MFE5130 – Financial Derivatives

First Term, 2019-20

Midterm Examination (Solution)

Question 1

(a)

We make use of put-call parity,

$$C(75,T)-P(75,T)=S-75e^{-rT}$$
$$P(75,T)+S=C(75,T)+75e^{-rT}.$$

When the strike price for a call is increased, its prices goes down, so

$$P(75,T)+S=C(75,T)+75e^{-rT}\geq C(80,T)+75e^{-rT}>C(80,T)+70e^{-rT}.$$

Similarly, we have

$$P(75,T)+S=C(75,T)+75e^{-rT}\leq C(60,T)+75e^{-rT}<C(60,T)+80e^{-rT}.$$

Therefore, the answer is (III) < (II) < (I).

Question 2

Let $X(t) = \max(S_1(t), S_2(t))$.

The payoff of the European option can be rewritten as

$$\max\left[\max\left(S_1(3), S_2(3)\right) - 25, 0\right] = \max\left[X(3) - 25, 0\right].$$

Hence, the European option can be regarded as a call option on asset X with strike price of 25.

Since

$$\max[S_1(3), S_2(3), 25] = \max[\max(S_1(3), S_2(3)), 25],$$

the payoff of the given financial claim can be rewritten as

$$\begin{aligned}\max\left[S_1(3), S_2(3), 25\right] &= \max\left[X(3), 25\right] \\ &= \max\left[X(3) - 25, 0\right] + 25.\end{aligned}$$

Hence, the given claim is made of

i. long one unit of 25-strike 3-year call option on asset X and
ii. long a 3-year zero-coupon bond with face value of 25.

Let $C(25, 3)$ be the current price of the 25-strike 3-year call option on asset X.

By no-arbitrage, we have

$$\begin{aligned}25.50 &= C(25,3) + 25e^{-3\times 0.08} \\ C(25,3) &= 25.50 - 25e^{-3\times 0.08} \\ C(25,3) &= 5.8343.\end{aligned}$$

Question 3

Let x_0 = €0.7 per 1 CAD and K = €0.625.

The euro-denominated CAD call is related to the CAD-denominated euro put by the equation

$$C_{\text{euro}}\left(x_0, K, T\right) = x_0 K P_{\text{CAD}}\left(\frac{1}{x_0}, \frac{1}{K}, T\right).$$

Thus,

$$\begin{aligned}
P_{\text{CAD}}\left(\frac{1}{0.7}, \frac{1}{0.625}, 0.5\right) &= \frac{1}{0.7\times 0.625}\times C_{\text{euro}}\left(0.7, 0.625, 0.5\right) \\
P_{\text{CAD}}\left(\frac{1}{0.7}, 1.6, 0.5\right) &= \frac{1}{0.7\times 0.625}\times 0.08 \\
&= 0.1829.
\end{aligned}$$

Using the put-call parity by treating the euro-denominated continuously compounded interest rate 8% as the dividend yield of the underlying asset (euro), we have

$$\begin{aligned}
C_{\text{CAD}}\left(\frac{1}{0.7}, 1.6, 0.5\right) - P_{\text{CAD}}\left(\frac{1}{0.7}, 1.6, 0.5\right) &= \frac{1}{x_0} e^{-r_{\text{euro}} T} - \frac{1}{K} e^{-r_{\text{CAD}} T} \\
&= \frac{1}{0.7} e^{-0.08\times 0.5} - \frac{1}{0.625} e^{-0.07\times 0.5} \\
C_{\text{CAD}}\left(\frac{1}{0.7}, 1.6, 0.5\right) &= P_{\text{CAD}}\left(\frac{1}{0.7}, 1.6, 0.5\right) + \\
&\quad \frac{1}{0.7} e^{-0.08\times 0.5} - 1.6 e^{-0.07\times 0.5} \\
&= 0.1829 + \frac{1}{0.7} e^{-0.08\times 0.5} - 1.6 e^{-0.07\times 0.5} \\
&= \text{CAD } 0.0105.
\end{aligned}$$

Question 4

Let $C(K)$ be the price of the K-strike call option.

