Given that,

$$E_{tot} = \frac{mL^2}{2} \omega^2(t) + \frac{mgL}{2} \theta^2(t) - mgL$$

So, we can write

$$E_n = \frac{mL^2}{2} \omega_n^2 + \frac{mgL}{2} \theta_n^2 - mgL \longrightarrow (1)$$

$$\text{and, } E_{n+1} = \frac{mL^2}{2} \omega_{n+1}^2 + \frac{mgL}{2} \theta_{n+1}^2 - mgL \longrightarrow (2)$$

Now,

$\{(2) - (1)\} \Rightarrow$

$$E_{n+1} - E_n = \frac{mL^2}{2} (\omega_{n+1}^2 - \omega_n^2) + \frac{mgL}{2} (\theta_{n+1}^2 - \theta_n^2)$$

$$= \frac{mL^2}{2} \{(\omega_{n+1} + \omega_n)(\omega_{n+1} - \omega_n)\}$$

$$+ \frac{mgL}{2} \{(\theta_{n+1} + \theta_n)(\theta_{n+1} - \theta_n)\}$$

Now, when we employ Euler's method to compute the motion, we get.

$$\theta_{n+1} - \theta_n = \Delta t \cdot \omega_n$$

$$\text{and, } \omega_{n+1} - \omega_n = \Delta t \cdot \alpha_n \quad [\alpha_n = \text{angular acceleration}]$$

$$= \Delta t \cdot (-g/L)\, \theta_n \quad [\text{small angle approximation}]$$

Therefore, we can write,

$$E_{n+1} - E_n = \frac{mL^2}{2} (\omega_{n+1} + \omega_n)\, \Delta t \cdot (-g/L)\, \theta_n$$

$$+ \frac{mgL}{2} (\theta_{n+1} + \theta_n)\, \Delta t \cdot \omega_n$$

$$\Rightarrow E_{n+1} - E_n = \frac{mgL}{2} \Delta t \left(\theta_{n+1} \omega_n - \theta_n \cdot \omega_{n+1} \right)$$

$$= \frac{mgL}{2} \Delta t \left[\theta_{n+1} \left(\frac{\theta_{n+1} - \theta_n}{\Delta t} \right) - \theta_n \left\{ \omega_n + \Delta t \left(-g/L \right) \theta_n \right\} \right]$$

$$= \frac{mgL}{2} \Delta t \left[\frac{1}{\Delta t} \left(\theta_{n+1}^2 - 2\theta_{n+1} \cdot \theta_n + \theta_n^2 \right) + \Delta t \cdot \theta_n^2 \left(g/L \right) \right]$$

$$\therefore E_{n+1} - E_n = \frac{mgL}{2} \left(\theta_{n+1} - \theta_n \right)^2 + \frac{mg^2}{2} (\Delta t)^2 \theta_n^2$$

Here, each term, in the right hand side of the equation above, is positive. That means, at each step E_{tot} is greater than that at immediate previous step.

So, we can conclude, E_{tot} increases monotonically with time, when Euler's method is used to compute the motion.

[Showed]