MINISTRY OF SCIENCE AND
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AHMET YESEVİ ÜNİVERSİTESİ

Halyk

Түркі әлемі математиктерінің
VII Дүниежүзілік Конгресі
(TWMS Congress-2023)

БАЯНДАМАЛАРЫНЫҢ ТЕЗИСТЕРІ

ABSTRACTS

of the VII World Congress of Turkic World Mathematicians
(TWMS Congress-2023)

ТЕЗИСЫ ДОКЛАДОВ

VII Всемирного Конгресса математиков тюркского мира
(TWMS Congress-2023)

20–23 қыркүйек, 2023, Түркістан, Қазақстан
September 20–23, 2023, Turkestan, Kazakhstan
20–23 сентября, 2023, Туркестан, Казахстан

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Дифференциалдық теңдеулер және математикалық физика теңдеулері

Differential equations and equations of mathematical physics

Дифференциальные уравнения и уравнения математической физики

ON THE ASYMPTOTIC STABILITY OF SOLUTIONS OF A THIRD-ORDER LINEAR VOLTAIRE INTEGRO-DIFFERENTIAL EQUATION WITH NONSMOOTH CUT-OFF FUNCTIONS

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All the functions involved are continuous and the relations take place when $t \geq t_0$, $t \geq \tau \geq t_0$; IDE is an integro-differential equation; the asymptotic stability of solutions of a third-order linear IDE is understood as the tendency to zero at $t \to \infty$ all solutions and their first and second derivatives.

Let [1]: $0 < W(t)$ - some weight function; $K(t,\tau) \equiv (W(t))^{-1} Q_2(t,\tau)W(\tau)$, $F(t) \equiv f(t)\,(W(t))^{-1}$; [2]: $K(t,\tau) \equiv \sum_{i=1}^{n} K_i(t,\tau)$,
$F(t) \equiv \sum_{i=1}^{n} F_i(t)$; $\psi_i(t)(i = 1..n)$ - some cutting functions,
$R_i(t,\tau) \equiv K_i(t,\tau)(\psi_i(t)\psi_i(\tau))^{-1}$, $E_i(t) \equiv F_i(t)(\psi_i(t))^{-1}(i = 1..n)$.

$R_i(t,\tau)$, $E_i(t)(i = 1..n)$ are called cut-off functions.

The problem. To establish sufficient conditions for the asymptotic stability of solutions of a third-order linear IDE of the Volterra type of the form

$$x'''(t) + \sum_{k=0}^{2} \left[a_k(t)x^{(k)}(t) + \int_{t_0}^{t} Q_k(t,\tau)x^{(k)}(\tau)d\tau \right] = f(t), \quad t \geq t_0$$

in the case when the cut functions $R_i(t,\tau)$, $E_i(t)(i = 1..n)$ they may be undifferentiable at some points of the semi-axis $t \geq t_0$.

This problem is being solved for the first time.

Keywords: integro-differential equation, asymptotic stability, weight functions, cutting functions.

AMS Subject Classification: 34K20, 45J05

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THE INVERSE PROBLEMS FOR THE HEAT EQUATION WITH INVOLUTION PERTURBATION

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We consider the equation of the type [1]

$$u_t(x,t) - u_{xx}(x,t) + \alpha u_{xx}(-x,t) + q(x)\,u(x,t) = f(x)\,, (x,t) \in \Omega,$$

with a complex-valued coefficient $q(x) = q_1(x) + iq_2(x)$, where $\Omega = \{-1 < x < 1,\ 0 < t < T\}$, $-1 < \alpha < 1$.

Problem: find a pair of functions $u(x,t)\,,\ f(x)$ satisfying equation and conditions

$$U_1(y) = y(-1) - y(1) = 0,\ \ U_2(y) = y'(-1) - y'(1) = 0;$$

$$u(x,0) = \varphi(x)\,,\ \ u(x,T) = \psi(x)\,,\ -1 \le x \le 1.$$

Let us introduce a non-self-adjoint second-order differential operator

$$L_{\alpha q} : D(L_{\alpha q}) \subset L_2(-1,1) \to L_2(-1,1)$$

by the formula

$$L_{\alpha q} y = -y''(x) + \alpha y''(-x) + q(x)\,y(x)$$

with the domain of definition

$$D(L_{\alpha q}) = \left\{ y(x) \in C^2[-1,1] : y(-1) = y(1)\,,\ \ y'(-1) = y'(1) \right\}.$$

Theorem 1. Let $q(x) \in C^2[-1,1]$, $\varphi(x)\,, \psi(x)$, $L_{\alpha q}\varphi(x)$, $L_{\alpha q}\psi(x) \in D(L_{\alpha q})$. Then problem has a unique solution $u(x,t)\,,\ f(x)$.

Keywords: heat equation with involution, Riesz basis, boundary value problem, inverse problem.

AMS Subject Classification: 34B09, 34K06, 35L05, 35L20

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FLUIDIZED BED REACTOR WITH RECYCLING

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Let a single-stage chemical reaction take place in a flow chemical reactor with an inhomogeneous fluidized bed. We propose that the transfer of matter in the dense and dilute phases, as well as the thermal energy in the diluted phase, are described by the complete displacement model, while the thermal energy of the dense phase is described by the complete mixing model. In the reactor, a first-order chemical reaction occurs, accompanied by the release (absorption) of heat according to the Arrhenius law. In addition, recycling is carried out, i.e. part of the flow leaving the reactor is returned to its inlet. Taking into account these assumptions, the equation for changing the mass and the temperature field in a dimensionless form for an adiabatic reactor can be written as follows [1]: for dense phase

$$\frac{\partial z_1}{\partial t} + U_1 \frac{\partial z_1}{\partial x} = (1 - z_1)\, g \exp\left(-\frac{\beta}{T_1}\right) - A\,(Z_1 - Z_2)\,, \tag{1}$$

$$\frac{dT_1}{dx} = U_1\omega(U_2 x\gamma T_2(1) + (1-\gamma)T_1 -$$

$$-(1 - U_1\gamma(1-x))T_1) + \omega g e^{-\frac{\beta}{T_1}} \int_0^1 (1 - Z_1)dx - B(T_1 - \int_0^1 T_2 dx); \tag{2}$$

for the diluted phase

$$\frac{\partial z_2}{\partial t} + U_2 \frac{\partial z_2}{\partial x} = A\,(z_1 - z_2) \tag{3}$$

$$\frac{\partial T_2}{\partial x} + U_2 \frac{\partial T_2}{\partial x} = A\,(T_1 - T_2) \tag{4}$$

The system of equations (1)-(4) is solved with the following initial and boundary conditions:

$$Z_1(x,0) = Z_{10}(x),\ z_2(x,0) = z_{20}(x),\ T_1(0) = T_{10}(x),\ T_2(x,0) = T_{20}(x)$$

$$Z_1(0,t) = Z_2(0,t) = \gamma\,(Z_1(1,t)U_1(1-x) + Z_2(1,t)U_2 x)\,,$$

$$T_2(0,t) = \gamma\,(T_1(t)U_1(1-x) + T_2(1,t)U_2 x) + (1-\gamma)T_i.$$

The solution of the problem is found and the influence of hydrodynamic parameters on the operating modes of the reactor is shown, and by selecting the values of the hydrodynamic parameters in the allowable range, it is possible to control the operating mode of the reactor with an inhomogeneous fluidized bed to achieve optimal conditions for the implementation of the technological process.

Keywords: chemical reactor, inhomogeneous fluidized bed,dense phase, diluted phase .

AMS Subject Classification: 35F16.

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HOMOGENIZATION OF ATTRACTORS TO GINZBURG-LANDAU EQUATIONS IN MEDIA WITH LOCALLY PERIODIC OBSTACLES

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Let $\Omega \subset \mathbb{R}^d, d \geq 2$, is a smooth bounded domain. Denote

$$\Upsilon_\epsilon = \{j \in \mathbb{Z}^d \,:\, \mathrm{dist}\,(\epsilon j, \partial\Omega) \geq \epsilon\sqrt{d}\}, \quad \Box \equiv \{\xi : -\frac{1}{2} < \xi_j < \frac{1}{2},\ j = 1, \dots, d\}.$$

Considering an 1-periodic in ξ smooth function $F(x,\xi)$, which satisfies $F(x,\xi)\Big|_{\xi\in\partial\Box} \geq \mathrm{const} > 0$, $F(x,0) = -1$, $\nabla_\xi F \neq 0$ as $\xi \in \Box\backslash\{0\}$, we define $D_j^\epsilon = \{x \in \epsilon\,(\Box + j) \mid F(x, \frac{x}{\epsilon}) \leq 0\}$. Perforated domain now is defined in the following way:

$$\Omega_\epsilon = \Omega \backslash \bigcup_{j\in\Upsilon_\epsilon} D_j^\epsilon.$$

Denote by ω the set $\{\xi \in \mathbb{R}^d \mid F(x,\xi) < 0\}$, and by S the set $\{\xi \in \mathbb{R}^d \mid F(x,\xi) = 0\}$. The boundary $\partial\Omega_\epsilon$ consists of $\partial\Omega$ and the boundary of the holes $S_\epsilon \subset \Omega$, $S_\epsilon = (\partial\Omega_\epsilon) \cap \Omega$. We study the asymptotic behavior of attractors to the problem

$$\begin{cases} \frac{\partial u_\epsilon}{\partial t} = (1+\alpha \mathrm{i})\Delta u_\epsilon + R(x,\frac{x}{\epsilon})\, u_\epsilon - \big(1+\beta(x,\frac{x}{\epsilon})\mathrm{i}\big)\, |u_\epsilon|^2 u_\epsilon + g(x), & x \in \Omega_\epsilon, \\ (1+\alpha \mathrm{i})\frac{\partial u_\epsilon}{\partial \nu} + \epsilon q(x,\frac{x}{\epsilon}) u_\epsilon = 0, & x \in S_\epsilon, t > 0, \\ u_\epsilon = 0, & x \in \partial\Omega, \\ u_\epsilon = U(x), & x \in \Omega_\epsilon, t = 0, \end{cases} \tag{1}$$

where α is a real constant, the vector ν is the outer unit vector to the boundary, $u = u_1 + \mathrm{i}u_2 \in \mathbb{C}$, $g(x) \in C^1(\Omega;\mathbb{C})$, nonnegative 1-periodic in ξ function $q(x,\xi)$ belongs to $C^1(\Omega;\mathbb{R}^d)$.

The Ginzburg-Landau equation is considered in locally periodic porous medium [1], with rapidly oscillating terms in boundary conditions. It is proved that the trajectory attractors of this equation converge in a weak sense to the trajectory attractors of the limit Ginzburg-Landau equation with an additional potential term.

Keywords: attractors, Ginzburg-Landau Equations, perforated domain, rapidly oscillating terms.

AMS Subject Classification: 35B20, 35B27.

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THE CONJUGATION PROBLEM FOR A MIXED PARABOLIC-HYPERBOLIC EQUATION OF THE THIRD ORDER

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In an area D, bounded by line segments $AC: x+y=0, CB: x-y=l, BB_0: x=l, B_0A_0: y=l, B_0A: x=0 (l>0)$, for the equation

$$L_1L_2u=0 \tag{1}$$

where

$$L_1=\begin{cases}\frac{\partial^2}{\partial x^2}-\frac{\partial}{\partial y}+c_1, y>0,\\ \frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}+c_1, y<0,\end{cases}\quad L_2=\frac{\partial}{\partial x}+\frac{\partial}{\partial y}, c_1, c_2-constant,$$

$D=D_1\cup AB\cup D_2$, $D_1=D\cap(y>0), D_2=D\cap(y<0), AB=\{(x,y):0<x<l, y=0\}$ the existence and uniqueness of the solution of the following conjugation problem is proved.

Problem 1. Need to define a function $u(x,y)$, which is the solution of the equation (1) in an area $D\setminus(y=0)$ and having the following properties:

$$u, u_y, u_{yy}\in C(\overline{D})\quad u\mid_{AA_0}=\varphi_1(y), u\mid_{BB_0}=\varphi_2(y),\quad u\mid_{AA_0}=\varphi_3(y), 0\le x\le\frac{l}{2},$$
$$u\mid_{BB_0}=\varphi_4(y), 0\le y\le h, \frac{\partial u}{\partial n}\mid_{AC}=\psi(x), 0\le x\le\frac{l}{2},$$

where $\varphi_i(y)(i=\overline{1,4}, \psi(x))$ given smooth functions, n internal normal, and $\varphi_i(y)\in C^1[0,h](i=1,2), \varphi_j(y)\in C[0,h](j=3,4), \psi(x)\in C^2[0,\frac{l}{2}].\varphi_1'(0)+\varphi_3(0)=\sqrt{2}\psi(0)$.

From the statement of problem 1 the following conditions of conjugation on the line of change of the equation type follow $y=0:$

$$u(x,-0)=u(x,+0), u_y(x,-0)=u_y(x,+0), u_{yy}(x,-0)=u_{yy}(x,+0).$$

Note, that the equation (1) it has the following characteristics: $y=const$ 2-fold, $x-y=const$ 1-fold (in an area D_1); $x+y=const$ 2-fold, $x-y=const$ 1-fold (in an area D_2).

An overview of boundary value problems for equations of mixed parabolic-hyperbolic type can be found in monographs [1-3].

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ON ONE INVERSE PROBLEM FOR INTEGRO-DIFFERENTIAL EQUATIONS OF THE FIFTH ORDER

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Investigating the inverse problem

$$u_{tt} = u_{ttxx} - u_{xxx} + b(t,x)u + \int_0^t K(t-s)u(s,x)ds + \varphi(t)f(t,x) + F(t,x), (t,x) \in \Omega, \quad (1)$$

$$u(0,x) = \psi_1(x),\ u_t(0,x) = \psi_2(x),\ x \in [0,1], \quad (2)$$

$$u(t,0) == 0,\ u_x(t,0) = 0,\ u_x(t,1) = 0,\ t \in [0,T], \quad (3)$$

$$u(t,x_1) = g_1(t),\ u(t,x_2) = g_2(t),\ t \in [0,T],\ x_1,\ x_2 \in (0,1),\ x_1 \neq x_2, \quad (4)$$

here $b(t,x)$, $f(t,x)$, $F(t,x)$, $\psi_1(x)$, $\psi_2(x)$, $g_1(t), g_2(t)$ are sufficiently smooth known functions, and $\psi_1(0) = \psi_1'(0) = \psi_1'(1) = 0$, $g_i(0) = \psi_1(x_i)$, $g_i'(0) = \psi_2(x_i)$, $i = 1,2$, $\Omega = \{(t,x)|0 \le x \le 1, 0 \le t \le T\}$.

Required to find functions:$u \in C^{2,3}(\Omega),\ K \in C[0,T]$ and $\varphi \in C[0,T]$.

Let

$$\begin{vmatrix} m(x_1) & c(t,x_1) \\ m(x_2) & c(t,x_2) \end{vmatrix} \neq 0, t \in [0,T], \quad (5)$$

where $m(x) = \int_0^1 G(x,\xi)\psi_1(\xi)d\xi,\ c(t,x) = \int_0^1 G(x,\xi)f(t,\xi)d\xi$,

$$G(x,\xi) = \begin{cases} \frac{1}{3}e^{x-\xi} - \frac{2}{3}e^{(\xi-x)/2}sin(\frac{\sqrt{3}}{2}(x-\xi)+\frac{\pi}{6}) - G_1(x,\xi), & 0 \le \xi \le x \le 1, \\ -G_1(x,\xi), & 0 \le x \le \xi \le 1. \end{cases}$$

The following theorem is proved

Theorem. If condition (5) is satisfied, then the inverse problem (1)-(4) is solvable and the solution $u \in C^{2,3}(\Omega)$, $K \in C[0,T]$, $\varphi \in [0,T]$ exists and is unique.

The solution can be constructed by the method of successive approximations.

AMS Subject Classification: 34K29.

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ON THE BASICITY OF EIGENFUNCTIONS AND ASSOCIATED FUNCTIONS OF THE STURM-LIOUVILLE OPERATOR WITH MULTIPLE SPECTRUM

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Consider in the space $H = L_2(0,1)$ the Sturm-Liouville operator

$$Ly = -y''(x);\; x \in (0,1). \tag{1}$$

$$U_i[y] = a_{i1}y(0) + a_{i2}y'(0) + a_{i3}y(1) + a_{i4}y'(1) = 0\ (i = 1,2) \tag{2}$$

with two $(i = 1,2)$ linearly independent boundary conditions. This condition means that of the six determinants $\Delta_{ij} = a_{1i}a_{2j} - a_{2i}a_{1j}$ $(i = 1,2,3,4)$ contained in the matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{pmatrix}$$

not all are zero. The eigenvalues of this operator coincide with the squares of the roots of its characteristic function [1]

$$\Delta(\lambda) = \Delta_{12} + \Delta_{34} + \Delta_{13}\frac{sin\sqrt{\lambda}}{\sqrt{\lambda}} + (\Delta_{14} + \Delta_{32})cos\sqrt{\lambda} - \Delta_{42}\sqrt{\lambda}sin\sqrt{\lambda}, \tag{3}$$

where $\Delta_{ij} = a_{1i}a_{2j} - a_{2i}a_{1j}$ $(i = 1,2,3,4)$ are minors of matrix A.
Problem statement. Let us assume that the function (3) has a countable set of multiple zeros. The question is what form does have the boundary conditions of the operator and whether the Riesz basis forms the eigen and associated functions of such a problem.

Theorem 1. The system of eigenfunctions and associated functions of the Sturm-Liouville operator $L_1(k)y = -y''(x);\; x \in (0,1)$

$$y'(0) + y'(1) = 0,\; (k \in C\ k \neq -1)\;\; y(0) + ky(1) = 0$$

forms a Riesz basis in the space $L_2(0,1)$.

Theorem 2. If $k \neq -1$, then the eigenfunctions and associated functions of the operator $L_2(k)y = -y''(x);\; x \in (0,1)$

$$ky'(0) - y'(1) = 0,\; (k \in C)\;\; y(0) - y(1) = 0$$

form a Riesz basis in the space $L_2(0,1)$.

Keywords: Sturm-Liouville operator, boundary conditions, eigenfunctions, associated functions, Riesz basis.
AMS Subject Classification: 34B24, 34L05, 34L10

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INVESTIGATION OF SOLUTIONS OF PARTIAL INTEGRO-DIFFERENTIAL EQUATION OF FOURTH ORDER BY MEANS OF A NEW WAY

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In [1,2] initial value problem for differential equation of the second order is considered. In [3] a new way to build solutions of partial differential equations of the fourth order of hyperbolic type is considered. We use results of this work.

We use denotations, classes and spaces of functions [1]: $G_2(T) := [0, T] \times R$; $G := [0, T] \times [0, T]$; $\overline{C}^{(k)}$ be the class of functions being continuous and bounded together with their derivatives up to the k-th order.

In this work we consider the following task

$$u_{tttt}(t, x) - 2a^2 u_{ttxx}(t, x) + a^4 u_{xxxx}(t, x) = bu_{ttt}(t, x) + cu_{ttx}(t, x) + du_{txx}(t, x)+$$

$$eu_{xxx}(t, x) + \int_0^t K(t, s)u(s, x)ds + f(t, x, u(t, x)), \quad (t, x) \in G_2(T) \tag{1}$$

with the initial conditions

$$\frac{\partial^k u(t, x)}{\partial t^k}|_{t=0} = u_k(x), \quad k = 0..3, \quad x \in R, \tag{2}$$

where

$$a, b, c, d = a^2 b, e = a^2 c \ - \ const, \ f(t, x, u) \in \overline{C}^4(G_2(T) \times R),$$

$$K(t, s) \in C(G), \int_0^T |K(t, s)|ds < \gamma = const.$$

In this work we do not reduce the initial value problem (1)-(2) to a canonical form, we reduce it to a system of integral equations. The developed scheme of using the method of additional argument for partial integro-differential equation of fourth order can be applied to partial integro-differential equations of higher order of other classes.

Keywords: method of additional argument, fourth order, partial derivatives, initial value problem, integral equation, integro-differential equation.

AMS Subject Classification: 35G55

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SOLVING OF SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS OF SECOND ORDER BY MEANS OF THE METHOD OF ADDITIONAL ARGUMENT

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Investigations of various classes of systems of partial differential equations of the first order on the base of the method of additional argument were considered in [1-3].

Considered the system of partial differential equations of the second order of type

$$u_{tt} = {p_1}^2(t,x)u_{xx} + a_1(t,x)u_t + b_1(t,x)u_x + f_1(t,x,u,\omega),$$

$$\omega_{tt} = {p_2}^2(t,x)\omega_{xx} + a_2(t,x)\omega_t + b_2(t,x)\omega_x + f_2(t,x,u,\omega), \quad (t,x) \in G_2(T) := [0,T] \times R, \quad (1)$$

with the initial conditions

$$\frac{\partial^k u}{\partial t^k} = u_k(x)|_{t=0}, \quad k = 0..1, \quad x \in R, \quad (2)$$

$$\frac{\partial^k \omega}{\partial t^k} = \omega_k(x)|_{t=0}, \quad k = 0..1, \quad x \in R, \quad (3)$$

where

$f_1(t,x,u,\omega) = \psi_1(t,x,u,\omega) + \int_0^t K_1(t,s)u(t,s)ds,$
$f_2(t,x,u,\omega) = \psi_2(t,x,u,\omega) + \int_0^t K_2(t,s)\omega(t,s)ds.$

Let $\overline{C}^{(k)}$ be the class of functions being continuous and bounded together with their derivatives up to the k-th order.

$u_k(x), \omega_k(x) \in \overline{C}^{(2-k)}(R), p_k(t,x), a_k(t,x), b_k(t,x) \in \overline{C}^2(G_2(T)),$
$\psi_k(t,x,u,\omega) \in \overline{C}^2(G_2(T) \times R \times R),$
$K_k(t,s) \in \overline{C}(R \times R), \int_0^T |K_k(t,s)|ds < \gamma = const, k = 0..1.$

In this work the system (1)-(2)-(3) is reduced to a type suitable to use the method of additional argument by a new way. Further, the obtained system is reduced to a system of integral equations by the method of additional argument.

Keywords: method of additional argument, system of equations, second order, partial derivatives, initial value problem, integral equation, contracting mapping. **AMS Subject Classification:** 35G55

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GREEN'S FUNCTION OF A BOUNDARY VALUE PROBLEM WITH INVOLUTION

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In this paper, we study a spectral problem for a second-order differential operator with involution and Neumann-type boundary conditions. The Green's function of the boundary value problem under study is constructed.

Consider the following boundary value problem

$$-u''(-x) = \lambda u(x) + f(x), -1 < x < 1, u'(-1) = 0, u'(1) = 0 \tag{1}$$

where $f(x)-$ continuous function.

Theorem 1. If λ is not an eigenvalue of the homogeneous boundary value problem (1), then for any continuous function $f(x)$ its solution can be represented as

$$u(x) = \frac{1}{8\rho}\left\{\frac{e^{\rho}-e^{-\rho}}{e^{\rho}+e^{-\rho}}\left(e^{\rho x}-e^{-\rho x}\right)\int\limits_{-1}^{1}\left(e^{\rho t}-e^{-\rho t}\right)f(t)\,dt\right.$$

$$-i\frac{e^{\rho i}+e^{-\rho i}}{e^{\rho i}-e^{-\rho i}}\left(e^{\rho i x}+e^{-\rho i x}\right)\int\limits_{-1}^{1}\left(e^{\rho i t}+e^{-\rho i t}\right)f(t)\,dt$$

$$+\int\limits_{-1}^{-x}\left[-i\left(e^{\rho i x}+e^{-\rho i x}\right)\left(e^{\rho i t}-e^{-\rho i t}\right)+\left(e^{\rho x}-e^{-\rho x}\right)\left(e^{\rho t}+e^{-\rho t}\right)\right]f(t)\,dt$$

$$+\int\limits_{-x}^{x}\left[i\left(e^{\rho i x}-e^{-\rho i x}\right)\left(e^{\rho i t}+e^{-\rho i t}\right)-\left(e^{\rho x}+e^{-\rho x}\right)\left(e^{\rho t}-e^{-\rho t}\right)\right]f(t)\,dt$$

$$\left.+\int\limits_{x}^{1}\left[i\left(e^{\rho i x}+e^{-\rho i x}\right)\left(e^{\rho i t}-e^{-\rho i t}\right)-\left(e^{\rho x}-e^{-\rho x}\right)\left(e^{\rho t}+e^{-\rho t}\right)\right]f(t)\,dt\right\}$$

The important point follows from the theorem.

Corollary 1. If λ is not an eigenvalue of the homogeneous boundary value problem (1), then there exists a unique Green's function of problem (1)

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$$G(x,t,\lambda)=\frac{1}{8\rho}\left\{\frac{e^{\rho}-e^{-\rho}}{e^{\rho}+e^{-\rho}}\left(e^{\rho x}-e^{-\rho x}\right)\int\limits_{-1}^{1}\left(e^{\rho t}-e^{-\rho t}\right)f(t)\,dt\right.$$

$$\left.-i\frac{e^{\rho i}+e^{-\rho i}}{e^{\rho i}-e^{-\rho i}}\left(e^{\rho ix}+e^{-\rho ix}\right)\int\limits_{-1}^{1}\left(e^{\rho it}+e^{-\rho it}\right)f(t)\,dt+g(x,t,\lambda)\right\}$$

$$g(x,t,\lambda)=\begin{cases}-i\left(e^{\rho ix}+e^{-\rho ix}\right)\left(e^{\rho it}-e^{-\rho it}\right)+\left(e^{\rho x}-e^{-\rho x}\right)\left(e^{\rho t}+e^{-\rho t}\right), -1\le t<-x\\ i\left(e^{\rho ix}-e^{-\rho ix}\right)\left(e^{\rho it}+e^{-\rho it}\right)-\left(e^{\rho x}+e^{-\rho x}\right)\left(e^{\rho t}-e^{-\rho t}\right),\ -x\le t\le x\\ i\left(e^{\rho ix}+e^{-\rho ix}\right)\left(e^{\rho it}-e^{-\rho it}\right)-\left(e^{\rho x}-e^{-\rho x}\right)\left(e^{\rho t}+e^{-\rho t}\right),\ x<t\le 1\end{cases}$$

In the case of boundary conditions of Dirichlet type, the solvability of the problem was considered in [1].

Keywords: differential equation with involution, eigenfunction expansions, basis

AMS Subject Classification: 65N80 Fundamental solutions, Green's function methods, etc. for boundary value problems involving PDEs

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THE DIRICHLET PROBLEM FOR AN ELLIPTIC EQUATION WITH THREE SINGULAR COEFFICIENTS WITH NEGATIVE PARAMETERS

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Consider the equation

$$L_{\alpha\beta\gamma}u \equiv u_{xx} + u_{yy} + u_{zz} + \frac{2\alpha}{x}u_x + \frac{2\beta}{y}u_y + \frac{2\gamma}{z}u_z = 0 \tag{1}$$

in domain $\Omega = \{(x,y,z):\, x \in (0,a)\,,\, y \in (0,b)\,,\, z \in (0,c)\}$, where $a,b,c \in R^+$.

In the domain Ω for equation (1) we study the following problem.

Problem D. Find a function $u\,(x,y,z)$ that satisfies the following conditions:

$$u\,(x,y,z) \in C\left(\bar{\Omega}\right) \cap C^{2,2,2}_{x,y,z}\,(\Omega)\,, x^{2\alpha}u_x,\ y^{2\beta}u_y,\ z^{2\gamma}u_z \in C\left(\bar{\Omega}\right);\ \ L_{\alpha\beta\gamma}u = 0,\ (x,y,z) \in \Omega;$$

$$u\,(0,y,z) = u\,(a,y,z) = 0,\ y \in [0,b],\ z \in [0,c];\ u(x,y,0) = u(x,y,c) = 0,\ x \in [0,a],\ y \in [0,b];$$

$$u\,(x,0,z) = \psi_1\,(x,z)\,,\ \ u\,(x,b,z) = \psi_2\,(x,z)\,,\ x \in [0,a]\,,\ z \in [0,c]\,,$$

where $\psi_1\,(x,z)$ and $\psi_2\,(x,z)$ are given functions.

The problem when $\alpha,\ \beta,\ \gamma \in (0,1/2)$ studied in [1]. In this paper, problem D is studied in the case when the parameters of the equation satisfy the conditions $\alpha,\beta,\gamma \in (-\infty,0]$. At the same time, the following theorem is proved.

Theorem. Let $\alpha,\gamma \in (-2k-2,\,-2k]\,,\ \ k = 0,1,2,...,\ \beta \in (-\infty,0]$ and functions $\psi_1\,(x,z)$ and $\psi_2\,(x,z)$ satisfy the following conditions:

I. $\psi_l\,(x,z) \in C^{2k+4,2k+4}_{x,z}\left(\overline{\Pi}\right)$, $l = \overline{1,2}$, where $\Pi = \{(x,z):\ 0 < x < a,\ \ 0 < z < c\}$;

II . $\frac{\partial^j}{\partial x^j}\ \psi_l(x,z)|_{x=0} = \frac{\partial^j}{\partial x^j}\ \psi_l(x,z)|_{x=a} = 0,\ \frac{\partial^j}{\partial z^j}\ \psi_l(x,z)|_{z=0} = \frac{\partial^j}{\partial z^j}\ \psi_l(x,z)|_{z=c} = 0,\ j = \overline{0,2k+2}.$

Then a solution to Problem D exists and is defined by the formula

$$u\,(x,y,z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4X_n\,(x)\,Z_m\,(z)\,\omega_{nm}\,(y)}{\left[acJ_{3/2-\alpha}\,(\sigma_{\alpha n})\,J_{3/2-\gamma}\,(\sigma_{\gamma m})\right]^2},$$

where $J_\nu\,(x)$ is the Bessel function of the first kind [2], $X_n\,(x) = x^{1/2-\alpha}J_{1/2-\alpha}\,(\sigma_{\alpha n}x/a)\,,\ n \in N;$ $Z_m\,(z) = z^{1/2-\gamma}J_{1/2-\gamma}\,(\sigma_{\gamma m}z/c)\,,\ m \in N;$ $\omega_{nm}(y) = [\bar{K}_{1/2-\beta}(\sqrt{\lambda_{nm}}y) - P_{nm}(y)\bar{K}_{1/2-\beta}(\sqrt{\lambda_{nm}}b)]$ $\times\psi_{1nm} + P_{nm}\,(y)\,\psi_{2nm};\ \psi_{lnm} = \int_0^c\int_0^a \psi_l\,(x,z)\,x^{2\alpha}X_n\,(x)\,z^{2\gamma}Z_m\,(z)\,dxdz,\ l = \overline{1,2};\ P_{nm}\,(y) = (y/b)^{1/2-\beta}\,I_{1/2-\beta}\left(\sqrt{\lambda_{nm}}y\right)/I_{1/2-\beta}\left(\sqrt{\lambda_{nm}}b\right),\ \bar{K}_\nu\,(x) = 2^{1-\nu}x^\nu K_\nu\,(x)\,/\Gamma\,(\nu)\,,\ \nu > 0,\ K_\nu\,(x)$ is the MacDonald function of order ν[2], $\lambda_{nm} = (\sigma_{\alpha n})^2 + (\sigma_{\gamma m})^2$, $\sigma_{\alpha n}$ and $\sigma_{\gamma m}$ are the positive roots of the equations $J_{1/2-\alpha}\,(x) = 0$ and $J_{1/2-\gamma}\,(x) = 0$ respectively.

Keywords: Dirichlet problem, Bessel function, singular coefficient, elliptic type.

AMS Subject Classification: 35J25

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INTEGRATION THE MODIFIED KORTEWEG-DE VRIES-LIOUVILLE-SINE GORDON (MKDV-L-SG) EQUATION IN THE CLASS OF PERIODIC INFINITE-GAP FUNCTIONS

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Abstract. In this thesis, the method of the inverse spectral problem is used to integrate the nonlinear modified Korteweg-de Vries-Liouville-sine Gordon (mKdV-L-sG) equation in the class of periodic infinite-gap functions.

Statement of the problem. Consider the Cauchy problem for the modified Korteweg-de Vries-Liouville-sine Gordon (mKdV-L-sG) equation of the following form

$$\begin{cases} q_{xt} = a(t)\left\{q_{xxxx} - \frac{3}{2}q_x^2 q_{xx}\right\} + b(t)e^q + c(t)e^{-q}, q = q(x,t),\ x \in R,\ t > 0 \\ q(x,t)|_{t=0} = q_0(x), \quad q_0(x+\pi) = q_0(x) \in C^6(R) \end{cases} \tag{1}$$

in the class of real infinite-gap π periodic with respect to x functions:

$$q(x+\pi,t) = q(x,t), \quad q(x,t) \in C^{4,1}_{x,t}(t>0) \cap C(t \geq 0). \tag{2}$$

In this paper, we propose an algorithm for constructing exact solution $q(x,t), x \in \mathbb{R}, t > 0$ of problem (1)-(2) by reducing it to an inverse spectral problem for the following Dirac operator:

$$L(\tau,t)y \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2}q_x'(x+\tau,t) \\ \frac{1}{2}q_x'(x+\tau,t) & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix},$$
$$x \in R,\ t > 0,\ \tau \in R. \tag{3}$$

In this thesis proved the following theorem:

Theorem. If initial function $q_0(x)$ satisfies the following condition:

$$q_0(x+\pi) = q_0(x) \in C^6(\mathbb{R})$$

then the problem (1)-(2) has unique solution, which defined sum of a uniformly convergent functional series constructed by solving the system of Dubrovin equations and the first trace formula satisfies the mKdV-L-sG equations.

Keywords: Modified Korteweg-de Vries-Liouville-sine Gordon (mKdV-L-sG) equation, Dirac operator, spectral data, system of Dubrovin equations, trace formula.

AMS Subject Classification: 34A34, 34A55, 34B05, 34C25, 34L05, 34L40, 35A09, 35B10, 35J10, 35G20, 35Q53 .

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THE CRITERION OF INVERTIBILITY OF THE MIXED TYPE MINIMAL OPERATOR

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Let $\Omega \subset R^2$ is a domain, bounded by segment $AB: y=0,\ 0<x<1$, and by characteristics $AC:\ x-\frac{2}{m+2}(-y)^{\frac{m+2}{2}}=0$ and $BC:\ x+\frac{2}{m+2}(-y)^{\frac{m+2}{2}}=1$ of the Gellerstedt equation

$$Lu \equiv -(-y)^m u_{xx} + u_{yy} = f(x,y), \tag{1}$$

The overdetermined Cauchy problem is considered: to find a regular solution of (1) in the Ω domain, satisfying the conditions:

$$u|_{y=0} = 0, \quad \left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, \tag{2}$$

$$u\big|_{AC:\, x-\frac{2}{m+2}(-y)^{\frac{m+2}{2}}=0} = 0, \quad u\big|_{BC:\, x+\frac{2}{m+2}(-y)^{\frac{m+2}{2}}=1} = 0. \tag{3}$$

Theorem 1. The minimal operator ((1), (2), (3) problem) is invertible in $L_2(\Omega)$ if and only if next conditions are met

$$\int_0^{\xi} d\xi_1 \int_1^{\xi} \frac{(\eta_1-\xi_1)^{2\beta}}{(\xi-\xi_1)^{\beta}(\eta_1-\xi)^{\beta}} \cdot f_1(\xi_1,\eta_1)\, d\eta_1 = 0, \tag{4}$$

$$\int_0^{\xi} d\xi_1 \int_1^{\xi} \frac{\eta_1-\xi_1}{(\xi-\xi_1)^{1-\beta}(\eta_1-\xi)^{1-\beta}} \cdot f_1(\xi_1,\eta_1)\, d\eta_1 = 0, \tag{5}$$

where the function f_1 is determined by a given function $f(x,y)$ from (1). In this case, the solution of overdetermined Cauchy problem is representable by the formula:

$$u(\xi,\eta) = \int_0^{\xi} d\xi_1 \int_1^{\eta} R(\xi,\eta,\xi_1,\eta_1) \cdot f_1(\xi_1,\eta_1)\, d\eta_1, \tag{6}$$

where $R(\xi,\eta,\xi_1,\eta_1)$ is the Riemann function of the Goursat problem.

Keywords: minimal differential operator, Gellerstedt equation, criterion, boundary condition, hypergeometric function.

AMS Subject Classification: 35L80, 35M10, 35N30, 33C05

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HIGH-ORDER ACCURACY COMPACT FINITE DIFFERENCE SCHEMES

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Fourth order differential equations subject to some boundary conditions arise in the mathematical description of some physical systems. For example, mathematical models of deflection of beams. These beams, which appear in many structures, deflect under their own weight or under the influence of some external forces(see, [2], [3], [4]). A brief and easily acces- sible discussion and the physical interpretation for some of the boundary conditions associated with the linear beam equation can be found in the work of Zill and Cullen [1]. In the paper [5] Taylor's decomposition on five points was presented. This approach permits us to construct the four-step difference schemes of the sixth order of accuracy for the approximate solutions of initial and boundary value problems for the fourth order ordinary differential equations generated by Taylor's decomposition on five points. The theoretical statements for the solution of these difference schemes were supported by the results of numerical experiments. In the present paper high-order accuracy compact finite difference schemes generated by the new Taylor's decomposition on five points are used for solving fourth-order differential equations. The theoretical statements for the solution of these difference schemes are supported by the results of numerical experiments.

Keywords: A new Taylor's decomposition on five points, compact difference scheme, approximation.

AMS Subject Classification: 65M, 65J.

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ON THE ASYMPTOTICS OF THE ATTRACTORS OF THE 2D NAVIER–STOKES SYSTEM IN A MEDIUM WITH OBSTACLES

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Let Ω be a smooth bounded domain in $\mathbb{R}^2$ and $\varepsilon > 0$ is a small parameter. Denote

$$\Upsilon_\varepsilon = \left\{ r \in \mathbb{Z}^2 : \operatorname{dist}(\varepsilon r, \partial\Omega) \geq \sqrt{2}\varepsilon \right\}, \quad \square \equiv \left\{ \xi : -\frac{1}{2} < \xi_i < \frac{1}{2}, i = 1, 2 \right\}.$$

Given an 1-periodic in ξ smooth function $F(x, \xi)$ satisfying $F(x,\xi)|_{\xi\in\partial\square} \geq \text{const} > 0$, $F(x, 0) = -1$ and $\nabla_\xi F \neq 0$ for $\xi \in \square \setminus \{0\}$, we define

$$G_\varepsilon^r = \left\{ x \in \varepsilon(\square + r) \mid F\left(x, \frac{x}{\varepsilon}\right) \leq 0 \right\}, \qquad G_\varepsilon = \bigcup_{r\in\Upsilon_\varepsilon} G_\varepsilon^r$$

and the perforated domain $\Omega_\varepsilon = \Omega \setminus G_\varepsilon$. We study the asymptotic behavior of the trajectory attractors to the following initial boundary value problem for an autonomous two-dimensional Navier–Stokes system of equations:

$$\begin{cases} \dfrac{\partial u_\varepsilon}{\partial t} - 2\nu \operatorname{div} \overline{e}(u_\varepsilon) + (u_\varepsilon, \nabla)u_\varepsilon + \nabla p_\varepsilon = g\left(x, \dfrac{x}{\varepsilon}\right), & x \in \Omega_\varepsilon, \\ (\nabla, u_\varepsilon) = 0, & x \in \Omega_\varepsilon, \\ \sigma_n^\varepsilon + \alpha_n \varepsilon(u_\varepsilon, n) = 0, & x \in \partial G_\varepsilon, t \in (0, +\infty), \\ \sigma_\tau^\varepsilon + \alpha_\tau \varepsilon(u_\varepsilon, \tau) = 0, & x \in \partial G_\varepsilon, \\ u_\varepsilon = 0, & x \in \partial\Omega \\ u_\varepsilon = U(x), & x \in \Omega_\varepsilon, t = 0. \end{cases} \tag{1}$$

Here $p_\varepsilon(x,t)$ is a pressure, $u_\varepsilon = u_\varepsilon(x,t) = (u_\varepsilon^1, u_\varepsilon^2)$, $2\overline{e}(u_\varepsilon) = \nabla u_\varepsilon + (\nabla u_\varepsilon)^T$, $g_\varepsilon(x) = g\left(x, \frac{x}{\varepsilon}\right) = (g^1, g^2) \in \left(L_2(\Omega; \mathbb{R})\right)^2$, $\sigma_n^\varepsilon(u_\varepsilon, p_\varepsilon) = -p_\varepsilon + \nu((\overline{e}(u_\varepsilon)n), n)$, $\sigma_\tau^\varepsilon(u_\varepsilon) = \nu((\overline{e}(u_\varepsilon)n), \tau)$, $n = (n_1, n_2)$ is the vector of the unit external normal to the boundary. Such boundary conditions, together with the system of Navier–Stokes equations, were proposed in [1] as the simplest model for steam-water condensation on cold walls. It is shown that the trajectory attractors of problem (1) converge in a weak topology to the trajectory attractors of the homogenized problem defined in the domain without obstacles, as $\varepsilon \to 0$.

Keywords: attractors, Navier-Stokes system, perforated domain, rapidly oscillating terms.

AMS Subject Classification: 35B20, 35B27.

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INVERSE PROBLEM OF PARABOLIC TYPE IN A BOUNDED DOMAIN

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Below we consider issues related to the integration method based on the Green function and the uniqueness of the solution of a nonlocally inverse parabolic problem in a bounded domain. When the problems under study have been transformed to the first kind integral equations, then methods related to regularization algorithms [1] and others can be applied.

We study the inverse problem of the form:

$$L_0W = \frac{\partial}{\partial t}(W_t + W) - \frac{\partial}{\partial x^2}(W_t + W) = f(x,t), (x,t) \in \overline{D},(D = (0,T) \times (0,T)) \tag{1}$$

$$\begin{cases} W(x,0) = g(x); AW|_{t=0} = 0, \forall \in [0,X], (AW = W_t + W, \forall (,t) \in \overline{D}), \\ (\frac{\partial}{\partial x}AW)|_{x=0} = (Hz)(t) \equiv \psi(t); AW|_{x=X} = 0, \forall t \in [0,T], (\psi(0) = 0), \\ p(t)(\frac{\partial}{\partial x}AW)|_{x=0} = \sum\limits_{j=1}^{n} \alpha_j\,(t)\,AW|_{x=x^j} + \varphi_0\,(t)\,;\ (\varphi_0(0) = 0; x^j \in (0,X)), \\ Hz \equiv \int\limits_0^t K(t,\tau)\gamma(\tau)z(\tau)d\tau, (z(0) = 0; K(t,t), p(t) \neq 0, \forall t \in [0,T]), \end{cases}$$

where under certain restrictions with respect to given functions, it is necessary to find the function $\Phi = (W,z)$. The problem under consideration, taking into account the $u(x,t)$ and Green function $G(\xi,\tau;x,t)$, is transformed into equations:

$$\begin{cases} W(x,t) = e^{-t}g(x) + \int\limits_0^t e^{-(t-\tau)}u(x,\tau)d\tau \equiv (Bu)(t), \\ u\,(x,t) = \int\limits_0^t \psi\,(\tau)\,G\,(0,\tau;x,t)\,d\tau - \int\limits_0^t\int\limits_0^X G\,(s,\tau;x,t)\,f(s,\tau)dsd\tau, \\ \psi(t) = p^{-1}(t)\,\{\,\sum\limits_{j=1}^{n} \alpha_j\,(t)\,[\,\int\limits_0^t \psi\,(\tau)\,G\left(0,\tau;x^j,t\right)d\tau - \int\limits_0^t\int\limits_0^X G\left(s,\tau;x^j,t\right)f(s,\tau)dsd\tau\,] + \varphi_0(t)\,\}\,, \\ \vartheta(t) = (K(t,t))^{-1}\,[\,\int\limits_0^t K_\tau(t,\tau)\vartheta(\tau)d\tau + \psi(t)\,]\,, (K \in C^{0,1}(D_0), D_0 = \{\,(t,\tau) : 0 \le \tau \le t \le T\,\}), \\ \int\limits_0^t \gamma(\tau)z(\tau)d\tau = \vartheta(t), (\vartheta(0) = 0; \vartheta \in^1_\phi [0,T]; \phi = \int\limits_0^t \gamma(\tau)d\tau, 0 < \gamma(t) \in L^1(0,T)). \end{cases}$$

So, on the basis of the uniqueness and regularizability of the solution of system (3), the regularizability of the inverse problem (1), (2), moreover, in the domain D there is a unique generalized solution of the problem under study, constructed according to the rule: $W = Bu$.

Keywords: Volterra equation of the first kind, inverse problem, regularization method, algorithm.

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CAUCHY PROBLEM FOR A LOADED HYPERBOLIC EQUATION

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Currently, studies in the field of loaded equations and their applications are relevant. The intensive and deep research of the last decades resulted in numerous works, references to some of them can be found in the monographs [1]–[4].

The report will present some results for the loaded hyperbolic equation

$$u_{xx} - u_{yy} + a(x,y)u_x + b(x,y)u_y + c(x,y)u = \lambda \sum_{i=1}^{n} a_i(x,y)u(x_i,y) \tag{1}$$

with given data on line $y = kx$, $0 < k \leqslant \infty$, where $a(x,y)$, $b(x,y)$, $c(x,y)$, $a_i(x,y)$ are smooth enough functions, $0 < x_i < l$, $\lambda = \text{const}$.

It will be shown that the domain of uniqueness of the problem essentially depends on the location of the point $\widetilde{x} = \min_{i=\overline{1,n}}(x_i, l - x_i)$ and coincides with the domain of uniqueness of the formulated Cauchy problem for the equation (1) for $\lambda = 0$, only in exceptional cases.

Keywords: loaded equations, Cauchy problem, hyperbolic equation, smooth functions.

AMS Subject Classification: 35L10

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INVERSION FORMULA FOR A DISTRIBUTED ORDER DIFFERENTIATION OPERATOR

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Consider the operator

$$\mathcal{D}_x^{[\mu]} f(x) = \int_{\mathbb{R}} \frac{d^t}{dx^t} f(x)\, \mu\,(dt)\,, \tag{1}$$

where $\frac{d^t}{dx^t}$ stands the fractional derivative of order t with respect to x; and μ is a non-negative Lebesgue–Stieltjes measure on $\mathbb{R}$ with a compact support. It is assumed that $\sup \operatorname{supp} \mu > 0$.

Operators of the form (1) belong to the class of fractional differentiation operators of distributed order [1, 2]. In the report we discuss the issue of constructing an inverse operator for the operator (1).

Keywords: fractional derivative, distributed order differentiation operator, inversion formula.

AMS Subject Classification: 26A33

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CONJUGATION PROBLEM FOR 4TH ORDER PARTIAL DERIVATIVE EQUATIONS WITH DIFFERENT REAL CHARACTERISTICS

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In the area bounded by straight line segments $AC : x + y = 0, CB : x - y = l, BB_0 : x = l,$ $B_0A_0 : y = h, A_0A : x = 0$, consider the conjugation problem for the equations

$$0 = \begin{cases} u_{xyyy} + a(x)u_x, & (x, y) \in D_1, \\ u_{xyyy} - u_{xxyy}, & (x, y) \in D_2, \end{cases} \tag{1}$$

where $a(x)$ is a given function, $D_1 = D \cap (y > 0), D_2 = D \cap (y < 0)$.

Note that the lines $y = const, x = const$ are simple and triple characteristics of equation (1) for $y > 0$, and $y = const$ by double, $x \pm y - const$ simple real characteristics of equation (1) for $y < 0$.

Equation (1) is a canonical form of 4th order hyperbolic equations with respect to higher derivatives according to the classification of work [1].

The unique solvability of the following conjugation problem for equations (1) is proved in this work.

Problem 1. *Find the function* $u(x, y) \in C(\overline{D}) \cap C^2(D) \cap [C^{(1+3)}(D_1) \cup C^{(2+2)}(D_2) \cup C^{(4+0)}(D_2)]$, *satisfying in the region* $D \setminus (y = 0)$ (1) *boundary conditions*

$$u|_{AC} = \psi_1(x), 0 \le x \le \frac{l}{2}, \quad \frac{\partial u}{\partial n}|_{AC} = \psi_2(x), 0 \le x \le \frac{l}{2}, \quad \frac{\partial u}{\partial n}|_{BC} = \psi_3(x), \frac{l}{2} \le x \le l,$$

$$u|_{BB_0} = \varphi(y), 0 \le y \le h, u|_{A_0B_0} = \varphi_4(x), 0 \le x \le h$$

and conjugations and conditions

$$u(x, +0) = \alpha(x)u(x, -0), u_y(x, +0) = \beta(x)u_y(x, -0), u_{yy}(x, +0) = \gamma(x)u_{yy}(x, +0), 0 \le x \le l$$

where $\psi_i(x)(i = \overline{1, 4}), \varphi(y), \alpha(x), \beta(x), \gamma(x)$ *are given functions,* n- *internal normal.*

Conjugation problem for equation partial derivatives of the fourth order with lower terms was studied in [2].

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ONE PROBLEM FOR INTEGRO-DIFFERENTIAL EQUATION

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In the last years, the theory of fractional powers of operators on Banach spaces aroused great interest due to the increasing need for modeling in Peridynamics,Biology, Finance, etc .(See [1], [2], [3] and the references therein.) The fractional Laplacian refers to anomalous diffusion that uses fractional derivatives, to peridynamical equation [4], [5] and a fractional heat equation [6]. We consider equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} + (-\Delta)^s u = f(x,t), \quad x \in \Omega, \ t > 0 \tag{1}$$

with initial data

$$u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x), \quad x \in \Omega, \tag{2}$$

where $s \in (0, 1/2)$, $\Omega \subset \mathbb{R}^n$ $-$ domain with piecewise smooth boundary and $n \geq 3$.

We suppose, that $u : \Omega \times [0,T] \to \mathbb{R}$ is unknown function, kernel $K : \Omega \times \Omega \to \mathbb{R}$ and function $f : \Omega \times [0,T] \to \mathbb{R}$ are scalar functions.

According to definitions from [1], equation (1) can be rewrite as

$$\frac{\partial^2 u(x,t)}{\partial t^2} + \int\limits_{\Omega} \frac{[u(x,t) - u(y,t)]}{|x-y|^{n+2s}} \, dy = f(x,t), \quad x \in \Omega, \ t > 0, \tag{3}$$

Now we formulate the main result of the paper on the solvability of problem (1)-(2).

Theorem 1. Let $0 < \beta < 2s/n$. Then for any $T > 0$ and $\varphi \in W_2^{2s}(\Omega)$, $\psi \in W_2^{2s}(\Omega)$ and $f \in C\big\{[0,T] \to W_2^{2s}(\Omega)\big\}$ problem (1)-(2) has a unique solution from $C^2\big\{[0,T] \to H^{\beta}(\Omega)\big\}$.

Keywords: peridynamics, integro-differential equation, initial problem, Sobolev space .

AMS Subject Classification: MSC2010 45K05, 47G20

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STUDY OF SOLUTIONS OF ONE CLASS OF VOLTERRA INTEGRAL EQUATIONS WITH A SINGULAR POINT

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The analytical structure of vanishing solutions of a class of Volterra integral equations with a regular singular point is studied. Consider the Volterra integral equations with a singular point

$$tu = \int_0^t \left[\lambda u(s) + \nu u^{\alpha}(s)\right] ds + \int_0^t K\left(t, s, u(s), u^{\alpha}(s)\right) ds, \tag{1}$$

where $K(t, s, u, w)$ is a holomorphic function in a neighborhood $t = s = u = w = 0$, and

$$K(t, s, 0, 0) \equiv 0, \quad \frac{\partial K(0,0,0,0)}{\partial u} = \frac{\partial K(0,0,0,0)}{\partial w} = 0,$$

λ, ν – complex parameters, $0 < \alpha < 1$.

We will be interested in vanishing solutions of equation (1) [1-2].

Sufficient conditions are found under which equation (1) has solutions in the form, in particular

$$u = P_{000} t^{\frac{1}{1-\alpha}} \left(1 + \sum_{i+j+k \geq 1} P_{ijk} t^{i + \frac{j}{1-\alpha} + \frac{k\alpha}{1-\alpha}}\right).$$

The convergence of formal solutions is proved by the implicit function theorem and the majorant method [3].

Keywords: Volterra integral equations, singular point, vanishing solutions, formal solutions, majorant method, implicit function theorem..

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NUMERICAL SOLUTION OF A SINGULARLY PERTURBED HYPERBOLIC SYSTEM

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The numerical solution of the Cauchy problem for a singularly perturbed system of hyperbolic equations is constructed:

$$\varepsilon\left(\partial_t u + A(x,t)\partial_x u\right) + B(x,t)u = f(x,t), \forall (x,t) \in \Omega, u(x,t,\varepsilon)|_{t=0} = u^0(x). \tag{1}$$

where $\Omega = [0,1] \times (0,T], \varepsilon > 0$ is a small parameter, $A(x,t), B(x,t), f(x,t), u^0(x)$ matrices-functions have the required number of derivatives, the $n \times n$ matrix $B(x,t)$ has eigenvalues with a positive real part. Following the Lomov method [1], problem (1) is reduced to a regular problem with respect to a small parameter, then the obtained regular problem is solved by the finite difference method. Since the initial condition is set only at $t = 0$, only one boundary condition arises along this line. By virtue of the above, we introduce only one regularizing variable $\xi = \frac{t}{\varepsilon}$. For an extended function $\tilde{u}(x,t,\xi,\varepsilon)|_{\xi=\frac{t}{\varepsilon}} \equiv u(x,t,\varepsilon)$ we pose the extended problem:

$$\partial_\xi \tilde{u} + B(x,t)\tilde{u} + \varepsilon(\partial_t \tilde{u} + A(x,t)\partial_x \tilde{u}) = f(x,t), \forall (x,t,\xi) \in \Omega, \tilde{u}(x,t,\xi,\varepsilon)|_{t=\xi=0} = u^0(x). \tag{2}$$

This problem is regular in ε as $\varepsilon \to 0$, so we can use any numerical method to solve this problem numerically. Differential problems are solved by the finite difference method on a uniform grid, i.e. derivatives are replaced by finite differences. The accuracy of the constructed solution is estimated as a value of the order of $O(\varepsilon(\tau + h))$.

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ASYMPTOTICS OF THE SOLUTION OF A MIXED PROBLEM FOR A HYPERBOLIC EQUATION WITH A POWER-LAW BOUNDARY LAYER

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We consider the problem

$$(\varepsilon+t)\partial_t u+\varepsilon A(x)\partial_x u+B(x,t)u=f(x,t),\ (x,t)\in\Omega$$
$$u(x,t,\varepsilon)|_{t=0}=u^0(x),\ u(x,t,\varepsilon)|_{x=0}=u^1(t), \tag{1}$$

here $\varepsilon>0$ is a small parameter, $A(x)\in C^\infty[0,1],\ B(x,t),f(x,t)\in C^\infty(\overline{\Omega}),\ \Omega=\{0<x<1,\ 0<t\leq T\}$.

The problem is studied for $B(x,0)>0,\forall t\in[0,T]$. The degenerate ($\varepsilon=0$) equation has a singularity at $t=0$, this leads to the appearance of a power boundary layer. The power boundary layer [1] is described by the function

$$\Pi(t,\varepsilon)=(\frac{\varepsilon}{t+\varepsilon})^\lambda,\ \lambda>0-const,$$

in addition, problem (1) has a discontinuity along the characteristic.

We have constructed a continuous asymptotic solution that contains regular, power, and angle boundary layer functions. Earlier, the problem of solving which contains the power-law boundary layer functions studied in [1] - [5].

Let us perform regularization [1] of problem (1), for which we introduce the functions

$$\xi_i=\varphi_i(x,t,\varepsilon),\nu_i=\phi_i(x,t,\varepsilon),\ \varphi_i(x,0,\varepsilon)=0,\phi_i(0,t,\varepsilon)=0 \tag{2}$$

and extended function

$$\widetilde{u}(x,t,\xi,\nu,\varepsilon)|_{\xi=\varphi(x,t,\varepsilon)}\equiv u(x,t,\varepsilon),\xi=(\xi_1,\xi_2,...,\xi_n),\varphi=(\varphi_1,\varphi_2,...,\varphi_n), \tag{3}$$
$$\nu=(\nu_1,\nu_2,...,\nu_n),\phi=(\phi_1,\phi_2,...,\phi_n).$$

Keywords: Asymptotics of the solution, mixed problem, hyperbolic equation, power-law boundary layer.

AMS Subject Classification: 35B40, 35F45.

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AN INVERSE PROBLEM FOR A PERTURBED FOURTH-ORDER EQUATION WITH INVOLUTION

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For a one-dimensional perturbed fourth-order equation with involution

$$u_{tt}\left(x,t\right)+u_{xxxx}\left(x,t\right)+\alpha\cdot u_{xxxx}\left(-x,t\right)=f\left(x\right)\ \ (1)$$

in a rectangular region $(x,t)\in\Omega=\{-1<x<1,\ 0<t<T\}$, consider the problem of finding a pair of functions $u(x,t), f(x)$ satisfying equation (1). In this case, the function $u(x,t)$ must satisfy the initial conditions

$$u\left(x,0\right)=\varphi\left(x\right),u\left(x,T\right)=\psi\left(x\right),u_{t}\left(x,0\right)=0,\in\left[-1,1\right],$$

and boundary conditions of the Dirichlet type

$$u\left(-1,t\right)=0,(1,t)=0,u_{xx}\left(-1,t\right)=0,u_{xx}\left(1,t\right)=0,t\in\left[0,T\right],$$

where $-1<\alpha<1$, $\varphi(x)$ and $\psi(x)$ are given sufficiently smooth functions.
With the help of the Fourier method, the existence and uniqueness of the solution of the inverse problem is proved.

Theorem. Let $\varphi\left(x\right),\ \psi\left(x\right)\in C^{6}\left[-1,1\right]$, $\varphi^{VI}\left(x\right),\ \psi^{VI}\left(x\right)\in L_{2}\left(-1,1\right)$, $\cos\sqrt{1+\alpha}\left(\pi k-\frac{\pi}{2}\right)^{2}T\leq\delta_{0}<1$, $\cos\sqrt{1-\alpha}(\pi k)^{2}T\leq\delta_{1}<1$, $k\in N$ and $\frac{d^{j}\varphi(\mp1)}{dx^{j}}=0$, $\frac{d^{j}\psi(\mp1)}{dx^{j}}=0, j=0,2,4$. Then there is a unique solution of the inverse problem in the form of a Fourier series.

Direct problems for a perturbed fourth-order equation with involution were considered in [1].

Keywords: An inverse problem, a perturbed equation, equation with involution, partial differential equations.

AMS Subject Classification: 35-11 Research data for problems pertaining to partial differential equations

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DECAY ESTIMATES FOR THE DEGENERATE TIME-FRACTIONAL EVOLUTION EQUATIONS

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The main goal is to study the next time-fractional degenerate evolution equation

$$\begin{cases} \partial^{\alpha}_{0+,t}u(t,x) - a(t)\mathcal{A}(u(t,x)) = 0, \ (t,x) \in \mathbb{R}_+ \times \Omega := \Omega_+, \\ u(0,x) = u_0(x), \ x \in \Omega, \\ u(t,x) = 0, \ t > 0, \ x \in \partial\Omega, \end{cases}$$

where $0 < \alpha \leq 1$, $a(t) \in L^1(\mathbb{R}_+)$, $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$, and $\partial^{\alpha}_{0+,t}$ is the Caputo fractional derivative [1, P. 97]

$$\partial^{\alpha}_{0+,t}u(t,x) = \begin{cases} \frac{1}{\Gamma(1-\alpha)} \int\limits_0^t (t-s)^{-\alpha} \partial_s u(s,x) ds, & \text{if } \alpha \in (0,1), \\ \partial_t u(x,t) = \frac{\partial u}{\partial t}(t,x), & \text{if } \alpha = 1 \end{cases}$$

and $\mathcal{A}(u)$ is one of the following linear and nonlinear operators: p-Laplacian, the porous medium operator, degenerate operator, mean curvature operator, and Kirchhoff operator.

Keywords: Caputo derivative, sub-diffusion equation, Kilbas-Saigo function, decay estimate.

AMS Subject Classification: 35R11, 35C10.

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A NEW FUNCTIONAL RELATION FOR THE RIEMANN ZETA FUNCTION

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In the paper the Riemann's functional equation reduced to a Riemann–Hilbert boundary value problem, and the integral Hilbert transforms arising in its solution allow the calculation of an exact lower bounds for the zeta function. This study is concerned with the properties of modified zeta functions. Riemann's zeta function is defined by the Dirichlet series

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s},\quad s=\sigma+it, \tag{1}$$

which is absolutely and uniformly convergent in any finite region of the complex s-plane for which $\sigma \geq 1+\epsilon, \epsilon>0$. If $\sigma>1$, then ζ is represented by the following Euler product formula

$$\zeta(s)=\prod_{j\in N}\left[1-\frac{1}{p_j^s}\right]^{-1}, \tag{2}$$

where p_j runs over all prime numbers. $\zeta(s)$ was first introduced by Euler in 1737 [1], who also obtained formula (2). Dirichlet and Chebyshev considered this function in their study on the distribution of prime numbers [2]. However, the most profound properties of $\zeta(z)$ were only discovered later, when it was extended to the complex plane. $\zeta(s)$ is a regular function for all values of s, except $s=1$, where it has a simple pole with residue 1; it satisfies the following functional equation:

$$\pi^{-s/2}\Gamma(s/2)\zeta(s)=\pi^{-(1-s)/2}\Gamma((1-s)/2)\zeta(1-s) \tag{3}$$

it equation is called Riemann's functional equation.

Keywords: Euler product, Dirichlet, Riemann, Hilbert, Poincaré, Riemann hypothesis, zeta function.

AMS Subject Classification: MSC[2010] 11M26

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ABOUT SYSTEM OF MOMENT EQUATIONS AND MOMENT BOUNDARY CONDITIONS DEPENDING ON THE SPEED AND SURFACE TEMPERATURE OF THE AIRCRAFT

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The prediction of aerodynamic characteristics of aircraft at very high speeds and high altitudes is an important problem in aerospace engineering. The aerodynamic characteristics of aircraft at such conditions can be determined using the methods of rarefied gas theory [1]. Describing a rarefied gas using the particle distribution function refers to the transition region. A correct description of the gas flow near the surface should be based on solving the kinetic Boltzmann equation. When calculating the aerodynamic characteristics of an aircraft in the high-velocity flow of rarefied gas, it is necessary to supplement the Boltzmann equation with a term that depends on the speed of the aircraft. Furthermore, the condition on the moving boundary has to contain a parameter that depends on the surface temperature of the aircraft.

A one-dimensional non-stationary nonlinear moment system of equations and an approximation of the microscopic Maxwell's boundary condition are presented. The initial and boundary value problem for the one-dimensional non-stationary nonlinear moment system of equations in the third approximation under the macroscopic boundary conditions is formulated. The existence and uniqueness of the solution to the aforementioned problem in the space of functions that are continuous on time and square summable by spatial variable are proven [2].

An iterative numerical method is designed for solving direct and inverse problems for a non-stationary nonlinear system of moment equations in the third approximation, reduced to canonical form. The algorithms are implemented in software, and their application for solving inverse problems of determining the speed and surface temperature of an aircraft is demonstrated. The results of numerical experiments are presented.

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APPROXIMATE SOLUTION OF NONLINEAR VOLTERRA INTEGRAL EQUATIONS OF THE SECOND KIND

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Various numerical methods for solving integral equations are studied and developed by many authors, for example [1],[2]. Here, we consider the nonlinear integral equation of the second kind

$$u(x) = \int_a^x K(x,t,u(t))dt + f(x),\ x \in [a,b], \tag{1}$$

where $K(x,t,u)$ is a given continuous function on $G \times R$, $G = (x,t) : a \le t \le x \le b$, $f(x)$ is a given continuous function on $[a,b]$ and $u(x)$ is the sought function on $[a,b]$. We shall assume that

$$K(x,t,u) = P(x,t)u + Q(x,t,u),\ (x,t,u) \in G \times R. \tag{2}$$

Theorem. Let $P(x,t) \in C(G)$, $Q(x,t,u) \in C(G \times R)$, $Q(x,x,u) = 0$ for all $(x,u) \in [a,b] \times R$, $Q(x,t,0) = 0$ for all $(x,t) \in G$ and for each (x,t,u_1), $(x,t,u_2) \in G \times R$ the following estimate holds

$$|Q(x,t,u_1) - Q(x,t,u_2)| \le L\,|u_1 - u_2|\,,$$

where L is a positive constant. If $f(x) \in C[a,b]$, then the integral equation (1) has a unique solution $u(x)$ in the space $C[a,b]$ and the estimate holds

$$\|u(x)\|_C = \sup_{x\in[a,b]} |u(x)| \le l_1, \tag{3}$$

where l_1 is some constant.

Further, using the trapezoid rule, the numerical solution of equation (1) is obtained. Also, an error estimate between the exact solution and the approximate solution for the given integral equation is investigated.

Keywords: Volterra nonlinear integral equation, unique solution, numerical solution, estimation error.

AMS Subject Classification: 45G10, 65R20

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PROPORTIONAL LEARNING AND FORGETTING MODEL ON TIME SCALES

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Fractional calculus, a non-integer generalization of classical calculus, is used in many theories, including time scales ([1]-[3]). Proportional derivative is a concept that emerged later. It has a different unit operator than the usual conformable fractional derivative. Fractional dynamical equations have been studied in many studies. In this study, we'll consider learning and forgetting model. The act of forgetting is the loss of knowledge that we have learned before. In classical case, there are some models containing many differential equations on this subject. One of these models is

$$\frac{dy}{dt} = S - fy, \tag{1}$$

where S is learning rate, f is forgetting rate and $y(t)$ is the amount of knowledge a person has at t time [4]. Equation (1) is considered as

$$D^{\alpha}y(t) = S - fy, \tag{2}$$

with the help of proportional derivative on time scales. In this study, equation (2) is handled with the established initial conditions and solved with the related methods of the proportional derivative on time scales. The solutions were handled in various time scales and their meanings were examined.

Keywords: Time scale, Fractional analysis.

AMS Subject Classification: 34N05, 26A33

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BOUNDARY VALUE PROBLEM FOR AN EQUATION OF HIGH EVEN ORDER INVOLVING A FRACTIONAL DERIVATIVE

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In the region $\Omega = \Omega_x \times \Omega_y$, $\Omega_x = \{x : 0 < x < 1\}$, $\Omega_y = \{y : 0 < y < 1\}$, consider the equation

$$_C D_{0x}^{\alpha} u(x,y) + x^{\beta} K(y)\, l(u(x,y)) = 0, \tag{1}$$

where

$$l(u(x,y)) = (-1)^s \frac{\partial^{2s} u(x,y)}{\partial y^{2s}} + \frac{\partial^{s-1}}{\partial y^{s-1}} \left((-1)^{s-1} p_{s-1}(y) \frac{\partial^{s-1} u(x,y)}{\partial y^{s-1}} \right) + \ldots$$

$$+ \frac{\partial}{\partial y} \left(-p_1(y) \frac{\partial u(x,y)}{\partial y} \right) + p_0(y)\, u(x,y),$$

$$p_j(y) \in C^j\left(\overline{\Omega}_y\right), \; j = 0, 1, ..., s-1, \; s \in N,$$

$$K(y) > 0, y \in (0,1]; K(0) = 0; K^{(i)}(y) = O\left(y^{m-i}\right), y \to +0; 0 \le m < s, i = 0, 1, ...,$$

$0 < \alpha < 1; -\alpha < \beta \in R;$

$_C D_{0x}^{\alpha}-$ fractional Caputo differentiation operator of order α

$$_C D_{0x}^{\alpha} u(x,y) = \frac{1}{\Gamma(1-\alpha)} \int\limits_0^x \frac{\frac{\partial}{\partial \tau} u(\tau,y)\, d\tau}{(x-\tau)^{\alpha}}.$$

Problem A. Find a solution to (1) from the class

$$_C D_{0x}^{\alpha} u(x,y) \in C(\Omega),$$

$$\frac{\partial^{2s-1} u(x,y)}{\partial y^{2s-1}} \in C\left(\overline{\Omega}\right), \; \frac{\partial^{2s} u(x,y)}{\partial y^{2s}} \in C(\Omega),$$

satisfying the conditions

$$\frac{\partial^j u(x,0)}{\partial y^j} = \frac{\partial^j u(x,1)}{\partial y^j} = 0, \; 0 \le x \le 1, \; j = 0, 1, ..., s-1, u(0,y) = \varphi(y),$$

here the function $\varphi(y)-$ is sufficiently smooth and satisfies the matching conditions. Sufficient conditions for the unique solvability of the formulated problem are obtained.

Keywords: fractional derivative, eigenvalue, eigenfunction, Kilbas-Saigo function.
AMS Subject Classification: 35R11

ON FREDHOLM SOLVABILITY AND ON THE INDEX OF THE GENERALIZED NEUMANN PROBLEM FOR AN ELLIPTIC EQUATION OF THE HIGH-ORDER ON A PLANE

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In simply connected region D in the plane bounded by the simple smooth contour Γ, we consider the elliptic equation

$$\sum_{r=0}^{2l} a_r \frac{\partial^{2l} u}{\partial x^{2l-r} \partial y^r} + \sum_{0 \le r \le k \le 2l-1} a_{rk}(x,y) \frac{\partial^k u}{\partial x^{k-r} \partial y^r} = f(x,y), (x,y) \in D \tag{1}$$

with real coefficients $a_r \in \mathbb{R}$ and $a_{rk} \in C^\mu(\overline{D})$, $\Gamma = \partial D \in C^{2l,\mu},\ 0 < \mu < 1$.

Problem S. The generalized Neumann problem consists in finding the solution $u(x,y)$ of equation (1) in the domain D by boundary conditions

$$\left. \frac{\partial^{k_j - 1} u}{\partial n^{k_j - 1}} \right|_\Gamma = g_j,\ j = 1, \ldots, l, \tag{2}$$

where $1 \le k_1 < k_2 < \ldots < k_l \le 2l$ and $n = n_1 + in_2-$ the unit external normal.

For a polyharmonic equation, this problem was studied by A.V. Bitsadze [1]. In [2], problem (1), (2) was investigated for $a_{kr} \ne 0$ and $f \ne 0$ in the space of functions $C_a^{2l-1,\mu}(\overline{D})$.

The report established: a sufficient condition for the Fredholm property of problem (1), (2); equivalence of the Fredholm condition of the problem to the complementarity condition (or Shapiro– Lopatinsky). A formula for the index of the problem $\operatorname{ind} S$ is calculated.

The condition of Fredholm property of various problems for equations of the fourth and sixth orders is established in detail, and formulas for the indices of the corresponding problems are described in explicit form.

Keywords: high order elliptic equations, boundary value problem, normal derivatives, Fredholm solvability of the problem, formula for problem index.
AMS Subject Classification: 35J30, 35J40, 35J37.

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DIRECT AND INVERSE PROBLEMS FOR NONLOCAL ANALOGUES OF EQUATIONS OF PARABOLIC AND HYPERBOLIC TYPES

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Let $x = (x_1, x_2, ..., x_n) \in R^n, n \geq 2$ and $S_1, S_2, ..., S_l, l \geq 1$ be a set of real symmetric commutative matrices $S_iS_j = S_jS_i$ such that $S_iS_j = S_jS_i$. For example, the matrix S_j may be the matrix of the following linear mapping $S_ix = (x_1, ..., x_{i-1}, -x_i, x_{i+1}, ..., x_n)$.

If we introduce the notation of the summation index $i$in the binary number system $i \equiv (i_l, i_{l-1}, ..., i_1)_2$, where $i_k = 0, 1$ for $k = 1, 2, ..., l$, we can consider mappings of the form $S_l^{i_l} \cdot ... \cdot S_1^{i_1}$. Using these mappings, we introduce the following non-local analogue of the Laplace operator:

$$L_l u(x) = \sum_{i=0}^{2^l-1} a_i u\left(S_l^{i_l} \cdot ... \cdot S_1^{i_1} x\right),$$

where $a_0, a_1, ..., a_{2^l-1}$ is a set of real numbers.

Let Ω be a unit sphere in R^n and $Q = \{(t, x) : (0, T) \times \Omega\}$. Consider in the domain Q the following equation

$$D_t^\alpha u(t, x) = L_l u(t, x) + F(t, x), (t, x) \in Q, \tag{1}$$

where $0 < \alpha \leq 2, D_t^\alpha$ - is a derivative of fractional order in the sense of Caputo [1].

In this paper, for equation (1), we study the solvability of direct and inverse problems. Theorems on the existence and uniqueness of solutions to the problems under consideration are proved. Note that the main boundary value problems for the nonlocal Poisson equation $L_l u(x) = f(x), x \in \Omega$ and the corresponding spectral problems were studied in [2,3].

Keywords: Nonlocal equation, parabolic equation, hyperbolic equation, fractional derivative, initial boundary value problem, inverse problem.

AMS Subject Classification: 35K15, 35L15, 35R30.

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TRAVELING WAVE SOLUTIONS OF THE SPACE-TIME FRACTIONAL BURGERS AND MODIFIED BURGERS EQUATION WITH VARIABLE COEFFICIENTS

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In this paper, we use the functional variable method to solve the space-time fractional Burgers and modified Burgers equation with variable coefficients. The obtained solutions show the efficiency of the considered method. It is worth to mention that among the obtained solutions there exist new soliton solutions which have its great importance to reveal the internal mechanism of the physical phenomena. Three-dimensional graphs of solutions are drawn via the mathematical software Matlab. The graphical representations of some obtained solutions are demonstrated to better understand their physical features. The obtained solutions have several applications in a recent area of research in mathematical physics. This method is effective to find exact solutions of many other similar fractional wave equations.

Keywords: the space-time fractional Burgers equation, the functional variable method, the space-time fractional modified Burgers equation, periodic wave solutions, variable coefficients, soliton wave solutions.

AMS Subject Classification: 34A34, 34B15, 35Q51, 35J60, 35J66.

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INTEGRATION OF A FINITE COMPLEX TODA CHAIN WITH A SELF-CONSISTENT SOURCE

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The finite complex Toda chain represents a discrete lattice of interacting particles with complex-valued coordinates and momenta [1]. These chains exhibit remarkable integrability properties, allowing for the explicit construction of conserved quantities and exact solutions [2], [3]. By introducing a self-consistent source into the Toda chain, we investigate the interplay between the chain's dynamics and the influence of the source term, leading to a deeper understanding of the system's behavior [4], [5].

We consider the following system of equations

$$\begin{cases} \dot{a}_n = a_n(b_n - b_{n+1}) + a_n \sum\limits_{i=1}^{N} \Big((g_n^i)^2 - (g_{n+1}^i)^2 \Big), \\ \dot{b}_n = 2(a_{n-1}^2 - a_n^2) - 2\sum\limits_{i=1}^{N} g_n^i (a_n g_{n+1}^i - a_{n-1} g_{n-1}^i), \\ J g_n^k = \lambda_k g_n^k, \;\; k = 1, ..., p, \;\; n = 0, 1, ..., N-1 \end{cases} \tag{1}$$

with the boundary conditions

$$a_{-1} = a_{N-1} = 0 \; . \tag{2}$$

The system (1), (2) is considered subject to the initial conditions

$$a_n(0) = a_n^0, \quad b_n(0) = b_n^0, \; n = 0, 1, ..., N-1 \; , \tag{3}$$

where a_n^0, b_n^0 are given complex numbers such that $a_n^0 \neq 0$ $(n = 0, 1, ..., N-2)$, $a_{N-1}^0 = 0$, $\lambda_1, \; \lambda_2, ..., \lambda_k$ all the distinct eigenvalues of the matrix J and by $m_1, ...m_p$ their multiplicities, respectively, so that $1 \le p \le N$ and $m_1 + m_2 + ... + m_p = N$.

The main aim of this work is to derive representations for the solutions of the finite complex Toda chain with a self-consistent source by means of the inverse spectral problem for the complex Jacobi matrices.

Keywords: Jacobi matrix, integrable systems, Toda chain, self-consistent source, soliton dynamics.
AMS Subject Classification: 35C08, 35G31, 37K60, 39A36.

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BISINGULARLY BOUNDARY VALUE PROBLEMS FOR THE ODE SECOND ORDER

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Let us investigate the boundary value problems generated by the equation

$$\varepsilon\, y''_\varepsilon(x) + xp(x)y'_\varepsilon(x) - q(x)y_\varepsilon(x) = f(x), \qquad 0 < x < 1, \tag{1}$$

and one of the boundary conditions of the form

$$y_\varepsilon(0) = a, \;\; y_\varepsilon(1) = b, \tag{2}$$

$$y'_\varepsilon(0) = a, \;\; y'_\varepsilon(1) = b, \tag{3}$$

$$y_\varepsilon(0) - h_1 y'_\varepsilon(0) = a, \;\;\; y_\varepsilon(1) + h_2 y'_\varepsilon(1) = b, \tag{4}$$

here $0 < \varepsilon << 1$, a, b, $0 < h_1$, $0 < h_2$ are given constants, $p(0) = 1, q(0) = 2$, $0 < p$, $0 < q$, $f \in C^\infty[0,1]$, $f''(0) \neq 0$, $y_\varepsilon(x)$ is the required function depending on the small parameter ε and on the independent variable x.

Uniform asymptotic expansions of the solution of two-point boundary value problems of Dirichlet, Neumann and Robin for a linear inhomogeneous ordinary differential equation of the second order with a small parameter at the highest derivative are constructed. A feature of the considered two-point boundary value problems is that the corresponding unperturbed boundary value problems for an ordinary differential equation of the first order has a regularly singular point at the left end of the segment. Asymptotic solutions of boundary value problems are constructed by the modified method of boundary functions [1]. Asymptotic expansions of solutions of two-point boundary value problems are substantiated. We propose a simpler algorithm for constructing an asymptotic solution of bisingular boundary value problems with regular singularpoints, and our boundary functions constructed in a neighborhood of a regular singular point have the property of "boundary layer that is, they disappear outside the boundary layer [2]-[3].

AMS Subject Classification: 34B05, 34D05, 34D15, 34E05, 34E10, 34E20.

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CRITICAL EXPONENTS FOR THE QUASILINEAR PARABOLIC PROBLEMS

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This paper studies the large-time behavior of solutions to the quasilinear inhomogeneous parabolic equation with combined nonlinearities. This equation is a natural extension of the heat equations with combined nonlinearities considered by Jleli-Samet-Souplet [1] (Proc AMS, 2020).

Firstly, we focus on an interesting phenomenon of discontinuity of the critical exponents. In particular, we will fill the gap in the results of [1] for the critical case. We are also interested in the influence of the forcing term on the critical behavior of the considered problem, so we will define another critical exponent depending on the forcing term.

The main results of this abstract were recently accepted for publication in [2].

Keywords: p-Laplace heat equation, combined nonlinearities, critical exponents, global solutions.

AMS Subject Classification: Primary 35K92; Secondary 35B33, 35B44

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INVERSE PROBLEMS OF A FOURTH-ORDER EQUATION WITH AN INVOLUTION WITH A FRACTIONAL DERIVATIVE

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The work is devoted to the study of inverse problems for a differential equation of the form

$$ {}_CD_{0t}^{\beta}u + u_{xxxx}(x,t) + \alpha \cdot u_{xxxx}(-x,t) = f(x), \quad (x,t) \in E, \tag{1}$$

where the real number is $-1 < \alpha < 1$, in a rectangular area $E = \{-1 < x < 1, \ 0 < t < T\}$. Here ${}_CD_{0t}^{\beta} = I_{0t}^{1-\beta}$, $\beta \in (0,1]$ – is the Caputo derivative of order β [1].

Similar problems are studied for fourth-order partial differential equations without involution in [2].

IPP: Find a pair of functions $u(x,t)$ and $f(x)$ in the area E that satisfy equation (1), initial conditions

$$u(x,0) = \varphi(x), \ \ u(x,T) = \psi(x), \ \ x \in [-1,1], \tag{2}$$

and boundary conditions of the period

$$\begin{gathered} u(-1,t) = u(1,t), \ u_x(-1,t) = u_x(1,t), \ u_{xx}(-1,t) = u_{xx}(1,t), \\ u_{xxx}(-1,t) = u_{xxx}(1,t), \ t \in [0,T], \end{gathered} \tag{3}$$

where $\varphi(x)$ and $\psi(x)$ are given sufficiently smooth functions.

A pair of functions $u(x,t)$ and $f(x)$, such that $u(x,t) \in C_x^3\left(\overline{E}\right) \cap_x^4 (E)$, $f(x) \in C[-1,1]$, satisfying equation (1), conditions (2) and (3) we call the solution of problem (1)–(3).

Theorem 1. Let $|\alpha| < 1$ and the following conditions are satisfied:

1) $\varphi(x), \ \psi(x) \in C^4\left(\bar{E}\right) \cap C^5(E)$;

2) derivatives of functions $\varphi(x)$ and $\psi(x)$ have properties $\frac{d^j\varphi(\mp 1)}{dx^j} = \frac{d^j\psi(\mp 1)}{dx^j}$, $j = 0,1,2,3,4$.

Then there is a unique solution to the inverse problem (1)–(3).

Keywords: fractional differential equations with involution, inverse problem, eigenvalue, eigenfunction.

AMS Subject Classification: 34, 35.

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SYMMETRY EQUIVALENCES OF BOUNDARY VALUE PROBLEMS FOR THE NON-UNIFORM BEAMS

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Consider a spectral problem with boundary conditions on transverse vibrations of a non-uniform beam:

$$\frac{d^2}{dx^2}\left(E\,I(x)\frac{d^2Y(x)}{dx^2}\right)-T\frac{d^2Y(x)}{dx^2}+k(x)Y(x)=\lambda\,A(x)Y(x) \tag{1}$$

where $Y(x)$ are eigenfunctions of transverse static beam deflection; $EI(x)$ is the flexural rigity; $\rho\,A(x)$ is mass of the beam per unit length; T is axial tension; $\lambda=\rho\,\omega^2$ are eigenvalues; ω is circular frequency; ρ is material density; $k(x)$ is variable base factor.

Spectral problems with different boundary conditions for equation (1) were studied in connection with practical applications in [1], [2], [4]. In [4], a closed form of the eigenfrequencies of various boundary value problems for equation (1) was obtained for $k(x)=0$, $E\,I(x), A(x)$ and the well-known results from [1] were modified. Symmetry equivalence of boundary value problems was also investigated in [4]. Symmetry equivalence of boundary value problems for the uniform beams with a variable coefficient of foundation without axial load was studied in [3].

In this talk, we study the symmetry equivalence of boundary value problems for equation (1). This method allows one to verify the models given on the entire interval using known models on half the interval.

Keywords: non-uniform beam, symmetry equivalence, natural frequency, Euler-Bernoulli equation.

AMS Subject Classification: 34B09, 35P15, 47A75, 93B60.

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CONFLUENT TRIPLE HYPERGEOMETRIC SERIES ARISING IN THE SOLUTIONS OF SOME BOUNDARY VALUE PROBLEMS FOR THE DEGENERATE PDE

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Consider the degenerate hyperbolic equation of second kind with the spectral parameter

$$L(U) \equiv x^n U_{xx} - y^m U_{yy} + \mu U = 0 \ \ (0 \le n < m < 1)$$

in a finite simply connected domain D, bounded by characteristics

$$\frac{2}{2-n} x^{(2-n)/2} - \frac{2}{2-m} y^{(2-m)/2} = 0 \ \text{ and } \ \frac{2}{2-n} x^{(2-n)/2} + \frac{2}{2-m} y^{(2-m)/2} = 1$$

of equation $L(U) = 0$ for $y \ge 0$. Here $\mu = \lambda$ or $\mu = \lambda i$, where λ is a real number.

Cauchy problem. Find a function $U(x, y) \in C^2(D) \cap C(\bar{D})$ satisfying equation $L(U) = 0$ and the following initial-value conditions

$$U(x,0) = \tau(x), \ 0 \le x \le 1; \quad U_y(x,0) = \nu(x), \ 0 < x < 1,$$

where $\tau(x)$ and $\nu(x)$ are sufficiently smooth given functions and $\tau(x) \in C^3[0,1]$, $\nu(x) \in C^2[0,1]$.

In solving the Cauchy problem by Riemann method the following confluent triple hypergeometric series

$$F^{(3)} \left[\begin{matrix} a :: -; -; (b'') : (c) \, ; c' ; (c''); \\ e :: -; -; a, (g'') : (h) \, ; -; (h''); \end{matrix} \ x, y, z \right]$$

$$= \sum_{m,n,k=0}^{\infty} \frac{(a)_{m+n+k} \prod\limits_{j=1}^{B''} \left(b_j''\right)_{k+m} \prod\limits_{j=1}^{C} (c_j)_m \cdot (c')_n \cdot \prod\limits_{j=1}^{C''} \left(c_j''\right)_k}{(e)_{m+n+k} (a)_{m+k} \prod\limits_{j=1}^{G''} \left(g_j''\right)_{k+m} \prod\limits_{j=1}^{H} (h_j)_m \prod\limits_{j=1}^{H''} \left(h_j''\right)_k} \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^k}{k!}. \tag{1}$$

is arising (for the general definition of such series, see [1], p. 44).

Proving new properties of the above introduced triple hypergeometric series, we obtain the classic solution of the Cauchy problem for equation $L(U) = 0$ in an explicit form.

Keywords: Cauchy problem; Riemann function; classic solutions; degenerate hyperbolic equation of second kind with the spectral parameter, confluent multiple hypergeometric series.

AMS Subject Classification: Primary 33C20, 33C65; Secondary 44A45.

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CONTROLLED EQUATIONS TO PRESENT TRANSFORMING VERBS WITH FUNCTIONAL RELATIONS

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In 1878 M.Berlitz proposed "immersive teaching method" to teach languages without other languages as media. Combining this idea with computers' capacities and presenting verbs as actions yielded the following definition ([1] and other publications):

Let any "notion" (word of a language) be given. If an algorithm: performs (generating randomly) sufficiently large amount of situations covering all essential aspects of the "notion" to the user; gives a command involving this "notion" in each situation; perceives the user's actions; detects whether a result fits the command, then such algorithm is said to be a computer interactive presentation of the "notion".

Some verbs *ber*(give), *koy*(put), *al*(take), *türt*(push), were implemented by parallel shifts of solids. To present more complex notions we proposed to use functional relations among points of an object.

Consider the verb *tart*(draw) for an object with two marked points. The user is to draw it by the first point to a spot (in screen). We have the system

$z_1'(t) = w(t), z_2'(t) = f(z_1(t), z_2(t), w(t))$

where $z_k(t) = (x_k(t), y_k(t)), k = 1..2$ are vector-functions of marked points, $w(t) = (u(t), v(t))$ is the user's control, the vector-function $f(z_1, z_2, w)$ is defined from conditions: $|z_1(t) - z_2(t)| = c = const$; motion of object is close to natural one.

Suppose that an infinitesimal shift of $z_2(t)$ is along the vector $z_1(t) - z_2(t)$. Let $p_k, k = 1..2$ be infinitesimals. We obtain the equation

$|(z_1(t) + p_1 w) - (z_2(t) + p_2(z_1(t) - z_2(t))| = |z_1(t) - z_2(t)|$.

Without loss of generality, $z_2(t) = 0$ at this moment.

$((x_1 + p_1 u) - p_2 x_1)^2 + ((y_1 + p_1 v) - p_2 y_1)^2 = x_1^2 + y_1^2$;

$2x_1(p_1 u - p_2 x_1) + 2y_1(p_1 v - p_2 y_1) = 0$; $p_1(ux_1 + vy_1) = p_2 c^2$.

Hence, $f(z_1, z_2, w) = \langle w, z_1 - z_2 \rangle \, c^{-2}(z_1 - z_2)$.

Corresponding systems of equations can be constructed for the verbs *tüzdö*(straighten the chain $z_1 - z_2 - z_3 - ...$), *iiy*(bend a stick $z_1 - z_2 - z_3$) and other transforming verbs. Difference approximation of such systems would be used for computer presentations.

Keywords: controlled equation, mathematical model, language, algorithm, computer model, independent presentation, functional relation

AMS Subject Classification: 34H05

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ON A METHOD FOR SOLVING A LINEAR BOUNDARY VALUE PROBLEM FOR IMPULSIVE DIFFERENTIAL EQUATIONS WITH PARAMETER

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We consider the linear boundary value problem for impulsive differential equations with parameter

$$\frac{dx}{dt} = A(t)x + \int_0^T K(t,s)x(s)ds + B(t)\mu + f(t),\ t \in (0,T)\backslash\{\theta_1, \theta_2, ..., \theta_m\}\ x \in \mathbb{R}^n,\ \mu \in \mathbb{R}^l, \quad (1)$$

$$x(\theta_i) - x(\theta_i - 0) = p_i, \quad p_i \in \mathbb{R}^n, \quad i = \overline{1,m}, \quad (2)$$

$$B_0\mu + Bx(0) + Cx(T) = d, \quad d \in \mathbb{R}^{n+l}. \quad (3)$$

Here the $(n \times n)$ -matrices $A(t)$, $K(t,s)$, $(n \times l)$- matrix $B(t)$ and n- vector-function $f(t)$ are continuous on $[0,T]$ and $[0,T] \times [0,T]$ respectivelly; the $((n+l) \times n)$- matrices B, C, the $((n+l) \times m)$ - matrix B_0 are constants.

In present report the new algorithm of parametrization method [1] for solving the linear boundary value problem for impulsive integro-differential equation with parameter is proposed. This method is based on a partition of the interval and the introduction of auxiliary parameters as the values of the solution at the initial points of subintervals. Sufficient conditions for solvability of the problem are obtained.

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Keywords: boundary value problem, impulsive differential equation with parameter, parametrization method, solvability.

AMS Subject Classification: 35R12, 34B07

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REDUCING OF PARTIAL INTEGRO-DIFFERENTIAL EQUATION WITH MANY VARIABLES TO SOLVING OF INTEGRAL EQUATION

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Applying of the method of additional argument for equation of hyperbolic type were considered in [1-3].

Let $\overline{C}^{(k)}$ be the class of functions being continuous and bounded together with their derivatives up to the k-th order.

Considered the initial value problem

$$\partial^2 u(t,x)/\partial t^2 = a^2(t,x)\Sigma\{\partial^2 u(t,x)/\partial x_i \partial x_j : i,j = 1..n\} + b(t,x)\partial u(t,x)/\partial t+$$
$$+c(t,x)\Sigma\{\partial u(t,x)/\partial x_i : i = 1..n\} + F(t,x;u), \quad (t,x) \in G_{n+1}(T) := [0,T] \times R^n, \tag{1}$$

$$u|_{t=0} = u_0(x),\ \partial u/\partial t|_{t=0} = u_1(x),\ x \in R^n, \tag{2}$$

where $a(t,x), b(t,x), c(t,x) \in \overline{C}^{(2)}(G_{n+1}(T))$, $a(t,x) \neq 0$, $F(t,x;u)$ is an operator containing the unknown function in general and under the sign of integral.

Denote $D_n[\omega] = \partial/\partial t + \omega\Sigma\{\partial/\partial x_i : i = 1..n\}$, $\vartheta(t,x) = D_n[-a(t,x)]u(t,x)$, $f(t,x) =$ $= (c(t,x) - D_n[a(t,x)]a(t,x))/a(t,x)$, $\beta_1(t,x) = \beta(t,x) + f(t,x)\, \beta_2(t,x) = \beta(t,x) - f(t,x)$, $\beta_3(t,x) = D_n[a(t,x)]\beta_1(t,x)$.

Lemma. The IVP (1)-(2) is equivalent to the integral equation
$\vartheta(t,x) = \varphi_1(q(0,t,x))/2 + \beta_1(t,x)(u_0(p(0,t,x)) + \int_0^t \vartheta(t,p(s,t,x))ds)/2+$
$+\int_0^t \beta_2(s,q)\vartheta(s,q)ds)/2 - \int_0^t \beta_3(s,q)(u_0(p(0,s,q(s,t,x))) + \int_0^s \vartheta(v,p(v,s,q))dv)ds/2+$
$+\int_0^t F(s,q,(u_0(p(0,s,q)) + \int_0^s \vartheta(v,p(v,s,q))dv)ds$
where $\varphi_1(x) = 2\vartheta(0,x) - \beta_1(0,x)u_0(x)$,
$p(s,t,x) = \{p_i(s,t,x) : i = 1..n\}$, $q(s,t,x) = \{q_i(s,t,x) : i = 1..n\}$ are solutions of corresponding systems of integral equations
$p_i(s,t,x) = x_i + \int_s^t a(v,p(v,t,x))dv, i = 1..n,$
$q_i(s,t,x) = x_i - \int_s^t a(v,q(v,t,x))dv, i = 1..n,\ 0 \le s \le t \le T, x \in R^n.$

Keywords: method of additional argument, second order, partial derivatives, integro-differential equation, integral equation.

AMS Subject Classification: 35G55

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ABOUT BOUNDARY CONDITIONS CONTAINING DERIVATIVES OF ORDER GREATER THAN THE ORDER OF THE EQUATION

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In [1], a boundary value problem for the heat equation was studied, when the boundary condition contained high-order normal derivatives.

In this paper, we consider the boundary value problem for the parabolic equation:

$$L(D_x, D_t)U(x,t) = 0 \tag{1}$$

in $Q_T \equiv \left\{(x', x_n, t) : x' \in R^{n-1},\ x_n \in R_+,\ t \in]0, T[\right\}$, that satisfies the initial condition:

$$U(x,t)|_{t=0} = 0 \tag{2}$$

and the boundary condition:

$$L_m(D_x)U(x,t)|_{x_n=0} = \varphi(x', t), \quad (x', t) \in Q_T^{(1)} \equiv Q_T \setminus x_n, \quad m \in N, \tag{3}$$

where $L(D_x, D_t) = \frac{\partial}{\partial t} - B(D_x)$ and $B(D_x)$ are second-order parabolic and elliptic operators with constant coefficients, respectively; $L_m(D_x) = \sum\limits_{|k| \le m} a_k D_x^k$, $k = (k_1, k_2, \ldots, k_n)$, $|k| = k_1 + k_2 + \cdots + k_n$; $x = (x_1, x_2, \ldots, x_n)$, $D_x^k = D_{x_1}^{k_1} D_{x_2}^{k_2} \ldots D_{x_n}^{k_n}$, $D_{x_i}^{k_i} = \frac{\partial^{k_i}}{\partial x_i^{k_i}}$, $i = \overline{1, n}$; a_k are given constants, $a_{0,0,\ldots,m} \ne 0$; $\varphi(x', t) \in C_{x'}^2\left(Q_T^{(1)}\right)$.

The solution of the boundary value problem (1)-(3) is sought in the form of a thermal potential with an unknown density. The Lemma on finding the limits of the m-th order derivatives of the function $U(x,t)$ in a neighborhood of the hyperplane $x_n = 0$ is presented. Using the Lemma and the boundary condition (3), an integro-differential equation (IDE) in R^{n-1} is obtained, when the order of the derivative under the integral sign is greater than the order of the derivative outside the integral. The IDE is reduced to the Volterra-Fredholm integral equation by the regularization method.

Theorem 1. If $\varphi(x', t) \in C_{x'}^2\left(Q_T^{(1)}\right)$ and the roots of the characteristic equation $\sum\limits_{l=0}^{m} b_l(\sigma')\lambda^{m-l} = 0$, $\sigma' = \frac{s'}{|s'|}$, $s' = (s_1, s_2, \ldots, s_{n-1})$ satisfy the inequality $Re\ q_l^2(\sigma') > -1$, then there is a function $U(x,t) \in C_x^m(Q_T)$, which is the solution of the boundary value problem (1)-(3).

Keywords: Parabolic equation, boundary value problem, integro-differential equation, regularization.

AMS Subject Classification: 35K45, 58J35.

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MIXED PROBLEM FOR A PARABOLIC INTEGRO-DIFFERENTIAL EQUATION WITH INVOLUTION

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We consider an inhomogeneous parabolic type partial integro-differential equation with degenerate kernel and involution. With respect to spatial variable x is used Dirichlet boundary value conditions and spectral problem is obtained. Eigenvalues and eigenfunctions of the spectral problems are found. The Fourier method of separation of variables is applied. The countable system of functional-integral equations is obtained. Theorem on a unique solvability of countable system of integral equations is proved. The method of successive approximations is used in combination with the method of contraction mapping. The unique solution of the mixed problem is obtained in the form of Fourier series. Absolutely and uniformly convergence of Fourier series is proved. Investigations on ordinary and partial integro-differential equations of the Fredholm type are great interest from the point of theoretical research and applications in the different fields of physics, mechanics, engineering and chemistry [1, 2]. Today some new problems are posed for ordinary and partial integro-differential equations, and a large number of papers devoted to study of boundary value and mixed problems for the parabolic integro-differential equations are published [3, 4]. In [5, 6, 7], integro-differential equations with a degenerate kernel were considered.

We consider an inhomogeneous parabolic type partial integro-differential equation with degenerate kernel and involution. With respect to spatial variable x is used Dirichlet boundary value conditions and spectral problem is obtained. Eigenvalues and eigenfunctions of the spectral problems are found. The Fourier method of separation of variables is applied. The countable system of functional-integral equations is obtained. Theorem on a unique solvability of countable system of integral equations is proved. The method of successive approximations is used in combination with the method of contraction mapping. The unique solution of the mixed problem is obtained in the form of Fourier series. Absolutely and uniformly convergence of Fourier series is proved. Investigations on ordinary and partial integro-differential equations of the Fredholm type are great interest from the point of theoretical research and applications in the different fields of physics, mechanics, engineering and chemistry [1-2]. Today some new problems are posed for ordinary and partial integro-differential equations, and a large number of papers devoted to study of boundary value and mixed problems for the parabolic integro-differential equations are published [3-4]. In [5-7], integro-differential equations with a degenerate kernel were considered.

In this paper, we study the solvability of the mixed problem for a parabolic integro-differential equation with a degenerate kernel and involution.

This work was partially supported by the Foundation for Basic Research of the Republic of Uzbekistan (Grant No. FZ–20200929224).

In the rectangular domain, $\Omega = \{0 < t < T,\ \ -1 < x < 1\}$ we consider the following partial integro-differential equation

$$U_t(t,x) - \omega^2\left[U_{xx}(t,x) + \varepsilon\, U_{xx}(t,-x)\right] = \nu \int_0^T K\,(t\,,\,s)\,U\,(s,x)\,d\,s + \alpha(t)\,U(t,x), \qquad (1)$$

where $0 < \alpha(t) \in C[0,T]$, T is given positive number, ε is parameter, $|\,\varepsilon\,| < 1$, ω is positive parameter, ν is nonzero real parameter, $K\,(t\,,\,s) = \sum_{r=1}^{k} a_r\,(t\,)\,b_r\,(s)$, $a_r\,(t\,)\,,\ b_r\,(s) \in C\ [0;\,T]$. It is assumed that the systems of functions $\{a_r\,(t\,)\}_{r=1}^{k}$ and $\{b_r\,(s\,)\}_{r=1}^{k}$ are linear independent.

In solving partial integro-differential equation (1), with respect to spatial variable x we use Dirichlet boundary value conditions

$$U(t,-1) = U(t,1) = 0, \qquad 0 \le t \le T. \qquad (2)$$

With respect to time variable t we use initial value condition

$$U(0,x) = \varphi(x), \qquad -1 \le x \le 1, \qquad (3)$$

where $\varphi(x)$ is enough smooth on the segment $[-1,1]$. For this function the following conditions are fulfilled

$$\varphi(-1) = \varphi(1) = 0.$$

We search solutions in the form of Fourier series:

$$U_i(t,x,\,\nu,\omega) = \sum_{n=1}^{\infty} \vartheta_{\,i\,n}(x)\left[\varphi_{in}\,h_{\,i\,1n}\,(t,\,\nu,\omega) + h_{\,i\,2n}\,(t,\,\nu,\omega,u_{\,in}) + \right.$$

$$\left. + \int_0^t \exp\left\{-\lambda_{\,i\,n}\,\omega\,(t-s)\right\}\,\alpha(s)\,u_{\,i\,n}(s,\,\nu,\omega)\,ds\right], \quad (\nu,\omega) \in \Lambda_{i\,2}, \quad i = 1,2. \qquad (4)$$

and we get solutions we receive countable system of functional-integral equations for

he unknown functions $u_{\,i\,n}(t,\,\nu,\omega)$ are under the sign of the determinant and under the sign of integral. The existence and uniqueness of whose solution is proved by the method of successive approximations. The absolute convergence of the obtained Fourier series and their differentiability are proved.

Keywords: Mixed problem, parabolic equation, degenerate kernel, involution, unique solvability.

AMS Subject Classification: Primary: 81Q10, Secondary:35P20, 47N50

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PARTICULAR SOLUTIONS OF THE MULTIPLE EULER-POISSON-DARBOUX EQUATION

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In this work we construct particular solutions of following multiple Euler-Poisson-Darboux equation

$$L(u) \equiv \sum_{j=1}^{n-1}\left(\frac{\partial^2 u}{\partial x_j^2}+\frac{2\alpha_j}{x_j}\frac{\partial u}{\partial x_j}\right)-\frac{\partial^2 u}{\partial x_n^2}-\frac{2\alpha_n}{x_n}\frac{\partial u}{\partial x_n}=0 \quad \left(0<2\alpha_j<1,\ \ j=\overline{1,n}\right)$$

in an explicit forms expressed by the Lauricella hypergeometric function $F_A^{(n)}$ [1, p. 114].

A solution of the equation $L(u)=0$ is searched in the form

$$u(x;\xi)=r^{-2\beta}\omega\left(\sigma_1,...,\sigma_n\right),$$

where $\omega\left(\sigma_1,...,\sigma_n\right)$ is unknown function; $2\beta=n-2+2\alpha_1+...+2\alpha_n$;

$$r^2:=\sum_{j=1}^{n-1}\left(x_j-\xi_j\right)^2-\left(x_n-\xi_n\right)^2;\quad \sigma_j:=-\frac{4x_j\xi_j}{r^2},\ j=\overline{1,n-1};\ \ \sigma_n:=\frac{4x_n\xi_n}{r^2}.$$

Calculating necessary derivatives and substituting them into equation $L(u)=0$, we obtain a system of partial differential equations [1, p.117]

$$\sigma_i\left(1-\sigma_i\right)\omega_{\sigma_i\sigma_i}-\sum_{j=1,j\neq i}^{n}\left(\sigma_i\sigma_j\omega_{\sigma_i\sigma_j}+\beta_i\sigma_j\omega_{\sigma_j}\right)+\left[2\alpha_i-\left(\beta+\alpha_i+1\right)\sigma_i\right]\omega_{\sigma_i}-\beta\alpha_i\omega=0, i=\overline{1,n}$$

which has 2^n linearly-independent solutions expressed by the multiple hypergeometric Lauricella function $F_A^{(n)}$ for different values of the numerical parameters of this function.

Note that particular solutions of of generalized Euler-Poisson-Darboux equation in the three-dimensional case were found in [2].

Keywords: multiple Euler-Poisson-Darboux equation; particular solutions; multiple hypergeometric Lauricella functions; systems of partial differential equations; linearly-independent solutions.

AMS Subject Classification: 35L80, 35C65, 35Q05.

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INVESTIGATION OF SYSTEM OF NONLINEAR PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS OF THE FIRST ORDER BY MEANS OF THE METHOD OF ADDITIONAL ARGUMENT

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We use results of papers [1-3].

Let $\overline{C}^{(k)}$ be the class of functions being continuous and bounded together with their derivatives up to the k-th order. Denote the differential operator $D[\omega_1, \omega_2] = \partial/\partial t + \omega_1 \partial/\partial x + \omega_2 \partial/\partial y$, the integral operator $I[\omega(..., \xi, \eta)] = \int_0^1 \int_0^1 \omega(..., \xi, \eta) d\xi d\eta$, the cortege of variables $\zeta = (t, x, y, u_1, u_2)$.

Considered the initial value problem

$$D[a_{11}(\zeta), a_{12}(\zeta)]u_1(t, x, y) = \Sigma\{a_{1k}(\zeta) : k = 1..2\} + I[u_1(t, \xi, \eta)],$$
$$D[a_{21}(\zeta), a_{22}(\zeta)]u_2(t, x, y) = b(\zeta),\ (t, x, y) \in G_3(T) := [0, T] \times {R_+}^2, \tag{1}$$

$$u_1(0, x, y) = x + y,\ u_2(0, x, y) = \varphi(x, y),\ (x, y) \in {R_+}^2,\ T > 1. \tag{2}$$

The given functions are sufficiently smooth:

$\varphi(x, y) \in \overline{C}^{(1)}({R_+}^2)$, $b(\zeta), a_{ij}(\zeta) \in \overline{C}^{(1)}(G_3(T) \times {R_+}^2), i, j = 1..2$.

