# ASSIGNMENT 2

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- Question: *Find the Inverse and QR Decomposition of the following.*

*Exercise 2.97:*

$$A = \begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$$

- **Solution:**

  INVERSE OF A:

  We are given with a matrix

  $$A = \begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$$

  Now,

  DetA=
  $$21 - 20 = 1$$

  Also,

  $$Adj A = \begin{pmatrix} 7 & -10 \\ -2 & 3 \end{pmatrix}$$

  Now,

  $A^{-1}, can \quad be \quad calculated \quad by \quad the \quad formula,$

$A^{-1} = Adj A / Det A$

Therefore,

$$A^{-1} = \begin{pmatrix} 7 & -10 \\ -2 & 3 \end{pmatrix}$$

QR DECOMPOSITION OF A:

The QR Decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$b = \begin{pmatrix} 10 \\ 7 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1$$

$$b = s_1 u_1 + t_2 u_2$$

where,

$$t_1 = ||a||$$

$$u_1 = a/t_1$$

$$s_1 = u_1^T * b/||u_1||^2$$

$$u_2 = b - s_1 * u_1/||b - s_1 u_1||$$

$$t_2 = u_2^T b$$

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The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

*Now, using the equations of $t_1, u_1, s_1, u_2$ and $t_2$ we get the values of $t_1, u_1, s_1, u_2$ and $t_2$*

$$t_1 = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$u_1 = 1/\sqrt{13} * \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 3/\sqrt{13} & 2/\sqrt{13} \end{pmatrix} * \begin{pmatrix} 10 \\ 7 \end{pmatrix}$$

$$s_1 = 44/\sqrt{13}$$

$$u_2 = 1/\sqrt{13} \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$t_2 = \begin{pmatrix} -2/\sqrt{13} & 3/\sqrt{13} \end{pmatrix} * \begin{pmatrix} 10 \\ 7 \end{pmatrix}$$

$$t_2 = -20/\sqrt{13} + (21\sqrt{13})$$

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$$t_2 = 1/\sqrt{13}$$

*substituting the values of* $t_1, u_1, s_1, u_2$ *and* $t_2$ *in the matrix*

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

*we get the required QR decomposition of A.*

$$\begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 3/\sqrt{13} & -2/\sqrt{13} \\ 2/\sqrt{13} & 3/\sqrt{13} \end{pmatrix} * \begin{pmatrix} \sqrt{13} & 44/\sqrt{13} \\ 0 & 1/\sqrt{13} \end{pmatrix}$$

*Exercise 2.98:*

$$A = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$$

- **Solution:**

INVERSE OF A:

We are given with a matrix

$$A = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$$

Now,

DetA=

$$6 - 4 = 2$$

Also,

$$Adj A = \begin{pmatrix} 2 & 1 \\ -4 & 2 \end{pmatrix}$$

Now,

$A^{-1}, can \quad be \quad calculated \quad by \quad the \quad formula,$

$A^{-1} = Adj A / Det A$

Therefore,

$$A^{-1} = \begin{pmatrix} 1 & 1/2 \\ 2 & 3/2 \end{pmatrix}$$

QR DECOMPOSITION OF A:

The QR Decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$b = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1$$

$$b = s_1 u_1 + t_2 u_2$$

where,

$$t_1 = ||a||$$

$$u_1 = a/t_1$$

$$s_1 = u_1^T * b/||u_1||^2$$

$$u_2 = b - s_1 * u_1/||b - s_1 u_1||$$

$$t_2 = u_2^T b$$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

*Now, using the equations of* $t_1, u_1, s_1, u_2$ *and* $t_2$ *we get the values of* $t_1, u_1, s_1, u_2$ *and* $t_2$

$$t_1 = \sqrt{3^2 + 4^2} = 5$$

$$u_1 = 1/5 * \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 3/5 & -4/5 \end{pmatrix} * \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$s_1 = -11/5$$

$$u_2 = 1/5 * \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$t_2 = \begin{pmatrix} 4/5 & 3/5 \end{pmatrix} * \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$t_2 = 4/5 + 6/5$$

$$t_2 = 2/5$$

*substituting the values of $t_1, u_1, s_1, u_2$ and $t_2$ in the matrix*

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

*we get the required QR decomposition of A.*

$$\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{pmatrix} * \begin{pmatrix} 5 & -11/5 \\ 0 & 2/5 \end{pmatrix}$$