**MFE5130 – Financial Derivatives**

**First Term, 2015-16**

**Midterm Examination (Solution)**

Question 1

Let *P*(*K*) is the price of the *K*-strike put option.

The prices of the options **violate** the following inequality



Because:



Based on the given option prices, we have



So, the arbitrage is available using the following transactions (asymmetric butterfly spread):

Buy *λ* of the 50-strike put options

Sell 1 of the 55-strike put options

Buy (1 – *λ*) of the 61-strike put options

where 

In the payoff table below, we have scaled the strategy up by multiplying 11:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | ***t* = 1 year** | | | |
| **Transaction** | ***t* = 0** | *S*1 < 50 | 50 ≤ *S*1 ≤ 55 | 55 < *S*1 ≤ 61 | 61 < *S*1 |
| Buy 6 of *P*(50) | –6(3.00) | 6(50 – *S*1) | 0.00 | 0.00 | 0.00 |
| Sell 11 of *P*(55) | 11(7.00) | –11(55 – *S*1) | –11(55 – *S*1) | 0.00 | 0.00 |
| Buy 5 of *P*(61) | –5(11.00) | 5(61 – *S*1) | 5(61 – *S*1) | 5(61 – *S*1) | 0.00 |
| Total | 4.00 | 0.00 | 6*S*1 – 300 | 305 – 5*S*1 | 0.00 |

If the final stock price is $52, then the arbitrage profits are



If the final stock price is $60, then the arbitrage profits are



Hence, 

*Alternative Solution*

In particular for this question, the arbitrage can also be achieved by using the following transactions:

Buy 0.5 of the 50-strike put options

Sell 1 of the 55-strike put options

Buy 0.5 of the 61-strike put options

In the payoff table below, we have scaled the strategy up by multiplying 2:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | ***t* = 1 year** | | | |
| **Transaction** | ***t* = 0** | *S*1 < 50 | 50 ≤ *S*1 ≤ 55 | 55 < *S*1 ≤ 61 | 61 < *S*1 |
| Buy 1 of *P*(50) | –3.00 | 50 – *S*1 | 0.00 | 0.00 | 0.00 |
| Sell 2 of *P*(55) | 2(7.00) | –2(55 – *S*1) | –2(55 – *S*1) | 0.00 | 0.00 |
| Buy 1 of *P*(61) | –11.00 | 61 – *S*1 | 61 – *S*1 | 61 – *S*1 | 0.00 |
| Total | 0.00 | 1.00 | *S*1 – 49 | 61 – *S*1 | 0.00 |

If the final stock price is $52, then the arbitrage profits are



If the final stock price is $60, then the arbitrage profits are



Hence, 

Question 2

Let *x*0 = $0.011/¥ and *K* = $0.008.

By the put-call parity, we have



The dollar-denominated call option on yen is related to the yen-denominated put option on dollars by the equation



Thus,



Question 3

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Quarter | Swap price | Zero-coupon bond price |  | Forward price |  |
| 1 | 3.25 | 0.9712 |  | *F*0,0.25=3.25 |  |
| 2 | 3.52 | 0.9655 |  | *F*0,0.5 = 3.79 |  |
| 3 | 3.41 | 0.9508 |  | *F*0,0.75 = 3.19 |  |
| 4 | 3.34 | 0.9312 |  | *F*0,1 = 3.12 |  |









### Question 4

### The synthetic short forward position is created by buying one 55-strike put and selling one 55-strike call (see P.17 of Chapter 3 of the lecture notes or P.68 of the textbook). Hence, the cost of the synthetic short forward position is

### Cost at time 0 = $5.83 – $1.90 = $3.93.

### *FV*(Cost) at time six months =$3.93*e*3.92%×0.5 = $4.0078..

The payoff and profit of the synthetic short forward position at time six months is given by

|  |  |  |
| --- | --- | --- |
|  | *S*0.5 ≤ 55 | *S*0.5 > 55 |
| Buying one 55-strike put | 55 – *S*0.5 | 0 |
| Selling one 55-strike call | 0 | – ( *S*0.5 – 55) = 55 – *S*0.5 |
| Payoff of the synthetic short forward position | 55 – *S*0.5 | 55 – *S*0.5 |
| Profit of the synthetic short forward position | 55 – *S*0.5 – 4.0078  = $50.9922 – *S*0.5 | 55 – *S*0.5 – 4.0078  = $50.9922 – *S*0.5 |

