COMPUTER ENGINEERING

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EMBEDDED SYSTEM DESIGN

Extended and Time Automata

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Objectives

- FSM with input variable
- Continuous FSM with time domain

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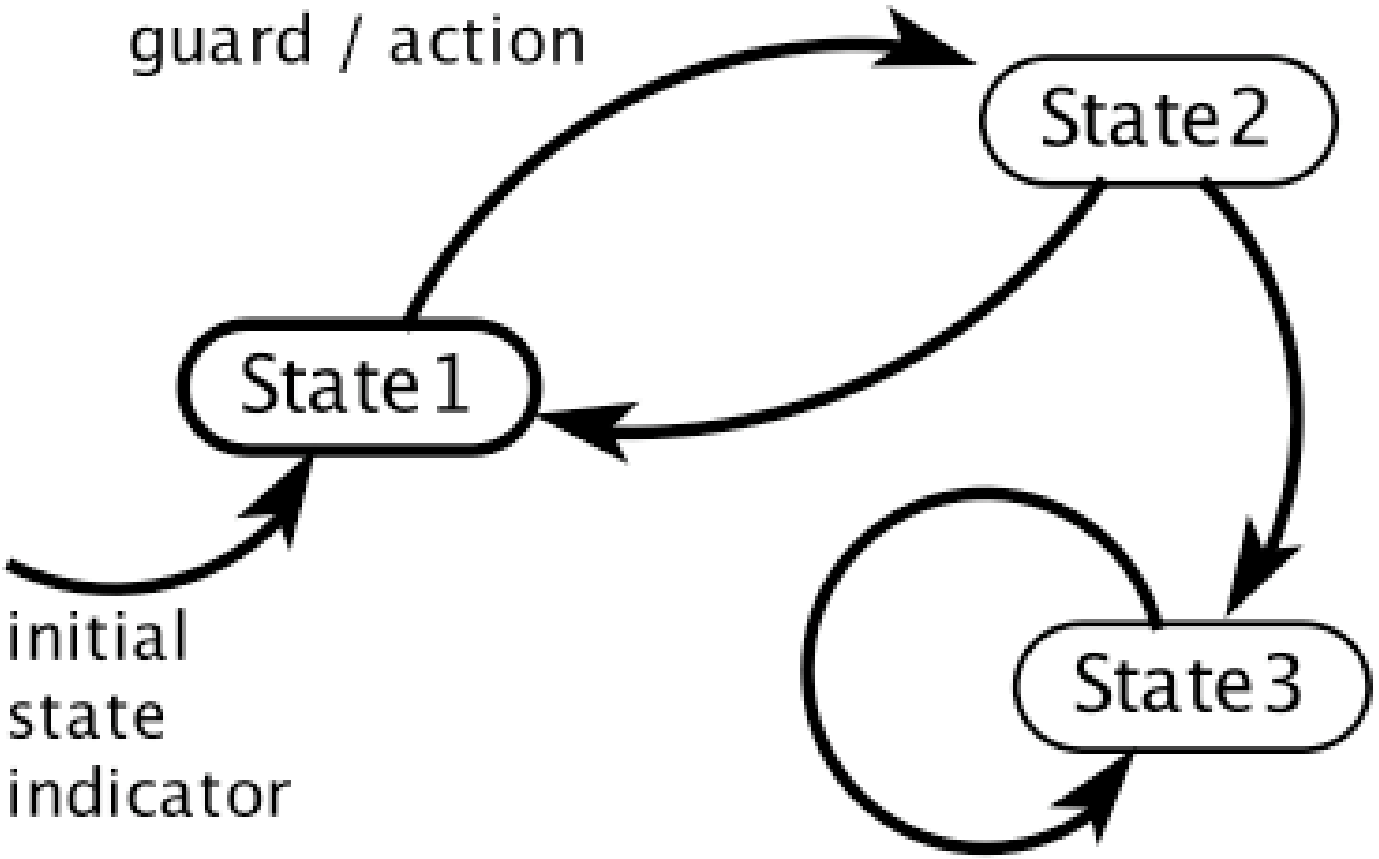
Contents

1. Introduction to Extended FSM
2. Continuous FSM in time domain
3. Example of continuous FSM

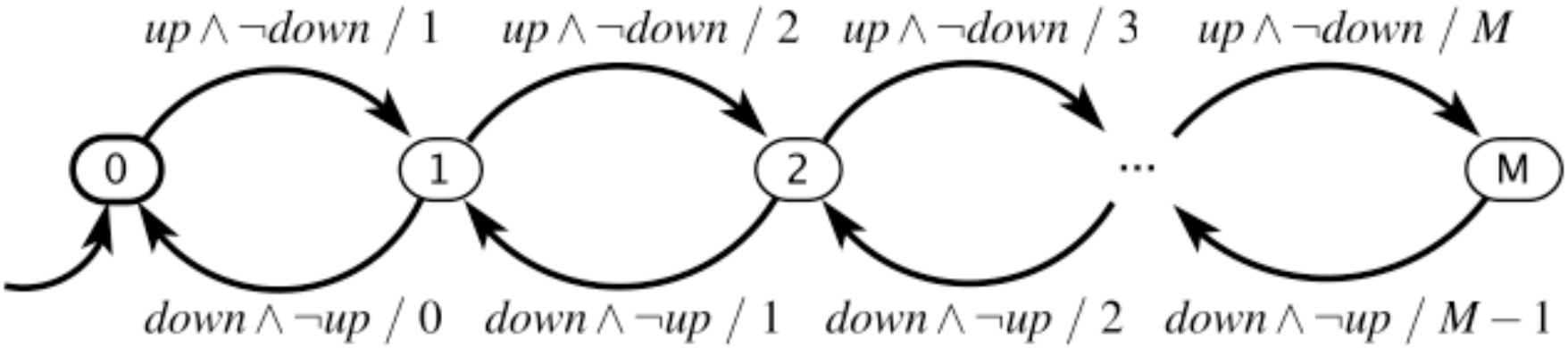
Extended FSM

FSM Notation (recall)



Garage Counter Example (recall)

inputs: $up, down \in \{present, absent\}$
output $\in \{0, \cdots, M\}$

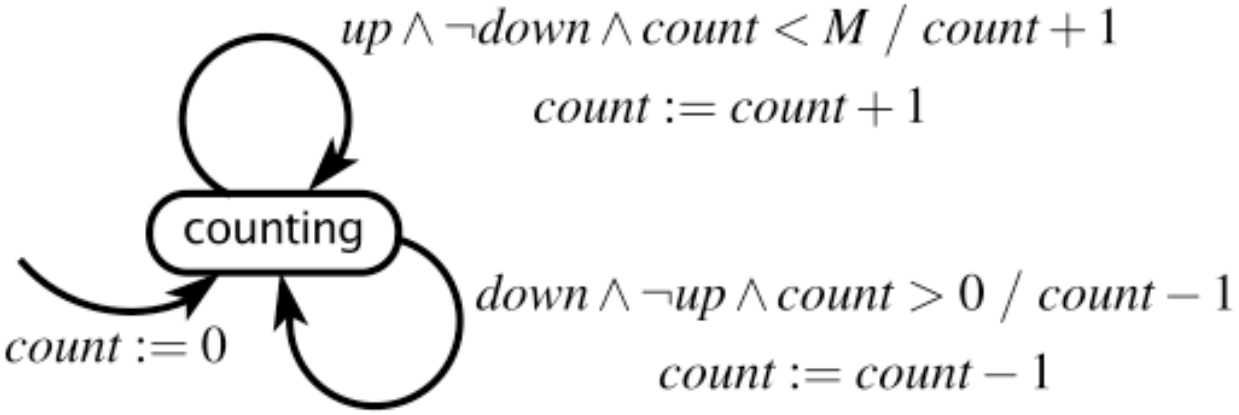


Extended State Machine (ESM)

Question: What is the size of the state space?