From the first equation of (1), by means of the method of additional argument and the principle of contraction mappings we have found: $u_1(t, x, y) = x + y + \exp(t) - 1$. Substituting in the second equation of (1) we obtain

$$D[\widetilde{a}_{21}(t, x, y, u_2), \widetilde{a}_{22}(t, x, y, u_2)]u_2(t, x, y) = \widetilde{b}(t, x, y, u_2), \tag{3}$$

where $\widetilde{b}, \widetilde{a}_{2i}(t, x, y, u_2) = b, a_{2i}(t, x, y, x + y + \exp(t) - 1, u_2), i = 1..2$. In its turn, the problem (3)-(2) is reduced to the following system of integral equations by means of our method:

$\omega(s, t, x, y) = \varphi(q_1(0, t, x, y)), q_2(0, t, x, y)) + \int_0^t \widetilde{b}(v, q_1(v, t, x, y)), q_2(v, t, x, y), \omega(v, t, x, y))dv,$

$q_1(s, t, x, y) = x - \int_s^t \widetilde{a}_{21}(v, q_1(v, t, x, y)), q_2(v, t, x, y))\omega(v, t, x, y)dv,$

$q_2(s, t, x, y) = y - \int_s^t \widetilde{a}_{22}(v, q_1(v, t, x, y)), q_2(v, t, x, y))\omega(v, t, x, y)dv,$

where $\omega(s, t, x, y) = u_2(s, q_1(s, t, x, y)), q_2(s, t, x, y))$.

Keywords: method of additional argument, system of equations, partial derivatives, integro-differential equation, non-linear equation, unique solution.

AMS Subject Classification: 35G55

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SINGULARLY PERTURBED EQUATIONS WITH MULTIPLE TURNING POINTS AND PULL-THROUGH LOSS OF STABILITY

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This work is devoted to the study of the asymptotic behavior of the singularly perturbed equations systems solution.

In the reference [1], [2], the solution of singularly perturbed problems in an unstable domain was studied.

A feature of the system under consideration is that the coefficient matrix for a linear unknown vector function has several linear conjugate eigenvalues. All eigenvalues have zeros belonging to the imaginary axis of the independent variable complex plane. The existent parts of all eigenvalues take negative values for negative values of the independent variable on the real axis, then positive values for positive values.

The problem of the phenomenon of delaying the loss of stability is posed and solved.

Keywords: singularly perturbed equation, asymptotic behavior, small parameter, unstable region, turning point, tightening buckling.

AMS Subject Classification: 34B99.

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INITIAL VALUE PROBLEM FOR NONLINEAR BOUSSINESQ EQUATION

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The subject of this paper is to study the local existence and uniqueness of solution of the nonlocal integral boundary value problem (NIBVP) for the Boussinesq – operator equation

$$u_{tt} - \triangle u_{tt} + Au = \triangle f(u), \; x \in R^n, \; t \in (0,T), \tag{1}$$

$$u(0,x) = \varphi(x) + \int_0^T \alpha(\sigma) u(\sigma,x)\,d\sigma, u_t(0,x) = \psi(x) + \int_0^T \beta(\sigma) u_t(\sigma,x)\,d\sigma, \tag{2}$$

where A is a linear operator in a Banach space E, $\alpha(s)$ and $\beta(s)$ are measurable functions on $(0,T)$, $u(x,t)$ denotes the E – valued unknown function, $f(u)$ is the given nonlinear function, $\varphi(x)$ and $\psi(x)$ are the given initial value functions and $\triangle$ denotes the Laplace operator in R^n. Note that, in this paper, we obtain the local existence and uniqueness of small- amplitude solution of the problem (1),(2) in [1, 2].

Lemma. Let E be an UMD space. Assume that $u \in L^p(R^n;;E)$, $D^m u \in L^q(R^n;;E)$, $p,q \in (1,\infty)$. Then for i with $0 \le i \le m$, $m > \frac{n}{q}$ we have

$$\left\|D^i u\right\|_r \le C\|u\|_p^{1-\mu} \sum_{k=1}^n \|D_k^m u\|_q^{\mu},$$

where $\frac{1}{r} = \frac{i}{m} + \mu\left(\frac{1}{q} - \frac{m}{n}\right) + (1-\mu)\frac{1}{p}$, $\frac{i}{m} \le \mu \le 1$.

Result. Let E be an UMD space. Assume that $u \in W^{m,p}(R^n;;E) \cap L^\infty(R^n;;E)$ and $f(u)$ possesses continuous derivatives up to order $m \ge 1$. Then $f(u) - f(0) \in W^{m,p}(\Omega;;E)$ and

$$\|f(u) - f(0)\|_p \le \left\|f^{(1)}(u)\right\|_\infty \|u\|_p,$$

$$\left\|D^k f(u)\right\|_p \le C_0 \sum_{k=1}^n \left\|f^{(j)}(u)\right\|_\infty \|u\|_\infty^{j-1} \left\|D^k u\right\|_p, \; 1 \le k \le m,$$

where $C_0 \ge 1$ is a constant.

Keywords: Boussinesq equations, Semigroups of operators, operator functions, operator valued L^p-Fourier multipliers.

AMS Subject Classification: 65J, 65N, 35J, 47D.

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INVESTIGATION OF THE SOLVABILITY OF BOUNDARY PROBLEM FOR SECOND ORDER DIFFERENTIAL EQUATIONS BY THE PARAMETERIZATION METHOD

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In this paper, we consider various boundary value problems for second-order differential equations on the interval $[0, 1]$

$$y''(x) + \lambda y(x) = f(x), \qquad x \in [0, 1]. \tag{1}$$

The parameterization method proposed by Professor D. Dzhumbaev [1] will be applied to these boundary value problems. Entering auxiliary parameters

$$\mu_1 = y(0), \ \ \mu_2 = y'(0), \tag{2}$$

and do the substitution

$$y(x) = u(x) + \mu_1 + \mu_2 x. \tag{3}$$

Successful introduction of parameter (2) and change of variables (3) splits the boundary value problem for equation (1) into two parts, i.e. Cauchy problem for equation (1) and a system of linear equations for determining the introduced parameters μ_1, μ_2.

The system of linear equations for determining the values of the parameters μ_1, μ_2 also sets the condition for the solvability of the boundary value problem under study.

Let for equation (1) is given boundary conditions

$$y(0) = \alpha y(1) + \beta \int_0^\theta y(\xi)\, d\xi, \quad y'(0) = 0, \ \ \alpha, \beta \in R, \ \ \theta \in [0, 1].$$

Introducing parameters (2) and performing substitution (3), we obtain the Cauchy problem $u(0) = 0, \ \ u'(0) = 0$ for the equation

$$u''(x) + \lambda u(x) = f^*(x), \qquad x \in [0, 1]$$

and a system of linear equations as to whether the introduced parameters

$$\alpha u(1) + \beta \int_0^\theta u(\xi)\, d\xi + \beta \mu_1 \theta = 0, \quad \mu_2 = 0.$$

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Keywords: second order equation, boundary value problem,Cauchy problem, parameterization method, unique solvability.

AMS Subject Classification: 45J05, 45B05, 34K28.

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BOUNDEDNESS OF SOLUTIONS OF A CLASS OF LINEAR INTEGRO-DIFFERENTIAL EQUATIONS OF THE SECOND ORDER ON THE SEMIAXIS

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In this paper based on a formula for one class of second order linear differential equations with variable coefficients were established sufficient conditions of exponential estimate of solutions for one class of second order linear integro-differential equations on the semi axis.

Consider the linear integro-differential equation

$$y'' + p(t)y' + q(t)y + \int_{t_0}^{t} K(t,s)u(s)\,ds = f(t), \;\; t \in [t_0, \infty), \tag{1}$$

where $K(t,s)$, $p(t)$, $q(t)$, $f(t)$ -known functions.

The questions of boundedness and asymptotic stability of solutions on the semiaxis for differential and integro-differential equations were studied, for example in [1]. In this paper, on the basis of the results of [2] and the method of transformations, we establish sufficient conditions for the boundedness of solutions of the integro-differential equation (1) on the semiaxis.

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ON THE THEORY OF LINEAR INTEGRAL EQUATIONS OF THE THIRD KIND

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The report considers a linear integral equation of the third kind [1]

$$(t-t_0)^n u(t) + \int_\Gamma K(t,\tau)u(\tau)d\tau = f(t), t \in \Gamma \tag{1}$$

where Γ – is a smooth closed contour in the complex plane z, given functions $f(t)$ and $K(t,\tau)$ from the class $H_\mu(\Gamma)$ and $H_\mu(\Gamma\times\Gamma)(\mu \in [0,1])$ of Helderovs functions. From the decomposition $(t-t_0)^n = a(z) + ib(z)$ with real functions $a(z)$ and $b(z)$,we arrive at the Riman boundary value problem of the theory of analytic functions, which is formulated as follows:

it is required to find a piecewise analytic function $\Phi(z)$ with a jump line Γ, which has a finite order at infinity, by the boundary condition

$$\Phi^+(t) = \frac{a(t)-b(t)}{a(t)+b(t)}\Phi^-(t) + \frac{f(t)}{a(t)+b(t)}, t \in \Gamma \tag{2}$$

Index of Riman boundary value problem (2) and singular integral equation with Cauchy kernel

$$a(t)u(t) + \frac{b(t)}{\pi i}\int_\Gamma \frac{\varphi(\tau)}{\tau - t}d\tau = f(t), t \in \Gamma \tag{3}$$

as well as the integral equation of the third kind (1) is the same.

Theorem. *It is assumed that $a^2(t) - b^2(t) \neq 0, t \in \Gamma$. Then the three problems (1), (2), and (3) under consideration are completely equivalent and they are simultaneously Noetherian. For equations (1) and (3), this result does not depend on completely continuous terms [2].*

Keywords: linear integral equation of the third kind, piecewise analytic function.

AMS Subject Classification: 45A05

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OPTIMAL DIFFERENCE METHOD IN A HILBERT SPACE

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In this work, the problem of constructing new optimal difference formulas is considered for finding an approximate solution to the initial-value problem for ordinary differential equations in a Hilbert space.

Firstly, the construction of the explicit optimal difference formula is studied of the Adams-Bashforth type in a Hilbert space. Here, an expression of the square norm of the error functional with respect to the coefficients is found, and a system of linear algebraic equations for coefficients of the difference formula is obtained. Then this system to a system of equations in convolution is reduced and it is completely solved with the help of a discrete analog of the differential operator. Moreover, the implicit optimal difference formula of the Adams-Moulton type is constructed in a Hilbert space.

It is known that the solutions of many practical problems lead to solutions of differential equations or their systems. Although differential equations have so many applications and only a small number of them can be solved exactly using elementary functions and their combinations. Even in the analytical analysis of differential equations, their application can be inconvenient due to the complexity of the obtained solution. If it is very difficult to obtain or impossible to find an analytic solution to a differential equation, one can find an approximate solution.

In the present work we consider the problem of approximate solution to the first order linear ordinary differential equation

$$y' = f(x, y), \ \ x \in [0, 1] \tag{1}$$

with the initial condition

$$y(0) = y_0. \tag{2}$$

We consider a difference formula of the following form for the approximate solution of the problem (1)-(2)

$$\sum_{\beta=0}^{k} C\,[\beta]\,\varphi\,[\beta] - h\sum_{\beta=0}^{k} C_1\,[\beta]\,\varphi'\,[\beta] \cong 0, \tag{3}$$

where $h = \frac{1}{N}$, N is a natural number, $C[\beta]$ and $C_1[\beta]$ are the coefficients, functions φ belong to the Hilbert space. The space $W_2^{(m,m-1)}(0,1)$ is defined as follows

$$W_2^{(m,m-1)}(0,1) = \{\varphi : [0,1] \to \mathbb{R} | \varphi^{(m-1)} \text{ is abs.contunuous, } \varphi^{(m)} \in L_2(0,1)\}$$

equipped with the norm

$$\|\varphi|W_2^{(m,m-1)}\| = \Big\{ \int\limits_0^1 \Big(\varphi^{(m)}(x) + \varphi^{(m-1)}(x)\Big)^2 dx \Big\}^{1/2}. \tag{4}$$

Keywords: Hilbert space; initial-value problem; multistep method; the error functional; optimal explicit difference formula; optimal implicit difference formula
AMS Subject Classification: 65L05; 65L12; 65L20

ATTRACTORS OF REACTION–DIFFUSION SYSTEM IN ORTHOTROPIC POROUS MEDIUM

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Let $\Omega \subset \mathbb{R}^n, n \geq 3$ be a smooth bounded domain. Suppose $G_0 \subset Y := (-1/2, 1/2)^n$, $\varepsilon > 0$, $\varepsilon^{n/(n-2)} G_0 \subset \varepsilon Y$, $Y_\varepsilon^j = \varepsilon j + \varepsilon Y$, $G_\varepsilon^j = \varepsilon j + \varepsilon^{n/(n-2)} G_0$, $j \in \mathbb{Z}^n$. Denote $\widetilde{\Omega}_\varepsilon = \{x \in \Omega : \rho(x, \partial\Omega) > \sqrt{n}\varepsilon\}$ and $\Upsilon_\varepsilon = \{j \in \mathbb{Z}^n : G_\varepsilon^j \cap \overline{\widetilde{\Omega}}_\varepsilon \neq \varnothing\}$. Consider the domain $\Omega_\varepsilon = \Omega \setminus \overline{G}_\varepsilon$, where $G_\varepsilon = \bigcup_{j \in \Upsilon_\varepsilon} G_\varepsilon^j$. We study the attractors $\mathfrak{A}_\varepsilon$ as $\varepsilon \to 0$, for initial-boundary value problem

$$\begin{array}{ll}
\frac{\partial u_\varepsilon}{\partial t} = \lambda \Delta u_\varepsilon - a\big(x, x/\varepsilon\big) f(u_\varepsilon) + g\big(x, x/\varepsilon\big), & x \in \Omega_\varepsilon, \\
\frac{\partial u_\varepsilon}{\partial \nu} + \varepsilon^{n/(n-2)} B_\varepsilon^j(x) u_\varepsilon = 0, & x \in \partial G_\varepsilon^j,\ j \in \Upsilon_\varepsilon,\ t \in (0, +\infty), \\
u_\varepsilon = 0, & x \in \partial\Omega, \\
u_\varepsilon = U(x), & x \in \Omega_\varepsilon,\ t = 0,
\end{array}$$

where $u_\varepsilon(x,t) = (u_\varepsilon^1, \ldots, u_\varepsilon^N)^\top$ is the unknown vector function, $f = (f^1, \ldots, f^N)^\top$ and $g = (g^1, \ldots, g^N)^\top$ are given, and λ is the $N \times N$-matrix with constant coefficients having a positive symmetric part, ν is the outward normal to the boundaries G_ε^j. Here $a(x,y)$ and components of the diagonal matrix $B_\varepsilon^j(x)$ are bounded.

The homogenized (limit) problem for the considered reaction-diffusion system has the form

$$\begin{array}{ll}
\frac{\partial u}{\partial t} = \lambda \Delta u - \overline{a}(x) f(u) - V(x) u + \overline{g}(x), & x \in \Omega, \\
u = 0, & x \in \partial\Omega,\ t \in (0, +\infty) \\
u = U(x), & x \in \Omega,\ t = 0,
\end{array}$$

where $V(x)$ is a diagonal matrix. The functions $\overline{a}(x)$ and $\overline{g}(x)$ are the mean of the $a\big(x, x/\varepsilon\big)$ and $g\big(x, x/\varepsilon\big)$ as $\varepsilon \to 0$.

Theorem 1. Let $\overline{\mathfrak{A}}$ be the attractor of the homogenized problem. The following limit relation holds: $\mathfrak{A}_\varepsilon \to \overline{\mathfrak{A}}$ as $\varepsilon \to 0$.

Keywords: attractors, homogenization, reaction-diffusion systems, perforated domain.

AMS Subject Classification: 35B40, 35B41, 35B45, 35Q30

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THE BISINGULAR PROBLEM WITH BIBOUNDARY LAYER

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We study a two-point boundary value problem:

$$\varepsilon^n y''(x) + x^k p(x) y'(x) - (x^k q(x) + \varepsilon^m r(x)) y(x) = f(x),\ x \in [0,1], \tag{1}$$

$$y(0) = a,\ y(1) = b, \tag{2}$$

where a, b are known constant numbers, p, q, r, $f \in C^\infty[0,1]$, $f(0) \neq 0, 0 < p(0), 0 < q(0)$, $0 < r(0)$, $m < n$, $1 < k, n, m \in N$.

It is required to find the necessary and sufficient conditions under which ratios of the parameters n and m an additional (intermediate) boundary layer appears and for this case to construct a uniform asymptotic approximation of the solution of the Dirichlet problem (1), (2) on the interval $[0,1]$, when the small parameter ε tends to zero.

The following theorems is proved

Theorem 1. In problem (1), (2) for the existence of an intermediate (additional) boundary layer, in addition to the classical boundary layer, in the vicinity of the left boundary point $x = 0$, it is necessary and sufficient that the condition be satisfied:$m(k+1) < n(k-1)$.

Theorem 2. To solve the two-point Dirichlet boundary value problem (1), (2) on the segment $x \in [0,1]$ for $\varepsilon \to 0$, the following asymptotic expansion is valid

$$y(x) = \left(\sum_{j=0}^{s} \varepsilon^{jm} v_j(x) + \frac{1}{\mu^{(k-1)m}} \sum_{j=0}^{(k-1)m(s+1)} \mu^j w_j(t) + \frac{1}{\lambda^{2m}} \sum_{j=0}^{2m(s+1)} \lambda^j \pi_j(\tau) \right) e^{\int_1^x \frac{q(\xi)}{p(\xi)} d\xi} + O(\varepsilon^{ms}).$$

Keywords: additional (intermediate) boundary layer, three-zone problem, bisingular perturbed, Dirichlet problem, asymptotic solution.

AMS Subject Classification: 34B05, 34D05, 34D15, 34E05, 34E10, 34E20.

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DEVELOPMENT OF THE SINGULARLY PERTURBED EQUATIONS THEORY WITH ANALYTIC FUNCTIONS

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In this paper, singularly perturbed ordinary differential equations are considered. The work consists of two parts.

In the reference [1], [2], the solution of singularly perturbed problems in an unstable domain was studied.

The first part presents the most general results obtained by L.S. Pontryagin, A.N. Tikhonov, A.B. Vasilyeva, M.I.

Imanaliev and other authors concerning singularly perturbed equations with a really independent variable.

The second part is considering development dynamics of the singularly perturbed equations theory with analytic functions, i.e. equations whose right-hand sides are analytic functions in all variables. The independent variable belongs to

Keywords: singularly perturbed equation, asymptotic behavior, small parameter, unstable region, analytic function.

AMS Subject Classification: 34B99.

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INITIAL-BOUNDARY VALUE PROBLEM FOR THE HEAT EQUATION UNDER PERIODIC BOUNDARY CONDITIONS IN THE ABSENCE OF AGREEMENT BETWEEN THE INITIAL AND BOUNDARY DATA

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In the report, the initial-boundary value problem for the heat equation under periodic boundary conditions in the absence of agreement between the initial and boundary data is considered.

Problem P. Find in $\Omega = \{(x,t) : 0 < x < l, 0 < t < T\}$ a solution $u(x,t)$ of the heat equation

$$u_t - k^2 u_x x = f(x,t), \tag{1}$$

satisfying the initial condition

$$u(x,0) = \varphi(x), 0 \leq x \leq l, \tag{2}$$

and periodic boundary conditions

$$\begin{cases} u_x(0,t) - u_x(l,t) = 0, \\ u(0,t) - u(l,t) = 0. \end{cases} \tag{3}$$

It is well known that for the existence of a classical solution, the matching conditions must be satisfied. For example, the zero and first order matching conditions for problem (1)-(3) are

$$A_0 \equiv \varphi(0) - \varphi(l) = 0, \; A_1 \equiv \varphi'(0) - \varphi'(l) = 0.$$

The second-order matching condition arises when we consider solutions of the problem from the class $u \in C^{2,1}_{x,t}(\bar{\Omega})$. For functions from such a class, we can pass to the limit in equation (1) at $t \to 0$ at $x = 0$ and $x = l$. Then we get

$$A_2 \equiv -k^2[\varphi''(0) - \varphi''(l)] - [f(0,0) - f(l,0)] = 0.$$

For a problem with Dirichlet boundary conditions, solutions with a mismatch between the boundary and initial data were studied in [1]. In this report, we consider the problem (1)-(3) with nonlocal boundary conditions. Cases where the matching conditions of the zero, first and second orders are not met are considered in the report.

Keywords: heat equation, initial-boundary value problem, nonlocal boundary conditions, matching conditions. provide 4 to 6 keywords which can be used for indexing purposes.

AMS Subject Classification: 35K05, 335K15.

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ON EXACT SOLUTIONS TO THE MASS BALANCE EQUATION

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Let $t \in [0,T]$–the time variable, $\Omega \subset R^3$ is a one-coherent bounded region filled without voids with liquid. If $h(t,x,y,z)$ is a given intensity of mass sources, then the mass balance equation in the region $G = [0,T] \times \Omega \subset R^4$ has the following form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho U)}{\partial x} + \frac{\partial(\rho V)}{\partial y} + \frac{\partial(\rho W)}{\partial z} = h(t,x,y,z). \tag{1}$$

Here: $\rho(t,x,y,z), U, V, W(t,x,y,z)$ – unknown density and velocity components of the fluid.

Definition 1. The resolving parameters of equation (1) are dimensionless scalars (total – 12, of which free – 10) related by the following relations:

$$\sum_{k=1}^{4} \beta_k \alpha_k = 0; \sum_{m=1}^{4} \gamma_m = 1.$$

Definition 2. The four-dimensional regular function defined in the domain G is a vector-function $u = (u_1, u_2, u_3, u_4)$ with components satisfying generalized Cauchy-Riemann conditions:

$$\frac{\partial u_1}{\partial t} = \frac{\partial u_2}{\partial x} = \frac{\partial u_3}{\partial y} = \frac{\partial u_4}{\partial z}; \frac{\partial u_1}{\partial x} = -\frac{\partial u_2}{\partial t} = \frac{\partial u_3}{\partial z} = -\frac{\partial u_4}{\partial y};$$
$$\frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial z} = \frac{\partial u_3}{\partial t} = \frac{\partial u_4}{\partial x}; \frac{\partial u_1}{\partial z} = -\frac{\partial u_2}{\partial y} = \frac{\partial u_3}{\partial x} = -\frac{\partial u_4}{\partial t}.$$

The infinite-dimensional space of regular four-dimensional functions is a dense subset of the space of 4–vectors with continuously–differentiable components in the G. Further, let ρ_0, c, L – be the characteristic density, velocity, and size of the flow.

Theorem. Equation (1) has a continuum of exact solutions of the form:

$$\begin{aligned}
\rho(t,x,y,z) &= \beta_1 \rho_0 u_1 \left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L} \right) + \gamma_1 \int_0^t h(\tau,x,y,z) d\tau \\
\rho U(t,x,y,z) &= \beta_2 \rho_0 c u_2 \left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L} \right) + \gamma_2 \int_0^x h(t,\xi,y,z) d\xi \\
\rho V(t,x,y,z) &= \beta_3 \rho_0 c u_3 \left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L} \right) + \gamma_3 \int_0^y h(t,x,\eta,z) d\eta \\
\rho W(t,x,y,z) &= \beta_4 \rho_0 c u_4 \left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L} \right) + \gamma_4 \int_0^z h(t,x,y,\zeta) d\zeta
\end{aligned} \tag{2}$$

It is easy to see that the solution depends on 19 parameters: four arbitrary functions and 15 scalars.

Keywords: four-dimensional functions,resolving parameters,Cauchy-Riemann conditions,exact solutions

AMS Subject Classification: 76A02,76M40

ON THE CONTINUUM OF EXACT SOLUTIONS TO THE SYSTEM OF MAXWELL'S EQUATIONS

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As is known [1], the basic equations of electrodynamics in theoretical physics are given in the form of a system of Maxwell's equatins of the following form

$$\begin{cases} div(\varepsilon\vec{E}) = 4\pi\rho, \\ rot\vec{E} = -\dfrac{1}{c}\dfrac{\partial(\mu\vec{H})}{\partial t}, \\ div(\mu\vec{H}) = 0, \\ rot\vec{H} = \dfrac{1}{c}\dfrac{\partial(\varepsilon\vec{E})}{\partial t} + \dfrac{4\pi\vec{J}}{c}, \end{cases} \tag{1}$$

where c is the speed of light, $\varepsilon > 0, \mu > 0$ - are known, dimensionless constants, $\vec{E}(t,x,y,z), \vec{H}, \vec{J}$ - are unknown vectors of electric (magnetic) field strengths and saturation current density of electric charges. The system of equations (1) is considered key in theoretical physics. To date, only a few exact solutions of this system are known. For example, the motion of a plane electromagnetic wave it is described in various textbooks on theoretical physics.

Definition 1. *The four - dimensional potential of the electromagnetic field is the vector function $\Pi_{4d} = (\theta, u, v, \omega)$, and it satisfies the following continuity equation for the electromagnetic field:*

$$\frac{\varepsilon\mu}{c}\frac{\partial\theta}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial\omega}{\partial z} = 0 \tag{1}$$

Theorem 1. *(1) The continuity equation has a continuum of exact solutions of the following form:*

$$\begin{cases} \varepsilon\mu\theta(t,x,y,z) = \beta_1 u_1(\alpha_1 ct, \alpha_2 x, \alpha_3 y, \alpha_4 z) \\ u(t,x,y,z) = \beta_2 u_2(\alpha_1 ct, \alpha_2 x, \alpha_3 y, \alpha_4 z) \\ v(t,x,y,z) = \beta_3 u_3(\alpha_1 ct, \alpha_2 x, \alpha_3 y, \alpha_4 z) \\ \omega(t,x,y,z) = \beta_4 u_4(\alpha_1 ct, \alpha_2 x, \alpha_3 y, \alpha_4 z) \end{cases} \tag{2}$$

here: $\alpha_k, \beta_k, k = 1,2,3,4$ - is an arbitrary set of scalars satisfying condition $\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3 + \alpha_4\beta_4 = 0$, and $U(X) = (u_1(x_1,x_2,x_3,x_4), u_2, u_3, u_4)$ is four - dimensional arbitraty regular function in the domain $G \subset R^4$.

Theorem 2. *For each four - dimensional potential of the electromagnetic field $\Pi_{4d} = (\theta, u, v, \omega)$ an exact solution of Maxwell's equations of the following form corresponds:*

$$\begin{cases} \overrightarrow{E} = \nabla\theta + \dfrac{1}{c}\dfrac{\partial\overrightarrow{\varphi}}{\partial t} \\ \overrightarrow{H} = -\dfrac{1}{\mu} rot\overrightarrow{\varphi} \\ \rho = \dfrac{\varepsilon}{4\pi}\left(\Delta\theta - \dfrac{\varepsilon\mu}{c^2}\dfrac{\partial^2\theta}{\partial t^2}\right) \\ \overrightarrow{j} = \dfrac{c}{4\pi\mu}\left(\Delta\varphi - \dfrac{\varepsilon\mu}{c^2}\dfrac{\partial^2\overrightarrow{\varphi}}{\partial t^2}\right) \end{cases} \tag{3}$$

formulas (2) - (3) give a continuum of exact solutions to Maxwell's equations.

Keywords: Maxwell's equations, electromagnetic field, four - dimensional potential, continuity equation.

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ON INVERSE HELMHOLTZ PROBLEM IN THE PRESENCE OF RANDOM PERTURBATIONS

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We consider the problem of extending Hamilton's principle to the class of natural mechanical systems with random perturbing forces of white noise type. By the method of moment functions, we construct the functionals taking a stationary value on the solutions of a given stochastic equation of Lagrangian structure. The Helmholtz problem in the class of stochastic equations can be formally divided into two interrelated subproblems.

Problem 1. Given the second-order Ito stochastic differential equation

$$\ddot{x}_\nu = F_\nu(x,\dot{x},t) + \eta_{\nu j}(x,\dot{x},t)\dot{\xi}^j, \quad (\nu = \overline{1,n}; j = \overline{1,m}) \tag{1}$$

it is required to construct the equivalent stochastic equation of Lagrangian structure in the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_\nu}\right) - \frac{\partial L}{\partial x_\nu} = \eta^{'}_{\nu j}(x,\dot{x},t)\dot{\xi}^j. \tag{2}$$

Problem 2. It is required to construct a functional which takes a stationary value on the solutions of the given equation of Lagrangian structure (2).

Note that [1,2] solve mainly Problem 1. The present report is devoted to the solving the Problem 2. Similarly to the classical case, we consider the Hamiltonian action $W = \int_{t_0}^{t_1} L(q,\dot{q},t)dt$ and introduce the following notions : 1) $M\int_{t_0}^{t_1} Ldt$ is the mathematical expectation of the Hamiltonian action, 2) $\int_{t_0}^{t_1} \Gamma L dt \equiv M\int_{t_0}^{t_1} L^2 dt$ is the second-order initial moment of the Hamiltonian action, 3)$\int_{t_0}^{t_1} DLdt \equiv M\int_{t_0}^{t_1}(L - ML)^2 dt$ is the variance of the Hamiltonian action. Now, using these moment functions we formulate the Hamilton principle in the presence of random perturbations.

Keywords: Stochastic differential equation, Helmholtz problem, method of moment functions.

AMS Subject Classification: 60H10; 34F05, 34A55

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LARGE TIME BEHAVIOUR SOLUTIONS FOR TIME-SPACE FRACTIONAL DIFFUSION EQUATION WITH GRADIENT NONLINEARITY

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In this paper, we study the initial-boundary value problem for the nonlinear time-space fractional diffusion equation

$$\begin{cases} \partial^{\alpha}_{0|t}u + (-\Delta)^s_p u = \mu|u|^r + \nu|\mathbb{D}_s u|^q, \quad (x,t) \in \Omega \times (0,T), \\ u(x,t) = 0, \; x \in \mathbb{R}^N \setminus \Omega, \; 0 < t < T, \\ u(x,0) = u_0(x), \; x \in \Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a smoothly bounded domain; $s \in (0,1), p \geq 2, r, q \geq 1$, $\mu, \nu \in \mathbb{R}$ and $\partial^{\alpha}_{0|t}$ is the left Caputo fractional derivative of order $\alpha \in (0,1)$, which given by

$$\partial^{\alpha}_{0|t}u(t) = I^{1-\alpha}_{0|t}\frac{d}{dt}u(t) = \frac{1}{\Gamma(1-\alpha)}\int\limits_0^t (t-s)^{-\alpha}u'(s)\,ds, \quad \forall t \in (0,T]$$

The fractional p-Laplacian operator for $s \in (0,1), p > 1$ and $u \in W^{s,p}(\Omega)$, is defined by

$$(-\Delta)^s_p u(x) = C_{N,s,p}\,\mathrm{P.V.}\int_{\mathbb{R}^N}\frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{N+sp}}dy,$$

where

$$C_{N,s,p} = \frac{sp2^{2s-2}}{\pi^{\frac{N-1}{2}}}\frac{\Gamma(\frac{N+sp}{2})}{\Gamma(\frac{p+1}{2})\Gamma(1-s)}$$

is a normalization constant and P.V. is an abbreviation for in the principal value sense. sense.

We give a simple proof of the comparison principle for the considered problem using purely algebraic relations, for different sets of γ, μ, r and q.

The blow-up phenomena, existence of global weak solutions and asymptotic behavior of global solutions are classified using the comparison principle.

Keywords: quasilinear parabolic equation, fractional integral and derivative, comparison principle.

AMS Subject Classification: 35R11, 35B51, 35K55.

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THE PROPERTIES OF THE CAUCHY PROBLEM FOR A DOUBLE NONLINEAR TIME-DEPENDENT PARABOLIC EQUATION IN NON-DIVERGENCE FORM WITH A SOURCE TERM

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In the domain $Q = \left\{(t,x) : t \geq t_0 > 0, x \in R^N\right\}$, we consider the following problem:

$$|x|^{-n}\,\partial_t u = u^q div\left(|x|^{n_1} u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) + \varepsilon\,|x|^{-n}\, t^l u^\beta,\ (x,t) \in Q \tag{1}$$

$$u\,(0,x) = u_0\,(x) \geq 0, \qquad x \in R^N \tag{2}$$

where $k, m \geq 1, p \geq 2, 0 < q < 1, \varepsilon = \pm 1$ and non-negative n, n_1, l, β are given numerical parameters.
The (1)-(2) arises in different applications [1]. The equation (1) might degenerate at the points where $u = 0$ and $\nabla u = 0$. Therefore, in this case, we need to consider a weak solution from having a physical sense class. The problem (1)-(2) has been intensively studied by many authors (see [1]-[2] and the literature therein).

Let us take the function

$$z\,(t,x) = l_1 t^{\frac{1+l}{1-\beta_2}}\,(t)\,\overline{f}\,(\xi) \tag{3}$$

$$\overline{f}\,(\xi) = \begin{cases} \left(a - b\xi^{\frac{p}{p-1}}\right)_+^{\frac{p-1}{m_2+k_2(p-2)-1}}, & \text{if } m_2 + k_2\,(p-2) \neq 1, \\ e^{-b\xi^{\frac{p}{p-1}}}, \quad \text{if } m_2 + k_2\,(p-2) = 1, \end{cases} \tag{4}$$

where $m_2 = \frac{m}{1-q}, k_2 = \frac{k}{1-q}, \beta_2 = \frac{\beta-q}{1-q}, l_1 = \left[\frac{\varepsilon(1-q)(\beta_2-1)}{1+l}\right]^{\frac{1}{1-\beta_2}}, a = const > 0$ and b defined above constant.

Theorem 1. Let us $m_2 + k_2\,(p-2) - 1 \geq 0,\ p > n + n_1,\ u\,(0,x) \leq z\,(0,x)\,,\ x \in R^N$. Then for solution of the problem (1)- (2) an estimate

$$u\,(t,x) \leq z\,(t,x) \qquad \text{in } Q$$

hold.

Keywords: Degenerate parabolic equation, non-divergent, weak solution, critical Fujita.

AMS Subject Classification: 35D30, 35K55, 35B40 .

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SOME RESULTS M-SPECTRAL PROBLEM WITH NATURAL TRANSFORM

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Let us consider the M-Sturm-Liouville problem with related to boundary conditions as follows:

$$\left[-D_M^{\alpha,\gamma} D_M^{\alpha,\gamma} y(x) + [q(x)y(x)] = \lambda y(x) \right] \tag{1}$$

In this study, our purpose we acquire representation of solution for the M-Sturm-Liouville problem by taking advantage of natural transform.
Furthermore, we define M-Natural transform as follows:

$$\mathcal{N}_{\alpha,\beta}^{a}\{f(t)\}(s) = F_{\alpha,\beta}^{a}(s,u) = \left[\frac{\Gamma(\beta+1)}{u} \int_a^\infty e^{-s\frac{\Gamma(\beta+1)(t-a)^\alpha}{u\alpha}} f(t) d_\alpha t \right]. \tag{2}$$

This problem is solved with M-Natural transform and main results are showed by graphs [1-4].

Keywords: Natural transform, M-derivative, Sturm-Liouville Problem

AMS Subject Classification: 26A33

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SOLVING AN INITIAL-BOUNDARY VALUE PROBLEM FOR A LOADED FRACTIONAL-ORDER DIFFUSION EQUATION

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In a domain

$$G = \{(x, y; t) | -\infty < x, y < +\infty,\ t > 0\}$$

find a solution to the equation

$$_{RL}D^{\alpha}_{0t}u = u_{xx} + u_{yy} + \lambda\{_{RL}D^{\beta}_{0t}\}\Big|_{x-1=t,y-1=t,t>0} + f(x, y, t), \tag{1}$$

where $_{RL}D^{\sigma}_{0t}f(t)$ is a Riemann-Liouville derivative of the order $\sigma, 0 < \sigma < 1$, and $0 < \beta \le \alpha < 1$.

The solution of equation (1) must satisfy the initial condition:

$$\lim_{t\to 0} {_{RL}D^{\alpha-1}_{0t}}u = \varphi(x, y) \tag{2}$$

Let us denote

$$\mu(t) = \{_{RL}D^{\beta}_{0t}u\}\Big|_{x-1=t,y-1=t,t>0}. \tag{3}$$

Using the representation formula for the solution of Problem (1) – (2) [1], we obtain a Volterra-type integral equation with respect to the function (3). The solution of the integral equation is obtained by the Laplace transform method.

The following theorem is proved.

Theorem 1. A function

$$u(x, y, t) = \lambda\left(\psi(t) * \frac{t^{\alpha-1}}{\Gamma(\alpha)}\right)(t) + \lambda^2\left(\psi(t) * \left(t^{2\alpha-\beta-1}\ \mathcal{E}_{\alpha-\beta,2\alpha-\beta}(\lambda t^{\alpha-\beta})\right)\right)(t) +$$
$$\varphi_1(x, y, t) + f_1(x, y, t),$$

is a solution to Problem (1) – (2), where the functions $\varphi_1(x, y, t)$ and $f_1(x, y, t)$ are defined by the initial function $\varphi(x, y)$ from condition (2) and the right side $f(x, y, t)$ from equation (1), respectively. $\mathcal{E}_{a,b}(z)$ is a Mittag-Leffler function and

$$\psi(t) = {_{RL}D^{\beta}_{0t}}\left(\varphi_1(x, y, t) + f_1(x, y, t)\right)\Big|_{x-1=t,y-1=t,t>0}.$$

Keywords: loaded equation, fractional derivative, Wright function, Laplace transform.

AMS Subject Classification: 45D05, 35K20, 26A33.

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A FREE BOUNDARY PROBLEM FOR A DIFFUSIVE COMPETITION SYSTEM WITH NONLINEAR CONVECTION TERM

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The spread of new or invasive species is a central topic of ecology, and significant research has been devoted to better understanding the nature of such spread. Environmental problems require the use of a whole hierarchy of models capable of describing not only different levels of organization of systems, but also the interaction between these levels. However, significant advances have been made in species invasion studies through frontal spread studies (see [1], [2]).

In this article, we study a diffusive Lotka-Volterra-type competitive system with free boundary:

$$u_t - d_1 u_{xx} + m_1 v_x u_x = u\left(a_1 - b_1 u - c_1 v\right), \quad 0 < t \le T, \quad 0 < x < s(t), \tag{1}$$

$$v_t - d_2 v_{xx} + m_2 u_x v_x = v\left(a_2 - b_2 u - c_2 v\right), \quad 0 < t \le T, \quad 0 < x < \infty, \tag{2}$$

$$u\left(0, x\right) = u_0\left(x\right), \quad 0 \le x \le s\left(0\right) \equiv s_0, \tag{3}$$

$$v\left(0, x\right) = v_0\left(x\right), \quad 0 \le x < \infty, \tag{4}$$

$$u_x\left(t, 0\right) = 0, \quad 0 \le t \le T, \tag{5}$$

$$u\left(t, s\left(t\right)\right) = 0, \quad 0 \le t \le T, \tag{6}$$

$$s'\left(t\right) = -\mu u_x\left(t, s\left(t\right)\right), \quad 0 \le t \le T, \tag{7}$$

$$v_x\left(t, 0\right) = 0, \quad 0 \le t \le T, \tag{8}$$

$$u(t, x) \equiv 0, \quad s(t) \le x < \infty, \tag{9}$$

where $s\left(t\right)$ is a free boundary to be determined, s_0, μ, d_i, m_i, a_i, b_i, c_i (i=1,2) are given positive constants, and the initial functions $u_0(x)$ and $v_0(x)$ satisfies

$$u_0\left(x\right) \in C^2\left([0, s_0]\right), \quad 0 < u_0\left(x\right) \le \frac{a_1}{b_1} \quad \text{in} \quad [0, s_0), \quad u_0'\left(0\right) = u_0\left(s_0\right) = 0,$$

$$v_0\left(x\right) \in C^2\left([0, \infty)\right), \quad 0 < v_0\left(x\right) \le \frac{a_2}{c_2} \quad \text{in} \quad (0, \infty), \quad u_0'\left(0\right) = 0.$$

Keywords: free boundary, reaction-diffusion system, parabolic equation, aprior bounds, existence and uniqueness.

AMS Subject Classification: 35B45, 35K20, 35K57, 35K59

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AN EXPLICIT POSITIVITY-PRESERVING NONSTANDARD FINITE DIFFERENCE SCHEME

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Abstract. Solving problems in various fields of science and engineering, such as physical and biological systems, leads to solving parabolic equations of the advection-diffusion reaction (ADR) type. In these problems, usually the concentration of chemical compounds or the size of the population are our unknowns, which are positive in nature. In general, the use of common methods, such as the classical finite difference method, may produce numerical drawbacks such as spurious oscillations and negative values in the solutions due to truncation errors. By using the nonstandard finite difference (NSFD) method, a better finite difference model is constructed. The proposed NSFD scheme ensures that the solutions are positive and there are no spurious oscillations in the solutions.

Keywords: Advection-diffusion reaction equations, Nonstandard finite differences, Positivity preserving.

AMS Subject Classification: 65M06; 65M12.

THE l-APPROACH PROBLEM IN A LINEAR DIFFERENTIAL GAME WITH CONSTANT COEFFICIENT

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We assume that a player P (the pursuer) follows another player E (the evader) in the finite-dimensional space $\mathbb{R}^n$. Let their movements be described by the following linear equations:

$$P: \quad \dot{x} + ax = u, \quad x(0) = x_0, \tag{1}$$

$$E: \quad \dot{y} + ay = v, \quad y(0) = y_0, \tag{2}$$

where $x, y, u, v \in \mathbb{R}^n$, $n \geq 2$; $a \neq 0$ and $a \in \mathbb{R}$; x_0 and y_0 are the initial positions of the players for which it is presumed that $|x_0 - y_0| > l$, $l > 0$.

The controls u and v are regarded as measurable functions $u(\cdot) : [0, +\infty) \to \mathbb{R}^n$ and $v(\cdot) : [0, +\infty) \to \mathbb{R}^n$ accordingly, and they are subject to the constraints

$$|u(t)| \leq \alpha \ \text{ for almost every } t \geq 0, \tag{3}$$

$$|v(t)| \leq \beta \text{ for almost every } t \geq 0. \tag{4}$$

which are usually termed the geometrical constraints (in short, the G-constraints), where α and β are non-negative numbers which designate the maximal speeds of P and E.

Definition. For $\alpha \geq \beta$, we call the function

$$\mathbf{u}(z_0, v) = v + \lambda(z_0, v)\left(m(z_0, v) - z_0\right) \tag{5}$$

the l-approach strategy or Π_l–strategy for P in this differential game, where

$$\lambda(z_0, v) = \frac{1}{|z_0|^2 - l^2}\left[\langle v, z_0\rangle + \alpha l + \sqrt{(\langle v, z_0\rangle + \alpha l)^2 + (|z_0|^2 - l^2)(\alpha^2 - |v|^2)}\right],$$

$m(z_0, v) = -l\Big(v - \lambda(z_0, v)z_0\Big)/|v - \lambda(z_0, v)z_0|$.

Theorem. Let $\alpha > \beta$ and $a > 0$. Then the Π_l–strategy guarantees to occur the l-aproach from arbitrary point $z_0 \notin lS$ in the time $T(z_0, v(\cdot)) \leq \theta$, where

$$\theta = \frac{1}{a}\ln\frac{a|z_0| + \alpha - \beta}{al + \alpha - \beta}.$$

We say the number $T(z_0, v(\cdot))$ a guaranteed time of l-approach.

Keywords: Differential game, pursuer, evader, l-approach, guaranteed time of l-approach.

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ON THE POSSIBILITY OF CREATING A THERMAL FIELD USING ELECTROMAGNETIC FIELDS

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In works [1]-[4], we consider the possibility of using a laser heat source to create the necessary thermal field in given areas of the body. This need arises due to the fact that cancer cells die at a temperature of approximately 46 degrees Celsius, while most normal cells remain alive. It is known that the thermal field satisfies the parabolic equation, for which the maximum principle is satisfied - according to which the maximum and minimum are reached at the boundary.

Using "needles that bring heat or toxic chemicals" into the body to kill cancer cells certainly requires going inside. But it is possible to use electromagnets and create the desired thermal field. To do this, at the beginning we write down the system of equations of electromagnetic hydrodynamics we need. Then you can control the initial and boundary conditions. Such a problem is mathematically quite solvable. The use of electromagnetic fields to create a thermal field does not require entering the inside of the body (to bring heat or "chemistry" inside).

Keywords: heat equation, electromagnetic fields, laser heat source, boundary value problem.

AMS Subject Classification: 35R30, 35K05, 49N45, 47A05.

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BOUNDARY VALUE PROBLEMS FOR THE LAPLACE EQUATION IN THE HYPEROCTANT OF A MULTIDIMENSIONAL BALL

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The present work is devoted to the studying of a boundary value problems with the Dirichlet conditions on the lateral faces of the ball's hyperoctant and on the sphere's hyperoctant for multidimensional Laplace equation

$$\Delta u \equiv \frac{\partial^2 u}{\partial x_1^2} + ... + \frac{\partial^2 u}{\partial x_m^2} = 0$$

in the m-dimensional Euclidean space $\mathbb{R}^m$, where $m \geq 2$; $u = u(x)$ is the desired (required) function of variables $x = (x_1, ..., x_m)$.

Let $\Omega_0^m \subset \mathbb{R}^m$ be a ball of radius R centered at the origin and S_0^m be a boundary of the ball Ω_0^m. We denote the first 2^{-n}th part of the sphere S_0^m by S_n^m and call it the S_n^m-sphere:

$$S_n^m = \left\{ x : \, x_1^2 + ... + x_m^2 = R^2, \; x_1 > 0, ..., x_n > 0; 1 \leq n \leq m \right\}.$$

Denote a finite domain bounded by planes $x_1 = 0$, ..., $x_n = 0$ and the S_n^m-sphere by Ω_n^m which has n lateral faces:

$$D_{n,p}^m = \left\{ x : \sum\nolimits_{i=1, \, i \neq p}^{m} x_i^2 < R^2, \; x_1 > 0, ..., x_{p-1} > 0, \; x_{p+1} > 0, ..., x_n > 0 \right\}, \; p = 1, 2, ..., n.$$

The Dirichlet problem D_n^m. To find a harmonic solution $u(x, y) \in C\left(\overline{\Omega_n^m}\right) \cap C^2\left(\Omega_n^m\right)$ of Laplace equation $\Delta u = 0$, satisfying conditions:

$$u|_{x_p=0} = \tau_p\left(\tilde{x}_p\right), \; \tilde{x}_p \in \overline{D_{n,p}^m}, \; p = 1, 2..., n; \;\; u(x)|_{S_n^m} = \varphi(x), \; x \in \overline{S_n^m},$$

where $\tilde{x}_p = (x_1, ..., x_{p-1}, x_{p+1}, ..., x_m)$; $\tau_p\left(\tilde{x}_p\right)$ and $\varphi(x)$ are given sufficiently smooth functions with $\tau_p\left(\tilde{x}_p\right)|_{S_n^m} = \varphi\left(\tilde{x}_p\right)$ $(p = 1, 2, ..., n)$.

The Holmgren problem H_n^m. To find a harmonic solution $u(x, y) \in C\left(\overline{\Omega_n^m}\right) \cap C^2\left(\Omega_n^m\right)$ of Laplace equation $\Delta u = 0$, satisfying conditions:

$$u_{x_p}\big|_{x_p=0} = \nu_p\left(\tilde{x}_p\right), \; \tilde{x}_p \in D_{n,p}^m, \; p = 1, 2..., n; \;\; u(x)|_{S_n^m} = \varphi(x), \; x \in S_n^m,$$

where $\nu_p\left(\tilde{x}_p\right)$ and $\varphi(x)$ are given sufficiently smooth functions.

The main result is a proof of the unique solvability of the problems considered. An energy integral method and Green's function method were used as the main tools in the proof of the main result. All solutions are found in an explicit forms, which contain fundamental (singular) solution of Laplace equation.

Keywords: Laplace equation, fundamental (singular) solution; Green function; Dirichlet problem; Holmgren problem; hyperoctant of the ball.

AMS Subject Classification: 35A08, 35J05, 35J08, 35J25.

SOLUTION OF A PARABOLIC PROBLEM IN AN EVOLVING DEGENERATE DOMAIN

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We consider the following two-dimensional boundary problem on spatial variables in the domain $Q = \left\{(x, y, t) \mid \sqrt{x^2 + y^2} < kt^{\omega}, \omega > \frac{1}{2},\ t > 0\right\}$ with a lateral surface

$\Gamma = \left\{(x, y, t) \mid \sqrt{x^2 + y^2} = kt^{\omega},\ t > 0\right\}$ for the equation

$$\frac{\partial u}{\partial t} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - a^2\beta \left(\frac{1}{x} \cdot \frac{\partial u}{\partial x} + \frac{1}{y} \cdot \frac{\partial u}{\partial x}\right) \tag{1}$$

with a boundary condition

$$u\left(x, y, t\right)|_{\Gamma} = g\left(x, y, t\right) \tag{2}$$

where $0 < \beta < 1$, $g(x, y, t)$ – is a given function. It is necessary to find a function $u(x, y, t)$, satisfying the equation (1) in Q and the boundary condition (2).

For the problem (1)–(2), we proved the following theorem.

Theorem 0.1. *If the conditions $g \in L_{\infty}(Q)$, $(0 < \beta < 1)$, are satisfied, then the boundary value problem (1) – (3) has a solution $u(x, y, t) \in L_{\infty}(G)$.*

The results of this work will be used in solving a similar problem in a funnel-shaped degenerate domain, that is, when the boundary of the domain changes according to the law $r = \gamma(t)$, $\gamma(0) = 0$.

Keywords: heat equation, boundary value problem, degenerate domain, Volterra singular integral equation, regularization.
AMS Subject Classification: 35K05, 45D99.

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PROPERTIES OF SOLUTIONS OF MULTIVARIATE INTEGRO-FUNCTIONAL EQUATIONS OF VOLTERRA-FREDHOLM TYPE IN THE SPACE OF $C[-1, 1+a]$

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In this work in the following space

$$C[-1, 1+a], [-1, 1+a] = [-1, 1+a_1] \times \cdots \times [-1, 1+a_m],$$

sufficient conditions are found for the unique solvability and continuous dependence of the solution on the parameters of multivariate integro-functional equation of Volterra-Fredholm type in the space of continuous functions

$$x(t) = F(t, x(t), \int_{a_1-t_1}^{t_1} \cdots \int_{a_p-t_p}^{t_p} \int_{-1}^{1+a_{p+1}} \cdots \int_{-1}^{1+a_{m+1}} K(t, s, x(s)) ds_1 \cdots ds_1)$$

where $t = (t_1, t_2, \cdots, t_m), s = (s_1, s_2, \cdots, s_m), a = (a_1, a_2, \cdots, a_m) \geq 0.$

Keywords: integro-functional equations, fixed point theorem, inequalities, Multiple integral transforms

AMS Subject Classification: 45B05, 34A12, 45D05, 45D99

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ON THE COMPLETENESS OF THE ROOT VECTORS OF THE SINGULAR OPERATOR GENERATED BY THE LINEAR PART OF THE KORTEWEG-DE VRIES OPERATOR

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Considerable literature [1-5] and works cited there are devoted to the issues of solvability of boundary value problems for differential equations of odd order and, in particular, for the Korteweg-de Vries equation. In contrast to these interesting papers, in this paper, we consider the operator generated by the linear part of the Korteweg-de Vries operator in $L_2(\Omega)$. For this operator in an unbounded domain with an unbounded coefficient, the following questions will be studied:

- the existence of a resolvent;
- separability (maximum regularity of solutions);
- compactness of the resolvent;
- Estimates of singular numbers.

In addition to the above questions, the paper investigates the question of the completeness of root vectors.

Keywords: compactness, resolvent, root vectors, singularity, The Korteweg-de Vries operator.

AMS Subject Classification: 35Q53

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ON A SPECTRAL PROBLEM FOR A BIHARMONIC OPERATOR IN A RECTANGULAR DOMAIN

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In rectangular region $\Omega_a = \{x, y| - 1 < x < 1, -a < y < a, \ 0 < a = \text{const} \leq 1\}$ with boundary $\partial\Omega_a$ we study the spectral problem:

$$\Delta^2 w = \lambda^2(-\Delta w), \ (x, y) \in \Omega_a, \tag{1}$$

$$w = 0, \ \partial_{\vec{n}} w = 0, \ (x, y) \in \partial\Omega_a, \tag{2}$$

where $\Delta = \partial_x^2 + \partial_y^2$ is the Laplace operator, $\vec{n}$ is the unit vector of the outward normal to $\partial\Omega_a$, excluding the vertices of the rectangle Ω_a.

First, we consider the case of a square domain Ω_1, i.e. $a = 1$. In this case, problem (1)–(2) was the subject of study in works [1, 2], as well as monographs [3, 4]. It is called the problem of studying bending vibrations of a clamped square plate, in contrast to the problem for a fixed plate, where the second condition in (2) is replaced by the following:

$$\Delta w = 0, \ (x, y) \in \partial\Omega_1, \tag{3}$$

and the latter is called the basic spectral problem. The spectrum of the basic problem gives a lower bound for the eigenvalues of the original problem, which, generally speaking, can be quite rough. To improve this estimate, a family of special intermediate problems is used, which are obtained by successive "strengthening" of conditions (3) and obtaining conditions approximating the second condition from (2). This approach is called the Aronshine-Weinstein method of intermediate problems [3, 4]. Note that in [3, 4] the importance of problem (1)–(2) for structural mechanics, shipbuilding mechanics, etc. is also noted.

We arrived at problem (1)–(2) (exactly, without any artificial transformations) by studying the numerical solution of direct and inverse problems for the linearized two-dimensional Navier-Stokes equation. In this case, we used the well-known (for the two-dimensional case) concept of the stream function.

Earlier, the spectral problem (1)–(2) was studied by us in [5] for the case when the region Ω is a circle with unit radius.

Note. In ([3], chapter VII, item 2) the following is stated: "If the clamped plate has the shape of a square or any other shape other than a circle, then" ... eigenvalues ... "cannot be expressed exactly in terms of elementary or tabulated special functions."

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To formulate the basic problem, we will replace the second boundary conditions from (2) only partially. Namely, instead of conditions (2) we will have

$$\begin{cases} w(-1,y) = \partial_x w(-1,y) = w(1,y) = \partial_x w(1,y) = 0, \\ w(x,-1) = \partial_y^2 w(x,-1) = w(x,1) = \partial_y^2 w(x,1) = 0. \end{cases} \tag{6}$$

Secondly, the case $a \neq 1$, is considered separately, i.e. when the region Ω_a is a rectangle. Differences are noted in the lower estimates of the eigenvalues of problem (1)–(2) using intermediate spectral problems both in accordance with formulas (3) and – with formulas (6).

Keywords: Biharmonic operator, spectral problem, rectangular domain.

AMS Subject Classification: 35Q30, 35R30, 65N21

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ON THE SOLVABILITY OF THE BOUNDARY VALUE PROBLEM FOR A LOADED DIFFERENTIAL EQUATION WITH A DELAY ARGUMENT

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A linear two-point boundary value problem for a system of loaded differential equations with a delay is considered on the interval $(0, T)$

$$\frac{dx(t)}{dt} = A_1(t)x(t) + A_2(t)x(t-\tau) + \sum_{j=1}^{N} K_j(t)x(\theta_j) + f(t), \quad x \in R^n, \quad \theta_j \in (0, T), \quad \tau > 0, \tag{1}$$

$$x(t) = \text{diag}[x(0)] \cdot \varphi(t), \quad t \in [-\tau, 0], \tag{2}$$

$$Bx(0) + Cx(T) = d, g(x(0), \quad d \in R^n. \tag{3}$$

Here the $n \times n$ matrices $A_1(t)$, $A_2(t)$, $K_j(t), (j = \overline{1, N})$, and the vector function $f(t)$ are continuous on $[0, T]$, $\varphi : [-\tau, 0] \to R^n$ is a continuously differentiable vector function such that $\varphi_i(0) = 1$, $i = \overline{1, n}$; τ is a constant delay.

A solution to the boundary-value problem (1)-(3) is a continuous on $[-\tau, T]$, continuously differentiable on $(0, T)$ vector function $x^*(t)$, satisfying Eq.(1) and conditions (2), (3).

To solve the problem (1)-(3), the idea of the parameterization method [1] is used, namely: the interval at which the problem is being considered is divided into subintervals whose lengths do not exceed the values of the constant delay; constant parameters are introduced at the left ends of these intervals; a new unknown function is introduced at each subinterval. Thus, the problem under consideration is reduced to an equivalent multipoint boundary value problem for loaded differential equations with delay containing parameters. On each of the subintervals, the auxiliary Cauchy problems without delay with zero initial conditions at the left ends of the subintervals are considered sequentially. We propose an algorithm for finding the solution of a multipoint boundary value problem for loaded differential equations with delay containing parameters. At each step of the algorithm a system of linear algebraic equations is solved to determine the values of the parameters, and an analogue of the Cauchy formula is used to obtain solutions to auxiliary Cauchy problems.

Keywords: boundary value problem, loaded differential equation, delay argument, parameterization method.

AMS Subject Classification: 39A27

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ABELIAN TYPE INTEGRAL EQUATIONS WITH GAUSSIAN HYPERGEOMETRIC FUNCTION IN THE KERNEL AND THEIR APPLICATION TO THE BOUNDARY VALUE PROBLEMS

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Many problems of applied mathematics are reduced to the solution of integral equations with special functions in kernels, therefore the inversion formulas for such equations play an important role in solving various problems.

Theorem. Let $-1 < Re(\alpha) \leq 0$, $1 < Re(\beta) < 2$, $Re(\alpha+\beta) > 1$, $g(x) \in C^1[0,b]$, and $x^\alpha g(x) \to 0$ as $x \to 0$. Then the integral equation

$$\int_0^x (x-t)^{\beta-1} F\left(\alpha, 1-\alpha; \beta; \frac{x-t}{2x}\right) f(t)dt = x^\alpha g(x)$$

is invertible by

$$f(x) = \frac{\sin\beta\pi}{(1-\beta)\pi}\frac{d}{dx}\int_0^x (x-t)^{1-\beta} F\left(-\alpha, 1+\alpha; 2-\beta; \frac{t-x}{2t}\right) t^\alpha g'(t)dt$$

in the space of functions $f(x) \in C^1(0,b)$, such that $xf(x) \to 0$ as $x \to 0$.

Note, in case $0 \leq Re(\alpha) < Re(\beta) < 1$, $Re(\alpha+\beta) < 1$, a similar theorem was proved in [1].

In this work the inversion formula found is applied to solving Cauchy-Goursat problems for the generalized Euler-Poisson-Darboux equation with the negative parameters

$$\frac{\partial^2 u}{\partial\xi\partial\eta} + \left[\frac{q}{\eta+\xi} + \frac{p}{\eta-\xi}\right]\frac{\partial u}{\partial\xi} + \left[\frac{q}{\eta+\xi} - \frac{p}{\eta-\xi}\right]\frac{\partial u}{\partial\eta} = 0, \quad -1 < 2p < 2q \leq 0.$$

As an application of the results obtained explicit solutions of the problems posed are applied to finding functional relationships between the traces of the desired solution and its derivative brought to the line of degeneracy from the hyperbolic part of the mixed domain.

Keywords: integral equations with Gaussian hypergeometric function in the kernels, generalized Euler-Poisson-Darboux equation with the negative parameters, Cauchy-Goursat problems, generalized solutions;

AMS Subject Classification: 33C05, 35L80, 35Q05, 45D05.

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ON ONE SOLUTION OF AN INITIAL-BOUNDARY VALUE PROBLEM FOR A NONLINEAR DIFFERENTIAL EQUATION IN PARTIAL DERIVATIVES OF THE THIRD ORDER

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On $\Omega = [0, X] \times [0, Y]$ we consider a nonlocal boundary value problem for the nonlinear equation

$$\frac{\partial^3 w}{\partial x^2 \partial y} = \frac{\partial w}{\partial y} + w\frac{\partial w}{\partial x} + \frac{\partial w}{\partial x}, \quad (x, y) \in \Omega, \tag{1}$$

$$w(x, 0) = \varphi(x), \quad x \in [0, X], \tag{2}$$

$$\frac{\partial w(0, y)}{\partial y} = \alpha(y)\frac{\partial w(X, y)}{\partial y} + \psi(y), \quad y \in [0, Y], \tag{3}$$

$$\frac{\partial w(0, y)}{\partial x} = \theta(y), \quad y \in [0, Y], \tag{4}$$

where the functions $\psi(y), \theta(y)$ are continuously differentiable on $[0, Y]$, the function $\varphi(x)$ is continuously differentiable on $[0, X]$, $\alpha(y) \neq 1$.

Let $C(\Omega, R)$ - be the set of functions $w : \Omega \to R$ continuous on Ω.

A function $u(x, y) \in C(\Omega, R)$, with partial derivatives $\frac{\partial u(x,y)}{\partial x} \in C(\Omega, R)$, $\frac{\partial u(x,y)}{\partial y} \in C(\Omega, R)$, $\frac{\partial^2 u(x,y)}{\partial x \partial y} \in C(\Omega, R)$, $\frac{\partial^2 u(x,y)}{\partial x^2} \in C(\Omega, R)$, $\frac{\partial^3 u(x,y)}{\partial x^2 \partial y} \in C(\Omega, R)$ is called a solution to problem (1)–(4) if it satisfies equation (1), for all $(x, y) \in \Omega$, and boundary conditions (2)–(4).

In this paper, we investigate a nonlinear boundary value problem for a third-order pseudoparabolic equation or the Benjamin-Bona-Mahony equation with nonlocal conditions. An algorithm for searching for an approximate solution to this problem is proposed, and conditions for the convergence of the proposed algorithm are obtained.

Keywords: partial differential equation, Benjamin-Bona-Mahony equation, algorithm, approximate solution.

AMS Subject Classification: 35G31

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APPLICATION OF THE GRAPH METHOD IN SOLVING DIFFERENTIAL EQUATIONS

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Differential equations encountered in various applications can be interpreted as equations on graphs. Boundary value problems on graphs are defined by spaces of functions defined on graphs and differential systems. In this paper, we consider the transformation of a boundary value problem on a graph into a boundary value problem for a differential system; the differential equation on the graph is redefined by converting each edge of the graph into an interval (0, 1). The boundary conditions are also transformed into intervals, respectively, and a connection is established between the initial boundary value problem and the newly obtained boundary value problem. This allows us to consider a boundary value problem in a graph as a problem of finding the eigenvalue of a differential operator in a graph and to investigate its spectral structure.

Let the edges of G be graphs which length is finite, and their number is finite K. Denote the edges $e_i,\ i=1,...,K$ and their lengths, respectively, $l_i,\ i=1,...,K$. Consider a second-order differential operator on the graph G:

$$ly := -\frac{d^2y}{dx^2}+q(x)y=\lambda y, \tag{1}$$

where $q(x)$ is the actual function defined in G (and beyond). Equation (1) can be considered as system

$$-\frac{d^2y_i}{dx^2}+q_i(x)y_i=\lambda y_i, \quad x\in(0,l_i),\ i=1,...,K,$$

where $q_i = q\mid_{e_i}, y_i = y\mid_{e_i}$.

The representation of self-reports in graphs is considered in [1] and the class of self-related boundary conditions is described in [2] and [3].

Keywords: graph, differential equation, boundary value problem, differential operator, self-adjoint operator, eigenvalue.

AMS Subject Classification: 39

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ON A SYSTEM OF EIGENFUNCTIONS OF A REGULAR MULTIPLE DIFFERENTIATION OPERATOR UNDER INTEGRAL PERTURBATION OF ONE BOUNDARY VALUE CONDITION THAT DOES NOT HAVE BASIS PROPERTY

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We consider the spectral problem for the operator L_1 in $L_2(0,1)$ given by the expression:

$$l(u) \equiv -u''(x) = \lambda u(x), \quad 0 < x < 1, \tag{1}$$

and the "perturbed" boundary value conditions

$$\begin{gathered} U_1(u) \equiv u'(0) + u'(1) - \alpha u(1) = 0, \quad \alpha > 0, \\ U_2(u) \equiv u(0) = \int_0^1 P(x)u(x)dx, \quad P(x) \in L_2(0,1), \end{gathered} \tag{2}$$

that are regular but not strongly regular.

Feature of the considered problem is the absence of the basis property for the system of root vectors of the unperturbed problem $P(x) \equiv 0$. The characteristic determinant $\Delta_1(\lambda)$ of the spectral problem (1), (2) is represented as

$$\Delta_1(\lambda) = \Delta_0(\lambda)\left(1 + \alpha\left[\sum_{k=0}^{\infty}\alpha_k \cdot \left(\frac{C_k^{(2)}}{\lambda - (2\beta_k)^2} + \frac{C_k^{(1)}}{\lambda - ((2k+1)\pi)^2}\right) + \sum_{k=1}^{\infty}\alpha_k \cdot 2\delta_k \cdot \frac{C_k^{(2)}}{\lambda - (2\beta_k)^2}\right]\right),$$

where $\Delta_0(\lambda) = \alpha \sin\sqrt{\lambda} - \sqrt{\lambda}(1 + \cos\sqrt{\lambda})$ is the characteristic determinant of the unperturbed problem $P(x) \equiv 0$.

It is proved that the set of functions $P(x)$, for which the system of eigenfunctions of the problem (1), (2) does not form an unconditional basis in $L_2(0,1)$, is dense in $L_2(0,1)$. It is shown that the adjoint operator has a similar structure and is a loaded differential operator of the second order.

Keywords: differential operator, eigen functions, basis property, root vectors, spectral problem.

AMS Subject Classification: 34L10

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PROPERTIES OF THE SOLUTION OF SOME MULTIPOINT PROBLEMS FOR THE ANNUAL TRANSMISSION EQUATION

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In the task [1]

$$\frac{\partial^2 x(t,s)}{\partial t^2} = f\left(t, s, x(t,s)\right) + \int_c^d n\left(t, s, \delta, x(t,\delta)\right) d\delta + g(t,s)$$

for the equation

$$x(a,s) = \phi(s), \qquad x_t^{'}(a,s) = \psi(s)$$

the existence of a single solution satisfying the initial condition is studied.

In this task

$$\frac{\partial^2 u(x,t)}{\partial t^2} + a\frac{\partial u(x,t)}{\partial t} = f\left(x, t, u(x,t)\right) + \int_0^l K\left(x, t, \tau, u(\tau,t)\right) d\tau, (x,t) \in [0,l] \times [0,T] = D \tag{1}$$

of the equation

$$u(x,0) + \sum_{i=1}^{P} \alpha_i u\left(x, t_i\right) = \phi(x), \frac{\partial u(x,0)}{\partial t} + \sum_{i=1}^{P} \beta_i \frac{\partial u\left(x,t_i\right)}{\partial t} = \psi(x), \quad 0 \le x \le l, \tag{2}$$

sufficient conditions are found for the existence and uniqueness of the solution satisfying the multipoint conditions, and the continuous dependence of the solution on the parameters, thus

$$1 + \sum_{i=1}^{P} \alpha_i \left(1 + at_i\right) = A \neq 0, 1 + \sum_{i=1}^{P} \beta_i \left(1 - at_i\right) = B \neq 0{,}1 + \frac{a^2}{\mathrm{AB}} \sum_{i=1}^{P} \alpha_i t_i \sum_{j=1}^{P} \beta_j t_j = C \neq 0.$$

Keywords: integro-functional equations, fixed point theorem, inequalities, Multiple integral transforms

AMS Subject Classification: 45B05, 34A12, 45D05, 45D99

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A COMPLEX PROBLEM FOR THE VOLTERRA-FREDHOLM DIFFERENTIAL INTEGRODIFFERENTIAL EQUATION

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In the task [1]

$$x^{'}(t) = F\left(t, x(t), \int_0^t K\left(t, s, x(s)\right) \mathrm{ds}\right), \qquad t \in [0;\ +\infty)$$
$$\sum_{i=1}^{P} \alpha_i x\left(t_i\right) = A \tag{1}$$

the existence and uniqueness of the solution of the problem and the continuous dependence of the solution on the parameters are studied.

In the task

$$x^{'}(t) = f\left(t, x(t), \int_0^t K_0\left(t, s, x(s)\right) ds, \int_0^{t_1} K_1\left(t, s, x(s)\right) ds, ..., \int_0^{t_p} K_p\left(t, s, x(s)\right) \mathrm{ds}\right), \tag{2}$$
$$(0 \le t \le T),$$

$$\sum_{i=1}^{p} \alpha_i x\left(t_i\right) = A \tag{3}$$

sufficient conditions are found for the existence of a unique solution to the problem and the continuous dependence of the solution on the parameters, here

$t_i \in [0, T]$ $\left(i = \overline{1, p}\right)$ are free points, $\sum_{i=1}^{p} \alpha_i = B \neq 0$.

(1) equation is the usual state of the (2) equation. If

$$f\left(t, x, u_0, u_1, ..., u_p\right) = F\left(t, x, u_0\right)$$

then (2) equation gives (1) equation. The results obtained in the [2] work are directly derived from the results obtained in this study. Therefore, it is appropriate to consider (2), (3) works as well.

Keywords: integro-functional equations, fixed point theorem, inequalities, Multiple integral transforms

AMS Subject Classification: 45B05, 34A12, 45D05, 45D99

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SOLUTION OF SOME NON-LOCAL PROBLEM FOR MIXED EQUATION WITH SECOND ORDER

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It is considered hyperbolic equation

$$u_{xx} - u_{yy} = 0, \quad y < 0. \tag{1}$$

in the area which is bounded by characteristics

$$x + y = 0, \quad x - y = 1 \quad \text{and} \quad y = 0 \tag{2}$$

Theorem 1. The hyperbolic equation "(1)" with characteristics "(2)" can be solved with non-local boundary conditions

$$\begin{cases} u(x;0) = \tau(x) \\ \sum_{i=0}^{\lfloor\frac{n-1}{2}\rfloor} u(\frac{x+2i}{2n}; -\frac{x+2i}{2n}) + \sum_{i=0}^{\lfloor\frac{n-2}{2}\rfloor} u(\frac{-x+2n-2i-1}{2n}; -\frac{x+2i+1}{2n}) = \phi(x) \end{cases}$$

Keywords: Elliptic-Hyperbolic, Parabolic-Hyperbolic, non-local problems for mixed equation, micro-periodic functional equations.

AMS Subject Classification: 35L51, 35M30, 35M32

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INVERSE PROBLEM FOR THE SUBDIFFUSION EQUATION

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The inverse problem of determining the right-hand side of the subdiffusion equation with the fractional Caputo derivative is considered. The right-hand side of the equation has the form $f(x)g(t)$ and the unknown is function $f(x)$. The condition $u(x, t_0) = \psi(x)$ is taken as the over-determination condition, where t_0 is some interior point of the considering domain and $\psi(x)$ is a given function. It is proved by the Fourier method that under certain conditions on the functions $g(t)$ and $\psi(x)$ the solution of the inverse problem exists and is unique. An example is given showing the violation of the uniqueness of the solution of the inverse problem for some sign-changing functions $g(t)$. For such functions $g(t)$, we find necessary and sufficient conditions on the initial function and on the function from the over-determination condition, which ensure the existence of a solution to the inverse problem. This is a joint work with Marjona Shakarova of Institute of Mathematic.

Keywords: forward and inverse problems, the Caputo derivatives, Fourier method.

ON A NONLOCAL PROBLEM FOR A MIXED TYPE EQUATION WITH A FRACTIONAL ORDER OPERATOR

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Let $\Omega = \{(x,t) : 0 < x < 1,\, -a < t < b\}$, $\Omega_1 = \Omega \cap (t > 0)$, $\Omega_2 = \Omega \cap (t < 0)$, where a, b are positive real numbers. In the domain Ω, we consider the following nonlocal problem:

Problem A. It is required to find a function $u(x,t)$, from the class

$$u, u_x \in C(\bar{\Omega}_1), {}_C D^{\alpha}_{0+} u, u_{xx} \in C(\Omega_1), u \in C^1(\bar{\Omega}_2) \cap C^{2,2}_{x,t}(\Omega_2),$$

which satisfies the following equation in $\Omega_1 \cup \Omega_2$

$$0 = \begin{cases} {}_C D^{\alpha}_{0t} u(x,t) - u_{xx}(x,t), t > 0, \\ u_{tt}(x,t) - u_{xx}(x,t), t < 0, \end{cases}$$

with boundary value conditions

$$u(0,t) = 0, u_x(1,t) = u_x(x_0,t), -a \le t \le b,$$

$$u(x,-a) = h \cdot u(x,b) + \varphi(x),\ \ 0 \le x \le 1,$$

and also the gluing condition $\lim\limits_{t\to+0} {}_C D^{\alpha}_{0t} u(x,t) = \lim\limits_{t\to-0} u_t(x,t)$.

Here is a given function, are given real numbers, ${}_C D^{\alpha}_{0t}, 0 < \alpha \le 1$ is the Caputo operator [1]. In this paper, we study the solvability of one boundary value problem with nonlocal Bitsadze-Samarskii type conditions for a fractional parabolic-hyperbolic equation of mixed type [2]. The problem is solved by using the variable separation method. Theorems on the existence and uniqueness of solutions to the considered problem are proved.

Keywords: mixed type equation, Bitsadze-Samarskii type problem, equation of fractional order, Mittag-Leffler function

AMS Subject Classification: 34K37, 34L10, 35M12

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THE CAUCHY PROBLEM FOR THE MODIFIED KORTEWEG-DE VRIES EQUATION WITH FINITE DENSITY IN THE CLASS OF PERIODIC FUNCTIONS

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Abstract. In this thesis, the method of the inverse spectral problem is used to integrate the nonlinear modified Korteweg-de Vries equation with finite density in the class of periodic functions.

Statement of the problem. Consider the Cauchy problem for the modified Korteweg-de Vries equation with finite density of the following form

$$\begin{cases} q_t = q_{xxx} - 6(q^2 - \rho^2)q_x, \ 0 < \rho < \infty \\ q(x,t)|_{t=0} = q_0(x), q_0(x+\pi) = q_0(x) \in C^5(\mathbb{R}) \end{cases} \tag{1}$$

in the class of real infinite-gap π periodic with respect to x functions:

$$\begin{cases} q(x+\pi,t) = q(x,t), \ x \in \mathbb{R}, \, t > 0 \\ q(x,t) \in C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0) \end{cases}. \tag{2}$$

In this paper, we propose an algorithm for constructing exact solution $q(x,t), x \in \mathbb{R}, t > 0$ of problem (1)-(2) by reducing it to an inverse spectral problem for the following Dirac operator:

$$\mathfrak{L}(\tau,t) \equiv B\frac{dy}{dx} + \Omega(x+\tau,t)y = \lambda y, x \in \mathbb{R}, \tau \in R, t > 0, \tag{3}$$

where

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \Omega(x+\tau,t) = \begin{pmatrix} 0 & q(x+\tau,t) \\ q(x+\tau,t) & 0 \end{pmatrix}, \ y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}.$$

In this thesis proved the following theorem:

Theorem. If initial function $q_0(x)$ satisfies the following condition:

$$q_0(x+\pi) = q_0(x) \in C^5(\mathbb{R})$$

then the problem (1)-(2) has unique solution, which defined sum of a uniformly convergent functional series constructed by solving the system of Dubrovin equations and the first trace formula satisfies the mKdV equations with finite density and belongs to the class of $C_x^3(t > 0) \cap C_t^1(t > 0) \cap C(t \geq 0)$.

Keywords: modified Korteweg-de Vries equation with finite density, Dirac operator, spectral data, system of Dubrovin differential equations, trace formulas.

AMS Subject Classification: 35J10, 34A55.

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EXPLICIT SOLUTIONS OF THE MASS SCHRÖDINGER DIFFERENTIAL EQUATION

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1. Introduction

In this article, we will use the fractional operator to derive explicit solutions of the effective mass Schrödinger equation, a second-order singular differential equation. These solutions will be reached using certain well-known fractional calculus methods, including Leibniz's rule, index law, and power rule [1],[3].

2. Mathematical analysis

Think about the effective mass Hamilton in one dimension by [2]:

$$\mathscr{H}_{eff} = -\frac{d}{dr}\left(\frac{1}{\mu(r)}\frac{d}{dr}\right)\nu_{eff}(r), \tag{1}$$

where ν_{eff} has the form

$$\nu_{eff} = \nu(r) + \frac{1}{2}(\tau+1)\frac{\mu''}{\mu^2} - \left[\gamma\left(\gamma+\tau+1\right)+\tau+1\right]\frac{(\mu')^2}{\mu^3},$$

where γ and τ are uncertainty parameters and prime numbers. Then the Schrödinger equation takes the form:

$$\left(-\frac{1}{\mu}\frac{d^2}{dr^2} + \frac{\mu'}{\mu^2}\frac{d}{dr} + \nu_{eff} - \epsilon\right)\phi(r) = 0.$$

With the help of appropriate transformations, the second order efficient Mass Schrödinger equation with singular coefficients is obtained.

Keywords: Fractional analysis; Fractional operatör; Mass Schrödinger equation; Differential equations.

AMS Subject Classification: 26A33, 34A08

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ON A NONLOCAL PROBLEM FOR HYPERBOLIC EQUATIONS WITH PIECEWISE CONSTANT ARGUMENT OF GENERALIZED TYPE

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Differential equations with piecewise constant argument arise in the mathematical modeling of diverse processes in biology, chemistry, mechanics, electronics, etc. A new class of differential equations with piecewise constant argument was proposed by M.Akhmet [1]. He suggested the delay argument to be an arbitrary piecewise constant function as opposed to the greatest integer function. Thus, differential equations with piecewise constant argument of generalized type have proven to be more suitable models for studying and solving various application problems, including neural networks, discontinuous dynamical systems, hybrid systems, etc.

In our talk, we present an algorithm for solving a nonlocal problem for a system of second-order hyperbolic equations with piecewise constant time argument of generalized type [2]. The method we use is based on the introduction of functional parameters that are set as the values of the desired solution along the lines of the domain partition with respect to the time variable. With the aid of the functional parameters and new unknown functions, the considered problem is reduced to an equivalent problem for a system of hyperbolic equations with data on the interior partition lines and functional relations with respect to the introduced parameters. We developed a two-stage procedure to approximately solve the latter problem: firstly, we solve an initial-value problem for a system of differential equations in functional parameters; then, we solve a problem for a system of hyperbolic equations in new unknown functions with data on the interior partition lines. We derived some conditions for the convergence of approximate solutions to the exact solution of the problem under study in terms of input data and proved that these conditions guarantee the existence of a unique solution of the equivalent problem. Finally, we established coefficient conditions for the unique solvability of the nonlocal problem.

Keywords: Hyperbolic equation, piecewise-constant argument of generalized type, nonlocal problem, functional parameter, solvability conditions.

AMS Subject Classification: 35L53, 35L69, 35Q92, 39B72, 92B20, 34K43.

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CONSTRUCTION OF AUTOMATIC CONTROL SYSTEMS ACCORDING TO A GIVEN MANIFOLD

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Consider the problem of construction of control automatic systems by given $(n-s)$-dimensional program manifold $\Omega(t) \equiv \omega(t,x) = 0$, in the following form [1]:

$$\dot{x}(t) = f(t,x) - B_1\xi, \quad \xi = \varphi(\sigma), \quad \sigma = P^T\omega, \quad t \in I = [0,\infty), \tag{1}$$

where $x \in R^n$ is a state vector of the object, $f \in R^n$ is a vector-function, satisfying to conditions of existence of a solution $x(t) = 0$, and $B_1 \in R^{n\times r}, P \in R^{n\times r}$ are matrices, $\omega \in R^s (s \leq n)$ is a vector, $\xi \in R^r$ is a differentiable on σ vector-function, satisfying of local quadratic connection

$$\begin{gathered}\varphi(0) = 0 \wedge 0 < \sigma^T\varphi(\sigma) \leq \sigma^T K\sigma, \quad \forall\sigma \neq 0,\\ K_1 \leq \frac{\partial\varphi(\sigma)}{\partial\sigma} \leq K_2, \{K = K^T > 0\} \in R^{r\times r}, K_i = K_i^T > 0.\end{gathered} \tag{2}$$

This problem reduce to investigation of quality properties of the following system with respect to vector-function ω [2-3]:

$$\dot{\omega}(t) = A\omega - B\xi, \quad \xi = \varphi(\sigma), \quad \sigma = P^T\omega, \quad t \in I = [0,\infty), \tag{3}$$

Here nonlinearity satisfies to generalized conditions (2), and

$$F(t,x,\omega) = -A\omega, \quad A \in R^{s\times s}, \quad H = \frac{\partial\omega}{\partial x}, \quad B = HB.$$

Using the Lyapunov function "quadratic form plus the integral of nonlinearity", sufficient conditions for the absolute stability of the program manifold with respect to a given vector function ω are obtained.

Keywords: Stability, program manifold, control systems, Lyapunov function.

AMS Subject Classification: 34K20, 34K32, 93C15

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ON THE INFLUENCE OF PERTURBATIONS OF THE VOLTERRA TYPE TO THE BOUNDEDNESS OF SOLUTIONS OF FIRST-ORDER LINEAR DIFFERENTIAL EQUATIONS ON THE SEMI-AXIS

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All functions and their derivatives are continuous and the relations take place at $t \geq t_0$, $t \geq \tau \geq t_0$.

The problem. Establish sufficient conditions for the boundedness of all solutions on the half-interval $I = [t_0, -\infty)$ of linear implicit integro-differential equation of Volterra type:

$$x'(t) + a(t)x(t) + \int_{t_0}^{t} [K(t,\tau) + Q(t,\tau)]\, x'(\tau)d\tau = f(t) + q(t), \quad t \geq t_0 \tag{1}$$

in case, when all nonzero solutions of corresponding linear homogeneous differential equation:

$$x'(t) + a(t)x(t) = 0, \quad t \geq t_0$$

and all solutions of linear nonhomogeneous differential equation:

$$x'(t) + a(t)x(t) = f(t), \quad t \geq t_0$$

may are unbounded on I.

For kernel $K(t,\tau)$ and free member $f(t)$ is applied the method of cutting functions (Iskandarov S., 1980) and for kernel $Q(t,\tau)$ and free member $q(t)$ – the method of partial cutting (Iskandarov S., Shabdanov D.N., 2007).

An illustrative example will be constructed. The technology of applying the method of cutting functions and the method of partial cutting are reflected in lemmas 1 and 2, respectively, in article [1].

Keywords: integro-differential equation, differential equation, perturbation, influence.

AMS Subject Classification: 34K20, 45J05

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A CRITERION FOR CORRECT SOLVABILITY OF A FIRST ORDER QUASILINEAR THE VERHULST LOGISTIC GROWTH DIFFERENCE EQUATION

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Abstract. In this work we consider the first order quasilinear the Verhulst logistic growth difference equation:

$$-\frac{1-s_{n+1}}{s_{n+1}}+(1+r_n)\frac{1-s_n}{s_n}=f_n, n=0,\pm1,\pm2,\dots,$$

where $r_n\geq 0; (1-s)/s=\{(1-s_n)/s_n\}_{n=0,\pm1,\pm2,\dots}, 0<s<1$ values of stock of logistic growth, endogenous factor, desired solution of the equation; $f=\{f_n\}_{n=0,\pm1,\pm2,\dots}\in\ell_p, 1\leq p\leq\infty$ values of logistic growth flow, exogenous factor, right side of the equation and

$$\forall t=1,2,\dots<\infty|\prod_{j=n}^{n+t-1}(1+r_j)<\infty.$$

The equation is called correctly solvable in the given space $\ell_p, 1\leq p\leq\infty$ if for any $f\in\ell_p$ there is a unique solution $(1-s)/s\in\ell_p$ and the following inequality $||(1-s)/s||\ell_p\leq c(p)||f||_{\ell_p}$ for all $f\in\ell_p$ holds with absolute constant $0<c(p)<\infty$. We find a criterion for correct solvability of the above difference equation in space ℓ_p.

Keywords: The Verhulst logistic growth; the first order quasilinear; the difference equation; a criterion; correctly solvable; a unique solution.

ON THE UNIQUE SOLVABILITY OF A NONLOCAL BOUNDARY VALUE PROBLEM FOR FOURTH-ORDER HYPERBOLIC EQUATION WITH IMPULSIVE ACTIONS

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Many real processes and phenomena studied in economics, technology, biology, and mechanics are characterized by the fact that at certain moments of their development, the parameters of the system undergo rapid changes. When constructing mathematical models of such processes and phenomena, such systems arise, the solutions of which, generally speaking are discontinuous functions. Such objects are usually called impulse systems. During the last three decades, the theory of impulsive partial differential equations has been intensively studied [1-7]. In this article, we study the unique solvability of a nonlocal boundary value problem for a fourth-order hyperbolic equation with impulsive actions. Using the integral representation of the desired function, the formulated nonlocal boundary value problem is reduced to an equivalent integral equation. Next, using a priori estimates, we prove the uniqueness of the solution of the integral equation and an unique solvability of the formulated nonlocal boundary value problem.

Keywords: fourth-order hyperbolic equations, impulsive partial differential equations, non-local boundary, impulsive actions, impulse systems.

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THE SOLUTION OF THE CAUCHY PROBLEM FOR A LOADED MKDF EQUATION WITH A GENERAL SELF-CONSISTENT SOURCE

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In this paper, we consider the following system of equations

$$u_t + \beta(t)u(x_0,t)(6u^2u_x + u_{xxx}) + \gamma(t)u(x_1,t)u_x = \sum_{k=1}^{2N}(f_{k1}g_{k1} - f_{k2}g_{k2}) + i\int_{-\infty}^{+\infty}(\phi_1^2 - \phi_2^2)\,d\eta,$$

$$L(t)f_k = \xi_k f_k, \quad L(t)g_k = \xi_k g_k, \quad k = 1,2,...,2N, \tag{1}$$

where $L(t) = i\begin{pmatrix} \frac{d}{dx} & -u(x,t) \\ -u(x,t) & -\frac{d}{dx} \end{pmatrix}$ and $\beta(t)$,$\gamma(t)$ given continuously differentiable functions. The system of equations (1) is considered under the initial condition

$$u(x,0) = u_0(x),\ x \in R^1, \tag{2}$$

where initial function $u_0(x)$ $(-\infty < x < \infty)$ has the following properties:

1)

$$\int_{-\infty}^{\infty}(1+|x|)\ |u_0(x)|\,dx < \infty. \tag{3}$$

2) The operator $L(0) = i\begin{pmatrix} \frac{d}{dx} & -u_0(x) \\ -u_0(x) & -\frac{d}{dx} \end{pmatrix}$ has exactly $2N$ simple eigenvalues $\xi_1(0), \xi_2(0), .., \xi_{2N}(0)$.

In the problem under consideration, $f_k = (f_{k_1},\, f_{k_2})^T$ is the eigenfunction of the operator $L(t)$ corresponding to the eigenvalue ξ_k and $g_k = (g_{k_1},\, g_{k_2})^T$ is the solution of the equation $Lg_k = \xi_k g_k$, for which

$$W\{f_k,\, g_k\} = f_{k_1}g_{k_2} - f_{k_2}g_{k_1} = \omega_k(t) \neq 0,\ \ k = \overline{1, 2N}, \tag{4}$$

where $\omega_k(t)$ are the initially given continuous functions of $t > 0$, satisfying the conditions

$$\omega_n(t) = -\omega_k(t)\ for\ \xi_n = -\xi_k, \mathrm{Re}\left\{\int_0^{\mathrm{t}}\omega_{\mathrm{k}}(\tau)\mathrm{d}\tau\right\} > -\mathrm{Im}\left\{\xi_{\mathrm{k}}(0)\right\}, k = \overline{1,\, N}, \tag{5}$$

for all non-negative values of t. For definiteness, we assume that in the sum in the right-handside of (1), the terms with $\mathrm{Im}\,\xi_{\mathrm{k}} > 0,\ \mathrm{k} = \overline{1, \mathrm{N}}$ and $\phi = (\phi_1(x,\eta,t), \phi_2(x,\eta,t))$ required the following asimptotics for $x \to \infty$

$$\phi \to \begin{pmatrix} h\,(\eta,t)\ e^{-i\eta x} \\ h\,(\eta,t)\ e^{i\eta x} \end{pmatrix}, \tag{6}$$

where $h(\eta,t) = h(-\eta,t)$ continuous function and satisfy the following condition:

$$\int_{-\infty}^{\infty}|h(\eta,t)|^2\,d\eta < \infty\ for\ t \geq 0. \tag{7}$$

Let us assume that the function $u(x,t)$ has the required smoothness and rather quickly tends to its limits at $x \to \pm\infty$, i. e.,

$$\int_{-\infty}^{\infty}\left((1+|x|)\,|u(x,t)|+\sum_{k=1}^{3}\left|\frac{\partial^k u(x,t)}{\partial x^k}\right|\right)dx<\infty,\quad k=1,2,3. \tag{8}$$

The main purpose of this work is to obtain representations for the solution $u(x,t),\ f_k,\ g_k,\quad k=\overline{1,2N}$, $\phi_1(x,\eta,t)$,$\phi_2(x,\eta,t)$ of problem (1)-(8) in the framework of the inverse scattering method for the operator $L(t)$.

The main result of this work is the following theorem.

Theorem. If the functions $u\,(x,t),\ f_k(x,t),\ \ g_k(x,t),\ k=\overline{1,N},\ \phi_1(x,\eta,t),\ \phi_2(x,\eta,t)$ are a solution to the problem (1)-(8), then the scattering data of the operator $L(t)$ with the potential $u(x,t)$ satisfy the following differential equations

$$\frac{dr^+}{dt}=\left[8\,i\xi^3\beta(t)u(x_0,t)+\sum_{k=1}^{N}i\omega_k(t)\,\left(\frac{1}{\xi+\xi_k}+\frac{1}{\xi-\xi_k}\right)-2i\xi\gamma(t)u(x_1,t)\right]\,r^+ +$$

$$+\left[-2\pi i h^2(\xi,t)+\text{v.p.}\int_{-\infty}^{+\infty}\frac{2h^2(\eta,t)}{\eta+\xi}d\xi\right]\,r^+,\,(\mathrm{Im}\xi=0)$$

$$\frac{d\xi_n}{dt}=i\omega_n(t),\qquad n=\overline{1,N},$$

$$\frac{dC_n}{dt}=\left(8i\xi_n^3\beta(t)u(x_0,t)+i\beta_n(t)\omega_n(t)\right)\,C_n+$$

$$+\left(-2i\xi_n\gamma(t)u(x_1,t)+2\int_{-\infty}^{\infty}\frac{h^2(\eta,t)}{(1+r(\eta)r(-\eta))(\eta+\xi_n)}d\eta\right)C_n,\quad n=\overline{1,N}.$$

The obtained equalities completely determine the evolution of the scattering data, which makes it possible to apply the inverse scattering method to solve problem (1)-(8).

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Keywords: Jost solutions, eignvalue, eignfunction, evolution of scattering data, Gelfand-Levitan-Marchenko integral equation.

UPPER THRESHOLD ANALYSIS OF THE N-DIMENSIONAL ONE-PARTICLE DISCRETE SCHRO'DINGER OPERATORS WITH DELTA POTENTIALS

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On the right side of the essential spectrum of the n-dimensional one-particle discrete Schrödinger operator with $n+1$ delta potentials, eigenvalue behaviors is studied. It can be demonstrated that upper threshold resonance and upper threshold eigenvalue emerge for $n \geq 2$.

Theorem 1. Let $n \geq 2$.

(a) Assume that $(\lambda, \mu) \in D_k$, $k \in \{0, 1, 2, n, n+1\}$. Then $H^{\mathrm{e}}_{\lambda\mu}$ has k eigenvalues in $(2n, \infty)$. In addition $H^{\mathrm{e}}_{\lambda\mu}$ has neither a threshold eigenvalue nor a threshold resonance

(b) $2n$ is not a super-threshold resonance of $H^{\mathrm{e}}_{\lambda\mu}$ for any $(\lambda, \mu) \in \mathbb{R}^2$.

Keywords: Discrete Schrödinger operator, resonance, eigenvalue, Fredholm determinant, upper threshold eigenvalue.

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NECESSARY AND SUFFICIENT CONDITIONS FOR THE EXISTENCE OF AN "ISOLATED" SOLUTION OF A SEMIPERIODIC BOUNDARY VALUE PROBLEM FOR A NONLINEAR LOADED HYPERBOLIC EQUATION

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Problems for loaded hyperbolic equations have acquired particular relevance in connection with the study of the vibration stability of the wings of an aircraft loaded with masses, and in calculating the natural oscillations of antennas loaded with lumped capacitances and self-inductions [1]. Questions of existence and uniqueness of solutions to various problems for loaded partial differential equations of hyperbolic type and methods for finding their solutions are intensively studied by many authors [2-4].

In the $\bar{\Omega} = [0, \omega] \times [0, T]$, we consider a semiperiodic boundary value problem for a semilinear loaded hyperbolic equations:

$$\frac{\partial^2 u}{\partial x \partial t} = A(x,t)\frac{\partial u(x,t)}{\partial x} + A_0(x,t)\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=x_0} + f\Big(x,t,u(x,t),\frac{\partial u(x,t)}{\partial t}\Big), \tag{1}$$

$$u(x,0) = u(x,T), \qquad x \in [0,\omega], \tag{2}$$

$$u(0,t) = \psi(t), \qquad t \in [0,T], \tag{3}$$

In this report, we announce the receipt the necessary and sufficient conditions for the existence of an "isolated" solution of a semiperiodic boundary value problem for a nonlinear loaded hyperbolic equation with a mixed derivative in terms of initial data.

Keywords: Isolated solution, nonlinear hyperbolic equation, loaded hyperbolic equation, necessary and sufficient conditions.

AMS Subject Classification: The author(s) should provide AMS Subject Classification numbers using the link https://mathscinet.ams.org/msnhtml/msc2020.pdf.

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SOLUTION OF SYSTEMS OF HIGHER ORDER LINEAR ORDINARY DIFFERENTIAL EQUATIONS

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To integrate systems of linear ordinary differential equations of the first order, the method of Leonard Euler (1707 - 1783), based on the use of characteristic numbers of the matrix of system coefficients, and sometimes the method of integrable combinations of Jean Leron D'Alembert (1717 - 1783) are usually used. Combining these methods involves the use of characteristic numbers to obtain integrable combinations, as shown in the works of the authors, which creates a synergy effect - that can greatly simplify the solution process. It was previously shown by the authors that this effect is especially pronounced in the solution of inhomogeneous systems. In this paper, we show how a certain class of systems of linear higher-order differential equations can be integrated using the combined method, a system of the form

$$\begin{cases} y'' + sy' = ay + bz + f(x), \\ z'' + sz' = cy + dz + g(x), \end{cases}$$

where the coefficients a, b, c, d, p, q, s are constant numbers, and f and g are given functions.

Keywords: systems of linear ordinary differential equations; solution of systems of equations with constant coefficients, Euler's method, d'Alembert's method, higher-order equations, constant coefficients, synergy.

AMS Subject Classification: 34A05, 34A25, 34A30, 34A34.

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SYSTEMS OF PDE SATISFIED BY CONFLUENT HYPERGEOMETRIC FUNCTIONS OF THREE VARIABLES

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A hypergeometric functions in many variables are usually divided into two types: complete and confluent functions. A great merit in the development of the theory of the hypergeometric series in two variables belongs to Horn, who gave, in 1889, a general definition and order classification of double hypergeometric series. He has investigated the convergence of hypergeometric series of two variables and established the systems of partial differential equations which they satisfy. Horn investigated in particular hypergeometric series of order two and found that, apart from certain series which are either expressible in terms of one variable or are products of two hypergeometric series, each in one variable, there are essentially 34 (14 complete and 20 confluent) convergent series of order two.

Srivastava and Karlsson [1] have presented, in 1985, a table of 205 distinct complete triple Gaussian functions together with their sources, if known. Hasanov and Ruzhansky constructed, in 2019, Euler-type integral representations for all complete hypergeometric functions in three variables, compiled, in 2022, a systems of partial differential equations, which satisfy these functions, and found all linearly-independent solutions of each PDE-system near the origin, if exist.

In recent work [2], a table of 395 distinct confluent hypergeometric functions in three variables are presented. In present work, we will write 395 systems of partial differential equations, which satisfy new defined triple confluent hypergeometric functions, and find all linearly-independent solutions of each PDE-system near the origin, if exist.

Keywords: Complete and confluent hypergeometric functions in three variables; system of PDE; linearly-independent solutions of PDE-system;

AMS Subject Classification: Primary 33C20, 33C65; Secondary 44A45.

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DETERMINATION OF THE REGULARIZATION PARAMETER OF A NONLINEAR INTEGRAL EQUATION WITH AN INEXACT RIGHT HAND SIDE

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With the development of IT-technologies, in our time it becomes relevant to develop numerical methods for solving various problems suitable for application in applied purposes. In this case, it is important to ensure the required accuracy [2].

Methods Regularization refers to approximate methods for solving equations. In this paper, we study the question of choosing the regularization parameter for the solution of a nonlinear integral equation of the I-kind with two variable integration limits.

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NONLOCAL BOUNDARY VALUE PROBLEMS FOR FREDHOLM TYPE INTEGRO-DIFFERENTIAL EQUATIONS WITH PARAMETERS

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In this paper, we study a nonlocal nonhomogeneous problem for a second-order ordinary Fredholm integro-differential equation with a degenerate kernel, two nonhomogeneous integral conditions and two real parameters. The regular and irregular values of the parameters are calculated, for which the nonexistence, existence of a unique solution, and the existence of more solutions of the problem are established. The corresponding implicit forms of solutions are constructed.

Problem. It is required to find a function $u(t)$ on the interval $(0,T)$, that satisfies the nonhomogeneous equation

$$u''(t)+\lambda^2 u(t)=\nu\int\limits_0^T K(t,s)\left[s\,u(s)+(T-s)u'(s)\right]\,ds+f(t) \tag{1}$$

with the following two nonhomogeneous boundary conditions:

$$u(T)-\int\limits_0^T s\,u(s)\,ds=\varphi_1,\quad u'(T)-\int\limits_0^T (T-s)\,u'(s)\,ds=\varphi_2, \tag{2}$$

where $T>0$, λ is positive parameter, ν is nonzero real parameter, $f(t)\in C\,[0,T]$, $0\neq\varphi_1$, $\varphi_2=$ const, $K(t,s)=\sum\limits_{i=1}^k a_i(t)\,b_i(s)\neq 0$, $a_i(t)$, $b_i(s)\in C\,[0,T]$. It is assumed that the functions $a_i(t)$ and $b_i(s)$ are linear independent.

Since the boundary conditions (2) are not homogeneous, the nonhomogeneous integro-differential equation (1) never has trivial solution. Therefore, we investigate the nonexistence and existence of nontrivial solutions. We determine that for what values of the parameters λ and ν the problem (1), (2) has unique solution and for what values of the parameters λ and ν the problem (1), (2) has more solutions. We construct these solutions in implicit forms. The case, when the problem (1), (2) for some values of the parameters λ and ν has only trivial solutions, is impossible. The results of this work is formulated as a theorem.

Keywords: Nonhomogeneous equation, nonhomogeneous integral conditions, degenerate kernel, real parameters, solvability.

AMS Subject Classification: 34B08, 34B10.

BOUNDARY CRITERION FOR INTEGRAL OPERATORS AND ITS APPLICATIONS

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Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with a smooth boundary $\partial\Omega$. In Ω, we consider the following integral operator

$$u(x) = (Kf)(x) = \left(L_K^{-1} f\right)(x) = \int\limits_{\Omega} K(x,y) f(y) dy \tag{1}$$

where $K(x,y) \in C^2(\Omega \times \Omega)$.

Assume that $u(x)$ is a solution of the following equation

$$\Delta u = \sum_{|\alpha \leq 2|} a_\alpha(x) \cdot D^\alpha u(x) + f(x), \tag{2}$$

where $a_\alpha(x) \in C^{\alpha+\beta}\left(\overline{\Omega}\right), \ \beta > 0$.

In the work of T. Sh. Kal'menov and M. O. Otelbaev [1], using the theory of regular boundary extension and correct restriction of differential operators with certain conditions on kernel $K(x,y)$, a criterion for boundary properties of the integral operator $L_K^{-1} f$ was established. The investigation [2]-[4] uncovered the boundary conditions associated with classical potentials in mathematical physics equations, such as the Newton potential, heat potential, and wave potential. In this work, we present a method for representing the solution of the Laplace equation with an inhomogeneous potential boundary condition.

Keywords: elliptic equations, potentials, boundary value problem, Laplace operator.

AMS Subject Classification: 35J15, 35J25, 35J05.

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BOUNDARY VALUE PROBLEM FOR A THIRD-ORDER PARTIAL DIFFERENTIAL EQUATION WITH AN INTERFACE LINE $x = 0$

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In the area D, bounded by lines AB, BB_0, B_0A_0, A_0A, where$A(-l_1,0)B(l,0), B(l,0)B_0(l,h)$, $B_0(l,h)A_0(-l_2,h)$ - straight line segments $y = 0, x = l, y = h$ respectively, where AA_0 smooth curve $x = \chi(y)$, connecting the dots $A(-l_1,0)$ and $A_0(-l_2,0)$, where $l_1 = -\chi(0), l_2 = -\chi(h)$, and $-l_1 < -l_2 < 0, l, l_1$ where h - any positive numbers, $D_1 = D \cap (x > 0), D_2 = D \cap (x < 0)$, the conjugation problem is considered for the equations:

$$0 = \begin{cases} u_{xxy} - u_{yy} = 0, (x,y) \in D_1, \\ u_{xxy} + a(x,y)u_x + b(x,y)u_y + c(x,y)u = 0, (x,y) \in D_2, \end{cases} \tag{1}$$

Problem 1. Find a function $u(x,y) \in C(\overline{D}_1 \cap C^2(D_1) \cap C^{2+1}(D_1$, satisfying the equation (1) in the area D_1, initial conditions $u(x,0) = \psi_1(x), u_y(x,0) = \psi_2(x), 0 \le x \le l$ boundary condition $u(l,y) = \varphi_1(y), 0 \le x \le h$ and function $u(x,y) \in C(\overline{D}_2 \cap C^{2+1}(D_2)$, satisfying the equation (1) in the area D_2 , initial conditions $u(x,0) = \psi_3(x), -l \le x \le 0$, boundary condition $u(\chi(y),y) = \varphi_2(y), 0 \le y \le h$ as well as the pairing conditions $u(-0,y) + u(+0,y), u_x(+0,y) = u_x(+0,y), 0 \le y \le h$.

By the method of the Riemann and Green function [1], the solvability of problem 1 is equivalently reduced to solving the Volterra integral equation of the second kind, which has a unique solution.

Boundary value problems for partial differential equations of the third order of the form (1) are considered in [2, 3].

Problems arising in the study of heat transfer phenomena in a mixed medium of me-chanical and electrical phenomena in heterogeneous media with sharply different physical properties, as well as in the study of the problem of unsteady filtration in fractured rocks, are given to the conjugation problems for equations of various types.

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SOLUTION OF INITIAL-BOUNDARY VALUE PROBLEMS FOR THE HEAT EQUATION WITH DISCONTINUOUS COEFFICIENTS

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The initial-boundary value problem for the heat equation with discontinuous coefficients is considered.

$$\frac{\partial u_i}{\partial t} = k_i^2 \frac{\partial^2 u_i}{\partial x^2} \tag{1}$$

in domain $\Omega = \cup\Omega_i,\ \Omega_i = \{(x,t) : l_{i-1} < x < l_i, 0 < t < T\}$, where $(i = 1,\ 2,\ 3)$, with initial conditions

$$u(x, 0) = \varphi(x), l_0 \leqslant x \leqslant l_3 \tag{2}$$

boundary conditions of the form

$$\alpha_1 \frac{\partial u_1(l_0, t)}{\partial x} + \beta_1\ u_1(l_0, t) = 0, \alpha_2 \frac{\partial u_3(l_3, t)}{\partial x} + \beta_2\ u_3(l_3, t) = 0, \tag{3}$$

and pairing conditions

$$u_i(l_i - 0, t) =\ u_{i+1}(l_i + 0, t), k_i \frac{\partial u_i(l_i - 0, t)}{\partial x} =\ k_{i+1} \frac{\partial u_{i+1}(l_i + 0, t)}{\partial x}, \tag{4}$$

The coefficients α_i, β_i, are real numbers, $k_i > 0,\ (i = 1, 2)$.

