MFE5130 – Financial Derivatives
First Term, 2019 – 20

Assignment 1 (Solution)

Chapter 2:

Question 2.4

(a) The payoff to a long forward at expiration is equal to:

Payoff to long forward = Spot price at expiration – forward price = $S_{0.5} - K$

where $S_{0.5}$ is the spot price at expiration.

Therefore, we can construct the following table:

Price of asset in six months	Agreed forward price	Payoff to the long forward
40	50	−10
45	50	−5
50	50	0
55	50	5
60	50	10

b) The payoff to a purchased call option at expiration is:

Payoff to call option = max[0, spot price at expiration – strike price] = max[0, $S_{0.5} - K$]

The strike (K) is given: It is $50. Therefore, we can construct the following table:

Price of asset in six months	Strike price	Payoff to the call option
40	50	0
45	50	0
50	50	0
55	50	5
60	50	10

c) If we compare the two contracts, we immediately see that the call option has a protection for adverse movements in the price of the asset: If the spot price is below $50, the buyer of the call option can walk away and need not incur a loss. The buyer of the long forward incurs a loss, but he has the same payoff as the buyer of the call option if the spot price is above $50. Therefore, the call option should be more expensive. It is this attractive option to walk away that we have to pay for.

Question 2.14

In order to be able to draw profit diagrams, we need to find the future values of the put premia. They are:

a) 35-strike put: $1.53 × (1 + 0.08) = $1.6524

b) 40-strike put: $3.26 × (1 + 0.08) = $3.5208

c) 45-strike put: $5.75 × (1 + 0.08) = $6.21

Payoff@T

i) 35-strike put: max(35 – S_T, 0)

ii) 40-strike put: max(40 – S_T, 0)

iii) 45-strike put: max(45 – S_T, 0)