- Extended state machines augment the FSM model with *variables* that may be read or written. E.g.:

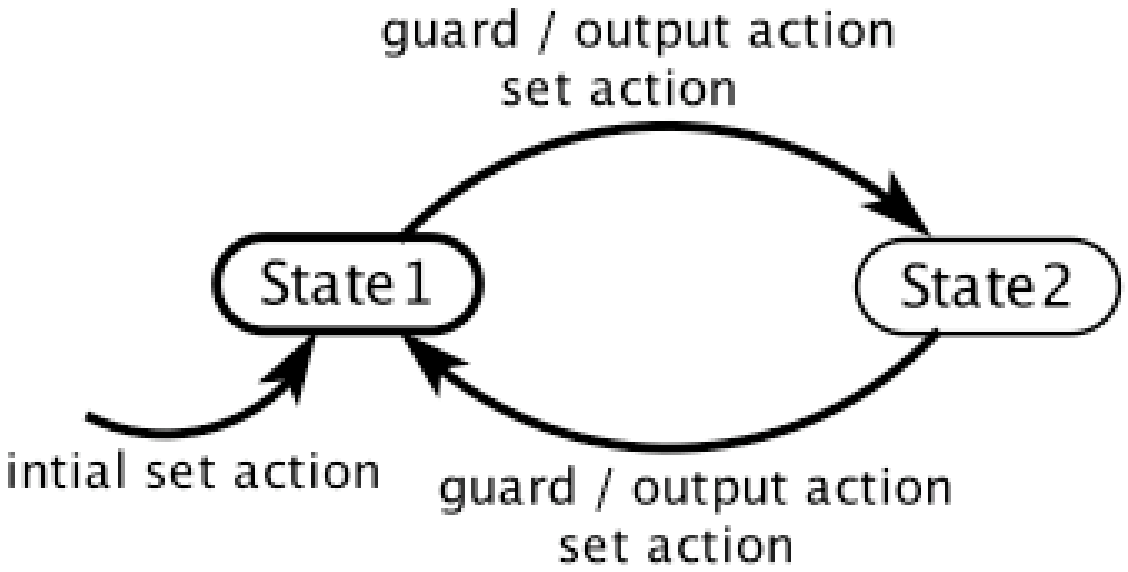
variable: $count \in \{0, \cdots, M\}$
inputs: $up, down \in \{present, absent\}$
output $\in \{0, \cdots, M\}$



ESM Notation

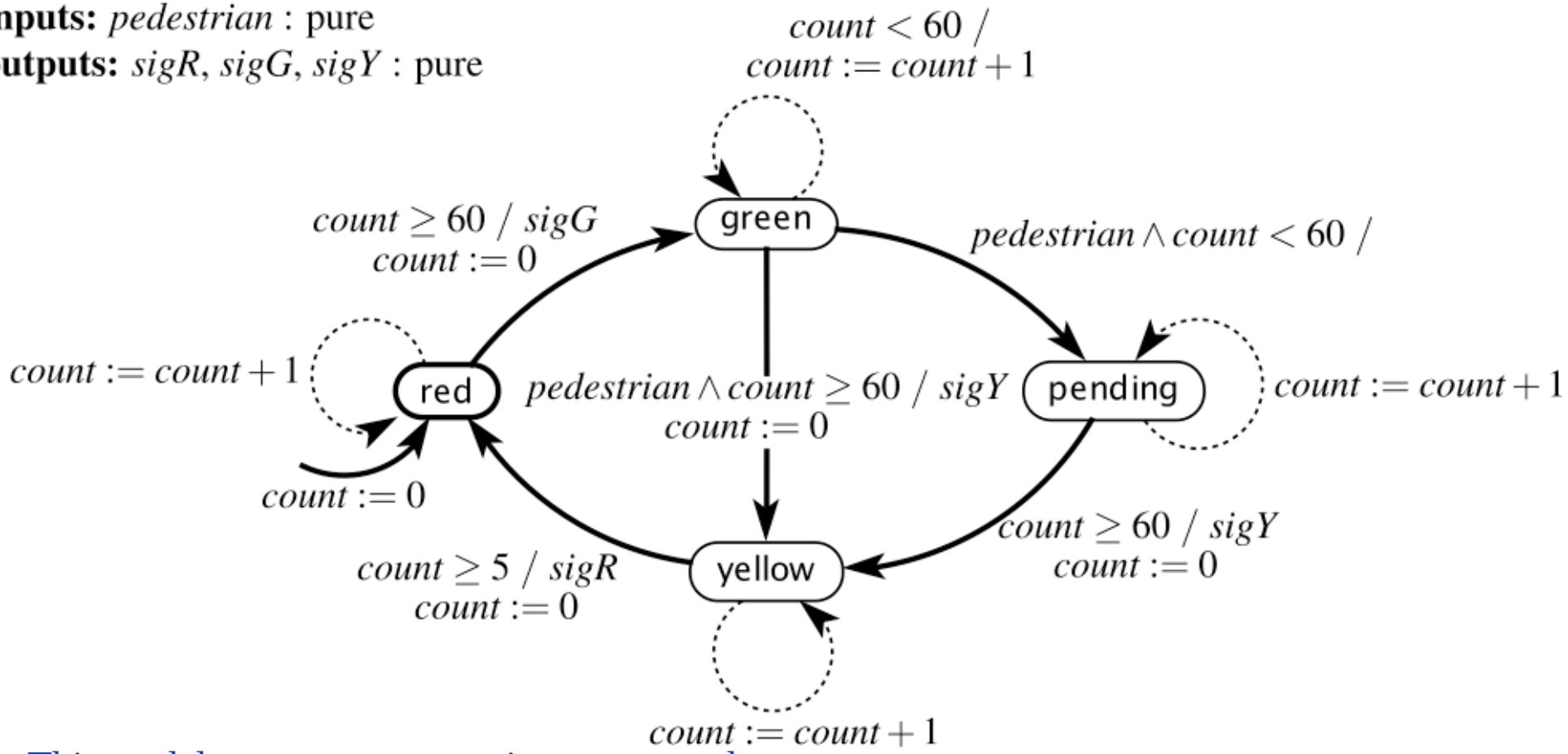
- We make explicit declarations of variables, inputs, and outputs to help distinguish the three.

variable declaration(s)
input declaration(s)
output declaration(s)



