Revision

Use of F.D. (Financial Derivative)

1) Protection

Protection ↓

a) Floor : Long the stock + Long a put

b) Cap : Short sell the stock + Long a call

c) Covered call : Sell a call + Long the underlying

d) Covered put : Sell a put + short the underlying

not very effective

Put - call parity ← synthetic forward (forward price = K) ← Not fair

$$\boxed{Call(K,t) - Put(K,t)} = PV(F_{0,T} - K)$$

Long

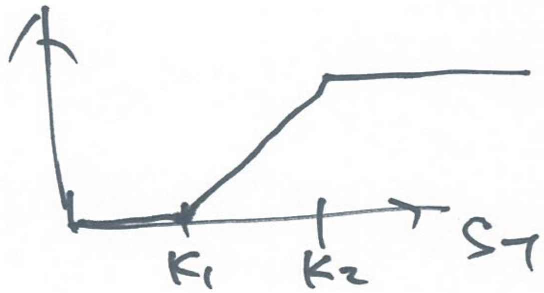
↑ cost for the forward contract $\neq 0$

↳ $= 0$ when $K = F_{0,T}$

2) Speculation

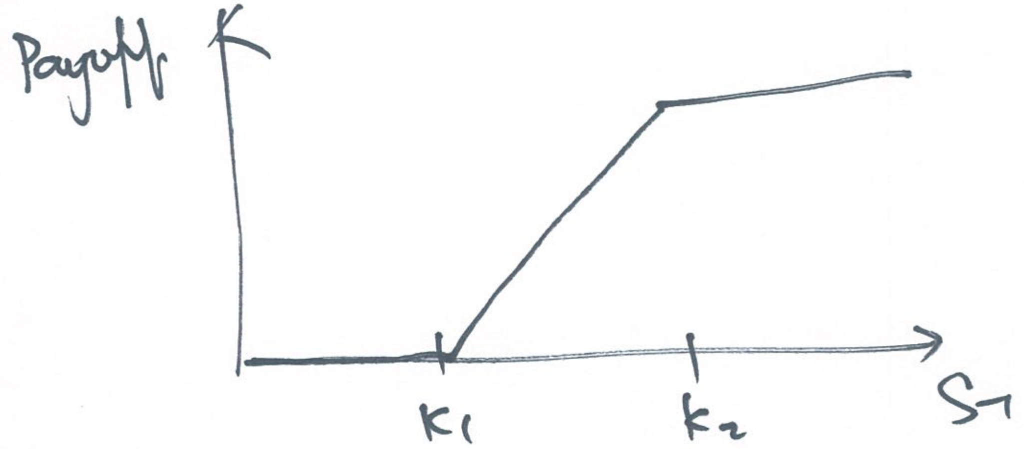
- spread strategy
  - vertical spread — Bull spread / Bear spread
    - same type of option (call / put)
    - same underlying.
    - same maturity
    - different strike

Payoff



Bull spread →
↳ call Bull spread
↳ put Bull spread

Call Bull Spread



Long T-year K_1 - Call

\+ Short T-year K_2 - call

$$\text{Payoff} = \max(S_T - K_1, 0) - \max(S_T - K_2, 0)$$

$$\text{Profit @ } T = \text{Payoff} - PV(\underbrace{\boxed{\text{call}(K_1) - \text{Call}(K_2)}}_{\text{cost}})$$

Put Bull Spread ← use put options only. Long K_1-put + Short K_2-put

$$\text{Call} = \text{put} + PV(F_{0,T} - K)$$

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TABLE 3.5 Profit at expiration from purchase of 40-strike call and sale of 45-strike call.

$-FV(C(40) - C(45))$

Stock Price at Expiration	Purchased 40-Call	Written 45-Call	Premium Plus Interest	Total
$35.0	$0.0	$0.0	−$1.85	−$1.85
37.5	0.0	0.0	−1.85	−1.85
40.0	0.0	0.0	−1.85	−1.85
42.5	2.5	0.0	−1.85	0.65
45.0	5.0	0.0	−1.85	3.15
47.5	7.5	−2.5	−1.85	3.15
50.0	10.0	−5.0	−1.85	3.15

