Fokker – Plank Eq$^{\underline{n}}$:

$$\frac{\partial n}{\partial t} = D \cdot \frac{\partial^2 n}{\partial x^2} + \frac{1}{\zeta} \frac{\partial}{\partial x}\left(n \frac{\partial U}{\partial x}\right)$$

with $U = -f(x)$, we obtain

$$\frac{\partial n}{\partial t} = D \cdot \frac{\partial^2 n}{\partial x^2} - \frac{f}{\zeta} \frac{\partial n}{\partial x}$$

For reflecting boundary condition, with $x \in [0, L]$ domain, we have

Just shifting the domain to have easier B/C

$$\left.\frac{\partial n}{\partial x}\right|_{x=0} = \left.\frac{\partial n}{\partial x}\right|_{x=L} = 0$$

$$n(x,0) = \delta(x)$$

Now, let. $n = X(x) \cdot T(t)$

It gives us,

$$X \frac{\partial T}{\partial t} = DT \frac{\partial^2 X}{\partial x^2} - \frac{fT}{\zeta} \cdot \frac{\partial X}{\partial x}$$

$$\Rightarrow \frac{1}{D} \cdot \frac{\dot{T}}{T} = \frac{X''}{X} - \frac{f}{D\zeta} \cdot \frac{X'}{X} = -K$$

Therefore, $\dot{T} = -KDT$

$$\therefore T(t) = e^{-KDt}$$

And, $$\frac{\partial^2 X}{\partial x^2} - \frac{f}{D\zeta} \frac{\partial X}{\partial x} = -KX$$

So, the eigenvalue problem becomes,

$$\lambda^2 - \frac{f}{D\zeta} \lambda + K = 0$$

$$\Rightarrow \lambda_1 = \frac{f + \sqrt{f^2 - 4KD^2\zeta^2}}{2D\zeta} \; ; \; \lambda_2 = \frac{f - \sqrt{f^2 - 4KD^2\zeta^2}}{2D\zeta}$$

And, the general solution takes the form.

$$X = e^{\frac{f}{2D\xi}} \left[A\cos\left(\frac{\sqrt{f^2 - 4KD^2\xi^2}}{2D\xi} x\right) + B\sin\left(\frac{\sqrt{f^2 - 4KD^2\xi^2}}{2D\xi} x\right)\right]$$

Now, $$X' = e^{\frac{f}{2D\xi}} \frac{\sqrt{f^2 - 4KD^2\xi^2}}{2D\xi} \cdot \left[-A\sin\left(\frac{\sqrt{f^2 - 4KD^2\xi^2}}{2D\xi} x\right) + B\cos\left(\frac{\sqrt{f^2 - 4KD^2\xi^2}}{2D\xi} x\right)\right]$$

Applying reflecting B/c's @ $x = 0$,

we find. $B = 0$

and applying reflecting B/c's @ $x = L$

we find. $-Ap\sin pL = 0$ $\left[\text{taking } \frac{\sqrt{f^2 - 4KD^2\xi^2}}{2D\xi} = p\right]$

$$\therefore p = n\pi/L = \frac{\sqrt{f^2 - 4KD^2\xi^2}}{2D\xi}$$

$$\Rightarrow K = \frac{f}{4D^2\xi^2} - n^2\pi^2/L^2$$

finally, we get,

$$X = e^{\frac{f}{2D\zeta}} \cdot \cos\left(\frac{\sqrt{f^2 - 4KD^2\zeta^2}}{2D\zeta}\right)x$$

And, $n(x,t) = X \cdot T$

$$= e^{f/2D\zeta}\left\{\cos\left(\frac{\sqrt{f^2 - 4KD^2\zeta^2}}{2D\zeta}\right)x\right\}e^{-KDt};$$

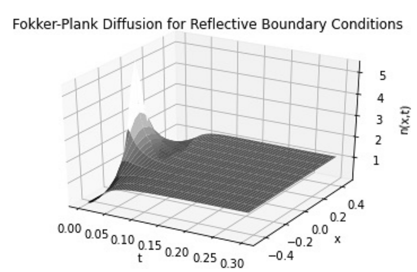
where, $K = \dfrac{f^2}{4D^2\zeta^2} - \dfrac{n^2\pi^2}{L^2}$

(also, when $K = 0$, $x_2 = 0$)

with $\lim\limits_{t\to\infty}$ we get the steady-state solution –

$$n_\infty(x) = e^{\frac{f}{2D\zeta}} \cos\left(\frac{\sqrt{f^2 - 4KD^2\zeta^2}}{2D\zeta}\right)x$$

And numerically we get the following plot:



and it depicts that, as time passes by the probability of finding the particle at any point of the domain is equally distributed