Besides, $|\alpha_i| + |\beta_i| > 0,\ (i = 1, 2)$

In the case without a discontinuity, the spectral theory of these problems is constructed almost completely. In [1], the heat equation with a discontinuous coefficient was considered under Sturm-type boundary conditions (separated boundary conditions), eigenvalues and eigenfunctions were found, and various special cases were investigated. In this paper, the spectral questions of problem (1)-(4) are investigated. The eigenvalues and eigenfunctions are found, and the existence and uniqueness theorem for the classical solution is proved.

Keywords: Heat equation, discontinuous coefficients, eigenvalues, eigenfunctions, spectral theory.
AMS Subject Classification: 35 Partial differential equations

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INTEGRATION OF THE HIROTA EQUATION WITH NONSTATIONARY COEFFICIENTS

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In this paper, we study the Hirota equation with nonstationary coefficients, namely, consider the following equation

$$iu_t + p(t)(u_{xx} + 2u|u|^2) + iq(t)(6|u|^2 u_x + u_{xxx}) = 0, \tag{1}$$

where $p(t)$, $q(t)$ are given continuously differentiable functions. Equation (**??**) is considered under the initial condition

$$u(x,0) = u_0(x), \tag{2}$$

where the initial function $u_0(x)$ $(-\infty < x < \infty)$ has the following properties:
1)

$$\int_{-\infty}^{\infty} (1+|x|)\,|u_0(x)|dx < \infty; \tag{3}$$

2) the operator $L(0) = i\begin{pmatrix} \frac{d}{dx} & -u_0(x) \\ -u_0^*(x) & -\frac{d}{dx} \end{pmatrix}$ in the upper half-plane of the complex plane has exactly N eigenvalues $\lambda_1(0), \lambda_2(0), \ldots, \lambda_N(0)$ with multiplicities $m_1(0), m_2(0), \ldots, m_N(0)$ and has no spectral singularities.

Let the function $u(x,t)$ have the required smoothness and tend to its limits fast enough as $x \to \pm\infty$, i.e.

$$\int_{-\infty}^{\infty} \left((1+|x|)|u(x,t)| + \sum_{k=1}^{3} \left| \frac{\partial^k u(x,t)}{\partial x^k} \right| \right) dx < \infty. \tag{4}$$

Thoerem 1. If the function $u(x,t)$ is a solution to problem (1)-(4), then the scattering data of the operator $L(t)$ with the potential $u(x,t)$ satisfy the following differential equations

$$\frac{dr^+}{dt} = \left(2i\lambda^2 p(t) + 4i\lambda^3 q(t)\right) r^+, \quad (\operatorname{Im}\lambda = 0)\ ; \qquad \lambda_k(t) = \lambda_k(0), \quad k = \overline{1,N}$$

$$\frac{d\varkappa_0^n}{dt} = \left(4i\lambda_n^2 p(t) + 8i\lambda_n^3 q(t)\right)\varkappa_0^n; \quad \frac{d\varkappa_1^n}{dt} = \left(4i\lambda_n^2 p(t) + 8i\lambda_n^3 q(t)\right)\varkappa_1^n + \left(8i\lambda_n p(t) + 24i\lambda_n^2 q(t)\right)\varkappa_0^n,$$

$$\frac{d\varkappa_2^n}{dt} = \left(4i\lambda_n^2 p(t) + 8i\lambda_n^3 q(t)\right)\varkappa_2^n + \left(8i\lambda_n p(t) + 24i\lambda_n^2 q(t)\right)\varkappa_1^n + \left(4ip(t) + 24i\lambda_n q(t)\right)\varkappa_0^n,$$

$$\frac{d\varkappa_l^n}{dt} = \left(4i\lambda_n^2 p(t) + 8i\lambda_n^3 q(t)\right)\varkappa_l^n + \left(8i\lambda_n p(t) + 24i\lambda_n^2 q(t)\right)\varkappa_{l-1}^n + \left(4ip(t) + 24i\lambda_n q(t)\right)\varkappa_{l-2}^n + 8iq(t)\varkappa_{l-3}^n,$$

$$n = \overline{1,N},\ \ l = 0, 1, \ldots, m_n - 1.$$

Keywords: inverse scattering method, scattering data, Hirota equation, Gelfand-Levitan-Marchenko integral equation.

AMS Subject Classification: 34L25, 35P25, 47A40, 37K15

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ANALYTICAL SOLUTIONS OF INTEGRAL EQUATIONS WITH NON-ANALYTICAL RIGHT HAND SIDE

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We considered equations with analytical functions [1,2]. Consider the example

$$tu(t)+\int_0^t u(s)ds=\sqrt{f}(t),\ t\in R \tag{1}$$

where $f(t)$ is an analytical function real coefficients. Substitute power series

$(t(u_0+u_1t+u_2t^2+...)+\int_0^t(u_0+u_1s+u_2s^2+...)ds)^2=f_0+f_1t+f_2t^2+....$

The necessary conditions for existing a solution are (*) $f_0=0$; $f_1=0$: $(u_0+u_1t+u_2t^2+...+u_0+u_1t/2+u_2t^2/3+...)^2=f_2+f_3t+f_4t^2+...;$

$(2u_0+3/2\cdot u_1t+4/3\cdot u_2t^2+...+(n+2)/(n+1)\cdot u_nt^n+...)^2=f_2+f_3t+f_4t^2+....$

Denote $v_n=(n+2)/(n+1)\cdot u_n, n=0..\infty:(v_0+v_1t+v_2t^2+...)^2=f_2+f_3t+f_4t^2+...;$

If the right hand part is positive then this equation has two analytical solutions $\pm g(t)$. We have $|u_n|\le|v_n|, n=0..\infty$.

Theorem 0.1. *If $f(t)=t^2f_p(t)$ where $f_p(t)$ is a positive analytical function then the equation (1) has two analytical solutions.*

Consider the second example

$$tu(t)+\int_0^t su(s)ds=\sqrt{f}(t),\ t\in R \tag{2}$$

Substitute:

$(t(u_0+u_1t+u_2t^2+...)+u_0t^2/2+u_1t^3/3+u_2t^4/4+...)^2=f_0+f_1t+f_2t^2+....$

Suppose (*):

$(u_0+u_1t+u_2t^2+...+u_0t/2+u_1t^2/3+u_2t^3/4+...)^2=f_2+f_3t+....$

Denote $w_0=u_0; w_n=u_n+u_{n-1}/(n+1), n=1..\infty.$

As well as above, if the analytical function with coefficients $\{w_n : n=0..\infty\}$ exists then $u_0=w_0; u_n=w_n-u_{n-1}/(n+1), n=1..\infty.$

Theorem 0.2. *If $f(t)=t^2f_p(t)$ where $f_p(t)$ is a positive analytical function then the equation 2) has two analytical solutions.*

Keywords: algorithm, evident, integro-differential equation, analytical function
AMS Subject Classification: 45J05

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SINGLE-PHASE NONLINEAR QUASI-STATIONARY STEFAN PROBLEM IN A SPHERICALLY SYMMETRIC FORMULATION

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The reduction of the single-phase Stefan problem for a nonlinear quasi-stationary heat equation with a spherically symmetric thermal field to an equivalent system of two nonlinear integro-differential equations is performed. At the initial moment, the domain of the problem solution is degenerate. An algorithm for solving a system of integro-differential equations has been developed. A computational experiment based on the exact solution of the nonlinear Stefan problem is investigated for convergence and stability.

BOUNDARY VALUE PROBLEMS FOR THE DEGENERATING PARABOLIC EQUATION OF THE FOURTH ORDER

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Consider the equation

$$Lu \equiv u_{tt} - t^n u_{xxxx} = 0,\ n > 0$$

in the domain $\Omega = \{(x,t) : 0 < x < l,\ 0 < t < t_0,\ l > 0,\ t_0 > 0\}$.

Problem 1. To find a function $u(x,y)$ from the class $u_{tt},\ u_{xxxx} \in C\left(\overline{\Omega}\right)$, satisfying in the domain Ω equation $Lu = 0$ and conditions:

$$\begin{aligned} &u(x,0) = 0, \qquad u\left(x,t_0\right) = 0,\ \ 0 \le x \le l,\\ &u(0,t) = \varphi_1(t), \quad u\left(l,t\right) = \varphi_2(t),\ \ 0 \le t \le t_0,\\ &u_x(0,t) = \psi_1(t),\ \ u_x\left(l,t\right) = \psi_2(t),\ \ 0 < t < t_0. \end{aligned}$$

Further, under certain conditions on given functions, uniqueness and existence theorems for the solution of problem are proved.

The uniqueness of the problem 1 is shown by the integral energy method. Using the following identity

$$\frac{\partial}{\partial t}(u_t u) + \frac{\partial}{\partial t}(t^n u_{xx} u_x - t^n u_{xxx} u) = (u_t)^2 + t^n (u_{xx})^2,$$

we get

$$\iint\limits_{\Omega} \left[(u_t)^2 + t^n (u_{xx})^2\right] dxdt = 0.$$

Hence, one can easily to show an uniqueness of the solution to the Problem 1.

An explicit solution to Problem 1 is found by the Fourier method:

$$u(x,t) = \sqrt{t} \sum_{n=1}^{\infty} \left\{ \left[A_n e^{-\mu_n x} + B_n e^{\mu_n x}\right] \sin\left(\mu_n x\right) \right.$$

$$\left. + \left[C_n e^{-\mu_n x} + D_n e^{\mu_n x}\right] \cos\left(\mu_n x\right) \right\} J_\alpha \left[2\alpha \left(-\lambda_n t^{n+2}\right)^{1/2}\right],$$

where A_n, B_n, C_n and D_n are known constants, expressed and calculated in terms of given functions of the Problem 1; $J_\alpha(z)$ is Bessel function;

$$\alpha = \frac{1}{n+2};\ \ \mu_n = \frac{\sqrt{2}}{2}\left(-\lambda_n\right)^{1/4},\ \ \ \lambda_n = -t_0^{-1/\alpha}\left[\frac{(2\alpha + 4n - 1)\pi}{8\alpha}\right]^2,\ \ n = 1, 2, \dots .$$

Keywords: Degenerating parabolic equation; equation of the higher order; integral energy method; Fourier method.

AMS Subject Classification: 35K25, 35K35, 35K65.

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ON THE DISCRETE SPECTRUM OF THE SCHRÖDINGER OPERATOR OF THE SYSTEM OF THREE-PARTICLES WITH MASSES $m_1 = m_2 = \infty$ AND $m_3 < \infty$ ON THE THREE DIMENSIONAL LATTICE

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In this paper, we study the spectrum of the discrete Schrödinger operator H corresponding to a system of three particles, with masses $m_1 = m_2 = \infty$ and $m_3 < \infty$, and interacting via short-range pair potentials in the three dimensional lattice. Using the integral decomposition, the study of spectrum of the operator H is reduced to investigating spectra of the more convenient fiber operators $H(K)$, depending on the quasi-momentum K. The essential spectrum $\sigma_{ess}(H(K))$ of $H(K)$ is the union of the spectra of the channel operators $H_\alpha(K), \alpha = 1, 2, 3$ (see, [1, 2]). We prove that the discrete spectrum of the corresponding Schrödinger operator is infinite (Theorem 1). Moreover, we show that an infinitely many eigenvalues may appear in the gap of the essential spectrum for some values of the interaction energy. In the one-dimensional case, a similar problem was considered in [3] and [4], and infiniteness of the discrete spectrum was proven.

The operator $H(K)$, $K \in \mathbb{T}^3 = (-\pi, \pi]^3$ associated with the current system is of the form

$$H(K) = H_0(K) - V_{12} - V_{23} - V_{31}, \tag{1}$$

where the operators $H_0(K)$ and $V_{\beta\gamma}$ are defined on the Hilbert space $L_2((\mathbb{T}^3)^2)$ by

$$(H_0(K)f)(p, q) = E(K; p, q)f(p, q), \quad f \in L_2((\mathbb{T}^3)^2),$$

$$(V_1 f)(p, q) = \frac{\mu_1}{(2\pi)^3} \int\limits_{\mathbb{T}^3} f(p, t)dt, \quad (V_2 f)(p, q) = \frac{\mu_2}{(2\pi)^3} \int\limits_{\mathbb{T}^3} f(t, q)dt,$$

$$(V_3 f)(p, q) = \frac{\mu_3}{(2\pi)^3} \int\limits_{\mathbb{T}^3} f(t, p + q - t)dt, \quad f \in L_2((\mathbb{T}^3)^2),$$

where

$$E(K; p, q) = \varepsilon(K - p - q)/m_3,$$

and the real-valued continuous function

$$\varepsilon_\alpha(p) = \frac{1}{m_\alpha}\epsilon(p), \quad \epsilon(p) = \sum_{j=1}^{3}(1 - \cos p^{(j)}), \quad p = (p^{(1)}, \ldots, p^{(d)}) \in \mathbb{T}^3, \tag{2}$$

is called the *dispersion relation of the α-th normal mode* associated with the free particle α ($\alpha = 1, 2, 3$).

Here $\mu_\alpha \geq 0$, $\alpha = 1, 2, 3$ is called an energy of the interactions.

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Let us define the following function

$$\Delta_\alpha(z) = 1 - \frac{\mu_\alpha}{(2\pi)^d} \int\limits_{\mathbb{T}^3} \frac{ds}{\varepsilon_3(s) - z}, \quad z \in \mathbb{C} \setminus [E_{\min}(K), E_{\max}(K)], \quad \alpha = 1, 2.$$

Set

$$\mu_\alpha^0 = \left(\frac{1}{(2\pi)^d} \int\limits_{\mathbb{T}^3} \frac{ds}{\varepsilon_3(s)} \right)^{-1}, \quad \alpha = 1, 2.$$

Lemma 1. *Let $\alpha = 1, 2$.*

(a) If $0 < \mu_\alpha \leq \mu_\alpha^0$, then $\Delta_\alpha(z)$ has no zeros in the interval $(-\infty, E_{\min}(K))$.

(b) If $\mu_\alpha > \mu_\alpha^0$, then $\Delta_\alpha(z)$ has a unique simple zero in the interval $(-\infty, E_{\min}(K))$, i.e., $\Delta_\alpha(z_\alpha^0) = 0$.

Lemma 2. *Let $\mu_\alpha > 0, \alpha = 1, 2$, and let $\mu_3 \geq 0$. Then, for every $K \in \mathbb{T}^3$, the following relations hold:*

For the essential spectrum of the main operator $H(K)$

$$\sigma_{ess}(H(K)) = [0, 6/m_3] \cup \left(\{z_1^0\} \cup \{z_2^0\} \cup [-\mu_3, 6/m_3 - \mu_3]\right).$$

Moreover, we have

$$\sigma_{ess}(H(K)) = \left(\{z_1^0\} \cup \{z_2^0\}\right) \cup [-\mu_3, 6/m_3], \quad 0 \leq \mu_3 \leq 6/m_3,$$
$$\sigma_{ess}(H(K)) = \left(\{z_1^0\} \cup \{z_2^0\} \cup [-\mu_3, 6/m_3 - \mu_3]\right) \cup [0, 6/m_3], \quad \mu_3 > 6/m_3,$$

where $\{z_\alpha^0\} = \varnothing$ if

Theorem 1. *Let $z_{\min} = \inf\{z_1^0, z_2^0\}$ and let $z_{\max} = \sup\{z_1^0, z_2^0\}$. Fix arbitrary μ_1, μ_2 and μ_3. Then, there exist infinite sets of eigenvalues $z_n \in (-\infty, z_{\min})$ and $\xi_n \in (z_{\max}, E_{\min}(K))$, $n \in \mathbb{Z}^3$, of $H(K)$ such that*

$$\lim_{n\to\infty} z_n = z_{\min} \quad \text{and} \quad \lim_{n\to\infty} \xi_n = z_{\max}.$$

Keywords: Schrödinger operator, dispersion functions, zero-range pair potentials, discrete spectrum, essential spectrum, cluster operators, bound states.

AMS Subject Classification: Primary: 81Q10, Secondary:35P20, 47N50

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IMPLEMENTATION OF SMALL PARAMETER METHOD FOR THE STUDY OF MULTIPERIODIC SOLUTIONS OF SYSTEMS WITH A DIAGONAL DIFFERENTIATION OPERATOR

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This report presents a small parameter method for investigating multiperiodic solutions of systems with a diagonal differentiation operator. It is based on the idea of the Poincare method for studying periodic solutions of systems of ordinary differential equations. Until now, the presented method has been applicable only to the issues of multiperiodic solutions of systems with the specified differentiation operator, when the corresponding linear systems had the property of dichotomy.

Such a significant generalization became possible due to the introduction of periodic characteristics of the diagonal differentiation operator by considering characteristic systems on the cylindrical surface of the space of time variables. Thus, as a field of differentiation along the diagonal, 1) taking a variety of the form of the Euclidean plane had linear δ-characteristics, parallel diagonals of the plane of time variables, and 2) taking a variety of the form of a cylindrical surface had β-characteristics of the form of a helix. These two approaches are equivalent, but they should be used based on their convenience when studying problems.

For example, when studying the stability of solutions of systems, one should mainly use the δ-characteristic method. In the case of studying problems related to the multiperiodicity of solutions, the β-characteristic method is more appropriate. We are convinced of this when extending the classical Poincare small parameter method to the case of multiperiodic solutions of systems with differentiation operators in the directions of vector fields, in particular, along the main diagonal of the space of time variables. Thus, in the report, using the β-characteristics of the differentiation operator, the small parameter method [1,2] is implemented to study multiperiodic solutions of systems with a small parameter.

Keywords: multiperiodicity, differentiation operator, characteristic method, vector field.

AMS Subject Classification: 35E15.

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CONSTRUCTION OF HYPERGEOMETRIC TYPE INHOMOGENEOUS SECOND-ORDER SYSTEM SOLUTIONS

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A regular system consisting of two differential equations in second-order partial derivatives of the form (0.0) is investigated

$$\begin{aligned} &x^2 g_0 p_{2,0} + xy g_1 p_{1,1} + x g_3 p_{1,0} + y g_4 p_{0,1} + g_5 p_{0,0} = g(x,y), \\ &y^2 q_0 p_{0,2} + xy q_1 p_{1,1} + x q_3 p_{1,0} + y q_4 p_{0,1} + q_5 p_{0,0} = q(x,y), \end{aligned} \tag{1}$$

where the coefficients of the system are polynomials in form

$$g_i(x,y) = r_{i,0} - \alpha_{i,0} x^h, q_i(x,y) = t_{0,i} - \beta_{0,i} y^h \tag{2}$$

$g(x,y)$ and $q(x,y)$ analytic functions of two variables; $p_{0,0} = Z(x,y)$ total unknown, $r_{j,k}, \alpha_{j,k}, t_{j,k}, \beta_{j,k}(j,k = \overline{0,2})$ unknown constants.

It is required to study the possibilities of constructing inhomogeneous system solutions (1) in form of orthogonal polynomials of two variables, when the corresponding homogeneous system is hypergeometric or reducible to them.

Many properties of inhomogeneous systems (1) with coefficients in form (2) remain poorly studied. Particularly heterogeneous systems associated with orthogonal polynomials of many variables. Let us return to the example of M.Ch. Hermite and move on to construct solutions to his heterogeneous system.

Theorem 1. An inhomogeneous system

$$\begin{aligned} &(1-x^2)p_{2,0} - xyp_{1,1} + (n-2)xp_{1,0} - (m+1)yp_{0,1} + (m+n)(m+1)p_{0,0} = g(x,y), \\ &(1-y^2)p_{0,2} - xyp_{1,1} - (n+1)xp_{1,0} + (m-2)yp_{0,1} + (m+n)(n+1)p_{0,0} = q(x,y), \end{aligned} \tag{3}$$

has a general solution, as the sum of the general decision corresponding to homogeneous system and the particular solution $Z_0(x,y)$ of the inhomogeneous system (3).

Keywords: system, regular, irregular, heterogeneous system, features.

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MULTISTABLITY IN NONSMOOTH SYSTEMS

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Many problems in engineering and applied science lead us to consider piecewise-smooth systems in the form of differential equations with discontinuous right-hand side or piecewise-smooth maps. Examples of such systems include relay and pulse-width modulated control systems [1], mechanical systems with dry friction or impacts [2].

In addition to the bifurcations occurring in smooth systems, piecewise-smooth systems also show a variety of border-collision related phenomena which occur when an invariant set such as, for example, a fixed point, collides with a switching set.

Moreover it is well known that the global stability is a rare property of nonlinear systems since it implies that there is single invariant set, which attracts all solutions of the system. Frequently more than one solution exists at the same values of parameters i.e. multistability. Multistability is one of the common phenomena in the theory of nonlinear dynamical systems. A distinguishing feature of multistable systems is their sensitivity to noise: an arbitrarily small level of noise may cause a sudden transition from one attractor to another. This implies that one can observe a hard transition from periodic solution to a chaotic or high-periodic attractor and vice versa.

In the present work, by means of detailed, numerically calculated phase portraits we present an example of the multistable system in which the complex behavior is associated with the interplay of the classic bifurcations with global (homoclinic) and border-collision bifurcations.

Keywords: Multistability, homoclinic bifurcation, border-collision phenomena, piecewise-smooth continuous maps.

AMS Subject Classification: 05.45.Gg, 05.45.Pq

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SYMMETRY OF THE DIFFERENTIAL EQUATION WITH DISCONTINUITY RIGHT HAND SIDE LEADS TO THE BISTABLILTY

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Considering a set of two coupled nonautonomous differential equations with discontinuous right-hand sides [1, 2, 3]

$$\dot{x}(t) = F(t, x), \tag{1}$$

$$F(t,x) = \begin{cases} F^{+}(x), & \varphi(t,x) > 0, \\ F^{-}(x), & \varphi(t,x) < 0, \end{cases}$$

$$F(t+\pi, x) = -F(t,x), \quad x, F^{\pm} \in \mathbb{R}^2, \quad t, \varphi \in \mathbb{R},$$

describing the behavior of a pulse frequency control system, we discuss how a symmetry of (1) can lead to the bistabilty. The Eq. (1) can be reduced to the discontinuity map. We demonstrate that there are two different reasons causing this map to be discontinuous, so that depending on parameters it may be either a gap map or a circle diffeomorphism. In both parameter domains the map exhibits a period adding structure. Using symbolic dynamics, we explain which of the periodic solutions are affected by bistability. This is done in terms of the associated symbolic sequence adding scheme, which is similar to the well-known Farey structure commonly used for rotation numbers.

Keywords: Differential equations with discontinuous right-hand side, periodic solutions, bistability, adding structure, symbolic dynamics.

AMS Subject Classification: 05.45.Gg, 05.45.Pq

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MULTIDIMENSIONAL NORMALLY REGULAR SOLUTIONS OF DEGENERATE SYSTEMS ASSOCIATED WITH BESSEL FUNCTIONS OF SEVERAL VARIABLES

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Until now, analogues of the Bessel equation have not been established in the form of degenerate systems whose solutions are multidimensional Bessel functions. The possibilities of the existence of normally regular solutions of related systems such as Horn, Whittaker and Bessel also remain unexplored.

In this paper, we study two Bessel-type systems whose solutions are multidimensional normally regular Bessel solutions and establish connections between the solutions of the above-mentioned related systems. A number of theorems are proved.

SPLITTING METHOD FOR STABILITY OF EQUATIONS WITH DELAY UNDER PERMANENTLY ACTING PERTURBATIONS

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The problem to stabilize an object $(u(t))$ under permanently acting perturbations $(f(t))$ by means of feedback $(-pu(t))$ is considered. It is known that rocking of $u(t)$ for too large value of p instead of stabilization takes place because of irremediable delay (h) of control. The product $\Delta := ph$ is an absolute constant.

To detect boundaries of such constants Lyapunov functions were to be used earlier. The method of splitting the space of solutions [1] is proposed here.

Theorem 0.1. *If $f(t) \in C(R_+), |f(t)| \leq f_0, \Delta < 1$ then a solution of the initial value problem*

$$u'(t) = -pu(t-h) + f(t), t \in R_+, \tag{1}$$

$$u(t) = 0, t \in [-h, 0] \tag{2}$$

is bounded: $|u(t)| \leq (1+2\Delta)f_0/p/(1-\Delta), t \in R_+$.

Remark 0.1. $(\sin t)' \equiv -sin(t-\pi/2)$; *hence, the upper boundary for Δ is $\pi/2 = 1.57...$.*

Proof. We use the splitting: $C[-h, 0] = R_+ \times \{u \in C[-h, 0] : u(0) = 0\}$.

Prove the following estimations by induction by steps of length h:

$|u(n)| \leq f_0/p/(1-\Delta); |u(n) - u(s)| \leq 2(n-s)f_0/(1-\Delta), n-h \leq s \leq n, n \in N_0$.

The shift operator:

$Su(\cdot)(t) := u(0) + \int_{-h}^{t}(-pu(s) + f(s))ds, -h \leq t \leq 0$.

Estimate:

$|Su(\cdot)(0)| = |u(0) + \int_{-h}^{0}(-pu(0) - p(u(s) - u(0)) + f(s))ds|$

$\leq |u(0)|(1-\Delta) + \int_{-h}^{0}(p|(u(s) - u(0)| + |f(s)|)ds \leq$

$\leq (1 - \Delta + \Delta^2 + (1-\Delta))f_0/p/(1-\Delta) = f_0/p/(1-\Delta)$;

$|Su(\cdot)(t) - Su(\cdot)(0)| \leq p|u(0)||t| + \int_{0}^{t}(p|(u(s) - u(0)| + |f(s)|)|ds| \leq$

$\leq (1/(1-\Delta) + ph/(1-\Delta) + 1)f_0|t| = 2f_0|t|/(1-\Delta)$.

Hence, at the next step

$|u(t)| \leq f_0/p/(1-\Delta) + 2f_0|t|/(1-\Delta) \leq f_0/p/(1-\Delta) + 2f_0ph/p/(1-\Delta)$.

Theorem is proven.

By means of introducing a trial function and applying validating computations the upper boundary for Δ was improved to 1.08.

Keywords: delay-differential equation, solution, stability, perturbation
AMS Subject Classification: 34K20, 34K27, 34K35

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NONLOCAL BOUNDARY VALUE PROBLEMS FOR FREDHOLM TYPE INTEGRO-DIFFERENTIAL EQUATIONS WITH PARAMETERS

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Integro-differential equations with a degenerate kernel were considered earlier by many authors. In cases, where the boundary of the region of a physical process is not available for measurements, nonlocal conditions in integral form can serve as an additional information sufficient for the unambiguous solvability of the problem. The papers [1, 2, 3, 4] considered nonlocal problems for a second-order integro-differential equation with the real parameters and with more simple integral condition.

In this paper, we study a nonlocal nonhomogeneous problem for a second-order ordinary Fredholm integro-differential equation with a degenerate kernel, two nonhomogeneous integral conditions and two real parameters. The regular and irregular values of the parameters are calculated, for which the nonexistence, existence of a unique solution, and the existence of more solutions of the problem are established. The corresponding implicit forms of solutions are constructed.

Problem. It is required to find a function $u(t)$ on the interval $(0, T)$, that satisfies the nonhomogeneous equation

$$u''(t) + \lambda^2 u(t) = \nu \int_0^T K(t,s) \left[s\, u(s) + (T-s) u'(s) \right] \, ds + f(t) \tag{1}$$

with the following two nonhomogeneous boundary conditions:

$$u(T) - \int_0^T s\, u(s)\, ds = \varphi_1, \quad u'(T) - \int_0^T (T-s)\, u'(s)\, ds = \varphi_2, \tag{2}$$

where $T > 0$, λ is positive parameter, ν is nonzero real parameter, $f(t) \in C\,[0,T]$, $0 \neq \varphi_1, \varphi_2 =$ const, $K(t,s) = \sum\limits_{i=1}^{k} a_i(t)\, b_i(s) \neq 0$, $a_i(t), b_i(s) \in C\,[0,T]$. It is assumed that the functions $a_i(t)$ and $b_i(s)$ are linear independent.

Since the boundary conditions (2) are not homogeneous, the nonhomogeneous integro-differential equation (1) never has trivial solution. Therefore, we investigate the nonexistence and existence of nontrivial solutions. We determine that for what values of the parameters λ and ν the problem (1), (2) has unique solution and for what values of the parameters λ and ν the problem (1), (2) has more solutions. We construct these solutions in implicit forms. The case, when the problem (1), (2) for some values of the parameters λ and ν has only trivial solutions, is impossible. The results of this work is formulated as a theorem.

Keywords: Nonhomogeneous equation, nonhomogeneous integral conditions, degenerate kernel, real parameters, solvability.

AMS Subject Classification: 34B08, 34B10.

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NONLINEAR n–VELOCITY SINGULARLY PERTURBED INVERSE PROBLEM OF KAC-TYPE

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Consider the n-velocity coefficient-inverse transport problem:

$$\varepsilon^{\beta}\left[\frac{\partial}{\partial t}\left(E^{1,1,\dots,1}_{(a_1,\dots,a_n)}U_\varepsilon\right)+U^2_\varepsilon(t,x_1,...,x_n)\right]+\lambda E^{1,1,..,1}_{(a_1,\dots,a_n)}U_\varepsilon+h_0(x_1,...,x_n)U_\varepsilon=Z_\varepsilon(x_1,...,x_n)f(t), \tag{1}$$

$$\begin{cases} (U^{(i)}_{\varepsilon t}(t,x_1,...,x_n)|_{t=0}=V^{(i)}_t(0,x_1,...,x_n)+(\sum_{j=1}^n 2a_jx_j\varepsilon^{-1})^i\exp\left(-\frac{1}{\varepsilon}\sum_{j=1}^n x_j^2\right), \\ V^{(i)}_t(0,x_1,...,x_n)=\varphi_i(x_1,...,x_n),\ (i=0,1),\ \ \forall(x_1,...,x_n)\in R^n, \end{cases} \tag{2}$$

$$\begin{cases} (U^{1,1,\dots,1}_{(a_1,\dots,a_n)})|_{t=T}\equiv(U_{\varepsilon t}+\sum_{j=1}^n a_jU_{\varepsilon x_j})|_{t=T}=g_0(x_1,...,x_n)+g_\varepsilon(x_1,...,x_n), \\ (V_t+\sum_{j=1}^n a_jV_{x_j})_{t=T}=g_0(x_1,...,x_n),\ \ \forall(x_1,...,x_n)\in R^n, \end{cases} \tag{3}$$

at the same time, information is entered regarding the initial data in the form:

$$\begin{cases} ||g_\varepsilon||_{L^p(R^n)}=\left(\int_{R^n}|g_\varepsilon(E_1,...,E_n)|^p\,dQ\right)^{\frac{1}{p}}\le\Delta_1(\varepsilon),(dQ=dx_1...dx_n), \\ ||g_\varepsilon(.)||_{L^p(0,T)}=\left(\sup\limits_{\bar\Omega_0}\int_0^t|g_\varepsilon(x_1-a_1(t-s),...,x_n-a_n(t-s))|^pds\right)^{\frac{1}{p}}\le\Delta_2(\varepsilon), \\ \qquad\qquad\qquad\qquad(\Delta_1,\Delta_2\le\Delta_0(\varepsilon)\underset{\varepsilon\to0}{\to}0), \\ ||U_\varepsilon(0,x_1,...,x_n)-V(0,x_1,...,x_n)||_{L^p(R^n)}\le\left(\pi p^{-1}\right)^{\frac{1}{p}}\varepsilon^{\frac{1}{p}}=\gamma_0\varepsilon^{\frac{1}{p}}, \\ \qquad\qquad\qquad\qquad(\,\mathrm{E}^{1,1,\dots,1}_{(a_1,\dots,a_n)}=\frac{\partial}{\partial t}+\sum_{j=1}^n a_j\frac{\partial}{\partial x_j}), \end{cases}$$

where $(U_\varepsilon,Z_\varepsilon)$ are unknown functions. The method for solving the original inverse problem modifies the method of work [1]. Under certain restrictions on the known data:
$0<h_0,\ f,\ \varphi_i,\ g_0,\ g_\varepsilon,\ 0<a_j,\ \lambda=const,\ (j=\overline{1,n}),\ \lambda^{-1}\ll1; 0<\beta<2^{-1}; f(0)=0, f(T)\ne0$ an estimate is obtained for the proximity of solutions to a singularly perturbed and degenerate inverse problem in the sense of $W^p_h(\Omega_0=(0,T_0)\times R^n)$, when a priori information is given from $L^p(R^n)$.

Keywords: singularly perturbed Kac-type equation, n-velocity transport problem, inverse problem, unbounded domain.

AMS Subject Classification: 35Q49, 35Q35, 35R30.

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ОБ ОДНОЙ ЗАДАЧЕ ДЛЯ УРАВНЕНИЯ ТРЕТЬЕГО ПОРЯДКА С КРАТНЫМИ ХАРАКТЕРИСТИКАМИ В ТРЕХМЕРНОМ ПРОСТРАНСТВЕ В ПОЛУОГРАНИЧЕННОЙ ОБЛАСТИ

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В области $D^{-}=\{(x,y,z):\,-\infty<x<0,\,0<y<q,\,0<z<r\}$ рассмотрим уравнения

$$L[u]\equiv\frac{\partial^3 u}{\partial x^3}-\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial z^2}=0, \tag{1}$$

где $q>0,\;\; r>0$ - постоянные вещественные числа, и для него исследуем следующую задачу.

Задача B**.** Найти решение уравнения (1) в области D^{-} из класса $C^{3,2,2}_{x,y,z}\left(D^{-}\right)\cap C^{2,1,1}_{x,y,z}\left(D^{-}\cup\Gamma\right)$, имеющего ограничение первой производной по x, y и z, второй производной по x при $x\to-\infty$, и $u_y,u_z\in L_2\left(D^{-}\right)$, удовлетворяющего следующими краевыми условиями

$$u_y\left(x,0,z\right)=u_y\left(x,q,z\right)=0,\;\;u_z\left(x,y,0\right)=u_z\left(x,y,r\right)=0,\,-\infty<x<0, \tag{2}$$

$$\begin{gathered}u\left(0,y,z\right)=\psi_1\left(y,z\right),u_x\left(0,y,z\right)=\psi_2\left(y,z\right),\\ \lim_{x\to-\infty}u\left(x,y,z\right)=\,0,0\le y\le q,\,0\le z\le r,\end{gathered} \tag{3}$$

где $\Gamma=\partial D^{-}$ - граница области D^{-}, $\psi_i\left(y,z\right),i=1,2$ - заданные достаточно гладкие функции, причем

$$\left\{\begin{array}{l}\dfrac{\partial\psi_i\left(0,z\right)}{\partial y}=\dfrac{\partial\psi_i\left(q,z\right)}{\partial y}=0,\dfrac{\partial^4\psi_i\left(y,0\right)}{\partial y^3\partial z}=\dfrac{\partial^4\psi_i\left(y,r\right)}{\partial y^3\partial z}=0,\\ \dfrac{\partial\psi_i\left(y,0\right)}{\partial z}=\dfrac{\partial\psi_i\left(y,r\right)}{\partial z}=0,\dfrac{\partial^4\psi_i\left(0,z\right)}{\partial z^3\partial y}=\dfrac{\partial^4\psi_i\left(q,z\right)}{\partial z^3\partial y}=0,\end{array}\right.\quad i=1,2. \tag{4}$$

Отметим, что в работах [1-2] в конечные области изучены краевые задачи в трехмерном пространстве.

Теорема 1. Если задача B имеет решение, то оно единственно.

Теорема 1 доказано с помощью методом интегралов энергии.

Теорема 2. Если функции $\dfrac{\partial^6\psi_i\left(y,z\right)}{\partial y^3\partial z^3}\in L_2\left[0<y<q,0<z<r\right],i=1,2$ и выполняются условия согласования (4), то решение задачи B существует.

Теорема 2 доказано с помощью методом Фурье.

Ключевые слова: Дифференциальное уравнение с частными производными, уравнение третьего порядка, кратные характеристики, краевая задача, единственность, существование, ряд, полуограниченная область, абсолютная и равномерная сходимость.

Предметная классификация AMS: 35G15

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РАЗРЕШИМОСТЬ КРАЕВОЙ ЗАДАЧИ ДЛЯ УРАВНЕНИЯ ЧЕТВЕРТОГО ПОРЯДКА

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В области $\Omega = \{(x,t) : 0 < x < 1, -\alpha < t < \beta\}$ рассмотрим уравнение

$$Lu \equiv u_{xxxx}(x,t) + \operatorname{sgnt} \cdot [u_t(x,t) - u_{tt}(x,t)] + b^2 u(x,t) = f(x,t),$$

где b - заданное число, $f(x,t)$ - заданная функция.

Задача. Найти функцию $u(x,t)$ удовлетворяющую следующим условиям:

$$u(x,t) \in C^{3,1}_{x,t}\left(\overline{\Omega}\right) \cap C^{4,2}_{x,t}\left(\Omega_+ \cup \Omega_-\right), \tag{1}$$

$$Lu(x,t) \equiv f(x,t), \ (x,t) \in \Omega_+ \cup \Omega_-, \tag{2}$$

$$u(0,t) = u(1,t), \ u_x(0,t) = 0, \ u_{xx}(0,t) = u_{xx}(1,t), \ u_{xxx}(1,t) = 0, \ -\alpha \le t \le \beta, \tag{3}$$

$$u(x,\beta) = \varphi(x), \ u(x,-\alpha) = \psi(x), \ 0 \le x \le 1, \tag{4}$$

где $\Omega_+ = \Omega \cap \{t > 0\}$, $\Omega_- = \Omega \cap \{t < 0\}$ и $\varphi(x)$, $\psi(x)$ - заданные достаточно гладкие функции.

Система функций

$$X_0(x) = 1, \ X_{2k-1}(x) = \frac{e^{\lambda_k x} + e^{\lambda_k (1-x)}}{e^{\lambda_k} - 1} + \sin \lambda_k x, \ X_{2k}(x) = 2\cos \lambda_k x, \tag{5}$$

$$Y_0(x) = 2x, \ Y_{2k-1}(x) = 2\sin \lambda_k x, \ Y_{2k}(x) = \frac{e^{\lambda_k x} - e^{\lambda_k (1-x)}}{e^{\lambda_k} - 1} + \cos \lambda_k x, \tag{6}$$

$$\lambda_k = 2\pi k, \ k = 1, 2, \ldots$$

биортогональная и образуют базис Рисса в $L_2(0,1)$ [1, 2].

Решение задачи найдено виде ряда составленных из базисных функций Рисса (5). Единственность решения задачи вытекает из полнота ортонормированных систем (6).

Теорема 1. *Если существует решение задачи* (1)-(4)*, то оно единственно только тогда, когда выполнены условия*

$$\mu_k \cos \frac{1}{2}\mu_k \alpha \cdot sh \frac{1}{2}\nu_k \beta + \nu_k \sin \frac{1}{2}\mu_k \alpha \cdot ch \frac{1}{2}\nu_k \beta \ne 0$$

при всех $k \in N \cup \{0\}$, $\nu_k = \sqrt{4\left(\lambda_k^4 + b^2\right) + 1}$, $\mu_k = \sqrt{4\left(\lambda_k^4 + b^2\right) - 1}$, $\lambda_0 = 0$.

Ключевые слова: уравнение четвертого порядка, базис Рисса, существование, единственность решение.

Предметная классификация AMS: 517.95.

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ПРИМЕНЕНИЕ СИСТЕМ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ В ЭПИДЕМИОЛОГИИ

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Хорошо известно, что математика универсальная наука, невозможно найти область, в которой она не применяется, она также играет важную роль в эпидемиологии, предоставляя инструменты для моделирования и анализа распространения инфекционных заболеваний. Так или иначе, человечество во все времена сталкивалось с массовыми заболеваниями неизвестной природы, которые позже были названы инфекционными. И, относительно, с недавнего времени для их изучения, прогнозирования и моделирования стали применяться математические методы, в частности дифференциальные уравнения и их системы.

В данной статье рассмотрены SIR –модель, модель «хищник-жертва» их фазовые портреты, а также общая модель эпидемии вида:

$$\dot{x}(t) = -2xy, \quad \dot{y}(t) = 2xy - y. \tag{1}$$

где x- число восприимчивых к болезни, y - число заболевших в соответствующих масштабах. Решаем данную систему следующим образом:

$$\dot{x}(t)/\dot{y}(t) = -1 + 1/2x \tag{2}$$

$$y = -x + 1/2 * lnx + c_1 \tag{3}$$

Итак,из формулы (3),видно, что число восприимчивых убывает, а число заболевших достигает некого максимума и убывает до нуля. Что отражает реальное протекание любой эпидемии.Далее, найдя первый интеграл системы (1),нарисуем её фазовый портрет на основе полученных данных рассмотрим, как развивается эпидемия.

Ключевые слова: система дифференциальных уравнений, моделирование, модель эпидемии, фазовый портрет.

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СИНГУЛЯРНО ВОЗМУЩЕННЫЕ УРАВНЕНИЯ С НЕСКОЛЬКИМИ ТОЧКАМИ ПОВОРОТА И ЗАТЯГИВАНИЕ ПОТЕРИ УСТОЙЧИВОСТИ

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Работа посвящена исследованию асимптотического поведение решение систем сингулярно возмущенных уравнений. Особенность рассматриваемой системы состоит в том, что матрица-коффициент при линейной неизвестной вектор-функции имеет несколько линейных сопряженных собственных значений. Все собственные значение имеют нулей, принадлежащие мнимой оси комплексной плоскости независимой переменной. Действительные части всех собственных значений принимают отрицательные значения при отрицательных значениях независимой переменной на действительной оси, далее положительные значения при положительных значениях. Поставлена и решена задача на явление затягивание потери устойчивости.

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АСИМПТОТИЧЕСКОЕ РЕШЕНИЕ ЗАДАЧИ КОШИ ДЛЯ ГИПЕРБОЛИЧЕСКОГО УРАВНЕНИЯ ЛАЙТХИЛЛА С МАЛЫМ ПАРАМЕТРОМ

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В данной статье методом униформизации получено равномерно пригодное представление решения модельного уравнение Лайтхилла гиперболического типа в случае экспоненциального роста решения на границе области.

Рассмотрим уравнение

$$\vartheta_{xy} = \varepsilon\left(\vartheta_x + \vartheta_y\right)\vartheta_{xx}, \qquad (\mathbf{1})$$

где $0 < \varepsilon << 1$ с начальными данными на характеристиках

$$\vartheta\left(x,1\right) = \varphi\left(x\right)e^{\alpha/x}\ ,\quad \vartheta\left(1,y\right) = \psi\left(y\right), \qquad (\mathbf{2})$$

здесь $\varphi\left(x\right), \psi\left(y\right) \in C^{\infty}\left(D\right)$,

где $D = \{(x,y) : 0 \le x \le 1, \qquad 0 \le y \le 1\}$

$\varphi_0 = \varphi\left(0\right) \ne 0,\ \ \varphi\left(1\right) = \psi\left(1\right) = 0, \qquad 0 < \alpha = const$

Доказана следующая теорема:

ТЕОРЕМА 1. Пусть $\varphi\left(x\right), \psi\left(y\right) \in C^{\infty}\left(D\right),\ \varphi_0 = \varphi\left(0\right) \ne 0,\ \varphi\left(1\right) = \psi\left(1\right) = 0$. Тогда решение (3) задачи (1) - (2), полученное методом малого параметра, является асимптотическим рядом на отрезке $\left[x_0\left(\varepsilon\right), 1\right]$ т.е. $\vartheta\left(x,y\right) = \vartheta_0\left(x,y\right) + \varepsilon\vartheta_1\left(x,y\right) + \varepsilon^2\vartheta_2\left(x,y\right) + \ldots + \varepsilon^n\vartheta_n\left(x,y\right) + O\left(\varepsilon^{(1-\delta)(n+1)}\right),\ \forall x \in \left[x_0\left(\varepsilon\right), 1\right]$

Доказательство этой теоремы проводится методом мажорант.

Равномерно пригодное разложение задачи (1) - (2) построим методом униформизации. Запишем (1) в виде системы

$$\vartheta_x = u, \qquad u_y = \varepsilon\left(u + \vartheta_y\right)u_x \qquad (\mathbf{3})$$

Ключевые слова: асимптотический ряд, метод униформизации, интегральное уравнение, метод мажорант, метод математической индукции, главный член асимптотики.

Предметная классификация AMS: 35M10

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РАЗРЕШИМОСТЬ ОДНОЙ КРАЕВОЙ ЗАДАЧИ ДЛЯ ВЫРОЖДАЮЩЕГОСЯ УРАВНЕНИЯ СМЕШАННОГО ТИПА ЧЕТВЁРТОГО ПОРЯДКА

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В работе доказана теорема о существовании и единственности решения краевой задачи в прямоугольной области для одного вырождающегося уравнения смешанного типа четвёртого порядка.

ВЕСОВЫЕ ОЦЕНКИ РЕШЕНИЯ ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ТРЕТЬЕГО ПОРЯДКА

КОРДАН ОСПАНОВ, АЙГЕРИМ ТОЛЕНОВА, ЖУЛДУЗАЙ УСПАНОВА

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Рассмотрим следующее дифференциальное уравнение

$$-\rho_1(x)(\rho_2(x)(\rho_3(x)y')')' + r\,(x)\,y' + q\,(x)\,y = f(x), \tag{1}$$

где $x \in R = (-\infty, +\infty)$ и $f \in L_2(R)$. Предположим, что $\rho_j > 0$ $(j = 1, 2, 3)$, ρ_1 и ρ_3 дважды непрерывно дифференцируемы, ρ_2 и r непрерывно дифференцируемы, а $q-$ непрерывная функция. Уравнение (1) описывает ряд математических моделей, представляющих интерес в технике, биологии и физике [1].

Пусть $L-$ замыкание в $L_2(R)$ дифференциального оператора $L_0y = -\rho_1(x)(\rho_2(x)(\rho_3(x)y')')' + r(\mathrm{x})\mathrm{y}' + q(\mathrm{x})\mathrm{y}$, определенного в $D(L_0) = C_0^{(3)}(R)$. Решением уравнения (1) назовем функцию $y \in D(L)$ такую, что $Ly = f$.

В докладе мы обсуждаем

а) достаточные условия корректности уравнения (1) и выполнения для его решения следующей оценки максимальной регулярности:

$$\|\rho_1(x)(\rho_2(\rho_3(x)y')')'\|_2 + \|ry'\|_2 + \|(1 + |q|)y\|_2 \leq C\|f\|_2, \tag{2}$$

где $\|\cdot\|_2-$ норма в $L_2(R)$;

б) достаточные условия компактности операторов вида $g(x)L^{-1}$, $h(x)\frac{d}{dx}L^{-1}$ и $\frac{d}{dx}\rho_2\frac{d}{dx}\rho_3L^{-1}$ в $L_2(R)$, где $L^{-1}-$ оператор, обратный к L, а g и $h-$ некоторые заданные функции.

Ранее для уравнения (1) вопросы a) и b) были исследованы в [2] в случае, когда $\rho_j = 1$ $(j = \overline{1,3})$ и в [3, 4] (когда $r = 0$).

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Ключевые слова: дифференциальное уравнение третьего порядка, обобщенное решение, корректность, оценка норм решения

Предметная классификация AMS: 34A30; 34C11

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РАЗВИТИЕ ТЕОРИИ СИНГУЛЯРНО ВОЗМУЩЕННЫХ УРАВНЕНИЙ С АНАЛИТИЧЕСКИМИ ФУНКЦИЯМИ

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В данной работе рассматриваются сингулярно возмущенные обыкновенные дифференциальные уравнения. Работа состоит из двух частей. В первой части изложены наиболее общие результаты полученные Л.С. Понтрягиным, А.Н. Тихоновым, А.Б. Васильевой, М.И. Иманалиевым и другими авторами касающиеся сингулярно возмущенных уравнений с действительно независимой переменной. Во второй части посвящена динамика развитие теории сингулярно возмущенных уравнений с аналитическими функциями, т.е. уравнения правые части которых являются аналитическими функциями по всем переменнным. Независимая переменная принадлежит некоторой области комплексной плоскости.

РАСЩЕПЛЕНИЕ РЕШЕНИЙ СЛАБО НЕЛИНЕЙНЫХ СИНГУЛЯРНО ВОЗМУЩЕННЫХ УРАВНЕНИЙ

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В данной работе рассматривается слабо нелинейное сингулярно возмущенное уравнение в комплексных областях. Поставлена задача о возможности расщепления уравнения на несколько составляющих, введением новых неизвестных функций, получена система из двух уравнений. Далее исследовано асимптотическое поведение решений полученных уравнений в комплексных областях. Доказано, решения каждого из этих уравнений является доминирующей в определенных частях рассматриваемых областей. Решение одной из этих уравнений определяет пограничные линии и области, а решение другой системы определяет регулярную область.

НАЧАЛЬНО-КРАЕВАЯ ЗАДАЧА ДЛЯ НАГРУЖЕННОГО ПСЕВДОПАРАБОЛИЧЕСКОГО УРАВНЕНИЯ ДРОБНОГО ПОРЯДКА

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В работе рассматривается разрешимость нагруженного уравнения по пространственной переменной для линейного псевдопараболического уравнения с начальным и вторым краевым условием (условие Неймана). Исследованию вопросов однозначной разрешимости задач для нагруженных уравнений посвящено множество работ, среди которых отметим работы [1-2], где в библиографии приведено достаточно литературы в этом направлении. Вопросы разрешимости задач с начальными и граничными условиями с операторами дробного дифференцирования для псевдопараболического уравнения изучены в [3-4].

В прямоугольной области $Q_T = \{x \in (0,1);\ t \in [0,T]\}$ рассмотрим нагруженное псевдопараболическое уравнение

$$D_{0,t}^{\alpha}u - D_{0,t}^{\alpha}u_{xx} - u_{xx} + cu = f(x,t) + b_1(x,t)D_{0,t}^{\alpha}u_{xx}(0,t) + b_2(x,t)u_{xx}(0,t), \quad 0 < x < 1,\ 0 < t < T, \tag{1}$$

с начальным

$$u(x,0) = 0, \ \ 0 \le x \le 1, \tag{2}$$

и краевыми условиями

$$u_x(0,t) = 0, \ u_x(1,t) = 0, \ \ 0 \le t \le T, \tag{3}$$

здесь $D_{0,t}^{\alpha}$ – дробная производная Капуто порядка $0 < \alpha \le 1$, $f(x,t)$, $b_i(x,t)\,(i = 0,\,1,\,2)$ и $u_0(x)$ – заданные функции, c – постоянная величина.

Налагая некоторые условия на данные задачи (1)-(3), доказаны существование и единственность сформулированной задачи.

Ключевые слова: Нагруженное псевдопараболическое уравнение, дробная производная Капуто, начально-кравевая задача.
Предметная классификация AMS: 35R11

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ОЦЕНКА ЧИСЛА ПРЕДЕЛЬНЫХ ЦИКЛОВ УРАВНЕНИЯ $\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)}$, ГДЕ P И Q ПОЛИНОМЫ

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Целью нашего исследования является оценка числа комплексных циклов, лежащих на интегральных кривых уравнения

$$\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)}, \ (1)$$

рассматриваемого в комплексном пространстве C^2 переменных x, y.P,Q - полиномы n-ой степени с комплексными коэффициентами. Интегральную кривую Ф уравнения (1) можно рассматривать как двумерную поверхность в четырёхмерном пространстве со своей группой гомологий. Вместе с (1) будем рассматривать и его параметрические представления

$$\frac{dx}{dt} = \frac{P(x,y)}{N(x,y)}, \ \frac{dy}{dt} = \frac{Q(x,y)}{N(x,y)}, \ (2)$$

$$\frac{dx}{dz} = \mathrm{P(x,y)}, \ \frac{dy}{dz} = \mathrm{Q(x,y)}, \ (2')$$

где N многочлен n-ой степени с комплексными коэффициентами. Даказано следующая

Теорема 6.. Число действительных предельных циклов уравнения (1) с действительными коэффициентами не превышает $2(2n^2+n+1)$.При этом число не алгебраических предельных циклов не превышает $2n^2+n+1$.

РЕШЕНИЕ ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ В ЧАСТНЫХ ПРОИЗВОДНЫХ С АВТОРЕГУЛИРОВАНИЕМ МЕТОДОМ ОСЦИЛЛИРУЮЩИХ ФУНКЦИЙ

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Рассмотрим дифференциальное уравнение в частных производных с авторегулированием вида:

$$\frac{\partial u(x,t)}{\partial t} = f\Big(x, t, u\,(x,t)\,, u\big(x, t-\tau\,(x,t,u\,(x,t))\,\big)\Big), \qquad t \geq t_0 \tag{1}$$

с условиями:

$$u(x,t) = \varphi(x,t) \quad \text{при} \qquad (x,t) \in E_{t,0} = [a,b] \times (-\infty, t] \tag{2}$$

где функции $\tau(x,t,u) \geq 0$ определены и имеют ограниченные частные производные по всем аргументам при $a \leq x, s \leq b, t_0 \leq t \leq T, \mid u \mid \leq r, \mid v \mid \leq r_1$, а начальная функция $\varphi(x,t)$ определена и имеет ограниченные частные производные в области E_{t_0}.

Кроме того, выполняются следующие условия (3):

a) $f(x,t,u_2,v_2) - f(x,t,u_1,v_1) \leq L_1\,(u_2 - u_1 + v_2 - v_1)$;

b) $\tau(x,t,u_2) - \tau(x,t,u_1) \leq L_2\,(u_2 - u_1)$;

c) $f \leq M; f_x \leq M_1; f_t \leq M_2; f_u \leq M_3; f_{u_t} \leq M_4$;

d) $\tau_x \leq M_5; \tau_u \leq M_6; u - \varphi\,(x,t_0) \leq \alpha + \varepsilon, \varphi' \leq Q$.

Решение задачи (1) – (3) существует и является единственным. Приближенное решения задачи (1)-(3) строится методом осциллирующих функций.Оценивается погрешность построенного приближенного решения, сходимость к точному решению. Основная сложность в построении приближенного решения заключается в определении области, в которой находится значение разности $t - \tau(x,t,u(x,t))$.

Ключевые слова: Дифференциальные уровнения в частных производных. Авторегулирование. Осциллирующие функции. Метод осициллирующих функций

Классификация предметов AMS: 35A22

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СПЕКТРАЛЬНЫЕ СВОЙСТВА ЗАДАЧ С УСЛОВИЯМИ БИЦАДЗЕ-САМАРСКОГО ДЛЯ ДИФФУЗИОННО-ВОЛНОВОГО УРАВНЕНИЯ ДРОБНОГО ПОРЯДКА

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В докладе будут изложены результаты исследования по вопросам разрешимости и спектральных свойств нелокальных задач для диффузионно-волнового уравнения. Установлена однозначная разрешимость задач, и доказаны теоремы о существовании собственных значений либо вольтерровости рассматриваемых задач. Рассмотрим уравнение

$$\begin{cases} D_{0x}^{\alpha} u\left(x,y\right) - u_{yy}\left(x,y\right) = f\left(x,y\right), \ (x,y) \in \Omega_0 \\ u_{xx} - u_{yy} = f\left(x,y\right), \ (x,y) \in \Omega_1 \end{cases} \tag{1}$$

где $D_{0x}^{\alpha} u\left(x,y\right)$ - интегро-дифференциальный оператор дробного порядка α в смысле Капуто, $0 < \alpha < 1, \Omega_0$ - прямоугольник ABB_0A_0 с вершинами $A\left(0,0\right), B\left(1,0\right), B_0\left(1,1\right), A_0\left(0,1\right)$ Ω_1 -область ограниченная отрезкамиAB и характеристиками $AC: x+y=0,\ BC:\ x-y=1$ уравнения (1), $f(x,y)$-заданная функция. Пусть гладкая кривая $BD:\ y=-\gamma\left(x\right), l \leq x \leq 1,\ \gamma\left(1\right)=0$ и $l-\gamma\left(l\right)=0$, если $0<l<0,5$, и $\gamma\left(l\right)=0$ если $l=0$ (в случае $D=A$) расположена внутри характеристического треугольника $0<x+y\leq x-y<1$. Относительно кривой $\gamma\left(x\right)$, предположим, что $\gamma\left(x\right)$ - дважды непрерывно дифференцируемая функция и $x\pm\gamma\left(x\right)$ – монотонно возрастающие функции, причем $0<\gamma'(x)<1,\ \gamma(x)>0,\ x>0$.

Задача M_2A**.**(один из задач) Найти решение уравнения (1), удовлетворяющее условиям

$$u\left(x,y\right)=0, \quad \left(x,y\right)\in AA_0\cup A_0B_0 \tag{2}$$

$$\left[u_x+u_y\right]\left[\theta\left(t\right)\right]+\mu\left(t\right)\left[u_x+u_y\right]\left[\theta^*\left(t\right)\right]=0,\ 0<t<1 \tag{3}$$

где $\theta\left(t\right),\left[\theta^*\left(t\right)\right]-$ аффикс точки пересечения характеристики AC (кривой AE) с характеристиками выходящей из точки $(t,0,)$ $0<t<1$, $\mu\left(t\right)$ – заданная функция. Справедлива следующая

Теорема. Пусть $\mu\left(t\right)\in C^2\left[0,1\right]$ и $\mu\left(t\right)\neq-1,\ 0\leq t\leq 1$ Тогда задача M_2A имеет собственное значение (хотя бы одно). Часть результатов доклада приведены в работах [1-2].

Данная работа профинансирована Комитетом науки МНВО РК (грант №AP 09058677)

Ключевые слова: Разрешимость краевой задачи, Собственные значения, уравнения дробного порядка.

Предметная классификация AMS: 35R11.

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ТРЕТЬЯ КРАЕВАЯ ЗАДАЧА ДЛЯ УРАВНЕНИЯ ТРЕТЬЕГО ПОРЯДКА С ПЕРЕМЕННЫМИ КОЭФФИЦИЕНТАМИ

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В области $D=\{(x,y):\ \ 0<x<p,\ 0<y<q\}$ рассмотрим следующее уравнение третьего порядка в вида

$$u_{xxx}-u_{yy}+a_1(x)\,u_x+a_2(x)\,u=g(x,y), \tag{1}$$

где $a_i(x)\,,\ i=\overline{1,2},\ g(x,y)$ заданные достаточно гладкие функции.

Задача B_3**.** Найти функцию $u(x,y)$ из класса $C^{3,2}_{x,y}(D)\cap C^{2,1}_{x,y}\left(\overline{D}\right)$, удовлетворяющую уравнению (1) и следующим краевым условиям:

$$\begin{cases}\alpha u(x,0)+\beta\, u_y(x,0)=0,\\ \gamma u(x,q)+\delta\, u_y(x,q)=0,\end{cases}\quad 0\le x\le p,$$

$$u(0,y)=\psi_1(y)\,,\quad u_x(p,y)=\psi_2(y)\,,\quad u_{xx}(p,y)=\psi_3(y)\,,\quad 0\le y\le q,$$

где $\psi_i(y)\,,\ i=\overline{1,3}$, заданные функции.

В работе [1] решение поставленной задачи для уравнения третьего порядка с постоянными коэффициентами было найдено с другими краевыми условиями.
Доказаны следующие теоремы:

Теорема 1. Если задача B_3 имеет решение, то при выполнении условий $a_1(x)\ge 0,\ \ a_2(x)-\frac{1}{2}a_1{}'(x)\ge 0,\ \alpha\beta\le 0,\ \gamma\delta\ge 0$ оно единственно.

Теорема 2. Если выполняются следующие условия:

1) $0\le C<\dfrac{\lambda_1^2}{Kp(1+\lambda_1)}$, 2) $a_1(p)=0$, 3) $\psi_i(y)\in C^4[0,q]\,,\ i=\overline{1,3}$,

$\alpha\psi_i(0)+\beta\psi_i{}'(0)=0,\ \ \gamma\psi_i(q)+\delta\psi_i{}'(q)=0,\ \ \alpha\psi''{}_i(0)+\beta\psi'''{}_i(0)\,,\quad \gamma\psi''{}_i(q)+\delta\psi'''{}_i(q)=0,$

4) $\dfrac{\partial^3 g(x,y)}{\partial x\partial y^2}\in C[0,q]\,,\ 0\le x\le p;\quad \alpha g(x,0)+\beta g'(x,0)=0,\quad \gamma g(x,q)+\delta g'(x,q)=0,$

то решение задачи B_3 существует.

Здесь $C=\max\{|a_1(x)|\,,|a_1{}'(x)-a_2(x)|\,,x\in[0,p]\}$, $\lambda_1=\sqrt[3]{\left(\frac{\pi}{q}\right)^2}$, $K=\frac{4}{3}\left(1-\exp\left(-\frac{2\sqrt{3}\pi}{3}\right)\right)^{-1}$.

Ключевые слова: Дифференциальное уравнение, третий порядок, кратные характеристики, функция Грина.

Предметная классификация AMS: 35G15

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КОРРЕКТНОСТЬ ОСНОВНОЙ СМЕШАННОЙ ЗАДАЧИ ДЛЯ ОДНОГО КЛАССА МНОГОМЕРНЫХ ГИПЕРБОЛО-ЭЛЛИПТИЧЕСКИХ УРАВНЕНИЙ

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Основная смешанная задача для многомерных гиперболических уравнений в пространстве обобщенных функций хорошо изучена. В работах доказана однозначная разрешимость этой задачи для многомерных гиперболических и эллиптических уравнений.

Для многомерных гиперболо-эллиптических уравнений эта задача еще не исследованы.

Пусть $\Omega_{\alpha\beta}$ – цилиндрическая область евклидова пространства E_{m+1} точек $(x_1, ..., x_m, t)$, ограниченная цилиндром $\Gamma = \{(x,t) : : |x| = 1\}$, плоскостями $t = \alpha > 0$ и $t = \beta < 0$, где $|x|-$ длина вектора $x = (x_1, ..., x_m)$.

Обозначим через Ω_α и Ω_β части области $\Omega_{\alpha\beta}$, а через Γ_α, Γ_β – части поверхности Γ, лежащие в полупространствах $t > 0$ и $t < 0$; $\sigma_\alpha -$. верхнее, а $\sigma_\beta-$ нижнее основание области $\Omega_{\alpha\beta}$.

В области $\Omega_{\alpha\beta}$ рассмотрим многомерные гиперболо-эллиптические уравнения

$$\Delta_x u - (sgnt)u_{tt} + \sum_{n=1}^{m} a_i(x,t)u_{x_i} + b(x,t)u_t + c(x,t)u = 0. \tag{1}$$

Задача 1. Найти решение уравнения (1) в области $\Omega_{\alpha\beta}$ при $t \neq 0$ из класса $C(\bar{\Omega}_{\alpha\beta})\cap$ $\cap C^1(\Omega_{\alpha\beta}) \cap C^2(\Omega_\alpha \cup \Omega_\beta)$, удовлетворяющее краевым условиям

$$u\Big|_{\Gamma_\alpha} = \psi_1(t,\theta), u\Big|_{\Gamma_\beta} = \psi_2(t,\theta),\ u\Big|_{\sigma_\beta} = \tau(r,\theta),\ u_t\Big|_{\Gamma_\beta} = \nu(r,\theta), .$$

Показано, что задача 1 имеет единственное решение и получен ее явный вид.

Ключевые слова: корректность, основная смешанная задача, гиперболо-эллиптические уравнения, цилиндрическая область, функция Бесселя.

Предметная классификация AMS: MSC 30C45.

ОБ ОЦЕНКЕ ГИПЕРСИНГУЛЯРНОГО ОПЕРАТОРА, СВЯЗАННОГО С ПЕРИДИНАМИКОЙ

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Рассмотрим в пространстве 2π-периодических вектор-функций сингулярный интегральный оператор

$$Af(x) \;=\; \int_{\mathbb{T}^n} \Omega(y)[f(x-y)-f(x)]\,|y|^{-n}\chi(|y|)\,dy,$$

играющий основную роль в уравнении перидинамики, предложенном в работе [2]. Здесь $\Omega(x)$ является $(n\times n)$-матрицей-функцией, гладкой в области $\mathbb{R}^n\setminus\{0\}$ и для любого $\lambda>0$ удовлетворяющей условию однородности: $\Omega(\lambda x) \;=\; \Omega(x),\; x\in\mathbb{R}^n\setminus\{0\}$. Функция $\chi(r)\geq 0$ предполагается принадлежащей $C^\infty(\mathbb{R})$, равной 1 в некоторой окрестности нуля и равной 0 при $r\geq\pi$.

Обозначим символом Ω^* постоянную матрицу, представляющую собой среднее значение матрицы $\Omega(x)$ по единичной сфере. В случае, когда $\Omega^*=0$, оператор A является оператором типа Кальдерона-Зигмунда (см. [1]).

Введем в рассмотрение оператор $\Lambda=\sqrt{1-\Delta}$, где $\Delta-$ оператор Лапласа, самосопряжённый в $L_2(\mathbb{T}^n)$.

Для любого $s\geq 0$ рассмотрим положительный оператор $\log^s(1+\Lambda)$. Область определения этого оператора обозначим символом $H^s_{\log}(\mathbb{T}^n)=D(\log^s(1+\Lambda))$.

Положим $\|f\|_s \;=\; \|\log^s(1+\Lambda)f\|_{L_2(\mathbb{T}^n)}$. Норму матрицы Ω^* обозначим $\|\Omega^*\|$.

Теорема. *Для любого $s\geq 1$ оператор A действует из $H^s_{\log}(\mathbb{T}^n)$ в $H^{s-1}_{\log}(\mathbb{T}^n)$ и удовлетворяет оценке*

$$\|Af\|_{s-1} \;\leq\; C\|\Omega^*\|\cdot\|f\|_s \;+\; C\|f\|_{s-1}.$$

Доказательство теоремы проводится методом, применённым в работе [3].

Ключевые слова: singular operators, Calderon-Zygmund inequality, peridynamics.

Предметная классификация AMS: 45E05, 45F15.

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ИНВОЛЮЦИЯСЫ БАР ДИФФЕРЕНЦИАЛДЫ ТЕҢДЕУ ҮШІН АНТИПЕРИОДТЫ ЕСЕПТІҢ МЕНШІКТІ ФУНКЦИЯЛАРЫ

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Инволюциясы бар екінші ретті дифференциалдық теңдеу үшін

$$-y''(x)+\alpha y''(-x)+q(x)\,y(x)=\lambda y(x)\,,\ -1<x<1,\qquad (1)$$

антипериодты есепті

$$y(-1)=-y(1)\,,\ y'(-1)=-y'(1)\qquad (2)$$

қарастырамыз, $-1<\alpha<1$. Бұл есеп өз-өзіне түйіндес емес спектрлік есеп болып табылады, өйткені $q(x)=q_1(x)+iq_2(x)$ коэффициенті комплекс мәнді функция болып табылады, $q(x)\in L_1(-1,1)$ Түйіндес спектрлік есеп келесі түрде болады:

$$-z''(x)+\alpha z''(-x)+\bar{q}(x)\,z(x)=\bar{\lambda}z(x)$$

Теңдеудің коэффициенті тұрақты болған жағдайда

$$-y''(x)+\alpha y''(-x)=\lambda y(x)\,,\ -1<x<1,\ y(-1)=-y(1)\,,\ y'(-1)=-y'(1)\qquad (3)$$

есебінің меншікті функциялары

$$\{y_k(x)\}=\left\{\ y_{k1}=sin(k+\frac{1}{2})\pi x,\ l=1,2,...;\ y_{k2}=\cos(k+\frac{1}{2})\pi x,\ \ k=0,1,2,...,\right\}$$

$L_2(-1,1)$ кеңістігінде толық ортонормаланған жүйе құрайды. Осы жүйе мен (1), (2) есебінің меншікті функцияларын салыстыру арқылы келесі нәтиже алынды.

1-теорема. Егер берілген (1), (2) және (3) спектрлік есептерінің барлық меншікті мәндері еселі емес болса, онда (1), (2) есебінің меншікті функциялар жүйесі $L_2(-1,1)$ кеңістігінің базисін құрайды.

Мұндай нәтиже өзгеше шеттік шарттар үшін [1], [2] жұмыстарында дәлелденген.

Бұл жұмыс Қазақстан Республикасы Ғылым және жоғары білім министрлігінің Ғылым комитеті тарапынан қаржыландырылған (грант AP13068539).

Кілттік сөздер: инволюция, меншікті функция, дифференциалдық теңдеу, спектралдық есеп.

AMS пәндік классификация: 39

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О РАЗРЕШИМОСТИ ТРЕТЬЕЙ КРАЕВОЙ ЗАДАЧИ ДЛЯ УРАВНЕНИЯ ЧЕТВЕРТОГО ПОРЯДКА С КРАТНЫМИ ХАРАКТЕРИСТИКАМИ

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Для уравнения

$$L[u] = u_{xxxx}(x,y) + a_1(x)u_{xx}(x,y) + a_2(x)u_x(x,y) + a_3(x)u(x,y) - u_{yy}(x,y) = f(x,y), \quad (1)$$

где $f(x,y)$, $a_i(x)$, $i=\overline{1,3}$ – заданные достаточно гладкие функции, в области $\Omega = \{(x,y) : 0<x<p,\ 0<y<q;\ p,q\in R\}$ изучим следующую задачу.

Задача A. Найти функцию $u(x,y)$ из класса $C^{4,2}_{x,y}(\Omega)\cap C^{3,1}_{x,y}(\bar{\Omega})$, удовлетворяющую уравнению (1) в области Ω и следующим краевым условиям:

$$\alpha u(x,0)+\beta u_y(x,0)=0, \quad \gamma u(x,q)+\delta u_y(x,q)=0, \quad 0\le x\le p,$$

$$u(0,y)=\psi_1(y), \quad u(p,y)=\psi_2(y), \quad u_{xx}(0,y)=\psi_3(y), \quad u_{xx}(p,y)=\psi_4(y), \quad 0\le y\le q,$$

где $\alpha,\beta,\gamma,\delta\in R\backslash\{0\}$ и $\psi_i(y)$, $i=\overline{1,4}$ – заданные достаточно гладкие функции.

Теорема 1. Если задача A имеет решение, то при выполнении условий $\alpha\beta<0$, $\gamma\delta>0$, $a_1(x)\le 0$, $a_1''(x)-a_2'(x)+2a_3(x)\ge 0$, оно единственно.

Теорема 2. Если выполняются следующие условия:

1) $C<\dfrac{\mu_1^3\left(1-e^{-2\mu_1 p}\right)^2}{p\left(2\mu_1^2+3\mu_1\left(1+e^{-4\mu_1 p}\right)+3\right)}$;

2) $\alpha\psi_i^{(j)}(0)+\beta\psi_i^{(j+1)}(0)=0$, $\gamma\psi_i^{(j)}(q)+\delta\psi_i^{(j+1)}(q)=0$, $\psi_i(y)\in C^3[0,q]$, $i=\overline{1,4}$, $j=0,2$;

3) $\alpha f(x,0)+\beta f_y(x,0)=0$, $\gamma f(x,q)+\delta f_y(x,q)=0$, $0\le x\le p$, $f_{xyy}(x,y)\in C(\bar{\Omega})$,

то решение задачи A существует, здесь

$$C=\max_{\xi\in[0,p]}\left\{\left|a_i^{(j)}(\xi)\right|,\ \left|a_1''(\xi)\right|,\ i=\overline{1,3},\ j=0,1\right\}, \quad \mu_1=\sqrt[4]{\lambda_1/4}.$$

Единственность решения поставленной задачи доказана методом интегралов энергии. Решение выписано через построенную функцию Грина.

Ключевые слова: Уравнение четвертого порядка, кратные характеристики, младшие члены, краевая задача, единственность, существование, функция Грина.

Предметная классификация AMS: 35C10, 35G15

Функциялар теориясы және функционалдық талдау

Theory of functions and functional analysis

Теория функций и функциональный анализ

VII World Congress of Turkic World Mathematicians
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QUASI-TRACES ON EXACT REAL C^*-ALGEBRAS

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It is shown that all 2-quasitraces on a unital exact real C^*-algebra are traces. As consequences one gets: (1) Every stably finite exact unital real C^*-algebra has a tracial state, and (2) any exact real AW^*-factor of type II_1 is a real W^*-factor of type II_1.

INTERPOLATION METHODS WITH PARAMETRIC FUNCTIONS

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In this work we construct interpolation methods with parametric functions that can be used to study the interpolation properties of spaces with mixed metrics.

Let $1 \leq \bar{q} = (q_1, q_2) \leq \infty$, $\bar{\varphi}(t) = (\varphi_1(t), \varphi_2(t)) \geq 0$. We define anisotropic Lorentz spaces as follows:

$$\Lambda_{\bar{q}}(\bar{\varphi}) := \left\{ f : \ \left(\int_0^{+\infty} \left(\int_0^{+\infty} (f^{*_1 *_2}(t_1, t_2) \varphi_1(t_1) \varphi_2(t_2))^{q_1} \frac{dt_1}{t_1} \right)^{\frac{q_2}{q_1}} \frac{dt_2}{t_2} \right)^{\frac{1}{q_2}} < \infty \right\},$$

where $f^{*_1 *_2} = f^{*_1 *_2}(t_1, t_2)$ is the nonincreasing permutation of a function f [1]. In the paper [2] were studied one-dimensional generalized Lorentz spaces.

In this work the interpolation theorem for Lebesgue and Lorentz spaces with mixed metrics are obtained.

Keywords: Lebesgue and Lorentz spaces, interpolation methods, interpolation theorem.

AMS Subject Classification: 46B70, 46E30.

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ULYANOV TYPE INEQUALITIES FOR THE MIXED MODULI OF SMOOTHNESS

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Let $L_{p_1p_2}, 1 \leq p_i \leq \infty, i = 1,2$ be the set of measurable functions of two variables $f(x_1, x_2)$, 2π - periodic in each variable, for which $\|f\|_{p_1p_2} = ||\{||f||_{p_1}\}||_{p_2} < \infty$, where

$$\|f\|_{p_i} = \left(\int\limits_0^{2\pi} |f|^{p_i} dx_i \right)^{\frac{1}{p_i}}, \text{ if } 1 \leq p_i < \infty,$$

$$\|f\|_{p_i} = \sup_{0 \leq x_i \leq 2\pi} |f|, \text{ if } p_i = \infty.$$

Let $L^0_{p_1p_2}$ be the space of functions $f \in L_{p_1p_2}$ such that $\int\limits_0^{2\pi} f(x_1, x_2)dx_1 = 0$ for almost all x_2 and $\int\limits_0^{2\pi} f(x_1, x_2)dx_2 = 0$ for almost all x_1.

For the function $f \in L_{p_1p_2}$, we define the fractional differences of positive order α_1 and α_2 with steps h_1 and h_2 respectively, by variables x_1 and x_2 as follows:

$$\Delta^{\alpha_1}_{h_1}(f) = \sum_{\nu_1=0}^{\infty} (-1)^{\nu_1} \binom{\alpha_1}{\nu_1} f(x_1 + (\alpha_1 - \nu_1)h_1, x_2),$$

$$\Delta^{\alpha_2}_{h_2}(f) = \sum_{\nu_2=0}^{\infty} (-1)^{\nu_2} \binom{\alpha_2}{\nu_2} f(x_1, x_2 + (\alpha_2 - \nu_2)h_2),$$

where $\binom{\alpha}{\nu} = 1$ for $\nu = 0$, $\binom{\alpha}{\nu} = \alpha$ for $\nu = 1$, $\binom{\alpha}{\nu} = \frac{\alpha(\alpha-1)...(\alpha-\nu+1)}{\nu!}$ for $\nu \geq 2$.

Denote (cf. [6]) by $\omega_{\alpha_1,\alpha_2}(f, \delta_1, \delta_2)_{p_1p_2}$ the mixed modulus of smoothness of positive orders α_1 and α_2, respectively, in the variables x_1 and x_2 of a function $f \in L_{p_1p_2}$, that is,

$$\omega_{\alpha_1,\alpha_2}(f, \delta_1, \delta_2)_{p_1p_2} = \sup_{|h_i| \leq \delta_i, i=1,2} ||\Delta^{\alpha_1}_{h_1}(\Delta^{\alpha_2}_{h_2}(f))||_{p_1p_2}.$$

The following (p, q)-inequality between moduli of smoothness in different metrics, nowadays called sharp Ulyanov type inequalities, is known (see [11]):

$$\omega_1(f, \delta)^{(1)}_q \ll \left(\int\limits_0^{\delta} \left(t^{-\theta} \omega_1(f, t)^{(1)}_p \right)^{q^*} \frac{dt}{t} \right)^{\frac{1}{q^*}},$$

where $1 \leq p < q \leq \infty, \theta = \frac{1}{p} - \frac{1}{q}$. For the Lebesgue spaces, Ulyanov type inequalities in the one-dimensional case have been studied for the moduli of smoothness of any positive order by

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many authors, in particular, Ulyanov [13], V.I. Kolyada [3], O. Domingues [1], B. Simonov [11], S. Tikhonov [12].

In the multidimensional case Ulyanov type inequalities were considered by many authors. Inequalities for the total moduli of smoothness of two variables were obtained in [1], [4], [5],[11], and [12]. Ulyanov type inequalities were also proved for mixed moduli of smoothness in ([6]-[10]). The (p,q) inequalities for the moduli of smoothness of derivatives in terms of moduli of smoothness of the function itself were established in [2], [4].

Theorem 1. Let $f \in L^0_{p_1p_2}$, where $1 < p_1 < q_1 < \infty$ or $1 = p_1 < q_1 = \infty$ and $1 < p_2 < q_2 < \infty$ or $1 = p_2 < q_2 = \infty$. Let for $\alpha_i > 0, \delta_i \in (0,1), i = 1,2$, we have

$$\omega_{\alpha_1,\alpha_2}\big(f,\delta_1,\delta_2\big)_{q_1q_2} \ll$$
$$\Big(\int_0^{\delta_2}\Big(\int_0^{\delta_1}\Big(t_1^{-\frac{1}{p_1}+\frac{1}{q_1}}t_2^{-\frac{1}{p_2}+\frac{1}{q_2}}\omega_{\alpha_1+\frac{1}{p_1}-\frac{1}{q_1},\alpha_2+\frac{1}{p_2}-\frac{1}{q_2}}(f,t_1,t_2)_{p_1p_2}\Big)^{q_1^*}\frac{dt_1}{t_1}\Big)^{\frac{q_2^*}{q_1^*}}\frac{dt_2}{t_2}\Big)^{\frac{1}{q_2^*}}. \tag{1}$$

Theorem 2. Let $f \in L^0_{p_1p_2}$, where $1 < p_1 < q_1 < \infty$ or $1 = p_1 < q_1 = \infty$ and $1 < p_2 < q_2 < \infty$ or $1 = p_2 < q_2 = \infty$. Let for $\alpha_i > 0, \rho_i \geq 0, \delta_i \in (0,1), i = 1,2.$ we have

$$\omega_{\alpha_1,\alpha_2}\big(f^{(\rho_1,\rho_2)},\delta_1,\delta_2\big)_{q_1q_2} \ll$$
$$\Big(\int_0^{\delta_2}\Big(\int_0^{\delta_1}\Big(t_1^{-\rho_1-\frac{1}{p_1}+\frac{1}{q_1}}t_2^{-\rho_2-\frac{1}{p_2}+\frac{1}{q_2}}\omega_{\alpha_1+\rho_1+\frac{1}{p_1}-\frac{1}{q_1},\alpha_2+\rho_2+\frac{1}{p_2}-\frac{1}{q_2}}(f,t_1,t_2)_{p_1p_2}\Big)^{q_1^*}\frac{dt_1}{t_1}\Big)^{\frac{q_2^*}{q_1^*}}\frac{dt_2}{t_2}\Big)^{\frac{1}{q_2^*}}.$$

Keywords: Mixed moduli of smoothness of positive order, Ulyanov type inequality.

AMS Subject Classification: 42A10, 41A25,26A48.

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BOUNDEDNESS OF A CLASS OF MATRIX OPERATORS IN WEIGHTED SEQUENCE SPACES

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Let $1 < p, q < \infty$, $\frac{1}{p} + \frac{1}{p'} = 1$. Let $u = \{u_i\}, v = \{v_i\}$ be the weight sequences, i.e. the sequences of positive numbers. Let l_{pv} the space of all sequences $f = \{f_i\}_{i=1}^{\infty}$ of real numbers such that $\|f\|_{pv} = \left(\sum\limits_{i=1}^{\infty} |v_i f_i|^p\right)^{\frac{1}{p}}$, $1 \le p < \infty$.

We consider the problem of boundedness of the following matrix operators

$$\left(A^+ f\right)_i = \sum_{j=1}^{i} a_{ij} f_j,\ i \ge 1, \tag{1}$$

$$\left(A^- f\right)_j = \sum_{i=j}^{\infty} a_{ij} f_i,\ j \ge 1 \tag{2}$$

from l_{pv} into l_{qu}, where $a_{ij} > 0, i \ge j \ge 1$, i.e. the vaidity of the inequality

$$\|A^{\pm} f\|_{qu} \le C\|f\|_{pv}, \forall f \in l_{pv}.$$

In this work we obtain necessary and sufficient conditions of boundedness of matrix operators (1) and (2) from l_{pv} into l_{qu} for $1 < q < p < \infty$, when matrix operators belong to the classes satisfying weaker conditions than Oinarov's condition.

Keywords: matrix operator, boundedness, weighted inequalities, weighted Lebesgue space, Oinarov's condition, Hardy operator, Hardy inequality.

AMS Subject Classification: 26D10, 26D15, 47B37

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ON THE INTERPOLATION PROPERTIES OF CERTAIN CLASSES OF DISCRETE NET SPACE

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Abstract. In this paper we study the interpolation properties of the net spaces $n_{p,q}(M)$, when M is the set of all segments from $\mathbb{Z}$. It is shown that in this case the scale of spaces is closed with respect to the real interpolation method. As a corollary, an interpolation theorem of Marcinkevich type is presented.

Let S be the set of all finite sets of indices from $\mathbb{Z}^n$. For a fixed set $M \subset S$ we define the space $n_{p,q}(M)$ $(0 < p, q \leq \infty)$ as the set of sequences $a = \{a_m\}_{m\in\mathbb{Z}^n}$ with quasinorm for $0 < p < \infty$, $0 < q < \infty$

$$\|a\|_{n_{p,q}(M)} = \left(\sum_{k=1}^{\infty} k^{\frac{q}{p}-1}(\bar{a}_k(M))^q\right)^{\frac{1}{q}},$$

and for $q = \infty$, $0 < p \leq \infty$

$$\|a\|_{n_{p,\infty}(M)} = \sup_{1\leq k<\infty} k^{\frac{1}{p}}\bar{a}_k(M),$$

where

$$\bar{a}_k(M) = \sup_{\substack{e\in M\\ |e|\geq k}} \frac{1}{|e|}\left|\sum_{m\in e} a_m\right|,$$

where $|e|$ is the number of indices in e.

These spaces were introduced in [6], and they were called net spaces.

Net spaces have found important applications in various problems of harmonic analysis, operator theory and theory of stochastic processes [1,2,3,7,8,9,10,11]. In this paper, we study the interpolation properties of these spaces. It should be noted here that net spaces are in a sense close to the discrete Morrey spaces:

$$m_p^{\lambda} = \left\{ a = \{a_k\}_{k\in\mathbb{Z}} : \sup_{m\in\mathbb{N}}\sup_{k\in\mathbb{Z}} \frac{1}{m^{\lambda}}\left(\sum_{r=k}^{k+m} |a_r|^p\right)^{\frac{1}{p}} < \infty \right\}.$$

In the case when $a = \{a_k\}_{k\in\mathbb{Z}}$, $a_k \geq 0$, for $\lambda = n\left(1 - \frac{1}{p}\right)$

$$\|a\|_{n_{p,\infty}(M)} \asymp \|a\|_{m_1^{\lambda}}.$$

The question of interpolation of Morrey spaces was considered in the works [5,12] and it was shown that this scale of spaces is not closed with respect to the real interpolation method.

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In this paper we show that if M is the set of all segments from $\mathbb{Z}$ the scale of spaces is closed with respect to the real interpolation method, i.e. the following relation holds

$$(n_{p_0,q_0}(M), n_{p_1,q_1}(M))_{\theta,q} = n_{p,q}(M). \tag{1}$$

Let (A_0, A_1) be a compatible pair of Banach spaces [4]. Let

$$K(t,a;A_0,A_1) = K(t,a) = \inf_{a=a_0+a_1}(\|a_0\|_{A_0} + t\|a_1\|_{A_1}),\ a \in A_0 + A_1,$$

be the functional Petre. For $0 < q < \infty$, $0 < \theta < 1$

$$(A_0,A_1)_{\theta,q} = \left\{ a \in A_0 + A_1 : \|a\|_{(A_0,A_1)_{\theta,q}} = \left(\int_0^\infty (t^{-\theta}K(t,a))^q \frac{dt}{t} \right)^{1/q} < \infty \right\},$$

and for $q = \infty$

$$(A_0,A_1)_{\theta,q} = \left\{ a \in A_0 + A_1 : \|a\|_{(A_0,A_1)_{\theta,q}} = \sup_{0<t<\infty} t^{-\theta}K(t,a) < \infty \right\}.$$

Theorem 0.1. *Let $1 \le p_0 < p_1 < \infty$ and $0 < q_0, q_1, q \le \infty$. Let M be the set of all segments from $\mathbb{Z}$. Then*

$$\left(n_{p_0,q_0}(M), n_{p_1,q_1}(M)\right)_{\theta,q} = n_{p,q}(M),$$

where $\frac{1}{p} = \frac{1-\theta}{p_0} + \frac{\theta}{p_1}$, $\theta \in (0,1)$.

Keywords: Net spaces, discrete Net spaces, Marcinkiewicz type interpolation theorem.

AMS Subject Classification: 46B70

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DESCRIPTION OF THE POLAR CAPTURE OPERATOR

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Consider a real Hilbert space H, a subset U of this space, and its polar:

$$U^{\circ} = \{x \in H : (u, x) \leq 1, \ \forall u \in U\}.$$

Denote by *conv* the collection of all convex closed subsets of this space containing zero. The polar U° of the set $U \in conv$ is denoted by $\pi(U)$. The same properties as $\pi(U)$ have the operator $-\pi$, where $(-\pi)(U) = \{y \in H : (y, x) \geq -1, \ \forall x \in U\}$.

Consider an operator $P : conv \to conv$ with the following properties:

$a)$ $P(B) = B$,

$b)$ $P(AU) = (A^{*})^{-1} P(U)$ for any $U \in conv$ and the following operators (here A^{*} is the operator adjoin to A):

$b_1)$ $A = cI$, where I is the identity operator, c is a number

$b_2)$ $A = A_v$, where A_v is the operator of orthogonal projection onto a one-dimensional subspace passing through the vector v. If $\|v\| = 1$ then $A_v x = (x, v)\, v$

$b_3)$ $A = \overline{A_v}$, where $\overline{A_v}$- is the orthogonal projection operator onto the hyperplane orthogonal to the vector v. If $\|v\| = 1$, then $\overline{A_v} x = x - (x, v)\, v$,

$b_4)$ $A = A_{u,v,\alpha}$, where $A_{u,v,\alpha}$ is the "two-dimensional rotation" operator: $A_{u,v,\alpha} = G_{\alpha} \circ \mathrm{Pr}_{u,v}$; here is $\mathrm{Pr}_{u,v}$- the orthogonal projection operator onto the plane spanned by the vectors u, v; G_{α}- the rotation operator by the angle α defined on the plane.

$c)$ İf $U_{\lambda} \in conv$ $(\lambda \in \Lambda)$, then

$c_1)$ $P\left(\overline{co} \bigcup\limits_{\lambda \in \Lambda} U_{\lambda}\right) = \bigcap\limits_{\lambda \in \Lambda} P(U_{\lambda})$ for any set of indices Λ,

$c_2)$ $P\left(\bigcap\limits_{\lambda \in \Lambda} U_{\lambda}\right) = \overline{co} \bigcup\limits_{\lambda \in \Lambda} P(U_{\lambda})$U for a finite set of indices Λ.

Theorem. Let the operator U have the properties a), b), c) . Then either $P = \pi$ or $P = -\pi$.

Keywords: Hilbert space, polar capture operator, two-dimensional rotation operator,hyperplane orthogonal .

AMS Subject Classification: 46C07 Hilbert subspaces (= operator ranges); complementation (Aronszajn, de Branges, etc.) [See also 46B70, 46M35]

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ABOUT THE MAIN PRINCIPLES AND SPACES OF FUNCTIONAL ANALYSIS

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Abstract. This paper discusses the principles of extending the class of Banach (normalized) spaces to the class of τ-Banach (τ-normalized) spaces, which in turn are the basic principles of functional analysis.

The general principles of functional analysis: the principle of openness of continuous linear operators, the principle of continuation of continuous linear functionals and the principle of boundedness of continuous linear operators play an important role in functional analysis and have numerous applications in various fields of mathematics.

Let $\mathbb{R}_+ = [0,\infty), \mathbb{R} = (-\infty,\infty)$ and τ an infinite cardinal number. Let $\mathbb{R}_+^\tau$ and $\mathbb{R}^\tau$ denote the Tichonoff product τ-pieces of copies of the spaces $\mathbb{R}_+$ and $\mathbb{R}$ corresponding to the natural topology. The τ-normalized space is defined as the definition of a norm with the replacement of R_+ by $\mathbb{R}_+^\tau$. A τ-Banach space is a complete τ-normed space.

Theorem 1 (on open mapping) [2]. *If $f : (L, \|\cdot\|) \to (L', \|\cdot\|_\tau{}')$ is a continuous linear function of the τ-Banach space $(L, \|\cdot\|)$ onto the τ-Banach space $(L', \|\cdot\|_\tau{}')$, then the mapping f is open.*

Theorem 2 (on the continuation of continuous linear τ-functionals) [2].

Let $(L, \|\cdot\|_\tau)$ be an arbitrary τ-normal space, L_0- be its subspace and f_0 be a continuous linear τ-functional on L_0. This continuous linear τ-functional can be extended to some continuous linear τ-functional f on the whole space $(L, \|\cdot\|_\tau)$ without increasing τ-norms, i.e. so that $\|f\|_\tau = \|f_0\|_\tau$.

Theorem 3 (on the uniform boundedness of continuous linear operators) [2]. *For convergence of a sequence of continuous linear operators $\{f_n\}$ that map the τ-Banach space $(L, \|\cdot\|_\tau)$ into τ-normal space $(L', \|\cdot\|_\tau')$, to a continuous linear operator f, it is necessary and sufficient that*

(1) τ-norms of operators f_n are limited in aggregate: $\|f_n\| \leqslant M, M \in \mathbb{R}_+^\tau (n = 1, 2, 3, ...)$;

(2) The sequence $\{f_n(x)\}$ converges to $f(x)$ for every $x \in L$.

Keywords: τ-Banach space, τ-normed space, factor space.

AMS Subject Classification: 54C05, 54C10, 54C20

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ABOUT τ–METRIC SPACES

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Abstract. In this paper, we considered of τ - metric spaces and their properties. The class of multimetric or τ-metric spaces is introduced and studied in [1], [2]. The most general metrics over topological semifields are considered and there are also a number of papers devoted to the generalization of metrics, in particular metrics over Banach spaces.

Let $\mathbb{R}_\tau = [0, \infty), \mathbb{R} = (-\infty, \infty)$ and τ an infinite cardinal number. Let $\mathbb{R}_+^\tau$ and $\mathbb{R}^\tau$ denote the Tichonoff product τ-pieces of copies of the spaces $\mathbb{R}_+$ and $\mathbb{R}$ corresponding to the natural topology. In the spaces $\mathbb{R}_+^\tau$ and $\mathbb{R}^\tau$ the operations of addition, multiplication and multiplication by a scalar, as well as the partial orderliness.

Definition 1. Mapping $\rho_\tau : X \times X \to \mathbb{R}_+^\tau$ is called a τ-metric or multimetric (if τ is not fixed) on X, if the following axioms are executed:

(1) $\rho_\tau(x, y) = \theta$ if and only if $x = y$, where θ is a point in the space $\mathbb{R}_+^\tau$, all coordinates of which consist of zeros;
(2) $\rho_\tau(x, y) = \rho_\tau(y, x)$ for all $x, y \in X$;
(3) $\rho_\tau(x, y) \leq \rho_\tau(x, z) + \rho_\tau(z, y)$ for all $x, y, z \in X$;
(4) for any $a_1, a_2 \in A$ there exists $a_3 \in A$ that $q_{a_3}(\rho_\tau(x, y)) = max\{q_1(\rho_\tau(x, y)), q_2(\rho_\tau(x, y))\}$ for all $x, y \in X$.

Definition 2. Projective spectrum $S = \{(X_a, d_a), \pi_a^b, A\}$ of length $\tau = |A|$ consisting of metric spaces (X_a, d_a) and continuous maps π_a^b over a directed set A is called the τ-spectrum τ-metric space (X, ρ_τ).

Theorem 1. Any complete τ-metric space (X, ρ_τ) is the limit of the projective spectrum $S = \{(X_a, d_a), \pi_a^b, A\}$ of length τ, $|A| = \tau$, composed ofcomplete metric spaces (X_a, ρ_a) and continuous mappings π_a^b.

Theorem 2. Every complete τ-metric space is a limit of the projective spectrum of length τ consisting of complete metric spaces.

Keywords: τ-metric space, τ-Banach space, τ-spectr.

AMS Subject Classification: 54C05, 54C10, 54C20

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MULTIPLIERS OF DOUBLE FOURIER-HAAR SERIES

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Let X, Y be the spaces of functions defined on the segment $[0,1]$, such that $X \hookrightarrow L_1$. Let $\{\varphi_k\}$ be a complete orthonormal system. Let the function $f \in X$ correspond to its Fourier series with respect the system $\{\varphi_k\}$

$$f \sim \sum_{k=1}^{\infty} a_k \varphi_k,$$

where a_k are the Fourier coefficients of the function f with respect the system $\{\varphi_k\}$. We say that a sequence of complex numbers $\lambda = \{\lambda_k\}$ is a multiplier of Fourier series with respect the system $\{\varphi_k\}$ from the space X to the space Y, if for a function $f \in X$ with Fourier series $\sum_{k=1}^{\infty} a_k \varphi_k$ there is a function $f_\lambda \in Y$, whose Fourier series coincides with the series

$$\sum_{k=1}^{\infty} \lambda_k a_k \varphi_k$$

and the operator $\Lambda f = f_\lambda$ is a bounded operator from X to Y.

The set $m(X \to Y)$ of all multipliers defined in this way is a linear space with the norm

$$\|\lambda\|_{m(X\to Y)} = \|\Lambda\|_{X\to Y}.$$

For trigonometric series the fundamental theorem of Marcinkiewicz is known [1]. Further development of the theory of multipliers of trigonometric Fourier series can be found in the works [2, 3, 4, 5, 6, 7]. Works [8]-[15] are devoted to the study of multipliers of Fourier series with respect the Haar system in the one-dimensional case. The aim of this work is to study the multipliers of double Fourier-Haar series in anisotropic Lorentz spaces.

Theorem 1. Let $1 < \bar{p} < \bar{q} < \infty$, $0 < \bar{r}, \bar{s} \le \infty$, $\frac{1}{\tau_i} = \left(\frac{1}{s_i} - \frac{1}{r_i}\right)_+ = \max\left\{\frac{1}{s_i} - \frac{1}{r_i}, 0\right\}$, $i = 1, 2$. Then the sequence of complex numbers $\lambda = \{\lambda_{k_1 k_2}^{j_1 j_2}\}_{(k_i, j_i)\in\Omega}$ is a multiplier from $L_{\bar{p},\bar{r}}[0,1]^2$ to $L_{\bar{q},\bar{s}}[0,1]^2$ if and only if

$$\left(\sum_{k_2=0}^{\infty}\left(\sum_{k_1=0}^{\infty}\left(2^{\left(\frac{1}{p_1}-\frac{1}{q_1}\right)k_1+\left(\frac{1}{p_2}-\frac{1}{q_2}\right)k_2} \sup_{\substack{1\le j_i\le 2^{k_i}\\ i=1,2}} \left|\lambda_{k_1k_2}^{j_1j_2}\right|\right)^{\tau_1}\right)^{\frac{\tau_2}{\tau_1}}\right)^{\frac{1}{\tau_2}} < \infty$$

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and the following equivalence is true

$$\|\lambda\|_{m(L_{\bar{p},\bar{r}}[0,1]^2 \to L_{\bar{q},\bar{s}}[0,1]^2)}$$
$$\asymp \left(\sum_{k_2=0}^{\infty} \left(\sum_{k_1=0}^{\infty} \left(2^{\left(\frac{1}{p_1}-\frac{1}{q_1}\right)k_1+\left(\frac{1}{p_2}-\frac{1}{q_2}\right)k_2} \sup_{\substack{1\le j_i\le 2^{k_i}\\ i=1,2}} \left|\lambda_{k_1k_2}^{j_1j_2}\right| \right)^{\tau_1} \right)^{\frac{\tau_2}{\tau_1}} \right)^{\frac{1}{\tau_2}}.$$

Here and below, in the case when $\tau_i = +\infty$, the corresponding sum in the expression on the right is replaced by the supremum.

Keywords: Fourier series, Haar system, Fourier series multipliers, anisotropic Lorentz spaces.

AMS Subject Classification:42B15, 42B05, 46E30

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THE INFINITE SUMMATION FORMULAS OF MULTIPLE LAURICELLA FUNCTIONS

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We introduce the following multivariable symbolic operators:

$$H_{\mathbf{x}}(\alpha,\beta) := \frac{\Gamma(\beta)\Gamma(\alpha+\delta)}{\Gamma(\alpha)\Gamma(\beta+\delta)} = \sum_{|\mathbf{k}|=0}^{\infty} \frac{(\beta-\alpha)_{|\mathbf{k}|}(-\delta_1)_{k_1}\cdots(-\delta_n)_{k_n}}{(\beta)_{|\mathbf{k}|}\,k_1!\cdots k_n!}$$

and

$$\bar{H}_{\mathbf{x}}(\alpha,\beta) := \frac{\Gamma(\alpha)\Gamma(\beta+\delta)}{\Gamma(\beta)\Gamma(\alpha+\delta)} = \sum_{|\mathbf{k}|=0}^{\infty} \frac{(\beta-\alpha)_{|\mathbf{k}|}(-\delta_1)_{k_1}\cdots(-\delta_n)_{k_n}}{(1-\alpha-\delta)_{|\mathbf{k}|}\,k_1!\cdots k_n!},$$

where α and β are complex numbers; $\Gamma(z)$ is famous Gamma function;

$$\mathbf{x} := (x_1,...,x_n)\,;\ \delta := \sum_{j=1}^{n}\delta_j,\ \ \delta_j := x_j\frac{\partial}{\partial x_j};\ |\mathbf{k}| := \sum_{j=1}^{n}k_j,\ \ k_j \geq 0,\ n = 1,2,\ldots.$$

Using the symbolic operators $H_{\mathbf{x}}(\alpha,\beta)$ and $\bar{H}_{\mathbf{x}}(\alpha,\beta)$, we will find more than twenty infinity summation formulas for Lauricella functions in n variables $F_A^{(n)}(a,\mathbf{b};\mathbf{c};\mathbf{x})$, $F_B^{(n)}(\mathbf{a},\mathbf{b};c;\mathbf{x})$, $F_C^{(n)}(a,b;\mathbf{c};\mathbf{x})$ and $F_D^{(n)}(a,\mathbf{b};c;\mathbf{x})$ (for definitions, see [**?**, p.33]), where a, b and c are complex numbers; $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are vectors with a complex components:

$$\mathbf{a} := (a_1,...,a_n)\,,\ \ \mathbf{b} := (b_1,...,b_n)\,,\ \ \mathbf{c} := (c_1,...,c_n)\,.$$

As an application of the new infinity summation formulas some integral representation formulas for multiple Lauricella functions are also presented.

Keywords: Appell functions; multiple Lauricella functions; the inverse pair symbolic operators; Poole's formula; integral representations.

AMS Subject Classification: Primary 33C20, 33C65; Secondary 44A45.

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INTEGRABILITY CONDITIONS OF TRIGONOMETRIC SERIES WITH GENERAL MONOTONE COEFFICIENTS

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This report is devoted to relation between integrability properties of functions and summability properties of their Fourier coefficients. In particular, we prove Hardy-Littlewood type theorem.

For functions $f(x) \in L_1([0,1])$ with Fourier series $\sum\limits_{k=-\infty}^{\infty} c_k e^{2\pi kx}$, where $\{c_k\}_{k=0}^{\infty}$ and $\{c_k\}_{-\infty}^{0}$ are nonincreasing sequences the Hardy-Littlewood theorem holds i.e., there exist $C_1, C_2 > 0$ such that

$$C_1\|f\|_{L_p} \leq \left(\sum_{k=-\infty}^{\infty} (|k|+1)^{p-2} c_k^p\right)^{\frac{1}{p}} \leq C_2\|f\|_{L_p}. \tag{1}$$

Equivalence (1) has a lot of generalizations, see for instance, [1], [2], [3] and references therein. In this work, we get a new generalizations of Hardy-Littlewood's theorem. In particular, we obtain the equivalence (1) for functions with Fourier coefficients $\{c_k\}_{k=-\infty}^{\infty}$ such that, for any $k \geq 0$ and some $C > 0$, the following inequality

$$\sum_{[2^{k-1}]\leq|m|<[2^k]} |a_m - a_{m+1}| \leq C \sup_{r\in\mathbb{N}} \min(1, 2^{r-k}) \sup_{2^{r-1}\leq|m|<2^r} \frac{1}{|m|}\left|\sum_{j=0}^{m} a_j\right|$$

holds, where $[x]$ is a integer part of number x.

Keywords: trigonometric Fourier series, Lebesgue spaces, general monotonicity, Hardy-Littlewood theorem.
AMS Subject Classification: 42A16, 42A32

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O'NEIL'S TYPE INEQUALITIES IN LOCAL MORRY SPACES

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Let $\lambda \in \mathbb{R}$, $0 < p, q \leq \infty$. The Local Morry Space

$$LM_{p,q}^{\lambda} = \left\{ f : \left(\int_0^{\infty} \left(t^{-\lambda} \|f\|_{L_p(B_t(0))} \right)^q \frac{dt}{t} \right)^{1/q} < \infty \right\}, \tag{1}$$

where $B_t(0) = \{y \in \mathbb{R}^n : |y| \leq t\}$ is the open ball centered at the point $x = 0$ of radius t, were introduced by Guliyev-Burenkov [1]. In this paper, we introduce spaces that generalize local Morrey spaces. We study the Riesz operators, which are dedicated in [2,3]. O'Neil-type inequalities are obtained for convolution operators in these spaces. The methods from [4] are used.

Keywords: Morry space, convolution operator, Young-O'Neill inequality, Riesz potential.

AMS Subject Classification: 47A30

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STABILITY OF EXCESSES FOR EXPONENTIAL SYSTEMS

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Let's analyze the main provisions on a case of the Banach space $C(S)$ of continuous functions on a Jordan rectified arc S in the complex plane $\mathbb{C}$ with the uniform norm $\|f\| := \sup_{s\in S}\bigl|f(s)\bigr|$.

Let $\Lambda := (\lambda_n)_{n=1,2,\dots} \subset \mathbb{C}$ be a sequence of pairwise distinct numbers (points). To this sequence, we assign the exponential system $\operatorname{Exp}^{\Lambda} := \Bigl\{z \underset{z\in\mathbb{C}}{\longmapsto} \exp(\lambda z) \Bigm| \lambda\in\Lambda\Bigr\}$. This system is *complete* in $C(S)$ if the linear span of $\operatorname{Exp}^{\Lambda}$ coincides with $C(S)$. The system $\operatorname{Exp}^{\Lambda}$ is *minimal* if the system $\operatorname{Exp}^{\Lambda\setminus\{\lambda_1\}}$ is incomplete in $C(K)$. The *excess* of $\operatorname{Exp}^{\Lambda}$ is equal to an integer $k \geq 0$ if the system $\operatorname{Exp}^{\Lambda\setminus\{\lambda_1,\dots,\lambda_k\}}$ is complete and minimal. The *excess* of $\operatorname{Exp}^{\Lambda}$ is equal to an integer $k<0$ if there are complex points $\lambda_{-1},\lambda_{-2},\dots,\lambda_k \notin \Lambda$ such that the system $\operatorname{Exp}^{\Lambda\cup\{\lambda_{-1},\dots,\lambda_k\}}$ is complete and minimal. When S is a *segment*, the excesses and their conservation at small perturbations of the sequence Λ are studied in detail in [1]–[2]; see also our survey in the monograph [3; Ch. 2]. The main results known today for the case of arcs and closed curves S, as well as Banach spaces of holomorphic functions, are established in our works [4]–[5]; see also [3; Ch. 3]. We intend to discuss new results in these directions, using, in particular, [6]–[7].

