MFE5130 – Financial Derivatives
First Term, 2019 – 20

Assignment 5

Additional Problem 1

Consider the following processes for two non-dividend paying stocks, Stock X and Stock Q:

$$\frac{dX(t)}{X(t)} = 0.05\,dt + 0.11\,dZ(t)$$

$$\frac{dQ(t)}{Q(t)} = 0.07\,dt + 0.12\,dZ(t)$$

$Z(t)$ is a standard Brownian motion. The continuously compounded risk-free rate of return is 3%. The current price of Stock X is \$182, and the current price of Stock Q is \$77. An investor buys or sells exactly 1 share of stock X as part of a strategy to earn arbitrage. Calculate the arbitrage profit over the time period with the length of dt.

Additional Problem 2

Given that $S(t)$ is a GBM (Geometric Brownian motion) which follows

$$\frac{dS(t)}{S(t)} = 0.05dt + 0.1dZ(t)$$

where $Z(t)$ is a standard Brownian motion under measure P.

Find another measure Q by specifying the Radon-Nikodym derivative of Q with respect to P, $\frac{dQ}{dP}$, such that $S(t)$ is governed by

$$\frac{dS(t)}{S(t)} = 0.09dt + 0.1d\tilde{Z}(t)$$

under Q, where $\tilde{Z}(t)$ is a standard Brownian motion under Q.

Additional Problem 3

Let $X(t)$ and $Y(t)$ be the price processes of the two stocks.

Suppose that $X(0) = 20$ and $Y(0) = 15$. Neither stock pays dividends. Under the risk-neutral measure Q, $X(t)$ and $Y(t)$ are governed by

$$\frac{dX(t)}{X(t)} = 0.08dt + 0.18dW_1(t)$$

$$\frac{dY(t)}{Y(t)} = \beta dt + 0.42dW_2(t)$$

where $W_1(t)$ and $W_2(t)$ are two correlated standard Brownian motions under Q and $dW_1(t)dW_2(t) = 0.2dt$.

a. Find β. What does β stand for?
b. Let $Z_1(t)$ and $Z_2(t)$ be two independent Brownian motions. The stochastic differential equations of $X(t)$ and $Y(t)$ can then be rewritten as

$$\frac{dX(t)}{X(t)} = 0.08dt + \alpha_1 dZ_1(t) + \alpha_2 dZ_2(t)$$

$$\frac{dY(t)}{Y(t)} = \beta dt + \alpha_3 dZ_2(t).$$

   Find the values of α_1, α_2 and α_3.

c. By choosing $X(t)$ as the numeraire, find the corresponding Radon-Nikodym derivative of Q_X with respect to Q, $\dfrac{dQ_X}{dQ}$.
d. Under Q_X, find $dY(t)$.

Additional Problem 4

Let $X(t)$ and $Y(t)$ be two stochastic processes.

Suppose that $X(0) = 100$ and $Y(0) = 100$. Under the measure P, $X(t)$ and $Y(t)$ are governed by

$$\frac{dX(t)}{X(t)} = 0.05dt + 0.27dZ_1(t) + 0.13dZ_2(t)$$

$$\frac{dY(t)}{Y(t)} = 0.08dt + 0.32dZ_2(t)$$

where $Z_1(t)$ and $Z_2(t)$ are two independent standard Brownian motions under P.

Let $\mathbf{I}_{\{Y(2)\le 100\}}$ be an indicator function which is defined as

$$\mathbf{I}_{\{Y(2)\le 100\}} = \begin{cases} 1 & \text{if } Y(2) \le 100, \\ 0 & \text{otherwise.} \end{cases}$$

Find $E^P\left[X(2)\mathbf{I}_{\{Y(2)\le 100\}}\right]$.

Additional Problem 5

Given that $S(t)$ is a GBM (Geometric Brownian motion) which follows

$$\frac{dS(t)}{S(t)} = 0.08dt + 0.3dZ(t), \quad S(0) = 8,$$

where $Z(t)$ is a standard Brownian motion under probability measure P.

Find $E^P\left[\max\left(S^2(2) - 60, 0\right)\right]$.

Additional Problem 6

Let $X(t)$ and $Y(t)$ be the price processes of the two stocks.

Suppose that $X(0) = 12$ and $Y(0) = 15$. Neither stock pays dividends. Under the risk-neutral measure Q, $X(t)$ and $Y(t)$ are governed by

$$\frac{dX(t)}{X(t)} = 0.04dt + 0.28dZ_1(t) + 0.12dZ_2(t)$$

$$\frac{dY(t)}{Y(t)} = 0.04dt + 0.31dZ_2(t)$$

where $Z_1(t)$ and $Z_2(t)$ are two independent standard Brownian motions under Q.

Consider a European type financial derivative with 3 years to expiration. The payoff at the maturiy is given by

$$\max\left[X(3)Y^2(3) - 235X(3), 0\right].$$

Find the price of the financial derivative at time 0.

Additional Problem 7

Let $S_1(t)$ and $S_2(t)$ be two stochastic processes. The dynamics of $S_1(t)$ and $S_2(t)$ under the probability measure P are governed by

$$\frac{dS_1(t)}{S_1(t)} = 0.08dt + 0.28dZ_1(t),$$

$$\frac{dS_2(t)}{S_2(t)} = 0.05dt + 0.35dZ_2(t).$$

where $Z_1(t)$ and $Z_2(t)$ are two correlated Brownian motions under P with $dZ_1(t)dZ_2(t) = -0.1dt$.

At time 0, we have $S_1(0) = 3$ and $S_2(0) = 5$.

Under P, find $\Pr(S_1(2)S_2(2) > 20)$.

Textbook Problems

14.20; 23.1