Revision (27-Sep-2018)

$F_{0,T}^{P}$: pre-paid forward price

$F_{0,T}$: forward price

$$F_{0,T} = FV(F_{0,T}^{P})$$

Synthetic product

$$\underbrace{\text{Payoff of Forward}}_{\text{Long 1 forward}} = \boxed{S_T - K}$$

$\Uparrow$

At maturity

$\boxed{\begin{array}{l} \to 1 \text{ unit of asset} \\ + \$(-K) \text{ cash} \end{array}}$

At today:

What is your portfolio at today to get $\boxed{}$ at maturity?

Divided paying stock

At today		At maturity
$e^{-\delta T}$ unit of stock	$\longrightarrow$	1 unit of stock
Borrow Ke^{-rT}	$\longrightarrow$	$-\$K$

Commodity vs Financial asset

(Storage cost) → Expense of owner

$\boxed{\text{production}}$ → Income of owner

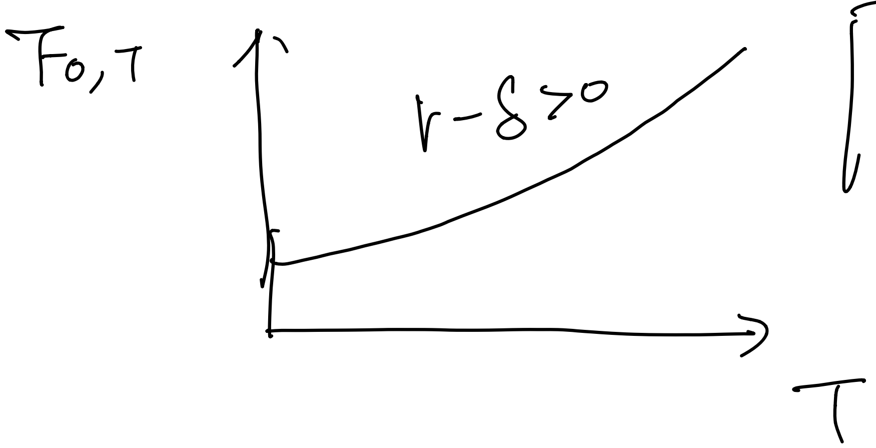
Expense of owner $\Rightarrow$ $\uparrow F_{0,T}$

Income of owner $\Rightarrow$ $\downarrow F_{0,T}$

Storage cost $\Rightarrow$ $\uparrow F_{0,T}$

Convenience yield $\Rightarrow$ $\downarrow F_{0,T}$

Shape of Forward curve



$\boxed{\text{Financial asset}}$

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

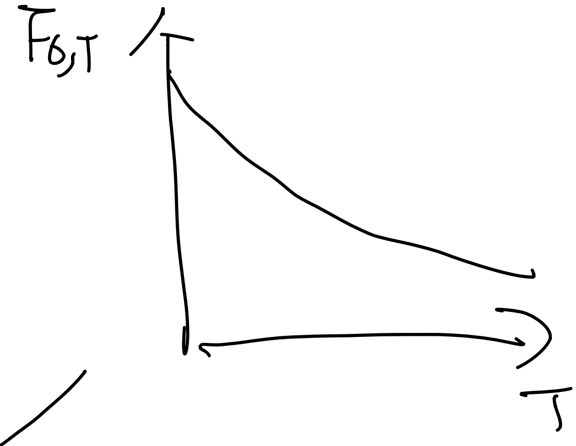
Introduction to Commodity Forwards (cont'd)

TABLE 6.1 Futures prices for various commodities, March 17, 2011.

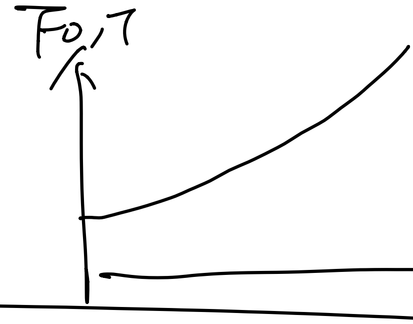
Expiration Month	Corn (cents/ bushel)	Soybeans (cents/ bushel)	Gasoline (cents/ gallon)	Oil (Brent) (dollars/ barrel)	Gold (dollars/ ounce)
April	—	—	2.9506	—	1404.20
May	646.50	1335.25	2.9563	114.90	1404.90
June	—	—	2.9491	114.65	1405.60
July	653.75	1343.50	2.9361	114.38	—
August	—	—	2.8172	114.11	1406.90
September	613.00	1321.00	2.8958	113.79	—
October		—	2.7775	113.49	1408.20
November	—	1302.25	2.7522	113.17	—
December	579.25	—	2.6444	112.85	1409.70

Data from CME Group.