profit @ T

Buy a 40-call

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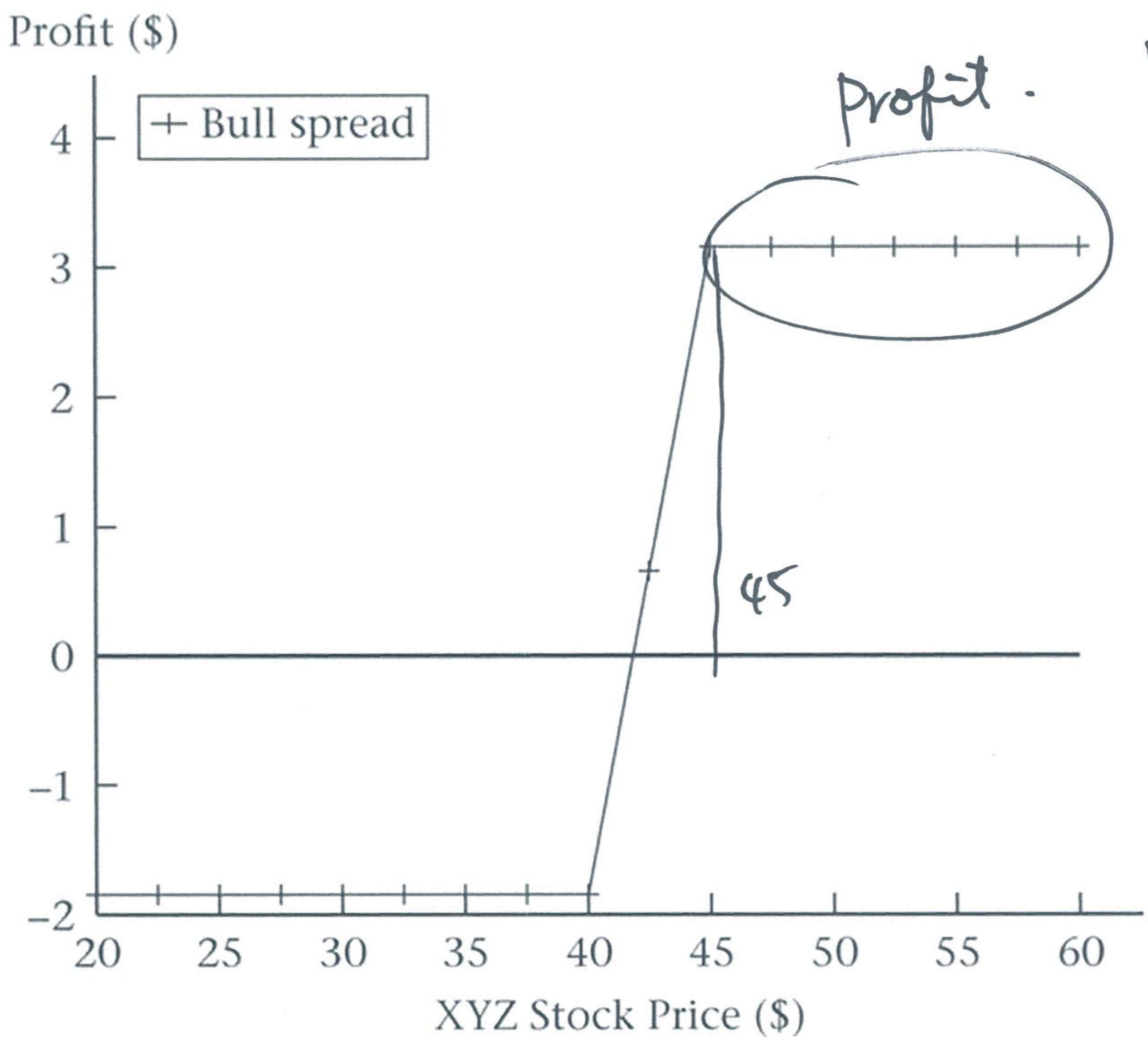
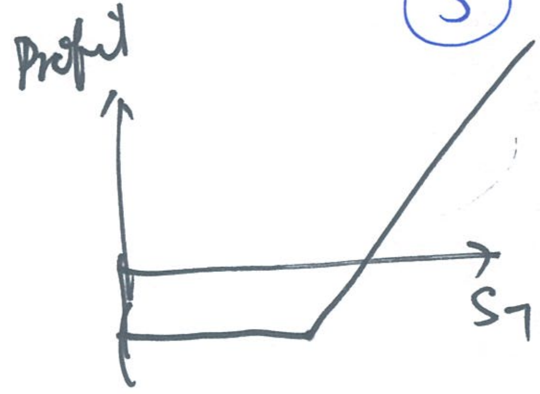
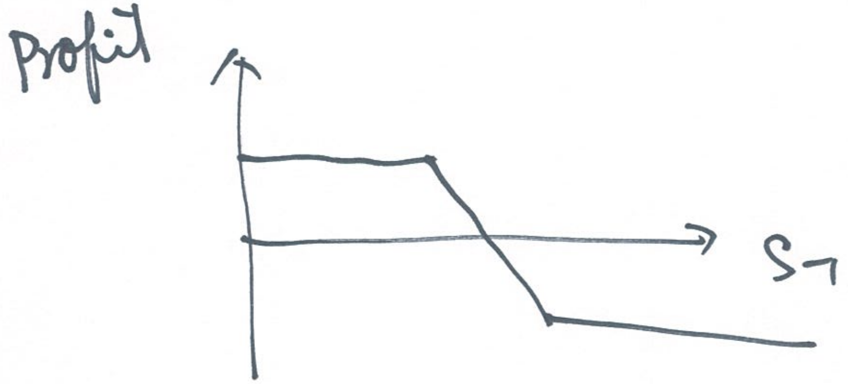




Bear spread Sell K_1- call + purchase K_2- call



(horizontal)
<u>Calendar spread</u>

Sell T_1-year K_1- call + purchase T_2-year K_1- call

- same type of option
- same underlying
- same strike
- different maturity

~~Dig~~ Diagonal Spread

↳ Different Strike + maturity

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Box spread (Borrowing + Lending)

Synthetic Long forward (K_1) + Synthetic short forward (K_2)

$\Updownarrow$

Long a call (K_1) + short a put (K_1)

$\Updownarrow$

short ~~Long~~ a call (K_2) + ~~short~~ Long a put (K_2)

	$t=0$	$t=T$
Long forward (K_1)	$-C(K_1)+P(K_1)$	$S_T - K_1$
short forward (K_2)	$+C(K_2) - P(K_2)$	$-(S_T - K_2)$