### Question 5

First, we need to find the fair value of the forward price. We plug the continuously compounded interest rate, the dividend yield and the time to expiration in years into the valuation formula and notice that the time to expiration is 1 year. We have:

*F*0,*T* = *S*0 × *e*(*r* −*δ*)×*T* = $1,100 × *e*(0.05−0.03) = $1,100 × 1.0202 = $1,122.22.

If we observe a forward price of $1,118, we know that the forward is too cheap, relative to the fair value we have determined. Therefore, we will buy the forward at $1,118, and create a synthetic short forward for $1,122.22, thus making an arbitrage profit as shown in the following table.

|  |  |  |
| --- | --- | --- |
| Description | Today | In 1 year |
| Long forward | 0 | *S*1 − $1,118 |
| Sell short tailed position in | $1,100 *e*−0.03 | −*S*1 |
| index | = $1,067.49 |  |
| Lend $1,067.49 | −$1,067.49 | 1067.49 *e*0.05  = $1,122.22 |
| TOTAL | 0 | $4.22 |

This position requires no initial investment, has no index price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy.

Question 6

1. Let us take the December Year 0 forward price as a proxy for the spot price in December Year 0. We can then calculate a reverse cash and carry arbitrage tableau:

|  |  |  |
| --- | --- | --- |
| Transaction | Time 0 | Time *T* = 6/12 |
| Long June of Year 1 forward | 0 | *ST* − 3.152 |
| Short widget | +3.00 | −*ST* |
| Lend money | −3.00 | +3*e*0.06×0.5 = 3.0914 |
| Lease payment to widget lender | 0 | −*L* |
| Total | 0 | −0.0606 – *L* |

To make the transaction fair to both parties, the lease payment from the widget borrower to the widget lender is −0.0606. We will receive 0.0606 from the widget lender.

b) Let us take the December Year 0 forward price as a proxy for the spot price in December Year 0. We can then calculate a reverse cash and carry arbitrage tableau:

|  |  |  |
| --- | --- | --- |
| Transaction | Time 0 | Time *T* = 15/12 |
| Long March of Year 2 forward | 0 | *ST* – 2.894 |
| Short widget | +3.00 | −*ST* |
| Lend money | −3.00 | +3*e*0.06×1.25 = 3.2337 |
| Lease payment to widget lender | 0 | −*L* |
| Total | 0 | 0.3397 − *L* |

To make the transaction fair to both parties, the lease payment from the widget borrower to the widget lender is 0.3397.

*Alternative Method 1*

We could use the equation for the amount of the lease payment (*L*) in the lecture notes (P.20 of Chapter 6) directly for both (a) and (b). However, it is **important** to pay attention to the sign of *L*. In the lecture notes, we use *L* in the lecture notes (P.19 of Chapter 6) instead of –*L*.

1. *L* = *F*0,0.5 – *S*0 *e*0.06×0.5 = 3.152 – 3*e*0.06×0.5 = 0.0606 > 0.

So, the widget borrower will receive 0.0606 from the widget lender.

1. *L* = *F*0,1.25 – *S*0 *e*0.06×1.25 = 2.894 – 3*e*0.06×1.25 = –0.3397 < 0.

So, the widget borrower will pay 0.3397 to the widget lender.

*Alternative Method 2*

You could find the lease rate *δl* from the following formula



and then use the equation in *Alternative Method 1* to find *L*. Since *F*0,*T* has already been given, it is *unnecessary* to find *δl* in order to use the equation for finding *L*.

1. *T* = 0.5 and *F*0*,*0.5 = 3.152.





So, the widget borrower will receive 0.0606 from the widget lender.

1. *T* = 1.25 and *F*0*,*1.25 = 2.894.





So, the widget borrower will pay 0.3397 to the widget lender.