ESM Example: Pedestrian Crossing Traffic Light

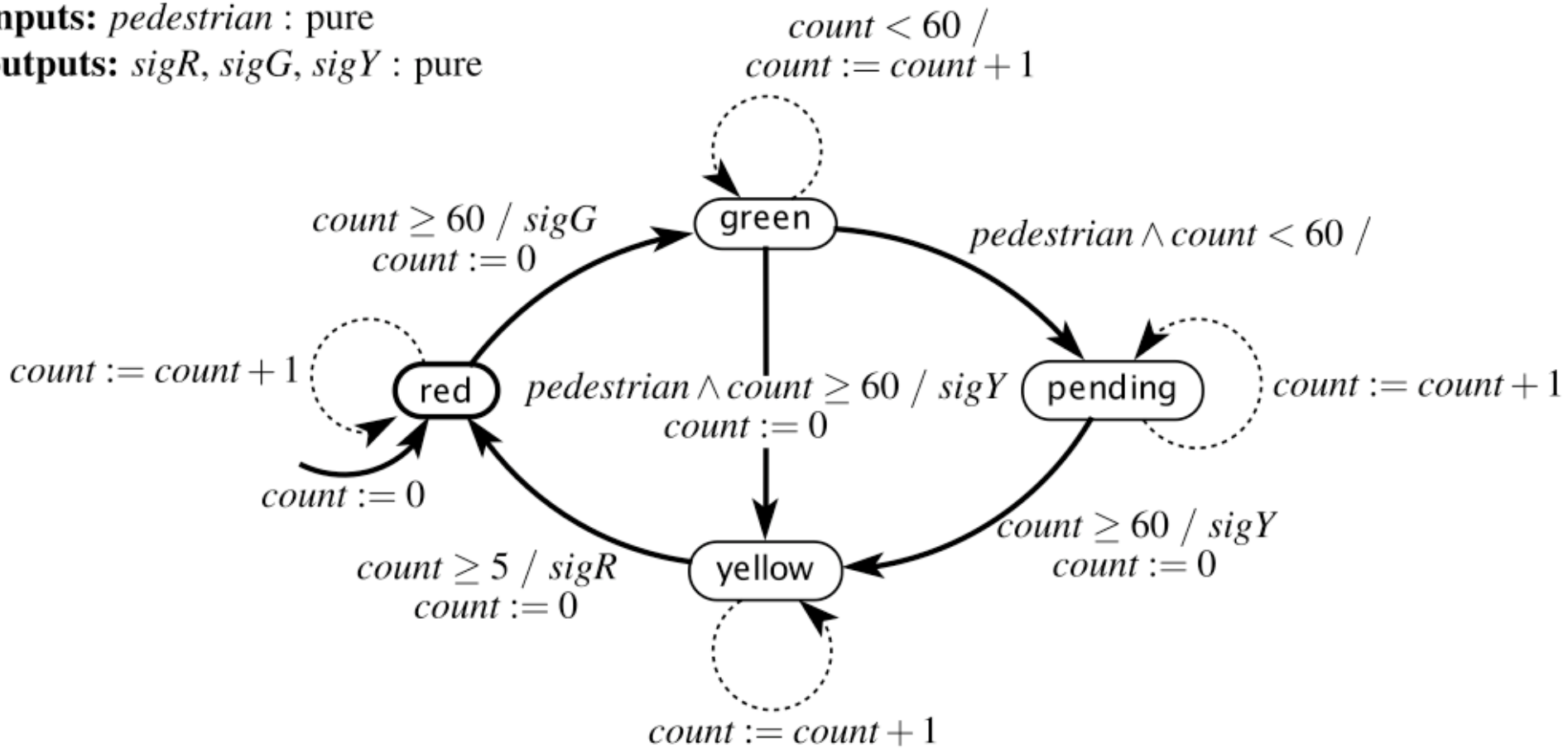
variable: $count$: $\{0, \cdots, 60\}$
inputs: $pedestrian$: pure
outputs: $sigR$, $sigG$, $sigY$: pure



This model assumes one reaction per second
(a *time-triggered* model)

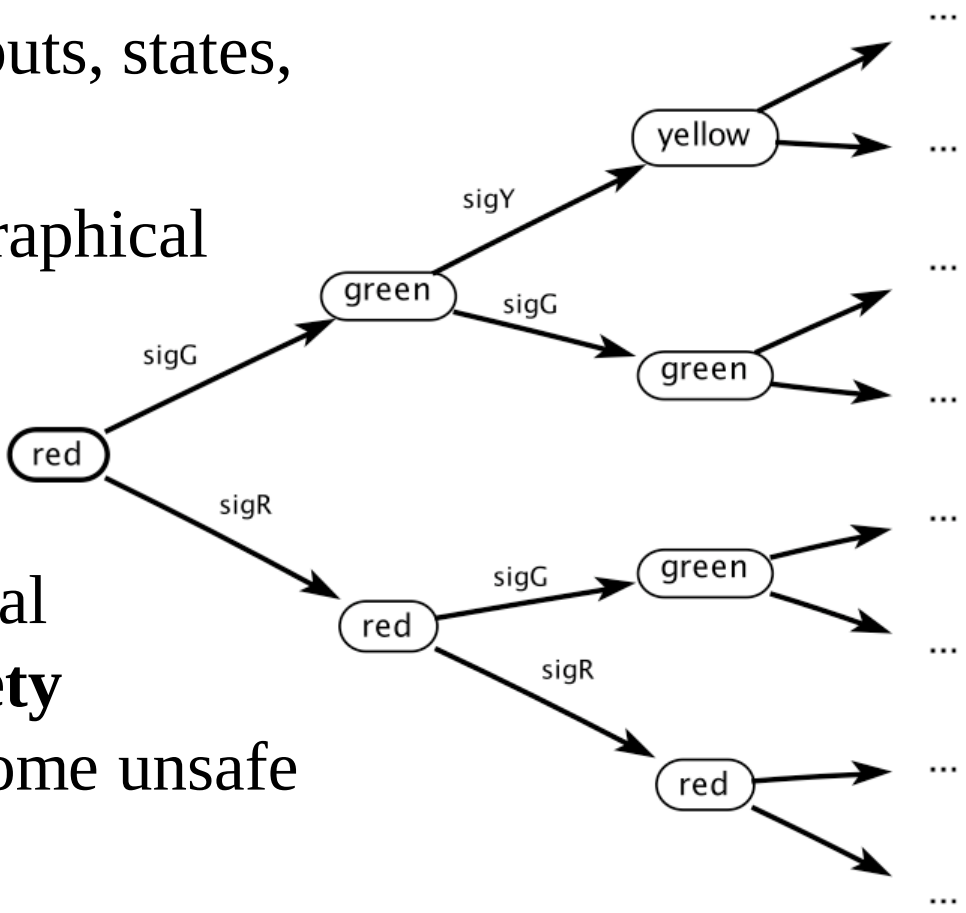
Quiz: What is the Size of the State Space?

variable: *count*: $\{0, \cdots, 60\}$
inputs: *pedestrian* : pure
outputs: *sigR*, *sigG*, *sigY* : pure



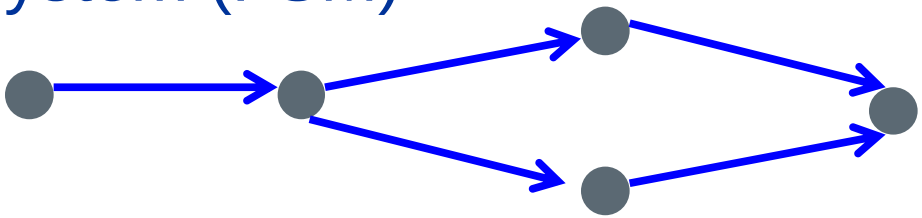
Behaviors and Traces

- FSM **behavior** is a sequence of (non-stuttering) steps.
- A **trace** is the record of inputs, states, and outputs in a behavior.
- A **computation tree** is a graphical representation of all possible traces.