Lending

Interest rate

Return

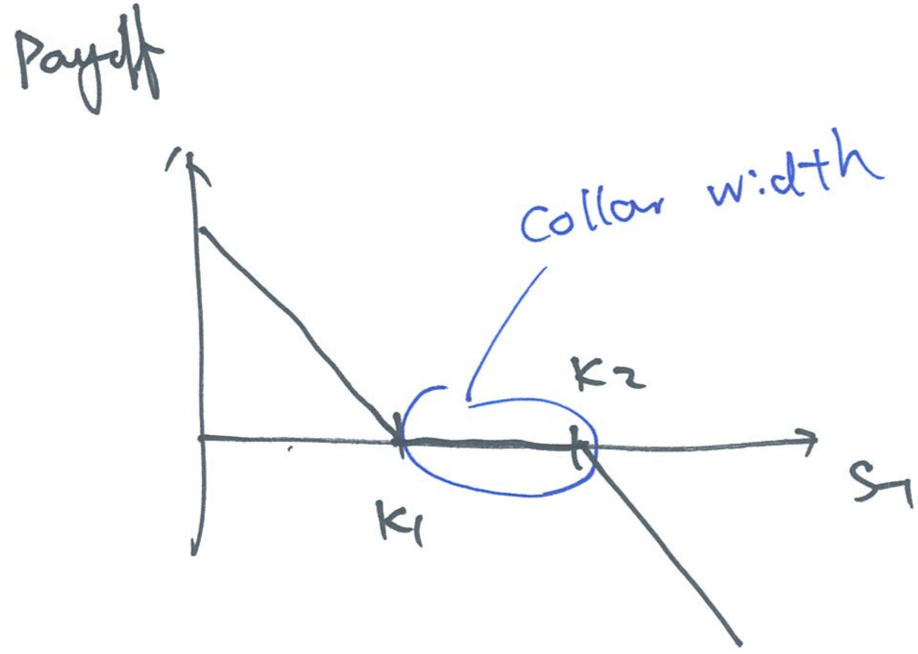
$C(K_2) < C(K_1)$

$<0 \Leftarrow \boxed{C(K_2) - C(K_1)} + (P(K_1) - P(K_2))$ $K_2 - K_1$

<0 <0

deterministic

Collar

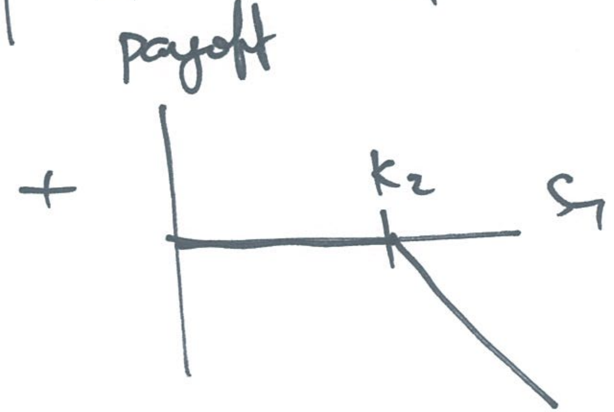


Use K_1- ~~call~~ put and K_2- ~~put~~ call

$\Rightarrow$

← Long K_1-put

+



Short K_2-call

Collar = Long K_1-put + Short K_2-call

Special case : $K_1 = K_2$, Collar $\Rightarrow$ synthetic forward (K_1)

$S_0 = K_1$

stock + a collar

Long a put (K_1) − Call (K_2)

floor

cost of protection

Floor

reduce cost of floor

collar (width $K_2 - K_1$)

Stock + Long put (K_1) + short a call (K_2)

= Collated stock

Callated stock

$S_0 = 40$

Stock + Long put (40) − Short $\boxed{\text{call } (41.72)}$

\$1.98   1.98

cost of insurance = 0

Puzzle : 0 cost insurance ??

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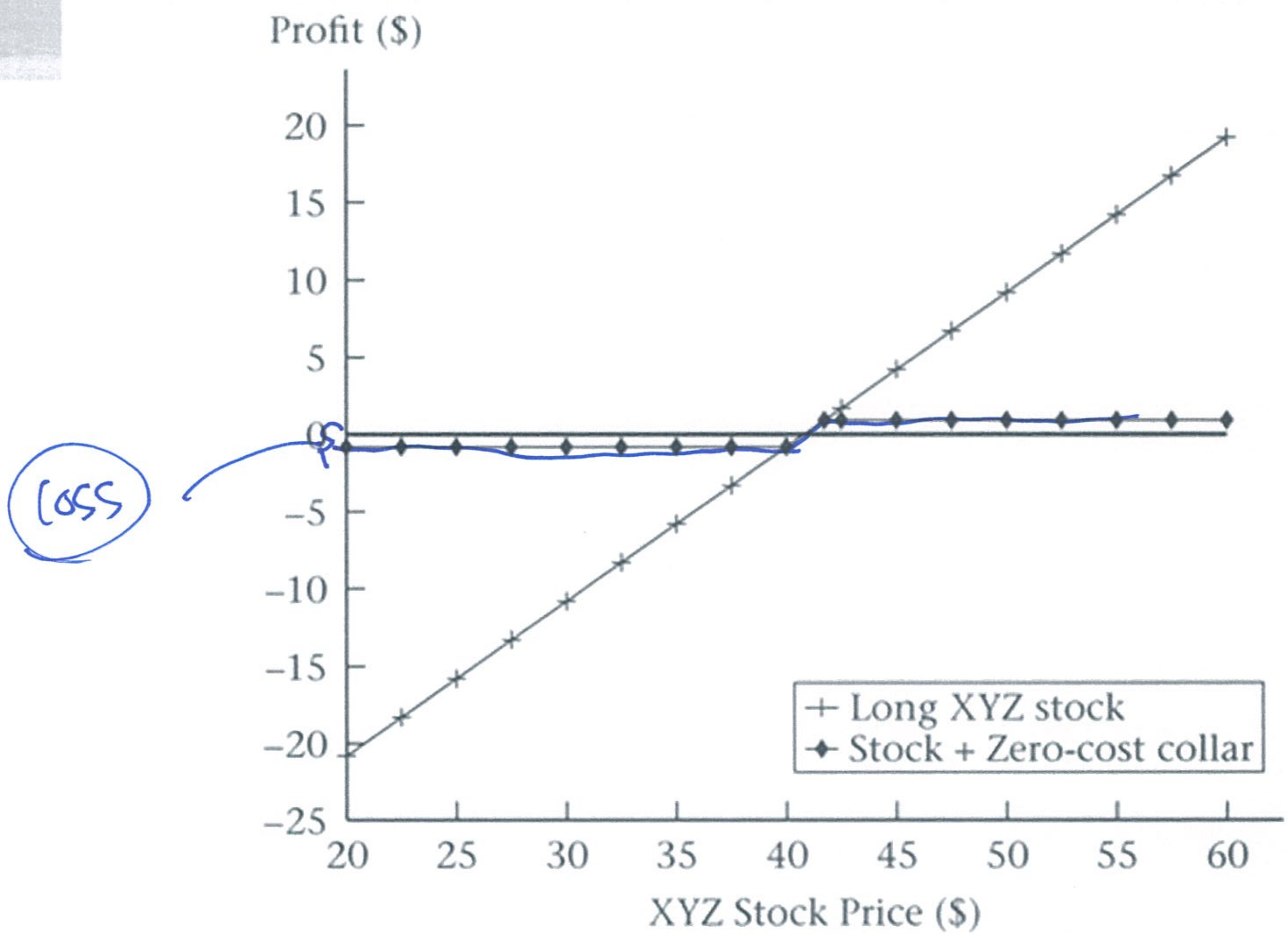
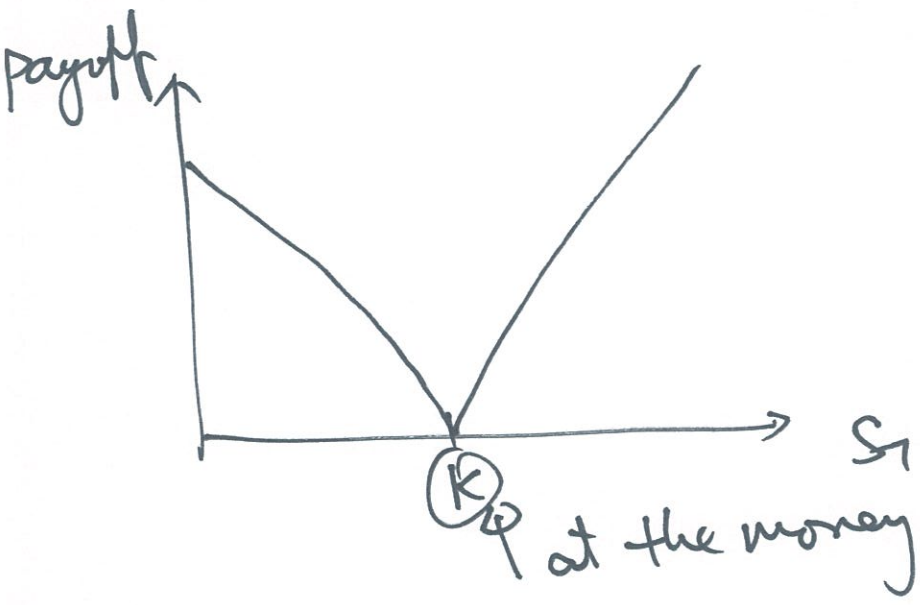
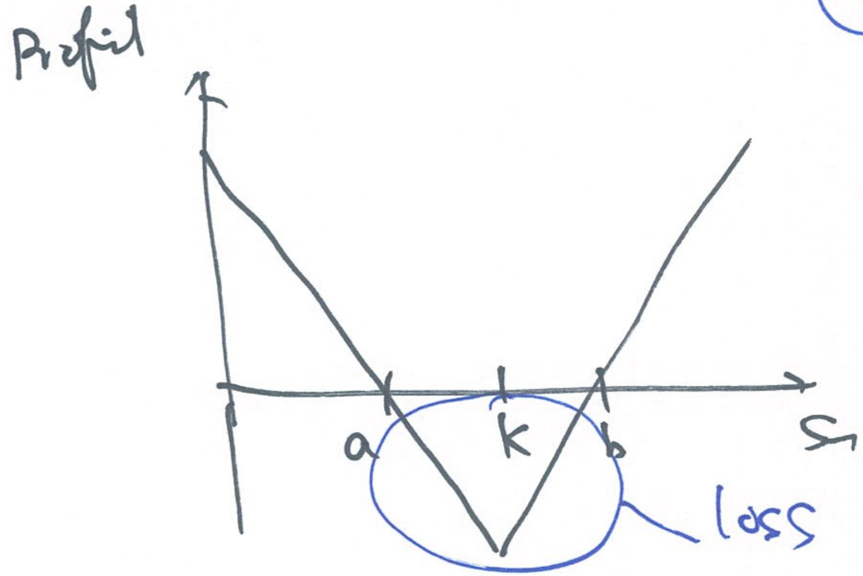
FIGURE 3.9