Keywords: exponential system, completeness, function space, entire function, Jordan curve, rectifiable arc

AMS Subject Classification: 42C30, 30B60, 30D15

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COMPACTNESS SETS AND COMMUTATORS FOR RIESZ POTENTIAL ON LOCAL MORREY-TYPE SPACES

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The paper considers Morrey-type local spaces from $LM^{w}_{p\theta}$ The main work is the proof of the commutator compactness theorem for the Riesz potential $[b, I_\alpha]$ in local Morrey-type spaces from $LM^{w_1}_{p\theta}$ to $LM^{w_2}_{q\theta}$. We also give new sufficient conditions for the commutator to be bounded for the Riesz potential $[b, I_\alpha]$ in local Morrey-type spaces from $LM^{w_1}_{p\theta}$ to $LM^{w_2}_{q\theta}$.

Let $1 \le p, \theta \le \infty$, w be a measurable non-negative function on $(0,\infty)$. The Local Morrey-type space $LM^{w}_{p\theta} \equiv LM^{w}_{p\theta}(\mathbb{R}^n)$ is defined as the set of all functions $f \in L^{loc}_{p}(\mathbb{R}^n)$ with finite quasi-norm $\|f\|_{LM^{w}_{p\theta}} \equiv \left\| w(r) \, \|f\|_{L_p(B(0,r))} \right\|_{L_\theta(0,\infty)}$, where $B(t,r)$ the ball with center at the point t and of radius r. In this paper we consider the Riesz Potential in the following form $I_\alpha f(x) = \int\limits_{\mathbb{R}^n} \frac{f(y)}{|x-y|^{n-\alpha}} dy$. For a function $b \in L_{loc}(\mathbb{R}^n)$ by M_b denote multiplier operator $M_b f = bf$, where f is measurable function. Then the commutator between I_α and M_b is defined by $[b, I_\alpha] = M_b I_\alpha - I_\alpha M_b = \int\limits_{\mathbb{R}^n} \frac{[b(x)-b(y)]f(y)}{|x-y|^{n-\alpha}} dy$.

Denote by $H^* g(t) := \int_t^\infty g(s)ds, \quad g \in \mathfrak{M}^+$, the Hardy operator. $W(t) := \int_0^t w(t)dw$, $U_*(t) := \int_t^\infty u(t)du$, $V_*(t) := \int_t^\infty v(t)dv$.

Theorem 1. (see. [1]) Suppose that $1 \le p \le \theta \le \infty$ and $w \in \Omega_{p\theta}$. Suppose that a subset S of $LM^{w}_{p\theta}$ satisfies the following conditions: $\sup\limits_{f\in S} \|f\|_{LM^{w}_{p\theta}} < \infty$, $\lim_{u\to 0} \sup_{f\in S} \|f(\cdot+u) - f(\cdot)\|_{LM^{w}_{p\theta}} = 0$, $\lim_{r\to\infty} \sup_{f\in S} \left\| f\chi_{{}^cB(0,r)} \right\|_{LM^{w}_{p\theta}} = 0$. Then S is a pre-compact set in $LM^{w}_{p\theta}(\mathbb{R})$.

Theorem 2. Let $1 < p \le q < \infty$, $0 < \alpha < n$ and $b \in VMO(\mathbb{R}^n)$. $1 < p < \frac{n}{\alpha}$, $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{n}$, $w_1, w_2 \in \Omega_\theta$ satisfy the conditions $\left\| w_2(r) \left(\frac{r}{t+r}\right)^{\frac{n}{p}} \right\|_{L_\theta(0,\infty)} \le \|w_1(r)\|_{L_\theta(t,\infty)}$, $A_0^* := sup_{t>0} (\int_t^\infty \int_\tau^\infty (1 + ln\frac{\tau}{r}) dr w(\tau) d\tau)^{\frac{1}{q}} [\int_t^\infty v(t)dv]^{-\frac{1}{p}}$, $A_1^* := sup_{t>0} W^{\frac{1}{q}}(t) (\int_t^\infty (\frac{U_*(\tau)}{V_*(\tau)})^{p'} v(\tau) d\tau)^{\frac{1}{p'}} < \infty$. Then the commutator $[b, I_\alpha]$ is a compact operator from $LM^{w_1}_{p\theta}$ to $LM^{w_2}_{p\theta}$.

Keywords: Compactness, Commutators, Riesz Potential, Local Morrey-type spaces.
AMS Subject Classification: 46B50, 47B47 .

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THE APPROXIMATION OF FUNCTIONS OF TWO VARIABLES OF BOUNDED P-VARIATION BY POLYNOMIALS WITH RESPECT TO HAAR SYSTEM.

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Let $f(x,y)$ be defined on the closed square $[0,1]^2$ and $\tau = \xi \times \eta$, where $\xi = \{x_0 < x_1 < ... < x_m = 1\}$, $\eta = \{y_0 < y_1 < ... < y_n = 1\}$- be an arbitrary partition of $[0,1]^2$.Let $(1 \leq \ p < \infty)$. The quantity $\aleph^p_{\xi,\eta}(f) = (\sum_{k=1}^n \sum_{l=1}^m |f(x_k,y_l) - f(x_{k-1},y_l) - f(x_k,y_{l-1}) + f(x_{k-1},y_{l-1})|^p)^{1/p}$. is called variational sum of order p of the function $f(x,y)$ with respect to the partitions τ.

The quantity $\omega_{1-1/p}(f,\delta_1,\delta_2) = \sup\limits_{\substack{|\xi|\leq\delta_1\\|\eta|\leq\delta_2}} \aleph^p_{\xi,\eta}(f)$, is called variational modulus of continity of order $1-\frac{1}{p}$ of the function $f(x,y)$. Here $|\xi| = \max\limits_{1\leq k\leq n}(x_k - x_{k-1})$,$|\eta| = \max\limits_{1\leq l\leq m}(x_l - x_{l-1})$.

We say that $f \in BV_p[0,1]^2$,$1 \leq \ p < \infty$, if $V_p\left(f,[0,1]^2\right) \equiv \omega_{1-1/p}(f,1,1) < \infty$, and if $f \in C_p[0,1]^2$, $1 < \ p < \infty$, if $\lim\limits_{\substack{\delta_1\to 0\\ \delta_2\to 0}} \omega_{1-1/p}(f,\delta_1,\delta_2) = 0$.

Endowed with the norm $\|f\|_{BV_p} = max\left\{\sup\limits_{(x,y)\in[0,1]^2} |f(x,y)|\,, V_p\left(f,[0,1]^2\right)\right\}$. This space is a Banach space. Let $h_n(x)$ is functions of the Haar system. For the case of $[0,1)^2$ we put $h_{m,n}(x,y) = h_m(x)h_n(y)$.

Denote by $E^h_{m,n}(f)_X$ the best approximation of $f \in X[0,1]^2$ by Haar polynomials of degree $\leq m \times n$ $(m,n \in N)$ in the norm of $X[0,1]^2$, where $X[0,1]^2 = C_p[0,1]^2$, $1 < p < \infty$ or $X[0,1]^2 = BV_p[0,1]^2$, $1 \leq p < \infty$ $E_{m,n}(f)_X = \sup\limits_{c_{ij}} \left\| f(x,y) - \sum_{i=1}^m \sum_{j=1}^n c_{ij}\chi_i(x)\chi_j(y) \right\|_X$.

Denote by $S^h_{m,n}(f)$ rectangular partial sum of Fourier series of f by Haar system. By $K_{\alpha,\beta,\gamma}$ we denote a constant whose value may be different at each occurrence.

Theorem 1. That $1 < p < \infty$ $f \in C_p[0,1]^2$, $m,n \in P$. Then $E^h_{m,n}(f)_{C_p} \leq K_p\omega_{1-\frac{1}{p}}\left(f,\frac{1}{m},\frac{1}{n}\right)$

Theorem 2. That $1 < p < \infty$ $f \in C_p[0,1]^2$, $m,n \in P$. Then $\omega_{1-\frac{1}{p}}\left(f,\frac{1}{m},\frac{1}{n}\right) \leq K_pE^h_{m,n}(f)_{C_p}$

Keywords: variational modulus of continity, Haar system, rectangular partial sum of Fourier series.

AMS Subject Classification: 46B50, 47B47 .

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LINEAR AND MULTILINEAR PSEUDO-DIFFERENTIAL OPERATORS ON MULTI-DIMENSIONAL TORUS

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In the talk, we will present several results on the boundedness of toroidal linear pseudo-differential operators of the form

$$T_\sigma(f;x) := \sum_{\xi\in\mathbb{Z}^m} \sigma(x,\xi)\widehat{f}(\xi)e^{2\pi i\,\xi x}$$

and multilinear pseudo-differential operators of the form

$$T_\sigma(f_1,\ldots,f_n;x) := \sum_{(\xi^1,\ldots,\xi^n)\in\mathbb{Z}^{nm}} \sigma(x,\xi^1,\ldots,\xi^n)\prod_{\nu=1}^{n}\widehat{f}_\nu(\xi^\nu)e^{2\pi i\,\xi^\nu x}$$

with symbols from the Hörmander classes and their multilinear analogs as operators from isotropic function spaces $B^s_{pq}(\mathbb{T}^m)$ of the Nikol'skii – Besov type or $L^s_{pq}(\mathbb{T}^m)$ of the Lizorkin – Triebel type on the $m-$dimensional torus and respectively from tensor products of such spaces into spaces of the same type.

Keywords: Pseudo-differential operator, multilinear operator, multi-dimensional torus, isotropic Nikol'skii – Besov/Lizorkin – Triebel function space.

AMS Subject Classification:42B15, 42B35

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ESTIMATES FOR CONVOLUTIONS IN MORREY-TYPE SPACES

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Let $\Omega \subset \mathbb{R}^n$, $0 < p < \infty$, $0 < q \leq \infty$ and $0 \leq \lambda \leq \frac{n}{p}$. We introduce the generalized Morrey-type spaces $M^{\lambda}_{p,q,\Omega}$ that are defined for $q < \infty$ as the spaces of all functions $f \in L^{loc}_p(\mathbb{R}^n)$ such that

$$||f||_{M^{\lambda}_{p,q,\Omega}} = \left(\int\limits_0^{\infty} \left(t^{-\lambda} \sup_{x\in\Omega} \|f\|_{L_p(B_t(x))} \right)^q \frac{dt}{t} \right)^{\frac{1}{q}},$$

and for $q = \infty$,

$$||f||_{M^{\lambda}_{p,\infty,\Omega}} = \sup_{x\in\Omega} \sup_{t>0} t^{-\lambda} \|f\|_{L_p(B_t(x))}.$$

In this paper we study of the boundedness of convolution operator in the case of Morrey-type spaces. In particular, by applying the Marcinkiewicz-type interpolation theorem we obtain estimates for the norm of convolutions in the Morrey-type spaces. The results of this abstract were formulated with proofs in [1].

Theorem 1. Let $\Omega \subset \mathbb{R}^n$, $1 < p < q < \infty$, $0 < \tau \leq \infty$, $0 < \nu \leq \lambda < \frac{n}{q}$ and $\mu > \nu$,

$$\frac{1}{\rho} = 1 + \frac{1}{q} - \frac{1}{p} - \frac{\lambda - \nu}{n} \quad \text{and} \quad \frac{1}{\sigma} = \frac{1}{\rho} - \frac{\mu}{n} > 0.$$

If $f \in M^{\nu}_{p,\tau,\Omega}$, $k \in L_{\rho,\infty}(\mathbb{R}^n)$ and

$$N(\lambda, \mu, \nu) = \sup_{m\in\mathbb{Z}} 2^{\mu m} \|k\|_{L_{\sigma,\infty}(D_m)} < \infty,$$

then the convolution

$$(k * f)(x) = \int\limits_{\mathbb{R}^n} k(x-y) f(y) dy$$

exists almost everywhere on $\mathbb{R}^n$ and

$$||k * f||_{M^{\lambda}_{q,\tau,\Omega}} \leq c(||k||_{L_{\rho,\infty}(\mathbb{R}^n)} + N(\lambda, \mu, \nu))||f||_{M^{\nu}_{p,\tau,\Omega}},$$

where $c > 0$ depends only on the parameters $n, p, q, \lambda, \mu, \nu$ and τ.

Theorem 2. Let $\Omega \subset \mathbb{R}^n$, $1 < p < q < \infty$, $0 < \nu \leq \lambda < \frac{n}{q}$, $0 < \tau \leq \infty$,

$$1 + \frac{1}{q} = \frac{1}{p} + \frac{1}{\rho} + \frac{\lambda - \nu}{n},$$

If $f \in M^{\nu}_{p,\tau,\Omega}$ and

$$M_\rho = \sup_{x\in\mathbb{R}^n} |x|^{n/\rho} |k(x)| < \infty,$$

then the convolution $k * f$ exists almost everywhere on $\mathbb{R}^n$ and the following inequality

$$\|k * f\|_{M^{\lambda}_{q,\tau,\Omega}} \leq c M_\rho \|f\|_{M^{\nu}_{p,\tau,\Omega}}$$

holds.

Keywords: Morrey-type spaces, convolution operator, interpolation theorems, Marcinkiewicz-type interpolation theorem.

AMS Subject Classification: 42B35, 47B38, 47G10.

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ON q- LACUNARY STRONG ALMOST SUMMABILITY OF WEIGHT g

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The purpose of this paper is to present the concept of q- lacunary almost statistical convergence of weight g and establish some connections between q-lacunary almost statistical convergence of weight g and q- lacunary strong almost summability of weight g, We also investigate the relations between the spaces $[\hat{N}^g, \theta, f, q, p]$ and $\hat{S}^g_{(\theta,q)}$.

Keywords: Weight function g, statistical convergence, strong almost convergence, modulus function.
AMS Subject Classification: 40A05, 40A35

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INTEGRAL COVERINGS AND ESTIMATES OF INTEGRAL MAJORANTS FOR CONES

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Keywords: integral coverings, estimates, majorants, integral majorants.

AMS Subject Classification: 46B50, 47B47 . Integral coverings and estimates of integral majorants for cones

Let K be a cone on the set of non-negative Lebesgue measurable functions on $(0,\ \infty)$ equipped with a positively homogeneous non-degenerate functional ρ_K; $q\ \in\ (0,\ \infty)$; $\mu-$ non-negative Borel measure on $(0,\ \infty)$, $0<\ M_q\ (t)\ :=\ \left(\int_{(0,\,t]} d\mu\right)^{1/q}<\infty,\quad t\in(0,\ \infty)$.

$$\lambda_{K,q}(t)=\sup\left\{M_q\,(t)^{-1}\left(\int_{(0,\,t]}h^q\,d\mu\right)^{1/q}:\quad h\in K;\ \rho_K\,(h)\le 1\right\},\quad t\in(0,\ \infty)\,.$$

Cone M covers the cone K $q-$integrally with the covering constant $c\in\mathbb{R}_+$ (designations: $K\underset{q}{\prec}M\,(c)$) if for any $h_1\in K$ there is $h_2\in M$:

$$\rho_M\,(h_2)\le\rho_K\,(h_1)\,;\left(\int_{(0,\,t]}h_1^q\,d\mu\right)^{1/q}\ \le\ \left(\int_{(0,\,t]}h_2^q\,d\mu\right)^{1/q}\quad t\in(0;\infty)\,.$$

Equivalence of cones $K\underset{q}{\approx}M$ means mutual covering of cones:

$$K\underset{q}{\approx}M\ \Leftrightarrow\ K\underset{q}{\prec}M\ (c_1)\,,\ M\underset{q}{\prec}K\ (c_2)\,;\ c_1,\ c_2\in\mathbb{R}_+\,.$$

Theorem. *Let $K\underset{q}{\prec}M\ (c)$. Then, for the integral majorants, the estimate $\lambda_{K,q}(t)\ \le\ c\,\lambda_{M,q}(t)$ $t\in(0,\ \infty)$. For equivalent cones, their integral majorants are order equivalent.*

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ON THE UPPER BOUNDS FOR THE NORM OF THE CONVOLUTION OPERATOR IN ANISOTROPIC MORREY SPACES

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Let $k \in \mathbb{Z}$, denote by G_k the set of all cubes in $\mathbb{R}^n$ of the form $[0, 2^k)^n + 2^k m,\ m \in \mathbb{Z}^n$. Consider the set $\mathbb{G} = \bigcup\limits_{k\in\mathbb{Z}} G_k$. Let $Q \in G_k$. Then the set of mutually disjoint cubes $\mathbb{T} = \{Q\} \subset \mathbb{G}$ is called a local partition of the space $\mathbb{R}^n$ if $\mathbb{R}^n = \overline{\bigsqcup\limits_{Q\in\mathbb{T}} Q}$ and $|\mathbb{T} \cap G_k| < \infty$.

Now let $\bar{n} = (n_1, ..., n_d)$: $n_i \in \mathbb{N}$, $|n| = n_1 + \cdots + n_d$, $\bar{k} = (k_1, ..., k_d)$: $k_i \in \mathbb{Z}$. Denote $G_{\bar{k}} = \{Q = Q_1 \times \cdots \times Q_d :\ Q_i \subset G_{k_i},\ i = 1, ..., d\}$. Mutually disjoint cubes $\mathbb{T}_i = \{Q_i\} \subset G_{k_i}$ are called local partitions of the space $\mathbb{R}^{n_i}$, the set $\mathbb{T}_1, \ldots, \mathbb{T}_d$ - local partitions of the spaces $\mathbb{R}^{n_1}, \ldots, \mathbb{R}^{n_d}$ respectively. Family of mutually non-intersecting parallelepipeds $\mathbb{T} = \mathbb{T}_1 \times \cdots \times \mathbb{T}_d = \{Q = Q_1 \times \cdots \times Q_d :\ Q_i \subset \mathbb{T}_i,\ i = 1, ..., d\}$ will be called a local partition of the space $\mathbb{R}^{|\bar{n}|}$.

Let $\bar{p} = (p_1, ..., p_d)$, $\bar{q} = (q_1, ..., q_d)$, $\bar{\lambda} = (\lambda_1, ..., \lambda_d)$ such that $0 < p_i \le \infty$, $0 < q_i \le \infty$, $0 < \lambda_i < \infty$. We define an anisotropic local Morrey space $LM^{\bar{\lambda}}_{\bar{p},\bar{q}}(\mathbb{T})$ as the set of measurable functions f for which

$$\|f\|_{LM^{\bar{\lambda}}_{\bar{p},\bar{q}}(\mathbb{T})} = \left(\sum_{k_d\in\mathbb{Z}} \cdots \left(\sum_{k_1\in\mathbb{Z}} \left(2^{-\langle \bar{k},\bar{\lambda}\rangle} \sum_{Q\in\mathbb{T}_{\bar{k}}} \|f\|_{L_{\bar{p}}(Q)}\right)^{q_1}\right)^{\frac{q_2}{q_1}} \cdots\right)^{\frac{1}{q_d}} < \infty.$$

The anisotropic classical Morrey space $M^{\bar{\lambda}}_{\bar{p}}$ is the set of Lebesgue measurable functions $f \in L^{loc}_{\bar{p}}(\mathbb{R}^{\bar{n}})$ for which

$$\|f\|_{M^{\bar{\lambda}}_{\bar{p}}} = \sum_{k_d\in\mathbb{Z}} \cdots \sum_{k_1\in\mathbb{Z}} \left(2^{\langle -\bar{k},\bar{\lambda}\rangle} \sup_{Q\in G_{\bar{k}}} \|f\|_{L_{\bar{p}}(Q)}\right) < \infty,$$

here $\langle \bar{k}, \bar{\lambda}\rangle = k_1\lambda_1 + \cdots + k_d\lambda_d$.

Consider the convolution operator

$$(Tf)(x) = (K * f)(x) = \int\limits_{\mathbb{R}^d} K(x-y) f(y) dy,$$

acting from one Morrey space to another Morrey space.

The main results of the work are the following theorems.

Theorem 1. Let $\mathbb{T}$ be some local partition of the space $\mathbb{R}^{|n|}$. Let $0 < \max(\bar{q}, 1) \le p \le \infty$, $0 < \bar{\lambda} < \frac{\bar{n}}{\bar{q}}$ and $0 \le \bar{\gamma} \le \frac{\bar{n}}{p}$, $0 < \bar{\alpha} = \bar{\gamma} - \bar{\lambda} + \frac{\bar{n}}{\bar{q}} < \frac{\bar{n}}{p}$.

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If $f \in M_p^{\bar\gamma}$ and $g \in LM_{p',\bar\infty}^{-\bar\alpha}(\mathbb{T})$, then $f * g \in M_{\bar q}^{\bar\lambda}$ and the inequality

$$\|f * g\|_{M_{\bar q}^{\bar\lambda}} < c\|f\|_{M_p^{\bar\gamma}}\|g\|_{LM_{p',\bar\infty}^{-\bar\alpha}(\mathbb{T})},$$

where the constant c depends only on the parameters $\bar n,\ \bar\lambda,\ \bar q, \bar\alpha,\ p$.

Theorem 2. Let $\mathbb{T}$ be some local partition of the space $\mathbb{R}^{|n|}$. Let $1 \le p < q \le \infty$, $0 < \bar\lambda < \frac{\bar n}{q}$ and $0 \le \bar\gamma \le \frac{\bar\lambda q}{p}$, $0 < \bar\alpha = \bar\gamma - \bar\lambda + \frac{\bar n}{q}$.

If $f \in M_p^{\bar\gamma}$ and $g \in LM_{p',\bar\infty}^{-\bar\alpha}(\mathbb{T})$, then $f * g \in M_q^{\bar\lambda}$ and the inequality

$$\|f * g\|_{M_q^{\bar\lambda}} < c\|f\|_{M_p^{\bar\gamma}}\|g\|_{LM_{p',\bar\infty}^{-\bar\alpha}(\mathbb{T})},$$

where the constant c depends only on the parameters $\bar n,\ \bar\lambda,\ q, \bar\alpha,\ p$.

Keywords: local and classical anisotropic Morrey spaces, convolution operator.

AMS Subject Classification: 46

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ON THE GROWTH OF MODULES OF ALGEBRAIC POLYNOMIALS IN DIFFERENT SPACES

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Let $\mathbb{C}$ be a complex plane and $\overline{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$; $G \subset \mathbb{C}$ be a bounded Jordan region with boundary $L := \partial G$ such that $0 \in G$; $\Omega := \overline{\mathbb{C}} \setminus \overline{G}$. Let $w = \Phi(z)$ be the univalent conformal mapping of Ω onto $\{w : |w| > 1\}$ such that $\Phi(\infty) = \infty$ and $\lim_{z\to\infty} \frac{\Phi(z)}{z} > 0$. $d(z, L) := \inf \{|\zeta - z| : \ \zeta \in L\}$

Let $0 < p \leq \infty$ and σ be the two-dimensional Lebesgue measure. For the Jordan region G, we introduce:

$$\begin{aligned} \|P_n\|_{A_p(h,G)} &:= \left(\iint\limits_G h(z) \left|P_n(z)\right|^p d\sigma_z \right)^{1/p}, \ 0 < p < \infty, \\ \|P_n\|_{A_\infty(1,G)} &:= \max_{z\in\overline{G}} |P_n(z)|, \ p = \infty, \end{aligned}$$

Well known Bernstein -Walsh Lemma [2] says that:

$$|P_n(z)| \leq |\Phi(z)|^n \|P_n\|_{C(\overline{G})}, \ z \in \Omega, \tag{1}$$

N. Stylianopoulos in [1] replaced the norm $\|P_n\|_{C(\overline{G})}$ with norm $\|P_n\|_{A_2(G)}$ on the right-hand side of (1) and found a new version of the Bernstein-Walsh Lemma for the some regions as follows:

$$|P_n(z)| \leq c\frac{\sqrt{n}}{d(z,L)} \|P_n\|_{A_2(1,G)} |\Phi(z)|^{n+1}, \quad z \in \Omega.$$

In this, we study for more general regions pointwise estimation in unbounded region Ω, for the $\left|P_n^{(m)}(z)\right|, m = 0, 1, 2, ...,$ in the following type:

$$\left|P_n^{(m)}(z)\right| \leq \eta_n(G, h, p, m, z) \|P_n\|_{A_p(h,G)}, \ z \in \Omega,$$

where $\eta_n(.) \to \infty$, as $n \to \infty$, depending on the properties of the G, h.

Keywords: Algebraic polynomial, quasicircle, smooth curve, inequalities..

AMS Subject Classification: 30C30, 30E10, 30C70.

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ON A GENERALIZATION OF CLASSES OF CONVEX IN THE DIRECTION AND TYPICALLY REAL FUNCTIONS

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We introduce a class of holomorphic in functions such that

$$|\arg\left[(1-\varepsilon z)^{1+\alpha}(1+\varepsilon z)^{1-\alpha}f'(z)\right]| \le \gamma\frac{\pi}{2}, \varepsilon = \lambda e^{-i\delta}, z \in E, \tag{1}$$

where $\lambda,\ \alpha, \gamma \in [0;1]$, $\delta \in [-\pi;\pi]$, which generalizes the class of functions convex in the direction of the imaginary axis [1] and a well-known class of functions with bounded turning [2]. Geometric properties of functions of this class are established, exact estimates of $|f'(z)|$, $\left|z\dfrac{f''(z)}{f'(z)}\right|$ are found and the radii of the convexity. The results obtained are transferred to the class $T_\delta(\lambda,\ \alpha,\gamma)$ of functions $F(z) = zf'(z)$, where $f(z) \in C_\delta(\lambda,\ \alpha,\gamma)$. In particular cases, previously known results are obtained for classes of convex in the direction of the imaginary axis [1], typically real [3-5] functions, functions with bounded turning [2,6] and others.

Keywords: estimates of regular functions, radii of convexity, close-to-convex functions, typically real functions.

AMS Subject Classification: MSC 30C45.

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ON FOURIER MULTIPLIER AND NIKOLSKII'S INEQUALITY ON QUANTUM TORI AND APPLICATIONS

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In this work, we study Hörmander type Fourier multiplier theorem and Nikolskii's inequality on quantum tori. On the way to obtain these results, we prove some classical inequalities such as Paley, Hausdorff-Young, Hausdorff-Young-Paley, and Hardy-Littlewood inequalities on quantum tori.

Keywords: quantum tori, Hörmander Fourier multiplier, Nikolskii's inequality, Hausdorff-Young inequality.

AMS Subject Classification: 46L51, 46L52, 58B34, 47L25, 11M55, 46E35, 42B05, 43A50, 42A16, 42B15 https://mathscinet.ams.org/msnhtml/msc2020.pdf.

ON THE ABEL BASIS PROPERTY OF THE ROOT FUNCTIONS SYSTEM OF THE STURM–LIOUVILLE OPERATOR ON A CURVE

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Let γ be a curve with parametrization $z = t + ih(t),\ t \in [0,1]$, where $h(0) = h(1) = 0$ and $M_1 := \inf\limits_{t_1 \neq t_2} \frac{h(t_2)-h(t_1)}{t_2-t_1} > -\infty, \quad M_2 := \sup\limits_{t_1 \neq t_2} \frac{h(t_2)-h(t_1)}{t_2-t_1} < \infty$. We put $\alpha_j = \arctan M_j\ (j = 1,2)$ and we assume that $-\pi/2 < \alpha_1 < \alpha_2 < \pi/2$. Further, let $l(y) = -y'' + qy$, where $q \in L^1(\gamma)$, and let L_U be an operator acting in $L^2(\gamma)$ by the rule $L_U y = l(y), \quad D(L_U) = \{y \in L^2(\gamma) :\ y, y' \in AC(\gamma), l(y) \in L^2(\gamma),\ U_1(y) = 0,\ U_2(y) = 0\}, \quad U_j(y) = y^{(j-1)}(0) - (l(y), \sigma_j)$, where $\sigma_j \in L^2(\gamma)$.

Theorem 1. *Let the functions $\tau_1(z) = \int_0^z \overline{\sigma_1}|dt|$ and $\tau_2(z) = \int_0^z \overline{\sigma_2}|dt|$ be differentiable at points 0 and 1 and $a := \tau_1'(1)(\tau_2'(0)+1) - \tau_1'(0)\tau_2'(1) \neq 0$. Then*

a) *outside the sector $\{-2\alpha_2 < \arg z < -2\alpha_1\}$ the spectrum L_U is finite,*

b) *the root functions system (RFS) of the operator L_U is complete in $L^2(\gamma)$.*

Theorem 2. *Let $U_j(y) = V_j(y) + \sum_{\nu=1}^{m_j} \int_0^1 y^{(\nu-1)}(x)d\sigma_{j\nu}(x)$, $V_j(y) = \sum_{\nu=1}^{m_j} \left(a_{j\nu-1}y^{(\nu-1)}(0) + b_{j\nu-1}y^{(\nu-1)}(1)\right)$ $(1 \leq m_j \leq 2)$, where $\sigma_{j\nu}$ are functions of bounded variation, continuous at the points $0, 1$, and the boundary conditions $V_j(y) = 0$ are regular in the sense of Birkhoff. Then the (RFS) of the operator L_U forms an Abel basis of order $\delta \in (1/2, \pi/2(\alpha_2 - \alpha_1))$.*

The completeness or basis property of the RFS of the operator L_U with $q \neq 0$ have been studied only for $h = \text{const}$ (that is, when γ is a segment): in [1] — the completeness, in [2] — the Riss basis property in the case when U_j have the same form as in Theorem 2. If $q = 0$ then root functions are expressed in terms of exponentials. Extensive literature is devoted to the issue of completeness of exponents [3].

Keywords: a curve of bounded slope, general boundary value problems, differential operators on a curve, Abel basis property, completeness.

AMS Subject Classification: 34L10,34L20, 34M45, 47B28,47A70.

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ASIMPTOTIC DISTRIBUTION OF EIGENVALUES OF OPERATOR DIFFERENTIAL EQUATIONS OF HIGH ORDER ON SEMIAXIS

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Let H be a separable Hilbert space. In the Hilbert space $H_1 = L_2 [H : 0 \leq x < \infty]$ consider a differential operator L generated by the expression

$$\ell (y) = (-1)^n y^{(2n)} + \sum_{j=2}^{2n} Q_j (x) y^{(2n-j)}, \quad 0 \leq x < \infty \tag{1}$$

and the boundary conditions

$$y^{(\ell_1)} (0) = y^{(\ell_2)} (0) = ... = y^{(\ell_n)} (0) = 0 \tag{2}$$

Where $0 \leq \ell_1 < \ell_2 < ... < \ell_n \leq 2n - 1,\ y \in H_1$, the derivatives are understood in the strong sense.

At some assumptions relative to operator coefficients $Q_j (x)\,,\ j = \overline{2, 2n - 1}$ that we will show below, the operator L has a pure discrete spectrum. Let $\lambda_1, \lambda_2, ..., \lambda_n,$ be eigenvalues of the operator L. By $N (\lambda)$ we'll denote the number of eigenvalues of the operator L, smaller than the given number λ, i.e.,

$$N (\lambda) = \sum_{\lambda_n < \lambda} 1.$$

Our principal problem is studying the asymptotic behavior of the function $N (\lambda)$ as $\lambda \to \infty$.

Note, that the asymptotics of the function $N (\lambda)$ for the Sturum-Liouville operator equation first was obtained by A.G.Kostyuchenko and B.M.Levitan [1]. M.Bayramoglu [2] and G.I.Aslanov's [3] papers were devoted to the construction of Green function for the equation of high order and investigations of the asymptotics function $N (\lambda)$.

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ON FRACTIONALANALYSIS OF CURVES

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The concept of fractional analysis emerges on the generalization of ordinary derivatives and integrals with real or complex numbers and is a field studied by famous mathematicians. In addition to gaining importance from the part of mathematics, it also tooks part in the solution of various problems in the field of application such as viscoelastic logic, hardware mechanics, dynamic systems.

The aim of this presentation is to carry out the Caputo fractional derivative, which is one of vairous fractional derivatives, on curves in differential geometry, and to exemplify its behaviours.

Keywords: Caputo fractional derivative, curvature, Frenet frame, plane curve.

AMS Subject Classification: 26A33,53A04

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SPECTRUM OF THE CESÀRO-HARDY OPERATOR IN REARRANGEMENT INVARIANT SPACES

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Let $f \in L_1(0,1)$ or $f \in (L_1 + L_\infty)(0,\infty)$ then, the continuous Cesàro-Hardy operator is defined by

$$Cf(t) = \frac{1}{t}\int_0^t f(s)\,ds. \tag{1}$$

The following theorems are the main results.

Theorem 1. Let $E(0,1)$ be a rearrangement invariant space over $(0,1)$ with $\underline{\alpha}$ and $\overline{\alpha}$ being its lower and upper Boyd indices, respectively, such that $0 \le \underline{\alpha} \le \overline{\alpha} < 1$. Let $C : E(0,1) \to E(0,1)$ be the Cesàro-Hardy operator mapping $E(0,1)$ into itself defined as in (1). Then, the spectrum of the operator C in $E(0,1)$ is the set

$$\sigma(C) = \left\{\lambda \in \mathbb{C} : \mathrm{Re}\left(\frac{1}{\lambda}\right) \ge 1 - \overline{\alpha}\right\}.$$

Theorem 2. Let $E(0,\infty)$ be a rearrangement invariant space over $(0,\infty)$ with $\underline{\alpha}$ and $\overline{\alpha}$ being its lower and upper Boyd indices, respectively, such that $0 \le \underline{\alpha} \le \overline{\alpha} < 1$. Let $C : E(0,\infty) \to E(0,\infty)$ be the Cesàro-Hardy operator defined as in (1). Then, the spectrum of the operator C in $E(0,\infty)$ is the set

$$\sigma(C) = \left\{\lambda \in \mathbb{C} : 1 - \overline{\alpha} \le \mathrm{Re}\left(\frac{1}{\lambda}\right) \le 1 - \underline{\alpha}\right\}.$$

Keywords: spectrum, Cesàro-Hardy operator, rearrangement invariant spaces, Boyd indices.

AMS Subject Classification: 46

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ON DEFERRED $f-$STATISTICAL BOUNDNESS OF DIFFERENCE SEQUENCE

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Study of difference sequences is a recent development in the summability theory. Sometimes a situation may arise that we have a sequence at hand and we are interested in sequences formed by its successive differences and in the structure of these new sequences. Studies on difference sequences was introduced in the 1980s and then many mathematicians studied on these kind of sequences and obtained some generalized difference sequence spaces. In this paper, using an unbounded modulus f and the difference operator Δ, we introduce the concept of Δ_f-deferred statistical boundedness and give some inclusion relations between Δ_f-deferred statistical convergence and Δ_f-statistical boundedness. Our results are more general than the corresponding results in the existing literature.

Keywords: Difference Sequence, Deferred Statistical Convergence, Deferred Statistically Cauchy.

AMS Subject Classification: 40A05, 40C05, 46A45.

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$G_k^{(2)}$-PERIODIC GIBBS MEASURES FOR THE POTTS-SOS MODEL

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The Cayley tree τ^k (see [1]) of order $k \geq 1$ is an infinite tree, i.e. a graph without cycles, from each vertex of which exactly $k+1$ edges issue. Let $\tau^k = (V, L, i)$, where V is the set of vertices of τ^k, L is the set of edges of τ^k and i is the incidence function associating each edge $l \in L$ with its endpoints $x, y \in V$.

We consider the model, where the spin takes values in the set $\Phi = \{0, 1, 2\}$.

The Hamiltonian of the Potts-SOS model with nearest-neighbor interaction has the form [2-4]

$$H(\sigma) = -J \sum_{\langle x,y\rangle \in L} |\sigma(x) - \sigma(y)| - J_p \sum_{\langle x,y\rangle \in L} \delta_{\sigma(x)\sigma(y)} \tag{1}$$

where $J, J_p \in \mathbb{R}$, $\sigma(x) \in \Phi$.

Let x_* be a fixed point $\psi(x) := \psi(x, \theta, r, k) = \left(\frac{2\theta + rx}{\theta^2 + \theta x + r}\right)^k$, where $\theta = \exp(J\beta)$, $r = \exp(J_p\beta)$ and also $\beta = 1/T$.

We have the following theorem.

Theorem. If

$$\frac{kx_*((2-r)\theta^2 - r^2)}{(2\theta + rx_*)(\theta^2 + \theta x_* + r)} > 1 \tag{2}$$

holds, then there are at least two non-translational-invariant $G_k^{(2)}$-periodic Gibbs measures for the model (1).

Keywords: Cayley tree, Potts-SOS model, Gibbs measure, periodic Gibbs measure, phase transition.

AMS Subject Classification: 82B26

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ON GENERALIZED SINGULAR NUMBERS OF DIFFERENCES OF POSITIVE MEASURABLE OPERATORS

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We denote by $\mathcal{M}$ a semi-finite von Neumann algebra on the Hilbert space $\mathcal{H}$ with a faithful normal semi-finite trace τ. The set of all τ-measurable operators is denoted by $L_0(\mathcal{M})$. For $x \in L_0(\mathcal{M})$, we let $\mu_t(x)$ $(t > 0)$ be the generalized singular numbers of x.

If $x, y \in L_0(\mathcal{M})$, then we shall say that x is submajorized by y, written $x \preccurlyeq y$, if and only if $\int_0^t \mu_s(x)ds \leq \int_0^t \mu_s(y)ds$ for any $t > 0$. Let $f, g \in L_0(0, \alpha)$. If $\int_0^t f^*(s)ds \leq \int_0^t g^*(s)ds$ for all $t \geq 0$, we call that f is *submajorized* by g, denote by $f \preccurlyeq g$.

If $x, y \in L_{\log_+}(\mathcal{M})$ such that

$$\int_0^t \log \mu_s(x)ds \leq \int_0^t \log \mu_s(y)ds, \qquad t > 0,$$

x is said to be logarithmically submajorized by y, denoted by $x \preccurlyeq_{\log} y$. By [3, Lemma 4.], we get that $x \preccurlyeq_{\log} y$ implies $x \preccurlyeq y$.

We extend the results in [1]) and [2] for τ-measurable operators associated with semi-finite von Neumann algebra.

Theorem 0.1. *Let $x, y \in L_{\log_+}(\mathcal{M})$.*

(1) *If x, y are Hermitian operators, then $x + y \preccurlyeq \sqrt{2}(x + iy)$.*
(2) *If x is positive and y is Hermitian, then $x + y \preccurlyeq_{\log} \sqrt{2}(x + iy)$.*
(3) *If x, y are positive operators, then $\mu_t(x + y) \leq \sqrt{2}\mu_t(x + iy)$ for all $t > 0$.*

Theorem 0.2. *Let $x, y \in L_{\log_+}(\mathcal{M})$ be positive operators. Then for any complex number z,*

$$x - |z|y \preccurlyeq_{\log} x + zy \preccurlyeq_{\log} x + |z|y.$$

Keywords: Submajorization, logarithmically submajorization, τ-measurable operator, accretive-dissipative operator matrix.

AMS Subject Classification: Primary: 46L51, Secondary: 46L52

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ON THE FOURIER TRANSFORM OF FUNCTIONS FROM A LORENTZ SPACE $L_{\bar{2},\bar{r}}$ WITH A MIXED METRIC

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Definition 1. Let $\bar{p} = (p_1, p_2)$, $\bar{r} = (r_1, r_2)$ and satisfy the following conditions: $0 < \bar{p} \leq \infty$, $0 < \bar{r} \leq \infty$. The Lorentz Space $L_{\bar{p},\bar{r}}[0,1]^2$ with a mixed metric is defined as the set of all measurable functions defined on $[0,1]^2$, for which the quantities are finite:

$$\|f\|_{L_{\bar{p},\bar{r}}} = \| \| \|f\|_{L_{p_1,r_1}} \|_{L_{p_2,r_2}} = \left(\int_0^1 \left(t_2^{\frac{1}{p_2}} \left(\int_0^1 \left(t_1^{\frac{1}{p_1}} f^{*_1}(t_1, \cdot) \right)^{r_1} \frac{dt_1}{t_1} \right)^{*_2}_{t_2} \right)^{\frac{r_2}{r_1}} \frac{dt_2}{t_2} \right)^{\frac{1}{r_2}},$$

in the case $0 < \bar{r} < \infty$, and

$$\|f\|_{L_{\bar{p},\infty}} = \sup_{t_1,t_2} t_1^{\frac{1}{p_1}} t_2^{\frac{1}{p_2}} f^{*_1 *_2}(t_1, t_2).$$

in the case $\bar{r} = \infty$.

Definition 2. Let $f \in L_1(\mathbb{R}^2)$. Its two-dimensional Fourier transform is defined by the following formula:

$$\hat{f}(\xi_1, \xi_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x_1, x_2) e^{-2\pi}.$$

The aim of this thesis is to show a two-dimensional analog of the Bochkarev type theorem for the Fourier transform.

Theorem. Let $\Phi_{m_1,m_2}(x_1, x_2) = \varphi_{m_1}(x_1) \cdot \psi_{m_2}(x_2)$, $m_1, m_1 \in \mathbb{N}$ is orthonormal bounded system of functions. Then for any $f \in L_{\bar{2},\bar{r}}[0,1]$, $2 < r_1, r_2 < \infty$ the inequality is fulfilled:

$$\sup_{\substack{|A_1| \geq 8 \\ A_1 \subset \mathbb{N},}} \sup_{\substack{|A_2| \geq 8 \\ A_2 \subset \mathbb{N},}} \frac{1}{|A_1|^{\frac{1}{2}} |A_2|^{\frac{1}{2}} (\log_2(|A_1|+1))^{\frac{1}{2} - \frac{1}{r_1}} (\log_2(|A_2|+1))^{\frac{1}{2} - \frac{1}{r_2}}} \times$$

$$\int_{A_1} \int_{A_2} \left| \hat{f}(\xi_1, \xi_2) \right| d\xi_1 d\xi_2 \leq C \|f\|_{L_{\bar{2},\bar{r}}}.$$

Keywords: The Lorentz Space, the Hausdorff-Young-Riesz theorem, the Bochkarev's theorem.

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FOURIER SERIES MULTIPLIERS IN THE GENERALISED HAAR SYSTEM

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This work is devoted to the study of the Fourier series multipliers in the generalized Haar system in Lebesgue and Lorentz spaces.

Let us consider the sequence $\lambda = \{\lambda_{l,h}^k\}_{k=1,\ l=0,\ h=1}^{\infty,\ m_k-1,\ p_{k+1}-1}$. Any sequence λ generates an operator Λ, called a multiplier, which is defined on polynomials in the generalized Haar system by the following way

$$\Lambda\left(\sum_{k=1}^{N}\sum_{l=0}^{m_k-1}\sum_{h=1}^{p_{k+1}-1} a_{l,h}^k(f)\chi_{l,h}^k(x)\right) = \sum_{k=1}^{N}\sum_{l=0}^{m_k-1}\sum_{h=1}^{p_{k+1}-1} \lambda_{l,h}^k a_{l,h}^k \chi_{l,h}^k(x),$$

here $\chi_n(x) := \chi_{l,h}^k(x)$ is the generalized Haar system, see e.g. [1] and [2].

A new theorem on sufficient conditions for the sequence to belong to the class $m(L_{p,r} \to L_{q,s})$ of multipliers of Fourier series with respect to the generalized Haar system is proved.

Theorem 1. Let $1 < p < \infty$, $0 < r, s \le \infty$, $\frac{1}{\tau} = \left(\frac{1}{s} - \frac{1}{r}\right)_+ = \max\left\{\frac{1}{s} - \frac{1}{r}, 0\right\}$ and $\{p_k\}$ be a sequence of natural numbers such that $p_0 = 1, 2 \le p_k \le c < +\infty$, $m_0 = 1$, $m_k = m_{k-1}p_k$, $k \in \mathbb{N}$. Let the sequence of complex numbers $\lambda = \{\lambda_k\}_{k\in N}$ satisfy the following condition

$$\left(\sum_{k=0}^{\infty}\left(m_k^{\left(\frac{1}{p}-\frac{1}{q}\right)} \sup_{m_k+1\le j\le m_{k+1}} \left|\lambda_k^j\right|\right)^{\tau}\right)^{\frac{1}{\tau}} \le A, \text{for } 0 < \tau < \infty \tag{1}$$

$$\sup_{\substack{0\le k\le\infty \\ m_k+1\le j\le m_{k+1}}} m_k^{\left(\frac{1}{p}-\frac{1}{q}\right)} |\lambda_k^j| \le A, \text{for } \tau = +\infty.$$

Then $\lambda \in m(L_{p,r} \to L_{q,s})$ and $\|\lambda\|_{m(L_{p,r}\to L_{q,s})} \le cA$.

Keywords: Fourier coefficients, generalized Haar system, Fourier multiplier, Lorentz spaces.

AMS Subject Classification: 42B15, 42B35

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HARDY AND POINCARE INEQUALITIES AND IDENTITIES FOR THE BAOUENDI-GRUSHIN OPERATOR

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The classical Hardy inequality was first extended by Garofalo [1] to the Baouendi–Grushin vector fields as follows

$$\int_{\mathbb{R}^n} \left(|\nabla_x f|^2 + |x|^{2\gamma} |\nabla_y f|^2 \right) dz \geq \left(\frac{Q-2}{2} \right)^2 \int_{\mathbb{R}^n} \left(\frac{|x|^{2\gamma}}{|x|^{2+2\gamma} + (1+\gamma)^2 |y|^2} \right) |f|^2 dz, \tag{1}$$

where $z = (x_1, ..., x_m, y_1, ..., y_k) = (x, y) \in \mathbb{R}^m \times \mathbb{R}^k$ with $n = m + k,\ m, k \geq 1,\ \gamma \geq 0,\ Q = m + (1+\gamma)k$ and $f \in C_0^\infty \left(\mathbb{R}^m \times \mathbb{R}^k \backslash \{(0,0)\}\right)$. Here, $\nabla_x f$ and $\nabla_y f$ are the gradients of f in the variables x and y, respectively.

In this talk, we show Hardy type identities for the Baouendi-Grushin operator involving radial derivatives in some of the variables leading to refinements of the above Hardy inequality. Moreover, we discuss stability type results. If time permits, we also discuss a sharp remainder formula for the Poincaré inequality. As an application, we show a blow-up result for solutions to the Dirichlet initial-boundary value problem for the Baouendi-Grushin heat operator.

This talk is based on the joint research with Ari Laptev (Imperial College London, UK) and Michael Ruzhansky (Ghent University, Belgium) [2]-[3], and with Durvudkhan Suragan (Nazarbayev University, Kazakhstan) [4].

Keywords: Hardy inequality, Poincare inequality, Baouendi-Grushin operator, Aharonov-Bohm magnetic field.

AMS Subject Classification: 26D10, 35P15

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CHARACTERISATION OF THE CONES OF MONOTONE FUNCTIONS GENERATED BY GENERALIZED FRACTIONAL-MAXIMAL FUNCTIONS

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Definition 1. Let $n \in \mathbb{N}$ and let $R \in (0;\infty]$. We say that a function $\Phi : (0;R) \to \mathbb{R}_+$ belongs to the class $B_n(R)$ if: (1) Φ decreases and is continuous on $(0;R)$; (2) There exists a constant $C = C_{\Phi,n} \in \mathbb{R}_+$ such that $\int\limits_0^r \Phi(\rho)\rho^{n-1}d\rho \le C\Phi(r)r^n,\ \ r \in (0,R)$.

Definition 2. Let $\Phi \in B_n(\infty)$. The *generalized fractional maximal function (GFMF)* $M_\Phi f$ is defined for the function $f \in L^1_{loc}(\mathbb{R}^n)$ by $(M_\Phi f)(x) = \sup_{r>0} \Phi(r) \int\limits_{B(x,r)} |f(y)|dy$, where $B(x,r)$ is the ball with the center at the point $x \in \mathbb{R}^n$ and radius $r > 0$.

Let $E = E(\mathbb{R}^n)$ be a rearrangement-invariant space (RIS) [1]. The space of GFMF $M^\Phi_E = M^\Phi_E(\mathbb{R}^n)$ is the set of all functions u, for which there is a function $f \in E(\mathbb{R}^n)$ such that $u(x) = (M_\Phi f)(x)$. We consider the following three cones generated by the non-increasing rearrangements of GFMF equipped with homogeneous functionals, respectively: $K_1 \equiv K^\Phi_E := \{h \in L^+(\mathbb{R}_+) : h(t) = u^*(t),\ u \in M^\Phi_E\},\ \rho_{K_1}(h) = \inf\{\|u\|_{M^\Phi_E} : u \in M^\Phi_E;\ u^*(t) = h(t),\ t \in \mathbb{R}_+\}$;

$$K_2 \equiv \widehat{K^\Phi_E} := \{h : h(t) = u^{**}(t),\ t \in \mathbb{R}_+,\ u \in M^\Phi_E\}, \rho_{K_2}(h) = \inf\{\|u\|_{M^\Phi_E} : u \in M^\Phi_E;\ u^{**}(t) = h(t),\ t \in \mathbb{R}_+\};$$

$$K_3 \equiv \widetilde{K^\Phi_E} := \big\{h \in L^+(\mathbb{R}_+) : h(t) = \sup_{t<\tau<\infty} \tau\Phi(\tau^{1/n})u^{**}(\tau),\ t \in \mathbb{R}_+,\ u \in E(\mathbb{R}^n)\big\},$$

$$\rho_{K_3}(h) = inf\big\{\|u\|_{E(\mathbb{R}^n)}, u \in E(\mathbb{R}^n) : \sup_{t<\tau<\infty} \tau\Phi(\tau^{1/n})u^{**}(\tau) = h(t),\ t \in \mathbb{R}_+\big\}.$$

Theorem 1. Let $\Phi \in B_n(\infty)$. Then $K_1 \approx K_2 \approx K_3$.

A criterion for embedding the space of GFMF in RIS $X(\mathbb{R}^n)$ is obtained. A description of the optimal RIS for such embedding is given. In the works [2-3] cones generated by generalized potentials were considered.

Keywords: rearrangement invariant spaces, generalized fractional maximal function, space of generalized fractional maximal functions, cones, covering of cones.

AMS Subject Classification: 42B25, 46E30, 47L07, 47B38.

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ON THE EMBEDDING OF THE SPACE OF GENERALIZED FRACTIONAL-MAXIMAL FUNCTIONS IN REARRANGEMENT-INVARIANT SPACES

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Let $R \in (0;\infty]$. A function $\Phi : (0;R) \to R_+$ belongs to the class $B_n(R)$ if the following conditions hold: Φ decreases and is continuous on $(0;R)$;There exists a constant $C \in R_+$ such that $\int\limits_0^r \Phi(\rho)\rho^{n-1}d\rho \le C\Phi(r)r^n, \quad r \in (0,R)$. Let $\Phi : R_+ \to \mathbb{R}$. The generalized fractional-maximal function $M_\Phi f$ is defined for the function $f \in E(\mathbb{R}^n) \cap L_1^{loc}(\mathbb{R}^n)$ by the equality $(M_\Phi f)(x) = \sup_{r>0} \Phi(r) \displaystyle\int_{B(x,r)} |f(y)|dy$. Let E is rearrangement-invariant space (briefly: RIS [1]) The space of generalized fractional maximal functions $M_E^\Phi = M_E^\Phi(R^n)$ as the set of the functions u, for which there is a function $f \in E(R^n)$ such that $u(x) = (M_\Phi f)(x),$, $\|u\|_{M_E^\Phi} = inf\{\|f\|_E : f \in E(R^n),\ \ M_\Phi f = u\}$. The cones: $K_1 := \{h \in L^+(\mathbb{R}_+) : h(t) = u^*(t),\ t \in \mathbb{R}_+,\ u \in M_E^\Phi\}$

Theorem 1. Let $\Phi \in B_n(\infty)$. The embedding $M_E^\Phi(\mathbb{R}^n) \hookrightarrow X(\mathbb{R}^n)$ is equivalence to the next embedding $K_1 M_E^\Phi(\mathbb{R}_+) \mapsto \widetilde{X}(\mathbb{R}_+))$.

Theorem 2. Let $\Phi \in B_n(\infty)$. The optimal RIS $X_0 = X_0(\mathbb{R}^n)$ for embedding (1) is defined by following norm:

$$\|f\|_{\widetilde{X}_0(0,\infty)} = \sup_{g^*}\left\{\int_0^\infty f^*(\tau)g^*(\tau)d\tau : g \in L_0(0,\infty), \sup_{\int_0^t h(s)ds \le \int_0^t g^*(s)ds} \Big\|\int_t^\infty \Phi(s^{1/n})sh(s)ds\Big\|_{E'} \le 1\right\}.$$

Note that in the [2,3,], the generalized Riesz potential was considered.

Keywords: rearrangement invariant spaces, generalized fractional maximal function, space of generalized fractional maximal functions, cones, covering of cones.

AMS Subject Classification: 42B25, 46E30, 47L07, 47B38.

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TRANSLATION-INVARIANT GIBBS MEASURES FOR THE HARD-CORE MODEL WITH A COUNTABLE SET OF STATES

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In this paper, we study the HC-model with a countable set $\mathbb{Z}$ of spin values on a Cayley tree of order two. This model is defined by a countable set of parameters (that is, the activity function $\lambda_i > 0$, $i \in \mathbb{Z}$). We consider the set $\mathbb{Z}$ as the set of vertices of a graph G. The activity set [1] for a graph G is a function $\lambda : G \to R_+$. For given G and λ we define the Hamiltonian of the $G-$HC model as $H_G^{\lambda}(\sigma) = J \sum\limits_{x\in V} \ln \lambda_{\sigma(x)}$, if $\sigma \in \Omega^G$, where $J \in R$.

Let $A \equiv A^G = (a_{ij})_{i,j\in Z}$ denote the adjacency matrix of G. We consider a specific graphs G_1 defined as $a_{i0} = 1$ for any $i \in \mathbb{Z}$, $a_{11} = 1$ and $a_{im} = 0$ otherwise, and G_2 defined as $a_{i0} = 1$ for any $i \in \mathbb{Z}$, $a_{11} = 1$, $a_{22} = 1$ and $a_{im} = 0$ otherwise. A functional equation is obtained that provides the consistency condition for finite-dimensional Gibbs distributions. Analyzing this equation, the following for $k = 2$ results are obtained :

- Let $\Lambda = \sum_i \lambda_i$. For G_1 and G_2 if $\Lambda = +\infty$ then there is no translation-invariant Gibbs measure (TIGM);
- For graph G_1 if $\Lambda < +\infty$, the uniqueness of TIGM is proved;
  Let $\Lambda_{cr} = 8\lambda^{3/2} - 10\lambda$ and $\Lambda^{(c)} = \frac{1}{1024}\Big((18\lambda^2 + 64\lambda)\sqrt{9\lambda^2 + 32\lambda} + 54\lambda^3 + 288\lambda^2 + 2304\lambda\Big)$.
- For G_2 if $\lambda \le \frac{49}{9}$ then for $\Lambda < \Lambda_{cr}$ there are exactly three TIGMs, for $\Lambda \ge \Lambda_{cr}$ there is exactly one TIGM;
- For G_2 if $\lambda > \frac{49}{9}$ then for $\Lambda \le \Lambda_{cr}$ there are exactly three TIGMs, $\Lambda_{cr} < \Lambda < \Lambda^{(c)}$ there are exactly five TIGMs, for $\Lambda = \Lambda^{(c)}$ there are exactly three TIGMs and $\Lambda > \Lambda^{(c)}$ there is exactly one TIGM.

Keywords: HC model, configuration, Cayley tree, Gibbs measure, boundary law.

AMS Subject Classification: 82B26 (primary); 60K35 (secondary)

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ON OSCILLATORY AND SPEKTRAL PROPERTIES OF A CLASS OF HIGHER-ORDER DIFFERENTIAL OPERATORS

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We study oscillatory properties of a $2n$th order differential equation and spectral properties of a $2n$th order differential operator. These properties are established by the variational method based on the validity of some nth order differential inequality, where its weights are the coefficients of the equation and the operator. In turn, the inequality is characterized when the weights satisfy certain conditions that guarantee for the function in this inequality the existence of boundary values at infinity and at zero.

Keywords: higher-order differential equation, differential operator, oscillation non-oscillation, variational method, weighted inequality.

AMS Subject Classification: 34C10, 26D10.

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SPECTRA OF THE ENERGY OPERATOR OF TWO-MAGNOM SYSTEMS IN THE FOUR-SPIN EXCHANGE HAMILTONIAN IN THE LATTICE

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We consider of the energy operator of two-magnon systems in the four-spin exchange Hamiltonian and investigated the considering system in the $\nu-$ dimensional lattice Z^ν. Hamiltonian of the considering system has the form

$$H = J\Sigma_{m,\tau}(\overrightarrow{S}_m\overrightarrow{S}_{m+\tau})(\overrightarrow{S}_{m+2\tau}\overrightarrow{S}_{m+3\tau}), \qquad (1)$$

where J is the four-spin exchange parameter between atoms, $\tau = \pm e_j$, $j = 1, 2, ..., \nu$; here e_j are unit mutually orthogonal vectors, $\overrightarrow{S}_m = (S^x_m; S^y_m; S^z_m)$ is the operator of the atomic spin $s, s > \frac{1}{2}$ the site m. Hamiltonian (1) acts in the symmetric Fock space $\mathcal{H}_{symm}$. We let φ_0 denote the vector, called the vacuum, uniquely defined by the conditions $S^+_m\varphi_0 = 0$, and $S^z_m\varphi_0 = s\varphi_0$, where $||\varphi_0|| = 1$. We set $S^\pm_m = S^x_m \pm iS^y_m$, where S^+_m and S^-_m are the magnon creation and annihilation operators at the site m. The vectors $S^-_m S^-_n\varphi_0$ describes the state of the system of two magnons at the sites m and n with the spin s. The vector space spanned by them is denoted by $\widehat{\mathcal{H}}_2$. It is a Euclidean space under the given inner product. We let $\mathcal{H}_2$ denote the closure of this space, called the space of two-magnon states of the operator H. We denote the restriction of H to the space $\mathcal{H}_2$ by H_2.

Theorem 1. The space $\mathcal{H}_2$ is invariant under the operator H. The operator H_2 is a bounded self-adjoint operator; it generates a bounded self-adjoint operator $\overline{H}_2$ acting in the space $l_2((Z^\nu)^2)$ as

$$(\overline{H}_2 f)(p,q) = J\sum_{p,q,\tau}\{[2s^2\delta_{p,q+2\tau} + 2s^2\delta_{p+2\tau,q} + s^2\delta_{p+\tau,q} + s^2\delta_{p,q+\tau} + s^2\delta_{p+3\tau,q} + s^2\delta_{p,q+3\tau}]\times \qquad (2)$$

$\times f(p,q) + (-s^2\delta_{p+3\tau,q} - 2s^2\delta_{p+2\tau,q} - s^2\delta_{p+\tau,q})f(p-\tau,q) + (-s^2\delta_{p,q+3\tau} - 2s^2\delta_{p,q+2\tau} - s^2\delta_{p,q+\tau})\times$ $\times f(p,q-\tau) + (-s^2\delta_{p+3\tau,q} - 2s^2\delta_{p+2\tau,q} - s^2\delta_{p+\tau,q})f(p+\tau,q) + (-s^2\delta_{p,q+3\tau} - 2s^2\delta_{p,q+2\tau} - s^2\delta_{p,q+\tau})\times$ $\times f(p,q+\tau) + 2s^2\delta_{p+2\tau,q}f(p-\tau,q-\tau) + (s^2\delta_{p+3\tau,q} + s^2\delta_{p,q+\tau})f(p+\tau,q-\tau) + (s^2\delta_{p,q+3\tau} + s^2\delta_{p+\tau,q})\times$ $\times f(p-\tau,q+\tau) + 2s^2\delta_{p+2\tau,q}f(p+\tau,q+\tau)\}$, where $\delta_{k,j}$ is the Kronecker symbol, and the summation over τ is over the nearest neighbors. The operator H_2 acts on the vector $\psi \in \mathcal{H}_2$ by the formula $H_2\psi = \sum_{p,q}(\overline{H}_2 f)(p,q)S^-_p S^-_q\varphi_0$.

Let $\nu = 1$.

Theorem 2. The spectra of operator H_2 are purely discrete, and consists of no more then six eigenvalues.

Keywords: Four-spin exchange Hamiltonian,two-magnon systems, spectra, discrete spectra, continuous spectrum.

AMS Subject Classification: 46L60, 47A75, 47N50, 47A10

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HAAGERUP NONCOMMUTATIVE ORLICZ SPACES

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We always assume that $\mathcal{M}$ is a σ-finite von Neumann algebra on a complex Hilbert space $\mathcal{H}$, equipped with a distinguished normal faithful state φ. Let $\hat{\varphi}$ be the dual weight our distinguished state φ. Then $\hat{\varphi}$ has the Radon-Nikodym derivative D with respect to τ (see [4]). Then

$$\hat{\varphi}(x) = \tau(Dx), \quad x \in \mathcal{N}^{+}.$$

Recall that D is an invertible positive selfadjoint operator on $L^2(\mathbb{R}, \mathcal{H})$, affiliated with $\mathcal{N}$.

For $x \in \mathcal{M}$, we define

$$\|\Phi^{-1}(D)^{\alpha} x \Phi^{-1}(D)^{1-\alpha}\|_{\Phi,\alpha} = \|\Phi^{-1}(D)^{\alpha} x \Phi^{-1}(D)^{1-\alpha}\|_{\Phi,\infty}.$$

Definition 0.1. *The completion of* $(\Phi^{-1}(D)^{\alpha}\mathcal{M}\Phi^{-1}(D)^{1-\alpha}, \|\cdot\|_{\Phi,\alpha})$ *is called the Haagerup noncommutative Orlicz space associted with* $\Phi, \mathcal{M}$ *and* φ*, we denote it by* $L^{\Phi,\alpha}(\mathcal{M}, \varphi)$.

Theorem 0.1. *Let* $0 \le \alpha \le 1$*. Then*

$$\|x\|_{\Phi,\alpha} = \frac{\mu_1(x)}{\Phi^{-1}(1)}, \qquad \forall x \in L^{\Phi,\alpha}(\mathcal{M}).$$

In addition the norm topology in $L^{\Phi,\alpha}(\mathcal{M})$ *is homeomorphic to the topology of convergence in measure inherited from* $L_0(\mathcal{N})$*.*

Keywords: Noncommutative Orlicz spaces; σ-finite von Neumann algebras; reduction; crossed products; Haagerup L^p-spaces.

AMS Subject Classification: Primary: 46L52; Secondary: 47L05

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ON THE RATIONALITY OF THE GENERATING FUNCTION FOR THE NUMBER OF ROOT FORESTS IN CIRCULANT GRAPHS

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Let $s_1, s_2, ..., s_k$ be natural numbers such that $1 \leq s_1 < s_2 < ... < s_k \leq \frac{n}{2}$. Graph $C_n(s_1, s_2, ..., s_k)$ on n vertices $0, 1, 2, ..., n-1$ is called *circulant* if vertex i, $i = 0, 1, ..., n-1$ is adjacent to vertices $i \pm s_1, i \pm s_2, ..., i \pm s_k (mod\, n)$. If $s_k < \frac{n}{2}$, then all vertices of the graph have an even degree $2k$. If n is even and $s_k = \frac{n}{2}$, then all vertices have an odd degree $2k-1$ ([1]).

Let $\Phi(x)$ be the generating function for the number $f_\Gamma(n)$ of rooted spanning forests in the circulant graph $\Gamma = C_n(s_1, s_2, ..., s_k)$ or $\Gamma = C_{2n}(s_1, s_2, ..., s_k, n/2)$. We will show that $\Phi(x)$ is a rational function with integer coefficients satisfying condition $\Phi(x) = -\Phi(\frac{1}{x})$. A *root tree* is a tree in which one vertex is highlighted. A *root forest* is a forest whose associated components are root trees. The *root spanning forest* in graph Γ is called the root forest containing all the vertices of graph ([2]). We consider ordinary graphs.

The main result is the following

Theorem 1. *Let $f_\Gamma(n)$ be the number of root spanning forests in a circulant graph $\Gamma = C_n(s_1, s_2, ..., s_k)$ of even valence or $\Gamma = C_{2n}(s_1, s_2, ..., s_k, n)$ of odd valence. Then*

$$\Phi(x) = \sum_{n=1}^{\infty} f_\Gamma(n) x^n$$

is a rational function with integer coefficients. Moreover, $\Phi(x) = -\Phi(\frac{1}{x})$.

Keywords: Root forest, root spanning forest, circulant graph, generating function.

AMS Subject Classification: 05C30, 39A12
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Scientific supervisor: Professor, Doctor of physico-mathematical sciences Alexander Dmitrievich Mednykh.

DYNAMICAL SYSTEMS OF AN INFINITE DIMENSIONAL OPERATOR

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Denote $l^1_+ = \left\{ x = (x_1, x_2, \dots, x_n, \dots) \, : \, x_i > 0, \, \|x\| = \sum_{j=1}^{\infty} x_j < \infty \right\}$.

Following [1] we consider discrete-time, infinite-dimensional dynamical systems (IDDS) generated by operator F defined on l^1_+ as

$$F : x'_{2n-1} = \lambda_{2n-1} \cdot \left(\frac{1 + \sum_{j=1}^{\infty} x_{2j-1}}{1 + \theta + \|x\|} \right)^2, \quad x'_{2n} = \lambda_{2n} \cdot \left(\frac{1 + \sum_{j=1}^{\infty} x_{2j}}{1 + \theta + \|x\|} \right)^2$$

where $n = 1, 2, \dots$, $\theta > 0$ and $\lambda = (\lambda_1, \lambda_2, \dots) \in l^1_+$.

The main problem for such a dynamical system (see Chapter 1 of [2]) is to study trajectory $t^{(m)} = F^m(t^{(0)})$, $m \geq 1$ for any $t^{(0)} \in l^1_+$.

We have proved the following

Lemma 1. If $\lambda = (\lambda_1, \lambda_2, \dots) \in l^1_+$ then F maps l^1_+ to itself.

Define two-dimensional operator $W : z = (x, y) \in \mathbb{R}^2_+ \to z' = (x', y') = W(z) \in \mathbb{R}^2_+$ by

$$W : x' = L_1 \cdot \left(\frac{1 + x}{1 + \theta + x + y} \right)^2, \quad y' = L_2 \cdot \left(\frac{1 + y}{1 + \theta + x + y} \right)^2,$$

where $\theta > 0$ and $L_i > 0$ are parameters.

Lemma 2. The IDDS generated by the operator F is fully represented by the two-dimensional DS generated by the operator W.

The following is main result

Theorem 1. If for $v^{(0)} = (x^{(0)}, y^{(0)}) \in \mathbb{R}^2$ the limit $\lim_{m\to\infty} W^m(v^{(0)}) = (a, b)$ exists then for each $t^{(0)} \in l^1_+$ with

$$\sum_{j=1}^{\infty} t^{(0)}_{2j-1} = x^{(0)}, \quad \sum_{j=1}^{\infty} t^{(0)}_{2j} = y^{(0)}$$

the following equality holds

$$\lim_{m\to\infty} F^m(t^{(0)}) = \left(\frac{a}{L_1}\lambda_1, \frac{b}{L_2}\lambda_2, \frac{a}{L_1}\lambda_3, \frac{b}{L_2}\lambda_4, \dots \right).$$

Keywords: Infinite dimensional operator, limit point, dynamical system.
AMS Subject Classification: 82B05, 82B20, 60K35.

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CHARACTERIZATION OF LIPSCHITZ FUNCTIONS VIA THE COMMUTATORS OF MAXIMAL FUNCTION IN ORLICZ SPACES ON STRATIFIED LIE GROUPS

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Let $\mathbb{G}$ be a stratified Lie group, $f \in L^1_{\rm loc}(\mathbb{G})$ and $0 \le \alpha < Q$, where Q is the homogeneous dimension of $\mathbb{G}$. The fractional maximal function $M_\alpha f$ is defined by

$$M_\alpha f(x) = \sup_{B \ni x} |B|^{-1+\frac{\alpha}{Q}} \int_B |f(y)| dy,$$

and the sharp maximal function of Fefferman and Stein $M^\sharp f$ is defined by

$$M^\sharp f(x) = \sup_{B \ni x} |B|^{-1} \int_B |f(y) - f_B| dy,$$

where the supremum is taken over all balls $B \subset \mathbb{G}$ containing x, and $|B|$ is the Haar measure of the $\mathbb{G}$-ball B. When $\alpha = 0$, we simply denote by $M = M_0$.

The maximal commutator generated by $b \in L^1_{\rm loc}(\mathbb{G})$ and M is defined by

$$M_b(f)(x) = \sup_{B \ni x} |B|^{-1} \int_B |b(x) - b(y)||f(y)| dy.$$

The commutators generated by $b \in L^1_{\rm loc}(\mathbb{G})$ and M, $M^\sharp$ are defined by

$$[b, M]f(x) = b(x)Mf(x) - M(bf)(x), \ [b, M^\sharp]f(x) = b(x)M^\sharp f(x) - M^\sharp(bf)(x).$$

We shall give some new characterizations of the Lipschitz spaces via the boundedness of commutators associated with the maximal operator in Orlicz spaces on stratified Lie groups. We give necessary and sufficient conditions for the boundedness of the maximal commutators M_b, the commutators of the maximal operator $[b, M]$ and the commutators of the sharp maximal operator $[b, M^\sharp]$ in Orlicz spaces $L^\Phi(\mathbb{G})$ on any stratified Lie group $\mathbb{G}$ when b belongs to Lipschitz spaces $\dot{\Lambda}_\beta(\mathbb{G})$, see [1,2].

Keywords: Stratified group, Orlicz space, maximal function, sharp maximal function, commutator, Lipschitz function.

AMS Subject Classification: 42B25, 42B35, 43A15, 46E30.

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WEAKLY PERIODIC GERENALUZED GIBBS MEASURE

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In the present paper is to show that the existence of H_A-weakly periodic generalized p-adic Gibbs measures for the p-adic Ising model on the Cayley tree of order two.

КОМПАКТНОСТЬ ИНТЕГРАЛЬНЫХ ОПЕРАТОРОВ, ПРЕДЕЛЫ КОТОРЫХ ЯВЛЯЮТСЯ ФУНКЦИЯМИ

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Пусть $1 < p, q < \infty$, $p' = \frac{p}{p-1}$, $I = (0, \infty)$, u и v -весовые функции, то есть неотрицательные измеримые на I – функции такие, что $u \in L_1^{loc}(I)$.

Рассмотрим вопрос о компактности из $L_{p,v} = L_p(v, I)$ в $L_{q,u} = L_q(u, I)$ интегрального оператора

$$S_{\alpha,\beta} f(x) = \int\limits_{\psi(x)}^{\varphi(x)} \frac{\left(ln\frac{W(x)}{W(x)-W(s)}\right)^{\beta}}{(W(x)-W(s))^{1-\alpha}} u(s) f(s) dW(s), \quad x \in I,$$

где $L_{p,v} = L_p(v, I)$ – пространство всех измеримых на I функций таких, что:

$$\|f\|_{p,v} = \left(\int\limits_0^{\infty} |f(x)|^p v(x) dx\right)^{\frac{1}{p}} < \infty, \ \ p > 1.$$

Положим $\frac{dW(x)}{dx} = w(x)$, для всех $x \in I$, где $\alpha > 0$, $\beta \geq 0$. При $\psi(x) = 0$ и $\varphi(x) = x$ критерий ограниченности и компактности такого оператора из одного весового пространство в другое весовое пространство получено в работе [1].

Теорема 1. Пусть $\frac{1}{\alpha} < p \leq q < \infty$, $0 < \alpha < 1$, $\beta \geq 0$. Тогда оператор $S_{\alpha,\beta}$ компактен из $L_{p,v} = L_p(v, I)$ в $L_{q,u} = L_q(u, I)$ тогда и только тогда, когда
$\lim\limits_{\varphi(z)\to 0} A_{\alpha,\beta}(\varphi(z)) = \lim\limits_{\varphi(z)\to\infty} A_{\alpha,\beta}(\varphi(z)) = 0$, и

$$A_{\alpha,\beta} = \sup_{\varphi(z)>0} \left(\int\limits_{\varphi(z)}^{\infty} W^{q(\alpha-\beta-1)}(x) v(x) dx\right)^{\frac{1}{q}} \left(\int\limits_0^{\varphi(z)} u^{p'}(s) W^{p'\beta}(s) w(s) ds\right)^{\frac{1}{p'}} < \infty.$$

Данная работа была выполнена при поддержке Министерства науки и высшего образования Республики Казахстан по направлению "Научные исследования в области естестованных наук"(номер гранта AP09259084).

Ключевые слова: компактность, интегральный оператор, весовые функции.

Предметная классификация AMS: 46B50, 47G10

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Алгебра, математикалық логика және геометрия

Algebra, mathematical logic and geometry

Алгебра, математическая логика и геометрия

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ON THE GEOMETRY OF VECTOR FIELDS

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Definition 1. A partition F of a riemannian manifold M by path-connected immersed submanifolds L_α is called a singular foliation of M if it verifies condition: for each leaf L_α and each vector $v \in T_pL_\alpha$ at the point p, there is $X \in XF$ such that $X(p) = v$, where T_pL_α is the tangent space of the leaf L_α at the point p , XF is the module of smooth vector fields on M tangent to leaves (XF acts transitively on each leaf) (see [1, 2, 3]). If the dimension of L is maximal, it is called regular, otherwise L is called singular. It is known that orbits of vector fields generate singular foliation (see [1, 2, 3]).

Let us consider a set of vector fields D, let $t \to X^t(x)$ be an integral curve of the vector field X with the initial point x for $t = 0$, which is defined in some region $I(x)$ of real line.

Definition 2. The orbit $L(x)$ of a system D of vector fields through a point x is the set of points y in M such that there exist $t_1, t_2, ..., t_k \in R$ and vector fields $X_{i_1}, X_{i_2}, ..., X_{i_k} \in D$ such that

$$y = X_{i_k}^{t_k}(X_{i_{k-1}}^{t_{k-1}}(...(X_{i_1}^{t_1})...)),$$

where k is an arbitrary positive integer.

The fundamental result in study of orbits is Sussmann theorem [3], which asserts that every orbit is an immersed submanifold of M.

Let us consider a family of $D = \{X_1, X_2\}$ vector fields on four-dimensional Euclidean space E^4 with Cartesian coordinates (p_1, p_2, q_1, q_2), where

$$X_1 = q_1\frac{\partial}{\partial p_1} + q_2\frac{\partial}{\partial p_2} + p_1\frac{\partial}{\partial q_1} + p_2\frac{\partial}{\partial q_2}, X_2 = q_1\frac{\partial}{\partial p_1} + p_1\frac{\partial}{\partial q_1}. \tag{1}$$

Theorem 1. The family of orbits of the vector fields (1) is a singular foliation regular leaf of which is a surface with a positive normal curvature and with non zero normal torsion.

Keywords: vector field; orbit; foliation; hamiltonian

AMS Subject Classification:57R27

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ON THE GEOMETRY OF HAMILTONIAN SYSTEMS

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Definition 1. Let M be a Poisson manifold with the Poisson bracket $\{\cdot,\cdot\}$ and $H : M \to \mathbb{R}$ a smooth function. The *Hamiltonian vector field* associated with H is the unique smooth vector field $sgradH$ on M satisfying

$$sgradH(F) = \{F, H\} = -\{H, F\} \tag{1}$$

for every smooth function $F : M \to \mathbb{R}$.

The equations governing the flow of $sgradH$ are referred to as *Hamilton's equations* for the *Hamiltonian function* H.

Definition 2. Let M^{2n} be a Poisson manifold and $sgradH$ Hamiltonian vector field with a smooth Hamiltonian function H.

Hamiltonian system $sgradH$ is called *completely integrable in the sense of Liouville*, if exists set of smooth functions $f_1, ..., f_n$ as:

1) $f_1, ..., f_n$ are first integrals of $sgradH$ Hamiltonian vector field,

2) they are functionally independent on M, that is, almost everywhere on M their gradients are linearly independent,

3) $\{f_i, f_j\} = 0$ for any i and j,

4) the vector fields $sgradf_i$ are complete, that is natural parameter on their integral trajectories is defined on the whole number line.

Definition 3. Partition of the manifold M^{2n} into connected components of joint level surfaces of the integrals $f_1, ..., f_n$ is called *The Liouville foliation* corresponding to the completely integrated system. Liouville foliation is consists of regular leaves (which fill almost all M) and special leaves (a subset of zero measure).

Theorem 1. Regular leaves of Liouville foliation generated by Hamiltonian

$$H(p_1, p_2, q_1, q_2) = \frac{1}{2}(p_1^2 + p_2^2 - q_1^2 + q_2^2)$$

are two dimensional surfaces of zero Gauss curvature and zero Gauss torsion.

Keywords: Poisson bracket, Hamiltonian system, Liouville foliation, Gauss curvature, Gauss torsion.

AMS Subject Classification: 37J35

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GENERALIZATION OF TIKHONOV'S THEOREM TO EXTRACT INFORMATION ON OBJECTS IN SPACES

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Tikhonov's theorem on continuity of reverse function with compact domain [1] provides conditions to restore (hidden) objects by their (observable) bijective images. On the one hand, some images are not bijective (for instance, projections), on the other hand we sometimes need not the whole object but any information on it (especially, if the object moves in a kinematical space).

We consider metrical spaces only. Let X be a compact space of "objects", Y be a space of their observable images, Z be a space of "information", $P : X \to Z$ be continuous.

Theorem 0.1. *If 1) $f : X \to Y$ is surjective and continuous;*
2) $(f(x_1) = f(x_2)) \Rightarrow (P(x_1) = P(x_2))$
then the assertion $(\exists x \in X)((y = f(x)) \wedge (z = P(x))$ defines a continuous function $g : Y \to Z$.

Proof. Let $y_0 \in Y$. Due to 1) we have $(\exists x_0 \in X)(y_0 = f(x_0))$ and $(\exists z_0 \in Z)(z_0 = P(x_0))$.

Hence, the function g is defined. If $((y_0 = f(x_1)) \wedge (y_0 = f(x_2))$ then due to 2) we have $P(x_1) = P(x_2)$. Hence, the function g is defined uniquely.

Suppose that (*) there exists such sequence $\{x_k | k \in N\} \subset X$ that the sequence $\{y_k := f(x_k) | k \in N\}$ converges to y_0 but the sequence $\{z_k := P(x_k) | k \in N\}$ does not converge to z_0.

Then there exists such positive number ε that any infinite subset $Z_1 \subset \{z_k\}$ is out of $E_0 :=$ ($\varepsilon-$neighborhood of z_0).

Consider the corresponding subset $X_1 \subset \{x_k\}$. By compactness there exists a subset $X_2 \subset X_1$ converging to any $x' \in X$.

Then the corresponding subset $Z_2 \subset Z_1$ converges to $z' := P(x') \notin E_0$. Hence, $P(x') \neq P(x_0)$. But $f(x') = f(x_0)$.

Therefore, the assumption (*) has implied a contradiction with 2). Consequently, the function g is continuous. Theorem is proven.

Example 0.1. *Let X be the set of segments in R^n_+ with Hausdorff metric, $Y := R^n_+, Z := R_+$. Denote*
$f(x) := \{projection\ of\ x\ onto\ Ox_j - axis | j = 1..n\}$,
$P(x)$ is the length of x.
Then, by Theorem, a continuous function $g : Y \to Z$ exists. Hence, the length of x can be found by its projections although x itself cannot be found.

. **Keywords:** topological space, information, projection, continuous function, motion

AMS Subject Classification: 54D05

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ON COSEMANTICNESS CLASSES OF A JONSSON SPECTRUM

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In this work, we study the properties of cosemanticness classes from a fixed Jonsson spectrum.

Let T be a Jonsson theory [1] of some countable language L. Let $K \subseteq E_T$. We consider a Jonsson spectrum $JSp(K)$ defined as follows [2]:

$$JSp(K) = \{T \mid T \text{ is a Jonsson theory and for all } A \in K \ A \models T\}.$$

Let us introduce the cosemanticness relation (defined by $\bowtie$) [3] on $JSp(K)$. Cosemanticness is an equivalence relation, and therefore divides the spectrum into cosemanticness classes. Thus, we get the factor set $JSp(K)_{/\bowtie}$. Next, we fix some cosemanticness class $[T]$. All the Jonsson theories in this class have the same semantic model, which is denoted by $C_{[T]}$.

Lemma 1. Let $[T] \in JSp(K)_{/\bowtie}$ be a cosemanticness class consisting of $\forall$-complete theories, and let $[T'] \in JSp(K)_{/\bowtie}$ be such that $[T']$ consists of extensions of the theories of $[T]$. Then if $p(\overline{x}) \cup T$ is consistent for each $T \in [T]$ then $p(\overline{x}) \cup T'$ is consistent for each $T' \in [T']$, where $p(\overline{x}) \in \exists_1$.

Definition 1. A class K of existentially closed models of the signature σ is called a J-class, if $Th_{\forall\exists}(K)$ is a Jonsson theory.

Theorem 1. Let $[T] \in JSp(K)_{/\bowtie}$ consists of $\forall$-complete theories. Then any class K' of existentially closed models of the theories from $[T]$ is a J-class.

Theorem 2. Let K' be a subclass of existentially closed models of theories from $[T] \in JSp(K)_{/\bowtie}$, $C_{[T]}$ be a semantic model of $[T]$. Then $[T'] \in JSp(K)_{/\bowtie}$, where

$$T' = T^0(K') \vee T^0(C_{[T]}) = \{\varphi \vee \psi \mid \varphi \in T^0(K'), \psi \in T^0(C_{[T]})\}.$$

Theorem 3. Let $[T] \in JSp(K)_{/\bowtie}$, $C_{[T]}$ be a semantic model of $[T]$. If $T^0(C_{[T]})$ is a finitely axiomatizable theory then there is a minimal Jonsson theory T' such that $[T'] \in JSp(K)_{/\bowtie}$.

Keywords: Jonsson theory, Jonsson spectrum, cosemanticness, cosemanticness class.

AMS Subject Classification: 03C65

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ON HOLOGRAPHICNESS IN THE FRAMES OF JONSSON THEORIES

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The class of any Jonsson theory can be conditionally divided into three classes. These are classes of the following models: finite (Fin_T), infinite (Inf_T), and existentially closed (E_T). That is, $Fin_T \cup Inf_T \cup E_T = ModT$ and $E_T \subseteq Inf_T$.

The following result shows that the countable categorical of the considered Jonsson theory is connected with the concept of holographicness and existential-closure of its models, while $Hol_T \subseteq ModT \setminus E_T$.

Lemma 0.1. *If T is a perfect, not ω-categorical Jonsson theory, then $Hol_T \cap E_T = \varnothing$*

The main question posed in [1] is to find other examples of holographic structures that differ from the examples given in [1].

Our goal is to find new examples of holographic structures that were not discussed in [1]. Our main idea for solving this problem is that such a property could be the concept of perfection of the considered Jonsson theory. To implement our conjecture, we proved the following result.

Theorem 0.2. *Let T be a κ-categorical, Jonsson theory, where $\kappa \geq \omega$, then T is perfect.*

If the theory T has holographic models, then by Hol_T we denote the set of holographic models of the theory T. The following result is an existence theorem for holographic models for a fairly wide subclass of inductive theories, namely the class of perfect Jonsson theories.

Theorem 0.3. *If T is a perfect Jonsson theory, then $Hol_T \neq \varnothing$, i.e. such a theory has a holographic model.*

Keywords: holographic structures, Jonsson theories, existentially closed models, perfectness, ω-categorical Jonsson theory. **AMS Subject Classification:** 03C10; 03C35; 03C50

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CRITERION OF ALGEBRAICALLY PRIMENESS FOR THE JONSSON ABELIAN GROUP

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In [1] a criterion for the existence of a prime model for an arbitrary abelian group was found. The concept of an algebraically prime model generalizes the concept of a prime model.

Definition 0.1. [2] *A model A of a theory T is called an algebraically prime model of this theory if it can be isomorphically embedded in every model of T.*

As shown in [3], no general criterion of algebraic primeness is known for an arbitrary theory. As is known from work [4], the theory of Abelian groups is a perfect Jonsson theory. The main result of this thesis is a criterion for the existence of an algebraically prime model for the theory of Abelian groups. In the work [5] gives criteria for the existence of different types of prime models and also for algebraically prime models for a particular case of Abelian groups, namely, for torsion-free Abelian groups. The results of works [1] and [5] are realized in the framework of the complete theories of the corresponding Abelian groups. Jonsson theory is, generally speaking, not complete.

The following theorem generalizes the main results from [1] and [5] on the language of JSP-cosemanticness ($\underset{JSp}{\bowtie}$), which generalized the notion of elementary equivalence.

Theorem 1. Let T be the theory of abelian groups. Then the theory T has an algebraically prime model if and only if at least one of the conditions is satisfied:

a) $C_T \underset{JSp}{\bowtie} \oplus_p \mathbb{Z}_{p^\infty}^{(\alpha_p)}$;

b) $C_T \underset{JSp}{\bowtie} \oplus \mathbb{Q}^{(\beta)}$ and T^* has an algebraically prime model,

where C_T is semantic model of Jonsson theory T, $T^* = Th(C_T)$, $\alpha_p, \beta \in \omega^+$, $|C_T| = 2^\omega$.

All information about Jonsson theory and its details linked with a JSP-cosemanticness one can extract from [4].

Keywords: Jonsson theory, semantic model, algebraically prime model, JSp-cosemanticness.

AMS Subject Classification: 03 Mathematical logic and foundations

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ON SEMANTIC JONSSON QUASIVARIETY OF ROBINSON UNARS

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This paper focuses on the study of model-theoretic properties of Robinson unars in terms of semantic Jonsson quasivariety and its Robinson spectrum. All undefined concepts regarding Robinson spectrum of semantic Jonsson quasivariety of unars can be found in work [1], regarding Jonsson theories in work [2].

Definition 1. [1] A set $RSp(JC_U)$ of Robinson theories of signature $\sigma =< f >$, where $RSp(JC_U) = \{\Delta_i \mid \Delta_i$ is Robinson theory of unars and $\forall C_{\Delta_i} \in JC_U, C_{\Delta_i} \models \Delta_i\}$, is said to be Robinson spectrum of class JC_U, where JC_U is semantic Jonsson quasivariety of Robinson unars, f is a unary function symbol.

Fact 1. There was obtained system of complete invariants of semantic model of cosemanticness class $[\Delta]_{\bowtie}$ in the work [1].

A class of Robinson theories $[\Delta_X^C]$ in the new enriched signature $\sigma_\Gamma(X) = \sigma \cup \{c_a, a \in X\} \cup \Gamma$, $\Gamma = \{P\} \cup \{c\}$ for each cosemanticness class $[\Delta]_{\bowtie} \in RSp(JC_U)$, where $\Delta_X^C \in [\Delta_X^C]$ is constructed as follows: $\Delta_X^C = \Delta \cup Th_{\forall\exists}(C,a)_{a\in X} \cup \{P(c_a), a \in X\} \cup \{P(c)\} \cup \{P, \subseteq\}$. P is a new 1-ary predicate symbol whose interpretations is an existentially closed submodel M of the semantic model C, i.e. $P(C) = M, M \in E_\Delta$, $\Delta \in [\Delta]_{\bowtie}$. Let $\Delta' = Th(JC_U)$ be a center for the class $[\Delta_X^C]$. Consider the theory Δ' in a restricted signature $\sigma_\Gamma(X)\backslash\{c\}$ so that Δ' becomes a complete type of c.

Definition 2. A complete type described above is called a central type for the Robinson theory of unars Δ' with respect to the Jonsson set X (denoted by p_X^C).

Theorem 1. Let JC_U be semantic Jonsson quasivariety of Robinson unars, $RSp(JC_U)$ be its Robinson spectrum, $[\Delta]_{\bowtie} \in RSp(JC_U)_{/\bowtie}$, $\Delta_i \in [\Delta]_{\bowtie} (i \in I)$, Δ^* is a center for the class $[\Delta]_{\bowtie}$. Let X_{Δ_i} be Jonsson sets for the theories Δ_i respectively, $dcl(X_{\Delta_i}) = M_{\Delta_i}$, $M_{\Delta_i} \in E_{\Delta_i}$. $[\Delta_X^C]$ is the class of the theories in the enriched signature as it is described above. If $\lambda \geq \omega$, then Δ^* is $J-\lambda$-stable if and only if Δ_S is λ-stable for any theory $\Delta_S \in [\Delta_X^C]$.

Keywords: Jonsson theory, unars, Robinson theory, semantic Jonsson quasivariety, Jonsson spectrum, Robinson spectrum, central type.

AMS Subject Classification: 03C05, 03C45, 03C60

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ON SEMANTIC JONSSON QUASIVARIETY OF UNDIRECTED GRAPHS

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This paper focuses on the study of model-theoretic properties of undirected graphs in terms of semantic Jonsson quasivariety and its Robinson spectrum. All undefined concepts regarding Robinson spectrum of semantic Jonsson quasivariety of graphs can be found in work [1], regarding Jonsson theories in work [2].

Definition 1. [1] A set $RSp(JC_G)$ of Robinson theories of signature $\sigma =< R >$, where $RSp(JC_G) = \{\Delta_i \mid \Delta_i$ is Robinson theory of undirected graphs and $\forall C_{\Delta_i} \in JC_G, C_{\Delta_i} \models \Delta_i\}$, is said to be Robinson spectrum of class JC_G, where JC_G is semantic Jonsson quasivariety of undirected graphs, R is binary symmetric relation.

A class of Robinson theories $[\Delta_X^C]$ in the new enriched signature $\sigma_\Gamma(X) = \sigma \cup \{c_a, a \in X\} \cup \Gamma$, $\Gamma = \{P\} \cup \{c\}$ for each cosemanticness class $[\Delta]_{\bowtie} \in RSp(JC_G)$, where $\Delta_X^C \in [\Delta_X^C]$ is constructed as follows: $\Delta_X^C = \Delta \cup Th_{\forall\exists}(C, a)_{a\in X} \cup \{P(c_a), a \in X\} \cup \{P(c)\} \cup \{P, \subseteq\}$. P is a new 1-ary predicate symbol whose interpretations is an existentially closed submodel M of the semantic model C, i.e. $P(C) = M, M \in E_\Delta$, $\Delta \in [\Delta]_{\bowtie}$. Let $\Delta' = Th(JC_G)$ be a center for the class $[\Delta_X^C]$. Consider the theory Δ' in a restricted signature $\sigma_\Gamma(X)\backslash\{c\}$ so that Δ' becomes a complete type of c.

Definition 2. A complete type described above is called a central type for the Jonsson theory of undirected graphs Δ' with respect to the Jonsson set X (denoted by p_X^C).

Theorem 1. Let JC_G be semantic Jonsson quasivariety of undirected graphs, $RSp(JC_G)$ be its Jonsson spectrum, $[\Delta]_{\bowtie} \in RSp(JC_G)_{/\bowtie}$, $\Delta_i \in [\Delta]_{\bowtie}\,(i \in I)$, Δ^* is a center for the class $[\Delta]_{\bowtie}$, and let $[\Delta_X]$ be as it is describe above. Then Δ_X^* is ω-categorical if and only if each $\Delta_S \in [T_X^C]$ is ω-categorical, $\Delta_S \in [\Delta_X^C]$.

Keywords: Jonsson theory, graphs, undirected graphs, semantic Jonsson quasivariety, Jonsson spectrum, Robinson spectrum, central type.

AMS Subject Classification: 03C05, 03C45, 03C60

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ON COUNTABLY PARACOMPACT EXTENSIONS

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In this work the set of all countably paracompact extensions of a topological space with the help of his uniformities has been constructed.

A uniform space is called sequentially complete if every Cauchy filter having countable base in it converges.

It is well known that every countably paracompact space is sequentially complete with respect to its universal uniformity U_X, and the system of all open covers of the normal space X forms the base of universal uniformity U_X[1], [2]. If Y is a dense subspace of the space X, and V is the uniformity on Y induced by the uniformity U, then the space (X, U) is the sequentially completion of the uniform space (Y, V). Thus, generally speaking, V is not a universal uniformity, but has a special feature which we call as countably preparacompactness. It turns out that by countably preparacompact uniform structures of space Y one can construct all his countably paracompact extensions namely, to obtain these extensions as sequentially completions of space Y countably preparacompact uniform structures. Therefore, the construction of countably paracompact extensions of the topological space by uniform structures is, in our opinion, the most convenient and natural.

Keywords: Countably paracompact extensions, countably preparacompact uniformity, sequentially completeness, sequentially completion.

AMS Subject Classification: 54E15, 54D20

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AUTOMORPHISMS OF SIMPLE QUOTIENTS OF THE POISSON AND UNIVERSAL ENVELOPING ALGEBRAS OF sl_2

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Let $P(\mathrm{sl}_2(K))$ be the Poisson enveloping algebra of the Lie algebra $\mathrm{sl}_2(K)$ over an algebraically closed field K of characteristic zero. The quotient algebras $P(\mathrm{sl}_2(K))/(C_P - \lambda)$, where C_P is the standard Casimir element of $\mathrm{sl}_2(K)$ in $P(\mathrm{sl}_2(K))$ and $0 \neq \lambda \in K$, are proven to be simple in [1]. Using a result by L. Makar-Limanov [2], we describe generators of the automorphism group of $P(\mathrm{sl}_2(K))/(C_P - \lambda)$ and represent this group as an amalgamated product of its subgroups. Moreover, using similar results by J. Dixmier [3] and O. Fleury [4] for the quotient algebras $U(\mathrm{sl}_2(K))/(C_U - \lambda)$, where C_U is the standard Casimir element of $\mathrm{sl}_2(K)$ in the universal enveloping algebra $U(\mathrm{sl}_2(K))$, we prove that the automorphism groups of $P(\mathrm{sl}_2(K))/(C_P - \lambda)$ and $U(\mathrm{sl}_2(K))/(C_U - \lambda)$ are isomorphic.

Keywords: universal enveloping algebra, Poisson enveloping algebra, Casimir element, free product, automorphism.

AMS Subject Classification: 16S30, 17B63, 16W20, 17B40, 17A36

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MODELLING A CONFLICT EVENT AS A RANDOM PROCESS BY A GENERAL LOGICAL-PROBABILISTIC METHOD

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Introduction: Modelling of conflict event as a random process using a logical-probabilistic approach includes the analysis and representation of the conflict in the form of random variables and their probabilistic distributions, the creation of a state model, where each state of the system is represented by Boolean variables.

Purpose of the study: The main purpose of the study is to model and calculate the reliability indicators of structurally complex systems using logical-probabilistic methods as a mathematical apparatus.

Methodology: I. Ryabinin methods of probability theory, mathematical logic and logical-probabilistic calculus were used to solve the set tasks. Using the logical-probabilistic method of the theory of reliability of monotone systems at the stage of probabilistic modelling, using special methods for transforming the health function system, a polynomial of calculated probability functions is constructed in the form:

$$P_F(\{p(t),q(t)\}, i=1,\dots,n,t), \implies P_F(t)=P_F(\{p(t),q(t)\})=P_{FX_y}(t)\vee P_{FX_1}(t)\vee P_{FX_2}(t)$$
$$P_{FX_y}(t)=P_1(t)Q_3\vee\ P_1(t)q_5(t)\vee\ P_1(t)Q_2(t)\vee\ P_2(t)q_4(t)$$
$$P_{FX_1}(t)=p_{x_{11}}+q_{x_{12}}+...+p_{x_{1n}}, P_{FX_2}(t)=q_{x_{21}}+q_{x_{22}}+...+p_{x_{2n}}$$

where $P_{FX_1}(t)$- polynomial of the calculated probability function X_1;
$P_{FX_2}(t)$ - is the polynomial of the calculated probability function X_2. [1]

The influence of various factors on the reliability of the system and their relative contribution is determined by the method of orthogonalization. Analysis of the results: The simulation results were analyzed and interpreted in the context of the conflict. Conclusion: The use of logical-probabilistic methods and orthogonalization of systems reliability analysis allow more accurate modelling and analysis of the influence of various factors on system reliability, taking into account their interaction and probabilistic characteristics.

Keywords: random process, conflict event, logical function, derivative of a Boolean function.

AMS Subject Classification: 03-04, 60A10, 60-08.

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VII World Congress of Turkic World Mathematicians
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ON THE MINIMIZATION OF k-VALUED LOGIC FUNCTIONS IN THE CLASS OF DISJUNCTIVE NORMAL FORMS

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The methods of k-valued logics are generally necessary for the study of a number of important problems from the most diverse fields: biology, medicine, military affairs, automation, control, planning of experiments etc., everywhere where not only the quantitative relationships between the quantities characterizing the processes under consideration are significant, but also the logical dependences connecting them. A multi-valued logical function can be represented as a disjunction (multi-place function "or") $K_1 \vee K_2 \vee ... \vee K_m$, where each term is a conjunction (multi-place function "and") of certain variables from the set $\{x_1, \ . \ . \ . \ , \ x_n\}$, taken with or without negation. k-valued function gives a description functioning of the control system, and the formula realizing it, in particular, the disjunctive normal form (d.n.f.), describes the scheme of this system, so that the nodes and elements of the scheme correspond to the terms and letters of the d.n.f. as a rule, k-valued function has many essentially different d.n.f. In mathematical logic they are considered from the qualitative side. With the development of cybernetics, the terms and letters of the d.n.f. began to reflect equipment costs in circuits and this drew attention to the quantitative side. Therefore, one of the problems of k-valued logics dictated by practice is the problem of minimizing multi-valued functions. The results of research in some areas in this area, in particular, minimization in certain systems multi-valued functions are quite widely displayed in the literature. Therefore, it should immediately be noted that we will only discuss the minimization of multivalued functions in the class of d.n.f.

Keywords: k-valued, minimization, disjunctive normal form, rank, abbreviated d.n.f., monotone function.

AMS Subject Classification: 03B50.

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THE PROBLEM ON THE COMPLETENESS OF CLASSES OF CORRECTING FUNCTIONS

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When solving a wide class of practical problems, algorithms are often used that make mistakes in calculating elementary properties or refuse to solve problems. In such cases, several incorrect algorithms are usually used to solve the same problem, and then a corrective function is built. Since the result of calculating an elementary property can be either 0 - refusal of the calculation, or 1 - the property is fulfilled, or 2 - the property is not fulfilled, then the corrective function is a three-valued logic function. For substantive reasons, no set of corrective functions should be restricted.

In this paper, special classes $\sigma_2,\ \sigma_3$ of corrective functions are considered. For the functions included in these classes, bases are constructed.

Let a family of $\{f\}$ three-valued logic functions be given, etc. such that it $f\left(x_1,\, x_2, ..., x_n\right) \in \{f\}$ follows from: $x_i \in \{0, 1,\, 2\}\,,\ f\left(x_1,\, x_2, ..., x_n\right) \in \{0, 1,\, 2\}\,.$

Take a set of L subsets of the $L_1,\ L_2, ..., L_k$ set $\{0, 1,\, 2\}\,.$ It is considered that the family $\{f\}$ preserves the set L if it $\alpha_i \in L_j,\ j = \overline{1, n}$ follows from the conditions that $f\left(\alpha_1,\, \alpha_2, ..., \alpha_n\right) \in L_j,\ j = \overline{1, k}$.

In what follows, as $\{f\}$ we will consider the set of functions of three-valued logic.

$$L^* = \left\{\{0\}\,, \{1\}\,, \{2\}\,,\ \{0, 1\}\,, \{0, 2\}\right\}.$$

Definition 1. A family of functions that preserve a set L^*, is called a class σ_i.

Keywords: Correcting functions, disjunctive normal forms (d.n.f.), linear Boolean functions, complex conjunctions, minimization.

AMS Subject Classification: 03B50.

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ON THE GROUP OF TOPOLOGICAL MAPS OF FOLIATED MANIFOLDS

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A manifold, with some fixed foliation on it, is called a foliated manifold. Theory of foliated manifolds is one of new fields of mathematics. It appeared in the intersection of Differential Equations, Differential Geometry and Differential Topology in the second part of 20^{th} century.

In this paper the group of topological maps of a foliated manifold with foliated compact-open topology is studied. The foliated compact open topology was introduced in the paper [1] and studied in [2].

Let M, N be n - dimensional smooth manifolds on which there are given $k-$ dimensional smooth foliations F_1, F_2 respectively (where $0 < k < n$).

If for the some homeomorphism $f : M \to N$ the image $f(L_\alpha)$ of any leaf L_α of foliation F_1 is a leaf of foliation F_2, we say that pairs (M, F_1) and (N, F_2) C^r- are homeomorphic. In this case the mapping f is called C^r- homeomorphism, preserving foliation and is written as $f : (M, F_1) \to (N, F_2)$.

In the case $M = N$, f is said homeomorphism of foliated manifold (M, F). Let us denote by $Top_F(M)$ the set of all topological maps of a foliated manifold (M, F). Topological maps, preserving foliation, are investigated in [3],[4].

In this article, the following results are obtained

Theorem 1. The set $Top_F(M)$ with $F-$ compact open topology is Hausdorff space.

Theorem 2. The space $Top_F(M)$ with an $F-$ compact open topology is a topological space with countable base.

Theorem 3. Let (M, F) be a smooth foliated manifold. Then the group $Top_F(M)$ is a topological group with respect to the F-compact open topology.

Keywords: topological map, foliated manifold, foliated compact-open topology, Hausdorff space.

AMS Subject Classification: 57R30, 53C12

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THE STUDY OF GEOMETRIC LAWS IN THE PATTERNS OF THE KHOJA AHMED YASAWI COMPLEX

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The article examines the history of geometric ornaments in the Islamic world characterized by an almost three-century break from the birth of Islam at the beginning of the VII century to the end of the XIX century. The use of geometric patterns has become one of the distinctive features of Islamic art. The significant intellectual contribution of Islamic mathematicians, astronomers and scientists was necessary to create this unique new style. In medieval oriental architecture, the period of its heyday is the doctrine of balance-the doctrine of harmony. Therefore, geometric statics becomes part of the doctrine of geometric harmony. It is here that the unity of the architectural and spatial format of the structure and the method of harmonizing its structure is manifested. The great encyclopedists of the medieval East considered harmony and balance as an aesthetic principle, which also extended to practical art. At that time, the architecture was based on geometric methods, which became the leading system of architectural ideas that were of great importance in the formation and development of medieval architecture in the Middle East. The works of architects, artists of the medieval East, engaged in ornament and geometric patterns, formed as a generalization of the experience formulated in connection with the new classification of mathematical sciences, had a decisive influence on the direction of the worldview of many generations. In addition, from the experience itself, from the experience of construction and art production, the foundations of the theory arose. The work and research of architects, they considered the basis of the architecture of their time as the lens of geometric technique.The range of application of geometric patterns was wide. The spatial structure of the building, the balance of architectural structures, the stability and strength of the structure as a whole, including geometric theory and practical issues such as geometric ornament. The Khoja Akhmat Yasawi complex in the medieval Middle East, which preserved the balance of such geometric ornament and architectural structures. This building was the center of propaganda and dissemination of Islam of the Iron State in the nomadic steppes, both ideological and political in nature. Like many buildings of Amir Temir, the Khoja Akhmat Yasavi complex is distinguished by its scale, grandeur of design, but the spatial interpretation of the complex is simple. The building rises above an elongated parallelepiped and two portals and two domes along its longitudinal axis.

Thus, the article examines the balance of a grandiose architectural object of the Middle Ages, that is, geometric ornament and architectural structures on the facades of the complex, built at the end of the XIV century to renovate the mausoleum of Khoja Ahmed Yasawi by order of ruler Amir Temur. According to al-Khorezmi, a square is what he considered a unit of surface measurement. He showed in his drawing, giving a special case of the proof of this theorem. At the same time, the problem of studying geometric patterns and other geometric patterns based on this theory, geometric patterns on the facades of the mausoleum of Khoja Ahmed Yasawi, is considered.