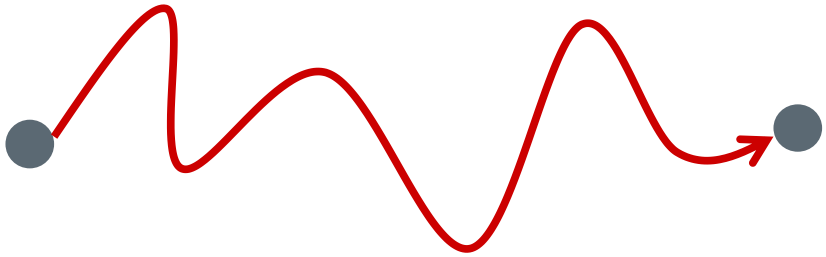


- FSMs are suitable for formal analysis. For example, **safety** analysis might show that some unsafe state is not reachable.

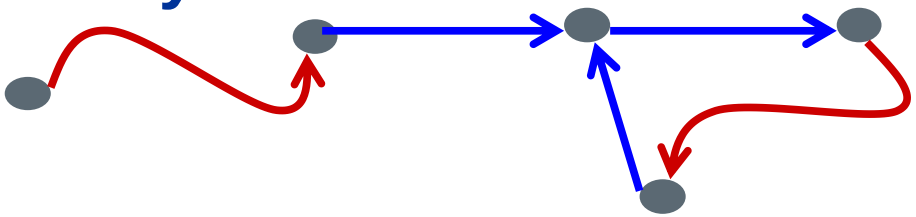
Discrete System (FSM)

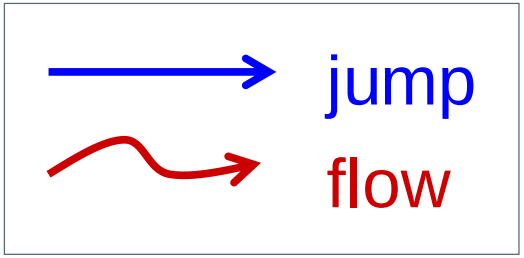


Continuous System



Hybrid System





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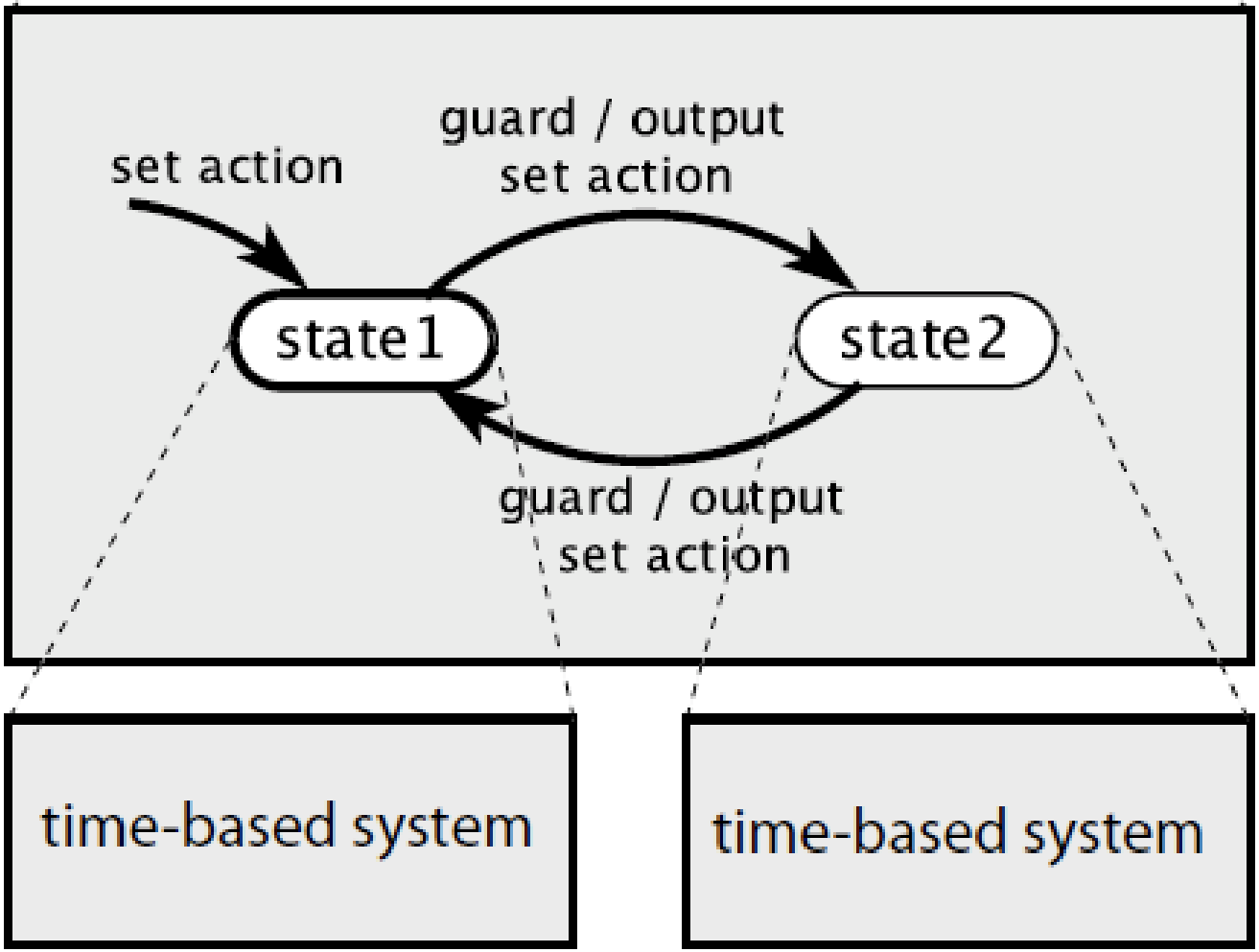
Where do Hybrid Systems arise?

