MFE5130 – Financial Derivatives

Class Activity (25-October-2018) (Solution)

Important Notes:

1. This class activity is counted toward to your class participation score. **Fail** to hand in this class activity worksheet in the class will receive **0 score** for that class.
2. **0 mark** will be received if you leave the solution blank.

Name:	Student No.:

Problem 1

The price of a share of stock is \$65. The stock pays dividends at a continuously compounded rate of 5%. The stock's volatility is 27%. The price evolution of the stock follows the forward tree with each period being 1 year in length.

The continuously compounded risk-free interest rate is 8%. A 1-year European put option on the stock has a strike price of \$63. The market price of the put option is \$6.00.

An arbitrageur constructs a strategy involving the purchase or sale of exactly one of the European put options.

Determine the accumulated arbitrage profits at the end of one year.

Solution

The values of u and d are

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.08-0.05)(1)+0.27\sqrt{1}} = 1.34986,$$
$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(0.08-0.05)(1)-0.27\sqrt{1}} = 0.78663.$$

The possible stock prices are therefore:

$$65u = 65(1.34986) = 87.7409,$$
$$65d = 65(0.78663) = 51.1310.$$

If the stock price moves up, then the option pays \$0. If the stock price moves down, then the option pays \$11.8692.

$$\begin{array}{ccccc} & 87.7409 & & & 0 \\ 65 & & & V & \\ & 51.1310 & & & 11.8690 \end{array}$$

To replicate the option, the investor must purchase delta shares of the underlying stock:

$$\Delta = e^{-\delta h}\frac{V_u - V_d}{S(u-d)} = e^{-0.05(1)}\frac{0-11.8690}{87.7409-51.131} = -0.3084.$$

To replicate the option, the investor must lend B:

$$B = e^{-rh}\frac{uV_d - dV_u}{u-d} = e^{-0.08(1)}\frac{1.34986(0)-0.78663(11.869)}{1.34986-0.78663} = 26.2587.$$

The purchase of the option can therefore be replicated by selling 0.3084 shares of stock and lending \$26.2587.

Therefore, the price of the put option should be:

$$V = S_0\Delta + B = 65(-0.3084) + 26.2587 = 6.2127.$$

But the market price of the option is \$6.00. Therefore an arbitrageur can earn an arbitrage profit by purchasing the option for \$6.00 and synthetically selling it for \$6.2127.

The synthetic sale of the option requires that the arbitrageur buy 0.3084 shares of stock and borrow \$26.2587.

		$t = 1$	
Transaction	$t = 0$	$S_1 = 87.7409$	$S_1 = 51.1310$
Buy the put option	−6.000	0	11.8690
Sell the synthetic put option	6.2127	0	−11.8690
Total	0.2127	0	0

The accumulated arbitrage profits at the end of one year = $0.2127e^{0.08} = 0.2304$.