The prices of the options **violate** the following inequality

$$\frac{C(K_1)-C(K_2)}{K_2-K_1} \geq \frac{C(K_2)-C(K_3)}{K_3-K_2}, \quad \text{where } K_1 < K_2 < K_3.$$

Because:

$$\frac{18-14}{55-50} < \frac{14-9.5}{60-55}$$
$$\frac{4}{5} < \frac{4.5}{5}.$$

Based on the given option prices, we have

$$\frac{C(50)-C(55)}{5} < \frac{C(55)-C(60)}{5}$$
$$-C(50)+2C(55)-C(60) > 0.$$

So, the arbitrage is available using the following transactions (symmetric butterfly spread):

Buy 1 of the 50-strike call option

Sell 2 of the 55-strike call options

Buy 1 of the 60-strike call option.

Let S_1 be the price of the underlying asset at t =1. The payoff table of the butterfly spread is given as:

		***t* = 1 year**			
Transaction	***t* = 0**	$S_1 < 50$	$50 \leq S_1 \leq 55$	$55 < S_1 \leq 60$	$S_1 > 60$
Buy 1 of $C(50)$	–18	0.00	$S_1 - 50$	$S_1 - 50$	$S_1 - 50$
Sell 2 of $C(55)$	2(14.00)	0.00	0.00	$-2(S_1 - 55)$	$-2(S_1 - 55)$
Buy 1 of $C(60)$	–9.5	0.00	0.00	0.00	$S_1 - 60$
Total	0.5	0.00	$S_1 - 50$	$60 - S_1$	0.00

If S_1 = \$53, then the arbitrage profits at the end of 1 year are

$$X = 0.5e^{0.08} + S_1 - 50 = 0.5e^{0.08} + 53 - 50 = 3.5416.$$

If S_1 = \$58, then the arbitrage profits at the end of 1 year are

$$Y = 0.5e^{0.08} + 60 - S_1 = 0.5e^{0.08} + 60 - 58 = 2.5416.$$

Question 5

(a)

Let $P(0, s)$ be the current price of a s-year zero-coupon bond with the face value of \$1.

From the given one-year implied forward rate, $r_0(t-1, t)$, we have

$$1+r_0(t-1,t)=\frac{P(0,t-1)}{P(0,t)}$$
$$P(0,t)=\frac{P(0,t-1)}{1+r_0(t-1,t)}.$$

Hence,

$$P(0,1)=\frac{P(0,0)}{1+r_0(0,1)}=\frac{1}{1+5.31\%}=0.9496$$
$$P(0,2)=\frac{P(0,1)}{1+r_0(1,2)}=\frac{0.9496}{1+6.81\%}=0.8891$$
$$P(0,3)=\frac{P(0,2)}{1+r_0(2,3)}=\frac{0.8891}{1+8.13\%}=0.8223$$

It is known that

$$F_{0,T}=\frac{S_0e^{(u-y)T}}{P(0,T)},$$

where S_0 is the current spot price of soybean, u is the continuously compounded storage cost of soybean and y is the convenience yield of soybean.

So,

$$F_{0,T}=\frac{5.2e^{(2.3\%-5.1\%)T}}{P(0,T)}=\frac{5.2e^{-(2.8\%)T}}{P(0,T)}.$$

T (in Years)	1	2	3
$F_{0,T}$	5.3248	5.5301	5.8142

(b)

Let R be the fixed swap price for one bushel of soybean.

$$R = \frac{\sum_{k=1}^{3} F_{0,k} P(0,k)}{\sum_{k=1}^{3} P(0,k)}$$

$$= \frac{5.3248(0.9496) + 5.5301(0.8891) + 5.8142(0.8223)}{0.9496 + 0.8891 + 0.8223}$$

$$= 5.5446.$$

Hence, the fixed swap price for one bushel of soybean is \$5.5446.