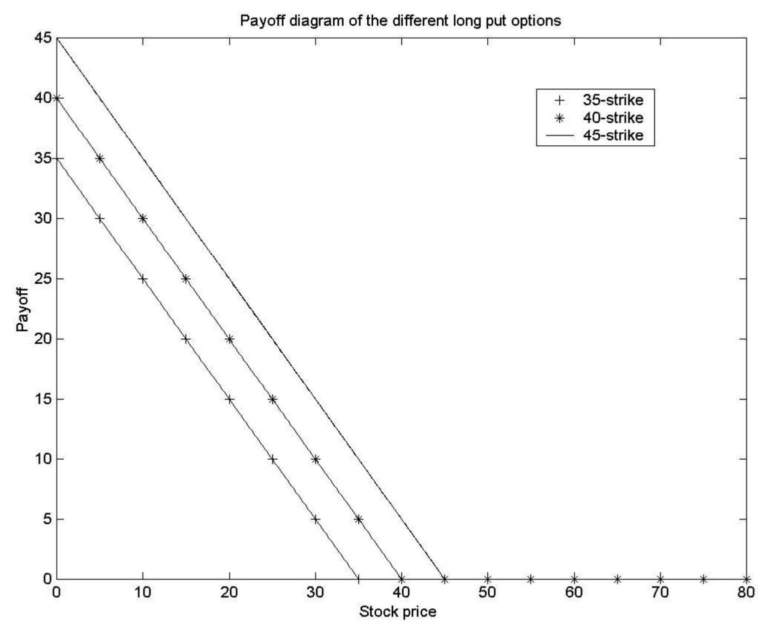
Profit@T

i) 35-strike put: max(35 – S_T, 0) – $1.6524

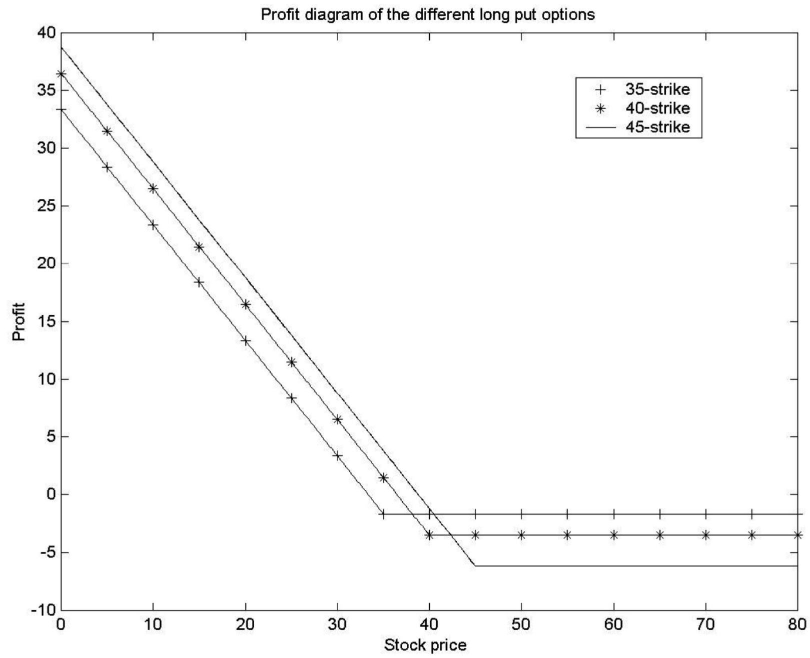
ii) 40-strike put: max(40 – S_T, 0) – $3.5208

iii) 45-strike put: max(45 – S_T, 0) – $6.21

We get the following payoff diagrams:



We get the profit diagram by deducting the option premia from the payoff graphs. The profit diagram looks as follows:



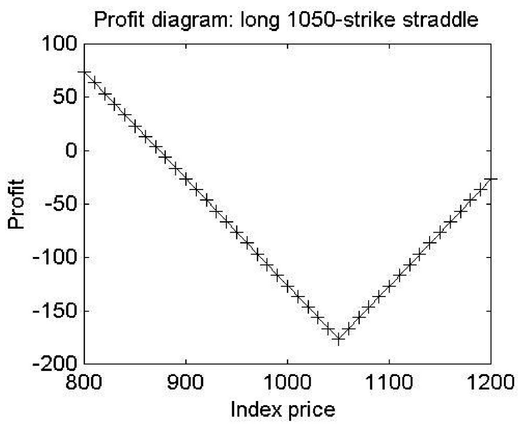
Intuitively, whenever the 35-strike put option pays off (i.e., has a payoff bigger than zero), the 40-strike and the 45-strike options also pay off. However, there are some instances in which the 40-strike option pays off and the 35-strike option does not. Similarly, there are some instances in which the 45-strike option pays off but neither the 40-strike nor the 35-strike pay off. Therefore, the 45-strike offers more potential than the 40- and 35-strike, and the 40-strike offers more potential than the 35-strike. We pay for these additional payoff possibilities by initially paying a higher premium. It makes sense that the premium is increasing in the strike price.

Chapter 3:

Question 3.13

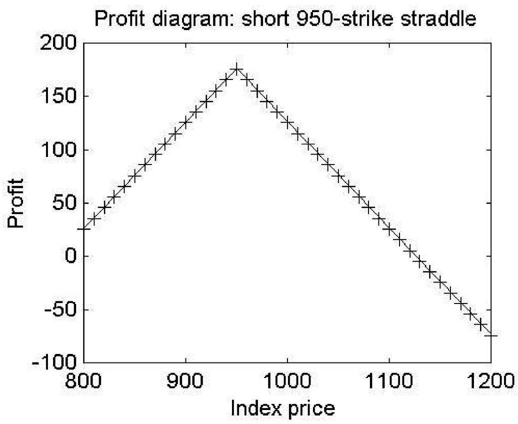
a) Payoff@6 months = max(1050 – $S_{0.5}$, 0) + max($S_{0.5}$ – 1050, 0)
Profit@6 months
= max(1050 – $S_{0.5}$, 0) + max($S_{0.5}$ – 1050, 0) – (101.214 + 71.802)(1.02)
= max(1050 – $S_{0.5}$, 0) + max($S_{0.5}$ – 1050, 0) – 176.48

