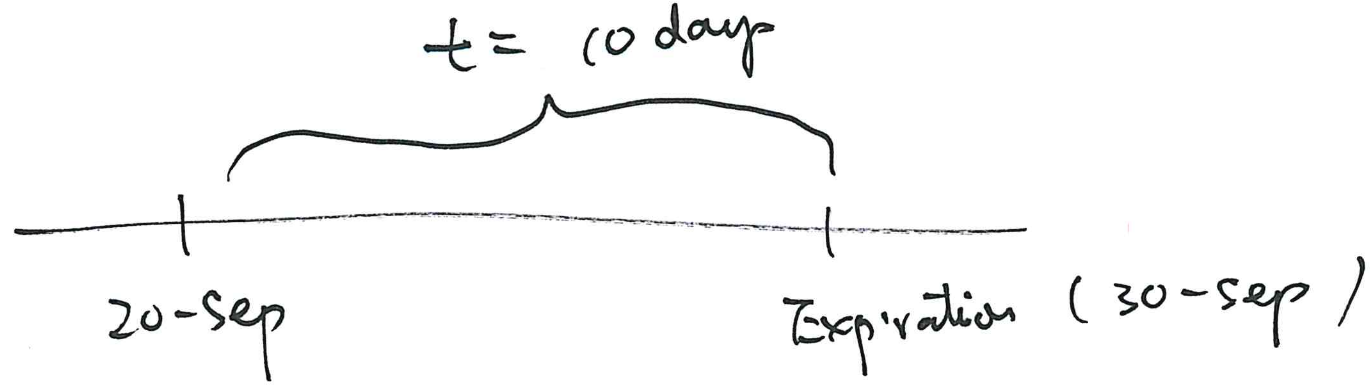
Revision

20-Sep-2018

Important relation : Put-call parity

$$Call(\underline{K}, t) - Put(\underline{K}, t) = PV(F_0(t) - \underline{K})$$

t: time to expiration (time to maturity)



$Call(K, 0) = \text{payoff} = \max(S_T - K, 0)$

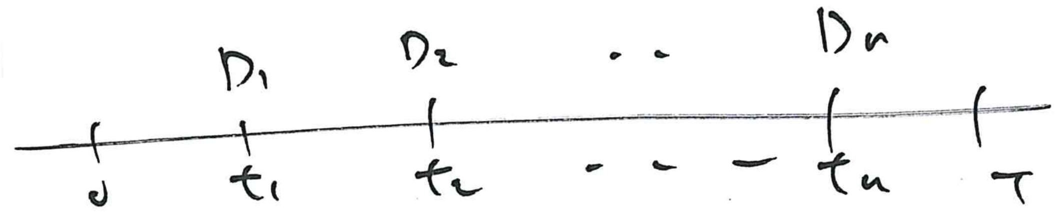
$Call(K, 1) =$

Pre-paid forward

$$F_{0,T} = \begin{cases} S_0 & \text{no-dividend} \\ \boxed{S_0 - \sum_{i=1}^{n} PV(D_i)} & \text{with discrete Dividend (deterministic)} \end{cases}$$

$$PV_{0,t_i}(D_i)$$

$$= S_0 - \sum_{i=1}^{n} e^{-rt_i} D_i$$



PV_{0,t_i}: present value at time 0 with payment at t_i.

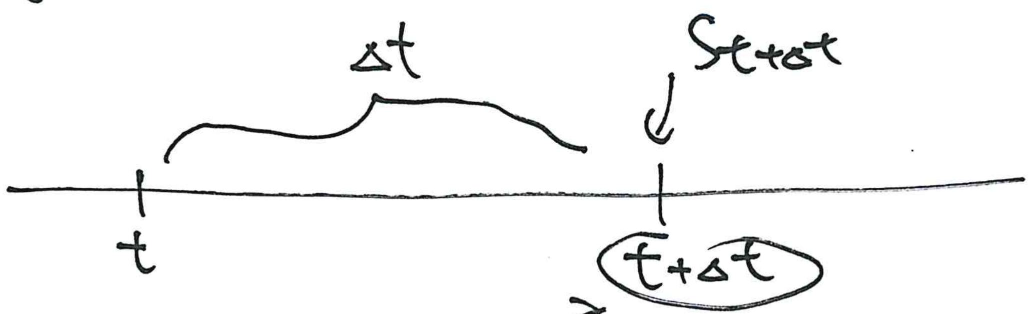
Continuous dividend

($\boxed{\text{stochastic}}$ dividend payment $\Rightarrow$ $\boxed{\text{deterministic}}$ no. of share of stock) ③