Zero-cost collar on XYZ, created by buying XYZ at $40, buying a 40-strike put with a premium of $1.99, and selling a 41.72-strike call with a premium of $1.99.



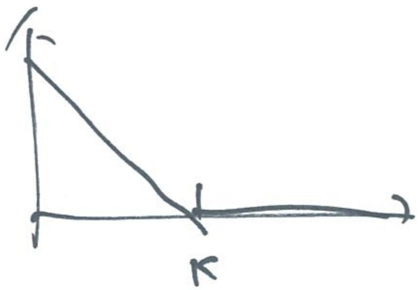
straddle



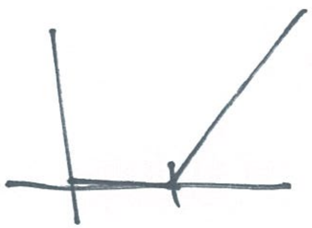


$S_T > b$ or $S_T < a$

$\Downarrow$



Long K-put

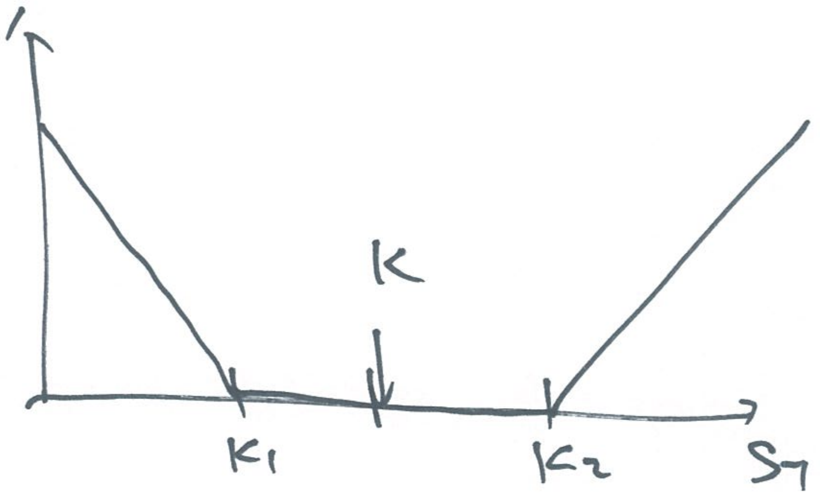


Long K-call

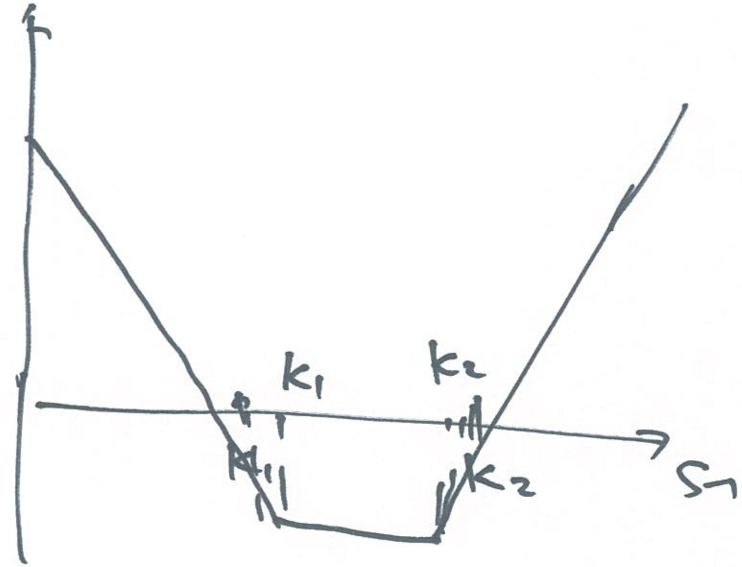
cost = Put(K) + Call(K)