	Payoff@6 months	Profit@6 months
$S_{0.5} < 1050$	$1050 - S_{0.5}$	$873.52 - S_{0.5}$
$S_{0.5} \geq 1050$	$S_{0.5} - 1050$	$S_{0.5} - 1226.48$



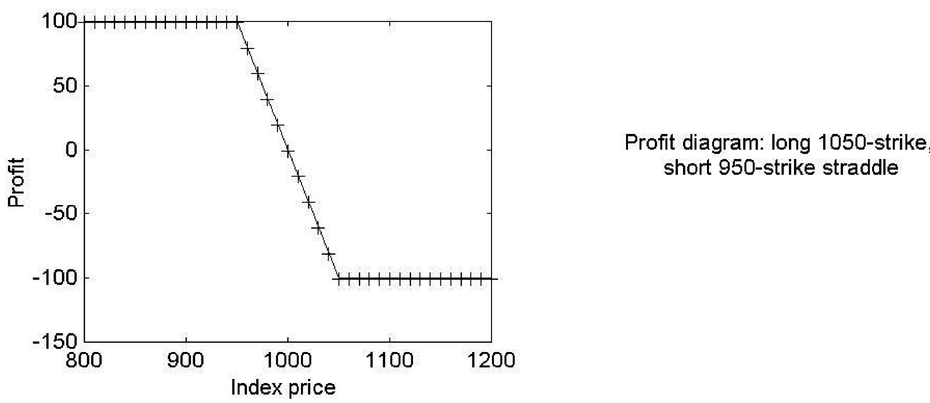
b) Payoff@6 months = –max(950 – $S_{0.5}$, 0) – max($S_{0.5}$ – 950, 0)
Profit@6 months
= –max(950 – $S_{0.5}$, 0) – max($S_{0.5}$ – 950, 0) + (51.777 + 120.405)(1.02)
= –max(950 – $S_{0.5}$, 0) – max($S_{0.5}$ – 950, 0) + 175.63

	Payoff@6 months	Profit@6 months
$S_{0.5} < 950$	$-950 + S_{0.5}$	$-774.64 + S_{0.5}$
$S_{0.5} \geq 950$	$-S_{0.5} + 950$	$-S_{0.5} + 1125.63$



(c)

	Payoff@6 months	Profit@6 months
$S_{0.5} < 950$	100	$100 - (176.48 - 175.63) = 99.15$
$950 \leq S_{0.5} < 1050$	$2000 - 2S_{0.5}$	$1999.15 - 2S_{0.5}$
$S_{0.5} \geq 1050$	-100	-100.85



We can see that the aggregation of the bought and sold straddle resembles a bear spread.

<u>Question 3.14</u>

a) This question deals with the option trading strategy known as Box spread. We saw earlier that if we deal with options and the maximum function, it is convenient to split the future values of the index into different regions. Let us name the final value of the S&R index S_T. We have two strike prices; therefore, we will use three regions: one in which $S_T < 950$, one in which $950 \leq S_T < 1,000$, and another one in which $S_T \geq 1,000$. We then look at each region separately and hope to be able to see that indeed when we aggregate, there is no S&R risk when we look at the aggregate position.

Instrument	$S_T < 950$	$950 \leq S_T < 1{,}000$	$S_T \geq 1{,}000$
Long 950 call	0	$S_T - \$950$	$S_T - \$950$
Short 1,000 call	0	0	$\$1{,}000 - S_T$
Short 950 put	$S_T - \$950$	0	0
Long 1,000 put	$\$1{,}000 - S_T$	$\$1{,}000 - S_T$	0
Total	\$50	\$50	\$50

We see that there is no occurrence of the final index value in the row labeled total. The option position does not contain S&R price risk.

