**MFE5130 – Financial Derivatives**

**First Term, 2016-17**

**Midterm Examination (Solution)**

Question 1

(a)



We could represent “The % gain of the S&P index at the end of year 3 to be capped at 105%” as



So,



|  |  |  |  |
| --- | --- | --- | --- |
|  | 0 < *S*3 < 1,200 | 1,200 ≤ *S*3 < 2,460 | *S*3 ≥ 2,460 |
| max(0, min(*S*3 – 1,200, 1,260)) | 0 | *S*3 – 1,200 | 1,260 |

The final payoff can then be rewritten as



Hence, *K*1 = 1,200 and *K*2 = 2,460.

(b)

A bull spread which is constructed by buying a 1,200-strike call and selling a 2,460-strike call.

(c)



Question 2

The prices of the European put options violate



where *P*(*K*, 1) is the price of the European put option with strike price *K* and 1 year until expiration,

because when *K*1 = 50 and *K*2 = 55, we have:



Arbitrage is available using a put bull spread:

Buy 50-strike put and sell 55-strike put.

The cost of the put bull spread = 4 – 8.75 = –4.75.

The strategy produces the following payoff table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | **Time 1** |  |
| **Transaction** | **Time 0** | *S*1 < 50 | 50 ≤ *S*1 ≤ 55 | 55 < *S*1 |
| Buy 50-strike put | –4.00 | 50 – *S*1 | 0 | 0 |
| Sell 55-strike put | 8.75 | –(55 – *S*1) | –(55 – *S*1) | 0 |
| Total | 4.75 | –5.00 | –(55 – *S*1) | 0 |

If the final stock price is $48, then the accumulated arbitrage profits are:

*X* = –5 – (–4.75 *e*0.09) = 0.1973.

If the final stock price is $52, then the accumulated arbitrage profits are:

*Y* = –(55 – 52) – (–4.75 *e*0.09) = 2.1973.

Hence,



Question 3

(a)

Let *R* be the fixed swap price per barrel of oil.



(b)

Let *S*1.5 be the spot price of the oil in 18 months.

The net cash flow of the transportation company in 18 months

= 1,000×(*S*1.5 – 59.18)

= 1,000×(57 – 59.18)

= –$2,180.

The **negative sign** means that $2,180 is the cash outflow of the transportation company.

(c)

Let *R*new be the new fixed swap price per barrel of oil in the swap.





Question 4

**Important Notes**:

|  |
| --- |
| In this question, since the underlying pays **discrete** dividend, we **CANNOT** short the tailed position (one unit of the underlying – PV(dividend)) at *t* = 0 and get one unit of the underlying at the maturity. By referring the detailed solution below, we must short sell one unit of underlying at *t* = 0. Also, the timing of the cash flow should be accurate such as the dividend, $3.2, is at *t* = 0.5. |

The theoretical forward price = (60.25 – 3.2*e*–4% ×0.5) *e*4% ×0.75 = 58.85.

Now, we have the observed market forward price, $52.13, is **less than** the theoretical forward price. So, the strategy to realize the arbitrage profit is to long the forward contract and short the synthetic forward.

|  |  |  |  |
| --- | --- | --- | --- |
| **Transactions** | **Cash Flows** | | |
| *t* = 0 | *t* = 0.5 | *t* = 0.75 |
| Long one forward | 0 | 0 | *S*0.75 – 52.13 |
| Short sell one share of the stock | 60.25 | 0 | –*S*0.75 |
| Lend $60.25 at *t* = 0 | −60.25 | 0 | 60.25*e*(0.04)(0.75) = 62.08 |
| Borrow $3.2 to pay the dividend to the stock lender at *t* =0.5 | 0 | 3.2 | −3.2*e*(0.04)(0.25) = −3.23 |
| Pay $3.2 to the stock lender as a dividend at *t* = 0.5 | 0 | −3.2 | 0 |
| Total | 0 | 0 | 6.72 |

This position requires no initial investment, has no stock price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy. The accumulated arbitrage profits at the end of 9 months is $6.72.

Question 5

To answer this question, we use put-call parity:



where *C*(*K*, *t*) and *P*(*K*, *t*) denote the premiums of call and put options with strike price *K* and **time *t* until expiration** respectively, and *F*0,*t* is the forward price of the underlying stock.

The current value of Set 1 is given by



The current value of Set 2 is given by



The current value of Set 3 is given by



The current value of Set 4 is given by



Let’s subtract the value of Set 2 from the value of Set 1 and use put-call parity to simplify:



Subtracting Set 4 from Set 3 and again using put-call parity to simplify, we obtain:

