

Given functions

$$F(x, y) = x^y e^{-x^y} + y^2$$

$$G(x, y) = \frac{x^4}{1+x^2 y^y}$$

$$\text{Now, } F(x^*, y^*) = G(x^*, y^*) = 1$$

$$\text{Here, } f(x^*, y^*) = f\{x - (x - x^*), y - (y - y^*)\}$$

$$\text{and } G(x^*, y^*) = G\{x - (x - x^*), y - (y - y^*)\}$$

from here, if we can express the system as -

$$x_n = x_{n-1} + \frac{G_y(x_{n-1}, y_{n-1}) - f_y(x_{n-1}, y_{n-1}) + G(x_{n-1}, y_{n-1}) \cdot f_y(x_{n-1}, y_{n-1}) - F(x_{n-1}, y_{n-1}) \cdot G_y(x_{n-1}, y_{n-1})}{F_x(x_{n-1}, y_{n-1}) G_y(x_{n-1}, y_{n-1}) - f_y(x_{n-1}, y_{n-1}) G_x(x_{n-1}, y_{n-1})}$$

and,

$$y_n = y_{n-1} + \frac{F_x(x_{n-1}, y_{n-1}) - G_x(x_{n-1}, y_{n-1}) + F(x_{n-1}, y_{n-1}) G_x(x_{n-1}, y_{n-1}) - f_x(x_{n-1}, y_{n-1}) \cdot G(x_{n-1}, y_{n-1})}{F_x(x_{n-1}, y_{n-1}) G_y(x_{n-1}, y_{n-1}) - f_y(x_{n-1}, y_{n-1}) G_x(x_{n-1}, y_{n-1})}$$

Language : MATLAB

Source Code : problem-2.m

Now, final $x_n = x^*$ and $y_n = y^*$

So, from the "Workspace" directory we get.

$$\begin{cases} x^* = 1.173977098108600266775112342294 \\ y^* = 0.80786881713905550481258636604448 \end{cases}$$

(up to tolerance of "eps")