b) The initial cost is the sum of the long option premia less the premia we receive for the sold options. We have:

$$\text{Cost } \$120.405 - \$93.809 - \$51.77 + \$74.201 = \$49.027$$

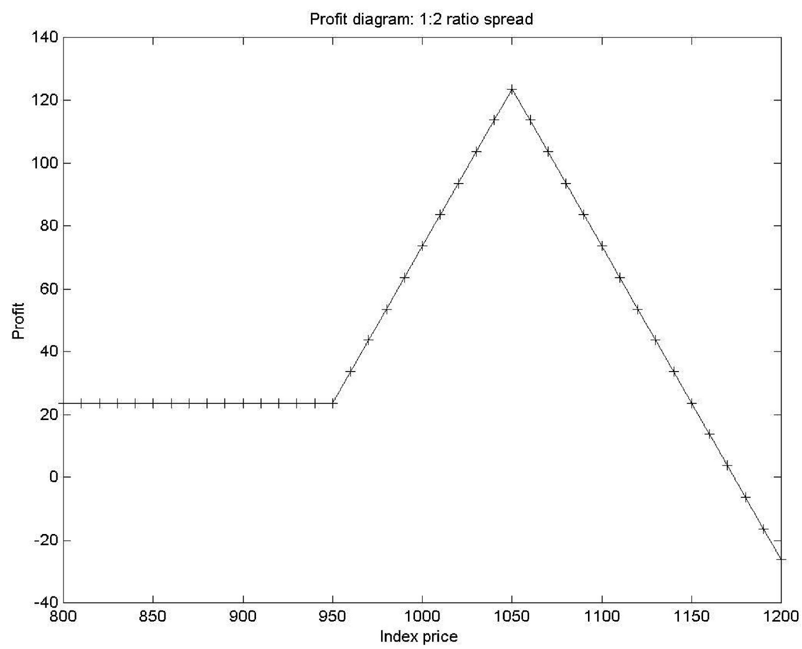
c) The payoff of the position after six months is \$50, as we can see from the above table.

d) The implicit interest rate of the cash flows is: \$50.00 ÷ \$49.027 = 1.019 ~= 1.02. The implicit interest rate is indeed 2%.

Question 3.15

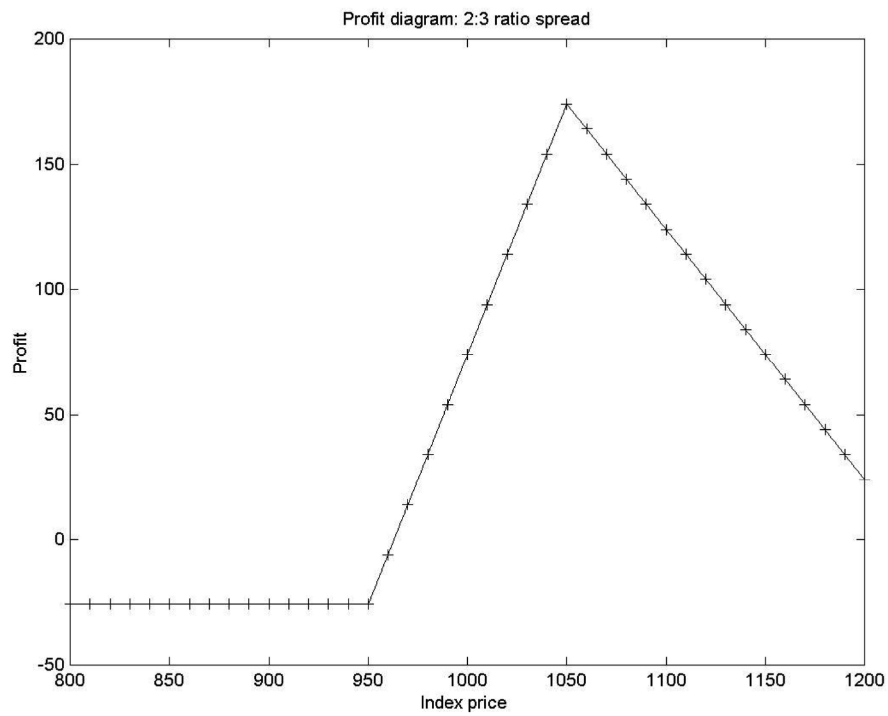
a) The future value of the cost of the 1:2 950-, 1050-strike ratio call spread @6 months $= (\$120.405 - 2 \times \$71.802) \times 1.02 = -\$23.66$.

