from (a), we've found $\underline{M} = \underline{V}\,\underline{L}\,\underline{V}^{-1}$

$$\Rightarrow \underline{M}^n = (\underline{V}\,\underline{L}\,\underline{V}^{-1})^n$$

$$\Rightarrow \underline{M}^n = [\underbrace{\underline{V}\,\underline{L}\,\underline{V}^{-1}}_{\substack{1st\\ term}}\ \underbrace{\underline{V}\,\underline{L}\,\underline{V}^{-1}}_{\substack{2nd\\ term}}\ \underbrace{\underline{V}\,\underline{L}\,\underline{V}^{-1}}_{\substack{3rd\\ term}} \dots \dots \underbrace{\underline{V}\,\underline{L}\,\underline{V}^{-1}}_{\substack{nth\\ term}}]$$

$$\Rightarrow \underline{M}^n = \underline{V}\,L^n\,\underline{V}^{-1}$$

$$\therefore \underline{M}^n = \underline{V} \begin{pmatrix} \lambda_1^n & & 0 \\ & \lambda_2^n & \\ 0 & \ddots & \lambda_n^n \end{pmatrix} \underline{V}^{-1}$$

that means, $\underline{M}^n$ has the eigenvalues λ_i^n with the same eigenvectors (showed)

Now, let λ_{max} be the largest eigenvalue of $\underline{M}$ in magnitude and its corresponding eigenvector is $\vec{v}_{max}$. The vector $\vec{b}$ can be written as a linear combination of the columns of $\underline{V}$.

$$\vec{b} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_{max}\vec{v}_{max} + \dots \dots + c_n\vec{v}_n$$

by assumption $\vec{b}$ has a non zero component in the direction of dominant eigenvalue λ_{max}; so $c_{max} \neq 0$

So, $\underline{M}^n\vec{b} = c_1\underline{M}^n\vec{v}_1 + c_2\underline{M}^n\vec{v}_2 + \dots \dots + c_{max}\underline{M}^n\vec{v}_{max} + \dots + c_n\underline{M}^n\vec{v}_n$

$$\therefore \boxed{\underline{M}^n\vec{b} = c_1\lambda_1^n\vec{v}_1 + c_2\lambda_2^n\vec{v}_2 + \dots \dots + c_{max}\lambda_{max}^n\vec{v}_{max} + \dots + c_n\lambda_n^n\vec{v}_n}$$

(showed)

Now, in indicial notations we can write this as —

$$M_{ij}^n b_j = c_i \lambda_i^n v_i$$

$$\Rightarrow \boxed{c_i = M_{ij}^n b_j \cdot \lambda_i^{-n} v_i^{-1}}$$

Again, we can write

$$\underline{M}^n \vec{b} = c_{max} \lambda_{max}^n \left\{ \frac{c_1}{c_{max}} \left(\frac{\lambda_1}{\lambda_{max}}\right)^n \vec{v}_1 + \frac{c_2}{c_{max}} \left(\frac{\lambda_2}{\lambda_{max}}\right)^n \vec{v}_2 + \dots \right.$$

$$\left. \dots + \vec{v}_{max} + \dots + \frac{c_n}{c_{max}} \left(\frac{\lambda_n}{\lambda_{max}}\right)^n \vec{v}_n \right\}$$

Here, $\left|\frac{\lambda_j}{\lambda_{max}}\right| < 1$

therefore, $\lim\limits_{n\to\infty} \underline{M}^n \vec{b} = \lim\limits_{n\to\infty} c_{max} \lambda_{max}^n \vec{v}_{max}$

$\therefore \boxed{\lim\limits_{n\to\infty} \underline{M}^n \vec{b} \propto \vec{v}_{max}}$; where the proportionality constant is $\boxed{c_{max} \lambda_{max}^n}$.