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ON GENERALIZED QUASI-TYPE RECTIFYING CURVES

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In this paper, we introduce quasi-type rectifying curves in Myller configuration. Firstly, we give the necessary and sufficient conditions for a curve in the Myller configuration to be a quasi-type rectifying curve. We obtain relationships between quasi-curvatures of quasi-type rectifying curves. Next, we state and prove some theorems according to special cases of quasi-curvatures by using the quasi-type frame. Finally, we express special cases corresponding to some selections of invartians given in Myller configuration.

Keywords: Quasi-type rectifying curves, Myller configuration, quasi-type frame, versor field.

AMS Subject Classification: 53A04.

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NEW VARIANT OF ORTHOGONALITY OF 1-TYPES IN WEAKLY O-MINIMAL THEORIES

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The present lecture concerns the notion of *weak o-minimality* originally studied by H.D. Macpherson, D. Marker and C. Steinhorn in [1]. A subset A of a linearly ordered structure M is *convex* if for all $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \ldots \rangle$ such that any definable (with parameters) subset of M is a finite union of convex sets in M.

Let T be a weakly o-minimal theory, $M \models T$, $A \subseteq M$, $p, q \in S_1(A)$ be non-algebraic. We say that p is not *weakly orthogonal* to q (denoting this by $p \not\perp^w q$) if there exist an L_A-formula $H(x, y)$, $\alpha \in p(M)$ and $\beta_1, \beta_2 \in q(M)$ such that $\beta_1 \in H(M, \alpha)$ and $\beta_2 \notin H(M, \alpha)$.

Here we study a new variant of orthogonality of non-algebraic 1-types in weakly o-minimal theories: almost quite orthogonality.

We need the notion of a (p, q)-splitting formula introduced in [2]. Let $A \subseteq M$, $p, q \in S_1(A)$ be non-algebraic, $p \not\perp^w q$. We say that an L_A-formula $\phi(x, y)$ is a *(p, q)-splitting formula* if there exists $a \in p(M)$ such that $\phi(a, M) \cap q(M) \neq \emptyset$, $\neg\phi(a, M) \cap q(M) \neq \emptyset$, $\phi(a, M) \cap q(M)$ is convex and $\inf[\phi(a, M) \cap q(M)] = \inf q(M)$.

Let T be a weakly o-minimal theory, $M \models T$, $A \subseteq M$, $p, q \in S_1(A)$ be non-algebraic. We say that p is not *almost quite orthogonal* to q if there exist a (p, q)-splitting formula $\phi(x, y)$ and an A-definable equivalence relation $E_q(x, y)$ partitioning $q(M)$ into infinitely many convex classes so that for any $a \in p(M)$ there is $b \in q(M)$ such that $\sup \phi(a, M) = \sup E_q(b, M)$. We say that T is *almost quite o-minimal* if the notions of weak and almost quite orthogonality of 1-types coincide.

Theorem 1. *Let T be a weakly o-minimal theory of finite convexity rank having less than 2^ω countable models, $\Gamma_1 = \{p_1, p_2, \ldots, p_m\}$, $\Gamma_2 = \{q_1, q_2, \ldots, q_l\}$ be maximal pairwise weakly orthogonal families of quasirational and irrational 1-types over $\emptyset$ respectively for some $m, l < \omega$. Then T has exactly $3^m 6^l$ countable models iff T is almost quite o-minimal.*

Keywords: weak o-minimality, orthogonality of 1-types, the countable spectrum of a theory, convexity rank.

AMS Subject Classification: 03C64, 03C15, 03C07, 03C50.

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SOME PROPERTIES OF SELECTION PRINCIPLES IN TOPOLOGICAL GROUPS

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In this work we study some properties of selection principles in topological group and transformation group by uniform structures.

A topological group G is said to be

(1) M-bounded if for each sequence $(U_n|n \in N)$ of neighborhoods of the neutral element $e \in G$ there is a sequence $(A_n|n \in N)$ of finite subsets of G such that $G = \bigcup\limits_{n \in N} A_n \cdot U_n$ [1].
(2) H-bounded if for each sequence $(U_n|n \in N)$ of neighborhoods of the neutral element $e \in G$ there is a sequence $(A_n|n \in N)$ of finite subsets of G such that each $x \in G$ belongs to all but finitely many $A_n \cdot U_n$ [1].
(3) R-bounded if for each sequence $(U_n|n \in N)$ of neighborhoods of the neutral element $e \in G$ there is a sequence $(A_n|n \in N)$ of elements of G such that $G = \bigcup\limits_{n \in N} x_n \cdot U_n$ [1].

In other words topological group G is said to be M-bounded (H-bounded, R-bounded), if G a uniformly Menger (uniformly Hurewicz, uniformly Rothberger) space with respect to the left group uniformity. The concept of M-bounded (H-bounded, R-bounded) transformation groups is defined similarly.

Note some important properties of M-bounded, H-bounded and R-bounded topological group:

(1) Any subgroup of M-bounded (H-bounded, R-bounded) topological group is a M-bounded (H-bounded, R-bounded) topological group.
(2) The product of a M-bounded topological group and a H-bounded topological group is M-bounded topological group.
(3) The product of a H-bounded topological group and a H-bounded topological group is H-bounded topological group.
(4) If a topological group G is contains everywhere-dense subgroup being a M-bounded (H-bounded, R-bounded) topological group then G itself is M-bounded (H-bounded, R-bounded).
(5) An image of a M-bounded (H-bounded, R-bounded) topological group by continuous homomorphism is a M-bounded (H-bounded, R-bounded) topological group.

Note that these properties are true for the case of a transformation group.

Keywords: Uniform space, topological group, transformation group, M-boundedness, H-boundedness, R-boundedness.
AMS Subject Classification: 54E15, 54D20

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TAME AUTOMORPHISM GROUP OF THE FREE ZINBIEL ALGEBRA OF RANK 2

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Let k be an arbitrary field. Let $\mathfrak{M}$ be an arbitrary homogeneous variety of algebras over the field k. We denote by $\mathfrak{M}\langle x_1, x_2, ..., x_n\rangle$ the free algebra of this variety in variables $x_1, x_2, ..., x_n$. We denote by deg the standard power function on $\mathfrak{M}$, i.e., $\deg(x_i) = i$ for i. A variety $\mathfrak{M}$ is called a $\circ$-variety if $\deg(f(h)) = \deg(f) \cdot \deg(h)$ for any nonzero $h \in \mathfrak{M}\langle x_1, x_2\rangle$ and for any nonzero $f \in \mathfrak{M}\langle x\rangle$.

Obvious examples of $\circ$-varieties of algebras are: the variety of associative-commutative algebras, the variety of associative algebras [1], the variety of Poisson algebras [2]. In [3] it was proved that the varieties of right-symmetric, non-associative, commutative algebras are also the $\circ$-varieties.

A linear space A over a field k equipped with a bilinear operation $x \cdot y$ is called a Zinbiel (dual Leibniz) algebra if $(xy)z = x(yz) + x(zy)$ holds for all $x, y, z \in A$.

Proposition 1. The variety of Zinbiel algebras is a $\circ$-variety.

Theorem 1. Let $A = Z\langle x, y\rangle$ be a free Zinbiel algebra in two variables x, y over a field k. The tame automorphism group $T(A)$ of the algebra A is the free product of subgroups of affine automorphisms $Af_2(A)$ and triangular automorphisms $Tr_2(A)$ with amalgamated subgroup $C = Af_2(A) \cap Tr_2(A)$, i.e.,

$$T(A) = Af_2(A) *_C Tr_2(A).$$

Corollary 1. Any reductive group of tame automorphisms of a two-generated free Zinbiel algebra $A = Z\langle x, y\rangle$ over a field k of characteristic zero is linearizable.

Theorem 2. Let D – any locally nilpotent derivation of a two-generated free Zinbiel algebra $A = Z\langle x, y\rangle$ over a field k of characteristic zero and let $expD \in T(A)$. Then D is triangulable.

Keywords: Zinbiel algebra, automorphism, amalgamated free product, linearization, triangulation.

AMS Subject Classification: 17A50, 17A36

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ON COMPLETION OF ORDERED UNIFORM SPACES

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In this work we study completion of ordered uniform spaces. Ordered uniform space was introduced by A.A. Borubaev [1].

Let $(X, <, U)$ be a ordered uniform space. An ordered uniformity U of the ordered space $(X, <)$ is called τ-prebounded, if it is preuniversal [1] and has a base consisting of interval covers cardinality $\leq \tau$.

By $U_\tau(X, <)$ ($U_L(X, <)$, $U_K(X, <)$) the set of all τ-prebounded (respectively preLindelöf, precompact) ordered uniformities of the ordered space $(X, <)$. These sets are partially ordered by inclusion $\supset$. By $(K_\tau(X, <), \geq)$ (($L(X, <), \geq)$, $(K(X, <), \geq)$) the set of all ordered index compactness $\leq$ extensions (respectively the set of all ordered Lindelöf extensions, the set of all ordered compact extensions).

Theorem 1. Let $(X, <)$ be an arbitrary ordered space. Then the partially ordered sets $(U_\tau(X, <), \supset)$ and $(K_\tau(X, <), \geq)$ are isomorphic. Moreover, the isomorphism takes place by association of each τ-prebounded ordered uniformity U with the completion of the ordered space $(X, <)$ by the uniformity U.

Corollary 1. Let $(X, <)$ be an arbitrary ordered space. Then the partially ordered sets $(U_L(X, <), \supset)$ and $(L(X, <), \geq)$ are isomorphic. Moreover, the isomorphism takes place by association of each preLindelöf ordered uniformity U with the completion of the ordered space $(X, <)$ by the uniformity U.

Corollary 2. Let $(X, <)$ be an arbitrary ordered space. Then the partially ordered sets $(U_K(X, <), \supset)$ and $(K(X, <), \geq)$ are isomorphic. Moreover, the isomorphism takes place by association of each precompact ordered uniformity U with the completion of the ordered space $(X, <)$ by the uniformity U.

Keywords: Ordered uniform space, τ-prebounded ordered uniformity, preLindelöf ordered uniformity, precompact ordered uniformity, ordered index compactness $\leq$ extensions, ordered Lindelöf extensions, ordered compact extensions.

AMS Subject Classification: 54E15, 54D20

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ON THE GEOMETRY OF SINGULAR FOLIATIONS

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The geometry of orbits of families of smooth vector fields is an important object of mathematics due to its importance in applications, in the theory of dynamic systems and in the foliation theory.

Definition 1. A partition F of a riemannian manifold M by path-connected immersed submanifolds L_α is called a singular foliation of M if it verifies condition: for each leaf L_α and each vector $v \in T_pL_\alpha$ at the point p, there is $X \in XF$ such that $X(p) = v$, where T_pL_α is the tangent space of the leaf L_α at the point p , XF is the module of smooth vector fields on M tangent to leaves (XF acts transitively on each leaf).

Definition 2. The orbit $L(x)$ of a system D of vector fields through a point x is the set of points y in M such that there exist $t_1, t_2, ..., t_k \in R$ and vector fields $X_{i_1}, X_{i_2}, ..., X_{i_k} \in D$ such that

$$y = X_{i_k}^{t_k}(X_{i_{k-1}}^{t_{k-1}}(...(X_{i_1}^{t_1})...)),$$

where k is an arbitrary positive integer.

The fundamental result in study of orbits is Sussmann theorem, which asserts that every orbit is an immersed submanifold of M. It is known orbits of a system of vector fields generate singular foliation.

Let us consider a family of $D = \{X_1, X_2\}$ vector fields on four-dimensional Euclidean space E^4 with Cartesian coordinates x_1, x_2, t, u, where

$$X_1 = t\frac{\partial}{\partial t} + u\frac{\partial}{\partial u}, X_2 = x_1\frac{\partial}{\partial x_1} + x_2\frac{\partial}{\partial x_2} - 2u\frac{\partial}{\partial u}. \qquad (1)$$

Theorem 1. The family of orbits of the vector fields (1) is a singular foliation regular leaf of which is a surface with a negative Gauss curvature and zero normal torsion.

Keywords: Vector field, orbit, singular foliation, Gauss curvature, normal torsion.

AMS Subject Classification: 57R30

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ON INNER AND LOCAL INNER DERIVATIONS OF FOUR-DIMENSIONAL JORDAN ALGEBRAS

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Jordan algebras are classical algebras widely studied by specialists. Jordan algebras turned out to be related to other classical algebras such as associative algebras and Lie algebras. The classification of finite-dimensional Jordan algebras is one of the primary areas of modern algebra and, as is known, was first studied by P. Jordan, J. von Neumann and E. Wigner [2]. The algebraic and geometric classifications of 3- and 4-dimensional Jordan algebras were constructed by I. Kashuba and M.E. Martin in [3], M.E. Martin in [6], I. Kashuba and I. Shestakov in [4], respectively.

The study of local derivations of Lie algebras began with the work [1]. They proved that every local derivation on a semisimple finite-dimensional Lie algebra is a derivation.

In the present paper we constructed the matrices of inner derivations and local inner derivations of four-dimensional Jordan algebras.

Definition 1. A Jordan algebra $\mathcal{J}$ is a vector space over a field $\mathbb{F}$ equipped with a bilinear operator $\cdot : \mathcal{J} \times \mathcal{J} \to \mathcal{J}$, satisfying the following identities:

$$x \cdot y = y \cdot x, (x^2 \cdot y) \cdot x = x^2 \cdot (y \cdot x) \tag{1}$$

for any x, $y \in \mathcal{J}$. An important special case of mapping derivation is the so-called "inner derivation". In this section, we introduce the notion of inner derivations of Jordan algebras and give their properties.

Let $\mathcal{J}$ be a Jordan algebra. Recall that a linear map $D : \mathcal{J} \to \mathcal{J}$ is called a derivation if $D(xy) = D(x)y + xD(y)$ for any two elements x, $y \in \mathcal{J}$.

Given subsets B and C of the Lie algebra with bracket $[\cdot, \cdot]$, denote by $[B, C]$ the set of all finite sums of elements $[b, c]$, where $b \in B$ and $c \in C$.

Consider the Jordan algebra $\mathcal{J}$ and let $m = \{xM : x \in \mathcal{J}\}$, where xM denotes the multiplication operator defined by $(xM)y := x \cdot y$ for all x, $y \in \mathcal{J}$. Let $aut(\mathcal{J})$ denote the Lie ring of all derivations of $\Re$. The elements of the ideal $int(\mathcal{J}) := [m, m]$ of the algebra $aut(\Re)$ are called inner derivations of $\mathcal{J}$. In other words, derivation of D by $\mathcal{J}$ is called inner derivation if there exist elements a_1, a_2, $\ldots$, a_m, b_1, b_2, $\ldots$, b_m into $\mathcal{J}$ such that

$$D(x) = \sum_{k=1}^{m} [a_k(b_k x) - b_k(a_k x)], x \in \mathcal{J}.$$

A linear mapping $\nabla : \mathcal{J} \to \mathcal{J}$ is called a local inner derivation if for every $x \in \mathcal{J}$ there exists an inner derivation $D : \mathcal{J} \to \mathcal{J}$ such that $\nabla(x) = D(x)$.

Let $E_{i,j}$ be a linear mapping of the algebra $\mathcal{J}$ with basis $\{e_1, \ldots, e_4\}$, which maps the basis element e_j to e_i and the rest basis elements to zero. Then, for example, the vector space of inner derivations of the algebra $\mathcal{J}_2$ admits a basis of the following form:

$$\{E_{3,1} + E_{3,2} - E_{1,4} - E_{2,4}, E_{4,1} + E_{4,2} - E_{1,3} - E_{2,3}, E_{3,3} - E_{4,4}\}.$$

Theorem 1. [5] *Let A be a four-dimensional Jordan algebra. Then it is isomorphic to one of the following pairwise non-isomorphic Jordan algebras:*

Let $\mathcal{J}$ — be a four-dimensional Jordan algebra. Then the following theorem takes place.

Theorem 2. *The description of inner derivations and local inner derivations of four-dimensional Jordan algebras are given as follows:*

Table 1

Inner derivations and local inner derivations on four-dimensional Jordan algebras.

$\mathcal{J}$	A basis of the vector space of inner derivations	A basis of the vector space of local inner derivation	Is each local inner derivation an inner derivation?
$\mathcal{J}_{25}$	$\{E_{3,1} + E_{3,2} + 2E_{4,3}, E_{3,1} + E_{3,2} - 2E_{4,3}, E_{4,1}\}$	$\{E_{3,1}, E_{3,2}, E_{4,1}, E_{4,3}\}$	–
$\mathcal{J}_{50}$	$\{E_{3,1} + 2E_{4,2}, E_{4,1}\}$	$\{E_{3,1}, E_{4,1}, E_{4,2}\}$	–
$\mathcal{J}_{55}$	$\{E_{2,1}, E_{3,1}, E_{4,1} + 2E_{3,4}\}$	$\{E_{2,1}, E_{3,1}, E_{4,1}, E_{3,4}\}$	–
$\mathcal{J}_{57}$	$\{E_{2,1}, E_{3,1}, E_{4,1} + 2E_{3,4}\}$	$\{E_{2,1}, E_{3,1}, E_{4,1}, E_{3,4}\}$	–
$\mathcal{J}_{58}$	$\{E_{2,1}, E_{3,1}, E_{4,1} + 2E_{2,4}\}$	$\{E_{2,1}, E_{3,1}, E_{4,1}, E_{2,4}\}$	–
$\mathcal{J}_{59}$	$\{E_{2,1}, E_{3,1} + 2E_{2,4}, E_{4,1} + 2E_{2,3} + 2E_{2,4}\}$	$\{E_{2,1}, E_{3,1}, E_{4,1}, E_{2,3}, E_{2,4}\}$	–
$\mathcal{J}_{60}$	$\{E_{2,1}, E_{3,1} - 2E_{4,2}, E_{4,1}\}$	$\{E_{2,1}, E_{3,1}, E_{4,1}, E_{4,2}\}$	–

Remark 1. *It is known that if the matrix of a local inner derivation coincides with the matrix of some inner derivation, then this local inner derivation is an inner derivation.*

The matrices of local inner derivations of Jordan algebras $\mathcal{J}_2$, $\mathcal{J}_9$, $\mathcal{J}_{16}$, $\mathcal{J}_{17}$, $\mathcal{J}_{18}$, $\mathcal{J}_{31}$, $\mathcal{J}_{32}$, $\mathcal{J}_{33}$, $\mathcal{J}_{36}$, $\mathcal{J}_{39}$, $\mathcal{J}_{42}$, $\mathcal{J}_{43}$, $\mathcal{J}_{45}$, $\mathcal{J}_{48}$, $\mathcal{J}_{49}$, $\mathcal{J}_{52}$, $\mathcal{J}_{53}$, $\mathcal{J}_{56}$, $\mathcal{J}_{61}$, $\mathcal{J}_{62}$, $\mathcal{J}_{63}$, $\mathcal{J}_{64}$, $\mathcal{J}_{65}$, $\mathcal{J}_{66}$, $\mathcal{J}_{69}$, $\mathcal{J}_{70}$ coincides with the common form of their inner derivations matrices.

Keywords: Jordan algebra, inner derivation, local inner derivation, nilpotent element.

AMS Subject Classification: 16W25; 46L57; 47B47; 17C65

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CLASSIFICATION OF INNER RICKART AND BAER ALGEBRAS

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In the present paper, we introduce and study counterparts of Rickart and Baer $*$-algebras, that is inner Rickart algebras and inner Baer algebras. We define an inner Rickart algebra as an associative algebra which is an inner RJ-algebra with respect to the Jordan multiplication $a \cdot b = \frac{1}{2}(ab + ba)$. Similarly, an inner Baer algebra is an associative algebra which is an inner BJ-algebra with respect to the Jordan multiplication. Inner RJ-algebras and inner BJ-algebras are introduced and studied in the papers [1], [2], [3].

The chosen notions were built around a (inner) quadratic annihilator. For each nonempty subset $\mathcal{S}$ of an associative algebra $\mathcal{A}$, the (inner) quadratic annihilator of $\mathcal{S}$ is defined by

$$^{\perp_q}\mathcal{S} := \{a \in \mathcal{A} : sas = 0 \text{ for all } s \in \mathcal{S}\}.$$

Thus, following by [4], an associative algebra $\mathcal{A}$ is called an inner Rickart algebra if, for each element $x \in \mathcal{A}$, there exists an idempotent $e \in \mathcal{A}$ such that $^{\perp_q}\{x\} \cap \mathcal{A}^2 = e\mathcal{A}e \cap \mathcal{A}^2$, where $\mathcal{A}^2 := \{a^2 : a \in \mathcal{A}\}$. There exist examples of inner Rickart algebras without unit element (cf. [2, Remark 1 in page 32]). An associative algebra $\mathcal{A}$ is called an inner Baer algebra if, for each subset $\mathcal{S} \subseteq \mathcal{A}$, there exists an idempotent $e \in \mathcal{A}$ such that $^{\perp_q}\mathcal{S} \cap \mathcal{A}^2 = e\mathcal{A}e \cap \mathcal{A}^2$. Note that, an associative algebra $\mathcal{A}$ is an inner Baer algebra if, and only if, it is an inner Rickart algebra and the set of all idempotents of $\mathcal{A}$ is a complete lattice.

Theorem 0.1. *Let $\mathcal{A}$ be an associative algebra with the degenerate radical $\mathcal{D}eg(\mathcal{A})$, which does not have nilpotent elements a such that $a^2 \neq 0$. Then, if the quotient $\mathcal{A}/\mathcal{D}eg(\mathcal{A})$ is an inner Rickart algebra, then $\mathcal{A}$ is an inner Rickart algebra.*

Corollary 0.2. *Let $\mathcal{A}$ be a finite dimensional associative algebra over a perfect field $\mathbb{F}$, with a nilpotent radical $\mathcal{N}il(\mathcal{A})$, which does not have nonzero nilpotent elements with a square root in $\mathcal{A}$. If the quotient $\mathcal{A}/\mathcal{N}il(\mathcal{A})$ does not have nonzero nilpotent elements, then $\mathcal{A}$ is an inner Rickart algebra, and there exist pairwise orthogonal idempotents $e_1, e_2, \dots, e_m$ in $\mathcal{A}$ such that*

$$\mathcal{A} = (e_1\mathbb{F} \oplus e_2\mathbb{F} \oplus \cdots \oplus e_m\mathbb{F}) \dot{+} \mathcal{N}il(\mathcal{A}).$$

Theorem 0.3. *Let $\mathcal{A}$ be a finite-dimensional inner Rickart algebra over a perfect field $\mathbb{F}$, with a nilpotent radical $\mathcal{N}il(\mathcal{A})$. Then there exist pairwise orthogonal idempotents $e_1, e_2, \dots, e_m$ in $\mathcal{A}$ such that*

$$\mathcal{A} = (e_1\mathbb{F} \oplus e_2\mathbb{F} \oplus \cdots \oplus e_m\mathbb{F}) \dot{+} \mathcal{N}il(\mathcal{A}).$$

Theorem 0.4. *Every finite-dimensional inner Rickart algebra over a perfect field is an inner Baer algebra.*

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Keywords: associative algebra; nilradical; nilpotent associative algebra; nilpotent element; idempotent.

AMS Subject Classification: 16W10, 16E50, 16N40 The author(s) should provide AMS Subject Classification numbers using the link https://mathscinet.ams.org/msnhtml/msc2020.pdf.

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ON QUASI-DOUBLE LINES OF A PARTIAL MAPPING OF EUCLIDE SPACE

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It is considered the problem related to partial mapping of 6- dimensional Euclidean space E_6.

A family of smooth lines is given in the domain $\Omega \subset E_6$ so that through each point $X \in \Omega$ passes one line of a given family. A movable frame is chosen so that it was Frenet's frame for the line of the given family. The integral lines of the coordinate vectors fields of this frame form a Frenet's net. On a tangent to the line ω^3 of this net a point F_3^2 is defined in an invariant way.

When the point X moves in the domain Ω the point F_3^2 describes its domain $\Omega_3^2 \subset E_6$ In this way we get a partial mapping $f_3^2 : \Omega \to \Omega_3^2$ such that $f_3^2(X) = \Omega_3^2$.

In [1], necessary and sufficient conditions were obtained for lines belonging to four-dimensional distributions to be quasi-double lines of the partial mapping under consideration.

In this paper, necessary and sufficient conditions are proved for lines belonging to five-dimensional distributions to be quasi-double lines of a partial mapping f_3^2.

Keywords: euclidean space, Frenet frame, net of Frenet, partial mapping, distribution, quasi-double line.

AMS Subject Classification: 53

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COMPUTER PROGRAM FOR OBTAINING A 3D MODEL OF THE SPATIAL FRACTAL LIMIT CUBE OF CUBES

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The concept of "fractal" was introduced by Benoit Mandelbrot in 1975. In his book Fractal Geometry of Nature, he writes that mathematicians of the past have always refused to study the forms that nature shows us, studying Euclidean geometric figures and inventing all kinds of theories that do not explain the reality around us. However, according to Mandelbrot, ... the new geometry is able to describe many of the irregular and fragmented forms in the world around us and give rise to completely complete theories, defining a family of figures that I call fractals [1].

A new figure - a fractal - can act as a model of complex natural systems, such as tree crowns, mountain ranges, coastlines, the surface of the moon, etc. Tree-like fractals are used to model not only plants, but also the bronchial tree, the functioning of the kidneys, and the circulatory system.

A geometric fractal Limit square of squares was constructed in the plane [2]. This paper considers the problem: if the above-mentioned fractal is a projection of some spatial fractal on one of its symmetry planes, then build a 3D model of this spatial fractal using computer technology. A 3D model of the spatial geometric fractal Limit Cube of Cubes has been obtained.

Key words: spatial geometric fractal, 3D model, computer technology, program

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ABOUT DISTRIBUTIONS DEFINED BY A GIVEN FRENET NET

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In the domain Ω of the Euclidean five-dimensional space E_5 a family of smooth lines is given such that one line of a given family passes through each point $x \in \Omega$. The movable orthonormal frame $R = (x, \vec{e_i})\,(i, j, k = \overline{1, 5})$ is chosen so that it is a Frenet [1] frame for a line of a given family. Integral lines of vector fields $\vec{e_i}$ form a Frenet net Σ_5. In the domain Ω we have p-dimensional $(p, q, s = 2, 3, 4)$ distributions, defined by vector fields $\vec{e_i}$.

It is proved

The theorem. If the Frenet net Σ_5 is a cyclic Frenet net [2], then the mean curvature vectors of the distributions $\Delta_3 = (x, \vec{e_1}, \vec{e_2}, \vec{e_3})$ and $\bar{\Delta}_3 = (x, \vec{e_2}, \vec{e_3}, \vec{e_4})$ are defined as follows, respectively:

$$\vec{M_3} = \frac{1}{3}\Lambda^4_{33}\vec{e_4},$$

where Λ^4_{33} is the first curvature of the line ω^3 of the cyclic Frenet net.

$$\vec{M_3} = \frac{1}{3}\Lambda^5_{44}\vec{e_5},$$

where Λ^5_{44} is the first curvature of the line ω^5 by the cyclic Frenet net.

Keywords: Euclidean space, distribution, Frenet network, mean curvature vector.

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ABOUT PARTIAL MAPPING, GENERATED BY A GIVEN DISTRIBUTION

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In the domain Ω of the Euclidean 5-dimensional space E_5, a 3-dimensional distribution Δ_3 is given. Then the 2-dimensional distribution Δ_2, orthogonally complementary to the given one, is defined invariantly.

In the paper [1] studies the properties of a partial mapping f of the Euclidean space E_n, which is defined using the mean curvature vectors of the considered distributions Δ_p, Δ_{n-p}, $(p < n)$.

In this paper, a partial mapping of the space E_5 is defined, when one of the considered distributions is minimal [2], and some of its properties are studied.

Keywords: Euclidean space, distribution, partial mapping, mean curvature vector.

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ON CATEGORICITY OF STRUCTURES WITH TWO EQUIVALENCE RELATIONS

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Our study concerns the theory T of the language $L = \{\eta_1, \eta_2\}$, where the binary symbols $\eta_i, i \in \{1,2\}$ are interpreted in models of the theory T as equivalence relations. Let M be a model of L-theories T. For the language L we can consider the following cases:

(1) $E \sim \eta_1 \wedge \eta_2$ implies $L = \{E\}$;

(2) $\forall a \in M$, $\eta_1(a) \supsetneq \eta_2(a)$;

(3) The intersection of η_1-classes and η_2-classes is not empty.

Algorithmic properties of structures with two equivalences were studied in [1]. The countably categoricity of a theory with condition (1) is shown in [2]. For case (2), consider the following invariants: denote by $Card = \{\kappa_j^i | i \in \{1,2\}, j \in \omega\}$ the set of cardinalities for $\eta_i, i \in \{1,2\}$ respectively and $Sp_i = (\kappa_j^i, \lambda(\kappa_j^i))$, where $\lambda(\kappa_j^i)$ is the number of classes of cardinality κ_j^i .

Theorem 1. *L-theory T with (2) is ω-categorical iff for arbitrary $Card = \{\kappa_j^1 | j \in \omega\}$ any countable models of T has the same $Sp_2 = (\kappa_j^2, \lambda(\kappa_j^2))$, where $j < \omega$.*

Now let us describe the structure of the models of the theory with condition (3).

Definition 1. The intersection of η_1-classes and η_2-classes is *completely cross-cutting* if any element of η_1-classes lies in η_2-classes.

For a L-theory T with completely cross-cutting equivalence relations we consider $Sp_{ccc} = (\kappa_t, \lambda(\kappa_t))$, $t \in \omega$.

Theorem 2. *L-theory T with completely cross-cutting equivalence relations is ω-categorical iff for $t < \omega$ any countable models of T has the same $Sp_{ccc} = (\kappa_t, \lambda(\kappa_t))$.*

Definition 2. The intersection of η_1-classes and η_2-classes is *partially cross-cutting* if in η_1-classes there are elements not in η_2-classes.

Naturally, a new spectrum is introduced to describe the cardinality and the number of finite and infinite intersections, as well as non-intersecting parts. Denote such a spectrum by $Sp_{pcc} = (Sp_\emptyset, Sp_\cap)$, where $Sp_\emptyset = (\kappa_j^\emptyset, \lambda(\kappa_j^\emptyset))$ spectrum to describe disjoint parts, and $Sp_\cap = (\kappa_j^\cap, \lambda(\kappa_j^\cap))$ spectrum to describe the intersection of η_1-classes with η_2-classes.

Theorem 3. *L-theory T with partially cross-cutting equivalence relations is ω-categorical iff for $j < \omega$ any countable models of T has the same $Sp_{pcc} = (Sp_\emptyset, Sp_\cap)$.*

Keywords: omega categorical theory, equivalence relation, structure with two equivalence relation.
AMS Subject Classification: 03C30, 03C15, 03C50, 54A05.

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STUCTURAL PROPERTIES OF ALMOST LIE ALGEBRAS

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A nonassociative algebra A over a field K is called a Almost Lie algebra if it satisfies the following identities:

$$(x \circ y) \circ z) + (y \circ z) \circ x + (z \circ x) \circ y = 0$$
$$x \circ (y \circ z) - (z \circ x) \circ y = 0$$

Almost Lie algebras were introduced by B. A. Kupershmidt in [1]. Every Lie algebra is Almost Lie algebra. We classify all two-dimensional Almost Lie algebras over field of characteristic $\neq 3$. We prove the following theorem

Theorem 1. *Let A be an Almost Lie algebra and $dimA = 2$. If A is not Lie algebra then A is commutative, associative and nilpotent of nilpotency length, equal to 3.*

Keywords: Nonassociative algebras, Almost Lie algebras, Nilpotency.

AMS Subject Classification: 16R10, 17A50, 17A30, 17D25, 17C50

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COMPUTABILITY IN REAL NUMBERS

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Real numbers are represented typically by Dedekind cuts, Cauchy sequences of rational numbers and binary or decimal expansions. The effectivization of these representations leads to equivalent definitions of computable real numbers.

This notion was first explored by Alan Turing in his famous paper A. M. Turing. On computable numbers, with an application to the Entscheidungsproblem, Proceedings of the London Mathematical Society, 42(2):230-265, 1936, where also the Turing machine is introduced. According to Turing, the computable numbers may be described briefly as the real numbers whose binary expressions are calculable by finite means.

In my report, I will talk about the latest achievements in the development of a structural theory of classes of real numbers partially ordered by the Turing reducibility relation. The smallest element of this partial order is the class of computable real numbers. All other classes consist of real numbers obtained by a number of natural generalizations of the concept of computability.

FIXED POINT THEOREMS FOR GENERALIZED COMPUTABLE NUMBERINGS

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In this talk, we provide criteria for noncomputability and highness of c.e. sets using (pre)complete computable numberings, which also generalize the Recursion Theorem. Also, for all families computable relative to Turing complete (non-computable) c.e. oracles, we provide their (weakly) precomplete numberings.

ON PSEUDOFINITENESS OF THEORIES WITH TWO EQUIVALENCE RELATIONS

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Our study concerns the theory T of the language $L = \{\eta_1, \eta_2\}$, where the binary symbols $\eta_i, i \in \{1,2\}$ are interpreted in models of the theory T as equivalence relations. Let M be a model of L-theories T. For the language L we can consider the following cases:

(1) $E \sim \eta_1 \wedge \eta_2$ implies $L = \{E\}$;

(2) $\forall a \in M$, $\eta_1(a) \supsetneq \eta_2(a)$;

(3) The intersection of η_1-classes and η_2-classes is not empty.

For case (2), consider the following invariants: denote by $Card = \{\kappa_j^i | i \in \{1,2\}, j \in \omega\}$ the set of cardinalities for $\eta_i, i \in \{1,2\}$ respectively and $Sp_i = (\kappa_j^i, \lambda(\kappa_j^i))$, where $\lambda(\kappa_j^i)$ is the number of classes of cardinality κ_j^i.

Theorem 1. Any L-theory T with condition (2) is pseudofinite.

Theorem 2. An L-theory T is smoothly approximable iff the set $Card = \{\kappa_j^i | i \in \{1,2\}, j \in \omega\}$ is finite and any models of T has the same $Sp_i = (\kappa_j^i, \lambda(\kappa_j^i))$.

Now let us describe the structure of the models of the theory with condition (3).

Definition 1. The intersection of η_1-classes and η_2-classes is *completely cross-cutting* if any element of η_1-classes lies in η_2-classes.

For a L-theory T with completely cross-cutting equivalence relations we consider $Sp_{ccc} = (\kappa_t, \lambda(\kappa_t))$, $t \in \omega$.

Theorem 3. Any L-theory T with completely cross-cutting equivalence relations is pseudofinite.

Theorem 4. An L-theory T with completely cross-cutting equivalence relations is smoothly approximable iff the set $Card = \{\kappa_j^i | i \in \{1,2\}, j \in \omega\}$ is finite and any models of T has the same $Sp_{ccc} = (\kappa_t, \lambda(\kappa_t))$, $t \in \omega$.

Definition 2. The intersection of η_1-classes and η_2-classes is *partially cross-cutting* if in η_1-classes there are elements not in η_2-classes.

Naturally, a new spectrum is introduced to describe the cardinality and the number of finite and infinite intersections, as well as non-intersecting parts. Denote such a spectrum by $Sp_{pcc} = (Sp_{\emptyset}, Sp_{\cap})$, where $Sp_{\emptyset} = (\kappa_j^{\emptyset}, \lambda(\kappa_j^{\emptyset}))$ spectrum to describe disjoint parts, and $Sp_{\cap} = (\kappa_j^{\cap}, \lambda(\kappa_j^{\cap}))$ spectrum to describe the intersection of η_1-classes with η_2-classes.

Keywords: pseudofinite theory, smoothly approximable structure, approximation of theory.

AMS Subject Classification: 03C30, 03C15, 03C50, 54A05.

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TYPES OF APPROXIMATION OF THE THEORIES OF UNARS

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We are dealing with pseudofinite unars [2, 3] and continue to study the types of approximations of unars theories [1]. A unar is a structure $\mathcal{U} = \langle U; f^{(1)}\rangle$, whose language consists of one single operation f. For any $u \in U$, let $f^0(u) = u, f^{n+1}(u) = f(f^n(u))$ for all $n \in \omega$, $f^{-1}(u) = \{w \in U | f(w) = u\}$. A unar $\mathcal{U}$ is called a *cycle* of length $n \in \mathbb{N}$ if there exists $u \in U$ such that $U = \{f^i(u) | 0 \leqslant i < n\}, f^n(u) = u, f^i(u) \neq f^j(u)$ for all different $i, j \in \{0, \dots, n-1\}$. The set $\{u_i | i \in \omega\} \subseteq U$ is called a *semichain* if $f(u_i) = u_{i+1}$ and $u_i \neq u_j$ for all distinct $i, j \in \omega$. The set $\{u_i | i \in \omega\} \subseteq U$ is called an infinite *antichain* if $f(a_{i+1}) = u_i$ and $a_i \neq u_j$ for all distinct $i, j \in \omega$. If $|f^{-1}(a)| = k$, we say that a is a k-*branching point*.

Definition. [1] Let $\mathcal{T}$ be a family of theories and T be a theory such that $T \notin \mathcal{T}$. The theory T is said to be $\mathcal{T}$-*approximated,* or *approximated by the family* $\mathcal{T}$, or a *pseudo-*$\mathcal{T}$*-theory,* if for any formula $\varphi \in T$ there exists $T' \in \mathcal{T}$ for which $\varphi \in T'$. If the theory T is $\mathcal{T}$-approximated, then $\mathcal{T}$ is said to be an *approximating family* for T, and theories $T' \in \mathcal{T}$ are said to be *approximations* for T. We put $\mathcal{T}_\varphi = \{T \in \mathcal{T} \mid \varphi \in T\}$. Any set $\mathcal{T}_\varphi$ is called the φ-*neighbourhood*, or simply a *neighbourhood*, for $\mathcal{T}$.

Definition. $\mathcal{T}$-approximated theory T is said to be *CYCLE-approximated,* if $\mathcal{T}$ is a family of theories of unars with cycles. Also, the $\mathcal{T}$-approximated theory T is said to be *FOREST-approximated,* if $\mathcal{T}$ is a family of theories of unars without cycles. In particular, if $\mathcal{T}$ is a family of the theory of connected unars, then T is said to be *TREE-approximated.*

Proposition 1. *The theory T of unlimited unar is CYCLE-approximated if and only if each connected component contains a semichain and only one antichain.*

Proposition 2. *The theory T of unar is FOREST-approximated if and only if T is the theory of a non-injective and non-surjective limited unar, each component containing an infinitely branching point.*

Keywords: pseudofinite theory, unar, approximation of theory.

AMS Subject Classification: 03C30, 03C15, 03C50, 54A05.

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ON SMOOTHLY APPROXIMABILITY OF REGULAR GRAPHS

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The study of countably infinite and countably categorical smoothly approximable structures is relevant in many areas of mathematics, including topology, analysis, and algebra.

Definition 1. [1] Let L be a countable language and let $\mathcal{M}$ be a countable and ω-categorical L-structure. L-structure $\mathcal{M}$ (or $Th(\mathcal{M})$) is said to be *smoothly approximable* if there is an ascending chain of finite substructures $A_0 \subseteq A_1 \subseteq \ldots \subseteq \mathcal{M}$ such that $\bigcup_{i\in\omega} A_i = \mathcal{M}$ and for every i, and for every $\bar{a}, \bar{b} \in A_i$ if $tp_{\mathcal{M}}(\bar{a}) = tp_{\mathcal{M}}(\bar{b})$, then there is an automorphism σ of M such that $\sigma(\bar{a}) = \bar{b}$ and $\sigma(A_i) = A_i$, or equivalently, if it is the union of an ω-chain of finite homogeneous substructures; or equivalently, if any sentence in $Th(\mathcal{M})$ is true of some finite homogeneous substructure of $\mathcal{M}$.

Theorem 1. [2]*Any theory T of regular graph is pseudofinite.*

Theorem 2. *A theory T of regular graph is smoothly approximable iff the diameter of $\Gamma \models T$ is totally bounded.*

Remark 1. Paper [3] states that a graph is countably categorical if and only if the diameter is bounded and realizes a finite number of 1-types (this means that for any model of this theory it is true that the set of degrees of vertices is finite and the number of vertices of such degree is the same). There are only two types of smoothly approximating regular acyclic graph: The trivial graph and the graph in which all components are edges.

Remark 2. It is known that a random graph is pseudofinite but not smoothly approximable. Pseudofiniteness is proved by the probabilistic method. Any sentence true in an infinite random graph is true (with probability 1) in a finite random graph. However, the random graph itself can be of different models. For example, the Erdős-Rényi models, the Rado graph, and there are also uncountable models. An interesting question is about the smooth approximability of random regular graphs.

Keywords: pseudofinite graph, regular graph, smoothly approximable structure.

AMS Subject Classification: 03C30, 03C15, 03C50, 54A05.

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INVERTIBILITY OF ALGEBRAIC POLYNOMIAL MAPPINGS

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Theorem 1. An algebraic polynomial P-endomorphism $\varphi \in End_p\left(P[x_1, ..., x_n]\right)$ over a field P of characteristic zero is an automorphism if and only if, when φ Keller.
Proof. It is clear that some polynomial automorphism of the Kellers. Conversely, let $\varphi \in End_p\left(P[x_1, ..., x_n]\right)$ be an algebraic endomorphism of Kellers.

$$\varphi^n + a_{n-1}\varphi^{n-1} + ... + a_1\varphi + a_0\varepsilon = 0$$

where a_0, a_1, ..., $a_{n-1} \in P$ and ε-identity automorphism. If $a_0 \neq 0$, but it is clear that φ-automorphism and

$$\varphi^{-1} = -\frac{1}{a_0}\left(\varphi^{n-1} + a_{n-1}\varphi^{n-2} + ... + a_1\varepsilon\right).$$

Let $a_0 = a_1 = ... = a_{i-1} = 0$ and $a_i \neq 0$, $0 < i < n$.
Then the polynomials from $\left(\varphi^{n-1} + a_{n-1}\varphi^{n-i-1} + ... + a_i\varepsilon\right) P[x_1, ..., x_n]$ lies in the kernel of the endomorphism φ^i. Since the Keller endomorphism φ is injective, then φ^i is also injective. From here

$$\varphi^{n-i} + a_{n-1}\varphi^{n-i-1} + ... + a_i\varepsilon = 0,\ a_i \neq 0$$

Will then,

$$\varphi^{-1} = -\frac{1}{a_i}\left(\varphi^{n-i-1} + a_{n-1}\varphi^{n-i-2} + ... + a_{i+1}\varepsilon\right)$$

Theorem 2 [2]. The Keller endomorphism of Yagzhev-Bass-Connell-Wright is algebraic.
Theorem 3 [1]. The invertibility of Keller polynomial endomorphism is equivalent to invertivility of Keller polynomial endomorphsim of Yagzhev-Bass-Connell-Wright.

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SOME HOMOTOPY PROPERTIES OF THE SPACE OF COMPLETE LINKED SYSTEMS

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A system $\xi = \{F_\alpha : \alpha \in A\}$ of closed subsets of a space X is called *linked* if any two elements from ξ intersect [1].

A.V. Ivanov defined the space $\mathcal{N}X$ of complete linked systems (CLS) of a space X in a following way:

Definition 1 [2]. A linked system $\mathfrak{M}$ of closed subsets of a compact X is called *a complete linked system* (a CLS) if for any closed set of X, the condition

"Any neighborhood OF of the set F consists of a set $\Phi \in \mathfrak{M}$"

implies $F \in \mathfrak{M}$.

A set $\mathcal{N}X$ of all complete linked systems of a compact X is called *the space* $\mathcal{N}X$ *of CLS of* X. This space is equipped with the topology, the open basis of which is formed by sets in the form of $E = O(U_1, U_2, \dots, U_n)\langle V_1, V_2, \dots, V_s\rangle = \{\mathfrak{M} \in \mathcal{N}X :$ for any $i = 1, 2, \dots, n$ there exists $F_i \in \mathfrak{M}$ such that $F_i \subset U_i$, and for any $j = 1, 2, \dots, s$, $F \cap V_j \neq \varnothing$ for any $F \in \mathfrak{M}\}$, where $U_1, U_2, \dots, U_n$, $V_1, V_2, \dots, V_s$ are nonempty open in X sets [2].

Theorem 1. If mappings $f, g : X \to Y$ are homotopic, then the mappings $\mathcal{N}f, \mathcal{N}g : \mathcal{N}X \to \mathcal{N}Y$ are also homotopic.

Proposition 1. Let for a subset $A \subseteq X$ the relation $\mathcal{N}A \subseteq \mathcal{N}X$ is correct. If a set A is a retract of a topological space X, then the set $\mathcal{N}A$ is a retract of the space $\mathcal{N}X$.

Theorem 2. The functor of complete linked systems $\mathcal{N}$ is a covariant homotopy functor.

Proposition 2. If a topological space X is contractible, then the space $\mathcal{N}X$ is also contractible.

Corollary. If spaces X and Y are homotopically equivalent, then the spaces $\mathcal{N}X$ and $\mathcal{N}Y$ are also homotopically equivalent.

Keywords: Functor, complete linked system, retract, contractible, homotopy.

AMS Subject Classification. Primary: 18F60; Secondary: 18A22, 54A25, 55P99.

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SIMPLE REPRESENTATIONS OF $\mathfrak{o}_I^{(1)}(5)$ WITH RESTRICTED HIGHEST WEIGHTS IN CHARACTERISTIC TWO

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Abstract. In this paper we give a complete description of simple representations with restricted highest weights of the simple ten-dimensional Lie algebra $o_I^{(1)}(5)$ over an algebraically closed field k of characteristic $p = 2$. We also prove that all fundamental representations have non-trivial cohomology. As an example, a complete description of the second cohomology of the standard fundamental module is given.

Keywords: simple Lie algebra, representation, simple module, characteristic two.

AMS Subject Classification: 17B10, 17B20, 17B50.

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MODIFICATION OF THE DANZIG-WOLF DECOMPOSITION METHOD FOR BUILDING HIERARCHICAL INTELLIGENT SYSTEMS

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The Dantzig-Wulf method was an important tool for solving large structured models of optimization problems that could not be solved using the standard simplex algorithm. This article illustrates the algorithm of the modified Dantzig-Wulf decomposition method with an efficient, in terms of speed and stability of the computational process, a coordinating task developed by the author for solving problems of a linear programming problem with a block-diagonal structure with binding constraints.

Recently, due to the fact that the implementation of complex algorithms for the study of hierarchical systems, which place great demands on the method, not only from the point of view of the pure speed of the computational process and, from the point of view of the availability of large amounts of memory and, to the speed of the computational process for the formation of recommendations management of complex hierarchical systems under conditions of uncertainty, which led to the fact that the Danzig-Wolfe method became less popular.

In the original Danzig-Wulf decomposition method, the coordinating problem contains the number of rows equal to the sum of the number of equations in the linking constraint - m_0 and the number of block constraints - q. In the developed modification of the decomposition method, the coordinating task contains with contain $(m_0 + 1)$ rows. Since it is the coordinating task that affects the solution of subtasks by changing the values of the coefficients of the objective function, then reducing the dimension of the coordinating task leads to an increase in the computational efficiency of the decomposition method in $(m_0 + q) / (m_0 + 1)$ times compared to the original decomposition method.

The advantage of this variant of the method is especially great, in comparison with the recommended one, when the number q of blocks is large, and each of the sets corresponding to these blocks can be specified by a small number of restrictions. It is these cases that are most often encountered in practice when modeling real processes.

The experience of practical application of the decomposition method for solving problems of high dimension was insignificant and, in many cases, unsuccessful. The performed computational experiments for problems with matrix order from 90 to 700 showed that, in terms of the number of iterations to obtain the optimal plan, the proposed modification of the Danzig-Wulf decomposition method has the same convergence as the simplex method, but the requirements for computer memory are reduced, and the computational efficiency is increased in $(m_0 + q) / (m_0 + 1)$ times.

ABOUT UNIFORMLY γ-SETS AND ITS MAPPINGS

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In this work we study uniform γ-sets of uniform spaces. Uniform γ-sets of uniform spaces was introduced by L.D.R. Kočinac [1].

Let (X, U) be a uniform space. A uniform space (X, U) is said to be a uniform γ-set if for each sequence $(\alpha_n | n \in N)$ of uniform covers of X there is a sequence $(x_n | n \in N)$ of elements of X such that each x is contained in $St(x_n, \alpha_n)$ for all but finitely many n [1].

Notice that each uniformly γ-set (X, U) is uniformly Rothberger space and each uniformly Rothberger space is uniformly Menger space, also each uniformly γ-set (X, U) is uniformly Hurewicz space and each uniformly Hurewicz space is uniformly Menger space but these four classes of space are distinct.

It follows from the fact that a Tychonoff space X is the γ-set if and only if its universally uniformity is uniformly γ-set.

Theorem 1. Any subspace of a uniformly γ-set (X, U) is uniformly γ-set.

Theorem 2. Let $f : (X, U) \to (Y, V)$ be a uniformly continuous mapping of a uniform space (X, U) onto a uniform space (Y, V). If a uniform space (X, U) is uniformly γ-set, then (Y, V) is also uniformly γ-set.

Theorem 3. Let $f : (X, U) \to (Y, V)$ be a uniformly perfect mapping of a uniform space (X, U) onto a uniform space (Y, V). If a uniform space (Y, V) is uniformly γ-set, then (X, U) is also uniformly γ-set.

Theorem 4. If a uniformly γ-set (X_0, U_0) is dense in a uniform space (X, U) then (X, U) is also uniformly γ-set.

Keywords: Uniform space, uniform γ-set, uniform Rothberger space, uniform Hurewicz space, uniform Menger space.

AMS Subject Classification: 54E15, 54D20

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ON THE TITS ALTERNATIVE FOR GENERALIZED TETRAHEDRON GROUPS OF TYPE (2,3,2,2,2,2)

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One says that a group G satisfies a Tits alternative if either G is virtually soluble, or G contains a non-abelian free subgroup. Following [1] we call a group G defined by a presentation

$$G = \langle x_1, x_2, x_3 \mid x_1^{k_1} = x_2^{k_2} = x_3^{k_3} = R_{12}^l(x_1, x_2) = R_{13}^m(x_1, x_3) = R_{23}^n(x_2, x_3) = 1\rangle, \qquad (1)$$

where $k_1, k_2, k_3, l, m, n \geq 2$, $R_{ij}(x_i, x_j)$ is a cyclically reduced word involving both x_i and x_j and $R_{ij}(x_i, x_j)$ is not a proper power in the free product on x_i and x_j. One says that G has a type (k_1, k_2, k_3, l, m, n).

Conjecture ([1]). The class of generalized tetrahedron groups satisfies the Tits alternative.

This conjecture has been proved [1, 2, 3, 4, 5] for all generalized tetrahedron groups except in the case

$$G = \langle x_1, x_2, x_3 \mid x_1^{k_1} = x_2^{k_2} = x_3^{k_3} = R_{12}^2(x_1, x_2) = (x_1^\alpha x_3^\beta)^2 = (x_2^\gamma x_3^\delta)^2 = 1\rangle, \qquad (2)$$

where $\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \geq \frac{1}{2}$. In [3] the conjecture is proved for groups of the form (2) in the case, when there exist k_i, k_j, $i \neq j$, such that $\frac{1}{k_i} + \frac{1}{k_j} < \frac{1}{2}$ except in the case $k_3 = 2$ and $(k_1, k_2) \in \{(3, 8), (3, 10), (4, 5), (4, 6), (4, 8), (5, 6)\}$. Cases $(k_1, k_2) = (3, 10), (4, 5), (5, 6)$ has been proved in [6].

Theorem 1. Let G be a generalized tetrahedron group with presentation

$$G = \langle x, y, z \mid x^2 = y^3 = z^k = R^2(x, y) = (xz^\alpha)^2 = (y^\beta z^\gamma)^2 = 1\rangle, \qquad (3)$$

where $k \leq 6$, $1 \leq \alpha, \gamma < k$, $1 \leq \beta \leq 2$, $R(x, y) = xy^{v_1} \dots xy^{v_s}$, $s \geq 1$ and $1 \leq v_i \leq 2$ for all i. Suppose that one of the following conditions holds:

1. s is even and $R(x, y)$ do not equal up to equivalence to 26 words in Table 2 [7];
2. s is odd, $v_1 + \dots v_s \not\equiv 0 \pmod 3$ and $R(x, y)$ do not equal up to equivalence to 31 words in Table 1 [8].

Then a group G contains a non-abelian free subgruop, hence the Tits aalternative holds for G.

Keywords: the Tits alternative, generalized tetraedron groups, free subgroups.
AMS Subject Classification: 20F05, 20E05, 20E07

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SPINOR REPRESENTATIONS OF A CURVE WITH FRENET-TYPE FRAME IN MYLLER CONFIGURATION FOR 3-DIMENSIONAL LIE GROUPS

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In this paper, we introduce the representation of a curve with Frenet-type frame in Myller configuration for 3-dimensional Lie groups by using the spinors, which are a widely used notion from mathematics to physics. Also, we get quite a general spinor representation and have some special cases. Moreover, we obtain spinor equations of a curve with Frenet-type frame in Myller configuration for 3-dimensional Lie groups. Then, we construct some geometric properties, theories, and results with respect to this concept.

Keywords: Spinors, Versor field, Frenet-type frame, Myller configuration, Lie groups.

AMS Subject Classification: 15A66, 22E15, 53A04.

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Ықтималдықтар теориясы және математикалық статистика

Probability theory and mathematical statistics

Теория вероятностей и математическая статистика

PROBABILITY MODELING OF FINANCIAL AND ECONOMIC PROCESSES WITH GRAPHS

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In financial and economic practice, there are often random processes that can be considered Markov with certain errors. To analyze discrete random processes occurring in the system, it is convenient to use graphs of its conditions.

Definition. The condition graph of a system means a set of squares (alternatively, you can also take rectangles or wheels) on the inner side of which there are many arrows representing possible direct transitions from one case to another in the designations and intervals of situations.

Example. In figure 1 shows a graph of the state of system $s_1 - s_8$ with eight cases with possible direct transitions $s_1 \to s_2$, $s_3 \to s_2$, $s_3 \to s_7$, $s_4 \to s_5$, $s_5 \to s_4$, $s_5 \to s_1$, $s_6 \to s_7$, $s_7 \to s_8$, $s_8 \to s_6$. For example, the transition from case s_3 to case s_8 is only possible through case s_7 , so it is indirectly indirect; direct transition $s_3 \to s_8$ is not possible because the corresponding arrow is not shown on the case graph.

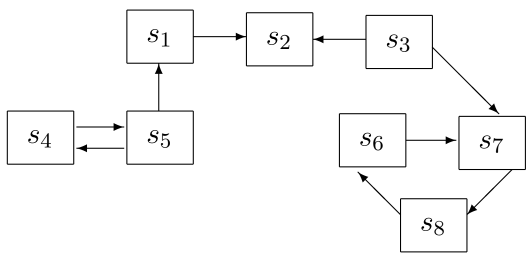

Figure 1. Graph of conditions of the system S, given by cases $s_1 - s_8$

Keywords: random variable, random process, random function, system, system state, random process occurring in the system, Markov process, system state graph.

AMS Subject Classification: 60

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PARABOLIC INTERPOLATION OF MORTALITY TABLES FOR FRACTIONAL AGES

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The article investigates the method of parabolic interpolation of mortality tables for fractional ages. Most of the interpolation methods considered for fractional ages have no concavity or are concave down [1],[2]. The parabolic interpolation method introduced by us has a concave up, which will show methods for calculating mortality tables. Let's now consider the interpolation method for fractional ages [3], which has a concave up.

Definition 0.1. *An interpolation formula for determining the survival function for fractional ages determined by the formula*

$$s^2(x+t) = (1-t)s^2(x) + ts^2(x+1) \tag{1}$$

*where $0 \leq t < 1, x \in Z^+$ will be called **parabolic interpolation**.*

Here $s(x)$ is known from the mortality table, for integer values of x.

Theorem 0.1. *Probability of surviving a person at age x to age $x+t$, considering parabolic interpolation (1) will determine by the formula*

$${}_tp_x = \sqrt{(1-t) + t \cdot p_x^2} \tag{2}$$

and the probability of death of a person at age x during a fractional time t

$${}_tq_x = 1 - \sqrt{(1-t) + t \cdot p_x^2} \tag{3}$$

The new interpolation formula can be used to solve problems of life insurance, and insurance annuities, also in education.

Keywords: actuarial mathematics, mortality table, interpolation of mortality tables, mortality tables for fractional ages, life insurance.

AMS Subject Classification: 62P05

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STRONG MIXING PROPERTIES OF CONVEX HULL IN A UNIT DISK

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The article is devoted to the study of the properties of convex hulls generated by the implementation of a inhomogeneous Poisson point process on the unit disk.

Efron B. [1], Reny A. and Sulanke R. [2] and other researchers were the first to study the functionals of the convex hull; they found the asymptotic behavior and revealed the connections between the asymptotic expressions for the mathematical expectations of the number of vertices, area and perimeter of the convex hull in the case when random points are uniformly distributed in the square. Carnal H. in [3], obtained asymptotic expressions for similar convex hull functionals generated by random points set in polar coordinates. Their components are independent of each other, the angular coordinate is uniformly distributed, and the tail of the distribution of the radial coordinate is a regularly varying function near the boundary of the support - the disk or at infinity.

Using the approximation of a binomial point process to a homogeneous Poisson process, Groeneboom P. in [4], managed to prove the central limit theorem for the number of vertices of a convex hull, for the case when the support of the original uniform distribution is a convex polygon or ellipse.

In this article, the property of strong mixing and the martingale property of functionals of vertex processes of the convex hull are established, in the case when the convex hull is generated from a inhomogeneous Poisson point process inside the disk.

Keywords: Convex Hull, Binomial Point Process, Poisson Point Process, Vertex Processes.

AMS Subject Classification: 60D05, 60F05 https://mathscinet.ams.org/msnhtml/msc2020.pdf.

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ON THE RATE OF CONVERGENCE IN LIMIT THEOREMS FOR FLUCTUATION CRITICAL BRANCHING PROCESSES WITH IMMIGRATION

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Let $\{\xi_{k,j}, k, j \geq 1\}$ and $\{\varepsilon_k, k \geq 1\}$ be two independent collections of independent, random variables taking non-negative integer values such that $\{\xi_{k,j}, k, j \geq 1\}$ identical distributed. Let the sequence of random variables $X_k, k \geq 0$ be defined by the following recursive relations:

$$X_0 = 0, \quad X_k = \sum_{j=1}^{X_{k-1}} \xi_{k,j} + \varepsilon_k, \ k = 1, 2, ... \tag{1}$$

Stochastic processes defined in this way often appear in population theory (see, for example, [1]) and are called branching processes with immigration.
Let $m = \mathrm{E}\xi_{1,1} < \infty$. The branching process (1) is called subcritical, critical and supercritical if $m < 1$, $m = 1$ and $m > 1$ respectively. We obtain estimates a for the rate of convergence of the distribution of fluctuations of a branching process with immigration to the normal law and the obtained estimates are applied to the case when the immigration flow $\{\varepsilon_k, k \geq 1\}$ is inhomogeneous and the mean value and variance of ε_n are regular and increasing.
Let us introduce the following notation: $m = \mathrm{E}\xi_{1,1}$, $\sigma^2 = D\xi_{1,1}$, $\gamma = \mathrm{E}\,|\xi_{1,1} - 1|^3 < \infty$, $\lambda_k = \mathrm{E}\varepsilon_k$, $b_k^2 = var\varepsilon_k$, $\theta_k = \mathrm{E}\,|\varepsilon_k - \lambda_k|^3$, $\Gamma_n = \sum_{k=1}^n (\varepsilon_k - \lambda_k)$, $\mathrm{T}_n^2 = D\Gamma_n$, $\mathrm{H}_n^2 = \sigma^2 \sum_{k=1}^n X_{k-1}$,
$A_n = \sum_{k=1}^n \lambda_k$, B_n^2- some sequence of positive numbers, $\Phi_\sigma(x)-$ normal distribution with mean zero and variance σ^2, $\Phi(x)-$ standard normal distribution,

$$\Delta_n = \sup_{-\infty<x<\infty} \left| \mathrm{P}\left(\frac{X_n - A_n}{B_n} < x \right) - \Phi(x) \right|.$$

Let us agree to denote by $C, C_1, C_2, ...-$ positive absolute constants.

Theorem 1. Let $m = 1$, $0 < \sigma^2 < \infty$, random variables ε_k, $k \geq 1$ are independent and $\theta_k < \infty$, $k \geq 1$. Then the following inequality is true

$$\Delta_n \leq C \frac{1}{\mathrm{T}_n^3} \sum_{k=1}^n \theta_k + 3 \left(\frac{\sigma^2 \sum_{k=1}^n A_{k-1}}{2\pi B_n^2} \right)^{1/3} + \frac{1}{\sqrt{2\pi e}} \left(\frac{B_n^2}{\mathrm{T}_n^2} - 1 \right).$$

Keywords: Branching process; generating function; weak convergence.

AMS Subject Classification: 60

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ON THE RATE OF CONVERGENCE IN THE LIMITING THEOREM FOR STOCHASTIC INTEGRALS OVER SEMI-MARTINGALES

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Let a sequence of semi-martingales $X^n = (X_s^n, \Im_s^n), X_0^n = 0, n \geq 1$ be given on some probability space $(\Omega, \Im, P)$ with selected filtration flows $F^n = (\Im_s^n)$ and satisfying the ordinary conditions.

Let, further, $W = (W_s, \Im_s)$ be a standard Wiener process (with respect to some flow $F = (\Im_s), s \geq 0$).

The main purpose of this article is to give an estimate for the Kolmogorov distance $R^n = \sup_{x \in R} |F^n(x) - F(x)|$ between distribution functions $F^n(x) = P\left(\int_0^1 f(s, X_s^n) dX_s^n \leq x\right)$ and $F(x) = P\left(\int_0^1 f(s, W(s)) dW(s) \leq x\right)$.

The estimate obtained in this article generalizes the results previously obtained by the author in [1] and [2] for the Kolmogorov distance $R_n = \sup_{x \in R} |G^n(x) - G(x)|$ between distribution functions $G^n(x) = P\left(\int_0^1 f(s, M_{s_-}^n) dM_{s_-}^n \leq x\right)$ and $G(x) = P\left(\int_0^1 f(s, W(s)) dW(s) \leq x\right)$, where $M^n = (M_s^n, \Im_s^n)$ are square integrable martingales, and $W = (W_s, \Im_s)$ are standard Wiener processes.

Keywords: Martingale, Semi-martingale, Wiener process, Kolmogorov distance .

AMS Subject Classification: 60G42, 60G48 https://mathscinet.ams.org/msnhtml/msc2020.pdf.

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ON ASYMPTOTICALLY PROPERTIES OF ESTIMATES OF THE RELATIVE-RISK FUNCTION BY STRONG MIXING

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Let $X_1, X_2, ..., X_n$ be survival times with continuous marginal distribution function (d.f.) $F(t),\ F(0) = 0$, censored from the right by nonnegative times $Y_1, Y_2, ..., Y_n$ with continuous d.f. $G(t),\ G(0) = 0$, such that instead of X_i we observe the pairs $(Z_i, \delta_i)\,,\ i = 1, ..., n$, where $Z_i = \min(X_i, Y_i)$ and $\delta_i = I(X_i \leq Y_i)$ with $I(A)$ standing for indicator of event A. We assume that survival times X_i are independent of censoring times Y_i and $\alpha-$mixing (or strong mixing) with mixing parameter

$$\alpha(n) = \sup_{k\geq 1}\left\{|P(A\cap B) - P(A)\cdot P(B)|\,,\ A \in A_1^k,\ B \in B_{k+n}^{\infty}\right\} \to 0,\ n \to \infty \qquad (1)$$

where A_i^k is the $\sigma-$algebra of events generated by random variables $\{X_j,\ i \leq j \leq k\}$. Among several mixing condition used in the literature, $\alpha-$mixing has many practical applications (see, Doukhan [3]). Under $\alpha-$mixing assumption we consider some asymptotic properties for relative – risk function (see for example [1,2], for no mixing situation).

In this article we consider the problems of estimation and studies of asymptotic properties of relative-risk ratio estimates in the case when the data is censored and strong mixing.

Keywords: Relative risk; censored data; strong mixing; cumulative hazard. .

AMS Subject Classification: 62H10, 62H12

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MARCINKIEWICZ–ZYGMUND STRONG LAW OF LARGE NUMBERS FOR WEAKLY DEPENDENT RANDOM VARIABLES IN BANACH SPACES

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We consider Marcinkiewicz–Zygmund strong law of large numbers for m -dependent random variables.
We assume that B is Rademacher type p $(1 \le p \le 2)$.

Definition. Let $(B, \|\bullet\|)$ be a Banach space and $1 \le p \le 2$. B is of Rademacher type p, if there exists a constant $c(p)$ such that for every finite sequence $X_1, X_2, ...X_n$ of centered random variables with $E\|X_i\|^p < \infty$, the following inequality holds

$$E\left\|\sum_{i=1}^{n} X_i\right\|^p \le c(p) \sum_{i=1}^{n} E\|X_i\|^p .$$

We consider Marcinkiewicz–Zygmund strong law of large numbers for m-dependent random variables with values in Rademacher type p $(1 \le p \le 2)$ Banach spaces B. For the independent case see [1].

Keywords: m-dependent random variable, Banach space, Rademacher type p.

AMS Subject Classification: 60F15

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ON THE MAXIMUM OF WEAKLY DEPENDENT RANDOM VARIABLES

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The distributions of the maximum of random variables have been studied by many authors. In the books [1]-[3], the results for the cases of independent and weakly dependent random variables are studied in detail and presented. We consider stationary sequences of random variables $\{X_n, n \in N\}$,that satisfy certain conditions. Let $\{\xi_i, i \in Z\}$ be a sequence of random variables. We will assume that the following representation holds

$$X_n = f\left(\{\xi_{n+i},\ i \in Z\}\right), n \in N,$$

where $f\ :\ R^z \to R$ measurable function. Also suppose that there are measurable functions $f^m\left(\xi_{-m},\ ...,\ \xi_0, ...,\ \xi_m\right):\ R^{2m+1} \to R,\ m = 1,\ 2,\ ...$ such that takes place

$$\lim_{n\to\infty} nP\left(X_n^{(m)} > a_n + b_n x\right) = u(x), m = 1, 2, ...,$$

where $X_n^{(m)} = f^{(m)}\left(\xi_{-m},\ ...,\ \xi_0, ...,\ \xi_m\right)$, $0 < u(x) < \infty$ on some interval of positive length and $e^{-\infty} = 0$,

$$\limsup_{n\to\infty} nP\left(\frac{1}{b_n}\left|X_1 - X_1^{(m)}\right| > \varepsilon\right) \to 0 \text{ at } m \to \infty \text{ for any } \varepsilon > 0.$$

We will consider limit teorems for

$$M_n = \max_{1\le i\le n}\{X_i\}$$

under some weakly dependence conditions on $\{\xi_i, i \in Z\}$.

Keywords: maximum; weakly dependent random variables; limit theorems.

AMS Subject Classification: 2020: 62G30, 62G32

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ON ESTIMATION OF SURVIVAL FUNCTION IN COX MODEL AND ROBUST INFERENCE

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The proportional hazards model was proposed by David Cox in 1972. Today, the Cox model is the most important model in survival analysis, reliability and quality of life research, clinical trials, epidemiology, and biomedical studies. There have also been widely applications of the Cox model in demography, econometrics, finance, pharmacology, biology, insurance, etc. One of the most widely used statistical tool for analyzing survival data is the famous Cox proportional hazard regression model, which helps us to examine the relationship of different important covariates with a randomly right censored response measuring some lifetime.

The random right censored observations are extremely common in different practical application including those from medical sciences. Mathematically, we can represent such randomly right censored observations as $X_i = \min\{T_i, C_i\}$ for each $i = 1, \ldots, n$, where n is the sample size, $T_1, \ldots, T_n$ are independent and identically distributed (IID) true lifetimes for the i-th entry (e.g., patients, item on test, etc.) and $C_1, \ldots, C_n$ are n IID censoring time following a distribution independent of that of T_is. We also know which observations are censored with data $\delta_i = I(T_i \leq C_i)$ for each $i = 1, \ldots, n$. When we additionally have p covarites corresponding to each entry as given by $\boldsymbol{Z}_1, \ldots, \boldsymbol{Z}_n \in \mathbb{R}^p$. The semi-parametric Cox regression model is then given by specifying the hazard of the i-th entry/lifetime as

$$\lambda_i(t) = \lambda(t|\boldsymbol{Z}_i) = \lambda_0(t)e^{\boldsymbol{\beta}^T \boldsymbol{Z}_i},\ i = 1, \ldots, n, \tag{1}$$

where $\boldsymbol{\beta} \in \mathbb{R}^p$ are the target parameter (called regression coefficients) indicating the covariate effects on censored response, but baseline hazard remains unspecified as an unknown function λ_0 making the model semi-parametric.

However, whenever there are evidences from the data about a possible parametric model for the baseline hazard (e.g., exponential, Weibull, etc.), we can make more efficient inference by assuming a fully parametric version of the Cox regression model in (1) which specifies the the hazard of the i-th entry/lifetime as

$$\lambda_{i,\boldsymbol{\theta}}(t) = \lambda_{\boldsymbol{\theta}}(t|\boldsymbol{Z}_i) = \lambda(t, \boldsymbol{\gamma})e^{\boldsymbol{\beta}^T \boldsymbol{Z}_i},\ i = 1, \ldots, n,\ \boldsymbol{\theta} = (\boldsymbol{\gamma}^T, \boldsymbol{\beta}^T)^T \in \mathbb{R}^{q+p}, \tag{2}$$

where $\lambda(t, \boldsymbol{\gamma})$ denotes the assumed parametric baseline hazard function. In the context of parametric proportional hazard model (2) with randomly right censored responses, the idea of the minimum density power divergence estimator has been developed and explored by Ghosh and Basu (2019).

Abdushukurov (2015) present three types of parametric non parametric estimators for conditional survival function in Cox proportional hazards regression model when the lifetime of

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interest is subjected to random censorship from both sides and proved consistency and asymptotic normality of estimators.

Keywords: Survival function; random censoring; covariates; robustness; Gaussian process.
AMS Subject Classification: 62H10, 62H12

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ON ESTIMATION OF MULTIVARIATE SURVIVAL FUNCTION IN THE PRESENT OF PROGNOSTIC FACTORS

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In such research areas as bio-medicine, engineering, insurance, social sciences researchers are interested in positive variables, which are expressed as a time until a certain event. For example, in medicine the survival time of individual, while in industrial trials, time until breakdown of a machine are non-negative random variables (r.v.-s) of interest. But in such practical situations, the observed data may be incomplete, that is censored. This is the case, for example, in medicine when the event of interest-death due to a given cause and the censoring event is death due to other cause. In industrial study, it may occur that some piece of equipment is taken away (that is censored) because it shows some sign of future failure. Moreover, the r.v.-s of interest (lifetimes, failure times) and censoring r.v.-s usually can be influenced by other variable, often called prognostic factor(covariate). In medicine, dose of a drug and in engineering some environmental conditions (temperature, pressure) are influenced to the observed variables. The basic problem consist in estimation of distribution of lifetime by such censored dependent data. The aim of paper is considering this problem in the case of right random censoring model in the presence of prognostic factor.

The problem of estimation of multivariate distribution (or survival) function from incomplete data is considered with beginning of 1980's (Campbell (1981), Campbell & Földes (1982), Hanley & Parnes (1983), Horváth (1983), Tsay, Leurgang & Crowley (1986), Burke (1988), Dabrowska (1988, 1989), Gill (1992), Huang (2000), Abdushukurov (2004) etc.). In the special bivariate case, there are the numerous examples of paired data representing the times to death of individuals (twins or married couples), the failure times of components of system and others which subject to random censoring with possible dependence between the two censoring variables. At present time there are several approaches to estimation of survival functions of vectors of life times. However, some of these estimators either are inconsistent or not fully defined in range of joint survival functions and hence not applicable in practice. Almost all of these estimators have an exponential or product structures. In this work we present other type of estimator of power structure for bivariate survival function $F(t,s)$ of bivariate lifetime vector, which is censored from the right by censoring vector of random variables. We prove weak convergence and strong consistency results for estimators. The authors consider the problem of estimation of multivariate survival functions in dependent models of random censoring using copula functions and in the presence of prognostic factors(covariates).

Keywords: Jointly survival function; random censoring; prognostic factors; copula function; Gaussian process.

AMS Subject Classification: 62H10, 62H12

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ON CRAMER-RAO TYPE INEQUALITY IN COMPETING RISKS MODEL UNDER RANDOM CENSORING FROM UNOBSERVATION INTERVALS, EFFICIENCY AND SEQUENTIAL ESTIMATION

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The Cramer-Rao information inequality makes it possible to evaluate the quality of statistical estimates of unknown parameters. The efficiency property of estimates is measured by the proximity of the standard deviation of the estimate to the lower Cramer–Rao bound. The variances of the effective estimates are inversely proportional to the Fisher information function. If the results of observations constitute a complete sample, then the corresponding results represent the basis of classical mathematical statistics. In the case of censored observations, information inequalities of the Cramer–Rao and Battacharya type were first established by the authors of [1]. Later, these results were generalized to the case of the Bayesian approach and also to samples of random size in [2-4]. In [5], the properties of Fisher information and the establishment of a Cramer–Rao-type inequality for observations that form a competing risks model were studied. In this paper, we consider competing risk models where random censoring occurs at unobservation intervals. For this model, we present some useful representations of the Fisher information matrix, establish an analog of the Cramer-Rao inequality and an efficiency measure for parameter estimation.

Keywords: random censoring, nonobservation intervals, Competing Risks Model, Fisher information, regularity conditions, Cramer-Rao inequality.

AMS Subject Classification: 62N01, 62N05.

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A FUNCTIONAL LIMIT THEOREM FOR BRANCHING PROCESSES WITH IMMIGRATION

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Let $\{\xi_{k,i}, k, i \geq 1\}$ and $\{\varepsilon_k, k \geq 1\}$ be two sequence of non-negative integer-valued random variables such that the two families $\{\xi_{k,i}, k, i \geq 1\}$ and $\{\varepsilon_k, k \geq 1\}$ are independent, $\{\xi_{k,i}, k, i \geq 1\}$ are independent and identically distributed (i.i.d.). We consider a sequence of branching processes with immigration $X_k, k \geq 0$, defined by recursion:

$$X_0 = 0, \quad X_k = \sum_{i=1}^{X_{k-1}} \xi_{k,i} + \varepsilon_k, \quad k \geq 1. \tag{1}$$

Intuitively, one can interpret $\xi_{k,i}$ as the number of offsprings produced by the i-th individual belonging to the $(k-1)$-th generation and ε_k is the number of immigrants in the k-th generation. We can interpret X_k as the number of individuals in the k-th generation.

Assume that $a := \mathbb{E}\xi_{1,1} < \infty$. Process X_k is called subcritical, critical or supercritical depending on $a < 1$, $a = 1$ or $a > 1$, respectively. We refer the reader to recent survey of [1] where one can find a historical overview of limit theorems for process (1).

The aim of this paper is to establish a functional limit theorem for fluctuations of critical process defined by (1) in the case when the immigration sequence $\{\varepsilon_k, k \geq 1\}$ is not necessarily identically distributed and generated by m-dependent random variables. Our result extends the previous known result [2] in the literature.

Keywords: Branching process, immigration, m-dependence, functional limit theorem.

AMS Subject Classification: 60J80

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THE METHOD OF ACCOMPANYING DISTRIBUTIONS AND THE LINDEBERG – FELLER THEOREM

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Let two sequences of probability distributions $\{F_n\}$ and $\{G_n\}$ be given. These sequences are said to be close if $\int\limits_{-\infty}^{\infty} u(x)d(F_n(x) - G_n(x)) \to 0$ for any continuous and bounded function $u(x)$ $(x \in \mathbb{R} = (-\infty, \infty))$. This convergence is written as $F_n - G_n \to 0$ properly or weakly (see [1]). Suppose we need to prove the theorem that $F_n \to F$ weakly. As it is shown in [1], it turns out that it is often easier to solve such problems not directly, but by choosing such distributions $\{G_n\}$, for which the convergence $G_n \to F$ is almost obvious, and then proving that $F - G_n \to 0$. This sequence $\{G_n\} = \{G_{n1}, \dots, G_{nn}\}$ is called accompanying distributions. The described way of proving limit theorems $F_n \to F$ is called, in total, the method of accompanying distributions.

Let for any $n \geq 1$, triangular arrays of independent random variables $X_{n1}, \dots, X_{nn}$, and their sums $S_n = \sum\limits_{j=1}^{n} X_{nj}$. Suppose that X_{nj} have the finite second moments $\sigma_{nj}^2 = \mathbf{E}X_{nj}^2 < \infty$ and, without any loss of generality, we will assume that $\mathbf{E}X_{nj} = 0$, $Var\ S_n = \sum\limits_{j=1}^{n} \sigma_{nj}^2 = 1$. Introduce the notations: let $\mathcal{B}$ be a class of such bounded nonnegative functions $b(x)$ on the line that $\lim\limits_{x\to 0} b(x) \to 0$, $m_b(\delta) = \inf\limits_{|x|>\delta} b(x) > 0$ for all $\delta > 0$, $R_b^n = \sum\limits_{j=1}^{n} \int\limits_{-\infty}^{\infty} |x|b(x)\,|F_{nj}(x) - \Phi_{nj}(x)|\,dx$, where $b(\cdot) \in \mathcal{B}$, $F_{nj}(x) = P(X_{nj} < x)$, $\Phi_{nj}(x) = \Phi\left(\frac{x}{\sigma_{nj}}\right)$, $\Phi(x)$ is the standard normal distribution with the parameter $(0, 1)$.

Theorem. $\sup\limits_{x} |P(S_n < x) - \Phi(x)| \to 0$ if and only if the condition $\sup\limits_{b\in\mathcal{B}} R_n^n \to 0, \ \ n \to \infty$ holds.

Keywords: the Lindeberg – Feller theorem, proper or weakly convergence, triangular array, accompanying distributions.

AMS Subject Classification: 60F05

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ON ASYMPTOTIC EXPANSION FOR A CERTAIN CLASS OF STATISTICS IN GENERALIZED URN MODELS

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Many combinatorial problems in probability and statistics can be formulated and better understood by using appropriate urn models, a topic that has been extensively studied in literature. We consider a Generalized Urn Model (GUM) defined as follows: Suppose a sample of size n is drawn from an urn containing N types of objects and η_m represents the frequency of m-th type of object appearing in the sample.We are interested in the asymptotic theory and higher-order expansions for the statistic $R_N = f_1(\eta_1) + ... + f_N(\eta_N)$, where $f_m(x)$ are Borel functions defined for non-negative axis.In many interesting schemes, one can express the random vector (r.vec.)$\eta = (\eta_1, ..., \eta_N)$ of frequencies through another r.vec. $\xi = (\xi_1, ..., \xi_N)$ with independent and non-negative integer components such that $\Im(\eta_1, ..., \eta_N) = \Im(\xi_1, ..., \xi_N \mid \xi_1 + ... + \xi_N = n)$, where $\Im(X)$ denotes the distribution of the r.vec. X . Here we study the asymptotic theory and present a unified approach to derive Edgeworth expansions for general classes of R_N. We consider the following three special cases of the GUMs along with applications (A) A Sample scheme with replacement, which corresponds to the case where the r.vec. η has the multinomial distribution. Here $\Im(\xi_m) = Poi(vp_m)$, where $v > 0$ is arbitrary,$p_m > 0$,$m = 1, ..., N$ and $p_1 + ... + p_N = 1$. The classical chi-square, likelihood-ratio statistic, and the empty-cells statistic are examples of R_N. (B) A Sample scheme without replacement from a stratified finite population of size $\Omega_N = \omega_1 + ... + \omega_N$ which corresponds to the case where the r.vec. η has the multivariate hyper-geometric distribution. Here $\Im(\xi_m) = Bi(\omega_m, v)$, where $\omega_m > 0$,arbitrary $v \in (0, 1)$; The sample sum for instance, is important variant of R_N. (C) Multicolor Pólya- Egenberger urn model which corresponds to the case where the r.vec.η has the generalized Pólya-Egenberger distribution. Here $\Im(\xi_m) = NBi(d_m, v)$,$d_m > 0$,arbitrary $v \in (0, 1)$. For instance, a sum of functions of “spacings-frequencies” (i.e. frequencies of one sample falling in-between the spacings made by the other) under the hypothesis of homogeneity of two samples can be formulated as a variant of R_N in this GUM.

Keywords: asymptotic expansions, Poisson, binomial, negative-binomial distribution, urn model.

AMS Subject Classification: 62G20, 60F05.

ЛОКАЛЬНАЯ АСИМПТОТИЧЕСКАЯ НОРМАЛЬНОСТЬ ЭКСПЕРИМЕНТОВ ПРИ НЕОДНОРОДНОМ СЛУЧАЙНОМ ЦЕНЗУРИРОВАНИИ ИНТЕРВАЛОМ НЕНАБЛЮДЕНИЯ

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Статистика отношения правдоподобия (СОП) играет ключевую роль в теории принятия решений. Так, например при проверке простой гипотезы H_0 против сложной альтернативы H_1 о неизвестном законе распределения критерии, построенные на основе СОП согласно фундаментальной лемме Неймана-Пирсона является равномерно – наиболее мощными при любом объёме n наблюдений (см. [1,2]). Довольно интересные задачи возникают, когда альтернативы H_1 зависят от n и являются «близкими» к H_0, т.е. $H_1 = H_{1n} \to H_0$ при $n \to \infty$. В таких ситуациях проявляются асимптотические свойства СОП, полезные в теории оценивания неизвестных параметров и проверки гипотез. К таким свойствам относятся локальная асимптотическая нормальность СОП. Имеется достаточное количество литературы, посвященной исследованию свойства локальной асимптотической нормальности СОП и его использованию в статистике. Следует особо отметить работы [3-7], в которых показано, что локальная асимптотическая нормальность даёт возможность развития асимптотической теории оценок максимального правдоподобия и байесовских оценок, а также контигуальности семейств вероятностных распределений. В работах авторов [8-10] свойства локальная асимптотическая нормальность для СОП установлены в некоторых моделях неполных наблюдений, получаемых случайным цензурированием наблюдений в модели конкурирующих рисков (МКР). В данной работе также исследованы свойства локальной асимптотической нормальности СОП в МКР при неоднородном случайном цензурировании интервалом ненаблюдения.

Ключевые слова: конкурирующие риски, случайное цензурирование, статистика отношения правдоподобия, локальная асимптотическая нормальность.
Предметная классификация AMS: 62F05

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Кері және қисынсыз есептер

Inverse and ill-posed problems

Обратные и некорректные задачи

NUMERICAL IMPLEMENTATION OF THE ONE-DIMENSIONAL INVERSE PROBLEM OF PROPAGATION OF THE ACTION POTENTIAL ALONG A NERVE FIBER

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In this article, an algorithm for the numerical implementation of the inverse problem is developed, based on the use of a mathematical model that describes the propagation of an action potential in a nerve fiber.

The mathematical model of the process of impulse propagation along the axon (along the nerve fiber) is described by the parabolic equation:

$$C_m(x)u_t^{'}(x,t) = \frac{r_a(x)}{2\rho_a(x)}u_{xx}^{''}(x,t) - \frac{u(x,t)}{\rho_m(x)\cdot l}, \quad x \in R_+, \quad t \in R_+, \tag{1}$$

$$u(x,t)|_{t<0} \equiv 0, \quad u_x^{'}(x,t)|_{x=0} = h_0\theta(t) + r_0\theta_1(t) + p_0\theta_2(t), \quad t \in R_+, \tag{2}$$

where, h_0, r_0, p_0 are positive constants, $\theta(t)$ - is the theta Heaviside function, $\theta_1(t) = t\theta(t)$, $\theta_2(t) = \frac{t^2}{2}\theta(t)$, $r_a(x)$, a, m, $\rho_m(x), \rho_a(x)$, $C_m(x)$, l - are parametres, $u(x,t)$ -electric current density at a point x in time t.

The inverse parabolic problem consists in determining one of the coefficients of equation (1) and a function with additional information of the form

$$u(x,t)|_{x=0} = g(t), \quad t \in [0,T], \quad T > 0. \tag{3}$$

Using the Kabanikhin method [2, p. 342] (Laplace transform), we reduce the inverse problem (1) – (3) to the inverse problem of an equation of hyperbolic type.

Using the finite-difference regularized method for inverse problems [3], we find the unknown coefficient and the function $u(x,t)$.

Conclusion. We used a mathematical model describing the propagation of an action potential in a nerve fiber and applied the method of regularized finite differences to solve the inverse problem. In the course of the work, we verified the numerical model by comparing the results with known analytical solutions. This allowed us to verify the effectiveness and accuracy of our method.

Keywords: Potential propagation, inverse problem, algorithm, numerical solution, computer implementation.

AMS Subject Classification: 35K10, 35K15, 35L10, 35Q90, 35Q92

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VII World Congress of Turkic World Mathematicians
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ON ONE INVERSE PROBLEM OF DETERMINING THE PERTURBATION COEFFICIENT, AN ELLIPTIC TYPE EQUATION

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The paper considers the equation

$$\Delta u(x) - k^2 u(x) + \lambda^2 a(x) u(x) = \delta(x - x_0), \quad k > \lambda, \quad x \in R^3, \tag{1}$$

where $a(x) = 1 + b(x)$, $b(x)$-continuous function with compact support contained in bounded domains $D \subset R^3$.

Let $u = u(x, x_0, k, \lambda)$-solution of equation (1), such that $u = u_0 + \vartheta$, where u_0-the fundamental solution of the methoharmonic operator $\Delta - (k^2 - \lambda^2)$.

The inverse problem for (1) has been studied in the following formulation.

Task. Restore function $b(x)$ knowing, that $\vartheta(x, x_0, k, \lambda)$ it satisfies the following condition

$$\vartheta(x, x_0, k, \lambda) = g(x, x_0, k, \lambda), \ \ x \in D_1, \ \ x_0 \in D_2, \tag{2}$$

here $D_i \in R_3 \ (i = 1, 2), \ D_1 \cap D = \emptyset, \ D_1 \cap D_2 = \emptyset, \ \lambda \in (\lambda_0, \lambda_1)$.

Using the method of potential "inverse squared distances" [1], the following statement is proved.

Theorem. In the class of continuous bounded functions independent of one of the spatial variables, the function $a(x)$ is uniquely restored from the given information (2).

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ON THE REGULARIZATION OF THE SOLUTION OF A NONLINEAR OPERATOR EQUATION OF THE FIRST KIND IN HILBERT SPACE

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This paper proposes a combined method of regularization of a new type, combining the ideas of the method of M.M. Lavrent'ev and the Newton-Kantorovich method for regularization of the solution of a non-linear operator equation of the first kind

$$K(z) = u \tag{1}$$

In Hilbert space, where $K : H \to H$, nonlinear operator, Frechet differentiable.

In space H we introduce the notation of the ball $S(z_0, r) = \{z : ||z - z_0|| <= r_z\}$

We assume, that when $u = u^*$ there exists unique z^* solution (1), i.e. there is an identity $K(z^*) \equiv u^*$

The element u^* is unknown to us, and instead of it is known an element u_δ such that $||u_\delta - u^*|| <= \delta$, where $\delta > 0$ - numeric parameter.

With K'^* we denote the operator, which is conjugate to the operator $K'(z_0)$.

To construct a regularizing operator, inside the ball $S(z_0, r_z)$ selecting a number z_α, form a ball $S(z_\alpha, r_z(\alpha))$, where the center of the ball as $\alpha \to 0$ tends to the center of the ball $S(z_0, r_z)$, i.e. $z_\alpha(u_0) \to z_0$, consequently $r_z(\alpha) \to 0$. The elements of the ball $S(z_\alpha, r_z(\alpha))$ satisfy the inequality $||z - z_\alpha|| <= r_z(\alpha)$, then between the elements z_α and z^* estimate takes place $||z^* - z_\alpha|| <= \gamma r_z(\alpha)$, where $0 < \gamma < 1/2$.

Along with equation (1) we consider the equation

$$\alpha z^\alpha + K'^*(z_\alpha)K(z^\alpha) = K'^*(z_\alpha)u \tag{2}$$

where $\alpha > 0$ is a small regularization parameter. The solution of equation (2) is constructed. An estimate is obtained between the solution of equation (1) and (2)

$$||z^{\alpha,\delta} - z^*|| <= C_1^{1-\sigma} C_2^\sigma (1 - \sigma)^{1-\sigma} \left(\frac{1}{\sigma}\right)^\sigma \delta^{1-\sigma} \tag{3}$$

When regularization parameter α satisfies the condition $\alpha(\delta) = \left(\frac{C_1}{C_2}\frac{\sigma}{1-\sigma}\delta\right)^\sigma$, where $C_1, C_2, \sigma = const$.

Keywords: nonlinear operator, regularization, Hilbert space, Frechet differential, linear operator, operator boundedness, Lipschitz conditions.

AMS Subject Classification: 47N20

IDENTIFICATION PARAMETER PROBLEM FOR HYPERBOLIC EQUATIONS WITH DISCRETE MEMORY

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On the domain $\Omega = [0,T] \times [0,\omega]$ we consider the identification parameter problem for a system of hyperbolic equations with discrete memory in the following form

$$\frac{\partial^2 u}{\partial t \partial x} = A(t,x)\frac{\partial u(t,x)}{\partial x} + B(t,x)\frac{\partial u(t,x)}{\partial t} + C(t,x)u(t,x) + f(t,x) +$$
$$+ A_0(t,x)\frac{\partial u(\gamma(t),x)}{\partial x} + B_0(t,x)\frac{\partial u(\gamma(t),x)}{\partial t} + C_0(t,x)u(\gamma(t),x) + D(t,x)\mu(x), \tag{1}$$

$$P(x)u(0,x) + S(x)u(T,x) = \varphi_1(x), \qquad x \in [0,\omega], \tag{2}$$

$$\int_\xi^\eta K(\tau,x)u(\tau,x) = \varphi_2(x), \qquad x \in [0,\omega], \tag{3}$$

$$u(t,0) = \psi(t), \qquad t \in [0,T], \tag{4}$$

where $u(t,x)$ and $\mu(x)$ are unknown vector functions on dimension n, the $n \times n$ matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $A_0(t,x)$, $B_0(t,x)$, $C_0(t,x)$, $D(t,x)$ and n vector function $f(t,x)$ are continuous on Ω; $\gamma(t) = \zeta_j$ if $t \in [\theta_j, \theta_{j+1})$, $j = \overline{0, N-1}$; $\theta_j \le \zeta_j \le \theta_{j+1}$ for all $j = 0,1,...,N-1$; $0 = \theta_0 < \theta_1 < ... < \theta_{N-1} < \theta_N = T$; the $n \times n$ matrix $P(x)$ and $S(x)$ and the n vector function $\varphi_1(x)$ are continuously differentiable on $[0,\omega]$; the $n \times n$ matrices $K(t,x)$ and n vector function $\varphi_2(x)$ are continuously differentiable on $[0,\omega]$, $0 \le \xi < \eta \le T$; the n vector function $\psi(t)$ is continuously differentiable on $[0,T]$.

In the present communication we propose a new approach for solving identification parameter problem (1)–(4) based on Dzhumabaev's parametrization method [1] and results in [2-3].

Keywords: Identification parameter problem, hyperbolic equations, discrete memory, integral condition.

AMS Subject Classification: 35L53, 34B08, 34K10, 35Q92.

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REGULARIZATION IN THE GENERALIZED SENSE OF THE LOADED INVERSE PROBLEM OF KORTEWEG DE VRIES TYPE DEGENERATING TO THE INCORRECT VOLTERRA EQUATION OF THE FIRST KIND

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In this thesis the inverse problem with a loaded differential operator of the Korteweg De Vries type in an unbounded frontier is studied [1], where the incorrect Volterra integral equation of the first kind degenerates [2,3]. Methods related to the Fourier transform and regularization in a certain space of the generalized functions are used to investigate this problem. Similar problems, in particular are found in the wave theory, in the problems of dispersing loaded waves and others, which is the relevance of this paper. For this purpose an inverse problem is set of the form:

$$U_t + \lambda U(U_x(t,x_0))^2 + U_{x^3} = \varphi(x)(J\theta)(t), \forall(t,x) \in \overline{D}, (D = (0,T) \times R), \tag{1}$$

$$U(x,0) = \varphi(x), \forall x \in R, \tag{2}$$

$$(U_t + U_{x^3})|_{x=0} = g(t), \forall t \in [0,T] \tag{3}$$

$$J\theta \equiv \sum_{i=0}^{1} \left(\lambda_i \int_0^t K_0(t,s)\theta(s)ds \right)^{i+1}, (\lambda_0 = 1), \tag{4}$$

where, $\lambda, \lambda_1, \varphi(x), g(t), K_0(t,s)$ - are known functions, and unknown are the vector function: $P = (U,\theta)$ from the space: $\tilde{G}^1_{[\tilde{W}^1(D);Z^1(0,T)]}(D) = \{(U(x,t),\theta(t)) : U \in \tilde{W}^1(D), \theta \in Z^1(0,T)\}$. Here $Z^1(0,T)$ – space with elements $L^1[0,T]$, and also singular generalized functions $\theta(t)$, focused at the beginning of a segment $[0,T]$ (with the conditions, if the testing function $\psi(0) = 0, then < \psi, \theta >= 0$).

Since from the problem (1)-(3) the incorrect Volterra equation of the first kind degenerates, it is necessary to prove the regularizability of the original problem in the generalized sense in $\tilde{G}^1_{[\tilde{W}^1(D);Z^1(0,T)]}(D)$.

Keywords: Inverse problem, incorrect problem, Volterra integral equation of the first kind, regularizability of inverse problem in generalized sense.

AMS Subject Classification: 45D05

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RECONSTRUCTION OF AN UNKNOWN IMPACT IN A DYNAMIC SYSTEM. NEW POSSIBILITIES OF THE DYNAMIC REGULARIZATION METHOD

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Let a controlled dynamical system be given by the equation $x' = f(t, x, u), \quad t \in [a, b] = T$, $x(a) = x_0$. Here x_0 – is a given element from R^n, the function $f(\cdot) : T \times R^n \times P \to R^n$ is continuous. The Lebesgue-measured function $u(\cdot) : T \to P$, where P –compact of R^q, we will call the impact, and the set of all impacts will be denoted by U. Function $x(\cdot) : T \to R^n$, that is the solution of the equation (according to Carateodory) and represented by the Lebesgue integral in the form $x(t) = x_0 + \int_a^t f(\tau, x(\tau), u(\tau))d\tau$, we will call the motion generated by the impact $u(\cdot)$. Let the compact $E \subset R^n$, set be such that $x(t) \in E$ for all $t \in T, u(\cdot) \in U$. The task of reconstructing an unknown impact from inaccurate information about the observed motion generated by it is generally ill-posed problem, since the set of its solutions may not be single-element. For the case when $f(t, x, u)$ is linear by u Yu.S. Osipov and A.V.Kryazhimsky proposed an efficient, stable to input data perturbation algorithm for solving this problem, called the dynamic regularization method. Its difference from the classical methods of regularization is that the data on the state of the system are available only in the nodes of the interval T with a known error. The formation of the desired approximation, constant at intervals between neighboring nodes, occurs in real time by performing a finite number of arithmetic operations on these data. These operations carry out the construction of the control of the auxiliary the model system according to the feedback principle. This control is taken as an approximation in the norm of the space $L_2(T, R^q)$ of the impact, which has the smallest norm in this space among all the impacts generating the observed motion $x(\cdot)$. The report considers a stable dynamic regularizing algorithm for constructing a piecewise linear approximation of the impact $u(\cdot)$ generating the observed motion $x(\cdot)$. In this case, the limited variation of the impact $u(\cdot)$ and its value at the point a are considered to be known.

Keywords: dynamic regularization method, ill-posed problem, restoring the impact in a dynamic system

AMS Subject Classification: 65L09

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ABOUT UNIQUENESS OF SOLUTIONS OF FREDHOLM LINEAR INTEGRAL EQUATIONS OF THE THIRD KIND ON THE SEGMENT

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In the present paper, we apply the method of integral transformation and the method of of non-negative quadratic form to prove uniqueness theorem for the new class of Fredholm linear integral equations of the third kind in the segment.

Consider the Fredholm linear integral equations of the third kind

$$m(t)(u(t) + \int_a^b K(t,s)u(s)\,ds = f(t), \;\; t \in [a,b], \tag{1}$$

where $m(t)$,$K(t,s)$ and $f(t)$ are given functions, $m(t) \in C[a,b]$, $0 \leq m(t)$ for all $t \in [a,b]$ and $m(t)$ equals zero at least at one point of the segment $[a,b]$, $u(t)$ is the unknown function.

$$K(t,s) = \begin{cases} A(t,s), & a \leq s \leq t \leq b, \\ B(t,s), & a \leq t \leq s \leq b. \end{cases}$$

Suppose that

$$A(t,s) + B(s,t) = \sum_{i=1}^{n} P_i(t)\; H_i(t,s)\; P_i(s),$$

where $P_i(t)$ and $H_i(t,s)$ are given continuous functions respectively on $[a,b]$ and $G = \{(t,s) : a \leq s \leq t \leq b\}$, $i = 1,2,...n$.

Using a modification of the approach proposed in [1], we prove uniqueness theorems for a solution in $C[a,b]$.

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REGULARIZATION AND STABILITY OF SOLUTIONS OF SYSTEM OF LINEAR INTEGRAL FREDHOLM EQUATIONS OF THE FIRST KIND ON A SEMI-AXIS

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In the present paper, we apply, the regularization method of M.M. Lavrentiev [1], for the regularization and stability of solutions of a new class of the system of linear Fredholm integral equations of the first kind on the semi-axis. Consider a system of linear Fredholm integral equations

$$Ku \equiv \int_{-\infty}^{a} K(t,s)u(s)ds = f(t), t \in (-\infty, a], \tag{1}$$

where

$$K(t,s) = \begin{cases} A(t,s), & -\infty < s \le t \le a, \\ B(t,s), & -\infty < t \le s \le a, \end{cases}$$

$$f(t) = (f_i(t)) = (f_1(t)...f_n(t))^T, u(t) = (u_i(t)) = (u_1(t)...u_n(t))^T.$$

$B(t,s)$ and $A(t,s)$ - given matrix functions, $f(t)$ - known vector function, $u(t)$ - unknown vector function. We will assume that $\|K(t,s)\| \in L_2(R \times R), \|f(t)\| \in L_2(R)$. Using a modification of the approach proposed in [2], we prove regularizations and stability of solutions of system linear integral Fredholm equations of the first kind in the semi-axis.

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ON PURSUIT-EVASION GAME PROBLEMS WITH La-CONSTRAINTS

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This work deals with investigating a pursuit differential game problem in the space $\mathbf{R}^n$. The La-pursuit game obeyed differential equations of the type:

$$\dot{z} = a(t)u - b(t)v, \quad z(0) = z_0, \tag{1}$$

where $z, u, v \in \mathbf{R}^n, n \geq 2$; z_0 are the initial positions of the players(pursuer and evader) and it is assumed that $z_0 \neq 0$; $a(t)$ and $b(t)$ are given scalar functions for all $t \geq 0$. Controllers u and v are subject to Langenhop type constraint (briefly, La-constraint)

$$|u(t)|^2 \leq \varrho^2 - 2\int_0^t k(s)|u(s)|^2 ds, \text{ for almost every } 0 \leq t < \bar{t}, \tag{2}$$

$$|v(t)|^2 \leq \varsigma^2 - 2\int_0^t k(s)|v(s)|^2 ds, \text{ for almost every } 0 \leq t < \bar{t}. \tag{3}$$

where $\int_0^t k(s) = K(t) \geq 0$, $a(t) \geq k(t)$, $b(t) \leq k(t)$

Definition 1. A function type : $\mathbf{u}_{La}(t, v) = (b(t)/a(t))\, v - \lambda_{La}(t, v(t))\zeta_0$, is called a Π_{La}-strategy of the Pursuer,in case $\varrho \geq \varsigma$,

where $\lambda_{La}(t, v) = \frac{b(t)}{a(t)}\langle v, \zeta_0\rangle + \sqrt{\left(\frac{b(t)}{a(t)}\langle v, \zeta_0\rangle\right)^2 + \omega e^{-2K(t)}}$, $\omega = \varrho^2 - \varsigma^2$ and $\zeta_0 = z_0/|z_0|$.

Theorem 1. The Π_{La}-strategy guarantees the completion of pursuit in the (1) La-Game on the time interval $[0, T_{La}]$, if and only if $\varrho > \varsigma$, and there exists the leats positive solution of the following equation: $\sqrt{F_P(t) + G_P(t)} - \sqrt{F_P(t)} = |z_0|$ with respect to t,

where $F_P(t) = \varsigma^2 K(t)\left(1 - e^{-2K(t)}\right)/2$, $G_P(t) = \omega\left(1 - e^{-K(t)}\right)^2$.

Keywords: Pursuer,evader,La-type constraint,pursuit-evasion problem.

AMS Subject Classification: 49N70,49N75,91A23,91A24

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INVERSE PROBLEMS FOR A PSEUDO-HYPERBOLIC EQUATION WITH INTEGRAL REDEFINITION

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Problem. Find triples of functions from conditions $\{u(x,t), q(t), f(t)\}$ from conditions

$$u_{tt} - \alpha u_{xxt} - \beta u_{xx} + q(t)u = f(t)h(x,t), (x,t) \in \Omega_T, \tag{1}$$

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x), 0 \leq x \leq l, \tag{2}$$

$$u_x(0,t) = u_x(l,t) = 0, 0 \leq t \leq T, \tag{3}$$

$$\int_0^l u(x,t)\omega(x)dx = \varphi(t), \int_0^l u_x(x,t)\omega(x)dx = \psi(t), 0 \leq t \leq T, \tag{4}$$

where $\Omega_T = \{(x,t) : 0 < x < l, 0 < t \leq T\}, \alpha, \beta > 0 - const, u_0, u_1, \omega, \psi, h$ - are given functions We will say that the inverse problem (1) -(4) satisfies the matching conditions, if

$$\begin{gathered} 1) u_0'(0), = u_0'(l) = 0, u_1'(0), = u_1'(l) = 0, \\ 2) \varphi(0) = \int_0^l u_0(x)\omega(x)dx, \varphi'(0) = \int_0^l u_1(x)\omega(x)dx, \\ 3) \psi(0) = \int_0^l u_0'(x)\omega(x)dx, \psi'(0) = \int_0^l u_1'(x)\omega(x)dx. \end{gathered} \tag{5}$$

Theorem 1. Let $\varphi, \psi \in C^2[0,T], u_0, u_1 \in C^1[0,l], f \in C(\overline{\Omega}_T), \omega \in C^2[0,l]$, $\omega'(0) = \omega'(l) = 0, |\varphi(t)| \geq \varphi_0 > 0, t \in [0,T], |h(0,T)| \geq h_0 > 0$ and the matching conditions (5) be satisfied. Then the inverse problem (1) -(4) for sufficiently small T has a unique solution. Inverse problems for pseudo hyperbolic equations were studied in [1].

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SOLVABILITY OF ONE COEFFICIENT INVERSE PROBLEM FOR THE BUSSINESK-LOVE EQUATION

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In the domain Ω_T , we consider the inverse problem of determining the functions $\{u(x,t), q(t)\} \in C^{(2,2)}(\Omega_T)\bigcap C^{(1,1)}(\overline{\Omega}_T)\times C[0,T]$ satisfying the equation

$$\frac{\partial^2}{\partial t^2}[u_{xx}-u]+u_{xx}+q(t)u=f(x,t), \tag{1}$$

with initial and boundary conditions

$$u(x,0)=u_0(x), u_t(x,0)=u_1(x), 0\le x\le l, \tag{2}$$

$$u_x(0,t)=\mu_1(t), u_x(l,t)=\mu_2(t), 0\le t\le T, \tag{3}$$

and override condition

$$u(x_0,t)=\psi(t), 0<x_0<l, 0\le t\le T, \tag{4}$$

where $\Omega_T=\{(x,t)|0<x<l, 0<t\le T\}, l>0, T>0-const.$

For the initial data, we will assume that the next smoothness and reconciliation have been made:

1) $u_0(x), u_1(x)\in C^2[0,l], f(x,t)\in(\overline{\Omega}_T), \mu_1(t), \mu_2(t)\in C^2[0,T]$,
$\psi(t)\in C^2[0,T], |\psi(t)|\ge\alpha>0, \forall t\in[0,T]$,
2) $u_0'(0)=\mu_1(0), u_0'(l)=\mu_2(0), u_1'(0)=\mu_0'(l), u_1'(l)=\mu_2'(0)$,
$u_0(x_0)=\psi(0), u_1(x_0)=\psi'(0)$.

