ASSIGNMENT 2

Showqeen Yousuf

January 13, 2021

- Question: *Find the Inverse and QR Decomposition of the following.*

Exercise 2.97:

$$A = \begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$$

- **Solution:**

INVERSE OF A:

We are given with a matrix

$$A = \begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$$

Now,

DetA=

$$21 - 20 = 1$$

Also,

$$AdjA = \begin{pmatrix} 7 & -10 \\ -2 & 3 \end{pmatrix}$$

Now,

$\mathrm{A}^{-1}, can \quad be \quad calculated \quad by \quad the \quad formula,$

$\mathrm{A}^{-1} = AdjA/DetA$

Therefore,

$$A^{-1} = \begin{pmatrix} 7 & -10 \\ -2 & 3 \end{pmatrix}$$

QR DECOMPOSITION OF A:

The QR Decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$b = \begin{pmatrix} 10 \\ 7 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1$$

$$\text{b=s}_1 u_1 + t_2 u_2$$

where,

$$t_1 = ||a||$$

$$\text{u}_1 = a/t_1$$

$$\text{s}_1 = u_1^T * b/||u_1||^2$$

$$\text{u}_2 = b - s_1 * u_1/||b - s_1 u_1||$$

$$\text{t}_2 = u_2^T b$$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

$Now,\ using\ the\ equations\ of\ t_1, u_1, s_1, u_2\ and\ t_2\ we\ get\ the\ values\ of\ t_1, u_1, s_1, u_2\ and\ t_2$

$$t_1 = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$u_1 = 1/\sqrt{13} * \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 3/\sqrt{13} & 2/\sqrt{13} \end{pmatrix} * \begin{pmatrix} 10 \\ 7 \end{pmatrix}$$

$$s_1 = 44/\sqrt{13}$$

$$u_2 = 1/\sqrt{13} \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$t_2 = \begin{pmatrix} -2/\sqrt{13} & 3/\sqrt{13} \end{pmatrix} * \begin{pmatrix} 10 \\ 7 \end{pmatrix}$$

$$t_2 = -20/\sqrt{13} + (21\sqrt{13})$$

$$\mathrm{t}_2 = 1/\sqrt{13}$$

$$substituting\ \ the\ \ values\ \ of\ \ t_1, u_1, s_1, u_2\ \ and\ \ t_2\ \ in\ \ the\ \ matrix$$
$$\begin{pmatrix} a & b \end{pmatrix} = QR$$
$$we\ \ get\ \ the\ \ required\ \ QR\ \ decomposition\ \ of\ \ A.$$

$$\begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 3/\sqrt{13} & 2/\sqrt{13} \\ -2\sqrt{13} & 3/\sqrt{13} \end{pmatrix} * \begin{pmatrix} \sqrt{13} & 44\sqrt{13} \\ 0 & 1\sqrt{13} \end{pmatrix}$$

Exercise 2.98:

$$A = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$$

- **Solution:**

INVERSE OF A:

We are given with a matrix

$$A = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$$

Now,

DetA=

$$6 - 4 = 2$$

Also,

$$AdjA = \begin{pmatrix} 2 & 1 \\ -4 & 2 \end{pmatrix}$$

Now,

A^{-1}, $can \quad be \quad calculated \quad by \quad the \quad formula,$

A$^{-1} = AdjA/DetA$

Therefore,

$$A^{-1} = \begin{pmatrix} 1 & 1/2 \\ 2 & 3/2 \end{pmatrix}$$

QR DECOMPOSITION OF A:

The QR Decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$b = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1$$

b=s$_1 u_1 + t_2 u_2$

where,

$$t_1 = ||a||$$

u$_1 = a/t_1$

s$_1 = u_1^T * b/||u_1||^2$

u$_2 = b - s_1 * u_1/||b - s_1 u_1||$

t$_2 = u_2^T b$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

$Now,\ using\ the\ equations\ of\ t_1, u_1, s_1, u_2\ and\ t_2\ we\ get\ the\ values\ of\ t_1, u_1, s_1, u_2\ and\ t_2$

$$t_1 = \sqrt{3^2 + 4^2} = 5$$

$$u_1 = 1/5 * \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 3/5 & -4/5 \end{pmatrix} * \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$s_1 = -11/5$$

$$u_2 = 1/5 * \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$t_2 = \begin{pmatrix} 4/5 & 3/5 \end{pmatrix} * \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$t_2 = 4/5 + 6/5$$

$$\mathrm{t}_2 = 2/5$$

$$substituting\ \ the\ \ values\ \ of\ \ t_1, u_1, s_1, u_2\ \ and\ \ t_2\ \ in\ \ the\ \ matrix$$
$$\begin{pmatrix} a & b \end{pmatrix} = QR$$
$$we\ \ get\ \ the\ \ required\ \ QR\ \ decomposition\ \ of\ \ A.$$

$$\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix} * \begin{pmatrix} 5 & -11/5 \\ 0 & 2/5 \end{pmatrix}$$