

Language : MATLAB

Source code : problem_1b.m

Here, the used equation to describe the Hopf model is—

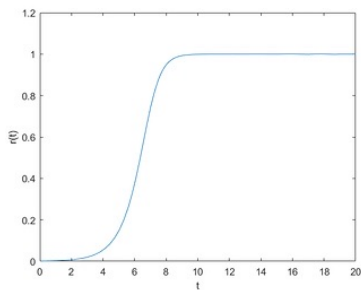
$$\dot{r} = ar - r^3$$

In the same code, I have used $a=1$, $a=0$ and $a=-1$ and captured the respective responses.

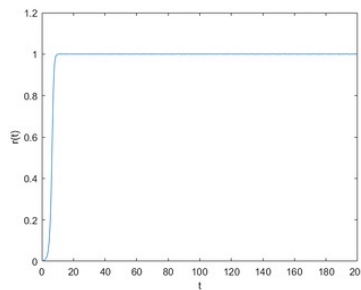
On the other hand, I have used the time intervals $[0, 20]$, $[0, 200]$, and $[0, 20000]$ and captured the respective responses.

All the following responses are captured with initial value $r(0) = r_0 = 0.001$

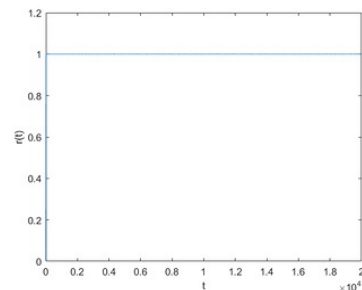
$$a=1$$



time interval
 $[0, 20]$

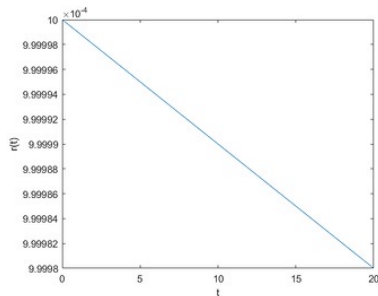


time interval
 $[0, 200]$

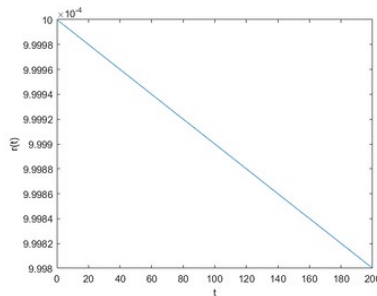


time interval
 $[0, 20000]$

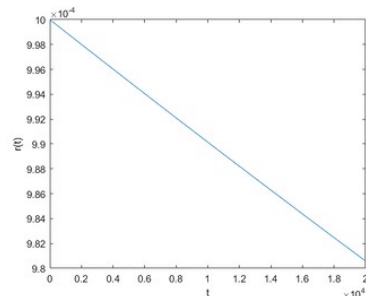
$$a=0$$



time interval
[0, 20]

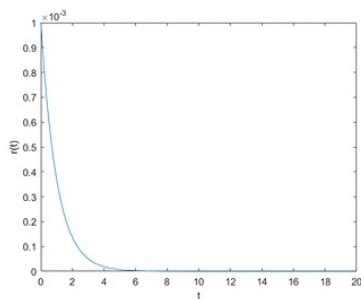


time interval
[0, 200]

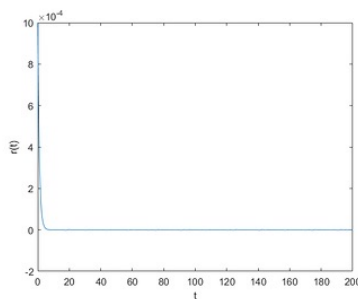


time interval
[0, 20000]

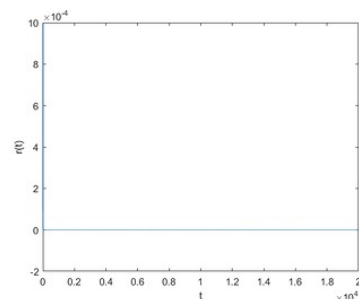
$$a=-1$$



time interval
[0, 20]

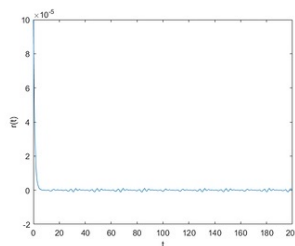


time interval
[0, 200]



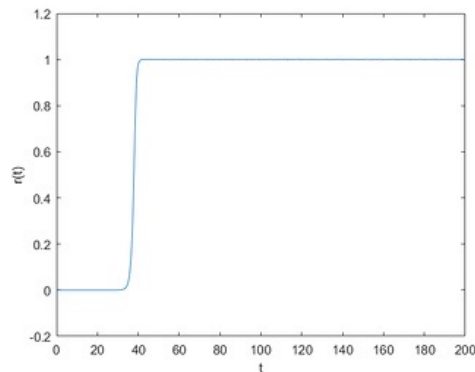
time interval
[0, 20000]

For $a=-1$, if we take the initial value $r(0) = r_0 = 0.0001$, we see a granular curve, for the time interval [0, 200]



If we want to observe a response that captures the step-up of the Hopf model, we can choose

$a=1$, time interval = $[0, 200]$ and $r_0 = 0.000\,000\,000\,1$
and then we find the following response—



$ar - r^3 = 0$, when $r = 0$, \sqrt{a} ; if we take $a=1$, the curve saturates at $\sqrt{a} = 1$. and also, with r_0 very small r^3 can be neglected in comparison to ar .