Theorem 1. Let the above conditions 1) and 2) be satisfied for the functions: $\{u_0(x), u_1(x), f(x,t), \mu_1(t), \mu_2(t)\}$. Then for sufficiently small $T>0$ there is exists an unique solution of the inverse problem (1)-(4).

The inverse problem of source recovery with integral overdetermination was studied in [1].

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INVERSE PROBLEM OF DETERMINING THE COEFFICIENT IN THE ALLER EQUATION WITH REDEFINITION AT INTERNAL POINT

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In the domain Q_T^+ , we consider the problem of determining the function $u(x,t)$, which is the solution of the first boundary value problem on the semiaxis for the Allerra equation

$$u_t(x,t) - \alpha^2 u_{xxt}(x,t) - u_{xx}(x,t) + q(t)u(x,t) = f(x,t), (x,t) \in Q_T^+, \tag{1}$$

$$u(x,0) = u_0(x), x \in [0,+\infty), \tag{2}$$

$$u(0,t) = h(t), 0 \le t \le T, \tag{3}$$

where $\alpha > 0$ is a small parameter. For the direct problem (1)-(3) is true for a given $q(t)$.

Theorem 1. Let $u_0(x) \in C^2_{M_\gamma}[0,+\infty)$, $f(x,t) \in C^{(1,0)}_{M_\gamma}(\overline{Q_T^+})$ for $\gamma < T-\alpha$ and $h(t) \in C^1[0,T]$, $q(t) \in C[0,T]$, then problem (1) -(3) in the domain Q_T^+ has a unique classical solution belonging to $C^{(2,1)}(Q_T^+)$.

Let us formulate the inverse problem. Let the functions $f(x,t)$, $u_0(x)$ and $h(t)$ be given and the function $q(t)$ unknown. It is required to determine a pair of functions $u(x,t)$ and $q(t)$ if additional information about the solution of problem (1)-(3) is known:

$$u(x_0,t) = \psi(t), 0 < x_0 < +\infty, 0 \le t \le T. \tag{4}$$

Theorem 2. Let $u_0(x) \in C^4_{M_\gamma}[0,+\infty)$, $f(x,t) \in C^{(2,0)}_{M_\gamma}(\overline{Q_T^+})$ for $\gamma < T-\alpha$, $\psi(t) \in C^1[0,T]$, $|\psi(t)| \ge \psi_0 > 0$, $0 \le t \le T$, and $f(0,T) = 0$, $u_0(0) = h(0) = 0$, then the inverse problem (1)-(4) in the region Q_T^+ has a unique solution belonging to $C^{(4,1)}(Q_T^+) \times C[0,T]$.

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ON AN INVERSE PROBLEM FOR A TWO-DIMENSIONAL FRACTIONAL PARABOLIC EQUATION WITH INVOLUTION

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In this paper, for a fractional parabolic equation with involution and with nonlocal Bitsadze-Samarskii type conditions [1], the inverse problem is studied in the spatial domain. To solve the problem, the spectral method is used, where the spectral questions of the obtained and adjoint problems are studied. The eigenvalues and eigenfunctions of these problems are found, and the Riesz basis property of systems of eigenfunctions is studied [2, 3]. Further, under certain conditions for the given functions, theorems on the uniqueness and existence of the solution of the problem are proved. In proving the uniqueness of the solution to the problem, the completeness of the system of eigenfunctions corresponding to the spectral problem is used, and the solution to the problem is constructed as an absolutely and uniformly convergent series.

Keywords: Bitsadze-Samarskii type problem, equation of fractional order, eigenvalues, eigenfunctions, completeness, Riesz basis

AMS Subject Classification: 35K65, 34R30, 34K37, 34L10

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THE LINEAR INVERSE PROBLEM WITH A NON-LOCAL BOUNDARY CONDITION OF PERIODIC TYPE FOR A MIXED TYPE EQUATION OF THE SECOND KIND OF THE SECOND ORDER IN AN UNBOUNDED PARALLELEPIPED

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In this article the correctness of a linear inverse problem with a non-local boundary condition of periodic type for a three-dimensional mixed-type equation of the second kind, second order in an unbounded parallelepiped. The existence and uniqueness theorems for a generalized solution to a linear inverse problem with a non-local boundary condition of periodic type are proved in a certain class of integralable function. The "$\varepsilon-$ regularization a priori estimates, approximation sequences and Fourier transform methods are applied.

In the domain

$$G = (0, 1) \times (0, T) \times R = Q \times R = \{(x, t, z); x \in (0, 1), 0 < t < T < +\infty, z \in R\}$$

we consider the three-dimensional mixed-type equation of the second kind of the second order:

$$Lu = k(t)u_{tt} - \Delta u + a(x, t)u_t + c(x, t)u = \psi(x, t, z), \tag{1}$$

where $\Delta u = u_{xx} + u_{zz}$ is the Laplace operator and let $k(0) \leq 0 \leq k(T)$, $\psi(x, t, z) = g(x, t, z) + h(x, t)f(x, t, z)$, the functions $g(x, t, z)$ and $f(x, t, z)$ are given and the function $h(x, t)$ is unknown.

Equation "(1)" refers to equations of mixed type of the second kind, since the sign of the function $k(t)$ by variable t inside the domain Q no restrictions are imposed [1, 2].

Let all the coefficients of equation "(1)" be sufficiently smooth functions in Q .

Linear inverse problem. Find the pair of functions $\{u(x, t, z),\ \ h(x, t)\}$ satisfying the equation "(1)" in the domain G, the following semi-nonlocal boundary conditions

$$\gamma u|_{t=0} = u|_{t=T}, \tag{2}$$

$$D_x^p u|_{x=0} = D_x^p u|_{x=1}; p = 0, 1, \tag{3}$$

where γ is some nonzero constant numbers, the values of which will be specified below.

In what follows, we assume that $u(x, t, z)$ and $u_z(x, t, z) \to 0$ when $|z| \to \infty$, $u(x, t, z)$ absolutely integrable with respect to z on R for any (x, t) in . (4)

In addition, the solution of problem "(1)"-"(4)" satisfies the additional condition

$$u(x, t, l_0) = \varphi_0(x, t), \tag{5}$$

where $l_0 \in R$ and functions $h(x, t)$ belongs to the following class

$$U = \{(u, h)|u \in W_2^{2,s}(G); h \in W_2^2(Q), s \geq 3 \in N.\}$$

where $W_2^{2,s}(G)$ is the Sobolev weight space with the norm

$$||u||^2_{W_2^{2,s}(G)} = (2\pi)^{-1/2} \int\limits_{-\infty}^{+\infty} \left(1+|\lambda|^2\right)^s ||\hat{u}(x,t,\lambda)||^2_{W_2^2(Q)} d\lambda,$$

where s are any finite positive integer, with $s \geq 3$.

Keywords: three-dimensional mixed-type equation of the second kind, the second-order, linear inverse problem with a semi-nonlocal boundary condition, "ε-regularization" method, a priori estimation method, approximation sequences method, Fourier transforms.

AMS 35M10.

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INTEGRATION OF THE PERIODIC HUNTER-SAXTON EQUATION WITH A SELF-CONSISTENT SOURCE

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The scale-invariant nondispersive wave equation

$$u_{xxt} + uu_{xxx} + 2u_x u_{xx} = 0 \tag{1}$$

was introduced by J.Hunter and R.Saxton [1] in connection with a model of nematic liquid crystals. J.Hunter and Y.Zheng [2],[3] have given a very beautiful and thorough treatment of (1), including the introduction of a Hamiltonian structure and proof of complete integrability.

We consider the Hunter-Saxton equation with a self-consistent source

$$u_{xxt} = -uu_{xxx} - 2u_x u_{xx} + \sum_{k=0}^{\infty} \alpha_k(t) s\left(\pi,\ \lambda_k, t\right) \left[q_x\left(x,t\right)\psi^2\left(x,\lambda_k,t\right) + 2q(x,t)\left(\psi^2\left(x,\lambda_k,t\right)\right)'\right] \tag{2}$$

in the class of real-valued π-periodic on the spatial variable x function $u = u\left(x,t\right)$ which satisfy the regularity of assumption

$$u \in C_x^3\left(t>0\right) \cap C_t^1\left(t>0\right) \cap C\left(t \geq 0\right)$$

with the initial condition

$$u(x,0) = u_0(x), x \in R, \tag{3}$$

where $q(x,t) = u_{xx}\left(x,t\right)$, $u_0\left(x\right) \in C^3(R)$ is the given real-valued π -periodic function and $\psi\left(x,\lambda_k,t\right)$ are the Floquet solution (normalized by the condition $\psi\left(0,\lambda_k,t\right) = 1$) of the weighted Sturm–Liouville equation

$$y'' = \lambda q(x,t) y, \quad x \in R. \tag{4}$$

The aim of this work is to provide a procedure for constructing the solution $u\left(x,t\right)$, $\psi\left(x,\lambda_k,t\right)$ of problem (2)-(4) using the inverse spectral theory for the weighted Sturm–Liouville equation (4).

Keywords: Hunter-Saxton equation, self-consistent source, trace formulas, inverse spectral problem, weighted Sturm - Liouville operator.

AMS Subject Classification: 34L25, 35P25, 37K15, 47F05.

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REVERSE PARABOLIC SOURCE IDENTIFICATION PROBLEM

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In this work, we study source identification problem to find a pair (v, p) for reverse parabolic equation with the following multipoint nonlocal conditions

$$\begin{cases} \frac{dv(t)}{dt} - Av(t) = p + f(t), 0 \leq t \leq 1, \\ v(0) = \sum\limits_{k=1}^{l} \mu_k v(\gamma_k) + \psi, \\ v(1) = \varphi. \end{cases} \tag{1}$$

Here $A : H \to H$ is a self-adjoint positive definite operator in a Hilbert space H , the function $f : [0,1] \to H$ and elements φ, $\psi \in H$ are given, $\gamma_1, \gamma_2, ..., \gamma_r, \mu_1, \mu_2, ..., \mu_r$ are known real numbers and

$$\sum_{k=1}^{r} |\mu_k| < 1, 0 < \gamma_1 < \gamma_2 < ... < \gamma_r \leq 1. \tag{2}$$

In paper [1] source identification problem for parabolic equation with multipoint nonlocal condition was studed. Papers [2]-[3] are devoted to well-posedness of direct nonlocal problems for reverse parabolic equation with integral type nonlocal condition.

The aim of current work is to investigate SIP (1) under the assumption (2). We obtained stability estimates for solution. In applications, we established well-posedness of four source identification problems for the multidimensional reverse parabolic equation.

Keywords: reverse parabolic equation, source identification problem, stability, inverse problem, well-posedness.

AMS Subject Classification: 35N25, 35N30

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ESTIMATION OF STABILITY OF A FINITE-DIFFERENCE ANALOGUE OF AN INTEGRAL GEOMETRY PROBLEM WITH A WEIGHT FUNCTION

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The theory and applications of inverse and ill-posed problems is of great importance in science and technology as a necessary tool for establishing a connection between a model and observations. Recently, however, inverse problems have taken center stage in many disciplines, a trend driven not only by advances in sensor technologies, wireless communications, and signal processing, but also by the need to obtain physically meaningful parameters and inputs for computational models with ever-increasing complexity.

There are many imaging applications that deal with the reconstruction of the internal structure of an object without damaging it. The basic mathematical idea common to many such reconstruction problems is based on integral geometry. In this paper, we consider a finite-difference analogue of the integral geometry problem for a family of curves satisfying certain regularity conditions. Integral geometry problems are related to ill-posed problems of mathematical physics and are associated with numerous applications. A wide range of works [1,2] is devoted to problems of integral geometry for various families of curves.

In [3,4], stability estimates were obtained for finite-difference and differential-difference analogues of two-dimensional and three-dimensional problems of integral geometry for a family of curves satisfying certain regularity-type conditions. The results obtained are based on reducing the problem under consideration to a boundary value problem for an equivalent second-order partial differential equation of mixed type.

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Keywords: inverse problems, integral geometry, stability estimates, finite-difference problem.

AMS Subject Classification: 53C65, 35R30.

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INVERSE PROBLEM ON THE DETERMINATION OF TWO COEFFICIENTS OF THE INTEGRO-DIFFERENTIAL EQUATION

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In the region $\Omega = \{(x, y, t) : (x, t) \in \mathbb{R}^2, y \in (0, l)\}$ consider the integro-differential equation

$$u_{tt} = \Delta u + p(x, y)u + \int_0^t m(\theta)u(x, y, t-\theta)\, d\theta, \tag{1}$$

with initial and boundary conditions

$$u|_{t<0} = 0, \qquad u_y|_{y=0} = -\delta(x)\delta'(t), \qquad u_y|_{y=l} = 0, \tag{2}$$

where $p(x, y)-$ is a coefficient characterizing the properties of the environment in which the wave process propagates; $m(t)-$ is a kernel characterizing the memory of the environment; $\delta(\cdot)-$ is the Dirac delta function, $l > 0-$ is a given fixed number.

Given the functions $p(x, y)$ and $m(t)$, finding the function $u(x, y, t)$ from the initial–boundary-value problem (1)–(2) is called direct problem. In the inverse problem, it is required to find the functions $p(x, y)(x \in \mathbb{R}, y \in (0, l))$ and $m(t)(t > 0)$ in (1), according to the available additional information about the solution of the direct problem (1)–(2) for $y = 0$:

$$u(x, 0, t) = f(x, t),\ x \in \mathbb{R}, t > 0. \tag{3}$$

The main result of this work is that construct a method for successively finding the kernel of the integral operator $m(t)$ and the coefficient $p(x, t)$ up to a value of order $O(\varepsilon^2)$ for $t \in [0, 2l]$, where $l > 0$ is any fixed number.

It is assumed that the desired coefficient weakly depends on the variable x and expands into a power series in powers of the small parameter ε. As in work [1], first, the solution of the direct problem in the zero approximation and the kernel of the integral term are determined by reducing the inverse problem to an equivalent system of nonlinear Volterra second integral equations kind. Further, assuming that the kernel $m(x, t)$ is known, the solution of the direct problem in the first approximation and the unknown coefficient $p(x, y)$ are simultaneously determined. In Theorems of local unique solvability of the posed inverse problems are proved.

Keywords: Integro - differential equation, inverse problem, the Dirac delta function, the kernel of the integral, the norm.

AMS Subject Classification: 35L10; 35L20; 35D99; 35H10; 35R30

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ON A LINEAR INVERSE PROBLEM FOR THE THREE-DIMENSIONAL TRICOMI EQUATION WITH A NONLOCAL BOUNDARY CONDITION IN THE UNLIMITED PARALELEPIPED

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This article considers the questions of the correctness of a linear inverse problem with a nonlocal boundary condition for the three-dimensional Tricomi equation in an unbounded parallelepiped. For this problem, the methods of "ε-regularization", a priori estimates, and a sequence of approximations using the Fourier transform are used to prove existence and uniqueness theorems for a generalized solution of the inverse problem in a certain class of integrable functions.

In the domain

$$G = (-1,1) \times (0,T) \times R = Q \times R = \{(x,t,z); x \in (-1,1), 0 < t < T < +\infty, z \in R.\}$$

we consider the three-dimensional Tricomi equation:

$$Lu = xu_{tt} - \Delta u + \ (x,t)\, u_t + c\,(x,t)\, u = \psi\,(x,t,z), \tag{1}$$

where $\Delta u = u_{xx} + u_{zz}$ is the Laplace operator. Here, $\psi\,(x,t,y) = g\,(x,t,y) + h\,(x,t) \cdot f\,(x,t,y)$, $g(x,t,y)$ and $f(x,t,y)$ are given functions, and the function $h(x,t)$ is to be defined.

Linear inverse problem. Find functions $(u(x,t,z),\ h(x,t))$ satisfying equation (1) in the domain G such that the function $u\,(x,t,z)$ satisfies the following non-local boundary conditions

$$\gamma D_t^p u|_{t=0} = D_t^p u|_{t=T}, \tag{2}$$

$$\eta D_x^p u|_{x=-1} = D_x^p u|_{x=1} \tag{3}$$

for $p = 0, 1$, where $D_t^p u = \frac{\partial^p u}{\partial t^p}$, $D_t^0 u = u$, $\gamma, \eta-$are some constant numbers other than zero, the values of which will be specified below.

$$\begin{gathered}\text{In what follows, we assume that } u(x,t,z) \text{ and } u_z(x,t,z) \to 0 \text{ when } |z| \to \infty, \\ u(x,t,z) \text{ absolutely integrable with respect to } z \text{ on } R \text{ for any } (x,t) \text{ in } \overline{Q}.\end{gathered} \tag{4}$$

In addition, the solution of problem "(1)"-"(4)" satisfies the additional condition

$$u(x,t,\ell_0) = \varphi_0\,(x,t),\ where\ \ell_0 \in R \tag{5}$$

and with functions $h(x,t)$ belongs to the class

$$U = \{\,(u,h)|\ \ u \in W_2^{2,3}(G\,);\, h \in W_2^2(Q\,).\}$$

Here, by $W_2^{2,3}(G)$ we denote the Banach space with the norm

$$\|u\|^2_{W_2^{2,3}(G)} = (2\pi)^{-1/2} \cdot \int\limits_{-\infty}^{+\infty} \left(1 + |\lambda|^2\right)^3 \cdot \|\hat{u}\,(x, t, \lambda)\|^2_{W_2^2(Q)}\, d\lambda.$$

Keywords: three-dimensional Tricomi equation, linear inverse problem, nonlocal boundary conditions, well-posedness of the problem, methods of "$\varepsilon-$regularization", a priori estimates, sequence of approximations and Fourier transforms.

AMS Subject Classification: 35M10

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INVERSE PROBLEMS FOR A PSEUDOPARABOLIC EQUATION WITH FRACTIONAL DERIVATIVE: WEAK SOLUTIONS

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Let $\Omega \subset \mathbb{R}^d$ $d \geq 1$, be a bounded domain with a smooth boundary $\partial\Omega$, and $Q_T = \{(x,t) : x \in \Omega,\ 0 < t \leq T\}$ is a cylinder with lateral Γ_T. We study the following inverse problem of determinig the pair of the functions $(u(x,t), f(t))$, which satisfy the pseudoparabolic equation with nonlinear damping term

$$D_t^{\alpha}(u - \Delta u) - \Delta u = \gamma\, |u|^{\sigma-2} u + f(t) \cdot g(x,t) \ \text{ in } \ Q_T, \tag{1}$$

the initial condition

$$u(x,0) = u_0(x) \text{ in } \Omega, \tag{2}$$

the boundary condition

$$u(x,t) = 0 \ \text{ on } \ \Gamma_T, \tag{3}$$

and the integral overdetermination condition

$$\int\limits_{\Omega} (u \cdot \omega + \nabla u \cdot \nabla \omega)\, dx = e(t), \ \ t \geq 0. \tag{4}$$

Here D_t^{α} is the Caputo fractional derivative of order with $\alpha \in (0,1)$ and the coefficient γ might be positive $\gamma > 0$ either negative $\gamma < 0$. The functions $g(x,t)$, $u_0(x)$, $\omega(x)$, and $e(t)$ are given. The exponent σ is given positive number, such that

$$1 < \sigma < \infty. \tag{5}$$

The problem (1)-(4), in the case $\alpha = 1$ was investigated in [1]. In this work, regarding to the sign of γ and the range of values of the exponent σ, and the suitable assumptions on the data, we establish the global and local in time existence and uniqueness of a weak solutions to the inverse problem (1)-(4).

Keywords: Inverse problem, pseudoparabolic equations, fractional derivative, existence, uniqueness, weak solution.

AMS Subject Classification: 35R30, 35K70, 35R11, 34A12, 35D30.

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NUMERICAL SOLUTION OF AN ILL-POSED BOUNDARY VALUE PROBLEM FOR A MIXED-TYPE SECOND-ORDER DIFFERENTIAL EQUATION WITH TWO DEGENERATE LINES

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This work is devoted to the numerical solution of an ill-posed boundary value problem for a second-order mixed type differential equation with two degenerate lines.

Ill-posed boundary value problems were studied by a number of authors, including A. L. Bukhgeim [1], V. Isakov, M. Klibanov, K. S. Fayazov. The works of K. S. Fayazov [2], K. S. Fayazov and I. O. Khajiev [3], K. S. Fayazov and Y. K. Khudayberganov [4], were dedicated to the construction of approximate solutions for non-classical equations.

Let $\Omega = \Omega_0 \times Q$, where $\Omega_0 = \left\{(x,y) : (-1;1)^2, x \neq 0, y \neq 0\right\}$, $Q = \{0 < t < T, T < \infty\}$.

Consider the equation

$$u_{tt}(x,y,t) + \operatorname{sgn} x u_{xx}(x,y,t) + \operatorname{sgn} y u_{yy}(x,y,t) = 0 \tag{1}$$

in the domain Ω.

Problem. Find a solution of the equation (1) satisfying the following initial

$$\left.\frac{\partial^i u(x,y,t)}{\partial t^i}\right|_{t=0} = \varphi_i(x,y), \ (x,y) \in [-1;1]^2, \ i = 0,1, \tag{2}$$

boundary

$$\begin{aligned} u(x,y,t)|_{x=\pm 1} &= 0, \ (y,t) \in [-1;1] \times \overline{Q}, \\ u(x,y,t)|_{y=\pm 1} &= 0, \ (x,t) \in [-1;1] \times \overline{Q}, \end{aligned} \tag{3}$$

and gluing conditions

$$\begin{aligned} \left.\frac{\partial^i u(x,y,t)}{\partial x^i}\right|_{x=-0} &= \left.\frac{\partial^i u(x,y,t)}{\partial x^i}\right|_{x=+0}, \ (y,t) \in [-1;1] \times \overline{Q}, \\ \left.\frac{\partial^i u(x,y,t)}{\partial y^i}\right|_{y=-0} &= \left.\frac{\partial^i u(x,y,t)}{\partial y^i}\right|_{y=+0}, \ (x,t) \in [-1;1] \times \overline{Q}, \end{aligned} \tag{4}$$

where $i = \overline{0,1}$ and $\varphi_i(x,y)$ are given sufficiently smooth functions and satisfied wherein $\varphi_i(x,y)|_{\partial\Omega_0} = 0$.

Using three various methods, we have constructed a sequence of approximate solutions that are stable on the set of correctness. Numerical calculations and their comparative analysis were carried out for all three methods.

Keywords: ill-posed problem, equation with two degeneration lines, regularization parameter, numerical solution analysis.

AMS Subject Classification: 35R25,35M10,65A05

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NONLOCAL BOUNDARY VALUE PROBLEM FOR A SYSTEM OF NONHOMOGENEOUS PARABOLIC TYPE EQUATIONS WITH TWO DEGENERATE LINES

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In this paper, we consider a nonlocal boundary value problem for a system of partial differential equations with changing the direction of time in space. Similar equations were considered by N. Kislov, S.G. Pyatkov, K.S. Fayazov, I.E. Egorov, S. Z. Djamalov, I.O. Khajiev and others. The correctness of nonlocal boundary value problems for some general differential and differential operator equations is studied in various aspects in the works of A.A. Dezin, V.K. Romanko, Y.I. Yurchuk and others.

Ill-posed problems in the sense of J. Hadamard were studied in the works of E.M. Landis, S.G. Krein, M.M. Lavrent'ev [1], V. A. Morozov, V. Ya Arsenin, V. G Romanov, S. I. Kabanikhin, B. A. Bubnov, H. A. Levine, I.E. Egorov and V.E. Fedorov, A.I. Kozhanov, S.G. Pyatkov, A.L. Buchheim, K.S. Fayazov [2], V.Isakov, M.Klibanov, A.Loiuse, P.Mass, E.Shock, A.Hasanugly, A.Amirov M.Kh. Alaminov, I.O. Khajiev, Y.K. Khudayberganov [3] and others.

Let $\Omega = \Omega_0 \times Q$, $\Omega_0 = \Omega_1 \times \Omega_2 \times \Omega_3$, $\Omega_1 = \{-1 < x_1 < 1\}, \Omega_2 = \{-1 < x_2 < 1\}, \Omega_3 = \{(x_3, ..., x_n): \ (0;\pi) \times ... \times (0;\pi)\}, Q = (0;T)$, $x = (x_1, x_2, ..., x_n)$.

Consider the system of equations

$$\begin{cases} u_{1t} + \operatorname{sgn}(x_1)\dfrac{\partial^2 u_1}{\partial x_1^2} + \operatorname{sgn}(x_2)\dfrac{\partial^2 u_1}{\partial x_2^2} + \displaystyle\sum_{i=3}^{n} \dfrac{\partial^2 u_1}{\partial x_i^2} + a_1 u_1 + b_1 u_2 = f_1, \\ u_{2t} + \operatorname{sgn}(x_1)\dfrac{\partial^2 u_2}{\partial x_1^2} + \operatorname{sgn}(x_2)\dfrac{\partial^2 u_2}{\partial x_2^2} + \displaystyle\sum_{i=3}^{n} \dfrac{\partial^2 u_2}{\partial x_i^2} + a_2 u_2 + b_2 u_1 = f_2, \end{cases} \tag{1}$$

in the domain $\Omega \cap \{x_1, x_1 \neq 0\}$, where the n-dimensional domain, a_j, b_j are some constants, $b_2 \neq 0$, $(a_1 - a_2)^2 + 4b_1 b_2 > 0$ and $j = 1, 2$, $f_j(x,t)$ are given sufficiently smooth functions.

Problem. Find a solution to the system of equations (1) satisfying the following conditions: nonlocal

$$u_j(x,t)\,|_{t=0} + \alpha u_j(x,t)\,|_{t=T} = \varphi_j(x),\ x \in \bar{\Omega}_0,\ j = 1, 2, \tag{2}$$

boundary

$$\begin{aligned} u_j(x,t)\,|_{x_1=\pm 1} &= 0,\ (x_2, x_3, ..., x_n, t) \in \bar{\Omega}_2 \times \bar{\Omega}_3 \times \bar{Q}, \\ u_j(x,t)\,|_{x_2=\pm 1} &= 0,\ (x_1, x_3, ..., x_n, t) \in \bar{\Omega}_1 \times \bar{\Omega}_3 \times \bar{Q}, \\ u_j(x,t)\Big|_{\substack{x_i=0 \\ x_i=\pi}} &= 0,\ (x_1, x_2, t) \in \bar{\Omega}_1 \times \bar{\Omega}_2 \times \bar{Q},\ \ i = \overline{3,n},\ j = 1, 2, \end{aligned} \tag{3}$$

and gluing

$$\begin{aligned}
&\left.\frac{\partial^i u_j(x,t)}{\partial x_1^i}\right|_{x_1=-0} = \left.\frac{\partial^i u_j(x,t)}{\partial x_1^i}\right|_{x_1=+0}, \ (x_2, x_3, ..., x_n, t) \in \bar{\Omega}_2 \times \bar{\Omega}_3 \times \bar{Q}, \\
&\left.\frac{\partial^i u_j(x,t)}{\partial x_2^i}\right|_{x_2=-0} = \left.\frac{\partial^i u_j(x,t)}{\partial x_2^i}\right|_{x_2=+0}, \ (x_1, x_3, ..., x_n, t) \in \bar{\Omega}_1 \times \bar{\Omega}_3 \times \bar{Q}\,, \ i = 0, 1.
\end{aligned} \tag{4}$$

conditions, where α is some constant, $\varphi_j(x)$ given sufficiently smooth functions.

In this work, we study the correctness of the desired problem depending on the value of the parameter α and obtain a representation of the solution, as well as an a priori estimate of the solution. In the case of ill-posedness, conditional correctness is proved.

Keywords: nonhomogeneous equation with two degeneration lines, ill-posed problem, a priori estimate, estimate of conditional stability, uniqueness of solution, set of correctness.

AMS Subject Classification (2020): 35R25,35R30,34B10

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CONDITIONAL STABILITY OF ILL-POSED PROBLEMS

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Time reversal problems for parabolic equations on general bounded regions $\Omega \subset \mathbb{R}^n$:

$$\begin{cases} u_t = \bigtriangledown \cdot (a(x,t) \bigtriangledown u), \quad x \in \Omega, t \in (0,T), \\ u(x,T) = \varphi(x), \quad x \in \bar{\Omega}, \\ u(x,t) = 0, \quad x \in \partial\Omega, t \in [0,T]. \end{cases} \tag{1}$$

Here coefficient (x,t) about time variables t is differentiable, and meets the following conditions.

Theorem 1. Properties of the eigenvalues of the symmetric elliptic operator L

i. All eigenvalues of the operator L are real numbers;

ii. Denoting $\sum = \{\lambda_k\}_{k=1}^{\infty}$ as the set of all eigenvalues of the operator L (identical eigenvalues are noted as one), and ordering them by size, we have

$$0 < \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \cdots, \tag{2}$$

and

$$\lambda_k \to \infty \ \ as \ \ k \to \infty, \tag{3}$$

iii. For $k = 1, 2, ...,$ if the eigenvalues ω_k corresponding to the characteristic function $\omega_k \in H_0^1(\Omega)$ satisfies the condition

$$\begin{cases} L\omega_k = \lambda_k \omega_k & \text{in } \Omega, \\ \omega_k = 0 & \text{on } \partial\Omega. \end{cases} \tag{4}$$

Then these characteristic function $\{\lambda_k\}_{k=1}^{\infty}$ constitute a set of standard orthogonal bases in the space $L^2(\Omega)$.

Keywords: ill-posed problems; Parabolic equation problems; regularization methods; conditional stability; variable coefficient problems; Logarithmic convex method.

AMS Subject Classification: 65L08, 65N20.

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CONDITIONS OF MONOTONICITY OF SOLUTIONS OF INTEGRAL EQUATIONS OF THE FIRST KIND IN THE SPACE OF ANALYTICAL FUNCTIONS

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In [2] we found some linear integral equations of the first kind which are correct in a space of analytical functions. In [3] it was explained as a consequence of "effect of analyticity" among other effects in mathematics [1].

Here we propose conditions for a solution of such equation to be an increasing function.

Consider the integral operator

$$K(x; w(s):s) := \int_{-\infty}^{\infty} \exp(-(x-s)^2) w(s) ds$$

in the space A_v of entire analytical functions with the norm

$$\|f\|_v := \sup\{|f^n(0)|v^{-n} : n = 0, 1, 2, ...\}, v > 0.$$

Denote $\widetilde{\pi} := 1/\sqrt{\pi}$. We proved that

$$K^{-1}(x; f(s):s) = \widetilde{\pi}\, \Sigma\{(-1)^k f^{(2k)}(x)/k! : k = 0..\infty\}, \ \|K^{-1}\|_v \le \widetilde{\pi} \exp(v^2/4).$$

Theorem 0.1. *If 1)* $(\forall x \in R)(f'(x) > 0)$,
2) $(\forall x \in R)(|f^{(2k+1)}(x)| \le k|f^{(2k-1)}(x)|/2, k = 1, 2, 3, ...)$
then the solution of the equation $K(x; w(s):s) = f(x)$ *increases for all* $x \in R$.

Proof. By induction, $|f^{(2k+1)}(x)| \le 2^{-k} k! |f'(x)|$. Hence,

$$w'(x) = \widetilde{\pi}\Sigma\{(-1)^k f^{(2k+1)}(x)/k! : k = 0..\infty\} \ge$$
$$\widetilde{\pi}(f'(x) - \Sigma\{|f^{(2k+1)}(x)|/k! : k = 1..\infty\}) \ge$$
$$\widetilde{\pi}(f'(x) - \Sigma\{|f'(x)|2^{-k}k!/k! : k = 1..\infty\}) = \widetilde{\pi} f'(x)(1 - \Sigma\{2^{-k} : k = 1..\infty\}) = 0.$$

Keywords: integral equation of the first kind, solution, correctness, analytical function, monotonicity
AMS Subject Classification: 45M99

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SOLVING OF SOME INVERSE PROBLEM OF EPIDEMIOLOGY BY STOCHASTIC NATURE-LIKE METHODS

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In the SEIR-HCD model (1), the asymptomatic population $E(t)$ transitions to the symptomatic population $I(t)$ after t_{inc} days. Infected people recover in t_{inf} days with probability β and are hospitalized $H(t)$ with probability $1-\beta$. The hospitalized person may then recover or may need to be placed on a ventilator $C(t)$. In the model, only critical cases can die $D(t)$ with probability μ [1].

$$\begin{cases} \frac{dS}{dt} = -\frac{5-a(t-\tau)}{5}\left(\frac{\alpha I(t)S(t)I(t)}{N} + \frac{\alpha E(t)S(t)E(t)}{N}\right), \\ \frac{dE}{dt} = \frac{5-a(t-\tau)}{5}\left(\frac{\alpha I(t)S(t)I(t)}{N} + \frac{\alpha E(t)S(t)E(t)}{N}\right) - \frac{1}{t_{inc}}E(t), \\ \frac{dI}{dt} = \frac{1}{t_{inc}}E(t) - \frac{1}{t_{inf}}I(t), \\ \frac{dR}{dt} = \frac{\beta}{t_{inf}}I(t) - \frac{1-\varepsilon HC}{t_{hosp}}H(t), \\ \frac{dH}{dt} = \frac{1-\beta}{t_{inf}}I(t) + \frac{1-\mu}{t_{crit}}C(t) - \frac{1}{t_{hosp}}H(t), \\ \frac{dC}{dt} = \frac{\varepsilon_{HC}}{t_{hosp}}I(t) - \frac{1}{t_{crit}}C(t), \\ \frac{dD}{dt} = \frac{\mu}{t_{crit}}C(t) \end{cases} \tag{1}$$

The Firefly algorithm is used to restore model parameters in order to accurately predict epidemic dynamics [2]. As a result of experiments on simulation data, it is shown that the proposed SEIR-HCD model using the Firefly algorithm shows a high accuracy of restoring model parameters using stochastic methods of global optimization.

Keywords: Inverse problems, optimization algorithms, Firefly algorithm, epidemiology.

AMS Subject Classification: 34A55, 65L09

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FUNDAMENTALS OF MATHEMATICAL MODELING OF MOISTURE TRANSFER IN THE SOIL

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Mathematical modelling of many natural processes and phenomena is based on the concept of a continuous medium. The term “continuous medium” does not mean that there are no pores or cracks that contain moisture, gas, or a mixture of fine particles [1]. The solid phase of a continuous medium can be nonporous (faintly porous), porous, or capillary.

At present, methods of mathematical modelling of moisture transfer are widely spread abroad [2, 3]. It has been noted that experimental methods and approaches need to be improved [3].

Moisture transfer in saturated soil can be written by using Darcy’s law [1]. In this case, the moisture transfer velocity is proportional to the pressure gradient

$$\frac{\partial W}{\partial t} = div(K(W) * grad(H)) \tag{1}$$

where K(W)-moisture conductivity coefficient that depends on coordinates x, y,z; W-volumetric humidity of soil; H-pressure, t-time.

The humidity of soil can vary depending on the movement. If at the initial time, the soil has an uneven distribution of moisture along the depth, then with time the moisture will increase in drier layers according to the diffusion law. This phenomenon is called the Haller effect, which uses the concept of fractured porous soil to describe the noted fact. A correction term is included in the moisture transfer equation, which takes into account moisture transfer in soils. Hence Haller’s model has the following form

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial x}(D\frac{\partial W}{\partial x} + A\frac{\partial W}{\partial x \partial t}) \tag{2}$$

where A,D-proportionality coefficient

AMS Subject Classification: 35,65

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NUMERICAL SOLUTION OF THE INVERSE PROBLEM OF MAGNETOTELLURIC SOUNDING

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Magnetotelluric sounding is one of the methods of geophysics used to study the structure of the earth's crust and search for mineral deposits. In the framework of geophysical research, two main problems are usually distinguished: direct and inverse. The inverse problem, in turn, is associated with the determination or refinement of the parameters of the environment model based on the registered or model data. In the case of magnetotelluric sounding, the inverse problem is to determine the physical properties of geological structures, such as conductivity or resistivity, based on measured or modelled electromagnetic fields. This allows you to refine the understanding of underground structures and helps in the search and exploration of mineral deposits.

Within the framework of this study, two inverse problems are considered: a single-layer model and a multi-layer model. Inverse problems are aimed at determining the parameters of the medium, such as conductivity, apparent electrical resistivity, and layer thickness. To solve inverse problems, an objective functional is constructed, which is a criterion for assessing the quality of the solution obtained. Then proceeds to find the optimal solution by minimizing this functional. In this study, various minimization methods are considered, including gradient methods, brute-force method, and others. For the numerical solution of inverse problems, these methods are used. As a result of the numerical experiments carried out, the results were obtained, which are numerical solutions of inverse problems.

Keywords: inverse problems, numerical solution, magnetotelluric sounding, gradient method, finite difference scheme.

AMS Subject Classification: 65L08, 65N20, 35R30

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TRANSFORMATION OF THE PROBLEM OF EXTENDING THE GRAVITATIONAL POTENTIAL FROM PERTURBING MASSES TO THE FIRST KIND FREDHOLM INTEGRAL EQUATION

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Consider the problem of extending the gravitational potential towards perturbing masses. This potential $U(x,z)$ satisfies the Laplace equation in the zone outside of anomalies [1].

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial z^2}=0,\ \ x>-H \tag{1}$$

At the Earth's surface, the following conditions are imposed:

$$\partial U/\partial z(x,0)=\varphi(x), \tag{2}$$

At the bottom, the boundary condition is

$$U(x,\infty)=0. \tag{3}$$

Problem (1) - (3) for $z>0$ is a regular boundary value problem. However, there is a problem with extending the solution of this well-posed problem to the adjacent region where $-H<z<0$.

The need to solve problems of extending gravitational fields towards perturbing masses arise in the presence of anomalies at points (x^i,z^i), $i=1,2$. It is well known [2] that the solution of problem (1) - (3) is a harmonic function. If we shift the origin to points (x^i,z^i), equation (1) will remain unchanged, and therefore, function $U(x-x^i,z-z^i)$ will also be a harmonic function of the form $U(x-x^i,z-z^i)=c_i\cdot ln\sqrt{(x-x^i)^2+(z-z^i)^2}$, where c_i- is the thickness of anomalies.

For an approximate solution, we use a computational algorithm based on the application of the first kind Fredholm integral equation.

Keywords:Laplace equation, anomalies,boundary condition, gravitational fields

AMS Subject Classification:35J05, 45B05

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REGULARIZATION METHODS FOR ILL-POSED PROBLEMS - FROM LINEAR ALGEBRA TO NEURAL NETWORKS

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The problem $A(q)=f$ is well-posed if it is proved that for any right-hand side f from F there is a solution q from Q, this solution is unique and continuously depends on the data. A problem is called ill-posed if at least one correctness condition is violated. Regularization methods are used to find normal, pseudo-, quasi-, and other solutions. Starting with the works of A.N. Tikhonov, regularization was based on methods of minimizing the functional $J(q)=\|A(q)-f\|^2$, to which the so-called penalty functions were added (the norm of the solution and/or its derivatives, a priori information about the solution, and others). Works on regularization theory cover various variants of sets Q and F (natural numbers N,{0,1}, Hilbert, Banach, metric and/or topological spaces), operators A (from matrices to compact operators), as well as functionals (L_1, L_2 and many others). Bayesian regularization, compression of data and/or a set of required parameters, decomposition of the operator A significantly expanded the possibilities of using regularization, including for machine learning problems. Important examples are iterative regularization (search for the minimum of the functional $J(q)$ using the gradient descent method), as well as data approximation (of the operator A and/or the right side of f) such that the problem $A(q)=f$ becomes correct.

The report contains a brief review of works on regularization methods and their justification.

Keywords: ill-posed problems, regularization methods, iterative regularization, machine learning problems.

DIFFERENTIAL-DIFFERENCE ANALOGUE OF THE THREE-DIMENSIONAL PROBLEM OF INTEGRAL GEOMETRY

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The wide range of works [1,2] is devoted to the problems of integral geometry, i.e. problems of determining a function based on known integrals of that function over a certain family of manifolds. The monograph [2] proves the uniqueness theorem and obtains an estimate of the stability of the solution for the three-dimensional problem of integral geometry. The proof of the theorem is based on the consideration of an auxiliary differential equation.

This work studies the differential-difference analog of the problem of integral geometry. The main result is an estimate of the conditional stability of the solution for the differential-difference problem. It should be noted that from the estimate of the conditional stability of the solution for the differential-difference problem, the uniqueness theorem, as well as the stability estimate for the solution of the problem of integral geometry, follows.

The obtained estimate of the stability of the solution for the differential-difference problem can be used to justify the convergence of numerical algorithms for solving the problem of integral geometry [3]. Similar results can be obtained in the case of anisotropic [4] and Finsler metrics [5].

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Keywords: inverse problems, integral geometry, stability estimates, differential-difference problem.

AMS Subject Classification: 53C65, 35R30.

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MULTIDIMENSIONAL ANALOGUE OF KREIN EQUATION FOR THE INVERSE ACOUSTIC PROBLEM

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Let us consider inverse problem of finding density $\rho(x, y)$ in next inverse acoustic problem:

$$L_\rho u^{(k)} = \left[\frac{\partial^2}{\partial t^2} - \Delta_{x,y} + \nabla_{x,y} \ln \rho(x, y) \cdot \nabla_{x,y}\right] u^{(k)} = 0, \tag{1}$$

$$x > 0, \quad y \in \mathbb{R}, \quad t > 0, \quad k \in \mathbb{Z};$$

$$u^{(k)}\Big|_{t<0} \equiv 0; \tag{2}$$

$$\frac{\partial u^{(k)}}{\partial x}(+0, y, t) = e^{iky} \cdot \delta(t); \tag{3}$$

$$u^{(k)}\Big|_{y=\pi} = u^{(k)}\Big|_{y=-\pi}. \tag{4}$$

We suppose that the trace of forward problem solution (1)-(4) exists and can be measured:

$$u^{(k)}(0, y, t) = f^{(k)}(y, t). \tag{5}$$

The main goal is to reduce multidimensional nonlinear inverse problem to the system of linear integral equations (multidimensional analogue of Krein equation [1-3]).

Numerical experiments by GLK-method determine the solution of inverse problem in particular point x_0 in depth without any special calculations of unknown coefficients on the interval $(0, x_0)$.

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Keywords: inverse acoustic problem, system of linear integral equations, solution of inverse problem.

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SOLVABILITY OF THE INVERSE PROBLEM FOR A PSEUDOPARABOLIC EQUATION

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In this paper, the inverse problem for a pseudo-parabolic equation with gradient nonlinearity is considered, the integral redefinition condition is considered as additional information. The existence and uniqueness of the inverse problem for a nonlinear pseudoparabolic equation is proved. Inverse problems of determining the right side of the differential equation arise in the mathematical modeling of some physical processes in the case when, in addition to solving the equation, it is necessary to restore the action of external sources. Pseudo-parabolic equations arise in the description of heat and mass transfer processes, processes of motion of non-Newtonian fluids, wave processes and in many other areas. The paper [1] provides generalized information on the distribution of powers between executive authorities in order to increase the efficiency of their work with the p-Laplacian and the non-local integral, taking into account redefinition. The solvability of inverse problems with local and non-local redefinition conditions for Sobolev type equations has been investigated in many papers (see [1] and the literature therein).

Keywords: Inverse problems, solvability, existence, uniqueness.

AMS Subject Classification: 35R30; 35A01

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ON A LINEAR TWO-POINT INVERSE PROBLEM FOR A MULTIDIMENSIONAL WAVE EQUATION WITH SEMI-NONLOCAL BOUNDARY CONDITIONS

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In this paper, we study the correctness of one linear two-point inverse problem for a multidimensional wave equation. Using the methods of a priori estimates, Galerkin, a sequence of approximations and shrinking mappings, we prove the unique solvability of a generalized solution of a linear two-point inverse problem for a multidimensional wave equation.

In connection with the significantly increased capabilities of computer technology in recent decades, complex mathematical models that take into account a much larger number of physical factors are beginning to be used in applied mathematics [1, 3]. In this regard, it should be especially noted that vibration processes are closely related precisely to multipoint inverse problems for hyperbolic equations [1, 3]. For this purpose, in this paper, using the results obtained in [2, 3], we study the unique solvability of a certain linear two-point inverse problem (LDIP) for a multidimensional wave equation.

Let Ω-be a simply connected region in space R^n with a sufficiently smooth boundary $\partial\Omega$. Let us consider multidimensional wave equations in the region

$$G=(0,T)\times\Omega\times(0,l)=Q\times(0,l)\subset R^{n+2},$$

$$Lu=u_{tt}-\sum_{m=1}^{n}u_{x_mx_m}-u_{yy}+\alpha(x,t)u_t+c(x,t)u=g(x,t,y)+\sum_{i=1}^{2}h_i(x,t)f_i(x,t,y), \tag{1}$$

here $\alpha(x,t)$, $c(x,t)$, $g(x,t,y)$ and $f_i(x,t,y)$, i=1,2$are given functions,$$h_1(x,t)$, $h_2(x,t)$ is a unknown functions.

Linear two-point inverse problem (LTPIP).

Find functions $(u(x,t,y),h_1(x,t),h_2(x,t))$ satisfying equation (1) in the domain G such that the function $u(x,t,y)$ satisfies the following semi-nonlocal boundary conditions:

$$\gamma D_t^p u\,|_{t=0}=D_t^p u\,|_{t=T}, \tag{2}$$

$$u\,|_{\partial\Omega}=0 \tag{3}$$

$$u\,|_{y=0}=u\,|_{y=l}=0 \tag{4}$$

For $p=0,1$, where $D_t^p u=\frac{\partial^p u}{\partial t^p}, D_t^0 u=u$, γ is some constant number different from zero, the value of which will be specified below.

In addition, the solution of problem (1)-(4) satisfies the additional conditions

$$u(x,t,\ell_j)=\varphi_j\,(x,t), \tag{5}$$

where $\ell_j \in (0,\ell)$, $j = 1,2$ such that $0 < \ell_1 < \ell_2 < \ell < +\infty$, and functions $u(x,t,y)$ and $h_i(x,t)$, $i = 1,2$ belong to the class

$$U = \left\{(u, h_i,\ i = 1,2);\, u \in W_2^2(G),\, D_y^3(u_{tt}, u_{xx}, u_{xt}) \in L_2(G),\, h_i \in W_2^2(Q)\right\}.$$

Keywords: multidimensional wave equation, linear two-point inverse problem, unique solvability of a generalized solution, methods of a priori estimates, Galerkin, sequences of approximations and contracting mappings..

AMS Subject Classification: 35M10

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REGULARIZATION OF LINEAR MACHINE LEARNING PROBLEMS

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In this paper, theoretical knowledge about inverse and ill-posed problems is applied to linear neural networks (LNN) in combination with machine learning methods using the most common regularization methods (including L1, L2, and dropout) to achieve a reduction in the retraining of neural networks [1]. In the L1 regularization method, a penalty is added to the loss function—the absolute value of the model parameters. This leads to the sparsity of the model parameters, which can simplify and speed up their study. For L2 regularization, a penalty is added to the loss function, the square of the L2 norm, which leads to increased noise immunity and, consequently, reduces the likelihood of repeated learning [2]. Dropout means that during the learning process, there is a certain probability that the neurons of the neural network will be disabled, thereby reducing retraining [3]. These methods can be used to select the optimal value of a regularized hyperparameter.

Keywords: Inverse and ill-posed problems; machine learning; linear neural networks.

AMS Subject Classification: 65N20, 68T07.

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INVERSE PROBLEMS FOR EQUATIONS OF MIXED TYPE

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As is known, A.V. Bitsadze in his studies pointed out that the Dirichlet problem for a mixed-type equation, in particular for a degenerate hyperbolic-parabolic equation, is ill-posed. The question naturally arises: is it possible to replace the conditions of the Dirichlet problem with other conditions covering the entire boundary, which will ensure the well-posedness of the problem? For the first time, such boundary value problems (nonlocal boundary value problems) for a mixed-type equation were proposed and studied in the works of F.I. Frankl when solving the gas-dynamic problem of subsonic flow around airfoils with a supersonic zone ending in a direct shock wave. Problems close in formulation to a mixed-type equation of the second order were considered in the studies by A.N.Terekhov , S.N.Glazatov, M.G. Karatopraklieva and S.Z.Dzhamalov. In these papers, nonlocal boundary value problems in bounded domains are studied for a mixed-type equation of the second kind of the second order.

Such problems for a mixed-type equation of the first kind in the three-dimensional case (in particular, for the Tricomi equation) in unbounded domains are studied in the works of S.Z. Dzhamalov and H. Turakulov. For equations of mixed type of the second kind in unbounded domains, nonlocal boundary value problems in the multidimensional case are practically not studied.

In this article, nonlocal boundary value problem of periodic type for a mixed-type equation of the second kind of the second order, is formulated and studied in a unbounded parallelepiped. To prove the uniqueness of the generalized solution, the method of energy integrals is used. To prove the existence of a generalized solution, the Fourier transforms is used, then, a new problem is obtained in the plane, and for the solvability of this problem, the methods of "ε-regularization" and a priori estimates are used. The uniqueness, existence, and smoothness of a generalized solution of a nonlocal boundary value problem of periodic type for a three-dimensional for a mixed-type equation of the second kind of the second order are proved using these methods and Parseval equality.

Keywords: three-dimensional mixed-type equations of the second kind of the second order in a unbounded parallelepiped, non-local boundary value problem of periodic type, unique solvability and smoothness of the generalized solution in the Sobolev space.

ALGORITHM FOR SOLVING THE INVERSE PROBLEM FOR THE HELMHOLTZ EQUATION ON "DISCRETIZATION-OPTIMIZATION"

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In [1], the problem of determining two boundary conditions for the Helmholtz equation is considered, the authors present theoretical studies of the problem under consideration. The solution of the problem is considered according to the scheme "optimization discretization". The original problem is reduced to the inverse problem, which is written in operator form. We reduce the operator equation to the task of minimizing the target functional. We write out the gradient of the functional. We build an algorithm for solving the inverse problem. For the numerical solution, we use the discretization of the direct and conjugate problems.

In this paper, we consider another scheme for solving inverse problems, i.e. the "discretization optimization" scheme. We consider the direct problem in discrete form, calculate the gradient of the functional in discrete form using the summation formulas in parts, and obtain the formulation of the conjugate problem in discrete form. We build an algorithm for solving the inverse problem. We solve the inverse problem numerically.

Keywords: inverse problems, numerical solution, Helmholtz equation, method Landweber, finite difference scheme.

AMS Subject Classification: 65M32, 65N21, 35R30

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REGULARISATION OF ILL-POSED ALGEBRAIC SYSTEMS

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The solution of ill-posed systems of linear equations has been an active and important area of research.

We discuss:

$$Az = u, \quad z \in Z = R^n, \quad u \in U = R^n. \tag{1}$$

(1)If it is well-conditioned, i.e., the number of conditions is small, the solution of equation (1) is stable with respect to the perturbation of the right terminal term; thus the problem is well-posed.

(2)If it is ill-posed, i.e., when the condition number is very large, a small change in the right-hand term u will cause a large deviation of the approximate solution from the true solution, and thus equation (1) is ill-posed.

Theorem 1. The vector $x_T \in U$ is said to be a regular fitting solution to the system of equations (1), which satisfies

$$\|x_T\| = \min_{u \in U} \|u\|.$$

Theorem 2. Let A be a continuous operator from the metric space F to the metric space U. Then $\forall \alpha > 0$ and $\forall u \in U, \exists z_\alpha \in F_1$ such that the generalized function

$$M^\alpha [z, u] = \rho_u^2 (Az, u) + \alpha \Omega [z], u \in U, z \in F_1 \subset F$$

reaches its lower exact bound at z_α, i.e.

$$M^\alpha [z_\alpha, u] = \inf_{z \in F_1} M^\alpha [z, u].$$

Theorem 3. The eigenvalues of $\tilde{A}^{\mathrm{T}}\tilde{A}$ are $\bar{\lambda}_1, \bar{\lambda}_2, \cdots, \bar{\lambda}_n$. For $A^{\mathrm{T}}A$ the size of the eigenvalue perturbation is the same as $\| A - \tilde{A} \|$.

Keywords: Stabilization functional, Regularization, Inverse problems,Regularization parameter.

AMS Subject Classification: 86-08 65L08

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A PROBLEM FOR IMPULSIVE DIFFERENTIAL EQUATIONS WITH LOADINGS

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In this talk, we pay our consideration to the numerical solving the following linear two-point boundary value problem for impulsive differential equations with loadings:

$$\frac{dx}{dt} = A(t)x + \sum_{i=1}^{m} M_i(t) \lim_{t\to\theta_i+0} \dot{x}(t) + f(t), \quad t \in (0,T)\backslash\{\theta_1, \theta_2, ..., \theta_m\}, \tag{1}$$

$$B_0 x(0) + C_0 x(T) = d, \quad d \in R^n, \quad x \in R^n, \tag{2}$$

$$B_i \lim_{t\to\theta_i-0} x(t) - C_i \lim_{t\to\theta_i+0} x(t) = \varphi_i, \quad \varphi_i \in R^n, \quad i = \overline{1,m}, \tag{3}$$

where $(n \times n)$ -matrices $A(t)$, $M_i(t)$, $(i = \overline{1,m})$, and n-vector-function $f(t)$ are piecewise continuous on $[0,T]$ with possible discontinuities of the first kind at the points $t = \theta_i$, $(i = \overline{1,m})$; B_i and C_i, $(i = \overline{0,m})$ are constant $(n \times n)$-matrices, and φ_i, $(i = \overline{1,m})$ and d are constant n vectors, $0 = \theta_0 < \theta_1 < \ldots < \theta_m < \theta_{m+1} = T$.

Various problems for impulsive differential equations with loadings and methods for finding their solutions are considered in [1]- [3]. In the present work, linear two-point boundary value problem for impulsive differential equations with loadings (1) - (3) is investigated by the Dzhumabaev parameterization method [4]. A numerical algorithm is offered for solving the considering problem.

Keywords: loaded differential equation, impulse effect, parametrization method, algorithm.

AMS Subject Classification: 34K10, 34K45, 65Q99.

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ОБРАТНЫЕ ЗАДАЧИ УРАВНЕНИЯ ПЕРЕНОСА ИЗЛУЧЕНИИ В МНОГОЗОННОЙ ОБЛАСТИ

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В настоящей работе изучаются локальные свойства классических решений односкоростного нестационарного уравнения переноса, рассматриваемого в многозонной области. При этих предложениях уравнение переноса имеет вид:

$$\frac{\partial u}{\partial t} + Lu = Su + f \tag{1}$$

Здесь $u = u(t, \vec{r}, \vec{\omega})$ - функция распределения частиц, f = f(t, $\vec{r}, \vec{\omega}$) - функция источника, $\delta = \delta(\vec{r}), \delta_S = \delta_S(\vec{r}) - (4\pi)^{-1} g(\mu_0)$, - индикатриса рассеяния, $\vec{r} = (x, y, z)$ - пространственные координаты, $\vec{\omega} = (\xi, \eta, \zeta)$ - точки единичной сферы Ω со сферическими координатами $\xi = sin\Theta cos\varphi, \eta = sin\Theta sin\varphi, \zeta = cos\Theta$,

$$Lu = (\omega, gr\vec{a}du) + \delta(\vec{r})u, Su = \frac{\delta_S(\vec{r})}{4\pi} \int_\Omega g(\mu_0) u(t, \vec{r}, \vec{\omega}) d\omega'$$

Для однозначной разрешимости к уравнению (1.1) необходимо присоединить начальное распределение частиц

$$u(0, \vec{r}, \vec{\omega}) = \Phi(\vec{r}, \vec{\omega}) \tag{2}$$

и режимы на внешней границе и на границе раздела зон

$$u(t, \vec{r}, \vec{\omega}) = 0, \vec{r'} \in \partial \overline{G}, (\vec{n_{ri}}, \vec{\omega}) < 0 \tag{3}$$

$$\lim_{\tau \to t^*{}_m + 0} u(\tau, \vec{r}, \vec{\omega}(t-\tau), \vec{\omega}) = \lim_{\tau \to t^*{}_m - 0} u(\tau, \vec{r}, \vec{\omega}(t-\tau), \vec{\omega}), m = \overline{2, M}. \tag{4}$$

Пусть $\overline{\Pi}_j = [0, T] \times \overline{G}_j, \widetilde{\Pi}_j = \overline{\Pi}_j \setminus \{\{0\} \times \delta G_j\}, j = \overline{1, J}$. C($\widetilde{\Pi} \times \Omega$) - класс функции f(t,$\vec{r}, \vec{\omega}$) непрерывных в каждом множестве $\widetilde{\Pi}_j \times \Omega, j = \overline{1, J}$ и таких, что

$$\underset{j}{max} \underset{\widetilde{\Pi}_j \times \Omega}{sup} |f(t, \vec{r}, \vec{\omega})| = \overline{f} < \infty$$

Ключевые слова: уравнения переноса, многозонная область, граничное условие, обратные задачи.

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ЗАДАЧА ДИРИХЛЕ ДЛЯ ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ СМЕШАННОГО ТИПА ВТОРОГО ПОРЯДКА С ОДНОЙ ЛИНИЕЙ ВЫРОЖДЕНИЯ

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Пусть $\Omega = \{(x, y) \in Q,\ 0 < t < T\}$, $Q = \{|x| < \pi,\ 0 < y < \pi\}$.

Рассмотрим дифференциальное уравнение

$$u_{tt} + \operatorname{sgn}(x) u_{xx} + u_{yy} = f(x, y, t) \tag{1}$$

в области $\Omega \cap \{x \neq 0\}$.

Постановка задачи. Найти функцию $u(x, y, t)$, удовлетворяющую уравнению (1) в области $\Omega \cap \{x \neq 0\}$ и следующим условиям:

$$u|_{t=0} = \varphi(x, y),\ \ u|_{t=T} = \psi(x, y),\ \ (x, y) \in \bar{Q}, \tag{2}$$

$$\begin{array}{c} u|_{x=-\pi} = u|_{x=\pi} = 0,\ \ 0 \leq y \leq \pi,\ 0 \leq t \leq T, \\ u|_{y=0} = u|_{y=\pi} = 0,\ \ -\pi \leq x \leq \pi,\ 0 \leq t \leq T, \end{array} \tag{3}$$

$$u|_{x=-0} = u|_{x=+0},\ \ u_x|_{x=-0} = u_x|_{x=+0},\ \ 0 \leq y \leq \pi,\ 0 \leq t \leq T, \tag{4}$$

где $\varphi(x, y)$, $\psi(x, y)$, $f(x, y, t)$ достаточно гладкие функции, причем $\varphi(x, y)|_{\partial Q} = 0$, $\psi(x, y)|_{\partial Q} = 0$.

В данной работе установлен критерий единственности и устойчивости решения задачи Дирихле (1)-(4). Решение задачи построено в виде суммы ряда Фурье. Отсюда возникает проблема малых знаменателей, т.е. не для всех данных решение задачи существует и единственно.

Ключевые слова: уравнение смешанного типа, некорректная задача, априорная оценка, единственность, условная устойчивость.

Предметная классификация AMS: 35M10, 35R25.

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ЧИСЛЕННЫЕ АЛГОРИТМЫ РЕШЕНИЯ ОБРАТНОЙ ЗАДАЧИ ВОССТАНОВЛЕНИЯ МЛАДШЕГО КОЭФФИЦИЕНТА УРАВНЕНИЯ СУБДИФФУЗИИ

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Работа посвящена исследованию дифференциального уравнения с дробной производной по времени

$$\begin{aligned} &\frac{\partial^\alpha U(x,t)}{\partial t^\alpha} - \Delta U(x,t) + m(t)n(x,t)U(x,t) = f(x,t), \quad && x \in \Omega,\ 0 < t \le T, \\ &U(x,0) = 0, && x \in \Omega, \\ &U(x,t) = 0, && x \in \delta\Omega, 0 < t \le T, \end{aligned} \tag{1}$$

где производная $\partial^\alpha U(x,t)/\partial t^\alpha$ порядка $0 < \alpha < 1$ определяется в смысле Капуто, Ω — ограниченная область в $\mathbb{R}^d$, $d \le 3$ с достаточно гладкой границей $\delta\Omega$. Если функции $m(t)$, $n(x,t)$, $f(x,t)$ заданы, то задача нахождения неизвестной функции $U(x,t)$ является прямой задачей.

В работе рассматривается обратная задача, состоящая в одновременном нахождении неизвестной функции $U(x,t)$, а также зависящего от времени младшего коэффициента (потенциала) $m(t)$. В качестве дополнительной информации дается интегральное условие

$$R(t) = \int_\Omega U^2(x,t)dx, 0 < t \le T. \tag{2}$$

Подход к решению обратной задачи основан на сведении ее к решению нелинейного операторного уравнения, получаемого из условия (2). Для его решения используется итеративный метод Левенберга-Марквардта [2]. На каждой итерации для решения системы линейных алгебраических уравнений используются методы градиентного типа.

Работа поддержана Министерством науки и высшего образования Республики Казахстан (проект №AP09258836).

Ключевые слова: Дробные производные, уравнение субдиффузии, обратные задачи, младший коэффициент, метод Левенберга-Марквардта.

Предметная классификация AMS: 35R30; 65R32.

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Оңтайлы басқару

Optimal control

Оптимальное управление

SYNTHESIS OF DISTRIBUTED AND BOUNDARY CONTROLS IN THE TRACKING PROBLEM FOR NONLINEAR OPTIMIZATION OF OSCILLATORY PROCESSES

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Unlike the article[1], here we consider a controlled process described by a boundary problem

$$v_{tt} - Av = \mu \int_0^t K(t,\tau)v(\tau,x)d\tau + f[t,x,u(t,x)], \quad x \in Q, \ \ 0 < t < T, \tag{1}$$

$$v(0,x) = \psi_1(x), \ \ v_t(0,x) = \psi_2(x), \ \ x \in Q, \tag{2}$$

$$\Gamma v(t,x) \equiv \sum_{i,k=1}^{n} a_{ik}(x) v_{x_k}(t,x)\cos(v,x_i) + a(x)v(t,x) = b[t,x,\vartheta(t,x)], \ x \in \gamma, \ \ 0 < t < T, \tag{3}$$

where A is an elliptic operator, Q – region of euclidean space R^n with piecewise smooth boundary γ; $Q_T = Q \times (0,T)$; $f[t,x,u(t,x)] \in H(Q_T)\forall$ of distributed control $u(t,x) \in H(Q_T)$, $b[t,x,\vartheta(t,x)] \in H(\Upsilon_T)\forall$ of boundary control $\vartheta(t,x) \in H(\Upsilon_T), \Upsilon_T = \gamma \times (0,T).\mu$ - parameter, T - fixed moment of time. Regarding the functions of external and boundary action, we will assume that

$$f_u[t,x,u(t,x)] \neq 0, \forall (t,x) \in Q_T; \ \ b_\vartheta[t,x,\vartheta(t,x)] \neq 0, \forall (t,x) \in \Upsilon_T, \tag{4}$$

such that they are monotonic in the functional variable.
In the synthesis problem, it is required to minimize the integral quadratic functional

$$\begin{aligned} J[u(t,x),\vartheta(t,x)] = \int_0^T \int_Q \{[(v(t,x) - \xi_1(t,x)]^2 + [(v_t(t,x) - \xi_2(t,x))]^2\}dxdt + \\ \int_0^T (a \int_Q M^2[t,x,u(t,x)]dx + \beta \int_\gamma N^2[t,x,\vartheta(t,x)]dx)dt, \quad \alpha,\beta > 0. \end{aligned} \tag{5}$$

Here $\xi_1(t,x)) \in H(Q_T), \xi_2(t,x)) \in H(Q_T), \ M[t,x,u(t,x)] \in H(Q_T), N[t,x,\vartheta(t,x)] \in H(\gamma_T)$-given functions.
In this tracking problem, it is required to find such a distributed control $u^0(t,x) \in H(Q_T)$ and a boundary control $\vartheta^0(t,x) \in H(\gamma_T)$ are sought as a functions (functionals) of the state of the controlled process i.e. as

$$u^0(t,x) = u[t,x,v(t,x),v_t(x,t)], (t,x) \in Q_T,$$
$$\vartheta^0(t,x) = \vartheta[t,x,v(t,x),v_t(x,t)], (t,x) \in \gamma_T.$$

Keywords: integro-differential equations, Volterra operator, tracking problem, Bellman functional, Frechet differential, optimal control synthesis.

AMS Subject Classification: 49K20

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THE l-CAPTURE PROBLEM IN A DIFFERENTIAL GAME WITH INERTIAL PLAYERS

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This paper deals with considering the l-capture problem in a differential game with inertial players, whose movements be described by the following equation with initial values

$$\ddot{z} = u - v, \quad z(0) = z_0, \quad \dot{z}(0) = z_1, \tag{1}$$

where $z, u, v \in \mathbb{R}^n$, $n \geq 2$; z_0 is initial distance between players for which it is presumed that $|z_0| > l$, $l > 0$; z_1 is the difference in their initial velocity vectors and it is assumed $z_1 = kz_0, k \in \mathbf{R}$. The controls u and v are regarded as measurable functions $u(\cdot) : [0, +\infty) \to \mathbf{R}^n$ and $v(\cdot) : [0, +\infty) \to \mathbb{R}^n$ accordingly, and they are subject to the constraints

$$|u(t)| \leq \alpha \quad \text{for almost every } t \geq 0, \tag{2}$$

$$|v(t)| \leq \beta \quad \text{for almost every } t \geq 0. \tag{3}$$

which are usually termed the geometrical constraints (in short, the G-constraints), where α and β and they designate the maximal velocities of players.

Definition 1. For $\alpha \geq \beta$, the function $\mathbf{u}(z_0, v) = v - \lambda(z_0, v)\frac{\alpha z_0 + vl}{\alpha + \lambda(z_0, v)l}$ is called the l-approach strategy in the differential game (1)-(3), where

$$\lambda(z_0, v) = \frac{1}{h^2}\left[\langle v, z_0\rangle + \alpha l + \sqrt{(\langle v, z_0\rangle + \alpha l)^2 + h^2(\alpha^2 - |v|^2)}\right], h^2 = |z_0|^2 - l^2.$$

Theorem 1. Let one of the conditions a) $\alpha = \beta$, $k < 0$; b) $\alpha > \beta$, $k \leq \frac{(\alpha-\beta)\sqrt{\alpha-\beta}}{\sqrt{2\beta(\alpha|z_0|-\beta l)}}$ be valid. Then the l-approach strategy guarantees to occur l-capture on the time interval $[0, T_l]$, where

$$T_l = \begin{cases} \left(|z_0|k + \sqrt{|z_0|^2k^2 + 2(\alpha-\beta)(|z_0| - l)}\right) / (\alpha - \beta) & \text{if } k \neq 0,\ \alpha > \beta, \\ (l - |z_0|) / |z_0|k & \text{if } k < 0,\ \alpha = \beta, \\ \sqrt{2(|z_0| - l)/(\alpha - \beta)} & \text{if } k = 0,\ \alpha > \beta. \end{cases}$$

Keywords: Differential game, pursuer, evader, l-capture, geometrical constraint, strategy.

AMS Subject Classification: 49N70,49N75,91A23,91A24

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ON SOLVABILITY OF NONLINEAR OPTIMIZATION PROBLEM FOR OSCILLATION PROCESSES WHEN VECTOR CONTROL

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In the paper the non-linear optimization problem is investigated , where it is required to minimize following integral functional

$$J[\bar{u}(t,x),\bar{\vartheta}(t,x)] = \int\limits_Q \left\{[V(T,x)-\xi_1(x)]^2+[V_t(T,x)-\xi_2(x)]^2\right\}dx+$$
$$+\int\limits_0^T\left[\alpha\int\limits_Q h^2[t,x,\bar{u}(t,x)]dx+\beta\int\limits_\gamma b^2[t,x,\bar{\vartheta}(t,x)]dx\right]dt, \alpha,\beta>0, \tag{1}$$

on the set of solutions of following boundary value problem

$$V_{tt}-AV=\lambda\int\limits_0^T K(t,\tau)V(\tau,x)d\tau+f[t,x,\bar{u}(t,x)], \quad x\in Q\subset R^n, 0<t\le T, \tag{2}$$

$$V(0,x)=\psi_1(x), V_t(0,x)=\psi_2(x), x\in Q, \tag{3}$$

$$\Gamma V(t,x)\equiv\sum_{i,j=1,n}^{n} a_{ij}(x)V_{x_j}(t,x)\cos(\delta,x_i)+(x)V(t,x)=p[t,x,\bar{\vartheta}(t,x)], x\in\gamma, 0<t\le T. \tag{4}$$

Here, the function $V(t,x)\in Q_T=Q\times(0,T]$, describes the state of the controlled oscillation process; Q is a domain of n-dimensional Euclidean space bounded by a piecewise smooth boundary γ; $\xi_1(x)\in H_1(Q),\ \xi_2(x)\in H(Q,)\psi_1(x)\in H_1(Q),\psi_2(x)\in H(Q)$ are given functions; T is a fixed point in time; A is an elliptic operator; $K(t,\tau)\in D=\{0\le t\le,\quad 0\le\tau\le\}$ is an element of Hilbert space; λ is a parameter; $H(Q)$ is Hilbert space of quadratically summable functions in the domain Q ; $H_1(Q)$ is Sobolev space of first order; δ is the normal vector emanating from the point $x\in\gamma$; Scalar functions $f[t,x,\bar{u}(t,x)]$ and $p[t,x,\bar{\vartheta}(t,x)]$ non-linearly depend on vector -functions $\bar{u}(t,x)=(u_1(t,x),...,u_k(t,x))\in H^k(Q_T)$ and $\bar{\vartheta}(t,x)=(\vartheta_1(t,x),...,\vartheta_m(t,x))\in H^m(\gamma_T)$ respectively.
In the article,optimality conditions of the equality type and inequality type are obtained, on the basis of the maximum principle for distributed parameter system. It is established that equality-type optimality conditions have the property of equal ratios, which is a specific feature of the nonlinear optimization problem when vector control. An algorithm has been developed for constructing a solution to the non-linear optimization problem.

Keywords: vector optimal control, boundary value problem, functional, generalized solution, optimal conditions, property of equal ratios.

AMS Subject Classification: 49K20

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SOLVABILITY AND METHOD FOR CONSTRUCTING AN APPROXIMATE SOLUTION OF THE FREDHOLM INTEGRAL EQUATION OF THE FIRST KIND

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Let us consider the Fredholm integral equation of the first kind:

$$Ku = \int_a^b K(t,\tau)u(\tau)d\tau = f(t), t \in I_1 = [t_0, t_1], \tau \in I_2 = [a,b]. \tag{1}$$

Integral equations, let us consider a countable number of Fredholm integral equations of the first kind with a fixed parameter of the following form $\int_a^b L^{(k)}(\tau)u(\tau)d\tau = a^{(k)}, k = 0,1,2,...,$ where $L^{(k)}(\tau) = \|L_{ij}^{(k)}(\tau)\|$, $i = \overline{1,n}$, $l = \overline{1,m}$, are matrices of order $n \times m$ and $a^{(k)} = (a_1^{(k)}, ..., a_n^{(k)}) \in R^n$ is a vector of order $n \times 1$ for each value k, $L_{ij}^{(k)}(\tau) \in L_2(I_2, R^1)$, $u(\tau) \in L_2(I_2, R^m)$ is a desired function.

Theorem 1. Let $\{\varphi_k\}, k = 0,1,2,..., t \in [t_0, t_1]$ be complete orthonormal system in L_2 determined by formula. Then the Fredholm integral equation of the first kind (1) is equivalent to the Fredholm integral equation of the first kind with a fixed parameter

$$\int_a^b L^{(k)}(\tau)u(\tau)d\tau = a^{(k)}, k = 0,1,2,..., \tag{2}$$

where

$$\begin{gathered} L^{(k)}(\tau) = \|L_{ij}^{(k)}(\tau)\|, i = \overline{1,n}, j = \overline{1,m}, a^{(k)} = (a_1^{(k)}, ..., a_n^{(k)}) \in R^n, \\ L_{ij}^{(k)}(\tau) = \int_{t_0}^{t_1} K_{ij}(t,\tau)\varphi_k(t)dt, a_j^{(k)} = \int_{t_0}^{t_1} f_j(t)\varphi_k(t)dt, k = 0,1,2,.... \end{gathered} \tag{3}$$

Keywords: orthonormal system, the equivalence of function, Fourier coefficients, integral equation, solvability.

AMS Subject Classification: 45B05, 45B99, 58B15, 58J20. https://mathscinet.ams.org/msnhtml/msc2020.pdf.

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THE PROBLEM OF OPTIMAL CONTROL OF THE PROCESS IN A MODEL OF A CHEMICAL REACTOR

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Abstract. We consider an optimal control problem for some mathematical problem of a chemical reactor.
Keywords: mathematical model, chemical reactor, optimal control, functional keywords which can be used for indexing purposes.

Let $\Omega = \{x : 0 < x < 1\}$, $Q_T = \Omega \times (0, T)$, where T is some fixed number. In the domain Q_T, consider the system of differential equations that is some mathema tical model of a nonadiabatic tubular reactor (see [1]):

$$\left.\begin{array}{l} \frac{\partial v_1(x,t)}{\partial t} = a \cdot \frac{\partial^2 v_1(x,t)}{\partial x^2} - \frac{\partial v_1(x,t)}{\partial x} - c \cdot v_1 \cdot f(v_2), \\ \frac{\partial v_2(x,t)}{\partial t} = b \cdot \frac{\partial^2 v_2(x,t)}{\partial x^2} - \frac{\partial v_2(x,t)}{\partial x} + k \cdot v_1 \cdot f(v_2) + g \cdot (v_3(t) - v_2(x,t)), \\ \frac{dv_3(t)}{dt} = p \cdot (\int\limits_0^1 v_2(x,t)dx - v_3(t)) + u(t) \cdot (E - v_3(t)) \end{array}\right\} \tag{1}$$

with the boundary conditions

$$\left.\begin{array}{l} a \cdot \frac{\partial v_1(0,t)}{\partial x} - v_1(0,t) = -1, \frac{\partial v_1(1,t)}{\partial x} = 0, \\ b \cdot \frac{\partial v_2(0,t)}{\partial x} - v_2(0,t) = -1, \frac{\partial v_2(1,t)}{\partial x} = 0 \end{array}\right\} \tag{2}$$

and the initial conditions

$$v_1(x,0) = v_{10}(x), v_2(x,0) = v_{20}(x), v_3(0) = v_{30}, \tag{3}$$

where $f(v_2) = \exp(\Gamma - \Gamma/v_2(x,t))$; a, b, c, Γ, k, g, p, E, and v_{30} are constants, positive parameters of the system, $u(t)$ is the control function (a control); $v_1(x,t)$, $v_2(x,t)$, and $v_3(t)$ are the functions of the concentration of the reacting mixture, the reactor temperature, and the temperature of the coolant respectively.

We consider the problem of minimizing the functional

$$J(u) = \int\limits_0^T v_1(1,t)\, dt, \tag{4}$$

i.e., the total amount of nonreacted material in time T at the reactor output, under the conditions (1)-(3).

For system (1)-(3) proven [2] the existence and unigueness of classic solution of system, at arbitrary confined and measurable function $u(t)$. In the paper performed regularization of functional. This kind of regularization provides improvement of convergence sequence of control developed by method of successive approximations.

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OPTIMAL CONTROL PROCESSES DESCRIBED BY THE GOURSAT-DARBOUX EQUATION

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In the work, using the problems of mathematical programming for the problem of optimal control characterized by the system of the Goursat-Darboux equation, the necessary extremum condition is obtained. In this paper, the considered problem is quite general, where the boundary condition is given by a two-ordinary differential equation with an initial condition. Denote $X = A^n([0,\, T] \times [0,\, S]) \times L^r_\infty([0,\, T] \times [0,\, S]) \times L^{r_1}_\infty[0,\, T] \times L^{r_2}_\infty[0,\, S]$, $V = L^r_\infty([0,\, T] \times [0,\, S]) \times L^{r_1}_\infty[0,\, T] \times L^{r_2}_\infty[0,\, S]$ and $Y = L^n_1([0,\, T] \times [0,\, S]) \times L^n_1[0,\, T] \times L^n_1[0,\, S] \times R^n$. Let $U \subset R^r$, $U_1 \subset R^{r_1}$ and $U_2 \subset R^{r_2}$; $int\, U \neq \emptyset$, $int\, U_1 \neq \emptyset$ and $int\, U_2 \neq \emptyset$; U, U_1 and U_2 convex sets. Let the functions $f : [0,\, T] \times [0,\, S] \times R^{3n} \times R^r \to R$, $\varphi : R^{3n} \to R$, $\varphi_1 : [0,\, T] \times R^{2n} \times R^{r_1} \to R$, $\varphi_2 : [0,\, S] \times R^{2n} \times R^{r_2} \to R$, $F : [0,\, T] \times [0,\, S] \times R^{3n} \times R^r \to R$, $\psi_1 : [0,\, T] \times R^n \times R^{r_1} \to R$ and$\psi_2 : [0,\, S] \times R^n \times R^{r_2} \to R$ satisfy the Carathéodory condition, $x_0 \in R^n$. If $(t, s) \in [0,\, T] \times [0,\, S]$, then consider the problem:

$$\begin{array}{l} x_{ts}(t,s) = f(t,s,x(t,s),x_t(t,s),x_s(t,s),u(t,s)), \\ x_t(t,0) = \psi_1(t,\, x(t,0),u_1(t)), \quad x_s(0,s) = \psi_2(s,x(0,s),u_2(s)), \quad x(0,0) = x_0. \end{array} \tag{1}$$

A function $x(\,\cdot\,) \in A^n([0,\, T] \times [0,\, S])$ that satisfying (1) almost everywhere is called a solution of problem (1).
The function $(\bar{x}(\cdot), \bar{u}(\cdot), \bar{u}_1(\cdot), \bar{u}_2(\cdot)) \in X$ minimizing the functional on the set of solutions of problem (1)

$$J(x,u,u_1,u_2) = \varphi(x(T,0),x(0,S),x(T,S)) + \int_0^T \varphi_1(t,x(t,0),x_t(t,0),u_1(t))dt +$$

$$+ \int_0^S \varphi_2(s,x(0,s),x_s(0,s),u_2(s))ds + \int_0^T \int_0^S F(t,s,x(t,s),x_t(t,s),x_s(t,s),u(t,s))dtds \tag{2}$$

is called a solution of problem (1), (2). After transformation we have that problem (1), (2) is equivalent to minimizing the function $J(x,u,u_1,u_2)$on the set$(x,u,\upsilon_1,\upsilon_2) \in Q$ under restrictions $h(x,u,u_1,u_2) = 0$. In the work there are gradients of mappings $h(x,u,u_1,u_2)$ and $J(x,u,u_1,u_2)$ in space X and using the Theorem 4 [1] for problem (1), (2) we obtain the necessary extremum condition.

Keywords: Goursat-Darboux equation, mathematical programming, optimal control.

AMS Subject Classification: 05C35, 52A20.

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PROBLEMS OF OPTIMAL CONTROLS FOR THE MOTION OF OSCILLATORY SYSTEMS WITH LIQUID DAMPERS

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In the paper, optimization control problems for oscillatory systems [2, 6, 7] with liquid dampers by periodic boundary condition are considered, where the second term contains fractionally rational derivatives. Firstly, the equations are reduced to the normal form. To determine optimal control using construction of an extended functional, the Euler-Lagrange equation [1, 3, 4, 5] is found.Then the computational effective algorithm is proposed, which is formed using the Mittag-Leffler function. The results are illustrated on numerical example.

Keywords: Oscillating system, fractional derivative, Euler-Lagrange method, optimal control.

AMS Subject Classification: 26A33.

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EMPTY CORNERS PHENOMENON FOR MODEL OF REPULSIVE DISCRETE ELECTRICAL CHARGES WITHIN CONVEX POLYGONS

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Some observations, numerical experiments [1], [2] yielded

Hypothesis. A random process for sufficiently large number of components in a compact media propelled by outer energy may be self-organizing.

Considered the system of ordinary differential equations with discontinuous right hand parts presenting mutual repulsing of pointwise charges in very viscous media within a convex polygon P. The empty corners phenomenon (with only charges in the vertices) is found for numbers n of points exceeded 400 in a square P_4 and exceeded 100 in a regular triangle P_3 (constants of numerosity [3]).

Forces of inertia can be neglected and a force pushes a body in the media as well as it is immobile each moment. Also, suppose that the velocity of a body in the media is proportional to the force. Thus we obtain the following system of ordinary differential equations of the first order for (distinct) points $z_k(t) = (x_k(t), y_k(t))|k = 1..n, t \in [0, T]$.

The vector of repulsing force is

$F_k := \Sigma\{(x_k - x_i)^2 + (y_k - y_i)^2)^{-3/2}(x_k - x_i, y_k - y_i)|i = 1..n, i \neq k\}$

The system of equations

$z_k'(t) = \{F_k(t) \,|\, z_k(t)$ in the interior of P;

projection of $F_k(t)$ on a side of $P \,|\, z_k(t)$ is on a side of P;

$0 \,|\, z_k(t)$ is in a vertex of $P\}, k = 1..n, t \in [0, T]$.

Corresponding system of difference equations was implemented in *pascal*. Numerical experiments with various random initial conditions demonstrated that:

while n increases most of points concentrate on boundary of P (corresponds to the well-known physical fact that continuous charges always reside on the outer surface of the conductor);

for sufficiently large n there appear and increase empty sectors with only charges in vertices.

Remark 0.1. *A difference scheme implemented on a real object (computer) is not only an approximate solving of a differential equation but a self-standing process too.*

We hope to conduct corresponding physical experiments.

Keywords: mathematical model, repulsion, ordinary differential equation, difference equation, phenomenon, constant of numerosity

AMS Subject Classification: 70F99

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THE PROBLEM OF OPTIMAL DESIGN OF A CIRCULAR ARCH

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Statement of the problem of optimal control. The dynamics of mixing of $v = v(t,x)$a particle of a curved thin rod (a convex circular arch) under external pressure can be represented as:

$v_{tt} - Av = q_1(x)p_1(t) + q_2(x)p_2(t) + f(t,x) \equiv F(t,x)$,(1)

where is A the differential operator:

$Av \equiv v^{VI} + \alpha_1 v^{IV} + \alpha_2 v^{II}, \quad \alpha_1 = \frac{2}{a^2} + \frac{pa}{\alpha}\ ;\ \alpha_2 = \frac{1}{a^4} + \frac{p}{\alpha a}.$

As control functions, we take $p_1(t), \quad p_2(t),\ f(t,x)$.The functions $q_1(x)$and $q_2(x)$on the right side of (1) are considered as given and characterize the shape (geometric) of external forces acting on the arch along the axis Ox. The function $f(t,x)$expresses an arbitrary external force. Note that in many control problems with a boundary (nonhomogeneous boundary conditions), the problem is reduced to a homogeneous one with the help of a special substitution, but the right side of the type can be $F(t,x)$,considered such images that the case when the control is carried out from the boundary is also considered.

Initial-boundary conditions:

$$\begin{cases} v(0,x) = \varphi_1(x), \quad v_t^{'}(0,x) = \varphi_2(x), \\ v(t,0) = v_x(t,0) = v_{xx}(t,0) = 0, \quad x = 0, \quad (2) \\ v(t,l) = v_x(t,l) = v_{xxx}(t,l) = 0, \quad x = l \end{cases}$$

The operator A associated with the boundary value problem (1)-(2) is a self-adjoint operator in $L_2(0,l)$, it has a linearly independent orthonormal system of basis functions corresponding to its eigenvalues. As an optimality criterion, we take the integral:

$$I\left[t_0,\ u,\ u_t\right] = \int_{t_0}^{T}\int_0^l \left[\alpha_1 u^2 + \alpha_2 u_t^2 + \beta_0 f^2(t,x)\right] dxdt +$$
$$+\int_{t_0}^{T} \left[\beta_1 p_1^2(t) + \beta_2 p_2^2(t)\right] dt,\ \alpha_1^2 + \alpha_2^2 \neq 0,\ \beta_0^2 + \beta_1^2 + \beta_2^2 \neq 0. \qquad (3)$$

T it is required to find the control functions $f(t,x) = f(t,u)$, $p_1(t) = p_1(t,\ u)$, $p_2(t) = p_2(t,\ u)$ as functions of the state $u = u(x,t)-$of the solution to problem (1), (2) and such that functional (3) (T is fixed) took the minimum possible value.

In structures with a sufficiently large height or length, determining the parameters of stable regimes and studying the model for the optimal design of a circular arch is an important task of modern applied science.

To solve the problem of synthesizing optimal control (1)-(3), using the dynamic programming method and the spectral decomposition method, synthesizing optimal controls are found.

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THE NONLINEAR DIFFERENTIAL GAME IN THE CASE OF THE l-CATCH

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We consider the l-catch problem in a nonlinear differential game with two players (Pursuer and Evader) in space $\mathbf{R}^n$. Here, the dynamics of the players is described by following equations:

$$\dot{x} = u + f(t,x), \quad x(0) = x_0, \quad \dot{y} = v + g(t,y), \quad y(0) = y_0, \tag{1}$$

where $x, y, u, v \in \mathbb{R}^n$, $n \geq 2$; $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ $(g : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n)$ is an effective flow field for the Pursuer (for the Evader) and they may describe the exogenous dynamic flows; x_0, y_0 are the initial locations of the players, respectively that is $|x_0 - y_0| > l$, $l > 0$. The controls u and v are regarded as measurable functions $u(\cdot) : [0, +\infty) \to \mathbf{R}^n$ and $v(\cdot) : [0, +\infty) \to \mathbb{R}^n$ accordingly, and they are subject to the constraints

$$|u(t)| \leq \rho \ \text{ for almost every } t \geq 0, \quad |v(t)| \leq \sigma \ \text{ for almost every } t \geq 0. \tag{2}$$

which are usually termed the geometrical constraints (in short, the G-constraints), where $\rho, \sigma > 0$ designate the maximal velocities of players.

Definition 1. For $\rho \geq |\omega|$, the control function: $\mathbf{u}_l^*(z_0, \omega) = \omega + \gamma_l^*(z_0, \omega)\big(\xi_l^*(z_0, \omega) - z_0\big)$ is said to be a convergence (approach) strategy of the Pursuer (for brevity, Π_l^*-strategy),where
$\gamma_l^*(z_0,\omega) = \left[\langle \omega, z_0\rangle + \rho l + \sqrt{(\langle \omega, z_0\rangle + \rho l)^2 + (|z_0|^2 - l^2)(\rho^2 - |\omega|^2)}\right] / (|z_0|^2 - l^2)$, $z_0 = x_0 - y_0$,
$\xi_l^*(z_0,\omega) = -l(\omega - \gamma_l^*(z_0,\omega)z_0)/(|\omega - \gamma_l^*(z_0,\omega)z_0|)$, $\omega = \omega(t,x,y,v) = v + g(t,y) - f(t,x)$.

Conjecture. 1 Assume that we can find Lebesgue-integrable functions $k(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ and $h(\cdot) : \mathbb{R}_+ \to \mathbb{R}$, which $k(t) \leq k$, $k > 0$ and $h(t) \leq h$, $h \in \mathbb{R}$, $|f(t,x) - g(t,y)| \leq k(t)|x - y| + h(t)$ is fulfilled for any $x, y \in \mathbb{R}^n$.

Conjecture. 2 Let this conditions be valid: a) $t^* := \min\{t : \Lambda(t) = l, t \geq 0, l > 0\}$; b) $\rho > \sigma + k(t)\Omega(t) + h(t)$ for each $t \in [0, t^*]$, where $\Lambda(t) := |z_0| - \int\limits_0^t \exp\left(-\int\limits_0^s k(\eta)d\eta\right)(\rho - \sigma - h(s))ds$,
$\Omega(t) = \Lambda(t)\exp\int\limits_0^t k(s)ds$.

Theorem 1. If Conjectures 1–2 are satisfied, then the strategy $\mathbf{u}_l^*(z_0,\omega)$ is winning on the interval $[0, t^*]$ in the l-catch problem.

Keywords: Differential game, players, l-catch, convergence (approach) strategy.

AMS Subject Classification: 49N70,49N75,91A23,91A24

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Математикалық және компьютерлік моделдеу

Mathematical and computer modeling

Математическое и компьютерное моделирование

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INVESTIGATION OF THE THEORY OF OWN AND FORCED OSCILLATORY PROCESSES OF DIFFERENT TYPES OF FLAT ELEMENTS

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This paper presents the results of the study of natural and forced vibrations of flat elements, taking into account the layering of the element material, rheological viscous properties, environmental influences, deformable base, anisotropy, etc. Various formulations of boundary value problems of vibrations of a rectangular flat element, both taking into account viscosity, are considered. so, taking into account the above factors of a geometric and mechanical nature, which are transcendental frequency equations, which reduces to algebraic ones, and the influence of both boundary conditions along the edges of a rectangular plate and parameters of a geometric and mechanical nature on the natural oscillation frequencies of rectangular plane elements is considered, and the previous results for a rectangular plate, the material of which satisfies the viscoelastic Maxwell model, are generalized [1].

When studying vibration processes in a solid deformable body, it is advisable to take the nuclei of viscoelastic operators as regular, since only such operators describe instantaneous elasticity, and then viscous flow, which is characteristic of deformable solids. Integro-differential equations with regular kernels are known to be equivalent to partial differential equations.Depending on the particular types of the plane element under consideration, the main unknown functions are selected in the general solutions of the three-dimensional problem: displacement or deformation at points in the fixed plane of the plane element, in particular, in the median plane of the plate of constant thickness. Displacements and stresses at an arbitrary point of a plane element are expressed in terms of basic unknown functions, which are determined from boundary conditions on the surfaces of the plane element. The obtained equations for the basic unknown functions are the general equations of oscillation of a plane element containing derivatives of functions in coordinates and time of any arbitrarily large order. General solutions are presented in the form of power series over the thickness of a flat element. The general solution refers to an equation of hyperbolic type, which describes the oscillatory and wave process in a plane element. Limiting the series of the general equation to a finite number of the first terms, we obtain approximate equations of oscillation of a plane element.

Keywords: oscillation theories, boundary conditions, plane element, proper oscillation, displacement, deformation.

AMS Subject Classification: 70K05.

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MATHEMATICAL MODEL OF OPTIMAL MANAGEMENT OF THE CUSTOMS CLEARANCE PROCESS

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The purpose of this study is the optimal management of the customs clearance process, as well as minimizing the time spent on this process. Because, ultimately, this will lead to minimizing the time for the customs clearance process, as well as to emaxizing revenues to the state budget and minimizing the expenses of an entrepreneur - a participant in foreign trade.

The following mathematical model of the problem of optimal management of the process of multistage customs clearance (1) - (2) is proposed:

$$f(t) = \sum\nolimits_{k=1}^{n} r_k t_k \to min \tag{1}$$

$$\begin{cases} \sum\limits_{k=1}^{n} a_{jk} t_k \le b_1; \quad a_{jk} = 1, \quad if \quad j = (\overline{1, m_1}) \\ \sum\limits_{k=1}^{n} a_{jk} t_k \le b_2; \quad a_{jk} = 1, \quad if \quad j = (\overline{m_1 + 1, m_2}) \\ \sum\limits_{k=1}^{n} a_{jk} t_k \le b_3; \quad a_{jk} = 1, \quad if \quad j = (\overline{m_2 + 1, m_3}) \\ \sum\limits_{k=1}^{n} a_{jk} t_k \le b_4; \quad a_{jk} = 1, \quad if \quad j = (\overline{m_3 + 1, m_4}) \\ t_k > 0, k = (\overline{1, n}) \end{cases} \tag{2}$$

here: $n = 18$- number of stages of customs clearance process; $m_1 = 7$, $m_2 = 8$, $m_3 = 11$, $m_3 = 18$ - the stages of the customs clearance process, which are responsible for the participant in foreign trade, the carrier, the owner of the customs warehouse and the customs service, respectively;

$a_{jk} = 0$at the values of the index j that are not included in the conditions (2);

$r_k = r_k(X)$ level of risk of execution of k-stage;

$X = X(x_1, x_2, x_{58})$ vector, x_i elements which are determined depending on the value of the cargo customs declaration graph. The peculiarity of the problem of optimal management of the process of multistage customs clearance, which is described in (1) - (2) are variable coefficients of the target function. The function $r_k(X)$, which represents the risk level of the k-stage of the process, is a function of the variables of the Cargo Customs Declaration. This requires a special approach to solving this problem. The conducted research has led to the fact that all methods of solving the linear programming problem with variable coefficients, assume certain requirements for the ratio of the coefficients of the target function. First, an explicit form and smoothness of the parametric function is required. Second, some methods of solving this problem require additional conditions such as continuity and differentiability of the function

[1]. However, in the case of the problem of optimal management of the customs clearance process, the parametric coefficients of the target function have no explicit expression and the above conditions cannot be required [2]. On this basis, within the framework of this work, the definitions of customs risk, customs risk zones are introduced, the theorem of necessary and sufficient condition for ensuring the completeness of coverage of customs risks of the customs clearance process through the elements of the cargo customs declaration is given and developed an algorithm for calculating the risks of customs value of goods. According to the results of the research, 53 logical control conditions were developed, which allow to localize customs risk zones. The implementation of these criteria of customs risks in 2022 in 88 thousand 897 cases prevented the risk of "determination of the customs value of goods" and the state budget arrears in the equivalent of more than 9 million 968.8 thousand U.S. dollars

Keywords: customs clearance, mathematical modeling, linear optimization.

AMS Subject Classification: AMS subject classification: 49, 90.

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ALGEBRAIC METHODS FOR SOLVING RECOGNITION PROBLEMS WITH NON-CROSSING CLASSES

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In recent years, the solution of applied problems of recognition classification and prediction has received great development. In many real cases, the solution scheme remains the same, the set of possible solutions is divided into subsets in such a way that solutions that are close in some metric fall into one subset. In the future, solutions that fall into one subset do not differ, and all objects corresponding to these solutions are assigned to one class. The information described by past experience is presented in the following form: various objects are described in some way and their descriptions are divided into a finite number of non-overlapping classes. When a new object appears, a decision is made to enroll it in one or another class. It is proposed to choose such a generalized algorithm so that it achieves extreme forecasting quality. Consider algorithmic models for solving classification problems. It is possible to distinguish among these models the most frequently encountered in solving problems of an applied nature.

Keywords: Zhegalkin polynomial, linear Boolean functions, complex conjunctions, first-order neighborhood, disjunctive normal forms, minimization.

AMS Subject Classification: 68T10

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MODELS, METHODS AND ALGORITHMS FOR MONITORING ENVIRONMENTAL IMPACT ON AGRICULTURAL PRODUCTION

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At present, in order to solve the problems that arise in the development of agriculture and the rational use of natural resources, it is necessary to scientifically substantiate the development of mathematical models for solving problems of optimizing agricultural production, algorithmic and managerial decision-making methods, develop mathematical and software that takes into account soil erosion, weather -climatic conditions, physical and mechanical properties of aerosol particles and other factors. In order to predict the spread of aerosols in the environment, to find the number of aerosols in the considered area D=$\{0 < x < a, 0 < y < b, 0 < z < H\}$ and the amount of aerosols that have fallen on the underlying surface, we use a mathematical model[1]:

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + (w - w_g)\frac{\partial\theta}{\partial z} + \sigma\theta = \mu\Delta\theta + \frac{\partial}{\partial z}\left(k\frac{\partial\theta}{\partial z}\right) + Q\delta\,(x,\, y,\, z)\,; \tag{1}$$

$$\theta\,(x,\, y,\, z,\, 0) = \theta_0\,(x,\, y,\, z)\,; \tag{2}$$

$$-\mu\frac{\partial\theta}{\partial x}\bigg|_{x=0} = \gamma\,(\theta - \theta_a)\ ;\ \mu\frac{\partial\theta}{\partial x}\bigg|_{x=L_x} = \gamma\,(\theta - \theta_a)\,; \tag{3}$$

$$-\mu\frac{\partial\theta}{\partial y}\bigg|_{y=0} = \gamma\,(\theta - \theta_a)\ ;\ \mu\frac{\partial\theta}{\partial y}\bigg|_{y=L_y} = \gamma\,(\theta - \theta_a)\,; \tag{4}$$

$$-k\frac{\partial\theta}{\partial z}\bigg|_{z=0} = \gamma(\beta\theta - F_0);\ -k\frac{\partial\theta}{\partial z}\bigg|_{z=H} = \gamma\,(\theta - \theta_a)\,. \tag{5}$$

At $H = 0$, we have an elevated source at the level of $z = H$ $(F_0 = 0)$, and at above-ground sources $F_0 \neq 0$ $(Q = 0)$. Having calculated the value of the volumetric flow rate of particles then using the boundary condition (5) we can solve the problem of the transfer and diffusion of pollutants in the atmosphere.

Keywords: model, method, algorithm, erosion, soil, aerosol, diffusion..

AMS Subject Classification: 90 Operations research, mathematical programming.

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CONSTRUCTION OF HIGH ORDER MATHEMATICAL MODELS OF THE MAGIC SQUARE AND THEIR COMPUTER SIMULATION

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For construction of magic squares (hereinafter M-matrices) various methods have been proposed [1]. One of known methods of construction of M-matrices is - method of terraces. Earlier authors have implemented algorithmic and programming of construction of M-matrices of high order. However, this method has a disadvantage - the method works only for construction of M-matrices of odd order: $n = 2k - 1, k \in N$. This paper shows computer simulation of high order M-matrix (both odd and even order) by a new method - decomposition method [2,3]. Some properties of arithmetic progression are used. Let's note, that constants of squares forming arithmetic progression act as identifiers - operators.

The relevance of construction of high-order M-matrix is due to its application in cryptography: because there is a large variety of these matrices. For example, the total number of classical magic squares of size 5×5 is 68 826 306.

The construction of mathematical models of high order magic square and their computer simulation is implemented in C++.

Keywords: magic square, cryptography, decomposition method, magic square encryption, computer modeling, arithmetic progression

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ON THE STABILITY OF NONLINEAR DIFFERENCE EQUATIONS AND SOME OF THEIR APPLICATIONS

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One of the main issues in the theory of nonlinear difference equations is the study of the stability of their solution. We study the stability of the trivial solution to the following equation

$$y_{n+1} = S_n y_n + \tau f_n(y_n), \;\; y(0) = y_0, \;\; n = 0, 1, ... \tag{1}$$

Along with (1), we consider the homogeneous equation

$$y_{n+1} = S_n y_n, \;\; y(0) = y_0, \;\; n = 0, 1, ... \tag{2}$$

Nonlinear perturbation $f_n(y_n)$ satisfies the following conditions:

$$\|f_n(y_n)\| = K_n \|y_n\|^r, \;\; r > 1, \;\; f_n(0) = 0, \;\; \sum_{m=0}^{n-1} K_m \le M < \infty. \tag{3}$$

where M is some positive constant. The following theorems are proven.

Theorem 1. Let the following conditions be satisfied: a) the trivial solution to equation (2) is uniformly stable, i.e. $\forall j > 0, \;\; j \le n$estimates $\|y_n\| \le M \|y_j\|$; b) the nonlinear right side of equation (2) satisfies conditions (3); c) the initial perturbation y_0 is sufficiently small. Then the trivial solution to equation (1) is stable and the following estimate $\|y_n\| \le M \|y_0\|, \;\; \forall n = 0, 1, ...$ holds.

Let us consider the following difference equation

$$y_{n+1} = S_n y_n + f_n(y_n) + g_n, \;\; y(0) = y_0, \;\; g_n(0) \ne 0, \;\; n = 0, 1, ... \tag{4}$$

Theorem 2. Let the conditions of Theorem 1 be satisfied. In addition, a permanent perturbation satisfies condition $\sum\limits_{m=0}^{n=1} \|g_m\| \le \delta_0, \;\; \forall m, \;\; \delta_0 > 0$. Then the trivial solution to equation 4 is stable under permanent perturbations, and the following estimate is true for its solution

$$\|y_n\| \le M \left(\|y_0\| + \sum_{m=0}^{n=1} \|g_m\| \right).$$

The results are applied to study the stability of solving explicit and implicit difference schemes.

Keywords: difference equation, difference schemes, stability, uniform stability.

AMS Subject Classification: 65M06, 65N06.

TWO-DIMENSIONAL PROBLEM OF ANOMALOUS TRANSPORT IN A TWO-ZONE FRACTAL POROUS MEDIUM

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Two-zone media are very common in a macroscopically inhomogeneous media. In such a media, the process of solute transport proceed with the manifestation of internal mass transfer between different zones. This significantly changes the overall pattern of filtration and mass transfer [1] . The transfer equations here, unlike the previous ones, have fractional derivatives. Therefore, the object can be considered as a macroscopically inhomogeneous fractal medium.

In this case, the equation in the two-dimensional case is written as

$$\theta_m \frac{\partial c_m}{\partial t} + \gamma\theta_{im} \frac{\partial^\alpha c_{im}}{\partial t^\alpha} = \theta_m \left(D_{mx} \frac{\partial^\beta c_m}{\partial x^\beta} + D_{my} \frac{\partial^\beta c_m}{\partial y^\beta} \right) - v_{mx}\theta_m \frac{\partial c_m}{\partial x} - v_{my}\theta_m \frac{\partial c_m}{\partial y}, \quad (1)$$

where θ_m, θ_{im} - porosity; c_m , c_{im},-volumetric concentrations of the substance $\left(\frac{m^3}{m^3}\right)$; D_{mx}, D_{my} are the hydrodynamic dispersion coefficients in the moving zone $\left(\frac{m^{\beta+1}}{c}\right)$; v_{mx}, v_{my}-average solution velocity (m/c), the index m corresponds to the mobile zone, and im-to the immobile zone.

The presence of a stagnant (immobile) zone is taken into account on the basis of the kinetic equation

$$\gamma\theta_{im} \frac{\partial^\alpha c_{im}}{\partial t^\alpha} = \omega\left(c_m - c_{im}\right), \quad (2)$$

where is the γ mass transfer coefficient $[\gamma] = T^{\alpha-1}, [\omega] = T^{-1}, 0 < \alpha \leq 1, 0 < \beta \leq 2$.

The fields of pressures and filtration rates are also determined. For this, an anomalous filtration equation is used and it is derived on the basis of the anomalous Darcy law

$$\frac{\partial p}{\partial t} = X_x \frac{\partial^{1+\delta_1} p}{\partial x^{1+\delta_1}} + X_y \frac{\partial^{1+\delta_2} p}{\partial y^{1+\delta_2}}, \quad (3)$$

where χ_x and χ_y are the piezoconductivities in the directions x and y.

For equations (1) - (3), the problem of filtration and solute transport is fixed and solved numerically. The anomalous transfer is characterized by the order of the derivative in the diffusion term of the transfer equation and the mass transfer kinetics equation . Decreasing the order of the derivative β in the diffusion terms of the transport equation in both zones leads to "fast diffusion".

Keywords: Anomalous solute transport, diffusion, fractional derivative, porous media.
AMS Subject Classification: 76-10

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GENERALIZED RELAXATIONAL FRACTIONAL DIFFERENTIAL MODEL OF FLUID FILTRATION IN A POROUS MEDIUM

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The relaxation theory of fluid filtration as a non-classical, anomalous filtration was theory developed in [1] . In relaxation models of filtration the fractional differentiation tools were used in [2].

In this paper, in contrast to [2], we consider a generalized relaxation fractional differential model, where both relaxation phenomena with respect to filtration velocity and pressure gradient are used. On the basis of this generalized model, filtration equations are derived. A filtration problem for this equation is posed and numerically solved. The influence of the orders of fractional derivatives on the distribution of pressure in the medium at different moments of time is estimated.

The filtration model with double relaxation in one-dimensional case has the form

$$v + \lambda_v D_t^{\beta} v = -\frac{k}{\mu}\frac{\partial}{\partial x}\left(p + \lambda_p D_t^{\alpha} p\right), \tag{1}$$

where λ_v , λ_p are the relaxation time of the filtration velocity v and pressure p, respectively, k is the permeability of the medium, μ is the viscosity of the fluid, D_t^{β},$D_t^{\alpha}$are the Caputo fractional derivative operators.

