(a)

Let's begin by considering the equation of motion for the three body problem in an arbitrary inertial reference. $R_1, R_2, R_3 \in \mathbb{R}^3$ are the respective positions of three bodies with masses $M_1 = M_2 = M_3 = M = 1$ solar mass.

In the absence of any further simplifying assumptions, the eq^n of motion are:

$$\ddot{R}_1 = GM\left[-\frac{R_1-R_2}{|R_1-R_2|^3} - \frac{R_1-R_3}{|R_1-R_3|^3}\right]$$

$$\ddot{R}_2 = GM\left[-\frac{R_2-R_1}{|R_1-R_2|^3} - \frac{R_2-R_3}{|R_2-R_3|^3}\right]$$

$$\ddot{R}_3 = GM\left[-\frac{R_3-R_1}{|R_3-R_1|^3} - \frac{R_3-R_2}{|R_2-R_3|^3}\right]$$

Now, Let. $r_1 = R_1/R$; $r_2 = \frac{R_2}{R}$ and $r_3 = R_3/R$

So, $dR_1 = R\,dr_1$; $dR_2 = R\,dr_2$ and $dR_3 = R\,dr_3$

and, $\hat{t} = t/\tau$

$$\Rightarrow d\hat{t} = \frac{dt}{\tau}$$

$$\therefore dt = \tau \cdot d\hat{t}$$

Here, r_1, r_2, r_3 and $\hat{t}$ are all dimensionless quantity.

So, we can write

$$\frac{d}{\tau \cdot d\hat{t}}\left(\frac{R\,dr_1}{\tau \cdot d\hat{t}}\right) = \frac{GMR}{R^3}\left[-\frac{r_1-r_2}{|r_1-r_2|^3} - \frac{r_1-r_3}{|r_1-r_3|^3}\right]$$

$$\Rightarrow \frac{d^2 r_1}{d\hat{t}^2} = \frac{GM\tau^2}{R^3}\left[-\frac{r_1-r_2}{|r_1-r_2|^3} - \frac{r_1-r_3}{|r_1-r_3|^3}\right]$$

$$\Rightarrow \ddot{r}_1 = \frac{GM\tau^2}{R^3}\left[-\frac{r_1-r_2}{|r_1-r_2|^3} - \frac{r_1-r_3}{|r_1-r_3|^3}\right]$$

Here, $\frac{GM\tau^2}{R^3}$ is a dimensionless quantity. We can set the value of this constant term equal to "1" and finally get the non-dimensionalized form.

$$\ddot{r}_1 = -\frac{r_1-r_2}{|r_1-r_2|^3} - \frac{r_1-r_3}{|r_1-r_3|^3}$$

Similarly, we can write for r_2 & r_3.

$$\ddot{r}_2 = -\frac{r_2-r_1}{|r_1-r_2|^3} - \frac{r_2-r_3}{|r_2-r_3|^3}$$

and $$\ddot{r}_3 = -\frac{r_3-r_1}{|r_1-r_3|^3} - \frac{r_3-r_2}{|r_2-r_3|^3}$$

For Non-dimensionalization

$$\frac{GM\tau^2}{R^3} = 1$$

$$\Rightarrow R = \sqrt[3]{GM\tau^2} \quad [M=1]$$

So, velocity of 1 Km/s or 1000 m/s can be written as: $\boxed{0.4056}$

(b)

Given the first star is initially stationary at the origin.

So, $r_1 = 0$, $v_1 = 0$.

Again, $r_2 = -r_3$ & $v_2 = -v_3$.

this gives,

$$\ddot{r}_1 = \frac{0 - r_2}{|0 - r_2|^3} - \frac{0 - (-r_2)}{|0 + r_2|^3} = 0.$$

that means, $\dot{r}_1 = \text{constant}$

as $v_1(0) = 0$, this is the velocity for the first star, all throughout.

So, the first star will remain stationary for all time.

- If r_1 is perturbated, the solution becomes unstable.

(c)

As the problem is associated with Energy, I chose rK4 method, as it conserves energy.

In this problem, besides the storage of "r" at every timestep, the storage of "v" is also important so that the "Kinetic Energy" can be calculated.

Total Energy = Potential Energy + Kinetic Energy.

But sadly I couldn't figure out how to write the "deriv" function for this problem of 3-body system.

r_1, r_2, r_3 are all dependent on each other. so, unlike the problems discussed in class and practiced in Homework this one is not that straight forward.

(d)

same problem as 5(c)

(e)

Same problem as 5(c) & 5(d).

I'm so sorry for my shortcomings.