Strangle

Payoff



Profit

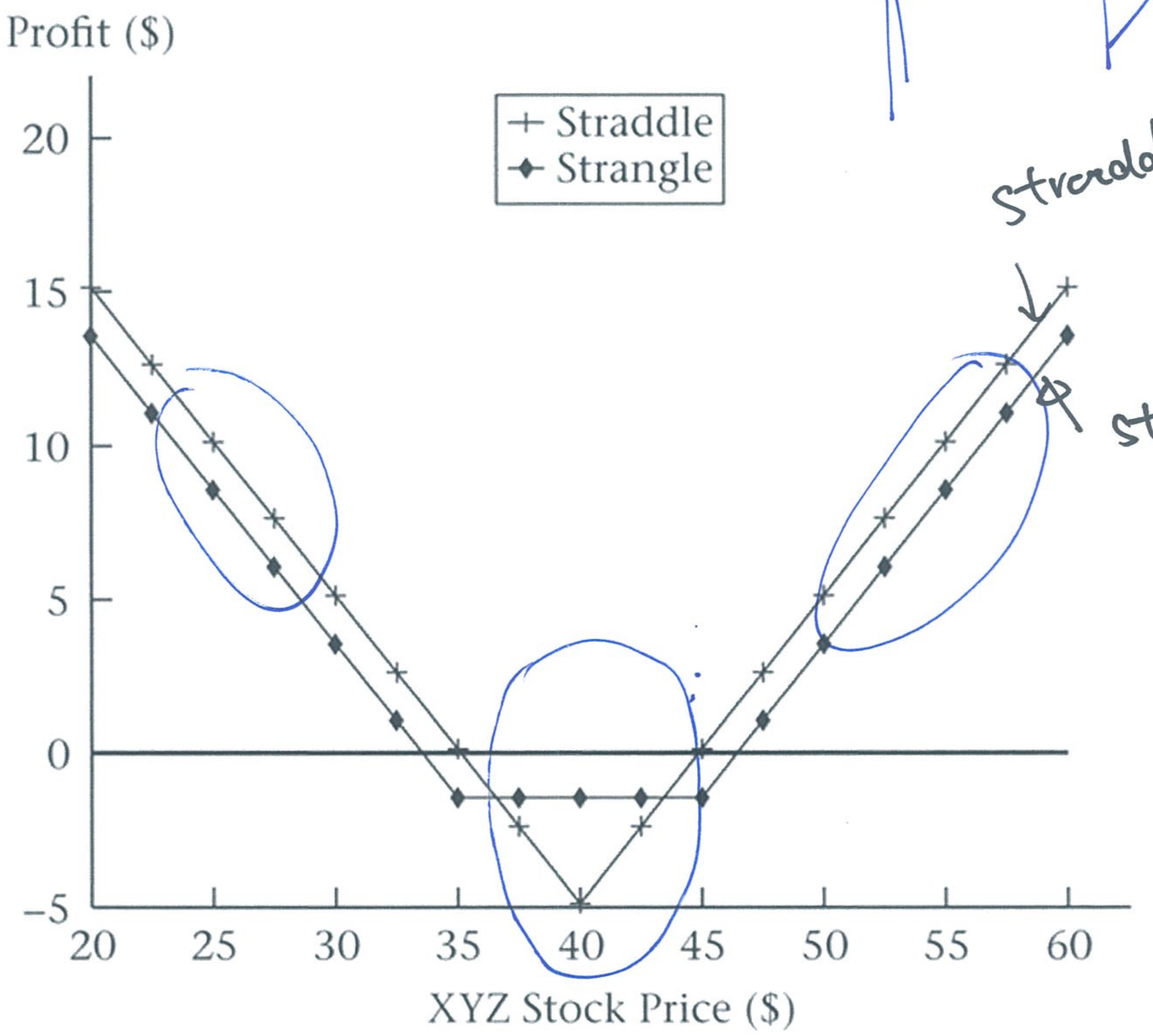


Long K_1 – ~~call~~ put

+ Long K_2 – call out-of-money out-of-money

$\Rightarrow$ cost Put (K_1) + Call (K_2)

(15)

