With the normalized height $u(x,t)$, the KdV equation is given as -

$$\frac{\partial u}{\partial t} + 6u(x,t)\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0 \longrightarrow (1)$$

We remember that, the simplest mathematical wave is a function of the form $u(x,t) = f(x-\alpha t)$, which is a solution to the simple PDE, $u_t + \alpha u_x = 0$, where α denotes the speed of the wave.

For the well known wave equation $u_{tt} - \alpha^2 u_{xx} = 0$, the famous d'Alembert solution leads to two wave fronts represented by terms $f(x-\alpha t)$ and $f(x+\alpha t)$.

Therefore, we can say that, for the KdV equation the solution $u(x,t) = v(x-ct) = v(y)$ is permitted where, the wave speed is denoted by 'c' here, instead of 'α' and the function is expressed as 'v'.

Also, $y = x - ct$.

Substituting the trial solution into (1), we are lead to the Ordinary Differential Equation,

$$-c \cdot \frac{dv}{dy} + 6v(y) \cdot \frac{dv}{dy} + \frac{d^3 v}{dy^3} = 0$$

$$\Rightarrow -cv'(y) + 6v(y)\, v'(y) + v'''(y) = 0$$

[Showed]