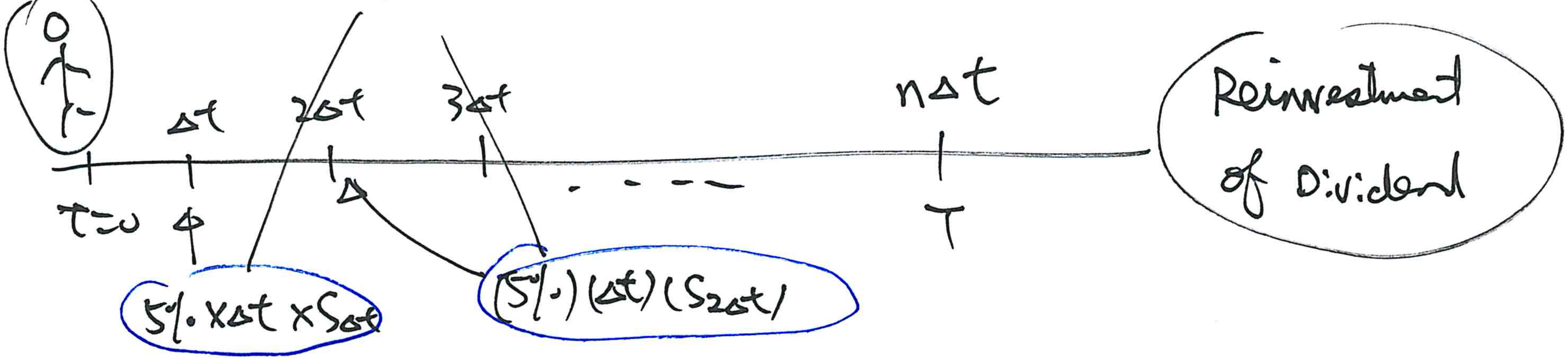
δ : dividend yield.

annualized dividend payment divided by stock price

$\delta = 5\%$



stochastic (dividend amt $= 5\% \times \Delta t \times S_{t+\Delta t}$



$$F_{0,T}^{P} = \begin{cases} S_0 & \text{no dividend} \\ S_0 - \sum PV(D_N) & \text{discrete} \;\;'' \\ S_0 e^{-\delta T} & \text{dividend yield} \end{cases}$$

Synthetic Forward

stock

cash

Payoff of Forward $= S_T - F_{0,T}$

$$= \boxed{S_T - S_0 e^{(r-\delta)T}}$$

How do you ~~crete~~ create this payoff?

synthetic forward

Cashflow table

	$t=0$	~~? Net ~~Cash~~ Cashflow~~	$t=T$
Buy $e^{-\delta T}$ units of stock	$-e^{-\delta T} S_0$		S_T
Borrow $e^{-\delta T} S_0$	$e^{-\delta T} S_0$		$-e^{(r-\delta)T} S_0$
	0	Net cashflow 0	$\boxed{S_T - e^{(r-\delta)T} S_0}$

=

Synthetic stock

Synthetic ~~Bond~~ zero-coupon bond

Payoff of Forward $= S_T - F_{0,T}$ (synthetic forward)

$S_T = \underline{\text{Payoff of Forward}} + F_{0,T}$ (synthetic stock)

$

$F_{0,T} = \underset{\text{stock}}{S_T} - \underset{\text{forward}}{\underline{\text{Payoff of Forward}}}$ (synthetic bond)

synthetic stock

TABLE 5.4 Demonstration that going long a forward contract at the price $F_{0,T} = S_0 e^{(r-\delta)T}$ and lending the present value of the forward price creates a synthetic share of the index at time T.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Long one forward	0	$S_T - F_{0,T}$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_0 e^{(r-\delta)T}$
Total	$-S_0 e^{-\delta T}$	S_T

$\Leftrightarrow$ buy $e^{-\delta T}$ units of stock at price S_0

Cash outflow

Synthetic zero-coupon bond

TABLE 5.5 Demonstration that buying $e^{-\delta T}$ shares of the index and shorting a forward creates a synthetic bond.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Buy $e^{-\delta T}$ units of the index	$-S_0e^{-\delta T}$	$+S_T$
$\boxed{\text{Short one forward}}$	0	$F_{0,T} - S_T$
Total	$-S_0e^{-\delta T}$	$F_{0,T}$

deterministic

cash outflow

cash inflow

$\Rightarrow$ Implied interest rate θ : $F_{0,T}$ ~~$= S_0e^{-\delta T})$~~

$F_{0,T} = (S_0e^{-\delta T})\, e^{\theta T}$ — loan rate

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Use of synthetic product:

(a) hedging

(b) ~~Arbitr~~ To get the arbitrage profit.