	Payoff@6 months	Profit@6 months
$S_{0.5} < 950$	0	23.66
$950 \le S_{0.5} < 1050$	$S_{0.5} - 950$	$S_{0.5} - 926.34$
$S_{0.5} \ge 1050$	$1150 - S_{0.5}$	$1173.66 - S_{0.5}$



b) The future value of the cost of the 2:3 950-, 1050-strike ratio call spread @6 months $(2 \times \$120.405 - 3 \times \$71.802) \times 1.02 = \$25.91$.

	Payoff@6 months	Profit@6 months
$S_{0.5} < 950$	0	-25.91
$950 \le S_{0.5} < 1050$	$2S_{0.5} - 1900$	$2S_{0.5} - 1925.91$
$S_{0.5} \ge 1050$	$1250 - S_{0.5}$	$1224.09 - S_{0.5}$



c) We saw that in part (a), we were receiving money from our position, and in part (b), we had to pay a net option premium to establish the position. This suggests that the true ratio n/m lies between 1:2 and 2:3.

Indeed, we can calculate the ratio n/m as:

$$
\begin{aligned}
& n \times \$120.405 - m \times \$71.802 && = 0 \\
\Leftrightarrow\ & n \times \$120.405 && = m \times \$71.802 \\
\Leftrightarrow\ & n/m && = \$71.802/\$120.405 \\
\Leftrightarrow\ & n/m && = 0.596
\end{aligned}
$$

which is approximately 3:5.

Question 3.16

A bull spread or a bear spread can never have an initial premium of zero because we are buying the same number of calls (or puts) that we are selling and the two legs of the bull and bear spreads have different strikes. A zero initial premium would mean that two calls (or puts) with different strikes have the same price—and we know by now that two instruments that have different payoff structures and the same underlying risk cannot have the same price without creating an arbitrage opportunity.

A symmetric butterfly spread cannot have a premium of zero because it would violate the convexity condition of options.

Question 3.17

From P.54 in Chapter 3 of the lecture notes we learn how to calculate the right ratio λ. It is equal to:

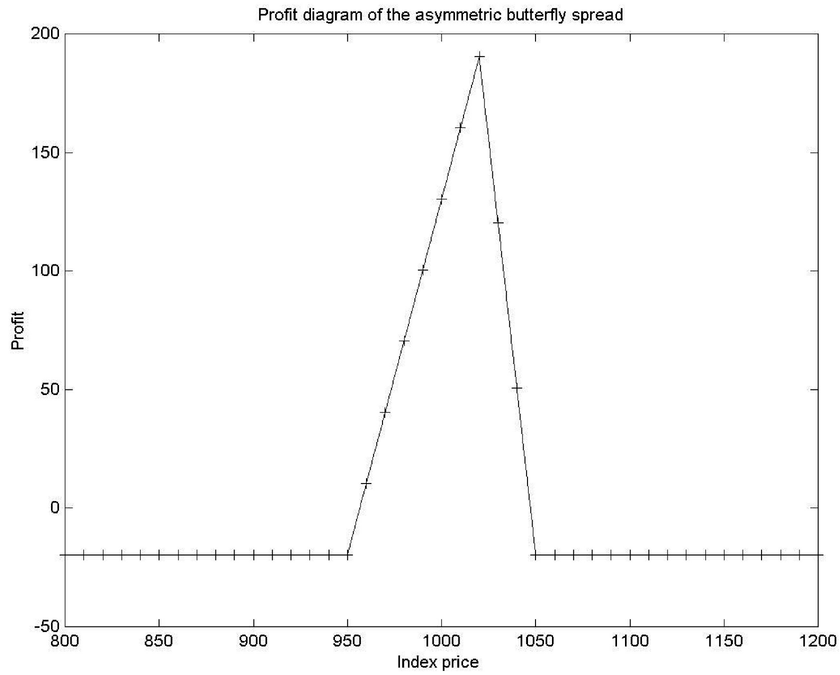
$$\lambda = \frac{K_3 - K_2}{K_3 - K_1} = \frac{1,050 - 1,020}{1,050 - 950} = 0.3$$

In order to construct the asymmetric butterfly, for every 1,020-strike call we write, we buy λ 950-strike calls and $(1 - \lambda)$ 1,050-strike calls. Since we can only buy whole units of calls, we will in this example buy three 950-strike and seven 1,050-strike calls, and sell ten 1,020-strike calls.