For (1) the following piezoconductivity equation is derived

$$\frac{\partial p}{\partial t} + \lambda_v D_t^{\beta+1} p = \kappa\left(\frac{\partial^2 p}{\partial x^2} + \lambda_p D_t^{\alpha}\left(\frac{\partial^2 p}{\partial x^2}\right)\right), \tag{2}$$

where $\kappa = \frac{k}{\mu\beta^*}$ is the coefficient of piezoconductivity, $0 < \alpha \leq 1,\ 0 < \beta \leq 1$.

To solve the equation (2) with the appropriate initial and boundary conditions, the finite difference method was used. Conclusions about the joint influence of α , β and λ_v , λ_p on the characteristics of filtration are given.

Keywords: fractional derivative, relaxation, filtration, porous medium, relaxation time.

AMS Subject Classification: 76-10

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ANOMALOUS SOLUTE TRANSPORT IN AN ELEMENT OF A FRACTURED-POROUS MEDIUM

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This paper considers the anomalous solute transport in an element of a fractured-porous medium (FPM), which consists of a fracture and a porous block (matrix) adjacent to it. A fracture is a semi-infinite one-dimensional object [1], so the distribution of the solute and the fluid flow over its cross-section is considered as uniform. We consider the transfer process in a porous block to be anomalous, while in a fracture the process occurs without the manifestation of anomalous phenomena.

The equations of solute transport and the flow of a fluid in the FPM element are taken as

$$\frac{\partial c_f}{\partial t} + v\frac{\partial c_f}{\partial x} = D_f \frac{\partial^2 c_f}{\partial x^2} + m_0 D_m \frac{\partial^{1-\gamma}}{\partial t^{1-\gamma}} \left(\frac{\partial^\delta c_m}{\partial y^\delta} \right)\bigg|_{y=0}, \tag{1}$$

$$\frac{\partial^\gamma c_m}{\partial t^\gamma} = D_m \frac{\partial^{1+\delta} c_m}{\partial y^{1+\delta}}, \tag{2}$$

where $c_f = c_f(t,x)$ is the solute volume concentration in the fracture, $\mathrm{m}^3/\mathrm{m}^3; c_m = c_m(t,x,y)$ is the solute volume concentration in the matrix, $\mathrm{m}^3/\mathrm{m}^3; D_f$ is a diffusion coefficient in the fracture, $\mathrm{m}^2/\mathrm{s}; v$ is the fluid velocity, m/s. D_m is the anomalous diffusion coefficient in the matrix, $\mathrm{m}^{1+\delta}/\mathrm{s}^\gamma$; m_0 is a matrix porosity, γ, δ are orders of the fractional derivatives with respect to the time and coordinate $(0 < \gamma, \delta \le 1)$, respectively, t is a time, s, x, y - coordinates.

The Equations (1), (2) with initial and boundary conditions is solved by the finite difference method [2, 3].

Numerical results show that the anomalous transport in a porous block of the FPM element affects the transport in the fracture in different ways. With "slow" diffusion in the porous block, the transport process in the fracture is intensified, on contrary, with "fast" diffusion in a porous block, it slows down.

Keywords: diffusion; fractional derivatives; porous block; fractured-porous medium, solute transport.

AMS Subject Classification: 76-10

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SOLID PARTICLE DYNAMICS SIMULATION BASED ON TLBM-IBB

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This research focuses on the mathematical and numerical simulation of non-isothermal two-phase flow containing suspended particle. Specifically, the study investigates the phenomenon of natural convection around a heated sphere located within a cubic cavity.

To model the thermal flows with particle loading, a combination of the thermal lattice Boltzmann equations method and the interpolated bounce-back scheme (TLBM-IBB) [1] is developed. The IBB method is employed to handle the interface between the fluid and solid boundaries, while TLBM is used to simulate the thermal flow of the liquid. The momentum exchange method [2] is utilized to compute the hydrodynamic force on the particle surface. The simulations cover a range of Rayleigh numbers 10^5 - 10^6.

The accuracy and efficiency of the proposed method are demonstrated by solving a test problem [3] involving the dynamics of a single solid particle in a viscous medium. Furthermore, the developed algorithm is applied to solve a three-dimensional problem of natural convection in an incompressible liquid-filled cubic cavity with a heated wall on the left, a cooled wall on the right, and two horizontal adiabatic walls. The obtained results exhibit excellent agreement with experimental and numerical findings reported by other researchers.

Keywords: TLBM, IBB, D3Q19, solid particle, cubical cavity, viscous medium.

AMS Subject Classification: 76-10.

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NUMERICAL STUDY OF TWO-COMPONENT SUSPENSION FILTRATION IN A POROUS MEDIUM WITH DYNAMICAL FACTORS

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The mathematical model of filtration of two-component suspensions in porous media is a generalization of the corresponding one-component model [1, 2]. The system of equations consists of balance equations for each component, Darcy's law and kinetic equation [3]

$$m\frac{\partial c^{(i)}}{\partial t}+v\frac{\partial c^{(i)}}{\partial x}+\frac{\partial \rho^{(i)}}{\partial t}=D^{(i)}\frac{\partial^2 c^{(i)}}{\partial x^2}, \tag{1}$$

$$v=K\left(m\right)\left|\nabla p\right|, \tag{2}$$

$$K\left(m\right)=k_0 m^3 \big/ \left(1-m\right)^2, \tag{3}$$

$$m=m_0-\left(\rho^{(1)}+\rho^{(2)}\right), \tag{4}$$

$$\frac{\partial \rho^{(i)}}{\partial t}=\begin{cases}\beta_r^{(i)} v c^{(i)}, & 0<\rho^{(i)}\le\rho_r^{(i)},\\ \beta_a^{(i)} v c^{(i)}-\beta_d^{(i)}\rho^{(i)}\left(1+\omega^{(i)}\left|\nabla p\right|\right), & \rho_r^{(i)}<\rho^{(i)}<\rho_0^{(i)},\\ 0, & \rho^{(i)}=\rho_0^{(i)},\end{cases} \tag{5}$$

where m is current porosity of media, $c^{(i)}$ is the concentration of the i^{th} component of the suspension (m^3/m^3), $\rho^{(i)}$ is the concentration of deposition of the i^{th} component of the suspension (m^3/m^3), $D^{(i)}$ is diffusion coefficient for each component (m^2/s), $K\left(m\right)$ is filtration coefficient, $\left|\nabla p\right|$ is the modulus of pressure gradient.

The initial and boundary conditions are in the form

$$c^{(i)}\left(x,\,0\right)=0,\;\;\rho^{(i)}\left(x,\,0\right)=0,\;\;c^{(i)}\left(0,\,t\right)=c_0^{(i)}=\text{const}. \tag{6}$$

To solve the problem (1) - (6), we use finite difference method [3]. The dynamics of particle deposition from both components of the suspension at various points in the medium has been analysed. It has been shown that at the points of the medium close to the inlet, the concentration of deposited particles can reach partial capacities. It has been established that for some set of initial data used in the calculations, the concentration of deposition of the second component can relatively quickly reach the partial capacities.

Keywords: deposition kinetics; filtration; porous media; two-component suspension.
AMS Subject Classification: 76S05

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OBTAINING THE SOLUTION OF THE ACOUSTIC WAVE EQUATION USING LAPLACE TRANSFORM

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In the current work, the problem of seismoacoustics is considered. Specifically, obtaining detailed data of inhomogeneities of the earth's soil using seismic screening. The mathematical model of these physical phenomena is described by the partial differential initial-boundary value problems.

The three-layered region is considered, each region has different physical parameters subsequently c_1, c_2, c_3 [1]. At the initial point, an acoustic wave is applied, which is described by the equations (1) - (3).

$$\frac{\partial\Theta}{\partial t^2} = c^2 \frac{\partial\Theta}{\partial x^2} \tag{1}$$

$$\Theta|_{x=0} = 0 \tag{2}$$

$$\frac{\partial\Theta}{\partial t}|_{t=0} = P_0 \tag{3}$$

P_0 - the magnitude of the pressure corresponding to the pressure during the explosion.

This model is solved using Laplace transforms, after which a system of equations is solved to identify the coefficients of each layer [2, 3, 4]. Based on the results obtained, the most efficient model for solving inverse problems of seismoacoustics in poroelastic media will be developed.

Keywords: acoustic wave, Laplace transforms, seismoacoustics, poroelastic medium.

AMS Subject Classification: 44A10

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STAGES OF SOLVING PROBLEMS BY THE METHOD OF ANALYSIS OF HIERARCHIES IN THE MPRIORITY PROGRAM

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Decision-making processes are at the heart of any purposeful activity. The need to make decisions under conditions of uncertainty arises in all areas of technology, economics and social life. Therefore, it is necessary to strive for the optimal use of the available information and, after weighing all possible solutions, try to find the best one among them. The Hierarchy Analysis Method (HAM) was developed by the American scientist T. Saaty [1]. It provides, with the help of simple and reasonable rules, the solution of multicriteria problems that contain qualitative and quantitative factors, while quantitative factors can have different dimensions. HAM is used to solve semi-structured and unstructured problems. The method is based on the decomposition of the task and its presentation in the form of a hierarchical structure. This allows you to include in the hierarchy all the knowledge on the problem being solved. As a result of the decision, the numerically expressed relative degree of interaction of elements in the hierarchy is determined. The solution of the problem with the help of HAM is carried out in stages.

The first stage involves the representation of the problem in the form of a hierarchy [2]. In the simplest case, the hierarchy is built starting from the goal, which is placed at the top of the hierarchy, through intermediate levels, on which the criteria are placed and on which subsequent levels depend, to the lowest level, which contains a list of alternatives.

Second stage. At this stage, it is necessary to prioritize the criteria and evaluate each alternative against the criteria to select the most important one.

Third stage. After the formation of the matrices of paired comparisons for all criteria and alternatives, it is necessary to determine the eigenvectors of the matrices, check the consistency of the matrices using their eigenvalues, and synthesize the global priorities of alternative solutions relative to the main goal.

The method of analysis of hierarchies allows modeling the psychological features of making expert decisions in multicriteria tasks. The proposed technique reveals the possibilities of the hierarchy analysis method and is quite simply implemented in the MPriority program.

Keywords: Multicriteria problems, hierarchical structure, criteria, the matrices of paired comparisons, alternatives, global priorities.

AMS Subject Classification: 93

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NUMERICAL MODELING OF DIFFUSION PROCESSES IN TWO-COMPONENT NONLINEAR MEDIA WITH VARIABLE DENSITY AND SOURCE IN CRITICAL CASES

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In this paper, consider the Cauchy problem in the domain $Q = \{(t, x) : \ 0 < t, \ \ x \in R^N\}$ for a reaction-diffusion system with a double nonlinearity with a variable density

$$\begin{aligned} |x|^{-l}\tfrac{\partial u}{\partial t} &= \nabla\left(|x|^n u^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u^{l_1}\right) + |x|^{-l}\varepsilon u^{p_1}v^{q_1}, \\ |x|^{-l}\tfrac{\partial v}{\partial t} &= \nabla\left(|x|^n v^{m_2-1}\left|\nabla v^k\right|^{p-2}\nabla v^{l_2}\right) + |x|^{-l}\varepsilon u^{p_2}v^{q_2} \end{aligned} \tag{1}$$

$$u(0,x) = u_0(x) \geq 0, \ \ v(0,x) = v_0(x) \geq 0, \ \ \ x \in R^N, \tag{2}$$

where, $k \in R, \ m_1, m_2 > 1, \ p_i, q_i \geq 1, \ p \geq 2$ are positive real numbers, $\nabla(\cdot) = grad_x(\cdot)$ and $u_0(x) \geq 0, v_0(x) \geq 0$- non-trivial, non-negative, bounded and sufficiently smooth function.

In this paper, the qualitative properties of solutions to system (1) are studied on the basis of a self-similar and approximately self-similar approach. We establish one boundary of the critical exponent, and the property of the finite perturbation velocity (FPV) for the system (1).

The existence of a solution with finite properties is proved. The asymptotics of the self-similar solution for fast diffusion in the case $(m_i + l_i + k(p-2) - 1 < 0, \ i = 1, 2)$ are obtained and also studied in the critical case $m_i + l_i + k(p-2) - 1 = 0, \ i = 1, 2$.

In the numerical solution of the problem, the equation was approximated on a grid using an implicit scheme of alternating directions (for the multidimensional case) in combination with the balance method. Iterative processes were built on the basis of the Picard and Newton methods. In a special case of linearization for iteration, the terms of system (1) are represented as

$$u_i^{p_1}v_i^{q_1} \sim u_{i-1}^{p_1-1}(t)u \ cdotv_{i-1}^{p_1-1}(t)v, \ \ u_i^{p_2}v_i^{q_2} \sim u_{i-1}^{p_2-1}(t)u \cdot v_{i-1}^{q_2-1}(t)v, \ i = 1, 2,$$

where $u_0(t), \ v_0(t)$ are solutions to the system of an ordinary differential equation

$$\frac{du_0}{dt} = -u_0^{p_1}v_0^{q_1}, \ \frac{dv_0}{dt} = -u_0^{q_2}v_0^{q_2}$$

The results of computational experiments show that all of the above iterative methods are effective for solving nonlinear problems and lead to nonlinear effects if solutions of self-similar equations constructed by the nonlinear splitting method and the standard equation method are used as the initial approximation [3, 4].

Keywords: variable density, critical cases, two-componential media, diffusion processes.

AMS Subject Classification: 90-10

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A MAXIMUM LIKELIHOOD SIMILARITY METHOD FOR IDENTIFYING COMMUNITIES IN A NETWORK IN GRAPHS

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Maximum likelihood method is one of the most effective methods for identifying communities in communication networks. This method is based on mathematical analysis of statistical data related to the level of interaction between network participants.

The main idea of this method is to find the most probable communities that possess certain characteristics. To achieve this, we need to determine the likelihood function that describes the probability that edges between network participants belong to the same community..

Consider a network $G = (N, E)$ in which the set of vertices has $N = \{1, 2, ..., n\}$ appearances. Let the number of edges of the network be $m = m\,(E)$, and the connection between vertices $E\,(i, j)$ i and j be

$$E(i,j) = \begin{cases} 1 & \text{If there is a connection between i and j teams} \\ 0 & \text{If there is no connection between teams i and j} \end{cases}$$

By community S we mean a non-empty subset of network vertices, and by partition ΠN we mean a set of disjoint communities whose union is exactly N sets: $\Pi N : \Pi(N) = \{S_1, S_2, ..., S_k\}$ here $\bigcup_{k=1}^{K} S_k = N$

Suppose that the real part of the network is $\Pi = \{S_1, S_2, ..., S_k\}$. Let the variables $n_k = n\,(S_k)$ and $m_k = m\,(S_k)$ denote the number of vertices and edges in the community $S_k, k = 1, ..., K$, respectively. Then $n = \sum_{k=1}^{K} n_k$ and $\sum_{k=1}^{K} m_k \leq m$. Let us express the conditions under which the division into teams is optimal. Check out the $S_k \in \Pi$ community. The probability of making m_k connections between n_k vertices in community S_k is equal to

$$p_{in}^{m_k}\,(1 - p_{in})^{\frac{n_k(n_k-1)}{2} - m_k}.$$

Each vertex i in a community S_k may have $n - n_k$ connections with vertices from other communities, but in fact it has $\sum_{j \notin S_k} E\,(i, j)$ connections with vertices from other communities have The probability of realizing a network with a given structure is equal to

$$L_\Pi = \prod_{k=1}^{K} p_{in}^{m_k} (1 - p_{in})^{\frac{n_k(n_k-1)}{2} - m_k} \prod_{i \in S_k} p_{out}^{\frac{1}{2}\sum_{j \notin S_k} E(i,j)} (1 - p_{out})^{\frac{1}{2}(n - n_k - \sum_{j \notin S_k} E(i,j))}. \quad (1)$$

Taking the logarithm of the probability function L_Π in (1) and simplifying it, we get

$$l_\Pi = log L_\Pi = \sum_{k=1}^{K} m_k log p_{in} + \sum_{k=1}^{K} \left(\frac{n_k(n_k - 1)}{2} - m_k \right) log(1 - p_{in}) + \left(m - \sum_{k=1}^{K} m_k \right) log p_{out} +$$

$$+ \left(\frac{1}{2} \sum_{k=1}^{K} n_k\,(n - n_k) - \left(m - \sum_{k=1}^{K} m_k \right) \right) log(1 - p_{out}). \quad (2)$$

The partition Π^* in which the function $l\Pi$ reaches its maximum over all possible partitions is called optimal. Note that there is still uncertainty in the choice of probabilities p_{in} and p_{out}. The function $l_\Pi = l_\Pi(p_{in}, p_{out})$ depends on the arguments of p_{in},p_{out}. By maximizing l with respect to p_{in} and p_{out}, these values can then be used in numerical calculations. Statement 1. For a fixed partition Π, the function $l\Pi pin, pout$ reaches a maximum in

$$p_{in} = \frac{2\sum_{k=1}^{K} m_k}{\sum_{k=1}^{K} n_k^2 - n}, p_{out} = \frac{2\left(m - \sum_{k=1}^{K} m_k\right)}{n^2 - \sum_{k=1}^{K} n_k^2}. \tag{3}$$

We create a model in Maple to calculate the function at each division

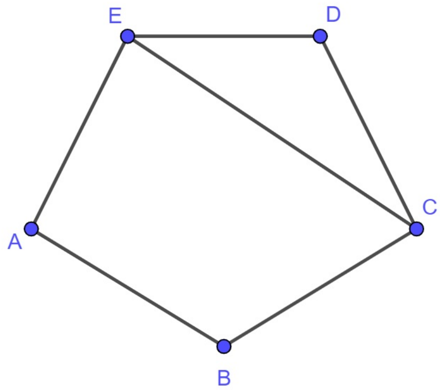

FIGURE 1. pentagonal network

Divisions	n_k,m_k	l_Π	p_{in},p_{out}	$l_\Pi(p_{in}, p_{out})$
$\{A, B, C, D, E\}$	$n_1 = 5$ $m_1 = 6$	$6 \log p_{in}]\!] + 4 \log(1 - p_{in})$	$p_{in} = \frac{3}{5}$	-6.73
$\{C, D, E\} \cup \{A, B\}$	$n_k = (3, 2)$ $m_k = (3, 1)$	$4 \log p_{in}]\!] + 2 \log p_{out}]\!] + 4 \log(1 - p_{out})$	$p_{in} = 1$ $p_{out} = \frac{1}{3}$	-3.81
$\{C, D, E\} \cup \{A, B\}$	$n_k = (3, 2)$ $m_k = (2, 1)$	$3 \log p_{in} + \log(1 - p_{in}]\!] +$ $3 \log p_{out}$ $]\!] + 3 \log(1 - p_{out})$	$p_{in} = \frac{3}{4}$ $p_{out} = \frac{1}{2}$	-3.81

It can be seen that partition $\Pi = \{C, D, E\} \cup \{A, B\}$ gives the most probable community structure for this network.

Keywords: maximum likelihood. Graps. Communication between teams. Teams section. Maple.

AMS Subject Classification: The author(s) should provide AMS Subject Classification numbers using the link https://mathscinet.ams.org/msnhtml/msc2020.pdf.

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VII World Congress of Turkic World Mathematicians
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MODELING OF HEAT TRANSFER PROCESSES IN A SOLAR THERMAL DESALINATION SYSTEM

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Drinking water scarcity is increasing day by day due to population growth and pollution of drinking water sources. By 2025, according to UN forecasts, more than 2.8 billion people will experience severe water shortages. The issue of shortage of drinking water did not pass by the regions of Kazakhstan. The southern, central and western regions are experiencing a shortage of drinking water. Where the south-western region (Mangistau region) is the only one with access to the Caspian Sea and where seawater desalination methods can be applied. In the Mangistau region there is a very good potential for solar energy, in this regard, it is necessary to develop autonomous, mobile installations using renewable energy sources for remote settlements and rural areas where there is no access to central communications.
In this paper, numerical models of the solar thermal system in the climatic conditions of the south-western region of Kazakhstan are considered. The numerical model is based on energy and mass balance. The numerical simulation results showed energy efficiency of solar still. Numerical results have shown that additional energy sources are required to increase the energy efficiency and performance of the solar style. The influence of the intensity of solar radiation on the temperature of steam and glass are discussed.

Keywords: desalination, mathematical model, solar energy, thermal desalination system.

AMS Subject Classification: 74A15, 80A05, 80-10.

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TURBULENT FLOW OF ISOTHERMAL FLUID IN A PIPE

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A simple mathematical model is proposed for estimating the turbulent flow of a fluid in a cylindrical pipe. The mathematical model is based on Prandtl's ideas. The flow region is conventionally divided into the turbulent flow region in the central part of the cylindrical pipe and the near-wall laminar boundary layer.

The model is based on the law of conservation of the mass of the liquid flowing through the section per unit time (mass flow rate of the liquid). Considering that the fluid flow in the central part of the pipe is turbulent, we assume that the speed of the turbulent flow is practically constant over the pipe section. The velocity distribution of a laminar near-wall flow is determined from the equation of motion of a viscous fluid. The total flow rate of the liquid of the two areas is given. The criterion for determining the thickness of the near-wall boundary layer is the equality of the axial velocities of laminar and turbulent flows at the boundary of the transition from laminar to turbulent.

By setting the flow rate of the liquid, other things being equal, we can estimate:
-boundary layer thickness;
-average speed of turbulent flow;
-distribution of axial velocity in the boundary layer; Reynolds number.
A comparison is made with laminar flow in a cylindrical tube under the same conditions.

Keywords: the turbulent flow, cylindrical pipe, Prandtl's ideas, laminar boundary layer.

AMS Subject Classification: 76F70.

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ABOUT NETWORK PARAMETERS OF LEARNING COMMUNITY SUSTAINABILITY FOR PROJECT TEAMS IN PROFESSIONAL TRAINING

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The article deals with the problem of sustainability for learning community of project teams as the successful blended learning condition. Project teams are widely used in professional training. Sustainable communities of student project teams are formed in university education, for example, when involving students in the development of digital educational resources.Social network analysis (SNA) models and methods are relevant for distance and blended learning; dynamic monitoring of the parameters of the network of connections between students makes it possible to predict and promptly support group work [1]. A network model of learning community in professional training can be described by knowledge flow in formal and informal learning processes [2]. Communications should be based on interactive knowledge flows predominantly, and not just on the formal participation in teams. Therefore, the network model of the learning community combines several tie markups, including Knowledge/Information Formal/Informal. The graph can be built using ego network analysis and student survey. Metrics of the community social network graph model can characterize various aspects of its functioning [3]. The main network parameters in the current study are range of vertexes' betweenness centrality, network graph closure, teams' hypergraph connectivity and density. As shown by comparative experimental work in engineering and pedagogical universities, significant for the learning community sustainability and endurance are students' involvement in triads of stable strong ties, formation of dyads in hypergraph with different teams' participants, equability of leaders' vertexes betweenness centrality in learning community.

Keywords: social network analysis, social network graphs, sustainable learning community parameters, learning community network graph endurance, project teams' community model, learning community sustainability.

AMS Subject Classification: 91D10 Models of societies, social and urban evolution, 91D15 Social learning.

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COMMUNICATING PROCESSES SIMULATION IN AGILE MANAGEMENT FOR STUDENT TEAMS PROJECTS OF SOFTWARE DEVELOPMENT

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The implementation of projects with structural dynamics by student teams, as unstable professional teams, requires agile management of blended interactions in conducting tasks timely by participants. The mechanism of communicating sequential processes (CSP) [1] is one of the ways to model concurrent and distributed processes. The experimental work on student projects management in software development showed that CSP model can be applied rather effectively in the management of individual and group project tasks [2]. Parallelism in this model is manifested in the possibility of simultaneous execution of distributed tasks by different participants. The disadvantage of CSP model is difficulty to conduct global analysis and monitoring in the practical management of several projects. CSP model allows transformation into Petri nets, which can also be used to manage student teams work [3]. The disadvantage of Petri nets for team projects, on the contrary, is the impossibility of delimiting the tasks of the personal responsibility of the participants. The possibility of combining two formalisms permits both to control and analyze the interaction of participants in joint project activities. In addition, software development learning with process modeling makes it possible to illustrate students' dynamic model application in social system automation.

Keywords: communication processes model, Petri nets, communicating sequential processes, agile project management, distributed processes, software development learning.

AMS Subject Classification: 90B10 Deterministic network models in operations research for network control, 68Q85 Models and methods for concurrent and distributed computing, 97P50 Programming techniques (educational aspects).

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COMPLEX RAYS AND THICK LENSES

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Complex rays are one of the mysterious subjects in modern optics and diffraction theory. In some cases, they allow for a more precise estimation of the behavior of diffracted waves compared to real geometric optics methods. As waves (acoustic, optics, etc.) propagate in a medium, diffraction effects cause the rays to broaden and diverge. It turns out that within the framework of geometric optics, it is possible to estimate this divergence without calculating diffraction integrals. This is achieved through the existence of complex solutions to the eikonal equation, the main equation of geometric optics. However, the main drawback of this method is that it requires techniques involving high-dimensional complex spaces. Unlike real solutions of the eikonal equation, complex solutions cannot be visually represented in physical space, which causes a significant loss of clarity in wave propagation. In [1], it has been shown that considering the eikonal equation in a non-Euclidean space, instead of the usual Euclidean space, might significantly simplify the geometric picture of propagation. This paper proposes another application of the complex. It is shown that the ray picture of the optical system involving thick lenses can be adequately described by using complex rays.

Keywords: complex rays, real rays, geometric optics, non-Euclidean space.

AMS Subject Classification: 78A45, 34E10

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THE STABILIZATION PROBLEMS OF QUADCOPTER MOTION BY GPS DATA

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In this work, using the method of designing optimal controllers and filters for the linear-quadratic Gaussian (LQG) problem, the quadcopter flight stabilization problem is solved using data from the GPS satellite navigation system. Based on the mathematical model of quadrocopter motion [3], three different discrete LQG problems are considered, which can be solved independently of one another in parallel. After discretization of the object motion equations and considering the random external perturbations, we have a discrete

$$x_{i+1}^j = \Phi^j x_i^j + \Gamma^j u_i^j + \omega_i^j, \; j = 1, 2, 3. \tag{1}$$

Further, let's supposed that in the time state i the measurements z_i^j are implemented, which are linearly related to the state of the trajectory x_i

$$z_i^j = H^j x_i^j +_i^j, \tag{2}$$

where ω_i is the vector of random external perturbations, υ_i is the vector of random measurement errors, which are assumed to be Gaussian random variables of the "white noise" type.

Let's assumed that mathematical expectations and correlation matrices for random variables x_0, ω_i, and υ_i have in the form as in [1, 2]. It is required to find such a control action as a function of the observation, which minimized the quadratic functional

$$J^j = E\left\{ \frac{1}{2} \sum_{i=1}^{\infty} \left[(x_i^j)^T (u_i^j)^T \right] \left[\begin{array}{cc} A^j & N^j \\ (N^j)^T & B^j \end{array} \right] \left[\begin{array}{c} x_i^j \\ u_i^j \end{array} \right] \right\}. \tag{3}$$

Further, giving the values Φ^j, Γ^j, H^j, A^j, B^j, N^j for $j = 1, 2, 3$ and each time solving the LQG problem according to algorithm [4], optimal controllers and filters are constructed.

Keywords: unmanned aerial vehicle, quadcopter, motion stabilization, GPS satellite navigation system, LQG problem.

AMS Subject Classification: 60G15, 93C95, 93E11.

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APPLICATION OF MACHINE LEARNING METHODS FOR THE ANALYSIS OF DATA FROM SOCIO-ECONOMIC SURVEY

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To ensure the content of decision-making information systems in the process of managing the assets of individuals, the development of mathematical models of complex social systems is required. An important priority throughout the life cycle of an individual is marriage. Growing up is closely related to the solution of the housing problem. The study of the expectations of young people in socio-economic issues is of great importance for understanding the future development of the state, for developing a strategy in the field of social policy. In this paper, using machine learning methods, socio-economic problems are studied from the point of view of first-year university students. The influence of various factors on the decision-making by students regarding the expected age of marriage and the solution of the housing problem, the expected income from work is studied. The data obtained as a result of an anonymous survey of students is processed by machine learning methods for analysis and development of forecasting models. A comparative analysis of the prediction accuracy of four classification methods is carried out: logistic regression, neural networks, support vector machine. Students are clustered by the K-means method. Preliminary processing of survey data is carried out by data mining methods. The resulting mathematical models make it possible to study behavior patterns and classify and cluster students depending on the expected age of entry into adulthood, the financial capabilities of the students' parents, expected income and other factors.

Keywords: machine learning, mathematical model, forecasting, behavior patterns, problems of youth

AMS Subject Classification: Computer science

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NUMERICAL SOLUTION OF ANOMALOUS FILTRATION PROBLEM IN A POROUS MEDIUM

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In this work, the problem of anomalous filtration taking into account the phenomenon of colmotation and suffusion in a two-dimensional porous medium consisting of two zones with inhomogeneous fluids is posed and numerically solved. It is considered that the medium has a fractal structure. The study area consists of two zones, one of which is highl permeable (zone $R_1\{0 \le x < \infty, 0 \le y \le l\}$) and the other is low permeable ($R_2\{0 \le x < \infty, l \le y \le \infty\}$). Initially the area R_1 and R_2 filled with liquid without substance [1].

$$\varepsilon_{0i}\frac{\partial c_i}{\partial t} = \varepsilon_{0i}\left\{D_{ix}\frac{\partial^{\beta_{ix}} c_i}{\partial x^{\beta_{ix}}} + D_{iy}\frac{\partial^{\beta_{iy}} c_i}{\partial y^{\beta_{iy}}}\right\} - \frac{\partial\left(v_{ix}c_i\right)}{\partial x} - \frac{\partial\left(v_{iy}c_i\right)}{\partial y} + \frac{\partial \varepsilon_i}{\partial t}, i = 1,2 \qquad (1)$$

$$\frac{\partial \varepsilon_i}{\partial t} = \omega_1\left(\varepsilon_{0i} - \varepsilon_i\right)\left|\nabla p_i\right| - \omega_2 \varepsilon_i c_i, i = 1,2 \qquad (2)$$

where C_i is the volumetric concentrations of solid particles in the liquid, $\varepsilon_{0i}, \varepsilon_i$ are the initial and current porosities, ω_1, ω_2 are the coefficients characterizing the intensity of colmotattion and suffusion of pores, $|\nabla p_i|$ are the absolute values of the pressure gradient p_i.

The filtration velocity components in R_1 and R_2 are defined as [2]

$$v_{ix} = -\frac{k_{ix}\left(\varepsilon_i\right)}{\mu}\frac{\partial^{\gamma_1} p_i}{\partial x^{\gamma_1}}, v_{iy} = -\frac{k_{iy}\left(\varepsilon_i\right)}{\mu}\frac{\partial^{\gamma_2} p_i}{\partial y^{\gamma_2}}, i = 1,2 \qquad (3)$$

where μ - fluid viscosity, $k_i\left(\varepsilon_i\right)$ are the permeability coefficients of the regions R_1 and R_2, which are functions of ε_i due to colmotation and suffusion effects .

Piezoconductivity equation is defined as follows

$$\frac{\partial p_i}{\partial t} = \chi_i^*\left(p_i\right)\left(\frac{\partial^{\gamma_{ix}+1} p_i}{\partial x^{\gamma_{ix}+1}} + \frac{\partial^{\gamma_{iy}+1} p_i}{\partial y^{\gamma_{iy}+1}}\right), \chi_i^*\left(p_i\right) = \chi_i\left(\varepsilon_{0i} + \beta_i^*\left(p_i - p_0\right)\right), i = 1,2 \qquad (4)$$

Problem (1), (2), (3), (4) is solved by the initial and boundary conditions by the finite difference method.

Based on the numerical results, the concentration of suspended particles, the porosity of the medium, the fields of filtration velocity and pressure are determined. The influence of orders of the fractional derivative and model parameters on the filtration characteristics of the medium is analyzed.

Keywords: Anomalous solute transport, filtration,colmotation, suffusion, fractional derivative, porous medium.
AMS Subject Classification: 76-10

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THE PROBLEM TO DETERMINE THE OPTIMAL SIZE OF CREDIT AND RENTED AREA IN THE PRODUCTION OF PRODUCTS

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Results of papers [1-3] are used in this article.

Formulation of the problem.

Farm does not have financial resources.

Farm has a cultivated area s. Farm plans to take a loan to grow n types of crops.

An agreement was concluded with a processing company for the sale of agricultural crops at an agreed price $c_j, j \in J = \{1, 2, ..., n\}$. The volume of production $x_j, j \in J$ is limited by the upper and lower bounds according to the agreement.

The paper developed a mathematical model for the problem to find:

the optimal amount of own arable land;

the optimal amount of rented arable land;

the necessary financial loan

for planting each crop with the criterion to maximize the net income of Farm.

A method to solve the problem is shown for the case when the volume of production is limited from both sides.

The adequacy of the mathematical model and the algorithm to solve the problem were checked on a specific example.

Keywords: mathematical model, sown area, agricultural crop, credit, economics, income, expense, production.
AMS Subject Classification: 91B86

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THE METHOD OF SUMMING DIVERGENT SERIES BY REDUCING TO A REPEATED SERIES

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Abstract. In this thesis, the problem of summing some divergent series by means of special repeated series is considered. If the sum of the series is determined differently from the traditional method, it is accepted to call it the generalized sum of the series.

It is known that divergent alternating series can be conditionally divided into the following three types:

1. Divergent series with a constant source: $\sum_{n=1}^{\infty} l \cdot (-1)^{n+1}$, $(l = const)$.

In this case the generalized sum of divergent series is equal to $\frac{l}{2}$;

2. Divergent series with a bounded source: $\sum_{n=1}^{\infty} a_n \cdot (-1)^{n+1}$, $\lim_{n\to\infty} a_n = l$, $l = const$;

3. Divergent series with an unbounded source: $\sum_{n=1}^{\infty} a_n \cdot (-1)^{n+1}$, $\lim_{n\to\infty} a_n = \infty$.

Theorem. The following equality holds:

$$\sum_{n=1}^{\infty}(-1)^{n+1} \cdot a_{n+1} = \frac{a_1}{2} + \frac{1}{2}\sum_{n=1}^{\infty}(-1)^{n+1} \cdot \Delta(a_n)$$

here $\Delta(a_n) = a_{n+1} - a_n$.

Using the theorem in the above, the sum of the following series can be easily find.

Example 1.

$$\sum_{n=1}^{\infty}(-1)^{n+1}(an^2 + bn + c) = \frac{b+2c}{4}.$$

Example 2.

$$\sum_{n=1}^{\infty}(-1)^{n+1} \cdot \alpha^n = \frac{\alpha}{\alpha+1}, \quad \alpha \geq 0.$$

Example 3.

$$\sum_{n=1}^{\infty}(-1)^{n+1} \cdot n \cdot \alpha^n = \frac{\alpha}{(\alpha+1)^2}, \quad \alpha \geq 0.$$

Keywords: divergent series, generalized sum, repeated series, alternating series.

AMS Subject Classification: 40A30, 40B05.

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TO NUMERICAL MODELING OF NONLINEAR DIFFUSION PROCESSES

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In this talk, we consider the following Cauchy problem

$$|x|^{-n}\frac{\partial u}{\partial t} = div(a(t)|x|^l[\left|\nabla u^k\right|^{p-2}\nabla u^m],\ (x \in R^N,\ t > 0)u(0,x) = u_0(x) \geqslant 0, \qquad (x \in R^N) \quad (1)$$

where

$$n > 0,\ l > 0$$

- numbers

$$p \geqslant 2,\ \mathrm{k}, m \geqslant 1$$

characterized nonlinear medium. This class of equation contains the linear diffusion equation

$$p = 2,\ m = 1$$

the nonlinear diffusion equation

$$p = 2,\ m > 1$$

known as the porous medium or the fast diffusion equation

$$p\ = 2,\ m < 1$$

and the gradient-dependent diffusion equation

$$p \neq 2,\ m =\ 1$$

that is the p-Laplacian equation when

$$p \neq 2$$

and

$$m \neq 1$$

Equation (1) - is called the doubly nonlinear diffusion equation with variable density, because its diffusion term depends on non-linearly on both the unknown density and its gradient

$$\nabla u$$

Different qualitative properties of the problem (1) intensively studied by many authors (see [1-4] and literature therein). In the research the qualitative properties such as an estimate of a weak solution and a free boundary, condition of a space localization, a finite speed of perturbation, asymptotic of self-similar solutions, depending on case

$$k\,(p-2) + m > 0$$

$$k\,(p-2) + m < 0$$

and critical case

$$k\,(p-2) + m = 0$$

for case

$$p > l + n$$

and

$$p = n + l$$

separately studied. In particular it is showed that condition of a space localization is

$$k(p-2) + m > 0, \int_0^\infty a(y)dy < \infty.$$

The problem a choosing an initial iteration for numerical analysis of solution to all considered cases when

$$p > l + n$$

in singular case

$$p = l + n$$

solved. The Zeldovich Barenblatt type fundamental solution when

$$u_0(x) = M\delta(x),\ M > 0$$

where

$$\delta(x)$$

-Dirac's delta function constructed. It is obtained an exact value of the front perturbation expressed via beta function. Results of numerical experiments showed an effectivity of suggested approach keeping new nonlinear effects.

Keywords: Cauchy problem, perturbation, numerical experiments, nonlinear effects, space localization, free boundary.

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MATHEMATICAL MODEL OF THE PROBLEM OF OPTIMIZING PRODUCTION TAKING INTO ACCOUNT RENTAL ACREAGE BY THE CRITERION OF MAXIMUM

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The paper considers one of the approaches to the organization of specialized household. To find the maximum

$$L(x,y,z) = \sum_{h\in H} a^h z^h - (\sum_{k\in K}\sum_{j\in J_0} c_{kj}x_{kj} + \sum_{h\in H}\sum_{l\in L} c_l^h y_l^h) - \sum_{k\in K}\varepsilon_k v_k \tag{1}$$

on the condition $\sum_{j\in J_0} x_{kj} \le s_k + v_k,\ k \in K\cup\bar{K},\ \sum_{k\in K} a_{kj}x_{kj} = \sum_{h\in H}\sum_{l\in L}\alpha_{jl}^h y_l^h, j \in J_0,\ \sum_{l\in L}\theta_l^h y_l^h = z^h, h \in H,\ \sum_{k\in\bar{K}}\varepsilon_k v_k \le D,\ 0 \le V_k \le R_k, k \in K,\ x_{kj} \ge 0, k \in K, j \in J_0, z^h \ge 0, h \in H, y_l^h \ge 0, h \in H, l \in L$, where j – is index of the type of crop production used in the daily diet of animals, $j \in J_0 = 1,2,\ldots,n$; k – is index of the category of acreage in the household, $k \in K$; h– is index of the type of livestock products produced on the household, $h \in H$; l – is index of the type of animal breed in the household, $l \in L$; s_k – is the size of the sown area of k-th categories in the household, $k \in K$; a_{kj}- is yield of j-th the type of culture on k-th categories of acreage of the household, $k \in K$, $j \in J_0$; α_{jl}^h - is annual demand of crop production of j-th types per animal of l-th breeds in production of h-th types of products, where $\alpha_{jl}^h = \beta_{jl}^h\gamma_{jl}^h,\ j \in J_0,\ l \in L,\ h \in H$; β_{jl}^h- is share of j -th crop production in the daily diet per animal of l-th breeds for production of h- th types of products, $j \in J_0, l \in L, h \in H; \gamma_{jl}^h$ - is the number of days in the diet of feeding with plant products of j -th type of l-th animal breeds in production of h-th types of products, $j \in J_0, l \in L, h \in H; \theta_l^h$ - is production volume of h-th of the type received by the household from one animal l-th types, $l \in L, h \in H; \varepsilon_k$ - is unit fee of k-th leased acreage, $k \in K; R_k$- maximum possible size k type of leased acreage, $k \in K; D$ - is the financial ability of the household to pay for rented space; a^h – is the selling price of the h-th type of livestock products on the household, $h \in H$; c_{kj} – are costs per unit size of k-th categories of acreage under j-th type of culture, $j \in J_0, k \in K; c_l^h$- is annual consumption per animal of l-th breeds in production of h-th types of livestock products, $h \in H, l \in L; x_{kj}$ – is size of k -th categories of acreage allocated for j-th type of culture, $j \in J_0, k \in K; y_l^h$ - is number of animals of l-th breeds on the household for production h-th types of products, $h \in H, l \in L; v_k$ – is size of k-th categories of leased acreage, $k \in K; z^h$ – the volume of livestock products sold of h-th type, $h \in H$.

MATHEMATICAL MODEL OF THE PROBLEM OF CONSUMER BEHAVIOR IN THE MARKET

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In this paper, we examine a crucial issue in modern economics: the behavior of buyers in the consumer market. The proposed model for understanding consumer behavior in the market distinguishes itself from other models by utilizing a quadratic utility function.

The application of the Lagrange multipliers method in the model of consumer behavior enables us to determine the buyer's demand function. Based on the found buyer demand function, you can get the results of the research, i.e. information about the products in question. We provide the necessary definitions and notations associated with this model.

Within this buyer behavior model in the consumer market, the goods under consideration are consistently valuable and lack a gross substitute.

The significance of these obtained results lies in the model's ability to predict consumer purchasing power. Through the utilization of mathematical tools, the proposed model can be generalized to accommodate an arbitrary number of goods.

Keywords: consumer behavior, mathematical model, market, valuable goods, gross substitution, demand function.

AMS Subject Classification: 91B42

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MATHEMATICAL MODELING AND ANALYSIS OF THE EXTREME DEFORMABILITY OF THE BOARD ROCK MASS

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Establishing the optimal parameters of open pit walls in the development of minerals in an open way is necessary to ensure the safety of mining operations, the safety of facilities and structures, transport and energy communications, mining and transport equipment, the completeness of the extraction of minerals and, finally, the high economic performance of mining enterprises.

Numerous natural observations of quarry wall deformations and studies of slope deformations on a model made of equivalent materials have established that the tool mass before collapse undergoes complex deformation: horizontal physical, vertical compression, shear. With a slope stability factor close to the limit and stress, to the maximum creep limit of rocks, deformations develop smoothly. In this paper, the results of assessing the state of slopes by stress, obtained on the basis of finite element methods, taking into account the elastic-plastic solution of the problem posed [1], are revealed.

Based on the analysis of the results of the tasks being solved in the work, taking into account the limiting deformability of the instrumental rock mass, it was found that in the models of the elasticity of the medium with the concentration of the pit wall, the shifts of the slope points and the surface of the slope zone are directed upward due to the elasticity of the restoration when the weight of the extracted layers is removed, and in models with taking into account inelastic disorders with a severe degree of career severity.

As a result of the analysis of the nature of the slope deformation in the development of the excavation for models of an elastic medium, an ideal plastic medium and a hardening medium, as well as with a comparison of laboratory modeling data, it was found that: - numerical analysis for the elastic medium model predicts that with the deepening of the excavation, the shifts of the slope points and the surface of the near-slope zone are directed upward due to elastic recovery when the weight of the layers being removed is removed; - in elastic-plastic solutions, the rise of the slope surface and the slope zone, as in the real model, is observed only at the first stages of excavation; from a certain depth, plastic deformations begin to prevail over elastic recovery deformations and the direction of displacements changes.

Keywords: Mathematical modeling, extreme deformability, board rock mass.

AMS Subject Classification: 74-10, 86-10, 74S05

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NUMERICAL STUDY OF THE TEMPERATURE VALUE AT NODAL POINTS OF A COMPLEX-SHAPED BODY

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In this article, the values of the temperature at the nodal points of the body of a complex shape are determined by the method of minimization based on the law of conservation of energy. In the process of determining their values, a calculation algorithm is created using a new mathematical model, and numerical studies are conducted based on temperature distribution. One part of this complex body is isolated from heat, one part receives heat flow, and another part has heat exchange with the external environment. The body is affected by several heat sources at the same time. The sources of heat here are heat flow and heat exchange (convective), temperature and heat insulation (insulation), heat conduction. Due to the size and configuration of the main structural elements, the distribution of temperature fields in its different parts is not uniform. Therefore, it is important to determine the value of the temperature in each part of the structural element (nodal points). This is because the body may be physically unusable (destroyed) due to the temperature in some part of the structural element. In order to understand the consequences of this and to save the main structural elements of the installations from damage, it is necessary to perform preliminary mathematical calculations and calculate the value of the temperature at each node point. In the article, the patterns of temperature distribution at the node points of the channel-like body (beam) taken as a structural element are determined and it is determined that the temperature distribution is symmetrical depending on the shape of the body.

Keywords: complex shaped body (beam), nodal point, temperature distribution laws, numerical study, heat flow, heat exchange (convective).

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APPLICATIONS OF THE METHOD OF TOPOLOGICAL ROUGHNESS TO THE STUDY OF ROUGHNESS, BIFURCATIONS AND CHAOS IN SYNERGETIC SYSTEMS

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In the theory of dynamical systems, there are two different approaches to the problem of roughness:

1) on the basis of the concept of roughness according to Peixoto or otherwise "structural stability";

2) based on the concept of roughness according to Andronov - Pontryagin, when, unlike the previous one, ε-proximity of the original and perturbed homeomorphisms is required [1 - 3].

In [4], on the basis of the concept of roughness according to Andronov - Pontryagin, the foundations of the "topological roughness method"were laid, which makes it possible to study the roughness (robustness) and bifurcations of dynamic systems of various nature, in particular, synergetic systems, as well as to synthesize rough (robust) control systems [5, 6].

Many fundamental results in the theory of roughness and bifurcation were obtained by A.A. Andronov and his school [1, 2].

In [4], on the basis of the concept of roughness according to Andronov-Pontryagin, the foundations of the “topological roughness method” are proposed based on the measure of roughness in the form of a condition number CM – matrices M - a normalized matrix for reducing the system to a canonical diagonal (quasi-diagonal) form at singular points of the phase space. Here, for the first time, the concept of maximum roughness and minimum non-roughness of systems is introduced.

The topological roughness method also makes it possible to determine the bifurcations of dynamical systems based on the criteria developed in the author’s works [5, 6]. Moreover, the method presents the possibilities of bifurcation prediction as well as control of bifurcation parameters.

Applications of the method to the study of roughness, bifurcations and chaos in synergetic systems have been used to study many systems, such as attractors of Lorenz and Rössler, Belousov-Zhabotinsky, Chua, "predator-prey Henon systems, Hopf bifurcations, etc. [5, 6] .

Keywords: synergetic systems, topological roughness, bifurcations, chaos, condition number of matrices.

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GENERALIZED STEFAN PROBLEM FOR THE MATHEMATICAL MODELING OF THE ELECTRICAL ARC TRANSITION FROM METALLIC TO GASEOUS PHASE

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The investigation of phenomena of the electrical arc transition from metallic to gaseous phase is very important for the optimal choice of parameters of switching apparatus. The initial arc stage is evaluated by vapors of ruptured bridge in the contact gap. Cathode and anode phenomena such as ion bombardment, thermoelectronic emission, inverse electrons flux, evaporation, radiation, heat conduction etc. are considered in dependence on time, current, opening velocity, parameters of the gas and contact materials. The conditions of the arc transition from one phase to another are formulated in terms of above characteristics and increasing of gas ionization level. This approach enables one to explain the transition from the metallic anode dominated arc to the gaseous cathode dominated arc and to estimate corresponding arc erosion during each phase. The mathematical model is based on the special Stefan problem for the short electrical arc, liquid and solid contact zones with nonlinear boundary conditions depending on the unknown temperature. The Stefan condition is written in the generalized form. It contains the special ejection coefficient which takes into account the ejection of a portion of the liquid metal from the melt due to the action of electromagnetic force, metallic vapor pressure and Marangoni effect. The values , and are in keeping with two-phase, one-phase and the phase with a partial ejection of the liquid phase correspondingly. To solve this problem, an iterative method of majorant functions has been developed, which makes it possible to obtain sequences of overestimated and underestimated temperatures in the liquid and solid zones of the contact area, as well as the corresponding values of the erosion values.

Keywords: Transient vacuum arc phenomena, mathematical modeling, anode and cathode dominated arcs.

AMS Subject Classification: 80A22, 35A35

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ON ESTIMATION OF DISTRIBUTION FUNCTION BY PRESMOOTHED RELATIVE-RISK FUNCTION

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In paper we present some comparison of distribution function estimators when the lifetime data subjecting to right random censoring. Nonparametric estimators based on conception of presmoothed estimation of relative-risk function. We give some numberical results also.

Keywords: Random, censoring, product-limit, relative risk, presmoothed, proportional hazards, asymptotic representation, strong consistency.

AMS Subject Classification (MSC 2020): 62N02, 62N05 (https://zbmath.org/static/msc2020.pdf).

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NON-CLASSICAL CYLINDRICAL STEFAN PROBLEM WITH A SOURCE (SINK) OF HEAT ON THE AXIS OF THE CYLINDER

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In this study we consider mathematical model of the heat process on the surface of the electrical contact material with heat source on the axis of the cylinder involving one-dimensional cylindrical heat equation with nonlinear thermal coefficients. The method of solution based on similarity variable considered in [1],[2]. Temperature distribution in liquid, solid phases and free boundary which describes the position of the melting interface between two phases are determined. The mathematical model of the problem is

$$c(\theta_i)\gamma(\theta_i)\frac{\partial\theta_i}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[\lambda(\theta_i)r\frac{\partial\theta_i}{\partial r}\right],\quad i=1:\quad 0<r<\alpha(t),\quad i=2:\quad \alpha(t)<r<\infty, \tag{1}$$

$$\lim_{r\to 0}\left(-2\pi r\lambda(\theta_1)\frac{\partial\theta_1}{\partial r}\right)=Q_0, \tag{2}$$

$$\theta_1(\alpha(t),t)=\theta_2(\alpha(t),t)=\theta_m, \tag{3}$$

$$-\lambda(\theta_1)\frac{\partial\theta_1}{\partial r}\bigg|_{r=\alpha(t)}=-\lambda(\theta_2)\frac{\partial\theta_2}{\partial r}\bigg|_{r=\alpha(t)}+l\gamma\frac{d\alpha}{dt}, \tag{4}$$

$$\theta_2(\infty,t)=0, \tag{5}$$

$$\theta_1(0,0)=\theta_2(r,0)=0,\quad \alpha(0)=0, \tag{6}$$

where $\theta_1(r,t)$ and $\theta_2(r,t)$ are temperature distribution in liquid and solid zones, $c(\theta_i),\gamma(\theta_i)$ and $\lambda(\theta_i)$ are specific heat, material's density and thermal conductivity depended on temperature, θ_m is a melting temperature, $Q_0>0$ is a axial heat source power, $l>0$ and $\gamma>0$ are latent heat and density of material at melting temperature, $\alpha(t)$ - free boundary describing the location of the melting interface.

Keywords: Stefan problem, nonlinear thermal coefficients, similarity substitution, melting, heat transfer.

AMS Subject Classification: 80A22, 80A23, 35C11.

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INVESTIGATION OF $\gamma * + \gamma \longrightarrow \eta(\eta')$ TRANSITIONS USING THE MESONS INFRARED RENORMALON CORRECTED DISTRIBUTION AMPLITUDES

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In this work the pseudoscalar $\eta(\eta')$ mesons transition form factors $F_{\eta(\eta')\gamma}$ [3-6]

$$F_{\eta(\eta')\gamma}\left(Q^2\right) = \int_0^1 dx \Phi_{\mathrm{M}}\left(x,\ \mu_F^2\right) T_{\mathrm{H}}\left(x, Q^2, \mu_F^2, \mu_R^2\right)$$

are calculated using the frozen coupling constant approximation and the mesons' infrared renormalon corrected distribution amplitudes:

$$\Phi_M\left(x, Q^2\right) = f_M [x(1-x)]^{1+\alpha} \sum_{n=0}^{\infty} b_n\left(Q^2\right) A_n\left(\alpha_S\right) C_n^{3/2+\alpha}(2x-1).$$

In calculations the usual $\eta - \eta'$ mixing scheme

$$|\eta\rangle = cos\theta\, |\eta_8\rangle - sin\theta\, |\eta_1\rangle\ ,$$

$$|\eta'\rangle = sin\theta\, |\eta_8\rangle + cos\theta\, |\eta_1\rangle$$

is employed. As input parameters the phenomenological values of the octet-singlet mixing angle $\theta = -15.4o$ and of the decay constants $f_1 \approx 0.108\ GeV$ and $f_8 \approx 0.116\ GeV$ are used. Comparison is made with the CLEO Collaboration data. The obtained results were comparised with results of [1,2].

Keywords: transition form factor, meson, distribution amplitudes, renormalon.

AMS Subject Classification: 81T15, 81T17.

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ON THE BEHAVIORS OF SOLUTIONS OF A NONLINEAR DIFFUSION SYSTEM WITH A SOURCE AND NONLINEAR BOUNDARY CONDITIONS

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We considered the doubly nonlinear degenerate parabolic equations with the source

$$\frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x}\left(\left|\frac{\partial u_i^k}{\partial x}\right|^{m-1}\frac{\partial u_i^k}{\partial x}\right) + u_i^{p_i},\ x \in R_+, t > 0,\ i = 1,\,2, \tag{1}$$

coupled through nonlinear boundary conditions:

$$-\left|\frac{\partial u_i^k}{\partial x}\right|^{m-1}\frac{\partial u_i^k}{\partial x}\bigg|_{x=0} = u_{3-i}^{q_i}(0,t), \qquad t > 0,\ \ i = 1,\,2, \tag{2}$$

where $m > 1$, $k \geq 1$ and q_i, $p_i > 0$ are numerical parameters. The following initial data should be taken into account

$$u_i|_{t=0} = u_{i0}(x),\ \ i = 1,\,2. \tag{3}$$

It is expected that the function and its corresponding first- and second-order derivatives conform to a set of criteria. Specifically, these derivatives should exhibit a degree of continuity, non-negativity, and compactness within the domain of R_+. Through the creation of numerous self-similar supersolutions and subsolutions, researchers have been able to derive the curve of critical global existence and Fujita type critical exponent. These self-similar solutions have proved to be invaluable in understanding the behavior of nonlinear diffusion systems.

Theorem 1. If $q_1 q_2 \leq \left(\frac{m}{m+1}\right)^2 (k+1-p_1)(k+1-p_2)$, then every nonnegative solution of the problem (1)-(3) is global in time.

Theorem 2. If $0 < p_i \leq 1$ and $q_i \geq \frac{m(p_{3-i}-1)(p_i+k)}{(p_i-1)(m+1)}$ or $p_i > 1$ and $q_i \leq \frac{m(p_{3-i}-1)}{(p_i-1)(m+1)} \times (p_i+k)$ then, each of the solutions to (1)-(3) blows up.

Keywords: nonlinear boundary condition; critical global existence curve; degenerate parabolic systems.

AMS Subject Classification: 35A01, 35B44, 35K57, 35K65

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A MATHEMATICAL MODEL OF TWO-PHASE FILTRATION IN A POROUS MEDIUM TAKING INTO ACCOUNT ITS DEFORMATION

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A mathematical model describing the mutual influence of a fluid flow and a change in the stressstrain state of a solid skeleton was first proposed in [1] to calculate the clay permeability coefficient. In this work, an effective stress tensor was introduced, which depends on the deformation of the skeleton and the fluid pressure. The basic equations of the mathematical model for describing the processes in a consolidated porous medium (the poroelasticity model) were then formulated in [2, 3].

Consider the filtration of oil and water in a homogeneous deformable oil reservoir penetrated by a system of producing wells. The wells are grouped into rows. We assume that oil and water are incompressible. A quasi-one-dimensional mathematical model of the filtration process in this system can be represented as follows.

Mass conservation law for phases

$$A\frac{\partial}{\partial t}(ms_\alpha)+\frac{\partial(Au_\alpha)}{\partial x}=A\sum_{k=1}^{n}Q_{\alpha k}\delta_k, \quad \alpha=1,2, \tag{1}$$

where s_α is saturation of phase $\alpha, m = m_0+\beta_m(p-p_0)$ is a porosity relation, m_0 is initial porosity, β_m is reservoir elasticity coefficient, p is reservoir pressure, p_0 is critical pressure, u_α is velocity of phase $\alpha, A=b(x)\cdot H(x)$ is cross section of the element, $b(x)$ is element width, $H(x)$ is reservoir thickness, δ is Dirac delta function, $Q_{\alpha k}$ are the debits of phases of the k-th row reduced to standard conditions, k is well row number, n is number of wells k th row, $\sum_\alpha s_\alpha=1$. Here $\alpha=1$ means oil phase, $\alpha=2$ is water phase.

(1) system of equations is solved on the initial and boundary conditions using large particles method and obtained results are analyzed.

Keywords: phase, deformation, mass conservation, filtration, saturation.

AMS Subject Classification: 76-10

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ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ЗАДАЧИ СТЕФАНА ДЛЯ СТАНДАРТНЫХ ОПЦИОНОВ ПОКУПАТЕЛЯ И ПРОДАВЦА

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В работе рассмотрены некоторые особенности расчетов цены опциона $V(t,x)$, цены акции $x(t)$ и оптимального момента τ ($\equiv t$) остановки (исполнения) на конечных и бесконечных временных интервалах. Затем рассматривается задача о нахождения рациональной цены опционов Американского типа за оптимальное время остановки (оптимальный момент остановки) на диффузионных (B, S)-рынках акций. Далее рассматривается задача о нахождения рациональной цены опционов Европейского типа. Сначала опцион рассматривается с точки зрения покупателя – опцион покупателя, затем – опцион продавца. Все рассматриваемые задачи сводятся к задаче Стефана. В математической физике задача Стефана возникает при изучении физических процессов, связанных с фазовым превращением вещества и состоит в том, чтобы найти функцию $u = u(t,x)$, описывающую температурный режим фаз, и границу $x = x(t)$, $t \geq 0$ разделения этих фаз.

В случае стандартных опционов покупателя и продавца также имеет место двухфазная ситуация – при отыскании оптимальных правил остановки можно ограничиться рассмотрением лишь двух односвязных фаз: области продолжения наблюдений C^T и области D^T. Все задачи решаются точно, если заранее найден оптимальный момент остановки или численно. Приведены результаты численного моделирования задачи Стефана методом прогонки и методом конечных элементов для стандартных опционов покупателя и продавца. А также сравнительный анализ численных результатов методом прогонки и методом конечных элементов.

Ключевые слова: цены опциона, цены акции, диффузионные рынки акций, опционы Американского и Европейского типов, задача Стефана, численное моделирование.

Предметная классификация AMS: 35R60, 39A50, 60G40, 65M06, 65M08

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D–ОПТИМАЛЬНЫЕ ПЛАНЫ В СЛУЧАЕ ФУНКЦИЙ ХААРА

АБИЛМАЖИН АДАМОВ, МАРС ГАББАСОВ, АЗАМАТ КУСЕБАЙ

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В статье рассматривается проблема проверки заданного плана на D-оптимальность. При этом в качестве базисных функций рассматриваются кусочно-постоянные функции – функции Хаара, и формулируются теоремы о распределении точек оптимального плана для D-критериев оптимальности. Рассмотрим линейную по параметрам регрессионную модель, в которой предполагается, что результаты измерений $y(x_j)$ в точках $x_j \in \chi$ представляются в виде

$$y_j = y(x_j, \omega_j) = \sum_{i=1}^{n} \theta_i \varphi_i(x_j) + \epsilon(x_j, \omega_j), j = 1, ..., N, \tag{1}$$

где $\varphi(x) = \{\varphi_1(x), ..., \varphi_m(x)\}^T$ – вектор базисных функций на χ, в нашем случае функции Хаара, θ_i - нейзвестные параматры, $\epsilon(x_j, \omega_j)$ - случайные ошибки, χ - множество планирования. Об ошибках предполагаются стандартные допущения. Регрессионная модель записывается в виде $(Y, F\theta, \sigma^2 I_n)$.

Пусть $k_j (j = 1, ..., n)$ количество точек, принадлежащих подмножеству $d(n, j_n)$ множества χ, где определяются функции Хаара и $\sum_{j=1}^{n} k_j = N$.

Как известно, если матрица системы нормальных уравнений $F^T F$ невырождена, то оценка МНК и дисперсионная матрица имеют вид

$$\hat{\theta} = (F^T F)^{-1} F^T Y, D\hat{\theta} = \sigma^2 (F^T F)^{-1} \tag{2}$$

Матрица $M = F^T F$ называется информационной матрицей для модели (1). Непосредственное вычисление определителя матрицы $F^T F$ дает

$$\Delta^2(Q) = det(F^T F) = C k_1 k_2 ... k_n, \tag{3}$$

и для невырожденности матрицы $F^T F$ необходимо и достаточно, чтобы $K_j \geq 1, j = 1, ..., n$.

Сформулированы теоремы о D–оптимальности плана в следующих случаях:
A) количество точек N кратно n, т.е. $N = l \cdot n$. Тогда определитель (3) достигает своего максимального значения, когда $k_1 = k_2 = ... = k_n$.
B) количество точек N не является кратным числу разбиения n, т.е. $N \neq l \cdot n$. Получено, что в оптимальном плане все k_i отличаются друг от друга не более, чем на единицу

ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССА ПОДЗЕМНОГО ВЫЩЕЛАЧИВАНИЯ ЦВЕТНЫХ МЕТАЛЛОВ

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В работе рассматривается численное моделирование процесса подземного выщелачивания цветных металл, на основе построенных вычислительных схем, где металлическая руда находится на глубине сотен метров в осадочных проницаемых породах и представляет собой мелкие зерна чистого металла или же их твердых соединений. Эти данные соединении, рассеянные на стенках пор, часто неравномерно по пространству, образуя высококонцентрированные зоны [1-2]. Во время добычи заливаемый водный раствор переносится по пласту, рефлектирует с металлом, позже растворяет его и выносится через откачные скважины. На поверхности происходит отделение чистого металла от раствора, где сама растворитель зависит от свойств и типа металла.

Разработанное численное моделирование для выщелачивания меди серной кислотой и трехвалентным железом учитывает растворение шестивалентного урана из окисла. На основе разработанной цифровой модели медь и четырехвалентный уран растворяются ионами трехвалентного железа. Данная приложение применительно к урану, к меди, а также к редким металлам. В данной разработанной цифровой модели описываются системой уравнений переноса с реакцией в пористых средах, где включены химические реакции как растворение руд металлов. А также учитывается вредные реакции как выпадение гипса снижающие поверхность контакта с рудой и нейтральные реакции, которые не производят негативного эффекта на выщелачивание, но на которые уходит часть растворителя.

Рассматриваемая численное моделирование для выщелачивания цветных металлов дает возможность прогнозировать дополнительную добычу металлов из непроточных зон, объем которой сравним с добычей в проточной зоне, тем самым получить данные о параметрах запаздывания решением обратных задач переноса и сопоставлением с экспериментальными данными.

Работа выполнена при финансовой поддержке гранта КН МНВО РК №AP09261179

Ключевые слова: численное моделирование, цифровые модели, пористые среды, выщелачивание.

Предметная классификация AMS: 86-10

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ДВУСТОРОННИЕ ОЦЕНКИ В МЕТОДАХ ФИКТИВНЫХ ОБЛАСТЕЙ

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Рассмотрим применение метода фиктивных областей для среды Максвелла. Получены двусторонние оценки по малому параметру α сходимости приближенного решения к точному решению. Рассмотрим постановку динамической вязкоупругой несжимаемой среды, построенной на основе модели Максвелла: в цилиндре $Q = \{D \times [0 \leq t \leq t_1]\}$ где $D \subset R^3$ ограниченная односвязная область с достаточно гладкий границей γ Вязкоупругая среда Максвелла является моделью, которая объединяет вязкость и упругость в материале. Она описывается с помощью уравнений, связывающих скорости деформации и соответствующие напряжения. Эти уравнения являются дифференциальными уравнениями, которые описывают реологическое поведение материала. Для определения материальных параметров вязкоупругой среды Максвелла требуются экспериментальные данные, которые могут быть получены, например, с помощью испытаний на растяжение или сжатие материала. Экспериментальные данные включают измерения скоростей деформации и соответствующих напряжений при различных условиях нагружения. Двусторонний метод оценки заключается в поиске материальных параметров, которые обеспечивают наилучшее соответствие между экспериментальными данными и результатами моделирования вязкоупругой среды Максвелла. Это достигается путем решения обратной задачи, которая заключается в нахождении оптимальных параметров модели, удовлетворяющих заданным экспериментальным данным. При использовании двустороннего метода оценки важно учитывать ограничения модели вязкоупругой среды Максвелла и возможные предположения, которые могут влиять на точность и достоверность оценок. Этот метод позволяет улучшить понимание реологического поведения материалов и применить их в различных инженерных приложениях.

Ключевые слова: метод фиктивных областей, скорости-напряжения, малый параметр.

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АНАЛИЗ И ОЦЕНКА АНАЛИТИЧЕСКИХ РЕШЕНИЙ МАТЕМАТИЧЕСКИХ МОДЕЛЕЙ ТАЯНИЯ МЕРЗЛОГО ГРУНТА

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Исследование температурного режима сооружений, расположенные в условиях вечной мерзлоты приведены в работах авторов [1,2]. Интересным оставалась вопрос - через какое времени процесс таяние перейдет в стационарный режим и какое будет величина глубины таяния за время установление процесса. Ответы на эти вопросы находятся через аналитические решения математических моделей процесса таяния мерзлого грунта.

В данной работе рассмотрены различные варианты математической модели, описывающие процесс таяния мерзлого грунта под основанием водоема и построены аналитические решения в виде двух слагаемых, отражающее стационарную и нестационарную часть исследуемого процесса. Оценка решений математических моделей проведен на основе изменения вида граничных и начального условия построением температурного поля до перехода в стационарный режим. Изменение температурного поля во времени и в пространстве приведены в виде графиков, в которых хорошо изображено процесс установление. Определено время установление процесса таяние и глубина таяния за время установление. На основе аналитико-численного эксперимента решений математических моделей сделаны следующие выводы: 1. изменение начального условия не влияет на время установление процесса и на глубину таяния[3]; 2. изменение вида граничного условия на нижней границе значительно влияет на время установление и на глубину таяния; 3. При долгосрочных прогнозах численные результаты трех разных математических моделей: модель учета теплообмена скелета грунта с водой в зоне талого грунта; модель определение подвижной границы между талым и мерзлым грунтом через условие Стефана; модель кондуктивного теплопереноса в заданной области, где граница таяния мерзлого грунта определяется местоположением нулевой изотермы, очень грубо не отличаются между собой. Поэтому при долгосрочных прогнозных расчетах процесса таяния можно отдать предпочтение модели кондуктивного теплопереноса, где используется только коэффициента температуропроводности среды. В других математических моделях требуется знание значения коэффициента теплообмена, содержание льда в мерзлом грунте, количества незамершей воды, влажности грунта, которые определяются экспериментально для каждого грунта отдельно.

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ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ДИНАМИКИ ПОГРАНИЧНОГО СЛОЯ АТМОСФЕРЫ, ПЕРЕНОСА ВРЕДНЫХ ПРИМЕСЕЙ И ДИФФУЗИИ

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Слабое проведение природоохранных мероприятий привели к сильному загрязнению атмосферного воздуха промышленных городов Республики Казахстан. Применяемые несовершенные технологии в теплоэнергетике, цветной и черной металлургии, нефтедобывающей и угле-добывающей отраслях, строительстве, авто- и железнодорожном транспорте являются причинами загрязнения атмосферного воздуха в Казахстане. В рамках выполняемой Программы проведены научно-исследовательские работы с использованием новых эффективных алгоритмов анализа больших данных, адаптированной к выбранному промышленному городу математической моделью переноса и трансформация примесей в атмосфере используя признанные международные индексы оценки качества атмосферы.

Разработанная геоинформационная система (ГИС) предназначенная для мониторинга атмосферы промышленных городов функционирует в режиме реального времени. Заложенный в основу метод позволит усваивать данные загрязнения атмосферного воздуха промышленных городов поступающих от экологических постов установленных в наиболее загрязненных районах города с учетом рельефа местности, гидрометеорологических данных. Геоинформационная система разработана с применением методов интеллектуального анализа данных, экспертной системы основанный на искусственном интеллекте и алгоритмах машинного обучения, современных инструментариев информационно-коммуникационных технологии (ИКТ) взаимосвязанный с погодными условиями.

Данные сети мобильных и стационарных датчиков мониторинга атмосферного воздуха ТОО «Экосервис-С» передаются в онлайн режиме на информационную систему, находящуюся на собственном сервере НИА РК размещенном в Дата-центре ТОО «Академсеть» с центром обработки данных (ЦОД) и обработанные данные отображаются на LED-экранах города. Для информирования населения создан специальный сайт ситуационного центра мониторинга и прогнозирования загрязнения атмосферного воздуха промышленного города и дополнительно будут использованы LED –экраны установленные на улицах города. На сайте и на LED -экранах будет круглосуточная информация об уровне загрязнения, определенных автоматическими экологическими постами и результаты более глубокого анализа проведенных в аккредитованных лабораториях и для сравнения ПДК.

Ключевые слова: Численное моделирование, геоинформационная система, анализ больших данных.

AMS Subject Classification: 68N01

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Computational and information technologies

Вычислительные и информационные технологии

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AN OPTIMAL QUADRATURE FORMULA IN THE SPACE $K_2^{(m)}$

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This paper studies the problem of construction of optimal quadrature formula in the sense of Sard for numerical calculation of Fourier integrals. Using the S.L.Sobolev's method we obtain new optimal quadrature coefficients. Moreover, explicit formula for the coefcients of the optimal formula is obtained. The obtained optimal quadrature formula is exact for hyperbolic functions. The accuracy of this optimal quadrature formula improves as the frequency of oscillations increases. Here, initially, in order to obtain a sharp upper bound for the error of the quadrature formula, the norm of the error functional is calculated. For this, the extremal function of the error functional for the quadrature formula is used. Then, by minimizing the norm of the error functional with respect to the coefficients, an optimal quadrature formula is obtained. Using the explicit form of the optimal coefficients, the norm of the error functional of the optimal quadrature formula is calculated.

Keywords: Error functional, the extremal function, optimal coefcients, optimal quadrature formula, oscillating functions.

OPTIMAL FORMULAS FOR THE APPROXIMATE SOLUTION OF THE GENERAL ABEL INTEGRAL EQUATIONS IN THE HILBERT SPACE

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In this paper, we consider the following integral equation

$$\int_0^x \frac{y(t)}{(x-t)^{\alpha}}dt = f(x), \tag{1}$$

where $0 < \alpha < 1$, $t \in [0;x)$, $x > 0$, $f(x)$ is a given function (sufficiently smooth function) and $y(x)$ is unknown function.

The analytical solution of equation (1) has the following form

$$y(x) = \frac{\sin(\pi\alpha)}{\pi} \cdot \left[\frac{f(0)}{x^{1-\alpha}} + \int_0^x \frac{f'(s)ds}{(x-s)^{1-\alpha}} \right]. \tag{2}$$

In the present paper, our goal is to calculate the integral in expression 2 with high accuracy. To do this, we calculate the approximate value of the definite integral using the quadrature sum.

Now we construct a quadrature formula in the space $W_2^{(2,1)}(0,t)$ of the form

$$\int_0^t \frac{\varphi(x)dx}{(t-x)^{1-\alpha}} \cong \sum_{\beta=0}^{N} C_\beta \varphi(h\beta), \tag{3}$$

where $0 < \alpha < 1$, C_β are the coefficients of formula (3), $h\beta$ are nodes, $h = \frac{t}{N}$, N natural number, $x \in [0,t)$, $t > 0$, $\varphi(x) \in W_2^{(1,0)}(0,t)$.

The function $\varphi(x)$ is an element of the space $W_2^{(2,1)}(0,t)$, which is defined as follows

$$W_2^{(2,1)}(0,t) = \left\{ \varphi \,|\, \varphi : [0,t] \to \mathbb{R},\ \varphi' - \text{absolutely continuous}\,,\ \varphi'' \in L_2(0,t) \right\}.$$

The difference between this integral and the quadrature sum is called the **error of the quadrature formula** (3)

$$(\ell, \varphi) = \int_0^t \frac{\varphi(x)dx}{(t-x)^{1-\alpha}} - \sum_{\beta=0}^{N} C_\beta \varphi(h\beta).$$

This difference corresponds to **error functional** of the form

$$\ell(x) = \frac{\varepsilon_{[0,t]}(x)}{(t-x)^{1-\alpha}} - \sum_{\beta=0}^{N} C_\beta \delta(x - h\beta),$$

where $\varepsilon_{[0,t]}(x)$ is the characteristic function of the interval $[0,t]$, $\delta(x)$ is the Dirac's delta function.

In this case, the main problems are as follows.

Problem 1. Find the norm of the error functional ℓ of the considered quadrature formula in the dual Hilbert space $W_2^{(2,1)*}(0,t)$.

By varying the coefficients C_β, to find the optimal quadrature formula in the space $W_2^{(2,1)}(0,t)$, then the following problem must be solved.

Problem 2. Find such values C_β which give the minimum value to the norm of the error functional.

Here we solve problems 1 and 2.

Keywords: Quadrature formula, Hilbert space, extremal function, error functional, error functional norm, optimal coefficients.

MSC 2020: 65D30, 65D32

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CONSTRUCTION OF BASIS FUNCTIONS FOR FINITE ELEMENT METHODS

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In this work, basis functions for finite element methods are constructed. Applying these basic functions, it is possible to solve the boundary value problems set for ordinary differential equations.

At the work [1] in. The Hilbert space $K_{2,\omega}^{(3)}$ the optimal interpolation formula of the form

$$\varphi(x) \cong P_{\varphi}(x) = \sum_{\beta=0}^{N} C_{\beta}(x) \cdot \varphi(x_{\beta}) \tag{1}$$

is constructed. We construct a set of basis functions using the coefficients of this optimal interpolation formula.

From the coefficients of the optimal interpolation formula (1) at $N = 2$ with nodes $x_0 = 0, x_1 = \frac{1}{2}, x_2 = 1$, we have the following functions

$$C_0(z) = \frac{\sin(\frac{\omega z-\omega x_2}{2})\sin(\omega z-\omega x_1)}{\sin(\frac{\omega x_0-\omega x_2}{2})\sin(\omega x_0-\omega x_1)}, C_1(z) = \frac{\sin(\omega x_0-\omega z)\sin(\omega z-\omega x_2)}{\sin(\omega x_0-\omega x_1)\sin(\omega x_1-\omega x_2)},$$

$$C_2(z) = \frac{\sin(\frac{\omega x_0-\omega z}{2})\sin(\omega z-\omega x_1)}{\sin(\frac{\omega x_0-\omega x_2}{2})\sin(\omega x_2-\omega x_1)}. \tag{2}$$

Now we construct a set of basis functions using (2). For this, we consider the interval $[0, 1]$ divided by $0 \le z_0 < z_1 < ... < z_n \le 1, z_i = ih, h = \frac{1}{n}(i = 0, 1, ..., n)$. We take basis function as follows.

Three non-zero local basis functions on the element $(y_{2i}, y_{2i+2})(i = 0, 1, ..., n-1)$ have the following form:

$$\xi_{2i}(z) = \begin{cases} 0, & z < y_{2i-2}, \\ C_2(z), & y_{2i-2} \le z < y_{2i}, \\ C_0(z), & y_{2i} \le z < y_{2i+2}, \\ 0, & y_{2i+2} \le z, \end{cases} \quad \xi_{2i+1}(z) = \begin{cases} 0, z < y_{2i}, \\ C_1(z), y_{2i} \le z < y_{2i+2}, \\ 0, y_{2i+2} \le z, \end{cases}$$

$$\xi_{2i+2}(z) = \begin{cases} 0, z < y_{2i}, \\ C_2(z), y_{2i} \le z < y_{2i+2}, \\ C_0(z), y_{2i+2} \le z < y_{2i+4}, \\ 0, y_{2i+4} \le z. \end{cases}$$

where

$$y_{2i} = z_i(\text{nodal points}), y_{2i+1} = \frac{z_i + z_{i+1}}{2}(\text{auxiliary points}).$$

Thus, we got a set of local trigonometric basis functions $\xi_i(z)(i = 0, 1, ..., 2n)$.

In our next works, we will study the properties, approximation procedure and practical applications of these basis functions.

Keywords: Basis functions, ordinary differential equation, boundary value problem, finite element, interpolation formula.

MSC 2020: 65D05, 74S05

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AN OPTIMAL QUADRATURE FORMULAS FOR NUMERICAL INTEGRATION OF RIEMANN-LIOUVILLE FRACTIONAL INTEGRAL

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In the present article, the problem of construction of the optimal quadrature formula is discussed for numerical integration of the right Riemann-Liouville integral in the Hilbert space $W_2^{(m,m-1)}[t,1]$ of real-valued functions. Initially, the norm of the error functional is found using the extremal function of the error functional of the quadrature formula. Since the error functional is defined on the Hilbert space, the quadrature formula that we are constructing is exact for zeros of this space, that is, we have the conditions that the influence of the error functional on these functions is equal to zero. Then, the Lagrange function is constructed to find the conditional extremum of the error functional. Thereby, a system of linear equations is obtained for the coefficients of the optimal quadrature formula. The existence and uniqueness of the solution of the obtained system are studied. This system of linear equations is solved by the Sobolev method. And the analytical form of the coefficients is obtained.

Keywords: Optimal quadrature formula, optimal coefficients, the error of the quadrature formula, the error functional, fractional calculus, fractional integral, Riemann–Liouville integrals.

AMS Subject Classification: 65D30, 65D32

ON OPTIMAL QUADRATURE FORMULAS FOR APPROXIMATION OF FOURIER INTEGRALS AND THEIR APPLICATIONS TO CT IMAGE RECONSTRUCTION

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This paper is devoted to the construction of optimal quadrature formulas in the Hilbert space $\widetilde{W_2}^{(m,m-1)}$ of complex-valued, periodic functions for the numerical calculation of the integral $\int_0^1 e^{2\pi i\omega x}\varphi(x)dx$ for $\omega \in \mathbb{Z}$. In the cases $m = 1$ and $m = 2$, the exponentially weighted integrals of some functions at the values of some N and ω are approximated using the constructed optimal quadrature formulas, and it is shown in numerical results that the orders of convergence of this formulas are $O\left(\frac{1}{N+|\omega|}\right)$ and $O\left(\left(\frac{1}{N+|\omega|}\right)^2\right)$, respectively. Also, in the space $\widetilde{W_2}^{(m,m-1)}$, the sharp upper bound of the error for the optimal quadrature formulas is obtained, and it is shown analytically that the order of convergence of the optimal quadrature formula is $O\left(\left(\frac{1}{N+|\omega|}\right)^m\right)$. Furthermore, in the case $\omega \in \mathbb{R}$, effective quadrature formulas for the approximate calculation of Fourier integrals are obtained and they used in the reconstruction of CT images.

Keywords: Strongly oscillatory integrals, optimal quadrature formulas, Hilbert space, periodic functions.

AMS Subject Classification: 65D30, 65D32

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STRIP TRANSFORMATIONS FOR EMBEDDING HIDDEN MESSAGES IN AN IMAGE

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To ensure the security of the communication channel, messages transmitted between two subscribers are converted in such a way that their interception by a third party is useless. When a message is received, for example, in the form of a file, the problem of its further protection is also relevant. Thus, an image file created by one person can be copied by another person or slightly modified and later illegally issued as an object of copyright. Then there is a need to create tools that allow you to uniquely identify the author when it comes to copyright, or identify the end user when it comes to finding the source of unlicensed copies of a file. Such tools are being developed and researched within the science of steganography. Steganography studies methods of creating a hidden communication channel by embedding secret messages in digital data objects called containers. In cryptography, access to a message is restricted if the secret key is unknown, and in steganography, the very existence of a secret message is hidden.

The rapid development of LSB implementation methods has led to the emergence of image steganalysis methods, i.e. methods for detecting the transmission of a secret message. This article explores the possibility of using the strip method for storing and noise-resistant transmission of images. This steganographic method uses the Hadamard transform and works with grayscale images. This method has been thoroughly tested on a variety of images with different textures and is reliable enough to avoid various attacks such as adding noise or compression. The experimental results show that the system under consideration successfully preserves the image quality and remains unnoticed by known methods of steganalysis.

Keywords: steganography, secret message, strip method, hadamard,steganalysis

AMS Subject Classification: 68U10, 94A08

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CREATION AND EVOLUTION OF THE MIRAS UNIVERSITY EDUCATIONAL PROCESS AUTOMATED CONTROL SYSTEM

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The development of educational technologies and the education system as a whole is ongoing. Both the modernization of the already familiar and the search for new innovative teaching methods are being carried out, and all this is aimed at improving the quality of education and its effectiveness. The trend of today is the active use of computers and computer technologies in education and training.

The development of automation at Miras University was started with the creation of an automated system for managing the educational process of a higher educational institution, which provides the implementation of the following functionality: the implementation of security and differentiated access rights through login and password; the availability of reference books for entering standard information; the ability to track information for each student, including passport and academic data, academic performance, the history of movement around the university by orders, automatic generation of document files, audit functions and automation of the bypass sheet, automation of academic mobility, notification of students; the ability to receive information in the context of academic groups, including lists of groups, journals, rating, examination and summary statements, as well as statements of academic history, automation of educational programs, including study schedules, a rating calculator, orders for referral to practice and contingent movements; automation of the testing process; implementation of the student portal; automation of the creation of reports.

The next step was the creation of a mobile application and the corresponding information and educational portal Miras.App that provides comprehensive automation of the process of credit technology training. The system consists of a centralized database in which the processes and information of the university are presented. A so called personal account (web page) is provided for each student and employee, which allows the university staff to automate their main tasks, students to see the necessary information, and remotely studying students to instantly get access to cases and knowledge control, communicate directly with the teacher in real time.

In connection with the development of distance education, including during the pandemic, Miras University has developed and implemented its own proctoring system Miras.Proctoring, which allows checking the level of academic integrity in the online testing process by analyzing video recordings using artificial intelligence algorithms.

Thus, the continuous development of software and information systems makes it possible to improve the quality and accessibility of education.

Keywords: automated control system, mobile application, proctoring, university, studying process.

AMS Subject Classification: 68N19, 68M11

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INTERNET OF THINGS IN EDUCATION: IMPACT AND EXPECTATIONS

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The introduction of the Internet of Things (IoT) into the education system creates new opportunities for effective and personalized learning, improves the learning process and expands access to educational resources around the world. However, the use of IoT in education must take into account the risks associated with data privacy and security, as well as accessibility issues for students from hard-to-reach areas.

The implementation of IoT in education involves the use of a variety of sensors and devices that can collect data on learning, behavior and health of students. This data can help educators tailor the educational process to the individual needs of students and make learning more effective. The IoT can also simplify the process of student assessment and parent feedback.

However, there are risks associated with the privacy and security of data collected by IoT devices in education. Some devices and sensors may collect personal information about students, which may be misused. Therefore, it is important to use only proven and secure devices, and to ensure that IoT solutions comply with data security standards.

Moreover, IoT in education can lead to accessibility issues for students in hard-to-reach areas who may not have reliable internet access or the right equipment to use these technologies. Therefore, it is necessary to carry out work to modernize the infrastructure of national educational systems in order to make IoT the most accessible and effective for all students, regardless of their financial situation.

In general, the introduction of IoT into the education system has the potential to change learning for the better and create more convenient and effective conditions for students and teachers in Kazakhstan, however, it requires a responsible approach to the development and use of technologies, as well as taking into account the risks and accessibility problems.

In general, IoT technology has a huge potential to improve the educational system in Kazakhstan. It will help to create a modern and effective educational environment that will meet modern needs and stimulate students and teachers to acquire new knowledge and skills. However, the introduction of IoT technology into the educational environment requires a systematic approach and decision-making at various levels, from experimental zones to national strategies and legislative acts.

An important aspect of introducing IoT into the educational environment is the development of human resources. Kazakhstani universities and colleges can create training programs for students and teachers on the implementation and use of IoT technologies in education. Such programs may include a wide range of courses, including programming, networking, data management, and so on. In addition, it is necessary to prepare teachers who will use IoT technologies in the educational process in such a way that teaching materials will be created that are easy to implement and use in the classroom.

In order to successfully implement IoT technologies in education, it is also necessary to conduct research and analyze the results of use. Companies and startups that develop IoT solutions can conduct pilot projects in educational institutions, which will be useful both for the companies themselves and for educational institutions. In doing so, social aspects should also be taken into account, such as accessibility for non-interacting students, as well as profitability and economic benefits for educational institutions.

In conclusion, the introduction of IoT into the education system of Kazakhstan is a promising step in the development of educational infrastructure. IoT technologies can help improve the quality of education to new heights, achieve high results in distance learning and fully adapt the learning process to the individual needs of students. However, for successful implementation, it is necessary to pay attention to a number of issues, such as training of human resources, social aspects, data protection and a systematic approach in all stages of implementation.

Keywords: IoT, education, sensors.

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NUMERICAL SOLUTION OF THE BURGERS EQUATION BY THE CONJUGATE OPTIMIZATION METHOD

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During last years, researchers have been increasingly interested in the idea of conjugate optimization for the numerical solution of hydrodynamic problems with parameter identification and flow control [1]. In this paper, the authors propose a method of fictitious domains with the idea of conjugate optimization for the Burgers equation, which allows us to build a homogeneous difference scheme in the entire extended domain. This method is very convenient in terms of programming automation. Consider the auxiliary problem of the method of fictitious domains in the domain $Q_T^\varepsilon = (0,1) \times (0,T)$ [2]:

$$\frac{\partial u^\varepsilon}{\partial t} + u^\varepsilon \frac{\partial u^\varepsilon}{\partial x} - \frac{\partial}{\partial x}\left(\mu^\varepsilon(x)\frac{\partial u^\varepsilon}{\partial x}\right) = f^\varepsilon(x,t) \qquad (x,t) \in Q_T^\varepsilon, \tag{1}$$

$$u^\varepsilon(0,t) = g_1(t), \quad u^\varepsilon(\xi,t) = g_2(t), \quad u^\varepsilon(1,t) = 0, \quad t \in (0,T), \tag{2}$$

$$u^\varepsilon(x,0) = v^\varepsilon(x), \qquad x \in (0,1), \tag{3}$$

The goal is to find a minimizing functional

$$I = \frac{1}{2}\int_0^T \left(u^\varepsilon(\xi,t) - g_2(t)\right)^2 dt \tag{4}$$

We will minimize the target functionality using the Landweber iteration method. The gradient of the functional is calculated by the following formula using the conjugate problem.

Keywords: Fictitious domain method; Navier-Stokes equations; Burgers equations; conjugate problem; Lagrange multiplier; difference schemes; iterative process; numerical implementation.
AMS Subject Classification: 35Q30, 65M85, 65Q10, 35A15

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FORMATION OF COMPETENCIES FOR THE USE OF AR TECHNOLOGIES IN COMPUTER SCIENCE EDUCATION

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Nowadays, the use of information technology is applied in teaching in various disciplines. Such examples of informatization tools are augmented reality technologies, augmented reality is becoming a completely new method and means of cognition of objective reality. The use of augmented reality in the educational process is already one of the most effective means of learning. For example, in computer science, AR can be both an object and a learning tool. We mean the educational application of augmented reality in computer science lessons, primarily at school. In education, immersive digital technology helps to activate thinking resources, concentrate attention, and better assimilate material. The overlay of an information layer on a picture of the real world — pictures, videos, text — is used in classes in physics, chemistry, mathematics, history and other subjects.

In the school curriculum, we see two ways of using augmented reality for computer science. It will help you to master difficult questions — and at the same time it becomes a separate topic for study. In addition to additional text data on any section, visual images superimposed on the camera image are used in laboratory work on the topics "Fundamentals of algorithmization and programming" or "Computer Architecture".

It is good to demonstrate the logical structure of a hard disk using its virtual 3D model, which can be viewed from all sides, scaled or viewed in a cross-section. Thus, learning turns into an exciting form of educational activity that increases motivation, allows students and teachers to develop ICT competencies, and form universal learning activities. In these cases, students are taught with the help of information technology, which helps in studying the material and self-development. This technology allows not only to attract the attention of students, but also to visually show students those things that cannot be used in the classroom due to their high cost, danger, inaccessibility. The need to use and train augmented reality technology is justified by two main reasons:the use of augmented reality technology can significantly increase the effectiveness of training, since this technology has a number of unique advantages, such as increasing visibility, carrying out previously impossible practical work, as well as increasing the degree of integration of information technologies in the educational process.

Keywords: information technology, augmented reality technologies, ICT competencies, learning tool.

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THE PROBLEM OF VECTORIZATION OF RUSSIAN AND TURKIC LANGUAGES

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Question answering systems aim to determine the semantic relatedness between a user's query and a pre-existing knowledge base [1]. This is achieved by measuring the vector distance between the query and the knowledge base. In practice, the cosine similarity measure is commonly used, which is calculated as the angle between the vectors A and B:

$$p(A,B) = \frac{\sum_{i=1}^{n} A_i * B_i}{\sqrt{\sum_{i=1}^{n} {A_i}^2} * \sqrt{\sum_{i=1}^{n} {B_i}^2}} \tag{1}$$

However, the main bottleneck in text classification is the task of text vectorization [2], which involves representing the text as a vector that takes into account all of its important features. One of the most significant challenges in this task is context modeling, which can be formulated as conditional probabilities of the system's answer A given the user's question Q and context C:

$$p(A|Q,C) \tag{2}$$

In this article, we describe and research vectorization algorithms for Russian and Kazakh languages in connection with different machine learning models. Our results can serve as a starting point for solving practical problems in natural language processing.

Keywords: machine learning, text classification, artificial intelligence, natural language processing

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APPLICATION OF THE "MULTISIM" SOFTWARE ENVIRONMENT WHEN PERFORMING LABORATORY WORK OF STUDENTS IN ELECTRODYNAMICS

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The article discusses the use of the software environment "Multisim" to enhance the independent work of students of engineering specialties in physics. Physics belongs to the field of fundamental sciences in which the processes of cognition and development require an inseparable connection of theoretical analysis and experimental research. Through the efforts of many specialists, a personal computer has become the main tool in the hands of a person, which is used to penetrate the secrets of nature. Moreover, it has now become obvious that further study of the physics that arose on its basis, practically makes little sense without the use of computer technology.

Consequently, the new paradigm of education considers one of the goals of professional training of a specialist to be his ability to self-study, to independently search for knowledge and to form his needs for professional and personal self-improvement. We consider the use of the "Multisim" software environment as a process that allows motivating a student to study a subject that will contribute not only to improving the quality of students' knowledge, but also integration into the global educational environment. The positive experience of using the "Multisim" software environment when performing laboratory work on the example of the Kyrgyz-Turkish Manas University for engineering specialties is given. The results of the study showed the effectiveness of the "Multisim" software environment and allowed to achieve a guaranteed pedagogical result.

Keywords:virtual lab, Physics, motivation, experience, attitude.

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CHATGPT IS AN EDUCATIONAL EMPOWERMENT TOOL

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In the modern scientific space, artificial intelligence is increasingly being studied, which leads to discussions about the future and the ethics of its use in various fields. ChatGPT is a digital transformation tool in education that provides unique opportunities to improve the learning process, individualize education and develop students' skills.

The article is devoted to the use of artificial intelligence in the educational environment. The author highlights the main advantages and disadvantages of integrating artificial intelligence into the educational process and concludes that artificial intelligence is a functional tool that allows optimizing many different operations that contribute to the organization of an effective educational process.

Keywords: ChatGPT, chatbot, artificial intelligence, education.

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PRIORITIES AND PROBLEMS OF STEAM EDUCATION IN CENTRAL ASIA

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In the first decade of the 21st century, the interdisciplinary learning approach STEM that integrates the areas of science, technology, engineering, and mathematics started to be popularized as a pedagogical approach that helps students to develop skills like problem-solving, critical thinking, and creativity. And today, STEM is one of the main trends in world education. Thanks to the rapid development of technologies, new professions are emerging, and the demand for STEM specialists is increasing everywhere. In countries that focus on high-tech production and innovative technologies, STEM education is a priority. The public sector and specialized IT companies are experiencing a shortage of scientific and engineering personnel who think creatively outside the norm. In many countries, mass digitization is prompting a rethinking of teaching methods.

Recently, STEAM education, where A stands for art, is being considered as an alternative approach that is used for the purpose of improving students' engagement in education and in developing students problem solving, leadership, and enterprising skills. The difference between STEAM from STEM education is that the latter considers both hard and soft skills development as a part of the educational process, unlike STEM which is more focused on technical knowledge teaching.

STEAM education is not just about pedagogical strategy, it is more about encouragement to transform the educational environment and development of community. Therefore, the process of STEAM education integration and the challenges faced during this process can vary depending on the region where this transformation is taking place. Considering this fact, the article discusses the priorities of STEAM education, and the main challenges in the application of this pedagogical approach in Central Asia, particularly in Kazakhstan and Uzbekistan.

Keywords: STEAM education, STEM education, skills, Central Asia, pedagogical approach

AMS Subject Classification: 94 Information and communication, circuits

MATHEMATICAL MODELS FOR INDEPENDENT COMPUTER PRESENTATION OF SCIENTIFIC NOTIONS

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By developing ideas [1] and [2] we [3] proposed independent computer interactive presentation of notions (without using other languages). In this method, verbs are presented as the user's actual actions and feedback confirms mastering of the notion.

We developed mathematical and computer models for some verbs, nouns and adjectives, proposed new classification of verbs and new "grammar" for notions, we with coauthors implemented some notions of Kyrgyz and English on computer.

We describe mathematical models for some *notions* in various sciences including topological *spaces* as it was proposed [4]. Background is in white ... black. Avatar object (A-object) is green. The result (F-object) of A-object is red. Target (T-object) for F-object is yellow. Approaching T-object (feedback) is accompanied by music of "hot-cold" type too. Tracks of A-object are light green and ones of F-object are light red.

(1) *Solving* of the *equation* $F(x) = 0, F : \mathbf{R} \to \mathbf{R}$. A-point can move along the abscissa axis only. T-object is the abscissa axis.

(2) Searching for $minF(x)$. A-point can move along the abscissa axis only. T-object is gradient of yellow color down.

(3) Presentations of non-Euclidean spaces filled with T-objects and brown Obstacles. The user drives a green car, with additional possibilities to put marks etc. The task is to find and erase T-objects without breaking Obstacles.

Moebius band: the user can verify that a right boot left on a street will be met as a left boot after passing half of the street;

Topological torus is a square with opposite sides glued. (This space used to be discovered by many programmers independently);

Riemann surface of the function $\sqrt{z}$, with the third coordinate up.

Projective plane with the third coordinate up. While motion along the street trees on this side move to us as usually but trees on the opposite side move from us.

It can be used for further development of such computer presentations and learning languages.

Keywords: mathematical model, language, computer model, independent presentation, learning

AMS Subject Classification: 97D80

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PROBLEMS OF TEACHING PROGRAMMING TO PRIMARY SCHOOL STUDENTS

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Every year, the number of scientific studies examining the influence of global information flow and widespread digitization on individuals' cognitive capacities is growing. The issue of teaching programming to children is becoming one of the most relevant topics in teaching computer science in schools. Gaining proficiency in programming aids primary students in comprehending the inner workings of various phenomena. Furthermore, it fosters the growth of computational, algorithmic thinking, and problem-solving abilities. In this endeavor, drawing upon national principles, we put forth a model for ongoing computer science education, encompassing programming as an integral component. We present a comprehensive system for nurturing computational thinking, providing methodological evidence that programming indeed influences its development.

The attained outcomes validate that students are encouraged to utilize the suggested tools for computational thinking and problem-solving, thereby amplifying their curiosity, motivation, and engagement in programming education.

Keywords: programming, computational thinking, algorithmic thinking, elementary school, information-educational environment.

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ABOUT AN OPTIMAL METHOD FOR THE APPROXIMATE SOLUTION OF SINGULAR INTEGRAL EQUATIONS

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Many problems of science and engineering are naturally reduced to singular integral equations. Moreover plane problems are reduced to one dimensional singular integral equations.

In the present paper, we will develop an optimal algorithm for the approximate solution of one dimensional singular integral equations with the Cauchy kernel. Here we are engaged in finding the analytical form of the coefficients of the optimal quadrature formula. We apply these coefficients to an approximate solution of the Fredholm singular integral equation of the first kind. Thus, we show the possibility of solving singular integral equations with higher accuracy using the optimal quadrature formula based on the Sobolev method.

Keywords: Singular integral equation, Cauchy kernel, optimal quadrature formula, Sobolev method.

AMS Subject Classification: 65D32

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MODELING OF THE BOUNDARY LAYER OF THE ATMOSPHERE OF AN INDUSTRIAL CITY WITH HARMFUL IMPURITIES

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In this paper, a model of the boundary layer of the atmosphere in a 3D region is considered to study the influence of anthropogenic heat sources and harmful substances of an industrial city. The equations of motion are considered:

$$\frac{\partial u_i}{\partial t}+\frac{\partial u_i u_j}{\partial x_i}=-\frac{\partial p_i}{\partial x_i}+a_i K_i+\frac{\partial}{\partial x_3}(\nu\frac{\partial u_i}{\partial x_3})+\Delta u_i,\sum_i\frac{\partial u_i}{\partial x_i}=0, i=1,2,3, \tag{1}$$

where t is time, p is pressure, in the second term in the left part and in the continuity equation summation is performed by repeating indices j, $u=(u_1,u_2,u_3)$ is a velocity vector, $x=(x_1,x_2,x_3)$ is cartesian coordinates, $a=(l,-l,\lambda)$, $K=(u_2,u_1,\theta)$. Heat inflow equation:

$$\frac{\partial\theta}{\partial t}+\frac{\partial\theta u_i}{\partial x_i}+Su_3=\frac{\partial}{\partial z}(\nu\frac{\partial\theta}{\partial z})+\Delta\theta, \tag{2}$$

here θ is background potential temperature, ν is vertical coefficient of turbulent exchange, S is stratification parameter. Equation of transfer of harmful substances in the atmosphere:

$$\frac{\partial\phi_q}{\partial t}+u_i\frac{\partial\phi_q}{\partial x_i}=\Delta\phi_q+\frac{\partial}{\partial x_3}(\nu\frac{\partial\phi_q}{\partial x_3})+\alpha_q\phi_q+\beta_q+f_q,\sum_q\phi_q=1, \tag{3}$$

ϕ_q is the proportion of concentrations of harmful substances, f_q - sources of substances at the level of roughness, α_q,β_q coefficients from the equations of transformations of impurities, q means the chemical formula of the substance [1,2]. These equations are considered with initial and boundary conditions depending on the climatic and geographical features of the industrial city. Work provides a strict mathematical justification of the model. Application is also considered.

Keywords: model of the boundary layer of the atmosphere, heat inflow equation, transfer equation, numerical methods.

AMS Subject Classification: 80A19, 80A30, 80A32.

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PREDICTING RESPIRATORY DISEASE RISK FROM ATMOSPHERIC POLLUTION USING RANDOM FOREST ALGORITHM: A CASE STUDY IN ALMATY, KAZAKHSTAN

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This study presents an analysis of the morbidity of the population in Almaty, Kazakhstan, and its correlation with atmospheric air pollution using machine learning algorithms. The study focuses on respiratory diseases of the population in Almaty from 2017 to 2022 and the levels of air pollution by suspended substances during the same period. The initial analysis is conducted to identify the priority pollutants [1]. The random forest method, which combines multiple decision trees to enhance the analysis [2], is utilized to determine the relationship between the morbidity rate and air pollution levels. Metric estimates are calculated to evaluate the quality of the constructed models, demonstrating the adequacy of these models [3]. The findings of this research will be used to develop strategies for reducing the adverse effects of air pollution on the health of the population in Almaty.

Keywords: machine learning, random forest, decision tree, determination coefficient, MAE, RMSE

AMS Subject Classification: 62P10, 62J15, 62J05.

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BUILDING RELIABLE AND SECURE FOG DATA SYSTEMS

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Building reliable and secure fog data processing systems is one of the key aspects of modern information technology. Fog Computing (FC) are a distributed architecture where computing resources and services are placed on edge devices, closer to data sources [1]. This allows you to process data more efficiently, reducing latency and improving performance. To implement effective FC, a number of basic requirements must be developed. Based on the study, the following requirements were drawn up:

(1) Redundancy and fault tolerance. This requirement is usually implemented through redundancy by duplicating critical components or services in the system. This redundancy ensures that in the event of a single component failure, a backup is available to keep the system running. However, this approach is not efficient enough. We propose methods to improve the efficiency of the system with a sufficient level of reliability and fault tolerance.
(2) Reliable security measures. This requirement includes the implementation of strong authentication and access control mechanisms, encryption of data in transit and at rest, secure communication protocols, and regular security checks and updates to address emerging vulnerabilities.
(3) Reliable network infrastructure. The requirement is related to redundant network connections, load balancing and Quality of Service (QoS) [2] mechanisms help ensure stable and efficient data transfer.
(4) Scalability and Flexibility: Fog systems are typically built with scalability and flexibility in mind to accommodate changing workloads and growing requirements. This may include the implementation of flexible resource allocation, dynamic load balancing, and adaptive processing capabilities.
(5) Monitoring and control. Real-time monitoring of system health, performance metrics, and network health helps you detect anomalies, diagnose problems, and take proactive action to ensure system reliability.
(6) Compliance with standards and regulations. This may include compliance with data protection regulations, industry security standards and privacy principles.

By meeting these fundamental requirements, fog (edge) systems can provide a higher level of security and reliability, allowing organizations to take advantage of edge computing while maintaining the integrity and security of their data and operations. In this paper, the solution paths for each developed requirement will be considered in detail.
Keywords: Fog computing, distributed systems, reliability, fault tolerance.
AMS Subject Classification: 68M14

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ADAPTATION OF ACTIVE SAFETY METHODS AND ALGORITHMS FOR USE IN FOG COMPUTING

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With the rapid growth of fog computing, there is an urgent need for active security. Fog computing is a distributed computing paradigm that extends the power of cloud computing. They aim to remove the limitations of traditional cloud computing and offer the following benefits: low latency, bandwidth optimization, scalability and reliability, privacy and security [1].

Active security in cryptography [2] is responsible for protecting the system during active attacks by intruders. The most popular methods are: public key infrastructure (PKI), digital signatures, message authentication cod (MAC), key-agreement protocol, public-key cryptography, Intrusion Detection System (IDS) and secure multi-party computation.

This paper explores the possibility of adapting the methods and algorithms of the active safety concept for use in fog computing. By integrating already known security techniques into fog computing, the goal is to increase the security and reliability of edge devices and their interactions, eliminating potential vulnerabilities and reducing the scale of active attacks.

Keywords: fog computing; active security; cryptography; cloud computing; public-key.

AMS Subject Classification: 68M14

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NEUTROSOPHIC TECHNOLOGIES

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In this article, the concept of neutrosoph was introduced. Some related theorems have also been established. Researchers in various fields such as medicine, economics and many other fields are daily faced with inaccurate and insufficient information when modeling uncertain data. The article discusses some problems and examples. A numerical example is given to test the practicality of the proposed strategy. A comparison of the proposed strategy with existing ones is given, with the aim of demonstrating effectiveness and practicality.

Neutrosophy means the study of ideas and notions that are not true, nor false, but in between. Each field has a neutrosophic part, i.e. that part that has indeterminacy. Thus, there were born the neutrosophic logic, neutrosophic set, neutrosophic probability, neutrosophic statistics, neutrosophic measure, neutrosophic precalculus, neutrosophic calculus, etc. There exist many types of indeterminacies – thats why neutrosophy can be developed in many different ways.

Keywords: Neutrosoph, underset, neutrosophic offlogic, neutrosophic overprobability, neutrosophic underlogic.

AMS Subject Classification: 68-04 Software, source code, etc. for problems pertaining to computer science.

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THE ROLE OF THE PROJECT METHOD IN THE DEVELOPMENT OF COGNITIVE NEEDS OF SCHOOLCHILDREN

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The article discusses the advantages and disadvantages of introducing the project method into the educational process and substantiates that project activity involves the movement of a child in a space of possibilities, where norms are not clearly defined, when children fall into conditions of uncertainty and that in project activity not only an understanding of the problem can be realized, but also the very idea of the child that project activity it has an addressable character.

Keywords: Project activity, information, project method, cognitive universal learning activities, uncertainty conditions, research.

COMMON ELEMENTS AND APPROACHES IN DEVELOPING SOFT SKILLS IN FUTURE IT PROFESSIONALS

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Recently, the trend in the demand for soft skills, also known as 21st-century skills, in the labor market of the IT sector is growing rapidly, which can be related to the fact that the world is undertaking rapid technological changes and matters like the creation of information society and shift towards the Industry 4.0. This affair was mentioned at the World Economic Forum 2020, where it was reported that the labor market will undergo a "reskilling revolution", where 94% of employees should gain new skills every year in order to be protected from instabilities. Considering this fact, specialists should gain adaptability and flexibility skills which again shows the importance of soft skills in the career life of people.