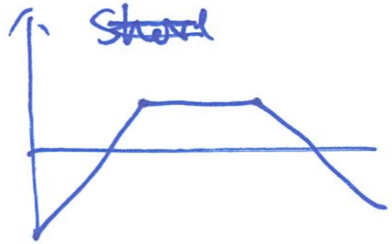


straddle

strangle

– short straddle

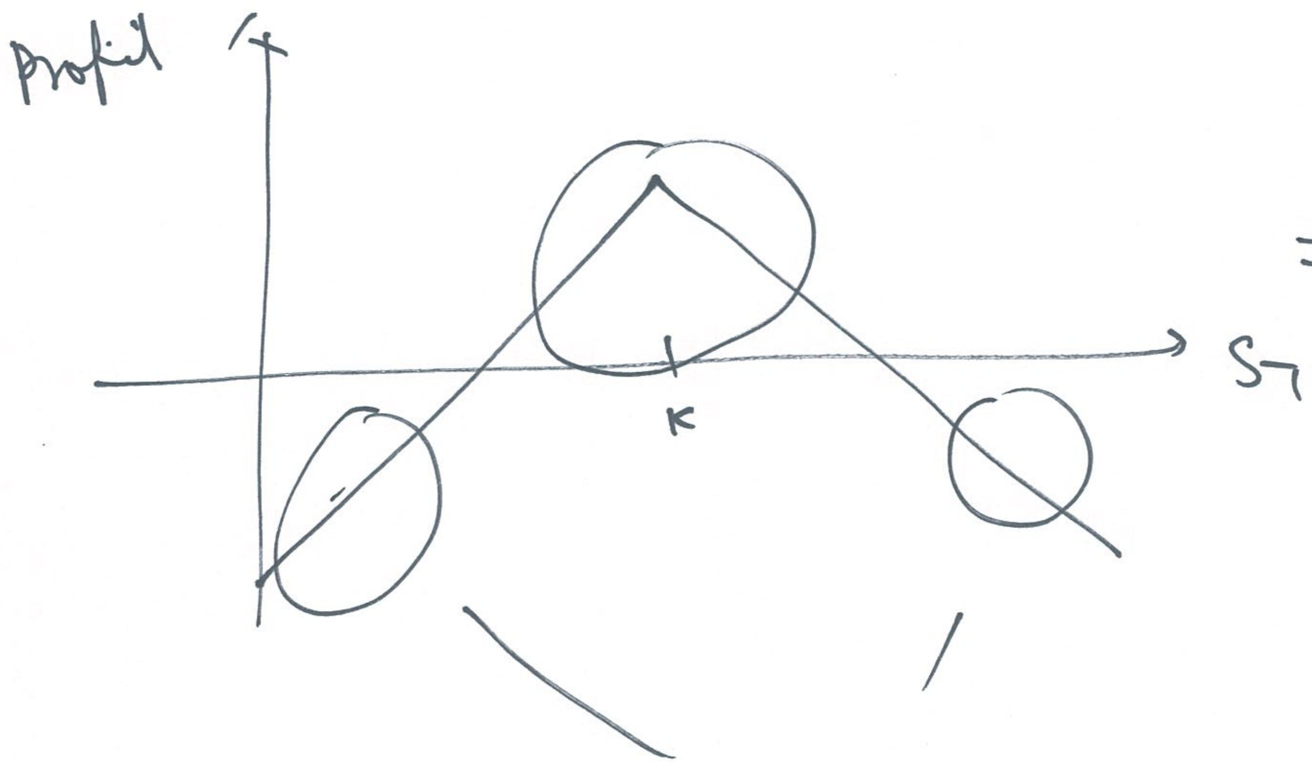
short K-call
+
short K-put



~~Short~~

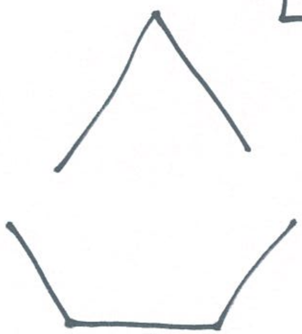
Short strangle

Short straddle



large loss

" Butterfly "



~~Long a~~
Short a straddle
+
Long a strangle

Butterfly

= ~~Stra~~ Short a K-straddle + Long a (K_1, K_2) strangle.

Short a K-put + Short a K-call

Long a K_1- ~~call~~ put

Long a K_2 - ~~put~~ call

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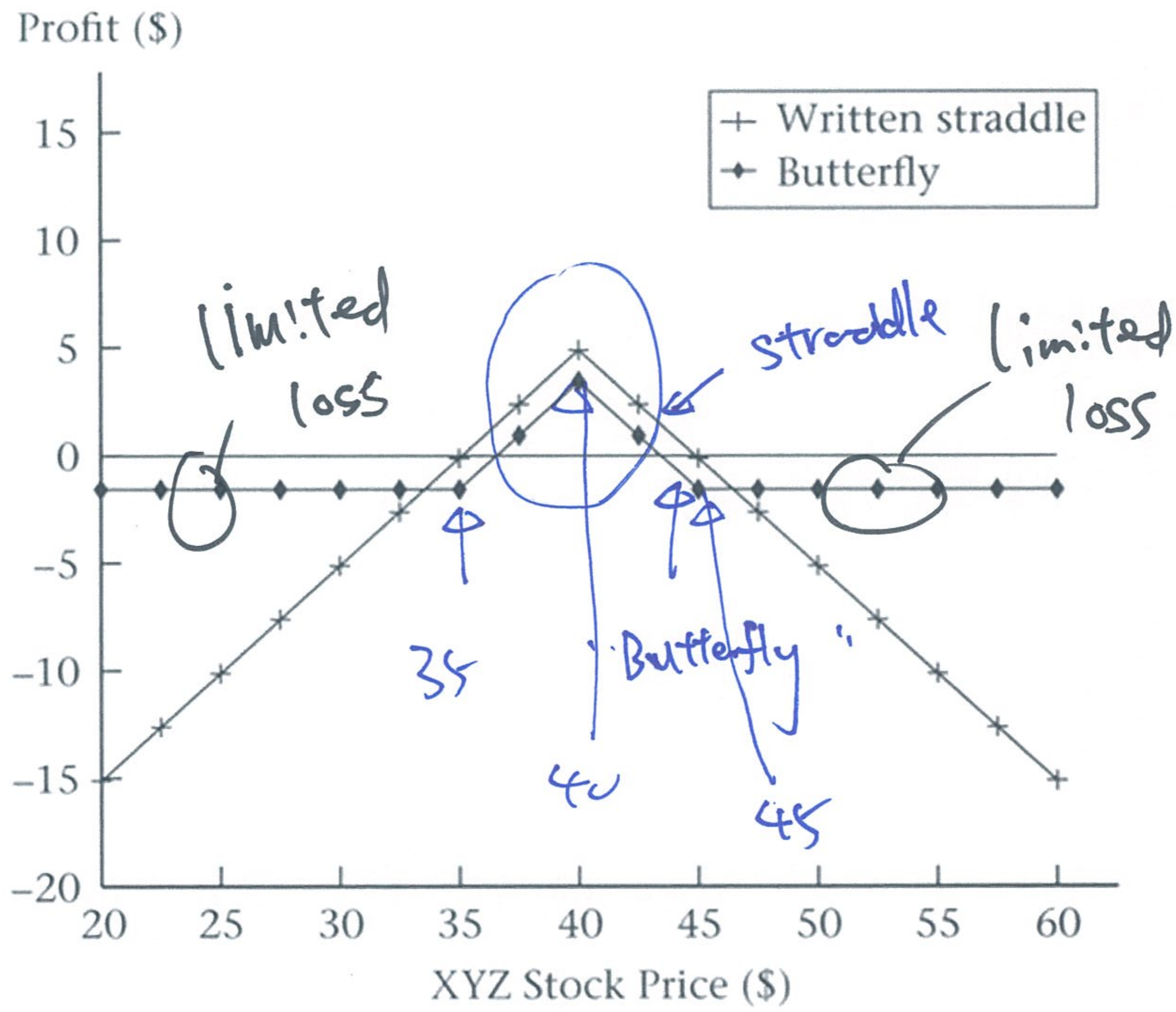
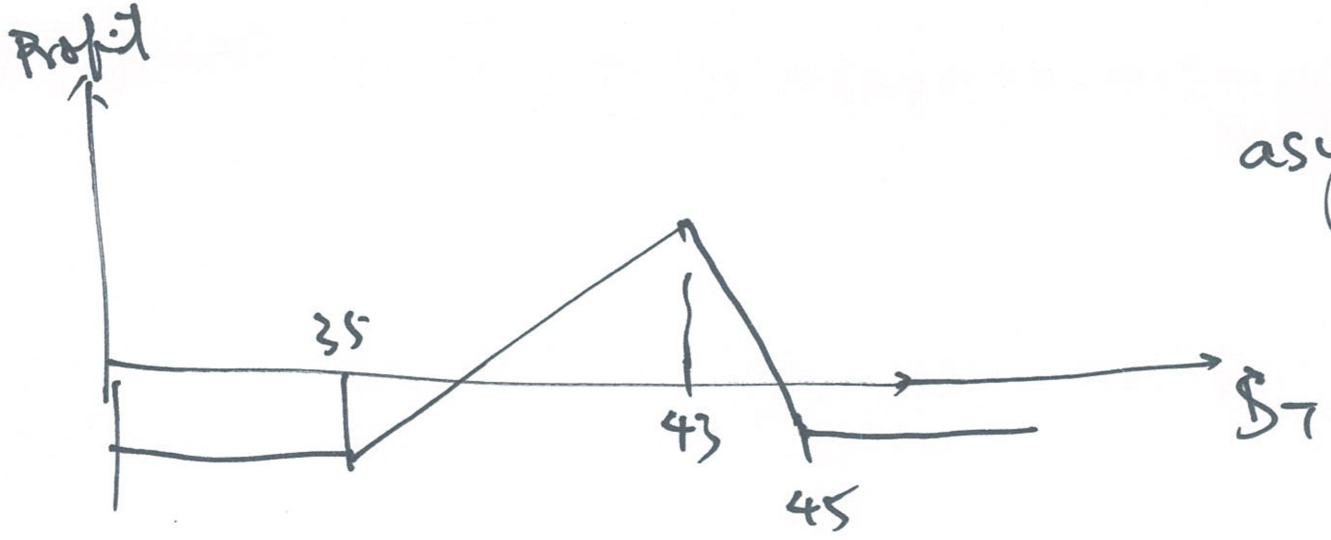
FIGURE 3.14

Comparison of the 35-40-45 butterfly spread, obtained by adding the profit diagrams in Figure 3.13, with the written 40-strike straddle.



asymmetric butterfly



"~~a call~~ buy "a" 35 - call

sell "b" 43 - call

buy "c" 45 - call

$a, b, c = ??$

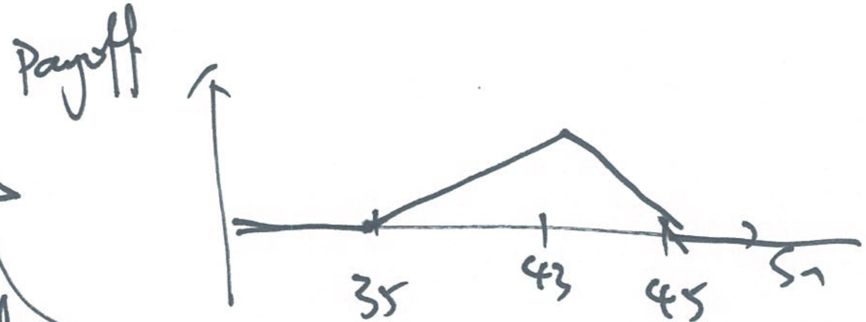
$43 = \lambda\, 35 + (1-\lambda)\, 45$

~~$\lambda = ?$~~

$43 = 45 - 10\lambda$

$\lambda = 0.2$

$\Rightarrow 43 = 0.2(35) + 0.8(45)$



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Sell 1 unit of 43-call

Buy 0.2 unit of 35-call

2

Buy 0.8 unit of 45-call

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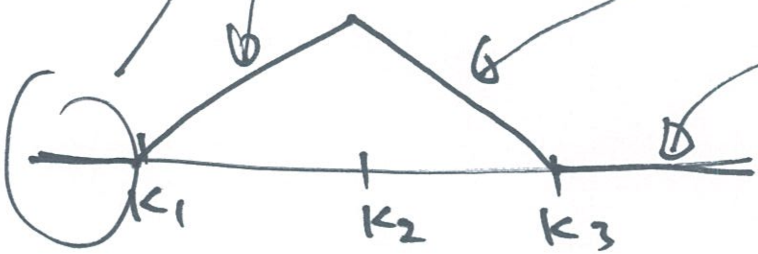
Payoff ~~Cashflow~~ table ($a=?$, $b=?$, $c=?$)

	$S_T < K_1$	$K_1 \le S_T < K_2$	$K_2 \le S_T \le K_3$	$S_T > K_3$
Buy a K_1-call	0	$a(S_T-K_1)$	$a(S_T-K_1)$	$a(S_T-K_1)$
Sell b K_2-call	0	0	$-b(S_T-K_2)$	$-b(S_T-K_2)$
Buy c K_3-call	0	0	0	$c(S_T-K_3)$
Total	0	$a(S_T-K_1)$	$(a-b)S_T + (bK_2-aK_1)$	① $S_T(a-b+c)$ + ② $(bK_2-aK_1-cK_3)$



~~Max(S_T-K, 0)~~

$\Rightarrow a-b+c=0$ and $bK_2-aK_1-cK_3=0$

$$a - b + c = 0 \Rightarrow c = b - a$$

$$bk_2 - ak_1 - ck_3 = 0 \Rightarrow bk_2 = ak_1 + ck_3$$

$$k_2 = \frac{a}{b}k_1 + \frac{c}{b}k_3$$

$$k_2 = \frac{a}{b}k_1 + \frac{b-a}{b}k_3$$

$$= \underbrace{\frac{a}{b}}_{\lambda}k_1 + \left(1 - \underbrace{\frac{a}{b}}_{\lambda}\right)k_3$$