$S_0 e^{(r-\delta)T}$ ← theoretical forward price

(b) Market forward price $F_{0,T}$ (market) $>$ $\cancel{F_{0,T}}$ ← Synthetic Forward

buy synthetic forward

sell market forward

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$F_{0,T}$

$F_{0,T}$ market forward price $> S_0 e^{(r-\delta)T}$

$\Rightarrow F_{0,T} - S_0 e^{(r-\delta)T} > 0$

TABLE 5.6 Transactions and cash flows for a cash-and-carry: A market-maker is short a forward contract and long a synthetic forward contract.

synthetic forward (Long)

buy synthetic forward

Transaction	Cash Flows: Time 0	Cash Flows: Time T (expiration)
① Buy tailed position in stock, paying $S_0e^{-\delta T}$	$-S_0e^{-\delta T}$	$+S_T$
② Borrow $S_0e^{-\delta T}$	$+S_0e^{-\delta T}$	$-S_0e^{(r-\delta)T}$
③ Short forward	0	$F_{0,T} - S_T$
Total	0	$F_{0,T} - S_0e^{(r-\delta)T}$

sell market forward

Cash and carry = ① + ③

$S_0 e^{(r-\delta)T}$

TABLE 5.7 Transactions and cash flows for a reverse cash-and-carry: A market-maker is long a forward contract and short a synthetic forward contract.

short synthetic forward

Transaction	Cash Flows: Time 0	Time T (expiration)
1 Short tailed position in stock, receiving $S_0e^{-\delta T}$	$+S_0e^{-\delta T}$	$-S_T$
Lend $S_0e^{-\delta T}$ 2	$-S_0e^{-\delta T}$	$+S_0e^{(r-\delta)T}$
Long forward 3	0	$S_T - F_{0,T}$
Total	0	$S_0e^{(r-\delta)T} - F_{0,T}$

reverse cash and carry = 1 + 3

Class Activity

Discrete Dividend

Theoretical forward price $= FV(F^P_{0,T}) = (S_0 - PV(1.2))\,e^{(5\%)(0.75)}$

$1.2\,e^{-(5\%)(0.25)}$

$F_{0,T}$ (market forward price) $= \$47.56$ $\quad = 45.84$

Theoretical forward price $< F_{0,T}$

(~~short~~/ Long) buy theoretical forward synthetic forward

(short) (~~Long~~) sell market forward

	$t=0$	$t=0.25$	~~$t=T$~~ $t=0.75$
short ~~Long~~ market forward	0	Dividend	
Buy 1 unit of stock	-45.34		
Borrow S_0	$+45.34$		
	0		

$$F_{0,T} = S_0 + \underbrace{\boxed{(r-\delta)}}_{\text{cost of carry}} \Delta t\, S_0 \qquad F_{0,T} = S_0\, e^{(r-\delta)T}$$

Forward price

(13)

$\approx$ when ~~Δt~~ T is small

$$e^{(r-\delta)T} = \left(1 + (r-\delta)T + \frac{(r-\delta)T^2}{2!} + \cdots\right)$$

T small $\quad e^{(r-\delta)T} \approx 1 + (r-\delta)T$

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Show Futures price = Forward price (interest rate is deterministic)

Table 5A.1 Investment strategy to show that futures and forward prices are equal.

Day	0	1	2	$n-1$	n
Futures price	F_0	F_1	F_2	F_{n-1}	F_n
Futures position	e^{δ}	$e^{2\delta}$	$e^{3\delta}$	$e^{n\delta}$	0
$\boxed{\text{Gain/loss}}$	0	$(F_1-F_0)e^{\delta}$	$\boxed{(F_2-F_1)e^{2\delta}}$		$(F_n-F_{n-1})e^{n\delta}$
Gain/loss compounded to day n	0	FV $(F_1-F_0)e^{n\delta}$	$(F_2-F_1)e^{n\delta}$		$(F_n-F_{n-1})e^{n\delta}$

margin A/C gain/loss

δ: interest rate.

FV $((F_2-F_1)e^{2\delta})$

$= (F_1 - F_0)e^{\delta} \cdot \boxed{e^{(n-1)\delta}}$

Total Gain/loss @ day n = Σ ( ) $= (F_n - F_0)e^{n\delta}$

Strategy

1) ~~Long e~~ Futures (T-0) + $\boxed{\text{Lend } F_0}$ cost $= F_0$

Total profit / Loss at day n $= (F_n - F_0)e^{n\delta} + F_0 e^{n\delta}$

$= F_n e^{n\delta} = S_n e^{n\delta}$

~~$F_n = S_n \Rightarrow$~~

2) Long $e^{n\delta}$ units of Forward contract with forward price $= G_0$ + lend G_0 $\boxed{\text{cost} = G_0}$

Payoff At day n $= G_0 e^{n\delta} + (S_n - G_0)e^{n\delta} = S_n e^{n\delta}$

$\Rightarrow \boxed{F_0 = G_0}$

Invest in Nikkei index

~~US$ → JPY~~ → Nikkei index Futures

exchange rate risk

risk of Nikkei index

FIGURE 5.2

Specifications for the Nikkei 225 index futures contract.

USD

Underlying	Nikkei 225 Stock Index
Where traded	Chicago Mercantile Exchange
Size	$5 × Nikkei 225 Index
Months	March, June, September, December
Trading ends	Business day prior to determination of settlement price
Settlement	Cash-settled, based upon opening Osaka quotation of the Nikkei 225 index on the second Friday of expiration month

"Quanto" ~~product~~ contract

exchange rate