- **Digital controller of physical "plant"**
  - thermostat
  - intelligent cruise/powertrain control in cars
  - aircraft auto pilot
- **Phased operation of natural phenomena**
  - bouncing ball
  - biological cell growth
- **Multi-agent systems**
  - ground and air transportation systems
  - interacting robots

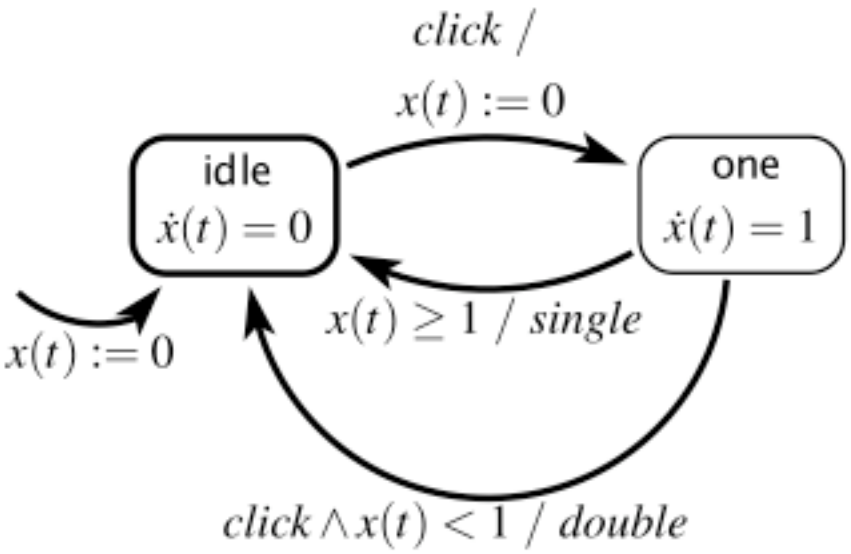
Timed Automata

Timed Automata: Special Case of Hybrid Systems



Mouse Double Click Detection

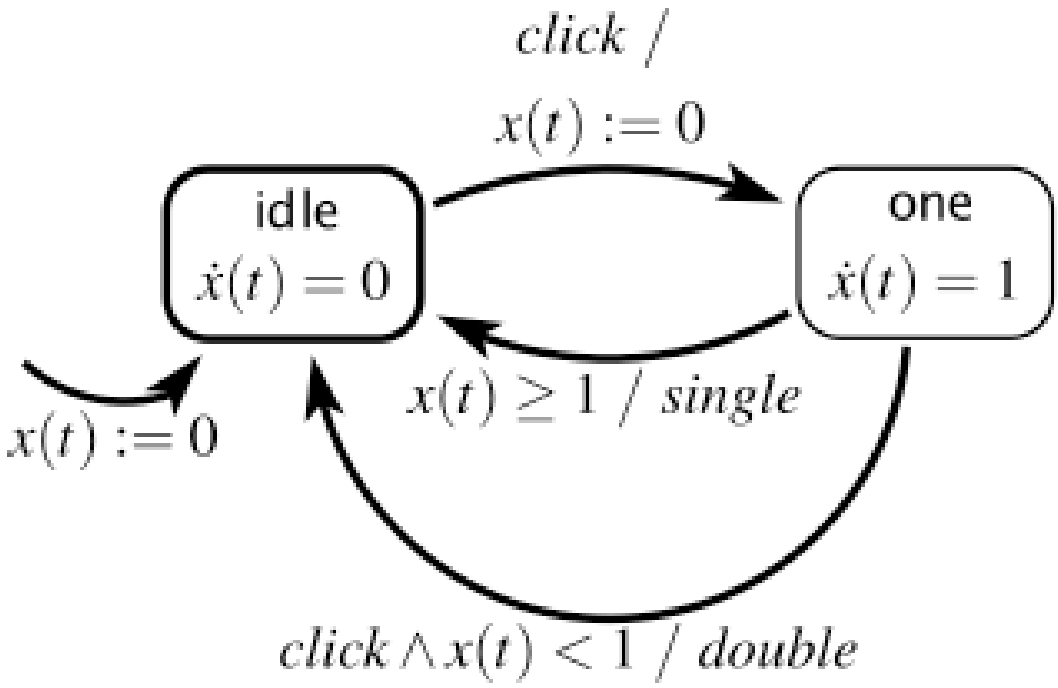
continuous variable: $x(t) \in \mathbb{R}$
inputs: $click \in \{present, absent\}$
outputs: $single, double \in \{present, absent\}$



This simple form of hybrid system is called a timed automaton, where the dynamics is just passage of time.

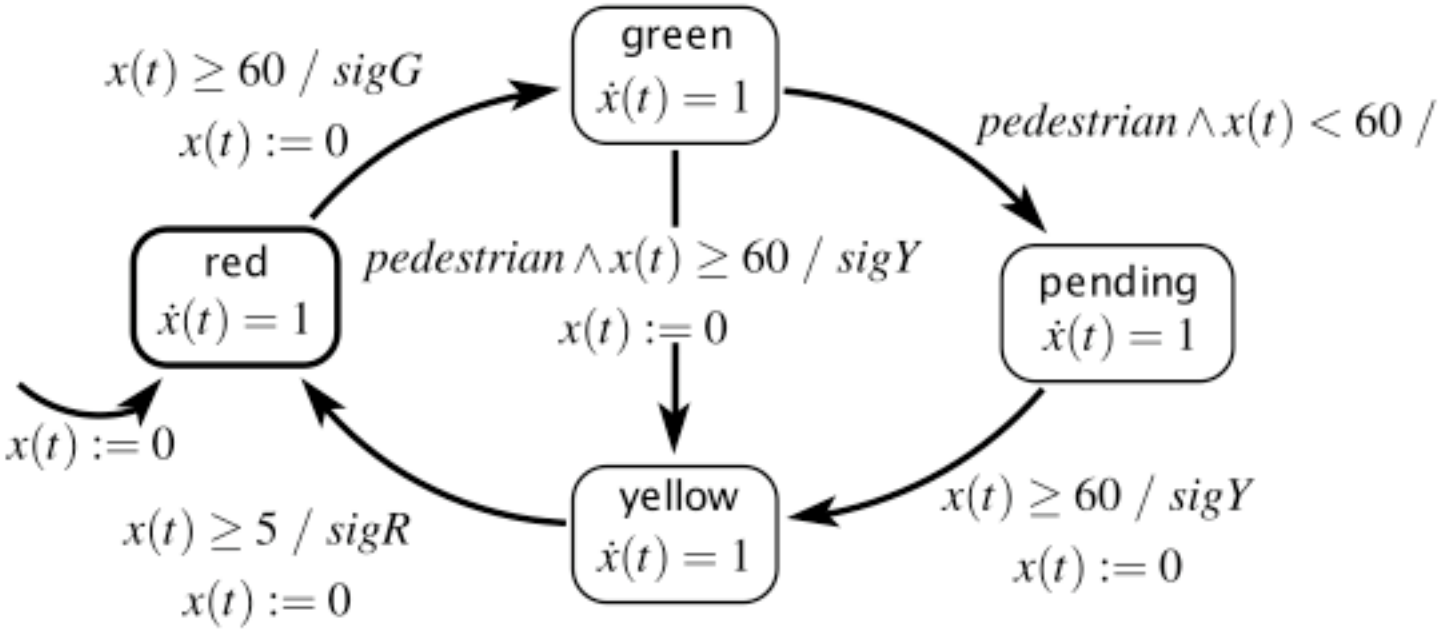
Quiz: How many states does this automaton have?

continuous variable: $x(t) \in \mathbb{R}$
inputs: $click \in \{present, absent\}$
outputs: $single, double \in \{present, absent\}$



