MFE5130 – Financial Derivatives

First Term, 2017-18

Midterm Examination (Solution)

Question 1

Let $C(K)$ and $P(K)$ be the premium of the K-strike call option and K-strike put option respectively.

We have

$$C_A(0) = C(108) - C(109)$$

$$C_B(0) = P(109) - P(108)$$

Adding $C_A(0)$ and $C_B(0)$ together, we have

$$\begin{aligned}
&C_A(0) + C_B(0) \\
&= C(108) - C(109) + P(109) - P(108) \\
&= \left[S_0 - 108e^{-r}\right] + \left[109e^{-r} - S_0\right] \\
&= e^{-r} = e^{-0.22} = 0.8025
\end{aligned}$$

where S_0 is the price of the underlying stock at time 0.

Now, we can find $C_B(0)$:

$$\begin{aligned}
C_A(0) + C_B(0) &= 0.8025 \\
0.3 + C_B(0) &= 0.8025 \\
C_B(0) &= 0.8025 - 0.3 = 0.5025.
\end{aligned}$$

Question 2

(a)

We make use of the version of the put-call-parity that can be applied to currency options. We have:

$$C_{\$}\left(x_0, K, T\right) = e^{-r_f T} x_0 + P_{\$}\left(x_0, K, T\right) - e^{-rT} K.$$

So,

$$\begin{aligned} C_{\$}\left(0.008, 0.008, 1\right) &= e^{-0.02(1)}\left(0.008\right) + 0.0009 - e^{-0.06(1)}\left(0.008\right) \\ &= 0.001207. \end{aligned}$$

(b)

The observed option price is too high. Therefore, we sell the call option and synthetically create a long call option, perfectly offsetting the risks involved. We have:

Transaction	$t = 0$	$t = 1$, $x < 0.008$	$t = 1$, $x > 0.008$
Sell Call	0.0019	0	$-(x - 0.008)$ $= 0.008 - x$
Buy Put	−0.0009	$0.008 - x$	0
Buy $e^{-r_f T}$ Spot	$-0.008e^{-0.02} = -0.007842$	x	x
Borrow PV(strike)	$0.008e^{-0.06}$ =0.007534	–0.008	–0.008
Total	0.000692	0	0

We have thus demonstrated the arbitrage opportunity.

(c)

By using

$$C_{\$}\left(x_0, K, T\right) = x_0 K P_{yen}\left(\frac{1}{x_0}, \frac{1}{K}, 1\right),$$

we have

$$\begin{aligned} C_{\$}\left(0.008, 0.008, 1\right) &= \left(0.008\right)\left(0.008\right) P_{yen}\left(\frac{1}{0.008}, \frac{1}{0.008}, 1\right) \\ 0.001207 &= \left(0.008\right)^2 P_{yen}\left(125, 125, 1\right) \\ P_{yen}\left(125, 125, 1\right) &= \frac{0.001207}{\left(0.008\right)^2} \\ &= 18.8594 \text{ yen.} \end{aligned}$$

Question 3

(a)

Let $F_{0,T}$ be the forward price on one troy ounce of gold with T years to expiration.

It is known that

$$F_{0,T} = \frac{S_0 e^{-\delta_l T}}{P(0,T)},$$

where S_0 is the current spot price of gold and δ_l is the lease rate of gold.

So,

$$F_{0,T} = \frac{1300e^{-0.025T}}{P(0,T)},$$

T (in Years)	1	2	3	4
$F_{0,T}$	1,273.2505	1,285.3116	1,281.5498	1,292.3409

Let R be the fixed swap price for one troy ounce of gold.

$$\begin{aligned} R &= \frac{\sum_{k=1}^{4} P(0,k) F_{0,k}}{\sum_{k=1}^{4} P(0,k)} \\ &= \frac{(0.9958)(1{,}273.2505)+(0.9621)(1{,}285.3116)+(0.9411)(1{,}281.5498)+(0.9102)(1{,}292.3409)}{0.9958+0.9621+0.9411+0.9102} \\ &= 1{,}282.9088. \end{aligned}$$

Hence, the fixed swap price for one troy ounce of gold is \$1,282.9088.

