ASSIGNMENT 6

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- **Question 2.81:**

  For what value of k is the following function continuous at the given point.

$$f(x) = \begin{cases} kx+1 & x \leq 5 \\ 3x-5 & x > 5 \end{cases}$$

Solution:

$$f(x) = \begin{cases} kx+1 & x \leq 5 \\ 3x-5 & x > 5 \end{cases}$$

Since $kx+1$ and $3x-5$ are linear polynomials(always continuous).So, for function f(x) to be continuous, it should be continuous at $x = 5$.

So,

$$lim_{x \to 5^+} f(x) = 3(5) - (5) = 10.$$

Also, $$lim_{x \to 5^-} f(x) = k(5) + 1.$$

Since for continuity,

$$lim_{x \to 5^+} f(x) = lim_{x \to 5^+} f(x).$$

or, $$10 = 5k + 1.$$

Hence, $$k = 9/5.$$

- **Question 2.82:**

  Prove that the function f given by $f(x) = |x - 1|$, $x \in R$ is not differentiable at $x = 1$.

  Solution:

  Here, $f(x) = |x - 1|,$ $x \in R$

$$f(x) = \begin{cases} x - 1 & x \geq 1 \\ -x + 1 & x < 1 \end{cases}$$

$$f(x) = \begin{cases} 1 & x \geq 1 \\ -1 & x > 1 \end{cases}$$

  Since at $x = 1$ LHS $\neq RHS(i.e, 1 \neq -1)$.

f(x) is not differentiable at $x = 1$.

- **Question 2.83:**

Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at $x = 1$ and $x = 2$.

Solution:

Since,

$f(x) = [x]$, $0 < x < 3$

$$f(x) = \begin{cases} 0 & 0 < x < 1 \\ 1 & 1 \leq x < 2 \\ 2 & 2 \leq x < 3 \end{cases}$$

$for x = 1:$

$$lim_{x\to 1-f(x)=0}.$$

$$lim_{x\to 1+f(x)=1}.$$

$$lim_{x\to 1-f(x)\neq lim_{x\to 1+f(x)}}.$$

f(x) is not continuous at 1.

Similarly,

$forx = 2 :$

$$lim_{x\to 2-f(x)=1}.$$

$$lim_{x\to 2+f(x)=2}.$$

$$lim_{x\to 1-f(x)\neq lim_{x\to 1+f(x)}}.$$

f(x) is not continuous at 2.

Since, f(x) is not differentiable at x=1 and x=2. So, it is not differentiable at x=1 and x=2.