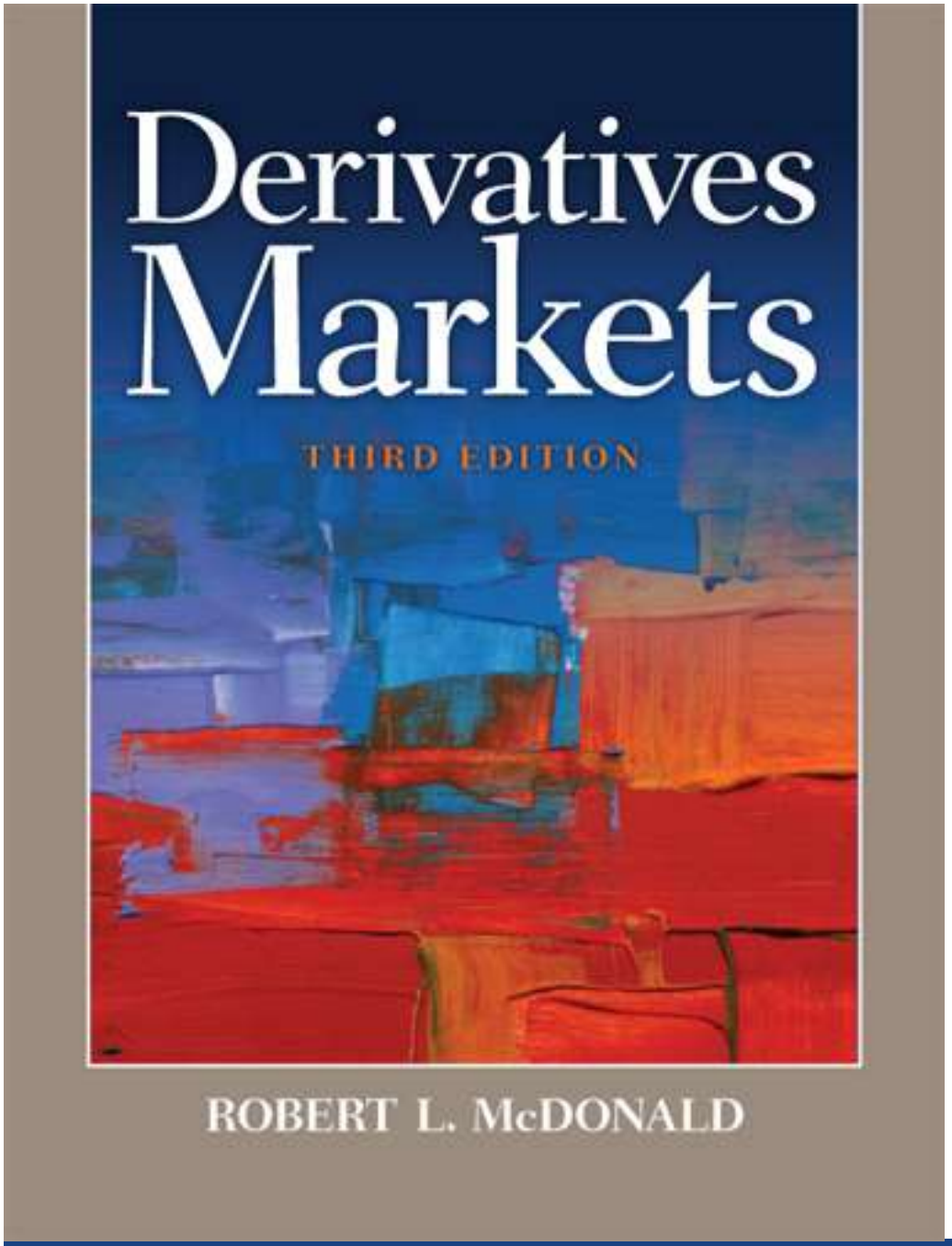

Chapter 5
(Chapter 6 in the textbook)

Commodity Forwards and Futures

Points to Note

1. What are the differences between the commodity and the financial asset? See P.3 to 5.
2. Definitions of backwardation and contango, see P.6 to 10.
3. How do the storage cost and convenience yield determine the forward price? (see $F_{0,T}$ on p.12 and 13)
4. What does the lease rate mean? See P.15.
5. What is the relationship among the lease rate, storage cost and convenience yield? See P.16.

Introduction to Commodity Forwards

- Differences between commodities and financial assets include
  - *Storage costs*

    The cost of storing a physical item such as corn or copper. It can be large relative to its value.
  - *Carry markets*

    A commodity for which the forward price compensates a commodity owner for *costs of storage* is called a *carry market*. In such a market, the return on a cash-and-carry, net of all costs, is the risk-free rate.