Timed automaton model of a traffic light controller

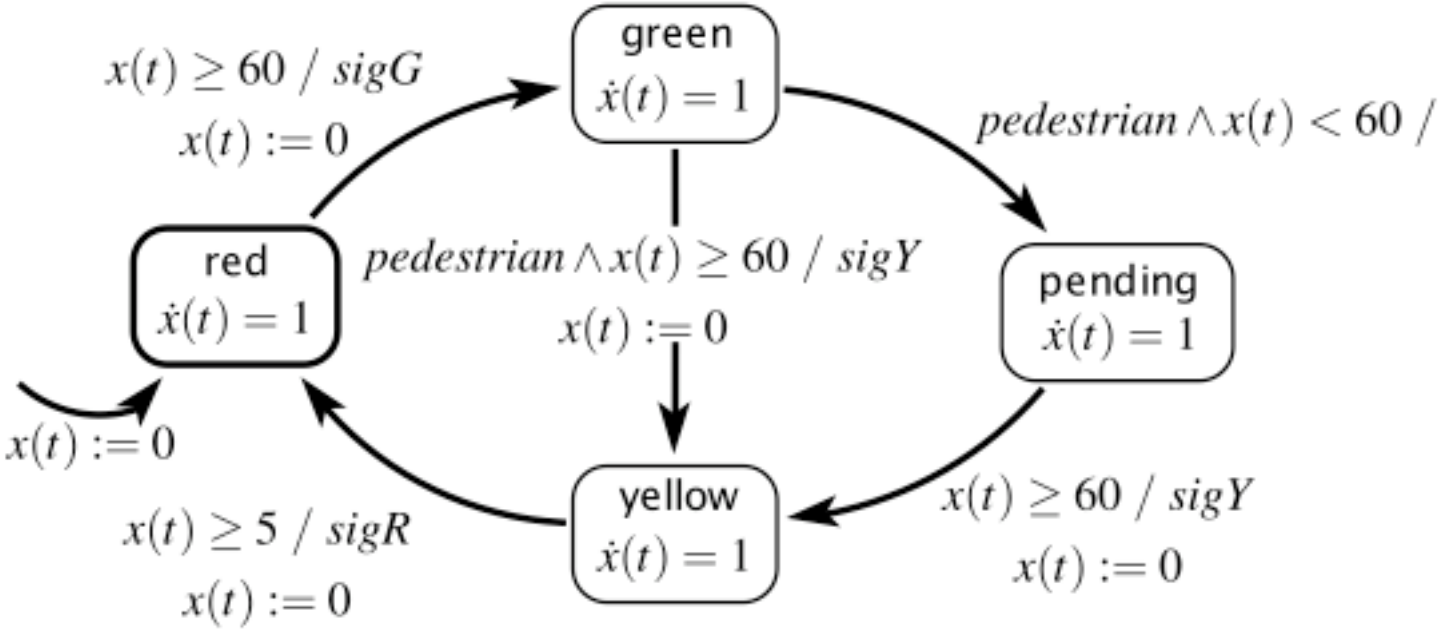
continuous variable: $x(t) : \mathbb{R}$
inputs: *pedestrian*: pure
outputs: *sigR*, *sigG*, *sigY*: pure



This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

When do reactions occur in a hybrid automaton?

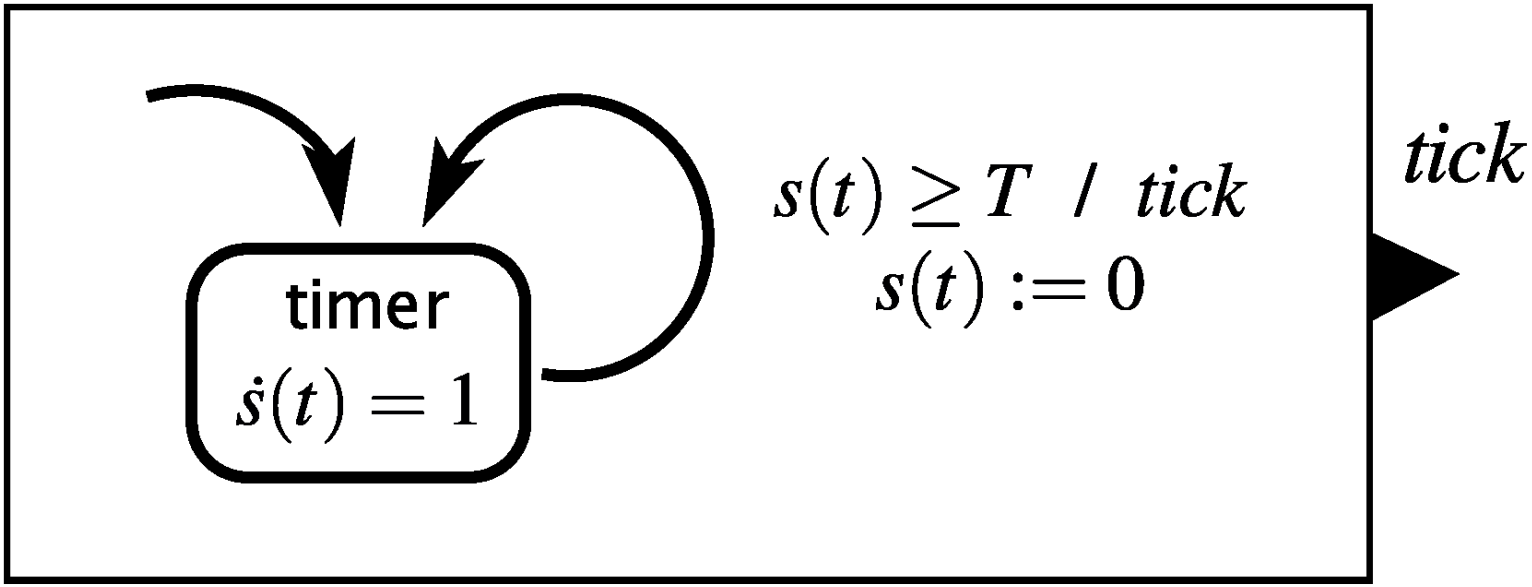
continuous variable: $x(t) : \mathbb{R}$
inputs: *pedestrian* : pure
outputs: *sigR*, *sigG*, *sigY* : pure



Reactions are occurring continually, with the continuous state variable *x* being continually updated.