Backwardation



Contango



Financial asset

dividend

↳ discrete D_i

↳ cts

$$F_{0,T} = FV\left[S_0 - \sum_{i=1}^{n} PV_{0,t_i}(D_i)\right]$$

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

Commodity

storage cost

↳ Discrete U_i

↳ cts storage cost

$$F_{0,T} = FV\left[S_0 + \sum_{i=1}^{n} PV_{0,t_i}(U_i)\right]$$

$$F_{0,T} = S_0 e^{(r+u)T}$$

$$u: \frac{\text{Storage cost per year}}{\text{commodity price}}$$

Introduction to Commodity Forwards (cont'd)

TABLE 6.1 Futures prices for various commodities, March 17, 2011.

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Data from CME Group.

Convenience yield : y

$$F_{0,T} = S_0 \, e^{(r+u-y)T}$$

$$F_{0,T} = \left[S_0 + \sum_{i=1}^{n} PV_{0,t_i}(U_i) \right] e^{(r-y)T}$$

Reference (Financial asset)

$$F_{0,T} = S_0 \, e^{(r-\delta)T}$$

Convenience yield plays the same role as the dividend yield.

Leasing market

"Gold miner"

lease rate.

$$\delta_\ell = y - u$$

$$F_{0,T} = S_0 e^{(r - \delta_\ell)T}$$

lease rate

$$F_{0,T} = S_0 e^{\int_0^T (r(s) - \delta_\ell(s)) ds}$$

deterministic

$\boxed{\text{Under stochastic interest rate}}$

Assume $\delta_\ell(s)$ is deterministic.

$$F_{0,T} = \frac{S_0 e^{-\int_0^T \delta_\ell(s) ds}}{P(0,T)}$$

$$P(0,T) = E^{Q}\left[e^{-\int_0^T r(s)\,ds}\right]$$

Cost of carry

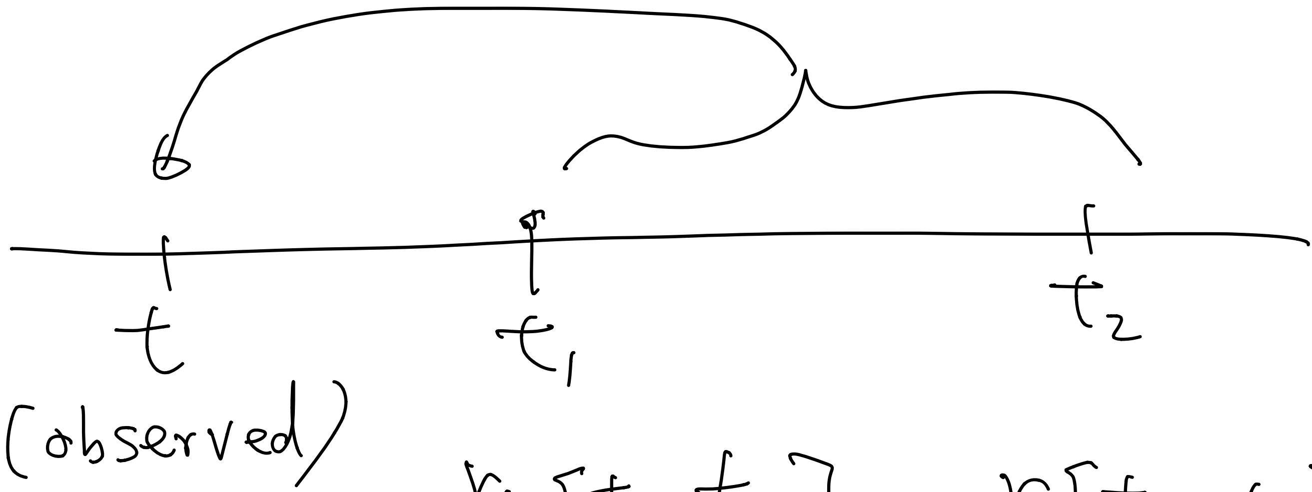
$$= r + u - y$$

r : borrowed interest

u : storage cost

y : convenience yield.