(b)

After 1 year, the forward prices of gold at that time are

T (in Years)	1	2	3
$F_{0,T}$	1,345.8358	1,356.8081	1,370.1482

Let R_{new} be the new fixed swap price for one troy ounce of gold in the swap.

$$R_{new} = \frac{\sum_{k=1}^{3} P(0,k) F_{0,k}}{\sum_{k=1}^{3} P(0,k)}$$

$$= \frac{(0.9852)(1,345.8358) + (0.9589)(1,356.8081) + (0.9335)(1,370.1482)}{0.9852 + 0.9589 + 0.9335}$$

$$=1,357.3789.$$

The market value of the swap in the perspective of the long party

$= 200(1,357.3789 - 1,282.9088)[P(0,1) + P(0,2) + P(0,3)]$

$= 200(1,357.3789 - 1,282.9088)[0.9852+0.9589+0.9335]$

$= 42,872.4366.$

Question 4

The theoretical forward price = $(30.58 - 1.8e^{-6\% \times 0.25} - 2.5e^{-6\% \times 0.5})\, e^{6\% \times (8/12)} = 27.4573$.

Now, we have the observed market forward price, \$29.15, is **higher than** the theoretical forward price. So, the strategy to realize the arbitrage profit is to short the forward contract and long the synthetic forward.

Transactions	**Cash Flows**			
	$t = 0$	$t = 0.25$	$t = 0.5$	$t = 8/12$
Short one forward	0	0	0	$29.15 - S_{8/12}$
Buy one share of the stock	–30.58	0	0	$S_{8/12}$
Borrow \$30.58 at $t = 0$	30.58	0	0	$-30.58e^{(0.06)(8/12)}$ $= -31.828$
Receive the dividend (\$1.8) at $t = 0.25$	0	1.8	0	0
Lend \$1.8 at $t = 0.25$	0	–1.8	0	$1.8e^{(0.06)(5/12)} = 1.8456$
Receive the dividend (\$2.5) at $t = 0.5$	0	0	2.5	0
Lend \$2.5 at $t = 0.5$	0	0	–2.5	$2.5e^{(0.06)(2/12)} = 2.5251$
Total	0	0	0	1.6927

This position requires no initial investment, has no stock price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy. The accumulated arbitrage profits at the end of 8 months is \$1.6927.

Question 5

Let $P(K)$ is the price of the K-strike put option.
The prices of Option A and Option B **violate** the following inequality

$$P(K_1) \leq P(K_2), \quad \text{for } K_1 \leq K_2.$$

This is because:

$$P(127) < P(120)$$
$$10 < 12.$$

Therefore, arbitrage profits can be earned by buying the 127-strike put and selling the 120-strike put. This is a put bear spread.

Let $S_{0.75}$ be the price of the underlying stock at the end of 9 months.

The payoff of the strategy is given as follows:

		t = 9 months (0.75 year)		
Transaction	$t = 0$	$0 < S_{0.75} < 120$	$120 \leq S_{0.75} < 127$	$127 \leq S_{0.75}$
Buy 1 unit of $P(127)$	–10	$127 - S_{0.75}$	$127 - S_{0.75}$	0
Sell 1 unit of $P(120)$	12	$-(120 - S_{0.75})$	0	0
Total	2	7	$127 - S_{0.75}$	0

If the final stock price is \$118, then the accumulated profits at the end of 9 months are

$$X = 2e^{(0.11)(0.75)} + 7 = 9.172.$$

If the final stock price is \$122, then the accumulated profits at the end of 9 months are

$$Y = 2e^{(0.11)(0.75)} + (127 - 122) = 7.172.$$

Hence,

$$\frac{X}{Y} = \frac{9.172}{7.172} = 1.2789.$$