Equation of motion for a harmonic oscillator,

$$\ddot{x} = -\omega^2 x$$

with solution, $x(t) = x_0 \cos(\omega t) + \frac{v_0}{\omega} \sin(\omega t)$

If we use natural units,

distance: $x = x\, x_c$ and

time: $\tau = \omega t \Rightarrow t = \tau/\omega$

plugging them, in the equation $\ddot{x} = -\omega^2 x$ gives us.

$$\frac{d}{\frac{1}{\omega} d\tau}\left(\frac{x_c\, dx}{\frac{1}{\omega} d\tau}\right) = -\omega^2 x_c x$$

$$\Rightarrow \omega^2 x_c \ddot{x} = -\omega^2 x_c x$$

$$\therefore \boxed{\ddot{x} = -x}$$

Therefore, using natural times $\tau = \omega t$, the equation can be non-dimensionalized.

Again, $x = x_c x$

$$\text{So, } \frac{dx}{dt} = x_c \frac{dx}{d\tau} \cdot \omega$$

$$\Rightarrow \dot{x} = x_c \cdot \dot{x} \cdot \omega$$

This gives us, initial velocity is 1 in these units (Answer.)