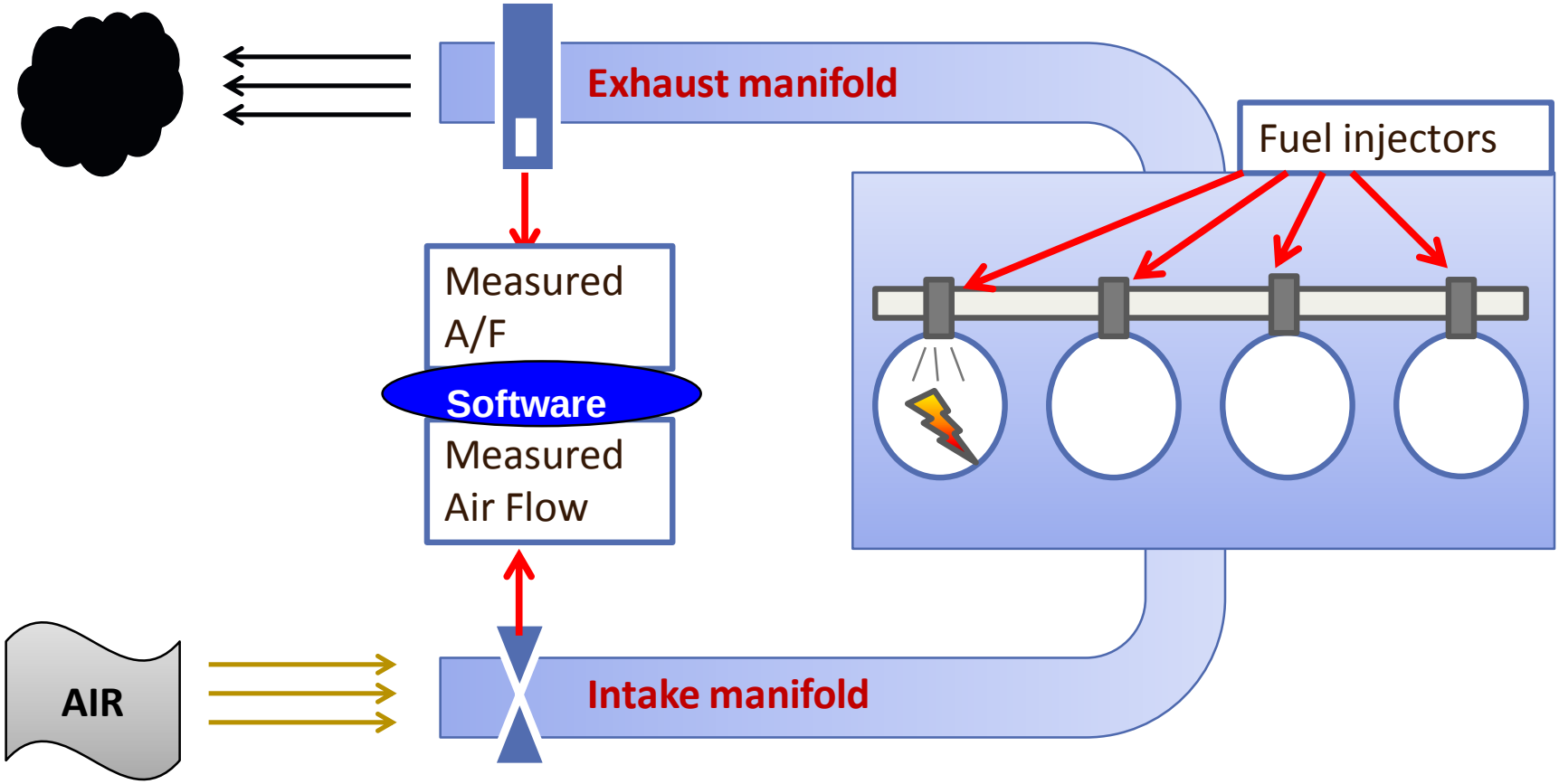
Example: "Tick" Generator (Timer)

How would you model a timer that generates a 'tick' each time T time units elapses?

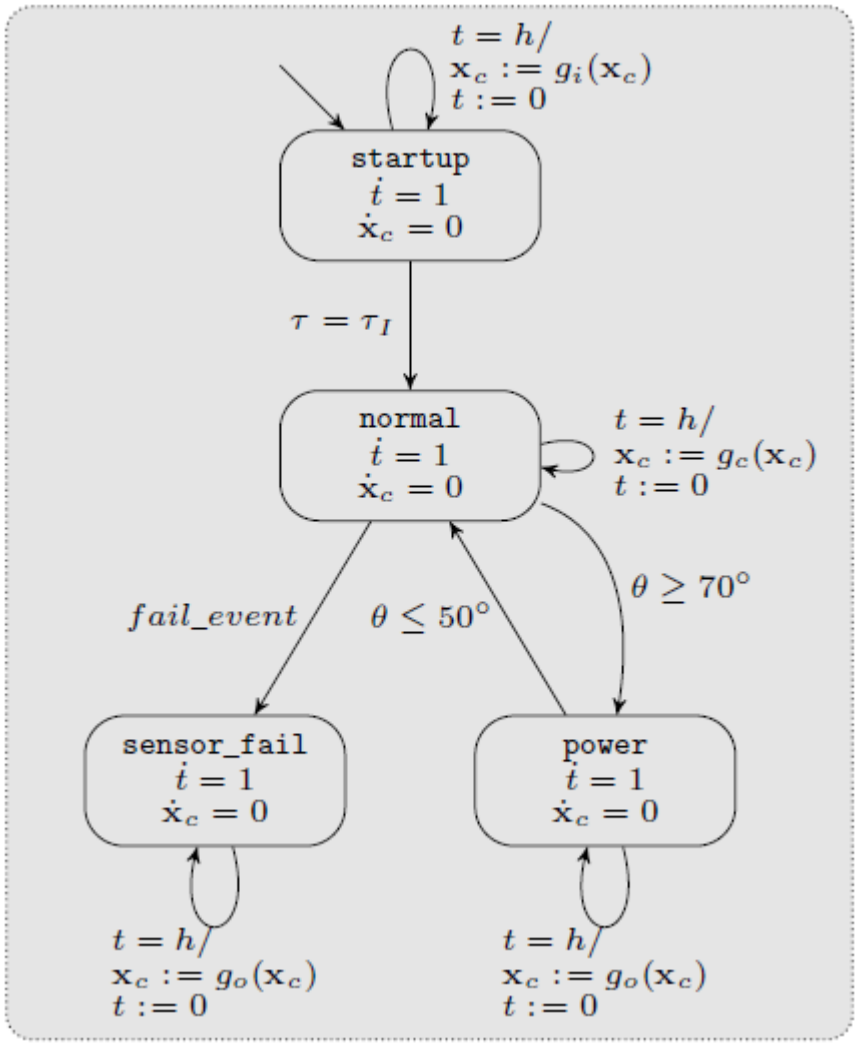


A similar timed automaton can model a generator of a timer interrupt.