The future value of the cost @6 months
$= (3 \times 120.405 - 10 \times 84.470 + 7 \times 71.802)(1.02) = 19.51$

	Payoff@6 months	Profit@6 months
$S_{0.5} < 950$	0	-19.51
$950 \leq S_{0.5} < 1020$	$3S_{0.5} - 2850$	$3S_{0.5} - 2869.51$
$1020 \leq S_{0.5} < 1050$	$7350 - 7S_{0.5}$	$7330.49 - 7S_{0.5}$
$S_{0.5} \geq 1050$	0	-19.51

The profit diagram looks as follows:



Additional Problem 1

We have the following for the different trades and different dealers:

	A	B	C
Trade 1	+9	–9	
Trade 2		+3	–3
Trade 3	–3		+3
Trade 4	+2		–2
Total	+8	–6	–2

Trading volume is equal to 9 + 3 + 3 + 2 = 17.
From the table in (a), we have the 8 outstand contracts (A long 8 contracts, B short 6 contracts and C short 2 contracts). So, the open interest is 8 contracts.

Additional Problem 2
Let Call(K, t) and Put(K, t) denote the premiums of options with strike price K and time to expiration of t, and $F_{s,t}$ be the forward price at time s with time to expiration of t.

At time 0, for the call, put and forwards with time to expiration of t, the put-call parity states that

$$\text{Call}(K,t) - \text{Put}(K,t) = PV\left(F_{0,t} - K\right).$$

Now, we take T be the maturity of the call, put and forwards.
At time T (maturity of the options and forwards), the time to expiration is 0.
At time 0, the time to expiration is T.

The profit at time T of the K_1–K_2–K_3 butterfly spread which is constructed by straddle and strangle is given by

$$
\begin{aligned}
&-\mathrm{Call}(K_2,0)-\mathrm{Put}(K_2,0)+\mathrm{Put}(K_1,0)+\mathrm{Call}(K_3,0)-\\
&FV\left(-\mathrm{Call}(K_2,T)-\mathrm{Put}(K_2,T)+\mathrm{Put}(K_1,T)+\mathrm{Call}(K_3,T)\right)\\
&=-\mathrm{Call}(K_2,0)-\left[\mathrm{Call}(K_2,0)+PV\left(F_{T,0}-K_2\right)\right]+\\
&\left[\mathrm{Call}(K_1,0)+PV\left(F_{T,0}-K_1\right)\right]+\mathrm{Call}(K_3,0)-\\
&FV\left(-\mathrm{Call}(K_2,T)-\left[\mathrm{Call}(K_2,T)+PV\left(F_{0,T}-K_2\right)\right]+\left[\mathrm{Call}(K_1,T)+PV\left(F_{0,T}-K_1\right)\right]+\mathrm{Call}(K_3,T)\right)\\
&=\mathrm{Call}(K_1,0)-2\mathrm{Call}(K_2,0)+\mathrm{Call}(K_3,0)+(K_2-K_1)-\\
&FV\left(\mathrm{Call}(K_1,T)-2\mathrm{Call}(K_2,T)+\mathrm{Call}(K_3,T)+PV(K_2-K_1)\right)\\
&=\mathrm{Call}(K_1,0)-2\mathrm{Call}(K_2,0)+\mathrm{Call}(K_3,0)-\\
&FV\left(\mathrm{Call}(K_1,T)-2\mathrm{Call}(K_2,T)+\mathrm{Call}(K_3,T)\right).
\end{aligned}
$$

Hence, we showed the result.