MFE5130 – Financial Derivatives
First Term, 2019 – 20

Assignment 4
Due date: 11:00pm, 9-December-2019

Important notes:
1. The assignment must be submitted via Blackboard.
2. Total: 8 Problems (Full Mark: 80).

Additional Problem 1

The price of a stock is \$50. The stock does not pay dividends. The continuously compounded risk-free rate of return is 8%. A \$50-strike, 3-month European call option has a price of \$3.48. The delta of the call option is 0.5824. The gamma of the call option is 0.0521. A \$55-strike, 4-month European call option has a price of \$2.05. The delta of the call option is 0.3769. The gamma of the call option is 0.0441.

A market-maker writes 100 of the the \$50-strike call options and delta-gamma hedges the position.

After 1 day, the new stock price is \$51, the new price of the \$50-strike call option is \$4.06, and the new price of the \$55-strike cal option is \$2.43.
Calculate the overnight profit for the market-maker.

Additional Problem 2

Assume that the Black-Scholes framework holds. Consider an option on a stock.
You are given the following information at time 0:

(i) The stock price is $S(0)$, which is greater than 75.
(ii) The option price is 5.92.
(iii) The option delta is -0.323.
(iv) The option gamma is 0.015.

The stock price changes to 86. Using the delta-gamma approximation, you find that the option price changes to 6.08. Determine $S(0)$.

Additional Problem 3

You are given the following information for a stock that has a lognormal distribution:

i. The annual continuously compounded expected return on the stock is $\alpha = 10\%$.
ii. The dividend yield is 0.
iii. The annual continuously compounded risk-free rate of return is $r = 8\%$.
iv. The current stock price is \$100.

Pete invests \$100 in zero-coupon bonds that mature in T years. Jim purchases 1 share of stock and holds the stock for T years. The probability that Pete's investment outperforms Jim's investment is 50%.

Determine the volatility of the stock, σ.

Additional Problem 4

You are given the following information about a stock:

i. The price of the stock is lognormally distributed.
ii. The expected return on the stock is $\alpha = 0.17$, and the dividend yield is $\delta = 0.02$.
iii. The probability that the stock price at time 3 months is greater than K is 60%.
iv. $\int_K^\infty S_{0.25} g(S_{0.25}; S_0) dS_{0.25} = 52.01$ where $g(S_{0.25}; S_0)$ is the probability density of $S_{0.25}$ conditional on S_0.
v. The conditional expectation of the stock price in 3 months, conditional on the stock price being less than K is:

$$E[S_{0.25} \mid S_{0.25} < K] = 67.69.$$

Calculate the current price of the stock, S_0.

Chapter 20:
Problem
20.1,

20.12

Additional Problem 5

You are given the following information:

i. $S(t)$ is the value of the British pound in U.S. dollars at time t.
ii. $\dfrac{dS(t)}{S(t)} = 0.05dt + 0.3dZ(t)$.
iii. The continuously compounded risk-free interest rate in the U.S. is $r = 0.06$.
iv. The continuously compounded risk-free interest rate in Great Britian is $r^* = 0.09$.
v. $G(t) = S(t)e^{(r-r^*)(T-t)}$ is the forward price in U.S. dollars per British pound, and T is the maturity time of the currency forward contract.

Based on Itô's lemma, Find $dG(t)$.

Additional Problem 6

The expression for the price of Stock X is:

$$X(t) = 15e^{0.14+0.07t}e^{0.42Z(t)}$$

The price of Stock Q is governed by the following Itô's process

$$dQ(t) = 0.08Q(t)dt + 0.13Q(t)dZ(t)$$

where $\{Z(t)\}$ is a standard Brownian motion.
Neither Stock X nor Stock Q pay dividends.
Calculate the continuously compounded risk-free rate of return.

End

When using the given standard normal distribution table, do not interpolate.

- Use the nearest z-value in the table to find the probability. Example: Suppose that you are to find $\Pr(Z < 0.759)$, where Z denotes a standard normal random variable. Because the z-value in the table nearest to 0.759 is 0.76, your answer is $\Pr(Z < 0.76) = 0.7764$.
- Use the nearest probability value in the table to find the z-value. Example: Suppose that you are to find z such that $\Pr(Z < z) = 0.7$. Because the probability value in the table nearest to 0.7 is 0.6985, your answer is 0.52.

NORMAL DISTRIBUTION TABLE

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z<z)

The value of z to the first decimal is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Values of z for selected values of Pr(Z<z)							
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995