Language used: MATLAB

Source code: ① problem_2b.m

② deriv.m

③ rk4.m

at $T=5$, for 6 different values of Δt, $x^{(verlet)}(T)$ and $x^{(rk4)}(T)$ is computed using the Verlet and Runge-Kutta 4th order methods. The values are reported below:

Δt	$x^{(verlet)}(T)$	$x^{(rk4)}(T)$
0.5	-0.9733	-0.9586
0.1	-0.9595	-0.9589
0.05	-0.9591	-0.9589
0.01	-0.9589	-0.9589
0.005	-0.9589	-0.9589
0.001	-0.9589	-0.9589

(Table-1)

produced plot: problem_2b.png

In the log-log plot, the difference between $\sin(T)$ and computed $x(T)$ is captured again Δt.

Here, $g_1(\Delta t) = |\sin(T) - x^{verlet}(T)|$

and, $g_2(\Delta t) = |\sin(T) - x^{rk4}(T)|$

As we can see in (Table-1), for even $\Delta t = 0.1$ $x^{(RK4)} = -0.9589$, whereas $x^{(verlet)}$ saturates at -0.9589 for $\Delta t = 0.01$ and smaller. Therefore, it's evident that this values will show non-identical differences with $\sin(T)$ at various Δt.

For, $\Delta t = 0.5$, curves g_1 and g_2 show maximum difference; with decreasing Δt, g_1 starts to drop dramatically. The drop of g_1 is linear upto $\Delta t = 0.1$. After that, as Δt decreases, the difference between g_1 and g_2 starts to drop more rapidly. ~~And~~ from $\Delta t = 0.01, 0.005, 0.001$ there is no difference observed for g_1 and g_2.