(0, 2)]^2$ or buying a 1-year bond paying $1 + r_0(0, 1)$ for 1 year, and reinvesting the proceeds at the implied forward rate, $r_0(1, 2)$, between years 1 and 2. The implied forward rate makes the investor indifferent between these alternatives.



Bond Basics (cont'd)

- In general

$$[1+r_0(t_1,t_2)]^{t_2-t_1} = \frac{[1+r_0(0,t_2)]^{t_2}}{[1+r_0(0,t_1)]^{t_1}} = \frac{P(0,t_1)}{P(0,t_2)}$$

- The implied forward zero-coupon price, $P_0(t_1, t_2)$, from t_1 to t_2 is given by

$$P_0(t_1,t_2) = \frac{1}{[1+r_0(t_1,t_2)]^{t_2-t_1}} = \frac{[1+r_0(0,t_1)]^{t_1}}{[1+r_0(0,t_2)]^{t_2}} = \frac{P(0,t_2)}{P(0,t_1)}$$

It is simply the ratio of the zero-coupon bond prices maturing at t_2 and t_1. It is actually the forward price of a t_1 -year forward contract with the underlying of t_2 -year zero coupon bond.

Bond Basics (cont'd)

- Example 7.1
  - What are the implied forward rate $r_0(2,3)$ and forward zero-coupon bond price $P_0(2,3)$ from year 2 to year 3? (use Table 7.1)

$$r_0(2,3) = \frac{P(0,2)}{P(0,3)} - 1 = \frac{0.881659}{0.816298} - 1 = 0.0800705$$

$$P_0(2,3) = \frac{P(0,3)}{P(0,2)} = \frac{0.816298}{0.881659} = 0.925865$$

Bond Basics (cont'd)

- Coupon bonds
  - The price at time of issue of t of a bond maturing at time T that pays n coupons of size c at time t_i where $t_i = t + i(T - t)/n$ and maturity payment of $1

$$B_t(t,T,c,n) = \sum_{i=1}^{n} cP_t(t,t_i) + P_t(t,T)$$

  - A **par bond** has $B_t = 1$, so the **par coupon** (the coupon rate at which the bond will be priced at par) is given by

$$c = \frac{1 - P_t(t,T)}{\sum_{i=1}^{n} P_t(t,t_i)}$$

Bond Basics (cont'd)

- Coupon bonds (cont'd)
  - Suppose the bond makes m payments per year. Denoting the per-period yield to maturity as y_m, we have

$$B_t(t,T,c,n)=\sum_{i=1}^{n}\frac{c}{(1+y_m)^i}+\frac{1}{(1+y_m)^n}$$

It is common to compute the quoted annualized yield to maturity, y, as $y = m \times y_m$.

Bond Basics (cont'd)

- Zeros from Coupons
  - **Bootstrapping**: the procedure in which zero coupon bond prices are deduced from a set of coupon bond prices.
  - From Column (4) in Table 7.1, we have

$$1 = (1 + 0.06)P(0,1)$$

$$P(0,1) = 0.943396$$

The second par coupon bond gives us

$$1 = 0.0648423P(0,1) + 1.0648423P(0,2)$$

$$P(0,2) = 0.881659$$

Bond Basics (cont'd)

– Similarly, we find

$$1 = 0.0695485P(0,1) + 0.0695485P(0,2) + 1.0695485P(0,3)$$

$$P(0,3) = 0.816298$$

Bond Basics (cont'd)

- Continuously Compounded Yields
  - In general, if we have a zero-coupon bond paying $1 at maturity, we can write its price in terms of an annualized continuously compounded yield, $r^{cc}(0,t)$, as

$$P(0,t) = e^{-r^{CC}(0,t)t}$$