Introduction to Commodity Forwards (cont'd)

- *Lease rate*

  A short-seller of an item may have to compensate the owner of the item for lending

Introduction to Commodity Forwards (cont'd)

– *Convenience yield*

The owner of a commodity in a commodity-related business may receive nonmonetary benefits from physical possession of the commodity

Introduction to Commodity Forwards (cont'd)

- The set of prices for different expiration dates for a given commodity is called the **forward curve** (or the **forward strip**) for that date
- If on a given date the forward curve is upward sloping, then the market is in **contango**. If the forward curve is downward sloping, the market is in **backwardation**
  - Note that forward curves can have portions in backwardation and portions in contango

Introduction to Commodity Forwards (cont'd)

- **More Commodity Terminologies**
  - Commodities can be broadly classified as *extractive* and *renewable*.
    - **Extractive commodities** occur naturally in the ground and are obtained by mining and drilling. Examples include metals (silver, gold, and copper) and hydrocarbons, including oil and natural gas.
    - **Renewable commodities** are obtained through agriculture, and include grains (corn, soybeans), livestock (cattle, pork bellies) and lumber.

Introduction to Commodity Forwards (cont'd)

- **More Commodity Terminologies**
  - Commodities can be further classified as *primary* and *secondary*.
    - **Primary commodities** are unprocessed; corn, soybeans, oil and gold.
    - **Secondary commodities** have been processed; gasoline.

Introduction to Commodity Forwards (cont'd)

TABLE 6.1 Futures prices for various commodities, March 17, 2011.

Expiration Month	Corn (cents/ bushel)	Soybeans (cents/ bushel)	Gasoline (cents/ gallon)	Oil (Brent) (dollars/ barrel)	Gold (dollars/ ounce)
April	—	—	2.9506	—	1404.20
May	646.50	1335.25	2.9563	114.90	1404.90
June	—	—	2.9491	114.65	1405.60
July	653.75	1343.50	2.9361	114.38	—
August	—	—	2.8172	114.11	1406.90
September	613.00	1321.00	2.8958	113.79	—
October	—	—	2.7775	113.49	1408.20
November	—	1302.25	2.7522	113.17	—
December	579.25	—	2.6444	112.85	1409.70

Data from CME Group.

Introduction to Commodity Forwards (cont'd)

- From Table 6.1, we have the following observations:
  - Contango: Near-term corn and soybeans, and with gold
  - Backwardation: Medium-term corn and soybeans, and with crude oil
  - Uncommon units: A barrel of oil = 42 gallons; A bushel ≈ 2,150 cubic inches; Troy ounce ≈ 1.097×1 avoirdupois ounce

The following sections are based on the materials of "Options, Futures and other Derivatives, 7 ed., by John C. Hull."

Storage Cost

Storage cost can be treated as negative income (or dividend). So,

$$F_{0,T} = (S_0 + U(0,T))e^{rT}$$

where is $U(0,T)$ the present value of all the storage cost at 0 over the period $[0, T]$, or

$$F_{0,T} = S_0\, e^{(r+u)T}$$

where u is the storage cost per annum as a proportion of the spot price.

Convenience Yield

- For consumption asset, users of this asset may feel that ownership of the physical asset provides benefits that can not be obtained by holding the futures contract. For example, oil refiner is unlikely to regard a futures contract on crude oil to be the same as crude oil held in inventory.

  So, we may have

$$F_{0,T} < (S_0 + U(0,T))e^{rT} \text{ or}$$

$$F_{0,T} < S_0\, e^{(r+u)T}$$

Convenience Yield (cont'd)

- Convenience yield (y) measures the amount of benefit that is associated with physically owning an asset, rather than owning a futures contract on it. y is defined as

$$F_{0,T} = (S_0 + U(0,T))e^{(r-y)T}$$

or

$$F_{0,T} = S_0\, e^{(r+u-y)T}.$$

y reflects the market's expectations concerning the future availability. Large $y \rightarrow$ higher chance that shortages will occur.

Lease Rate

- For a commodity owner who lends the commodity, the lease rate is like a dividend.
- With a commodity, the lease rate, δ_l, is the income earned only if the commodity is loaned. It is not directly observable, except if there is a lease market (e.g., http://www.kitco.com/commentaries/2016-05-06/Gold-Leasing-Explained.html)
- δ_l, y and u are related by

$$\delta_l = y - u$$

Cost of Carry

- For a commodity, the cost of carry is given by

$$cost\ of\ carry = r - q + u$$

where q is the rate of income provided by the commodity.