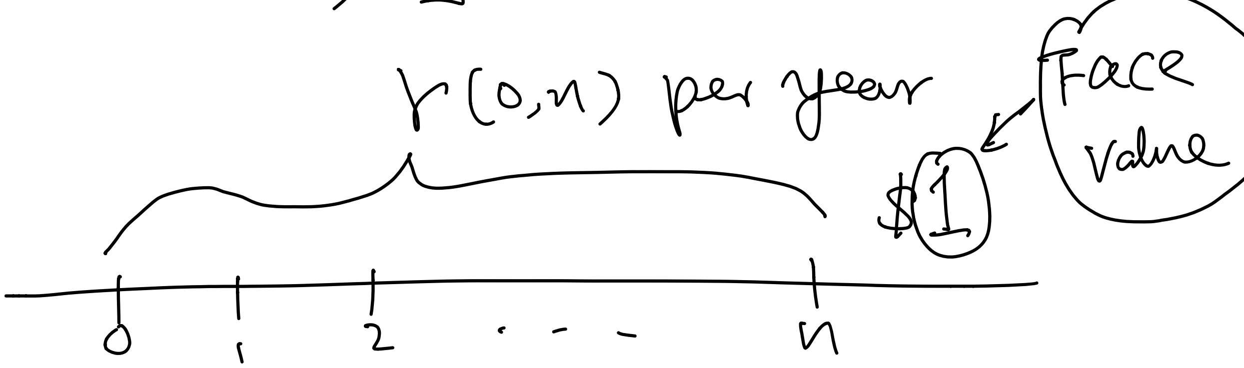
$r_t[t_1, t_2]$



$$r_{t_1}[t_1, t_2] = r[t_1, t_2]$$

Zero-coupon yield

$$r(0,n) = r_0[0,n]$$



$$PV_{0,n}(1) = \frac{1}{(1+r(0,n))^n} = P(0,n)$$

$r(0,n)$

zero-coupon yield

price of n-year zero-coupon bond with face value = $1

Bond Basics (cont'd)

- Zero-coupon bonds make a single payment at maturity

TABLE 7.1 Five ways to present equivalent information about default-free interest rates. All rates but those in the last column are effective annual rates.

Years to Maturity	(1) Zero-Coupon Bond Yield	(2) Zero-Coupon Bond Price	(3) One-Year Implied Forward Rate	(4) Par Coupon	(5) Continuously Compounded Zero Yield
1	6.00%	0.943396	6.00000%	6.00000%	5.82689%
2	6.50	0.881659	7.00236	6.48423	6.29748
3	7.00	0.816298	8.00705	6.95485	6.76586