Hybrid Automaton Model of Toyota Powertrain Control Example



Hybrid Automaton Model of Toyota Powertrain Control Example

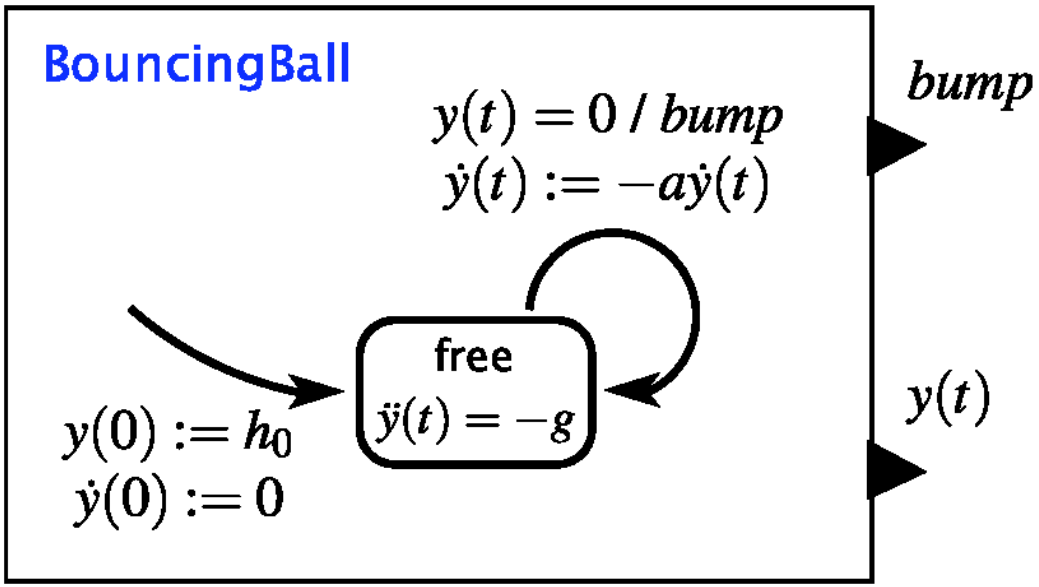


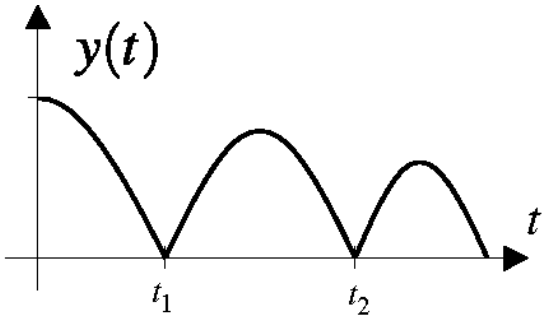
Four Operating Modes:

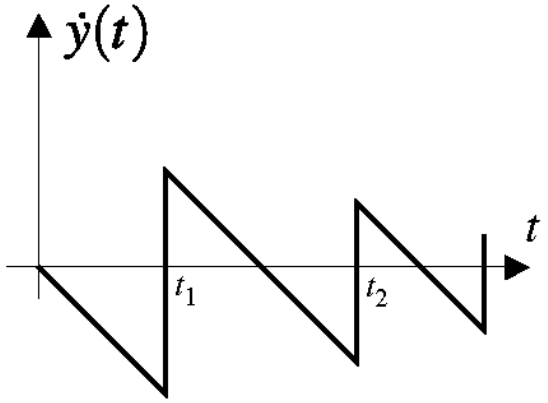
1. **Startup:** Wait for O2 sensors to start giving accurate readings (temp dependent), employ open-loop control
2. **Normal:** Use combination of feedback PI control and feedforward control to regulate A/F ratio
3. **Power:** Driver depresses gas pedal more (higher throttle angle) – switch to feedforward
4. **Sensor Failure:** switch to feedforward control

"Powertrain Control Verification Benchmark", Jin et al., HSCC 2014

Hybrid Automaton for Bouncing Ball







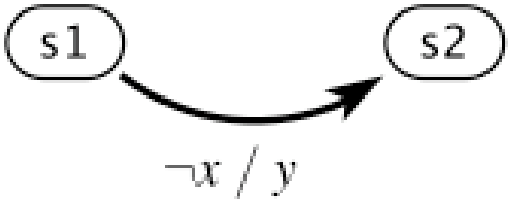
y – vertical distance from ground (position)

a – coefficient of restitution, $0 \cdot a \cdot 1$

If you plotted y(t), what would it look like?

When do reactions occur in a hybrid automaton?

input: $x \in \{present, absent\}$
output: $y \in \{present, absent\}$



- Suppose x and y are discrete and pure signals. When does the transition occur?

  Answer: at the earliest time t when x is absent after entering s1. This will always be the same time when s1 is entered. Why?

  If x is absent when s1 is entered, then the transition is taken then. If x is present when s1 is entered, then it will be absent at a time infinitesimally larger. How to model this rigorously?

Example: Newton's Cradle

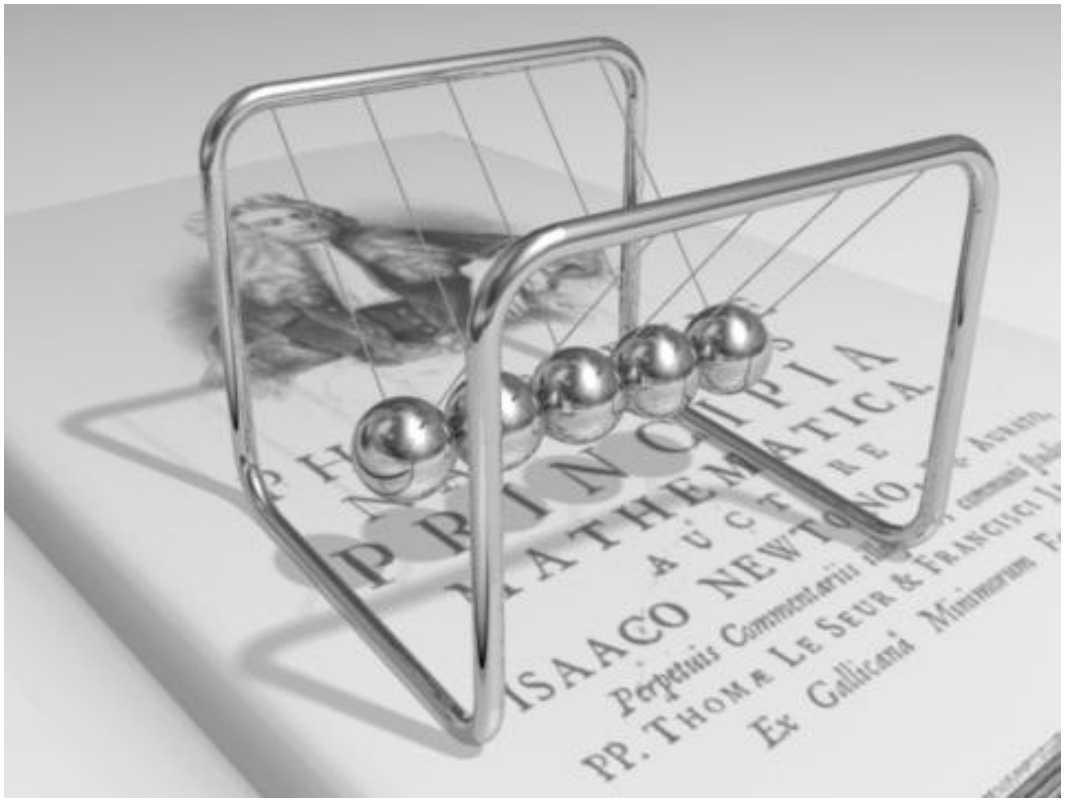


Image src: Wikipedia Commons

A middle ball does not move, so its momentum must be 0. But the momentum of the first ball is transferred somehow to the fifth. So there is an instant at which it is non-zero!

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Practice

- Hệ thống bơm nước
- Hệ thống cửa thang máy
- Hệ thống vòi nước tự động
- …

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Summary

1. What is Extended FSM?
2. What is Timed Automata?
3. Giving examples of ESM and timed automata

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Q&A