Pre-paid forward

$t=0$ $\qquad$ $t=T$

Long $\quad$ \$K $\qquad\qquad$ Long $\quad$ underlying

Short $\qquad\qquad\qquad$ Short

Forward

$\$K \neq \$\tilde{K}$ $\qquad$ $\tilde{K} = FV(K)$ or $K = PV(\tilde{K})$

$t=0$ $\qquad$ $t=T$

\$0 $\quad$ Long $\quad$ \$0 $\quad$ N~~o transaction~~ $\qquad$ $\$\tilde{K}$ $\quad$ Long $\quad$ underlying

Short $\qquad\qquad\qquad$ Short

keep to "T" 23

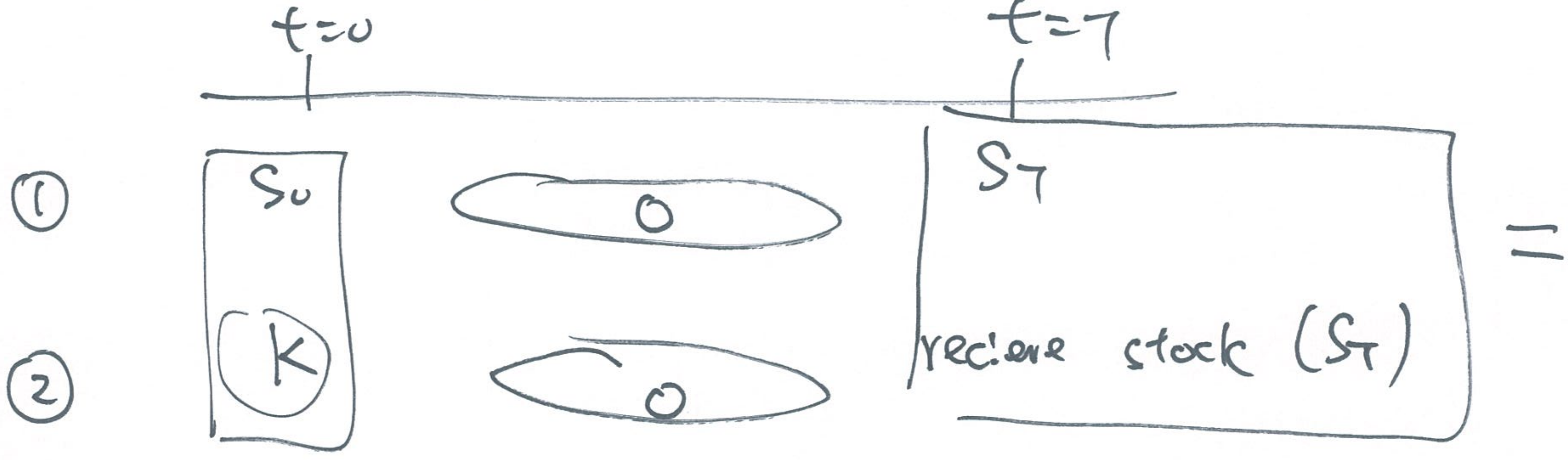
TABLE 5.1 Four different ways to buy a share of stock that has price S_0 at time 0. At time 0 you agree to a price, which is paid either today or at time T. The shares are received either at 0 or T. The interest rate is r.

Description	Pay at Time	Receive Security at Time	Payment
(1) Outright purchase	0	0	$\boxed{S_0 \text{ at time } 0}$
Fully leveraged purchase	T	0	$\boxed{S_0 e^{rT}}$ at time T
(3) Prepaid forward contract	0	T	? K = ?
Forward contract	T	T	$? \times e^{rT} = Ke^{rT}$ = FV(K)

K = ?

Suppose non-dividend paying stock.

Pricing by "analogy"



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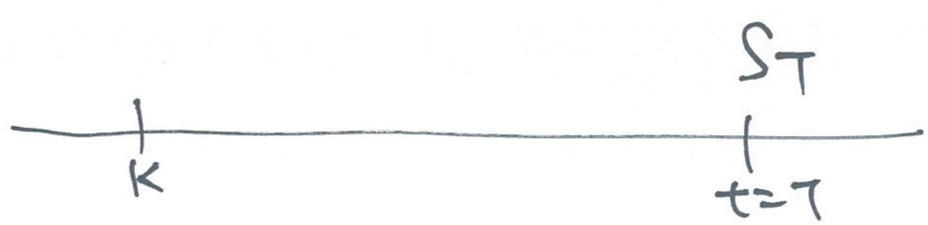
same cost $\Rightarrow$ $K = S_0$ $\quad K = F_{0,T}^{P}$ ← ~~pre for~~ pre-paid forward price

Forward price $= FV(K) = FV(S_0) = S_0 e^{rT} = F_{0,T}$

start end

Pricing by discounting



~~$F_{0,T}^P =$~~

$$F_{0,T}^P = PV\left(E[S_T]\right)$$

$$\cancel{= e^{-rT} E[S_T]}$$

$$= e^{-\alpha T}\left(E[S_T]\right)$$

$$= e^{-\alpha T} e^{\alpha T} S_0$$

$$= S_0$$

α : expected return of stock.

Pricing by no-arbitrage

$F_{0,T}^{P} \overset{?}{=} S_0$

$F_{0,T}^{P} > S_0$ (buy low sell high)

$\Rightarrow$ cost of stock < cost of pre-paid forward

Buy stock + sell pre-paid forward

$\Rightarrow$ arbitrage $\Rightarrow$ $F_{0,T}^{P} \leq S_0$

NO probability knowledge required

$\boxed{\Rightarrow F_{0,T}^{P} = S_0}$

Want to show $F_{0,T}^{P} \not< S_0$

Suppose $F_{0,T}^{P} < S_0$ $\Rightarrow$ buy pre-paid forward + sell the stock

$\Rightarrow$ arbitrage $\Rightarrow$ $F_{0,T}^{P} \not< S_0$

Dividend paying stock

① ↳ Discrete deterministic dividends.

② ↳ ~~Cts~~ Continuous deterministic dividend.

①

$t=0$	t_1	t_2	t_3	...	t_n	$t=T$	
$-S_0$	$+D_1$	$+D_2$	---		$+D_n$	S_T ← cost	outright purchase
$-F_{0,T}^P$	0	0	0	...	0	S_T	pre-paid forward

$$\left.\begin{aligned} &\text{Cost of outright} = S_0 - \sum_{i=1}^{n} PV(D_i) \\ &\text{Cost of pre-paid} = F_{0,T}^P \end{aligned}\right] \Rightarrow F_{0,T}^P = S_0 - \sum_{i=1}^{n} PV(D_i)$$