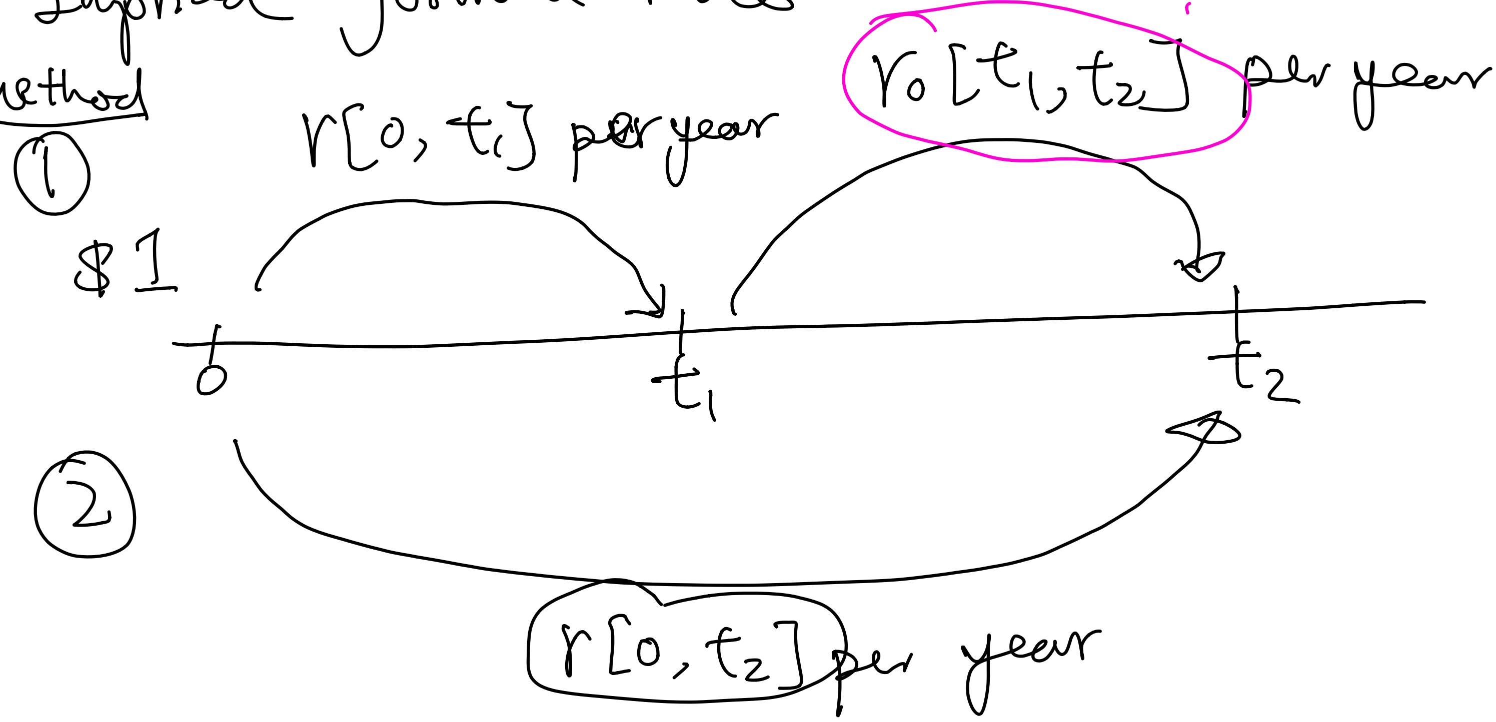
Column 1

$$0.943 = \frac{1}{1+r(0,1)} \Rightarrow r(0,1) = 6\%$$

$$0.8817 = \frac{1}{(1+r(0,2))^2} \Rightarrow r(0,2) = 6.5\%$$

Implied forward rate

method



Amt @ t_2 .

by method 1 $= 1\,(1+r(0,t_1))^{t_1}\,(1+r_0[t_1,t_2])^{t_2-t_1}$

by method 2 $= 1\,(1+r(0,t_2))^{t_2}$

$=$

$$(1+r_0(t_1,t_2))^{t_2-t_1} = \frac{[1+r(0,t_2)]^{t_2}}{[1+r(0,t_1)]^{t_1}}$$

$$= \frac{P(0,t_1)}{P(0,t_2)}$$ deterministic

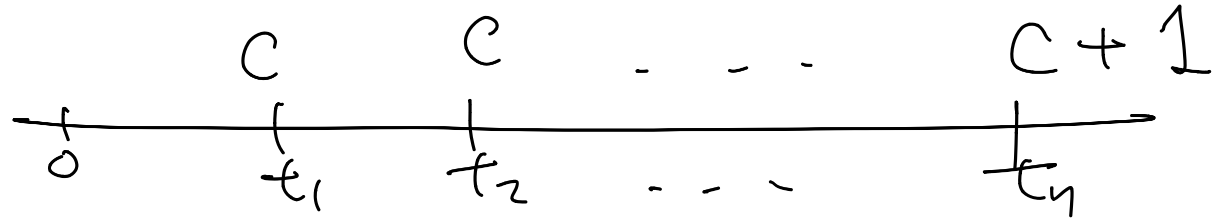
$\Rightarrow r_0[t_1, t_2]$ = deterministic number.

<u>Column 3</u>

$$(1+r_0(0,1))^1 = \frac{P(0,0)}{P(0,1)} \Rightarrow r_0(0,1) = 6\% = r(0,1)$$

$$(1+r_0(1,2)) = \frac{P(0,1)}{P(0,2)} \Rightarrow r_0(1,2) = 7.002\%.$$

Coupon-paying bond



$$\boxed{\text{Price of Coupon bond}} = \sum_{i=1}^{n} PV_{0,t_i}(C) + PV_{0,t_n}(1)$$

$$= \underbrace{\boxed{\sum_{i=1}^{n} C\,P(0,t_i) + P(0,t_n)}}_{\text{portfolio of zero-coupon bonds.}}$$

"STRIPS"

2-yr coupon bond

$$\text{Price} = C\,P(0,1) + C\,P(0,2) + P(0,2)$$

Buy a 2-yr coupon bond (C)

+ Sell a 1-yr zero-coupon bond with face value = C

$$= \begin{aligned} &C\,P(0,1) + C\,P(0,2) + P(0,2) \\ &- C\,P(0,1) = (C+1)P(0,2) \end{aligned}$$