The researchers in the field show that the behavioral and interpersonal skills of the employees are directly related to the higher education institutions from where they graduated. Based on this fact, there is a common opinion that soft skills should be formed during university life. Despite the fact that the period when future specialists should be introduced to soft skills is broadly-known, there is a lack of systematic knowledge about the ways and elements of the soft skills formation process. Therefore, the main goal of this article is to shed light on this issue and to find an answer to the following questions:

- What are the common elements of soft skills?
- What are approaches to soft skills development in the higher education environment?

What are the common elements of soft skills?] What are approaches to soft skills development in the higher education environment? The article reviews the research works studying pedagogical approaches to teaching soft skills in the university environment on the different higher education degrees.

Keywords: Soft skills, pedagogical approach, information society, IT sector, higher education

AMS Subject Classification: 94 Information and communication, circuits

PROPERTIES OF THE SOME DISCRETE ANALOGUE

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One can use the Sobolev method, which is based on the discrete analogue of the differential operator $\frac{d^8}{dx^8}+1$, to construct an optimal quadrature formula or an optimal interpolation spline in the space $L_2^{(4,0)}(0,1)$. Here $L_2^{(4,0)}(0,1)$ is an Hilbert space. Recently, in the work [1] the discrete operator $D_4[\beta]$ which satisfies the following equation was constructed

$$D_4[\beta]*G_4[\beta]=\delta[\beta],$$

where $G_4[\beta]=-\frac{\mathrm{sign}[\beta]}{8}\left[\sum\limits_{k=1}^{4}e^{[\beta]\cdot\cos\frac{(2k-1)\pi}{8}}\cdot\cos\left([\beta]\cdot\sin\left(\frac{(2k-1)\pi}{8}\right)+\frac{(2k-1)\pi}{8}\right)\right]$ and $\delta[\beta]=\begin{cases}1, & \beta=0,\\ 0, & \beta\neq 0.\end{cases}$

The explicit expession for $D_4[\beta]$ is

$$D_4[\beta]=-\frac{4}{K}\cdot\begin{cases} M_1-K_1+\sum\limits_{k=1}^{3}\frac{A_k}{\lambda_k}, & \beta=0,\\ 1+\sum\limits_{k=1}^{3}A_k, & |\beta|=1,\\ \sum\limits_{k=1}^{3}A_k\cdot\lambda_k^{|\beta|-1}, & |\beta|\geq 2.\end{cases} \tag{1}$$

Where $|\lambda_k|<1$ and quantities K, M_1, K_1 and A_k are known.

In the present work we investigate some properties of the discrete operator $D_4[\beta]$. We note that $D_4[\beta]$ has similar properties as the differential operator $\frac{d^8}{dx^8}+1$. Namely, the following holds.

Theorem. The discrete analogue (1) of the differential operator $\frac{d^8}{dx^8}+1$ satisfies the following equalyties:

- $D_4[\beta]*e^{[\beta]\cos\frac{(2k-1)\pi}{8}}\cos\left([\beta]\sin\frac{(2k-1)\pi}{8}\right)=0\quad k=\overline{1,4},$
- $D_4[\beta]*e^{[\beta]\cos\frac{(2k-1)\pi}{8}}\sin\left([\beta]\sin\frac{(2k-1)\pi}{8}\right)=0\quad k=\overline{1,4}.$

Keywords: Hilbert space; generalized function; operator; discrete analogue.

AMS Subject Classification: 65D25, 65D30, 65D32

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NUMERICAL MODELING OF THE STEFAN PROBLEM IN FINANCIAL MATHEMATICS

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The paper considers some features of calculating the option price $V(t, x)$, stock price $x(t)$ and the optimal stop (execution) moment τ ($\equiv t$) on finite and infinite time intervals. Then we consider the problem of finding a rational price of American-type options for the optimal stopping time (optimal stopping moment) in diffusion (B, S)-stock markets. Next, we consider the problem of finding a rational price for European-type options.

First, the option is considered from the point of view of the buyer – the option of the buyer, then – the option of the seller. All considered problems are reduced to the Stefan problem. In mathematical physics, the Stefan problem arises in the study of physical processes associated with the phase transformation of matter and consists in finding a function $u = u(t, x)$ that describes the temperature regime of the phases and the separation boundary $x = x(t)$, $t \geq 0$ of these phases.

In the case of standard buyer and seller options, a two-phase situation also takes place – when searching for optimal stopping rules, we can restrict ourselves to considering only two simply connected phases: the area of continuation of observations C^T and the area D^T.

All problems are solved exactly if the optimal stopping time is found in advance or numerically.

The results of numerical modeling of the Stefan problem by the sweep method and the finite element method for standard call and ask options are presented. As well as a comparative analysis of the numerical results by the sweep method and the finite element method.

Keywords: option prices, stock prices, equity diffusion markets, options of American and European types, Stefan's problem, numerical modeling.

AMS Subject Classification: 35R60, 39A50, 60G40, 65M06, 65M08

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COMPOUND CUBATURE FORMULAS ON A LATTICE

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In the present paper lattice optimal cubature formulas are constructed by the variational method in the Sobolev space. In addition, the square of the norm of the error functional of the constructed lattice optimal cubature formulas in the conjugate Sobolev space is explicitly calculated.

Keywords: Sobolev space, extremal function, composite lattice optimal cubature formulas, error functional.

AMS Subject Classification: 65D32

ON ONE ALGORITHM FOR FINDING THE FACES OF THE VORONOI DOMAIN OF THE SECOND PERFECT FORM

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The problem of classifying integer quadratic forms has a long history, during which many mathematicians contributed to its solution. Binary forms were comprehensively studied by Gauss. He and later researchers also outlined the main ways to solve the problem of classifying ternary forms and forms of higher dimensions

These are interesting and non-trivial problems of geometric number theory, which were dealt with by many mathematicians (Hermite, Gauss, Korkin, Zolotarev, Minkowski, Voronoi, Delaunay, Ryshkov, Malyshev, Barnes, Vladimirov, Scott, Larmut, Stacey, Baranovsky, Shushbaev, Anzin, Umarov and etc.). They also appeared in the works of S.L. Sobolev and Kh.M. Shadimetov in connection with the construction of lattice optimal cubature formulas.

The paper proposes an algorithm for calculating non-equivalent quadratic forms corresponding to the faces of the Voronoi domain of the second perfect form in many variables, and using this algorithm, all corresponding non-equivalent quadratic forms are calculated.

In this paper, using this algorithm, we prove the following proposition.

Theorem. Number of $20-$dimensional faces of the Voronoi domain $V^{21}(\varphi_1^6)$ perfect form φ_1^6, permutations of the variables $x_3, ..., x_6$ that are not equivalent with respect to the group S_4, equals 12.

Keywords: Limit forms, arithmetic minimum, perfect lattices, perfect forms, Voronoi region, Voronoi neighborhood.

MSC: 65D05.

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DESIGN AND DEVELOPMENT OF A MOBILE GAMING APPLICATION FOR STUDYING THE HISTORY OF KAZAKHSTAN

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Gamification is gaining wide popularity in modern education, where the game becomes the main method of teaching and preparing didactic materials.

Educational and gaming mobile applications have already become one of the optimal means of learning, in particular, means of control and self-control, verification, consolidation of learned material. They help to increase the motivation of students to study the subject. In addition, it becomes possible to optimize the learning process by ensuring the implementation of some of its components through modern information and communication technologies, namely, ensuring that tasks are completed in a playful way with automatic control, which can be applied during classes, during extracurricular hours or with mixed learning - at the request of the teacher.

In this regard, the purpose of our research is to analyze the prospects for using mobile educational and gaming applications as learning tools, to develop a mobile application for Android OS on the Unity platform for the purpose of fascinating study, consolidation and control of the assimilation of material on the history of Kazakhstan and knowledge diagnostics.

The subject of the research described in this paper is the development of a mobile educational and gaming application for an interesting and fascinating journey through the history of Kazakhstan. Overcoming the challenges of the virtual world here is connected with the ability to use the knowledge that was obtained during the course of the plot.

To achieve this goal, we have solved the following tasks:

1) a review of game development software for mobile OS;
2) existing gaming applications have been studied;
3) the architecture of the mobile application has been designed and developed;
4) the implementation of the game application has been implemented;
5) the developed game application is being tested.

The main expected results of using such an application are the optimization of the educational process, including extracurricular training, both for students and teachers; increasing the motivation of students to study the history of Kazakhstan; reducing the negative psychological effect of students' perception of the educational process.

Keywords: mobile gaming application, learning tool, game development software, Android operating system, Unity platform.

AMS Subject Classification: 68U05, 68N19.

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USE OF GAMIFICATION ELEMENTS IN COMPUTER SCIENCE

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Gamification can be seen as a learning process in which students solve problems by completing tasks in a playful way. There is competition here, not cooperation and teamwork. It is clear that competition, in turn, increases the mental abilities and motivation of students to study. This article suggests ways to use gamification elements in teaching computer science in a secondary school.

The purpose of the research work is to explore and present the benefits of using elements of gamification, as well as to offer some ideas for its implementation in education.

Keywords: gamification, computer science, education, gamification computer science.

WAYS TO IMPROVE SOFT SKILLS IN FUTURE IT SPECIALISTS

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The sharp alterations occurring in every aspect of life, such as the transition to Industry 4.0., globalization, the problems that occurred after COVID-2019, the formation of the information society, and rapid changes in technology, obviously, have a great impact on the education system. In this regard, more and more attention is paid to the formation of inter- and intra-personal and meta-subject skills, known as soft skills, in future specialists. Employers expect students to acquire soft skills during their higher education. Hence, universities should adjust their curricula to labor market requirements by incorporating soft skills into their program. The trend for soft skills development is clearly expressed in the fast-changing IT sector, and it seems that the demand will only grow in the near future.

Despite the strong interest in soft skills, there is still no commonly accepted approach to teaching them in the higher education system. Considering the fact of the high variability of the soft skills teaching approaches in the context of higher education and taking into account the importance of these "21st-century skills" in the IT sector, this article introduces the systematic literature review on ways to improve the soft skills in future IT professionals in the higher education environment.

The goal of this study is to shed some light on 2 main questions:

- What soft skills are being considered in IT sector?
- What practices are being considered to teach soft skills?

The results attained illustrate that the students are aware of the importance of soft skills for the success of their career life, and the list of practices being considered to be productive in teaching soft skills was obtained.

Keywords: soft skills, IT specialist, higher education, education, curricula.

AMS Subject Classification: 94 Information and communication, circuits

ANALYSIS OF CRM-SYSTEM APPLICATION

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The article analyzes the problem of increasing the efficiency of online stores. The paper identifies and substantiates the advantages and disadvantages of implementing a CRM system. The aspects influencing its successful functioning are singled out and described. The research task of the authors is to assess the feasibility of implementing a CRM system[1-3]. Based on the analysis of statistical data, as well as the involvement of expert opinions, the authors substantiate the idea that companies considering the implementation of CRM need to carefully weigh its advantages, disadvantages and clearly define the required functionality.

Keywords: CRM-system, implementation CRM-system, advantages and disadvantages of CRM-system, sales analysis post-imlementation CRM-system

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DEVELOPMENT OF RESULT-ORIENTED ASSESSMENT TOOLS IN PHYSICAL AND MATHEMATICAL EDUCATION

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The implementation of the competence approach to education is characterized by the achievement of learning outcomes.

The level of achievement of the specified learning outcomes of the educational program (discipline) of the student or graduate is assessed with the help of tools oriented to the result [1,2]. The analysis of scientific-methodical, scientific-pedagogical literature and practice shows that it is not advisable to organize a comprehensive evaluation of the results of students' education and the predominance of traditional evaluation systems in real pedagogical practice. The traditional system has limited ability to assess students' professional competencies. Competencies have an integrated, complex nature, so it is necessary to take it into account when developing assessment tools.

In the course of the study, tools for evaluating physical and mathematical education programs were analyzed and their quality was described. As a result, practical recommendations were given on the development of tools for evaluating the results of future mathematics teachers after completion of the educational program.

Keywords: assessment, assessment tools, learning outcomes, physical-mathematical education.

AMS Subject Classification: 97B50 Mathematics teacher education

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BASIC CONCEPTS OF STEM EDUCATION FORMATION AT SCHOOL

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In Kazakhstan, the abbreviation STEM has been spreading for several years. Compared to the traditional school system, STEM education is based on combining theory and practice with an emphasis on conducting experiments, creating models, performing creative work independently and turning your ideas into reality. In addition, STEM education allows you to identify gifted children and create the necessary conditions for their further development, creates conditions for the development and improvement of analytical and creative abilities of students.

The main concepts of STEM education formation at school include the following aspects:

- Integration of Subjects: STEM Education aims to integrate scientific subjects such as mathematics, physics, computer science, chemistry and biology to demonstrate their interrelationship and application in specific situations;

- Learning environment and equipment: For the successful implementation of STEM education at school, an appropriate learning environment and affordable equipment are required;

- Project activity: STEM training focuses on the practical application of knowledge through project activities. Students work in groups to solve specific problems or create specific products. This develops their problem solving, critical thinking, creativity and collaboration skills;

- Technology Application: STEM knowledge includes the use of modern technologies such as computer modeling, programming, robotics, 3D printing and other tools;

- Skill Development: STEM education focuses on developing the skills needed to successfully adapt to a rapidly changing world;

- Professional Orientation: STEM education helps students discover their interests and abilities in science, technology and engineering.

Summing up, there are three main characteristics that distinguish STEM education from the traditional education system. Firstly, with the help of STEM, students have more time and opportunities for independent preparation; secondly, by participating in teamwork, students get the opportunity to make creative discoveries; thirdly, within the framework of STEM education, mutual assistance in solving educational problems will be developed and digital literacy skills will be formed.

Keywords: STEM education, skill development, professional orientation, integration of subjects.

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ANALYSIS OF THE TEACHER ASSESSMENT SYSTEM

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One of the most pressing issues is the study of the rating assessment system of university teachers and the digitalization of this process. The personnel potential of the University reflects not only the readiness of teachers to perform their functions at the moment, but also the totality of their capabilities in the long term, taking into account scientific and pedagogical qualifications, level of motivation, age, practical experience, business activity, professional mobility and quality of service, including its effectiveness and innovation. Modern technologies are currently used to collect data in the fund, group and analyze it according to the characteristics. One of those areas is machine learning.

The article analyzes the system of evaluation of the results of the teaching activity of the teacher its functions and principles, the point-rating system for evaluating the teaching activity of teachers, the methodology for monitoring and evaluating the quality of teaching work of teachers and the mechanism for evaluating the teaching activity of teachers.

Data analysis is implemented in the Python programming language (version 3.9.7) in the Anaconda(Jupiter notebook) environment. The libraries Numpy, Scipy, Pandas, sklearn, matplotlib, seaborn are used.

Keywords: analysis, average value, visualization, critical value, deviation.

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THE USE OF AN ELECTRONIC CONCEPTUAL TERMINOLOGICAL DICTIONARY IN THE ICT COURSE

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The industrialization of the world and the introduction of new technologies are contributing to the emergence of new concepts in information and communication technologies and the computer industry. The ICT course leads to language development as a result of the adoption of terminological vocabulary in different languages in their own language, the establishment of interlingual correlation terms. Therefore, the article provides a brief overview of the formation of the system of concepts and terminology of ICT courses. The main task of information and communication technologies in modern education is to create an information and educational environment of the Internet with the help of computer tools and information resources. In this regard, one of the priorities of higher education is to be able to actively use not only the general and specialized professional knowledge and skills required in the specialty, but also a range of computer tools that are constantly updated. Modern society is characterized by the maturity of education closely related to it, the widespread use of information and communication technologies and the peculiarities of creating a conceptual and terminological dictionary of ICT courses and their use in the learning process. They are widely used in the current education system to disseminate information and facilitate student interaction. Currently, in order to form the digital literacy of students in educational institutions, ICT courses and computer science classes are organized and conducted in IT landfills, information zones.

Keywords: Information and communication technologies, computerization, system of ICT concepts, Infozone, mobile technologies, digitization.

AMS Subject Classification: 68

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MATHEMATICAL MODELING OF THE TRANSFER AND DISPERSION OF HARMFUL EMISSIONS OF THE SURFACE LAYER OF THE ATMOSPHERE AND THE TECHNIQUE OF NUMERICAL EXPERIMENTS IN ENVIRONMENTAL DISASTERS

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Ecological disasters caused by natural phenomena and human activity bring serious changes to the environment and pose a threat to the life of not only plants and animals, but also the person himself. The problems of the propagation of volley emissions with their constant ejection from industrial pipes are considered in many well-known works. In solving these problems, analytical and empirical research methods were used. The works devoted to the analysis of accidents and explosions are mainly of an experimental nature. An analytical model is presented to determine the distribution of volley emissions for 2 cases: a short-term release from a production pipe and a thermal explosion of chemicals in storage. The model includes the solution of the differential equation of turbulent diffusion in partial derivatives, which differs from the presence of a partial derivative of concentration with respect to time [1]:

$$\frac{\partial C}{\partial \tau} + \nu \frac{\partial C}{\partial x} = D_T \Big(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \Big) + \varphi, \tag{1}$$

The results of the calculation reveal the numerical dependence of the concentration of impurities in the air on the distance to the place of release, wind speed, pipe height, and others. Using calculated data, it is possible to determine the time and find the place of maximum content in the atmosphere, which is especially important in the course of work to eliminate the consequences of accidents, including for the timely protection of people and domestic animals from the harmful effects of salvo emissions, and in the future - for the ecological reconstruction of contaminated land.

Keywords: Atmosphere, dispersion of contaminants, ecology, emission, mathematical model, pollution.

AMS Subject Classification: 92D40, 86-10

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MACHINE LEARNING ALTORITHMS FOR PREDICTING PROPAGATION OF IMPURITIES IN ATMOSPHERE

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The paper proposes a new approach to predicting the distribution of harmful substances in the atmosphere which is based on the combined use of the parameter estimation technique and machine learning algorithms. The essence of the proposed approach is based on the assumption that the concentration values predicted by machine learning algorithms at observation points can be used to refine the pollutant concentration field when solving a differential equation of the convection-diffusion-reaction type. This approach reduces to minimizing an objective functional on some admissible set by choosing the atmospheric turbulence coefficient. We consider two atmospheric turbulence models and restore its unknown parameters by using the limited-memory Broyden–Fletcher–Goldfarb–Shanno algorithm. Three ensemble machine learning algorithms are analyzed for prediction of concentration values at observation points, and comparison of the predicted values with the measurement results is presented. The proposed approach has been tested on the example of the industrial city of Ust-Kamenogorsk, Kazakhstan. In addition, due to the lack of data on pollution sources and their intensities, an approach for identifying this information is presented.

Keywords: machine learning, inverse problem, atmosphere, finite element method.

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SOFT TO CONDUCT COMPETITIONS AND EXAMINATIONS IN MATHEMATICS

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Denote jury or teacher as Organizer, contestant or examinee as Student, competition or examination as Event. Some items below are well-known, some ones were proposed and implemented by us.

(1) Event and Study modes (Soft is a kind of text-book).

(2) Several languages (with Student's option in Study: soft is a kind of dictionary).

(3) Setting option for Organizer: select a scope of tasks for Event; set time; choose showing marks immediately or after Event etc.

(4) Forming a file with Student's name, marks, date, time and other information in ciphered form.

(5) Capacity of remote participation in Event (on-line or sending (4) to Organizer).

(6) Generativity or parameterized tasks: Task does not exist before Event and is generated randomly (not only numerical data, content of task may vary too, under demand "of same difficulty level").

(7) As a consequence, Uniqueness: all Students are given different Tasks.

(8) Presentation of Task in various forms: text, drawing, picture, short videoclip, sound (earphones are necessary).

(9) Student's respond in various forms: number(s), word, action by computer mouse.

Various types of tasks

(10) Tasks which can be solved without special knowledge. Also, wording of tasks without special terms is desirable.

(11) Tasks on measuring imagery [1] and demanding approximate answers "with given accuracy". They also provide (9): "give exact or approximate answer".

(12) Interactive tasks, cf. (8).

(13) Tasks "from outside" with real or virtual objects to check skills too.

(14) Simple tasks on physics, chemistry, geography, astronomy with strict mathematical solutions (for instance, "let atomic weights be natural numbers", "let the free fall constant be $9.8m/sec^2$").

Over-task: to create a soft issuing such a variety of tasks that the solution of one of them does not give a specific hint for other tasks. Such soft for official Events should not be kept secret and would be used for current Events and as a text-book.

Keywords: soft, mathematics, competition, examination, generativity, option.

AMS Subject Classification: 97D60

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COMPUTER SYSTEM FOR DIABETIC RETINOPATHY TREATMENT SUPPORT THROUGH EYE DATA PROCESSING AND ANALYSIS

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Diabetic retinopathy is a common complication of diabetes that affects the retina, leading to potential vision loss if left untreated. Timely diagnosis and effective treatment are crucial for managing this condition. In recent years, significant advancements have been made in the development of computer systems that leverage eye data processing and analysis to support the treatment of diabetic retinopathy. These systems offer immense potential in improving patient outcomes, enhancing diagnostic accuracy, and optimizing treatment strategies. As a leading scientist in this field, I will explore the key components and benefits of these innovative computer systems. The advancements in computer systems for diabetic retinopathy treatment support through eye data processing and analysis have significant relevance in Kazakhstan. Like many countries worldwide, Kazakhstan faces the growing challenge of managing diabetes and its complications, including diabetic retinopathy. The application of computer systems in diabetic retinopathy management can have several benefits in the context of Kazakhstan:

(1) Computer systems aid healthcare professionals in diagnosing diabetic retinopathy accurately. By analyzing retinal images and using advanced algorithms, these systems detect early signs of the disease and classify its severity precisely, enabling timely intervention and treatment planning.
(2) Improved Access to Specialized Care: Kazakhstan, a large country with remote regions, overcomes challenges in providing specialized healthcare by leveraging computer systems for telemedicine and remote monitoring. This enables remote patients to receive expert opinions and consultations from ophthalmologists, eliminating the need for travel. This enhances access to specialized care, benefiting underserved areas with advanced diagnostic and treatment support.
(3) Optimized Treatment Planning: Computer systems can aid in developing personalized treatment plans for patients with diabetic retinopathy. By analyzing patient data and considering factors such as medical history, demographics, and disease progression, these systems can recommend the most suitable treatment options. This can lead to more effective and targeted interventions, optimizing patient outcomes and resource utilization.
(4) Long-term Disease Monitoring: Computer systems enable the long-term monitoring of diabetic retinopathy progression and treatment outcomes. By analyzing and comparing retinal images over time, these systems can provide valuable insights into disease progression patterns and treatment efficacy. This information can guide clinicians in making informed decisions regarding adjustments to treatment plans and interventions.

In this paper, a neural network model such as a convolutional neural network (CNN) is developed and trained using labeled retinal image datasets. The neural network learns to recognize patterns and features associated with different stages of diabetic retinopathy using the extracted features as input. In summary, the advancements in computer systems for diabetic retinopathy treatment support have relevance in Kazakhstan by improving diagnostic accuracy, enhancing access to specialized care, optimizing treatment planning, facilitating efficient resource allocation, and enabling long-term disease monitoring. By embracing and implementing these innovative technologies, Kazakhstan can enhance its diabetic retinopathy management strategies and improve the quality of care for individuals affected by this condition.

Keywords: Computer system, diabetic retinopathy, treatment support, neural networks, medical images, research, recognizing.

ARTIFICIAL INTELLIGENCE IN EDUCATION

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Artificial Intelligence (AI) technologies are being applied in a wide range of human activity fields. Education plays a special role in the social life. The readiness of the education system to adapt to new technologies is crucial for the future of our civilization. This paper aims to examine the basic activities in the education sphere, including studies, scientific and methodological work, extracurricular activities, organizational work, knowledge assessment, and the possibility of implementing AI technologies. The main theses of our conception are presented as a tutorial aimed at students in the pedagogical field. The tutorial consists of several chapters. The first chapter provides an overview of main concepts and definitions from the AI field. It also describes the history of the formation and development of AI theory, conceptual foundations, and application directions of AI technologies. The second chapter describes the top-down AI paradigm, which includes logic programming based on the resolution principle and mathematical logic. This chapter deals with different tasks and applications of logic programming as the basis for constructing mathematical models. The third chapter focuses on the bottom-up AI paradigm. It contains a detailed discussion of Neural Networks technologies and their applications in various fields, including industry, economics, healthcare, and science. The fourth chapter examines the challenges of applying AI technologies to solve problems in education, such as control of the smart campus, creation of a methodical system for teaching certain disciplines based on AI methods, studying content creation based on a hierarchical model of the subject area, development of individual learning paths, and description of the methodology of teaching in a smart classroom. The fifth chapter is focused on researching the application of humanoid robots in education. The tutorial is supplemented by a complex of laboratory sessions and practical trainings that aim to consolidate the educational material presented in the tutorial.

Keywords: Artificial Intelligence, Resolution Method, Prolog, Neural Networks, Educational activities, Smart Campus, Individual learning path, Humanoid robot

THE USE OF ARTIFICIAL INTELLIGENCE SYSTEMS IN THE FRAMEWORK OF EDUCATIONAL DIALOGUES IN LEARNING PROCESS

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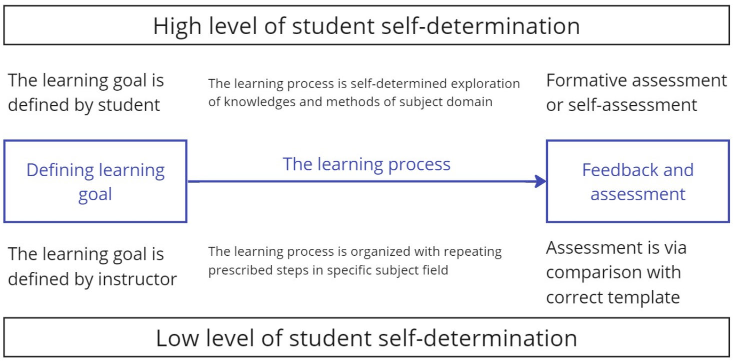
The process of technological development based on artificial intelligence is advancing rapidly, and many educational organizations are already trying to integrate systems based on artificial intelligence into the learning process.[1] The relevance of the study is related to the lack of scientific information about artificial intelligence systems application in education.

The educational process can be schematically represented as three components: goal setting, direct learning, feedback and assessment.[2] At the same time, there are different approaches to the organization of the educational process, which differ in the level of student autonomy. The traditional school class system based on the works of J.A. Comenius assumes a minimum level of student autonomy. On the other hand models of student-centered education, such as the approach based on the ideas of free education L.N. Tolstoy or A.S Neill suggest a high level of student autonomy.[3]



This research explores the application of artificial intelligence systems in learning (generative artificial intelligence and semantic analysis models) in three components of learning. For students with low levels of self-determination, the traditional approach to learning with semantic analysis AI models application is used. For students with a high level of self-determination, a student-centered approach with generative AI models application is used. The findings of the study relate to the questions of how the use of artificial intelligence systems will affect the subject and metacognitive learning outcomes in different approaches.

Keywords: Artificial intelligence, student-centered learning, self-determination level, generative AI, semantic analysis AI.

AMS Subject Classification: 97B40

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COMPUTER TEACHING OF TURKIC LANGUAGES: MATHEMATICAL MODELS OF SPATIAL NOTIONS

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Consider the phrase in Kyrgyz language "kuş taktanın *üstünön* uçup ketti" .

It can be translated word-by-word into other Turkic languages but cannot be translated into English (a computer translation "the bird flew over the board" is wrong), because its sense is

"the bird had flown away from the *upper space* of the board"

and the word "*üst*" and corresponding words in other Turkic languages are nouns, they are inflected by Genitive, Dative, Locative, Ablative cases. Also, it has subspaces: "kuş taktanın *üstünün* sol jagında" - "the bird is in the left part of the *upper space* of the board".

Before developing mathematical models [1] for notions "*üst*" , "*ast*" (lower space), "*iç*" (inner space), "*sol, ong jak*" (left, right space), "*art*" (behind space), "*ara*" (space between), "*jan*" (neighbor) we conducted experiments with native speakers [2]. (Experiments on witness'es actual meaning of words were conducted in courts but we did not find ones in philological publications).

We discovered that human's "models in mind" vary sufficiently.

Denote a point and a solid in space or in plane as $P = (x, y, z)$ or $P = (x, y)$ and S, a space of interviewees' responds as V.

The space for "*jan*" can be presented as $V = \{P|dist(P, S) < (0.2..0.4)S\}$, thus it is a fuzzy set by L.Zadeh.

The space for "*üst*": if S is a horizontal segment $\{0 \leq x \leq 1; y = 0\}$ then responds are evident: $V = \{(x, y)|0 < x < 1, y > 0\}$;

if S is a vertical segment $\{x = 0; -1 \leq y \leq 0\}$ then $V = \{(x, y)|y > 0; |x| < (0.1..3.0)y\}$ is a very fuzzy set;

for S be an inclined segment $\{0 \leq x \leq 1; y = x\}$ we discovered the alternation: $V = \{(x, y)|0 < x < 1; (y > 1)\, or\, (y > x)\}$. It can be explained by alternation of notions $\forall, \exists$ in mind.

Some interviewees included "*jan*" into "*sol jak*" and into "*ong jak*", other ones do not do it.

The lowers know that testimonies of honest witnesses can vary significantly. Our experiments confirmed it.

These results can be used for further development of such computer presentations and learning languages.

Keywords: mathematical model, language, computer model, independent presentation, teaching

AMS Subject Classification: 97D80

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ALGORITHM FOR ASSESSING THE CREDITWORTHINESS OF ENTERPRISE USING THE METHOD BASED ON FUZZY LOGIC

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This article considers the peculiarities of lending to small businesses. To define the expediency of small business crediting we considered the set of evaluation indexes: branch and regional specifics, activity of small business and financial-economic indexes, peculiar for service and trade sphere.

An approach to assessing creditworthiness using fuzzy logic mathematical apparatus is proposed, which allows taking into account approximate qualitative information about the characteristics of the borrower. The theory of fuzzy sets and fuzzy logic are an effective tool for formalizing qualitative and approximate concepts based on linguistic models and representing knowledge in the form of production rules "If ... then ...". In this case, knowledge-based inference is based on fuzzy inference.

The indicators take on rather arbitrary values. More precisely, each parameter changes in a certain interval inherent in it. Further, we assume that the parameters have been unified, i.e. the corresponding intervals are mapped onto the interval $[0, 1]$.

The rules on the basis of which decisions are made have the form $\varphi(x_1, \ldots, x_n) \to \psi(y_1, \ldots, y_m)$. Parameters $x_1, \ldots, x_n$ are obtained as a result of the analysis of the enterprise. These are the indicators mentioned in section 1. Parameters $y_1, \ldots, y_m$ are predicted. These are indicators such as: loan amount, interest rate, loan duration. In its most general form, one parameter is predicted called the credit index, varying from 0 to 1 and having a natural interpretation. Preference is given to enterprises with a higher credit rating

Keywords: SME lending, fuzzy logic, linguistic variable.

A NEW APPROACH TO THE APPROXIMATE SOLUTION OF DIFFERENTIAL EQUATIONS

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Analytical methods have a relatively low degree of universality, i.e. focused on solving rather narrow classes of problems. Approximate analytical methods (projection, variational methods, small parameter method, operational methods, various iterative methods [4,9]) are more universal than analytical ones. Numerical methods (finite difference method, method of lines, control volume method, finite element method, etc) are very universal methods. Probabilistic methods (Monte Carlo methods) are highly versatile. Can be used to calculate discontinuous solutions. However, they require large amounts of calculations and, as a rule, lose with the computational complexity of the above methods when solving such problems to which these methods are applicable.

Probabilistic methods (Monte Carlo methods) can be used to calculate discontinuous solutions. However, they require large amounts of calculations and, as a rule, lose with the computational complexity of the above methods when solving such problems to which these methods are applicable.

The proposed method of moving nodes combines numerical and analytical methods [1-3]. In this case, we can obtain, on the one hand, an approximately analytical solution of the problem. On the other hand, this method allows one to obtain compact discrete approximations of the original problem for simple cases. The aim of the study is to develop a computing technology based on the proposed method of moving nodes and develop some applications.

Applications of the method of moving nodes include: 1) approximate-analytical solution of initial and initial-boundary problems of differential equations; 2) construction of compact discrete circuits on their basis; 3) calculation of the approximation error of differential equations when they are replaced by discrete equations.

Keywords: moving node. differential equation, approximate, boundary value problem.

AMS Subject Classification: 65L05, 65N06 https://mathscinet.ams.org/msnhtml/msc2020.pdf.

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RESEARCH ON CRYPTOGRAPHIC SECRET SHARING SCHEMES

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This article presents a research study on cryptographic secret sharing schemes. Secret sharing schemes are an important area of cryptography designed for secure distribution of secret information among multiple participants. The aim of this research is to provide an overview of various cryptographic secret sharing schemes and analyze their advantages and limitations.

The paper presents a review of the basic concepts and definitions associated with secret partitioning schemes. A review of various existing cryptographic secret sharing schemes, such as Shamir's secret sharing scheme based on polynomial interpolation, Asmuth-Bloom secret sharing scheme based on Chinese remainder theorem, and Blackley's secret sharing scheme was performed. Algorithms were compiled for all schemes and implemented in the Python programming language.

In addition, the advantages and limitations of each secret sharing scheme are analyzed, including their resistance to different attacks, effectiveness, and applicability in different scenarios. Current challenges and future research directions for cryptographic secret sharing schemes are also discussed.

Keywords: Group Key Management, Secret Sharing, Key Information Recovery, Shamir's Scheme, Chinese Remainder Theorem.

AMS Subject Classification: 94A62

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AN EFFECTIVE METHOD OF CONVERSATION NUMBERS FROM RESIDUE NUMBER SYSTEM TO POSITIONAL NOTATION

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One of the most widely used non-positional number systems is the Residue Number System (RNS) [1]. It is especially effective in applications and systems where frequent addition, subtraction and multiplication operations are required [2]. This is due to the parallel execution of operations at the level of bits and lack of need for digit transfer between numbers.

In the Residue Number System, any positive integer A is represented by a set of non-negative residues $(x_0, x_2, \dots, x_n)$ from dividing this number by predefined positive integers $(p_0, p_2, \dots, p_n)$, called bases or modules.

Thus, the number X is written in the RNS in the following form:

$$X = (x_0, x_2, \dots, x_n)\,. \tag{1}$$

Reductions "(1)" x_i can be calculated by the formula:

$$x_i = X - \left\lceil \frac{X}{p_i} \right\rceil \cdot p_i. \tag{2}$$

To convert numbers from the Residue Number System back to the positional notation algorithms are required. This paper researches methods of converting numbers from RNS to the positional notation. The main methods are the CRT based method, the Interval Method, the method of conversion to the Weighted Number System (WNS), the Diagonal Function (DF) method and the Approximate Method (AM). To confirm our results, we present a study of the performance of the considered methods in Python. Our analysis has shown the advantage of our proposed method in $\frac{1}{n}$ times in comparison with the approximate method, where n is the number of bases of the system.

Keywords: residue number system; Chinese remainder theorem; approximate method; core functions; mixed number system; non-modular operations.

AMS Subject Classification: 11A07

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EIGENVALUES AND VECTORS COMPUTATION BY THE FADDEEV METHOD IN A COMPUTER MATHEMATICS SYSTEM

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In spite of the fact that the problem of eigenvalues has a rather simple formulation and the theory of its solution is known and has been developed in the works of J. López-Bonilla [1], and the algorithms of calculation methods are considered by J. C. Gower [2], Shui-Hung Hou [3], [4], G. Helmberg [5] determination of exact solutions makes a wide range of complex problems [6]. The paper proposes a code to calculate eigenvalues and of eigenvectors by D.K.Faddeev's method in the system of computer mathematics Maple. This system has been chosen on the basis of analysis and optimization of algorithms, oriented to the performance of computational operations, this system represents its highly efficient capabilities. The practical implementation of D.K.Faddeev's method in Maple demonstrates the effectiveness of the solution, automation and speed of computational operations, which gives an opportunity to application in numerous practical applications.

Keywords: fourth-order hyperbolic equations, impulsive partial differential equations, non-local boundary, impulsive actions, impulse systems.

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AL-FARABI'S PLACE OF HONOR IN THE SCIENTIFIC WORLD AND THE INFORMATIZATION OF MATHEMATICAL EDUCATION BY HIS SCIENTIFIC LEGACIES

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In the report, the role of Al-Farabi's scientific heritage in the development of mathematics and the formation of mathematical science is determined by the historical processes that took place in the Mediterranean basin. On their basis, it can be proved that with regard to the emergence of the first mathematical facts, concepts and methods, priority belongs to the Babylonian peoples and other people of Turkic-Persian origin bordering with them, who had a stone-stone script and a sexagesimal number system since the Stone Ages.

The second priority place for the development of mathematical art is given to the ancient Egyptian-Alexandrian peoples of the North African Mediterranean coast. The development of mathematical science originates again from the Babylonian Persian-Turkic peoples, but its rapid development and thorough formation takes place in the period from the VI century BC to the VI century AD in the Egyptian-Alexandrian kingdom, which replaced the Pharaonic government of the state. There were scientific schools of Pythagoras of Samos, Hippocrates of Chios, Euclid-Uklidis, etc.

Since the VII century AD, the heirs of the Babylonians have dominated the development of mathematical science: the Arab-Bagdat and Central Asian peoples of the Islamic faith. Islamic religious and scientific higher schools were formed - madrassas, observatories and the house of wisdom of Bagdat. The "Constellation of the East" appeared headed by Al-Khorezmi (IX), Al-Farabi (X-XI), etc. On the contrary, mathematical science did not develop in feudal Europe until the XV century AD.

In Western Europe in the XII–XV centuries, mathematical scientific thoughts "Constellation of the East" were translated from Arabic into Latin, universities were opened, situations of the emergence of modern mathematics Descartes- Newton-Leibniz, etc. were created.

Aristotle of the East, the scholar of the encyclopedist Al-Farabi is legitimately considered one of the leading founders of the "quadriruem" of scientific arts: 1) arithmetic, 2) geometry, 3) astronomy and 4) music, which were studied at the first universities in Europe. This determines the honorable place of Al-Farabi, a native of the Otrar Kazakh steppe, in the mathematical scientific world. His name is worthy of perpetuation in the Kazakh national mathematical education. The report reasonably lists the ways of informatization of education with its scientific legacies.

Keywords: al-Farabi, historical facts, history of mathematics and mathematicians.

AMS Subject Classification: 01A17.

TEACHING AND LEARNING IN THE ERA OF GENERATIVE ARTIFICIAL INTELLIGENCE

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The half-century history of the emergence of mobile phones, the early use of logarithmic scales in ancient times, and the recent experience of using calculators have shown numerous revolutionary drivers of scientific and technological progress. Despite some societal skepticism, these technologies have found their place in everyday life and the educational process. The evolutionary development and instantaneous impact of ChatGPT and other generative artificial intelligence models on various fields of activity highlight their relevance and the necessity of integrating them into the education system as EdTech. Therefore, it is important to comprehensively explore the opportunities provided and the influence of cutting-edge teaching and learning technologies. Adapting the learning process to the needs of the digital generation, and considering new educational technologies is unquestionably necessary.

This research presents the possibilities of EdTech based on generative artificial intelligence models. Using ChatGPT as an example demonstrates how artificial intelligence can be utilized in the pedagogical and research activities of educators. The teacher plays a crucial role in shaping and developing the personality of each student. They can bring personal perspectives, motivate, inspire, and provide personalized learning. Pedagogical professionalism, intuition, and adaptability to different situations remain integral parts of the educational environment.

Therefore, implementing artificial intelligence aims not to replace teachers completely, but to enrich their professional activities and create a more effective and personalized learning environment. The combination of teacher competencies and artificial intelligence capabilities enables optimal results in education and prepares students for the modern information society.

As a result of our research, the following tasks were accomplished: 1) identification of the most promising methods for integrating AI technologies into the professional activities of educators; 2) determination of the correlation between the teacher's professional activities and the solutions offered by artificial intelligence (AI); 3) development of approaches to implementing AI solutions in the professional activities of teachers.

Keywords: AI solutions, ChatGPT, higher education, education, professional activities of teachers.

AMS Subject Classification: 68

CURRENT ASPECTS AND TECHNOLOGIES OF ARTISTIC MODELING IN COMPUTER GRAPHICS

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In the conditions of the modern world, a person is increasingly faced with large volumes of information, which are quite difficult to interact with, if they are not presented in a convenient form, therefore today it is difficult to imagine life without tables, diagrams, graphs, charts, pictures and video materials, because these tools presentation information helps us to visualize any data. Graphic information in a computerized environment is the final product of computer graphics — an image. Computer graphics is a branch of computer science that studies methods and means of presenting and visualizing data in graphic form by means of hardware and software. This is the fastest growing segment in the field of information technologies. Digital graphic representation finds its application in various areas of human activity. Among them, it is possible to distinguish: researchers in various scientific and applied fields; for computer specialists; artists; designers; designers; developers of advertising production. Another example is the production of jewelry: with the help of computer graphics, it is possible to completely design the product, set its size, color, and indicate which metals will be used in the production of the final product. Modern technologies use various types of artistic modeling in the design of devices and appliances. This allows the use of computer programs and 3d technologies based not only on real calculations, but also on the imagination and ideas of the authors. In this way, images of items and accessories produced in various industries are created. Artistic videos are always in demand, and modeling allows you to see the finished image on the PC screen. Further changes may be made later.

Keywords: image, computer graphics,picture,icon

AMS Subject Classification:94A08 Image processing(compression, reconstruction in information and communication theory)[See also 68U10]

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IMPLEMENTATION OF THE ZABBIX SYSTEM FOR REMOTE MONITORING OF THE OPERATION OF THE EQUIPMENT OF THE TELEVISION AND RADIO BROADCASTING CENTER

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Annotation. Many modern television and radio broadcasting enterprises have local networks, which include: servers, access points, individual workstations and other network equipment that perform auxiliary and at the same time very important functions. As a rule, the local network of enterprises has one or more exit points to external global networks, including the internet. Many companies and enterprises are constantly developing their network infrastructure by adding new servers and network equipment to create additional information resources.

Every day, new, high-performance technical solutions appear that make it possible to increase the efficiency of the network infrastructure. The use of one high-tech solution in enterprises of medium and large sizes makes it possible to significantly reduce the costs of maintaining a bulky and scattered architecture, which is often created on the basis of various products with interaction problems. But along with the positive aspects of these systems, several negative factors can be distinguished:

- large financial costs for reorganization; - complexity of accident diagnostics; - the need for additional training of employees. Given the high cost of such solutions, it is necessary to create conditions for stable operation of the equipment and exclude the possibility of its failure, which will lead to large restoration costs.

These tasks can be solved by centralized monitoring and data transmission network management systems. There are many ready-made systems, both freely distributed and commercial, but before introducing any of them into the production process, it is necessary to conduct a thorough analysis and take into account all the risks associated with the use of such systems in their own infrastructure. This article will be discussed above about the introduction of the Zabbix system, which is used for remote monitoring at Radio and television broadcasting stations of the Aktobe region.

Keywords: TV and radio broadcasting system, remote monitoring, centralized monitoring, Zabbix system.

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СТОХАСТИКАЛЫҚ СТОКС-ДАРСИ МОДЕЛІН БӨЛШЕК РЕТТІ ЖАЛПЫЛАУДЫҢ САНДЫҚ ӘДІСІ

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Бұл жұмыс гидрологияда, мұнай өнеркәсібінде және биомедицинада маңызды ғылыми, қолданбалы және экономикалық мәні бар стохастикалық Стокс-Дарси моделінің бөлшек ретті жалпылауын тиімді сандық енгізуге бағытталған. Стокс-Дарси біріккен теңдеулерімен сипатталған модель жер үсті және жер асты ағындарының өзара әрекеттесу мәселелерінде, кәрісті кеуекті құрылымы бар мұнай қабаттарында, жер асты жүйелерінде болатын процестерді болжауда, кәрсті сулы горизонттары және т.б кеңінен қолданылуына байланысты е сұйықтық динамикасын есептеуде маңызды орын алады.

Бұл модельде сұйықтықтың еркін ағыны Стокс теңдеуімен, ал кеуекті ортадағы ағын Дарси теңдеуімен сипатталады . Стохастикалық Стокс-Дарси моделінің маңызды кемшілігі сұйықтық ағынының сипатына айтарлықтай әсер ететін кеуекті ортаның өте маңызды қасиетін - жадыны елемеу болып табылады [1-4]. Бұл қасиетті ескеру қажеттілігі сұйықтық ағыны кезінде ортаның кеуектілігі мен өткізгіштігінің өзгеруіне, уақыт өте келе ағын жылдамдығының баяулауына әкеледі. Сондықтан сұйықтық ағынының сипаты тек кеуекті ортаның ағымдағы күйімен ғана емес, сонымен бірге оның барлық бұрынғы күйлерімен де анықталады. Осы бағыттың маңыздылығын ескере отырып, бұл мақалада Стокс-Дарси стохастикалық моделінің одан әрі жалпылауы зерттеледі, ол меди қасиеттерінің ұзақ мерзімді өзгерістерін ескереді. Біздің жұмысымызда жадты есепке алу бөлшек дифференциалдық есептеуді қолдану арқылы, атап айтқанда Капуто анықтамасы мағынасында уақытқа бойынша бүтін ретті туындыларды бөлшек ретті туындылармен ауыстыру арқылы жүзеге асырылады. Соңғы 3-4 жылдағы зерттеу жұмыстары [5,6] бүтін уақыт туындылары бар Стокс-Дарси теңдеулері үшін жоғары ретті сандық схемаларды құруға бағытталған. Бұл жұмыста уақыт бойынша бөлшек ретті туындылы Стокс-Дарси теңдеулерін шешу үшін $3-\alpha$ ретті сандық схема құрылды және есептеуді тездету үшін параллел алгоритм жүзеке асырылды.

Бұл жұмыс Қазақстан Республикасының Ғылым және жоғарғы білім министрлігінің қаржыландыруымен жүзеге асырылған, грант ИРН AP14871299.

Кілттік сөздер: Стокс-Дарси моделі, бөлшек ретті туынды, параллель алгоритм, жинақтылық реті, ақырлы элементтер әдісі

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ҚОСЫМША БІЛІМ БЕРУДЕ ВИЗУАЛДЫ ПРОГРАММАЛАУҒА ОҚЫТУДЫҢ ЦИФРЛЫҚ ОРТАСЫ

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Мектепте қосымша білім беру оқушылардың әлеуметтік бейімделуін және бос уақытын тиімді ұйымдастыруды қамтамасыз ететін олардың бейінін, қабілеттері мен қызығушылықтарын, әлеуметтік және болашақ кәсіби іс-әрекеттерін өздігінен анықтауға мүмкіндік беретін факторлардың бірі болып табылады.

Информатикадан қосымша білім беру барысында оқушылардың бейімділігі, жеке тұлғалық ерекшеліктері мен қызығушылықтарына сәйкес келешек оқыту бейімін саналы таңдау; нақты бейін бойынша оқуға деген мотивациясы деңгейінің артуы; жоғары сыныптарда білім беру бейінін таңдауға бағытталған іс-әрекеттегі практикалық тәжірибені қалыптастыру жүзеге асырылады.

Информатикадан қосымша білім беру жүйесінде ойындарды жасауға оқытудың мақсаты – оқушыларға ойын ортасы мен интерактивті технологияларды пайдаланып, қызықты және тиімді оқу тәжірибесін құруға қажетті білім беріп, дағдыларын қалыптастыру болып табылады.

Бастауыш және орта мектеп оқушыларының программалау біліктіліктерін қалыптастыру мақсатында Scratch визуалды объектіге бағытталған программалау ортасы пайдаланылады.

Информатикадан қосымша білім беру жүйесінде үйретуші ойындарды жасауға оқыту үшін қазақ халқының тарихына, мәдениетіне оқушылардың қызығушылықтарын арттыру және ұлттық құндылықтарымызды дәріптей отырып, киіз үйдің көшпелі ұлтымыздың өміріндегі орыны мен оның пайда болу тарихы, сонымен қатар құрастыру технологиясы, оның бөліктері және әрқайсысының мәнін, түсіндіре отырып, Scratch-те цифрлық орта жасалды [1].

Оқушылар аталмыш цифрлық ортада киіз үйді жинап, оның бөліктерін құрастыру ретін түсініп, Scratch-те визуалды түрде құрастыруды үйренеді және олардың программалау біліктіліктері мен дағдысы қалыптасады.

Оқушылар тек визуалды программалаумен жұмыс жасап қана коймай, осы цифрлық орта арқылы өз қиялындағы ғажайыптарды шындыққа айналдырып, өзгелердің де сол қиялдың көрермені болуына мүмкіндік туады [2].

Keywords: визуалды орта, цифрлық орта, программалау, оқушылар

Әдебиеттер тізімі

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МОДЕЛИРОВАНИЕ ПОДГОТОВКИ ПЕДАГОГОВ К ИСПОЛЬЗОВАНИЮ ИММЕРСИВНЫХ СИСТЕМ В КАЧЕСТВЕ ОБЪЕКТА И СРЕДСТВА

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Иммерсивные технологии, включающие в себя дополненную реальность, дополненную виртуальность и виртуальную реальность позволяют организовать для человека погружение в недоступную среду или работу с недоступными объектами. Такие технологии могут существенно повысить эффективность различных ступеней и направлений образования, а также повысить производительность определенных видов деятельности.

Однако ключевым условием внедрения таких технологий в образовательный процесс является подготовка педагогов к использованию иммерсивных систем в качестве объекта и средства обучения. При этом, в зависимости от направления и ступени образования методики применения и обучения должны существенно отличаться.

Так, в школах целесообразно знакомить детей таким системам на уроках информатики для последующего применения в качестве средства обучения, а также для дальнейшего использования в быту и работе. Разработке иммерсивных систем целесообразно обучать на внеурочных занятиях, либо на специализированных направлениях ВУЗов и СПО. Учителей же следует готовить работе с такими средствами, методике использования и базовой разработке образовательного материала.

Keywords: информатизация образования, иммерсивные технологии, технология виртуальной реальности, технология дополненной реальности, технология дополненной виртуальности.

AMS Subject Classification: 68-00 General reference works (handbooks, dictionaries, bibliographies, etc.) pertaining to computer science

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ИНФОРМАТИЗАЦИЯ НАЧАЛЬНОГО ОБРАЗОВАНИЯ В КАЗАХСТАНЕ

АЛЬМИРА ИБАШОВА, ДАМИРА БЕЛЕСОВА

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Глобальная информатизация социального пространства является одной из закономерностей развития общества.

От степени технологического развития каждой страны зависит не только ее экономическое могущество и уровень жизни населения, но и положение этой страны в мировом сообществе, возможности экономической и политической интеграции с другими странами, а также решение проблем национальной безопасности. В то же время уровень развития и использования современных технологий в той или иной стране определяется не только развитием материальной базы, но, главным образом, уровнем интеллектуализации общества, его способностью производить, усваивать и применять новые знания. Все это самым тесным образом связано с уровнем развития образования в стране и с проблемами информатизации образования.

На современном этапе развития образования все больше внимания уделяется информатизации начального образования.

Одна из главных причин заключается в том, что современное общество предъявляет новые требования к вступающему в жизнь поколению. Нужно иметь навыки, чтобы планировать свою деятельность и находить информацию, необходимую для решения проблемы, а также строить информационную модель изучаемого объекта или процесса, эффективно использовать новые технологии. Развитие детей младшего школьного возраста с помощью работы на компьютере, как показывает отечественный и зарубежный опыт, является одним из важных направлений современной педагогики.

Целью информатизации начального образования в Казахстане является создание условий для повышения качества образования и развития информационной грамотности учащихся. Это позволяет расширить доступ к образованию и обеспечить учащимся современные навыки.

В целом, информатизация образования в начальных классах способствует развитию учащихся, обеспечивает им широкий доступ к знаниям и улучшает качество образования в соответствии с требованиями современного информационного общества.

Ключевые слова: информатизация, начальные классы, образование, информационная грамотность, информационное направление, алгоритмическое направление.

Предметная классификация AMS: 68

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ҚАЗАҚ ЕРТЕГІЛЕРІН ЦИФРЛАНДЫРУ – БАЛАЛАРДЫ АДАМИ ҚҰНДЫЛЫҚТАРҒА ТӘРБИЕЛЕУДІҢ БІР БАҒЫТЫ РЕТІНДЕ

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Ертегілердің тәрбиелік мәні балаларды ұлттық құндылықтарға, сыни тұрғыдан ойлауға, қиялға баулуға, үлкен және ақылды отбасы мүшелерін құрметтеуге және әлеуметтік өзара әрекеттесуге ықпал етеді.

Қазіргі кезде қазақ ертегілерінің ескіріп қалу себептерін айқындай отырып, ол себептерге қарамастан, ертегілер балаларды тәрбиелеу мен дамытудың маңызды көзі болып қала беретініне, сондықтан оларды оқуды және білім беруде пайдалануды кеңінен насихаттауды жалғастыру маңызды екені түсінікті.

Біздің жобамыздың мақсаты – қазақ ертегілерін цифрлық кітаптар, анимациялық туындылар, мультфильм ретінде балалар назарына ұсыну. Цифрлық кітапта ертегі мазмұны мәтінмен қоса суреттер, анимациялық суреттер, дыбыс қосу мүмкіндіктерімен ұсынылады. Цифрлық кітапты дайындауға қазіргі кезде көптеген мүмкіндіктері мол BookCreator, Canva, FlipPDF, т.с.с. платформалар мен сервистерді пайдалануға болады.

Ертегілерді анимациялық туынды ретінде дайындауға Scratch бағдарламалау ортасын және Animaker онлайн бейне жасау платформасын пайдаландық. Бұл платформалар – өзінің дайын кейіпкерлерімен, шаблондарымен анимациялық туынды жасаушының жұмысын бірнеше есе жеңілдетеді.

Ертегіні мультфильм түрінде ұсынуға FlipaClip, Adobe Animate анимация құралдары пайдаланылды.

Негізгі мақалада дайындалған цифрлық кітаптардың, анимациялық туындылардың, мультфильмнің дайындалу технологияларын сипаттаумен қатар, студенттердің педагогикалық практика кезінде оқушыларға осы бағытта үйірмеде бірлесе жұмыс жасағандары баяндалады.

Анимация дайындау сияқты шыдамдылықты талап ететін еңбекті үйренудің және ертегілерді цифрландырудың маңыздылығы айқын. Мақалада оқушылардың да, студенттердің де анимациялық туынды дайындауда ертегілердің тәрбиелік мәнін талқылаумен бірге, қызығушылықтары артып, нәтижеге қол жетуге ұмтылыстары туралы талқыланады.

Кілттік сөздер: Ертегі, ұлттық құндылық, тәрбие, білім беру, мультфильм.

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ОПЕРАЦИЯЛЫҚ ЖҮЙЕЛЕРДІ ОҚЫТУДА ВИРТУАЛИЗАЦИЯНЫ ҚОЛДАНУ ЕРЕКШЕЛІКТЕРІ

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Виртуализация – физикалық компьютерде бір уақытта бірнеше операциялық жүйелерді іске қосу тәсілі. Виртуализация чиптегі ендірілген жүйелерден бастап ірі деректер орталықтары мен бұлттық орталарға дейін кез келген дерлік машинада жасалуы мүмкін. Бір қарағанда виртуализация қарапайым болғанымен тым күрделі болатын мүмкіндіктерді ашады.

Виртуализация - виртуалды компьютерлік жүйелерді бір физикалық компьютерде іске қосу тәсілі. Қазіргі уақытта виртуализация негізгі процессор конструкциялары арқылы жүзеге асатын технология, сондай-ақ виртуализация үшін қажетті бағдарламалық жасақтама және қазірдің өзінде өндіріс ортасының ажырамас бөлігі болып табылады. Бұл жағдайда компьютерлік виртуализация болашақ информатика мұғалімдерін даярлау білімінің ажырамас бөлігі болуы керек [1]. Виртуализация классикалық аппараттық құралға қосымша бірнеше мүмкіндіктерді ұсынатын, бірнеше операциялық жүйелерді бір уақытта пайдалану және іске қосу; бағдарламалық құралды орнатуды жеңілдету; тестілеу және қалпына келтіру; инфрақұрылымды біріктіру және бұдан да басқа мүмкіндіктерді беретін технология [2].

Осыдан мақаланың мақсатын айқындауға болады. виртуализацияға және оның қосымшаларына шолу және талдау жасау, сондай - ақ оның болашақ информатика мұғалімін дайындау бағдарламасы бойынша операциялық жүйе пәнін оқытудағы артықшылықтарын көрсету.

Виртуализацияның артықшылықтарымен бірге болашақ информатика мұғалімдерін дайындайтын тіпті осы курсты өтетін барлық мамандықтағы студенттердің осы жаңа технологияны игеру ауыртпалығы туындайды [3]. Бұл тапсырмаларды сәтті орындау үшін оқытудың дұрыс платформаларын таңдамас бұрын көптеген ақпаратты талдау қажет және берілетін операциялық жүйе курсы бойынша оқу-әдістемелік ұсыным жасалу керек. Біз виртуализацияға және оның қосымшаларына қысқаша шолу мен талдау жасадық, сонымен қатар оның операциялық жүйе пәнін оқытуда қолдану жолдарын қарастырдық.

Кілттік сөздер: көптік операциялық жүйелер, виртуализация, деректер орталығы, бұлт, информатика білімі.

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КОЛЛЕДЖДЕ АҚПАРАТТЫҚ-КОМУНИКАЦИЯЛЫҚ ТЕХНОЛОГИЯНЫ ҚОЛДАНУ ЕРЕКШЕЛІГІ

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Білім беруді ақпараттандыру, білім салаларының барлық қызметіне ақпараттық технологияны енгізу және ұлттық модельді қалыптастыру қазақстандық білім беруді сапалы деңгейге көтерудің алғы шарты. Ақпараттандыру технологиясының дамуы кезеңінде осы заманға сай білімді, әрі білікті мамандар даярлау - мұғалімнің басты міндеті. Қоғамдағы ақпараттандыру процестерінің қарқынды дамуы жан-жақты, жаңа технологияны меңгерген жеке тұлға қалыптастыруды талап етеді.

Жоғары кәсіптік білім беруді ақпараттандыру мәселелеріне арналған ғылыми және ғылыми-әдістемелік әдебиеттердегі ақпараттық іздеу, осы кезге дейін «Оқытудың ақпараттық технологиясы» түсінігінің дәл, нақты анықтамасы жоқ екенін көрсетті. Осы түсініктерге байланысты әртүрлі дерек көздерден «Жаңа ақпараттық технологиялар», «Компьютерлік оқыту технологиясы», «Компьютерлік-педагогикалық технологиялар» және т.с.с. синоним сөздерді кездестіруге болады. «Жаңа» сөзі педагогикалық дерек сөздерде жиі қолданылады. Бұл жерде негізгі ой түрлі салалардың сонымен қатар педагогикалық саланың да құрамын түбегейлі өзгертетін жаңашылдық жайлы. Оқытудың компьютерлік технологиясына өту, оларды жасап шығаруға сынақтан өткізу мен енгізілуіне жағдай жасау, олардың дәстүрлі әдістермен сәйкес келу тәсілдерін іздеу өте күрделі болып, олар көптеген психологиялық-педагогикалық, оқу-әдістемелік және басқа да мәселелерді шешуді талап етеді. Олардың ішіндегі негізгі бағыттар: 1. Білім беру үдерісіне компьютерлік технологияларды енгізу мәселесіне қатысты бірыңғай ғылыми-әдістемелік тәсілді енгізу жолдары. 2. Практикалық тұрғыда оқу үдерісінде компьютерлік технологияны қолдану әдіс-тәсілдерін ұсыну. 3. Болашақ мұғалімдерді компьютерлік технологияны меңгеруге даярлау жолдарын теориялық және әдістемелік тұрғыда көрсету. 4. Пайдаланушылардың білім мен біліктілігін арттыру мақсатында, болашақ мұғалімдерді компьютерлік технологияларды оқытуға даярлау. 5. Білім беру мекемесін материалдық-техникалық базасын жабдықтау. 6. Қажетті әдістемелік құралдарды іздеу, өңдеу және жасап шығару. Электрондық оқулықты пайдалану мұғалімнің де ғылыми-әдістемелік потенциалын дамытып, оның сабақ үстіндегі еңбегін жеңілдетеді. Оқытудың әр сатысында компьютерлік тесттер арқылы оқушыны жекелей бақылауды, графикалық бейнелеу, мәтіндері түрінде, мультимедиалық, бейне және дыбыс бөлімдерінің бағдарламасы бойынша алатын жаңалықтарды іске асыруға көп көмегін тигізеді.

Кілттік сөздер: ақпараттандыру, компьютерлік технология, электрондық оқулық, білім беру, оқыту. **AMS пәндік классификация:** 68-00

ТУРБУЛЕНТНОЕ ТЕЧЕНИЕ ИЗОТЕРМИЧЕСКОЙ ЖИДКОСТИ В ТРУБЕ

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Для течения жидкости в длинных цилиндрических трубах критическое число принято равным $Re \approx 2300$. Установлено, что при значениях $Re < 2300$ режим течения жидкости ламинарный, течение при $2300 < Re < 10000$ называется неустойчивым турбулентным, при $Re > 10000$ – развитым турбулентным. Следует однако отметить, что экспериментально было определено, что критическое значение числа Re в цилиндрических трубах может доходить до $Re \approx 20000$. Такие высокие значения критического числа Re обусловлены особыми условиями проведения опытов: постоянной температурой, стабилизацией расхода, отсутствием возмущений потока, очень малыми значениями шероховатости стенок и т.д. Для идеально равномерного профиля скорости на идеально гладкой поверхности критическое число Re стремится к бесконечности. На практике принято считать турбулентным поток при $Re > 2300$, однако при наличии дополнительных источников возмущений ламинарное течение заканчивается при гораздо более низких значениях чисел Рейнольдса. Основные результаты исследований по переходу ламинарного течения в турбулентное получены экспериментальными работами. Теория перехода ламинарного течения жидкости в турбулентное развита слабо.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССА НЕРАВНОВЕСНОЙ ФИЛЬТРАЦИИ НЕСЖИМАЕМОЙ ЖИДКОСТИ

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Моделирование фильтрации многофазной жидкости имеет большую экономическую значимость в нефтяной промышленности, гидрологии, секвестрации углерода и управлении ядерных отходов. Данные модели лежат в основе гидродинамических симуляторов, используемых при разработке нефтяных месторождений, позволяя проводить прогнозные расчеты показателей разработки. В силу сложности данных моделей, вычисления для одного месторождения могут длиться от нескольких часов до нескольких недель. Несмотря на то, что высокопроизводительные параллельные вычисления успешно внедрены во все коммерческие гидродинамические симу-ляторы, исследования, направленные на создание и математическое обоснование параллельных алгоритмов решения задач фильтрации не перестают проводиться. Процессы фильтрации принято подразделять на равновесные и неравновесные. К равновесным относятся процессы, при которых отсутствуют межфазные переходы вещества и химические превращения. Неравновесная фильтрация сопровождается фазовыми переходами и химическими реакциями [1]. Динамика протекания фильтрационных течений многофазной жидкости нелинейным образом зависит как от структурно-механических свойств пористой среды и жидкостей, так и свойств окружающего скелета. Исследование процесса течения многофазной жидкости в пористой среде наиболее полно проведено в предположении о локальном фазовом равновесии. Однако, в реальных пластовых условиях существенное влияние на процесс фильтрации имеет свойство запаздывания насыщенности фазы, изучение которого привело к возникновению теории неравновесной фильтрации. Необходимость учета данного явления при разработке нефтяных месторождений обсуждается во многих работах [2, 3]. В настоящем докладе обсуждаются вопросы численного моделирования процесса неравновесной фильтрации. В общем случае данная задача нелинейна. Научная новизна заключается в построении двух конечно-элементных методов решения данной задачи. Получена новая постановка задачи, которая основанная на введении новой переменной «глобальное давление». В данной работе уравнение для давления и скорости фильтрации в обоих методах решается смешанным МКЭ.

Ключевые слова: метод конечных элементов, стабилизиро-ванный метод, неравновесная фильтрация.

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РАЗВИТИЕ ЦИФРОВОЙ СРЕДЫ В УСЛОВИЯХ ГЛОБАЛИЗАЦИИ ОБРАЗОВАНИЯ

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В настоящее время во всех сферах деятельности человека, включая систему математического и естественнонаучного образования, в большинстве стран накоплено огромное количество разрозненных цифровых ресурсов. Целесообразно сосредоточиться не столько на разработке новых средств и технологий, сколько на определении способов их объединения в единые комплексы не только в одном государстве, но и на региональном или мировом уровне. Неслучайно большинство ученых считают, что создание и внедрение цифровой образовательной среды (ЦОС) – это способ объединения разрозненных цифровых средств и систем, применяемых в системе образования. При подходе, основанном на унификации и интеграции разрозненных цифровых ресурсов в единые ЦОС, возникают преимущества, такие как сокращение сроков освоения цифровых средств, упрощение технической интеграции в рамках единых Интернет-порталов и коллекций, создание основы для универсальной подготовки кадров, появление новых технологий разработки, эффективных методов обучения и воспитания, ведение и информационная защита единых баз данных, экономический и управленческий эффекты. Для интеграции и унификации цифровых ресурсов можно предложить базирование цифровых средств на единых общих базах данных, принципах обмена информацией, унификацию содержания цифровых ресурсов, выработку формальных методов описания содержания образовательных областей, единообразное использование элементов математической теории графов, введение единой системы спецификаций и метаописания, создание единого комплекса требований к качеству и технологии экспертизы цифровых средств для образования. Проведение такой интеграционной деятельности способствует трансграничности образования, распространению и унификации информации о системах образования, координации развития и возможности сравнения систем образования разных стран, повышению доступности глобальных информационных ресурсов. Внедрение глобальной ЦОС будет способствовать расширению академической мобильности педагогов и обучающихся, непрерывности образования, информационному обмену. У педагогов и ученых появится возможность использовать не только национальные, но и мировые источники информации. Страны тюркского мира могли бы осуществлять фундаментальные научно-педагогические исследования путей обновления и интеграции информационных технологий, информационных ресурсов и их содержательного наполнения, унифицированных систем подготовки педагогов в области информатизации образования.

Ключевые слова:цифровые ресурсы, информатизация образования, глобализация образования, цифровая образовательная среда.

Предметная классификация AMS: 68

БОЛАШАҚ ИНФОРМАТИКА МҰҒАЛІМДЕРІНЕ ПРОГРАММАЛАУДЫ ОҚЫТУДА ВИЗУАЛДАУ ҚҰРАЛДАРЫН ҚОЛДАНУ

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Бұл мақаланың мақсаты - болашақ информатика мұғалімдерін оқытуда визуалдау құралдарын қолданудың артықшылықтары мен тиімділігін зерттеу. Болашақ информатика мұғалімдерінің программалау дағдыларын оқытуда визуализация құралдарын қолданудың әсерін олардың түсінігін жақсарту және программалау ұжырымдамаларын бекіту мақсатында талдауға баса назар аударылады. Информатика саласындағы білім берудің қазіргі дамып келе жатқан технологиялық ландшафтына көп көңіл бөлінеді. Оқушыларға программалау тұжырымдамалары мен дағдыларының берік негізін қамтамасыз етудің маңыздылығын мойындай отырып, Информатика бойынша білім беру саласындағы зерттеулер мен әзірлемелерді ілгерілетуге күш салынды. Визуалды программалауға кіріспе компьютерлік программалауды оқытудың тиімді ортасы екендігі дәлелденді және бұл тәсілді қолдау үшін көптеген білім беру құралдары әзірленді. Алайда, бұл құралдарда программалаудың маңызды аспектісі болып табылатын желілік өзара әрекеттесу тұжырымдамасы жиі болмайды. Болашақ информатика мұғалімдеріне программалауды оқытуда визуалидау құралдарын қолдану оқу процесінің тиімділігін арттыруға, программалаудың абстрактілі тұжырымдамаларын түсінуді жақсартуға және білімді білім беру қызметіндегі тәжірибеге тиімді көшіруге ықпал етеді.

Кілт сөздер: визуалдау құралдары, визуалдау, программалау.

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К ВОПРОСУ ВНЕДРЕНИЯ НАУЧНОГО НАСЛЕДИЯ АЛЬ-ФАРАБИ В СОВРЕМЕННОЕ ОБРАЗОВАНИЕ

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В своем выступлении на торжественном собрании, посвященном 75-летию Национальной академии наук, президент Республики Казахстан Касым-Жомарт Токаев, отметил, что «Наука, процветавшая на просторах Великой Степи, имеет глубокие исторические корни. Ярким тому подтверждением служит наследие Второго учителя человечества аль-Фараби».

В связи с этим не вызывает сомнений необходимость изучения богатого научного наследия аль-Фараби, его дидактического потенциала, внедрения его в современную систему образования в контексте духовной модернизации, инновационным образом обогащая содержание обучения научными достижениями ученого, прививая подрастающему поколению интерес к знаниям и шедеврам науки и привлекая их к научно-методическим исследованиям в этой области.

«Аль-Фараби правильно воссоздал философско-логический фундамент науки, обозначил порядок изучения и преподавания науки, попытался выделить предмет и содержание каждой из них. Он провел комплексные исследования музыки, сделал великие открытия в математике, оставил часть работ по астрономии, обогатил физику новыми идеями, написал работы по важнейшим областям естественных наук, таким как медицина, химия, минералогия, по мнению ученых древнего мира проанализировал передовые и непреходящие принципы. Метафизика аль-Фараби, лингвистика, логика, психология, география, этика и труды по другим наукам каждая в отдельности является вершиной достижения» [1]. Кроме того, аль-Фараби является одной из величайших фигур в истории педагогики. Он создал первую конструктивную педагогическую систему в странах Востока, написал немало работ по педагогике.

Особый интерес вызывает наследие ученого в области математики и математического естествознания, оказавшее огромное влияние на дальнейшее развитие мировой науки. В них приведены уникальные алгоритмы решения задач, важных в практической деятельности человека и достойных рассмотрения в современной системе образования. Алгоритмический подход к решению математических задач, присущий ученому, и экспериментальный подход при решении задач в области физики и естествознания, существенно облегчают их компьютерную реализацию и создание цифровых ресурсов для эффективного их усвоения.

Цель работы заключается в изучении вопросов информатизации математического наследия аль-Фариби и внедрения его в современную систему образования для обеспечения интеллектуального и духовно-патриотического развития и воспитания подрастающего поколения

Ключевые слова: научное наследие аль-Фараби, информатизация образования, цифровые образовательные технологии, алгоритмический подход.
Предметная классификация AMS:97-00

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ИНФОРМАТИЗАЦИЯ МАТЕМАТИЧЕСКОГО ОБРАЗОВАНИЯ: К СОЗДАНИЮ ЭЛЕКТРОННЫХ СРЕДСТВ ОБУЧЕНИЯ МАТЕМАТИКЕ

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На современном этапе развития общества, когда цифровизация стала неотъемлемой частью повседневной жизни человека, цифровая трансформация образования должна быть закономерным следствием изменений, протекающих в обществе. Система электронного обучения должна стать одним из основных инструментов модернизации образования.

Подобные преобразования должны привести к смене методов и технологий обучения, а также расширить перечень достигаемых результатов образовательной деятельности. Современная школа должна отвечать на вызовы информационного общества, подготавливая личностей готовых успешно взаимодействовать с обществом с целью успешной самореализации.

На данный момент система образования предусматривает развитие предметных, метапредметных и личностных результатов обучения, которых на данном этапе общественного развития уже недостаточно, так как деятельность человека становится все более интеллектуализированной. В связи с этим, необходимо обеспечить «формирование и развитие у обучаемых вычислительного, структурного, интуитивного и алгоритмического мышления», являющихся одними из важнейших навыков современного человека, обеспечивающих ему конкурентоспособное существование в общественной и профессиональной деятельности.

В последние годы происходит резкое падение мотивации молодежи к обучению с использованием традиционных бумажных и электронных учебников. В этой связи представляет интерес моделирование структуры и содержания учебных средств, адекватных когнитивным характеристикам современных учеников.

Цель – обосновать подход к созданию инновационных учебных пособий, опирающего на платформу структурно-ментальных схем и вопросно-задачный формат содержания учебной дисциплины (на примере темы школьной математики «Уравнения», 6-11 классы). Предложенный подход можно применить для других содержательных линий предметных дисциплин в школе и вузе.

Ключевые слова: Инновационные средства обучения математике, информатизация, ментальный подход, перевернутый формат, «прозрачный сундук», принцип «вертикального рычага».

Предметная классификация AMS: 94

ЦИФРОВОЙ УНИВЕРСИТЕТ: СОВРЕМЕННЫЕ ТЕНДЕНЦИИ

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Необходимость распространения информатизации по всем направлениям образовательной деятельности порождает основные задачи информатизации системы высшего педагогического образования, требующие первоочередного разрешения, обсуждаемые в настоящее время. Среди таких задач, важных для данного исследования, можно выделить следующую: формирование цифрового университета на основе разработки и внедрения цифровой образовательной среды системы высшего педагогического образования: обеспечение взаимодействия и взаимодействия информационных технологий и педагогических вузов в многоуровневом процессе подготовки педагогов, разработка научных основ управления учебным процессом и формирование распределенной базы данных по различным учебным дисциплинам и в различных предметных областях, характерных для педагогических вузов. Первые концептуальные идеи понятия «цифровой университет» начали озвучивать зарубежные ученые в начале XXI века, параллельно с появлением технологий Четвертой промышленной революции, а в Казахстане, в научных кругах начали обсуждать с 2017 года, с выходом государственной программы «Цифровой Казахстан». Ученые-исследователи оцифровки высшего образования представляют цифровой университет с точки зрения интегративного явления, которое включает в себя множество элементов. Очевидно, что цифровой университет нельзя интерпретировать только как основу «физического» университета, поскольку цифровые технологии позволяют создавать новые условия, вносить изменения в реальную среду, организовывать процессы обучения и управления. Кроме того, «физический» университет, существующий в реальном мире, а не в виртуальном, представляет собой целое пространство, в котором размещены технические и человеческие ресурсы. Другими словами, нельзя говорить о концепции цифрового университета только с точки зрения цифрового (виртуального) мира или только с точки зрения технической инфраструктуры.

Ключевые слова: цифровой университет, цифровизация образования, цифровая образовательная среда, интеграция информационных технологий.

Предметная классификация AMS: 68N99 https://mathscinet.ams.org/msnhtml/msc2020.pdf.

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УСТОЙЧИВОСТЬ ЧИСЛЕННОГО РЕШЕНИЯ РАЗНОСТНОЙ ЗАДАЧИ ДЛЯ УРАВНЕНИЯ СЕН-ВЕНАНА

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В данной работе рассматриваются вопросы, связанные с численной разрешимостью смешанной задачи для систем уравнений Сен-Венана. Для решения этой задачи применяется противопоточная разностная схема и метод расщепления по младшим членам. Была доказана теорема об экспоненциальной устойчивости численного решения разностной задачи в норме L_2. Это означает, что численный метод, обеспечивает устойчивость решении. Алгоритм численного расчета основан на использовании метода двухточечной прогонки, который позволяет эффективно находить численное решение. Этот метод широко применяется в расчетах систем уравнений Сен-Венана и позволяет получать надежные результаты.

Основное внимание уделено проблеме разработки алгоритма и исследованию устойчивости метода численного расчета, экспоненциально устойчивых решений уравнений Сен-Венана. Алгоритм численного решения, представленный в работе, основывается на методе матричной прогонки. Было проведено множество расчетов для задачи определения уровня и скорости воды на участке Большого Алматинского канала протяженностью 10000 метров, который находится в городе Алматы [1]. Результаты расчетов позволяют получить детальную информацию о поведении воды в данном участке канала и оценить его уровень и скорость.

В этих экспериментах была исследована устойчивость предложенной противопоточной разностной схемы для уравнения Сен-Венана[2]. Дополнительно был рассмотрен случай, когда в систему были включены произвольное трение и изменяющийся в пространстве наклон. Это привело к возникновению неоднородных установившихся состояний, что означает, что моделирование позволило изучить поведение системы при наличии этих факторов и наблюдать формирование различных стационарных режимов.

Работа выполнена при финансовой поддержке гранта КН МНВО РК №AP14872379.

Ключевые слова: уравнений Сен-Венана, гиперболическая система, устойчивость, противопоточная разностная схема.

Предметная классификация AMS: 65N12

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ОЦЕНКА ОБЪЕМОВ ВЫБРОСОВ ОТ АВТОТРАНСПОРТА ГОРОДА УСТЬ-КАМЕНОГОРСКА

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Рассмотрим вопросы оценки объема экологического воздействия автотранспортных потоков на регулярном клеточном разбиении территории города Усть-Каменогорска. Предполагаем, что в автотранспортных потоках участвует весь автопарк города, равномерно распределенный по улично-дорожной сети. Для начала наложим равномерную сетку на карту города, и используем клеточные функции улично-дорожной сети города Усть-Каменогорска, полученные в каждом квадрате разбиения. Применяя алгоритм построения клеточной функции, получим протяженности улично-дорожной сети в полосах [1]. По сведениям, полученным нами на 1 января 2021 года в городе Усть-Каменогорске зарегистрировано Общее количество автомобилей Ма.п.= 94934; Общая длина протяженности улично-дорожной сети по полосам LД=848,9 км; Предполагаем, что состав автопарка является однородным, состоящим из легковых автомобилей с бензиновыми и карбюраторными двигателями (ВМ1), средняя длина одного автомобиля Lбазы=5 м. Характеристики автотранспортных потоков клетки однозначно определяется ее функцией состояния. В свою очередь, функция состояния зависит от параметров: m – кратность узлов, q – времени красного сигнала светофора и l – расстояние между светофорами. Оценим величину li, основываясь, что где закрашенные клетки считаем загруженными участками города и расстояние между светофорами естественно меньше чем в остальных участках города. По экспериментальным данным среднее число светофоров на загруженный участок города характеризуется цифрой: 1,360937 шт./км [2]. Значит скорость движения по такой улично-дорожной сети, чем там, где участки свободного движения транспорта существенно больше. Будем считать, что для остальных клеток города справедливы оценки среднего расстояния между светофорами. По правилам дорожного движения в населенном пункте максимальную скорость берем 60 км/час. и закон «плотность-скорость» на одной полосе дороги является квадратичным. По сеточной функции мы можем определить, что по городу средний расход топлива Q=631536 л/час.≈ 467 336,7 кг/час. Расход топлива по городу за сутки – 5608040 кг/сут. Расход топлива по городу за год - 3344605,92 т/год.

Главной задачей данной исследовательской работы является, используя компьютерную программу [2], созданную на базе математической модели, создавать различные ситуации планирования правильных размещений дорог на улично-дорожной сети города и определить общий объем выбросов вредных веществ от городского автотранспорта.

Данная работа выполнена при финансовой поддержке Министерства науки и высшего образования Республики Казахстан, ИРН AP19679550.

Ключевые слова: загрязняющие вещества, линейный источник, автотранспорт, валовый источник.

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МОДЕЛЬ ДИАГНОСТИКИ STEAM-КОМПЕТЕНТНОСТИ БУДУЩИХ ПЕДАГОГОВ-БАКАЛАВРОВ ПО ПРОФИЛЮ «МАТЕМАТИКА»

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STEAM-подход в образовании является одним из мировых образовательных трендов.

Новая парадигма образования активно внедряется и в систему школьного образования на всех ее уровнях и базируется на междисциплинарном подходе объединяя естественнонаучные дисциплины, технологию, инжененрию, искусство и математику, активизирует интерес учащихся к ним, обеспечивает их подготовку к технико-технологическим инновациям жизни. В реализации STEAM-подхода в образовании математика, наряду с другими STEAM-дисциплинами становится одной из самых значимых учебных предметов [1].

Чтобы осуществить полноценную интеграцию STEAM подхода в образовательное пространство требуются педагоги нового формата, обладающие высоким уровнем профессиональной компетентности для обучения на базе STEAM.

Одной из значимых для успешной реализации STEAM образования в структуре профессиональной компетентности будущего педагога-бакалавра профиля «математика» выступает его STEM-компетентность – интегральное личностно-деятельностное качество учителя, проявляющееся в готовности и способности разрабатывать и применять проблемные, практико-ориентированные задачи, задачи исследовательского характера с межпредметным содержанием; организовать проектно-исследовательскую деятельность учащихся [2].

Вопросы формирования STEAM-компетентности будущих педагогов не могут быть решены без надежной методики ее диагностики.

STEAM-компетентность будущих педагогов на сегодняшний момент можно оценивать лишь качественно, например, по уровням: низкий, средний, высокий. В этой связи интервальные измерительные шкалы оценки STEAM-компетентности студента малопригодны, что определяет необходимость создания порядковой диагностической модели.

Цель работы заключается в обосновании и разработке модели диагностики STEAM-компетентности будущих педагогов профиля «математика» на основе критериального способа и порядковой измерительной шкалы.

Результаты исследования. Предложенная модель диагностики STEAM-компетентности будущего педагога может быть использована в качестве контролирующего оценочного средства по дисциплинам в рамках STEAM-обучения, а также для проведения психолого-педагогических исследований.

Модель может быть адаптирована к диагностике качества образовательных результатов, допускающих критериальную форму.

Ключевые слова: STEAM-образование, STEAM-компетентность учителя, модель диагностики, порядковая измерительная шкала

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БАСТАУЫШ СЫНЫП ОҚУШЫЛАРЫНА РОБОТОТЕХНИКАНЫ ОҚЫТУДА ВИРТУАЛДЫ ЛАБОРАТОРИЯЛАРДЫ ПАЙДАЛАНУДЫҢ МАҢЫЗДЫЛЫҒЫ

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Бұл мақалада бастауыш сынып оқушыларына робототехниканы оқытуда виртуалды зертханаларды пайдаланудың маңыздылығы қарастырылады. Технология қазіргі білім беруде ажырамас рөл атқара беретіндіктен, робототехниканы оқыту сыни ойлау, мәселелерді шешу және ынтымақтастық сияқты қажетті дағдыларды дамытудың маңызды құрамдас бөлігі болды. Дегенмен, дәстүрлі робототехника зертханалары ресурстардың шектеулі болуына, қауіпсіздік мәселелеріне және логистикалық шектеулерге байланысты жиі қиындықтар тудырады. Осы кедергілерді жеңу және оқу процесін жақсарту үшін виртуалды зертханалар нақты робототехника эксперименттерін имитациялайтын Имитациялық ортаны қамтамасыз ету арқылы перспективалы шешім ұсынады. Бұл мақалада виртуалды зертханалардың артықшылықтары, соның ішінде қолжетімділіктің жоғарылауы, белсенділіктің жоғарылауы, интерактивті оқу тәжірибесі және студенттердің ынтасының жоғарылауы қарастырылады. Білім беру саласындағы эмпирикалық деректер мен идеяларды ұсына отырып, бұл зерттеу жас оқушыларда STEM дағдыларының берік негізін қалыптастыру үшін виртуалды зертханаларды бастауыш мектепте робототехниканы оқытуға біріктірудің маңыздылығын көрсетеді.

Кілттік сөздер:виртуалды лаборатория, білім берудегі робототехника, бастауыш мектеп, STEM білім беру, эмпирикалық оқыту, педагогика.

AMS пәндік классификация: 68

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ИНФОРМАТИЗАЦИЯ ПЕДАГОГИЧЕСКОГО НАСЛЕДИЯ АЛЬ-ФАРАБИ

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Педагогическое наследие великого ученого-мыслителя и энциклопедиста Аль-Фараби как непреходящая историческая, общечеловеческая ценность не утратило своего значения до настоящего времени и может найти применение в современной педагогике [1].

Идеи мыслителя продолжают жить и сегодня, поэтому изучение и осмысление педагогических идей Аль-Фараби поможет глубже осознать современные проблемы воспитания и обучения подрастающего поколения. По его мнению, целью воспитания является формирование всестороннее развитой личности, именно это высказывание созвучно с требованиями, которые предъявляются к воспитанию современной молодежи, поэтому с уверенностью можем сказать, что его педагогические идеи приобретают новый облик.

В связи с этим, в настоящее время идет поиск эффективных способов обучения будущих учителей педагогическому наследию Аль-Фараби.

На наш взгляд, в современных условиях цифровизации образования приобщение студентов к педагогическому наследию Аль-Фараби немыслимо без использования информационных технологий. Информационные технологии будут способствовать пробуждению интереса к педагогическим взглядам великого мыслителя, сделают процесс обучения интересным и увлекательным. При изучении педагогического наследия мыслителя возможно применение различных видов компьютерных средств, в частности, таких как:

- мультимедийные презентации,
- программные тренажеры,
- интерактивный электронный учебник, включающий информационную часть с информацией о великом мыслителе, его взглядах о воспитании и обучении подрастающего поколения, о роли учителя в формировании личности, блок с заданиями для самостоятельной работы студентов по рассматриваемому вопросу, и контролирующую часть, содержащую тесты, творческие задания по изученной теме. Вышеуказанные компьютерные средства облегчат изучение педагогического наследия великого мыслителя, дадут возможность под руководством преподавателя или самостоятельно изучить учебный материал, выявить уровень усвоения рассматриваемой проблемы.

Ключевые слова: педагогические идеи, дидактические взгляды, воспитание, обучение, информационные технологии

Предметная классификация AMS: 68

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МЕТОДИЧЕСКИЕ ОСНОВЫ ПОДГОТОВКИ БУДУЩИХ УЧИТЕЛЕЙ ИНФОРМАТИКИ К ИСПОЛЬЗОВАНИЮ ОБЛАЧНЫХ ТЕХНОЛОГИЙ В ПРОФЕССИОНАЛЬНОЙ ДЕЯТЕЛЬНОСТИ

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Впервые разработанный профессиональный стандарт «Педагог» ориентирован на мотивацию профессиональной деятельности учителя и повышение качества системы образования в целом. От того какими компетенциями будут владеть учителя зависит качество системы образования. В Концепции развития образования Республики Казахстан на 2022-2026 годы говорится о том, что сегодня «особое внимание уделяется педагогическим направлениям подготовки, инженерным, обрабатывающим и строительным отраслям, IT направлениям».

Появление парадигмы облачных вычислений (облачных технологий) как технологии распределенной обработки данных, в которой компьютерные ресурсы и мощности предоставляются пользователю как Интернет-сервис, способствовало развитию облачных вычислений от «уровня инфраструктуры как услуги» до «уровня программного обеспечения как услуги», отказ от управления инфраструктурой облака пользователем. В связи с этим становится актуальным использование именно облачных сервисов.

Независимо от этого повышенное внимание к использованию облачных сервисов в учебном процессе остается и требует более глубокого исследования применения облачных технологий для формирования информационно-коммуникационной компетенции будущих учителей. Слабое учебно-методическое обеспечение обучения будущих учителей информатики по применению облачных технологий в профессиональной деятельности учителя, а также недостаточность разработки соответствующей методики использования облачных технологий для формирования ИКТ-компетентности будущих учителей информатики требуют научно-методического обоснования необходимости подготовки будущих учителей информатии к применению облачных технологий для формирования ИКТ-компетенций будущих учителей информатии и разработки соответствующей методической системы этой подготовки. Все это позволяет использовать облачные технологии в образовании, что влечет необходимость методической подготовки бакалавров информатики и будущих учителей информатики к использованию облачных технологий. Обучение. Для этого необходимо обучать будущих учителей информатики принципам облачных технологий, применению «облаков» при решении профессиональных задач. В связи с этим необходимо определить цель, задачи, содержание методической подготовки будущих учителей информатики (бакалавров) к использованию облачных технологий во всех направлениях их профессиональной деятельности

Ключевые слова: профессиональная подготовка, профессиональная подготовка будущих учителей информатики, облачные технологии, облако, использование облачных технологий.

КЛАССИФИКАЦИЯ МОБИЛЬНЫХ ЦИФРОВЫХ РЕСУРСОВ ИСПОЛЬЗУЕМЫХ В ОБУЧЕНИИ С УЧЕТОМ ПОТРЕБНОСТЕЙ ЛИЦ С ОГРАНИЧЕННЫМИ ВОЗМОЖНОСТЯМИ ЗДОРОВЬЯ

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В настоящий момент времени мобильное устройство все чаще используется не только для связи и развлечений, но и как важнейший образовательный инструмент, что дополнительно было форсировано переходом на смешанные и дистанционные формы обучения, в связи с пандемией COVID-19.

Популяризация мобильных образовательных технологий (m-learning) является одним из самых сильных трендов в образовании. Развитие направления m-learning также сказывается на инфраструктуре образовательных учреждений. Так, если ВУЗ активно развивает и внедряет образовательные цифровые ресурсы доступные на индивидуальных устройствах, то количество компьютеров, лабораторий и печатных изданий существенно уменьшается, по сравнению с ВУЗами с более традиционным подходом.

Однако стоит отметить, что качество образования, получаемого с применением мобильных устройств, существенно ниже, чем могло бы быть из-за качественного и количественного дефицита как программных средств, так и методической проработки материала. Для исправления данной ситуации в первую очередь необходимо исследовать особенности мобильных устройств с точки зрения их применения в образовательном процессе, определить какие типы цифровых ресурсов используются при обучении, а также оценить пригодность и адаптацию учебных материалов для детей с ОВЗ.

Ключевые слова: m-learning, мобильные цифровые ресурсы, дистанционное обучение, смешанное обучение, информатизация образования, обучение детей с ОВЗ..

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Теориялық және қолданбалы механика, тұтас орта механикасы

Theoretical and applied mechanics, continuum mechanics

Теоретическая и прикладная механика, механика сплошной среды

CFD ANALYSIS OF CENTRIFUGAL PUMP

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The centrifugal pump is widely utilized in industries, agriculture, and domestic settings as the most prevalent type of pump. The main components of a centrifugal pump include volute casing, rotor with blades (or impeller), suction and discharge nozzles. It functions by converting mechanical energy from the motor into increased pressure within the fluid through the rotation of its impeller. The fluid enters through the impeller's center and travels along its blades. As a result of the impeller's rotation, centrifugal force is generated which pushes liquid from the impeller to its periphery. This force enhances the fluid's velocity, ultimately converting kinetic energy into pressure.

The primary goal of this study is CFD analysis in examining and predicting the performance of centrifugal pumps. Using CFD techniques, it is possible to simulate the flow of fluid inside a pump and enable the study of pressure contours, velocity contours, flow streamlines, and other relevant factors.

To conduct CFD analysis of a centrifugal pump, a geometric model of the pump was created, including its volute casing, impeller and nozzles. Then the boundary conditions are set, such as the flow and pressure at the inlet and outlet of the pump. After setting up the model, the Navier-Stokes equations are numerically solved to describe the movement of fluid inside the pump. The Shear Stress Transport (SST) model has been identified as an appropriate turbulence model.

The results of CFD analysis allow to evaluate the efficiency and performance of the pump, identify potential problems such as backflow areas or eddy losses, and optimize the pump design to improve its performance.

Keywords: centrifugal pump, computational fluid dynamics, numerical simulation, turbulent flow, shear stress transport model.

AMS Subject Classification: 76D05, 76F10, 76F60.

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MELTING ENHANCEMENT OF PCM IN A TUBE LATENT HEAT THERMAL ENERGY STORAGE

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The sun is a source of enormous amounts of energy, and it is one of the most affordable and environmentally friendly sources of thermal energy on Earth. Daily and seasonal thermal energy storage is required to continuously meet the heat demand due to the intermittent nature of solar energy. A promising method for storing and using thermal energy for various purposes is the use of materials with a phase transition (PCM). In this study, we propose various mechanisms for accumulating solar thermal energy using phase transition materials for continental climate conditions. In a previous study, the authors developed a numerical algorithm using COMSOL Multiphysics software to calculate complex heat and mass transfer processes in phase transition materials.

The algorithm solves the coupled system of Navier-Stokes equations, the energy conservation equation and the enthalpy method. This numerical algorithm was applied to simulate latent heat accumulation processes using different reservoir models containing the same PCM. In this study, the efficiency of heat accumulation will be calculated in three different shapes of PCM containers will be explored: the first is a standard cylinder shape, the second is a wavy shell wall and the third is a wavy shape. In this regard, the numerical algorithm will be modified to take into account this new configuration: stratification of water in the tank under both laminar and turbulent flow regimes, modeling of a new models of containers located in tanks and internal ribbed structures of PCM cylinders were added for additional studies.

Keywords: solar thermal energy, phase change material, water storage tank, heat transfer, fluid flow

AMS Subject Classification: 76-05, 76-10, 76D05, 76F60, 76T06, 80-04.

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STUDY OF THERMAL COMFORT IN ELECTRIC VEHICLES

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Nowadays, individuals have a growing desire for comfortable driving conditions, leading to the widespread installation of air conditioning systems in cars. This has resulted in an increased variety of available air conditioners. The advantages of air conditioning have greatly benefited people by allowing them to achieve their desired temperature while on the road. Consequently, conducting a calculation and numerical study that considers the temperature inside the car compared to the surrounding environment is highly relevant.

The subject matter of this project is frequently encountered in everyday life, as car air conditioning is commonly used. This means that when we sit inside a car, we have the ability to adjust the climate to our preferred temperature, even if the outside temperature or internal climate of the car is uncomfortable. By activating the air conditioner, we can create the desired climate. The experimental results clearly demonstrate that the proposed system significantly benefits from the thermostatic control of the car's air conditioning system. It ensures that the average temperature on board remains within the range of 23°C to 25°C.

The main objective of this study is to establish a framework for assessing the heat load inside the cabin. By accurately calculating the heat load, it becomes possible to determine the cooling capacity required and the corresponding cooling tonnage needed.

Keywords: electric vehicles, thermal comfort, climate control, user experience, sustainable transportation.

AMS Subject Classification: 65-02, 80-02, 80A05, 80M50.

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SOLUTION OF ONE BOUNDARY VALUE TASK OF VISCOELASTICITY IN A NONLINEAR FORMULATION, IN THE CASE OF A CUBIC STRESS-STRAIN RELATION

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In this paper, the solution of a boundary value task in the nonlinear formulation is considered by the authors [1]. In spite of its proximity to linear theory, the nonlinear theory of viscoelasticity has not yet been fully developed. This issue is far from being fully completed, since the existing calculation methods do not yet provide a complete answer to the many different questions posed by practice. For this reason, in order to obtain a nonlinear law relating the strains σ_{ij} and deformations ε_{ij} a number of conditions are formed:

(1) The specific work of deformation A must be a function of the entire deformation history from the beginning of deformation to the current time t.
(2) The material of a viscoelastic body is homogeneous and isotropic.
(3) For very small deformations the nonlinear relation law between σ_{ij} and ε_{ij} in the limit should pass to relations in linear approximation.

Keywords: bulk compression modulus, linear integral operator, kernel of integral operator, nonlinear dependence, quadratic strain intensity, Fourier and Laplace transforms.

AMS Subject Classification: 74H45-Vibration in dinamical problem in solid mechanics.

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DESIGN OF THE IMPELLER OF A CENTRIFUGAL PUMP USING THE LAW OF AFFINITY

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This paper investigates the application of the law of affinity in the design of centrifugal pump impellers. It presents a comprehensive analysis of the relevant theory, models for calculating pump data and performance. Based on the law of affinity, an empirical mathematical model of the impeller design was developed and implemented using Matlab. The study demonstrates the effectiveness of similitude theory in impeller design and its potential in hydrodynamics.

By using the affinity law, designers can create efficient and economical impeller designs, reducing the need for physical testing. The developed Matlab program allows engineers to evaluate various impeller configurations and optimize performance. Integration of CFD methods using the law of affinity can optimize the design and optimization of the impeller.

This research serves as a valuable resource for researchers and engineers involved in centrifugal pump impeller design. By integrating the law of affinity, fluid dynamics research can be improved leading to improved pumping systems in various industries. Below is a table according to the law of affinity.

Diameter Change	Speed Change	Diameter and Speed Change
Flow $Q_2 = Q_1(\frac{D_2}{D_1})$	$Q_2 = Q_1(\frac{n_2}{n_1})$	$Q_2 = Q_1(\frac{D_2}{D_1})(\frac{n_2}{n_1})$
Head $H_2 = H_1(\frac{D_2}{D_1})^2$	$H_2 = H_1(\frac{n_2}{n_1})^2$	$H_2 = H_1(\frac{D_2}{D_1})^2(\frac{n_2}{n_1})^2$
Power $N_2 = N_1(\frac{D_2}{D_1})^5$	$N_2 = N_1(\frac{n_2}{n_1})^3$	$N_2 = N_1(\frac{D_2}{D_1})^5(\frac{n_2}{n_1})^3$
Generalized affinity law not applicable. Proportionality dependent on the specific characteristics of the pump.	$NPSHr_2 = NPSHr_1(\frac{n_2}{n_1})^2$	$NPSHr_2 = NPSHr_1(\frac{n_2}{n_1})^2$

1. Values at initial conditions.
2. Values at new conditions.

Keywords: centrifugal pump, impeller design, affinity law, geometric similarity, kinematic similarity, dynamic similarity, efficiency, Matlab program.

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ON SOME RESULTS OF SOLVING INVERSE DYNAMICAL PROBLEMS WITH REGARD FOR BAUMGARTE CONSTRAINT STABILIZATION

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While solving Couchy problem for system of ordinary differential equations (ODE) with mechanical constraints, deviations from initial data can cause an solution instability. Joachim Baumgarte presented in his work [1] a method of constraint stabilization that prevents numerical instability and pushes the solution to its undeviating state. This work is devoted to the implementing of this stabilization method for solving inverse dynamical problems.

Let a dynamical system be described by the set of generalized coordinates $\mathbf{q} = (q_1, \dots, q_n)$ and velocities $\dfrac{d\mathbf{q}}{dt} = \dot{\mathbf{q}}$. This system can be imposed with a set m of kinematic constraints:

$$a_{\mu i}\dot{q}_i = 0, \ \ \mu = 1, \dots, m. \tag{1}$$

Here we will assume summation under repeating indexes. We can treat system (1) as a system of linear algebraic equations with respect for generalized velocities. We can solve them as follows:

$$h_i = \dot{q}_i - v_i(\mathbf{q}), \tag{2}$$

where functions $v_i(\mathbf{q})$ contains some ambiguity. Baumgarte stabilization method defines the equations of constraints perturbations: $\dot{h}_i = F_i(\mathbf{h}, \mathbf{q}, \dot{\mathbf{q}}, t)$. The structure of functions $\mathbf{F}$ depends on the structure of the target system of ODE. Let's consider system of Lagrange equations of the second kind with Rayleigh dissipation function. In order for system (2) with perturbation to be reducible to such system, it must satisfy the generalized Helmholtz conditions [2]. In paper [3] it has been shown that in general function $F_i(\mathbf{h}, \mathbf{q}, \dot{\mathbf{q}}, t)$ must be linear with respect for $\mathbf{h}$.

Keywords: Baumgarte constraint stabilization, inverse dynamical problems, Helmholtz conditions, Lagrange equations.

AMS Subject Classification: 70F17

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SIMULATION OF THE EFFECT OF FRACTURES AND UNDISSOLVED POLYMERS ON THE INJECTIVITY OF THE POLYMER SOLUTION IN GRANULAR MEDIA

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The text of your abstract starts here. Polymer irrigation is a method of increasing oil yield used in order to increase the efficiency of reservoir coverage by pumping an aqueous solution into the underground oil reservoir. Increasing the viscosity of the water injected into the plast through the dissolved polymer provides an increase in the coefficient of displacement between the pumped water and the hydrocarbon in the plast. Theoretically, the high viscosity of the polymer reduces the receiver property in comparison with water. However, field studies have shown that the acceptability of the polymer is at a value higher than theoretical predictions. There are several factors that affect polymer receptivity. The purpose of this project is to predict the receptive properties of a polymer mixture introduced into a granular medium, taking into account the Rheology of cracks and insoluble polymer, water quality, polymer caused by high-viscosity additives. Modeling is done in the grain mash with the connection of hydrodynamics and granular mechanics. The liquid-particle bond is modeled by linking computational hydrodynamics (CFD) and the discrete elementals method (DEM). In the course of modeling, the geometry through which the polymer flow passes was built, and a mathematical model of the Navier-Stokes equation on computational hydrodynamics was built. Since the polymer flows in the direction of light in a granular medium, the direction of the crack affects the coverage area of the polymer. Polymer rheology, water quality, and undissolved polymers also affect the perception property of the polymer. This paper first shows the formation of gaps in the granular model and its effect on the receptivity property of the polymer.

Keywords: Enhanced Oil Recovery (EOR), Polymer Injection Technology, Computational Fluid Dynamic Modeling (CFD), Discrete element method (DEM), CFD-DEM.

BOUNDARY VALUE PROBLEMS OF THE DYNAMICS OF THERMOELASTIC RODS AND THEIR SOLUTIONS

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Space-one-dimensional boundary value problems of uncoupled thermoelasticity are considered, which can be used to study various bar structures under conditions of thermal heating. Here we propose a unified technique for solving various boundary value problems typical of practical applications.

Thermoelastic rod of length $2L$, which is characterized by density ρ, thermoelastic constants γ and κ. The displacements of the rod sections and the temperature field of the rod are described by a system of hyperbolic-parabolic equations of the form [1]:

$$\rho c^2 u_{,xx} - \rho u_{,tt} - \gamma\theta_{,x} + \rho F = 0\theta_{,xx} - \kappa^{-1}\theta_{,t} + F = 0 \tag{1}$$

Here $u(x,t)$ - components of longitudinal displacements, $\theta(x,t)$ - relative temperature, c - velocity of propagation of thermoelastic waves in the rod, ρ - density.

Initial conditions (Cauchy conditions): at $t = 0$ the displacements, velocities and temperature are known:

$$u(x,0)u_0(x),\, \theta(x,0) = \theta_0(x),\ |x| \leq L, \partial_t u(x,0)\dot{u}_0(x),\ |x| < L.$$

Boundary problems are solved when displacements, temperature, stresses and heat fluxes at the ends of the rod are known. It is required to determine the thermally stressed state of the rod. On the basis of the method of generalized functions, generalized solutions of non-stationary boundary value problems are constructed under the action of power and heat sources of various types, including those described by singular generalized functions, under various boundary conditions at the ends of the rod. Regular integral representations of generalized solutions are obtained, which give an analytical solution of the formulated boundary value problems. The peculiarity of the constructed solutions makes them convenient for studying network thermoelastic systems that can be modelled by thermoelastic graphs.

Keywords: thermoelastic rod, generalized functions, Green's function, boundary value problems, temperature stresses.

AMS Subject Classification: 74A15, 65R20, 35L05, 35K05.

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ON EXACT SOLUTIONS TO THE MASS BALANCE EQUATION

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Let $t \in [0,T]$–the time variable, $\Omega \subset R^3$ is a one-coherent bounded region filled without voids with liquid. If $h(t,x,y,z)$ is a given intensity of mass sources, then the mass balance equation in the region $G = [0,T] \times \Omega \subset R^4$ has the following form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho U)}{\partial x} + \frac{\partial(\rho V)}{\partial y} + \frac{\partial(\rho W)}{\partial z} = h(t,x,y,z). \tag{1}$$

Here: $\rho(t,x,y,z), U, V, W(t,x,y,z)$ – unknown density and velocity components of the fluid.

Definition 1. The resolving parameters of equation (1) are dimensionless scalars (total – 12, of which free – 10) related by the following relations:

$$\sum_{k=1}^{4} \beta_k \alpha_k = 0; \sum_{m=1}^{4} \gamma_m = 1.$$

Definition 2. The four-dimensional regular function defined in the domain G is a vector-function $u = (u_1, u_2, u_3, u_4)$ with components satisfying generalized Cauchy-Riemann conditions:

$$\frac{\partial u_1}{\partial t} = \frac{\partial u_2}{\partial x} = \frac{\partial u_3}{\partial y} = \frac{\partial u_4}{\partial z}; \frac{\partial u_1}{\partial x} = -\frac{\partial u_2}{\partial t} = \frac{\partial u_3}{\partial z} = -\frac{\partial u_4}{\partial y};$$
$$\frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial z} = \frac{\partial u_3}{\partial t} = \frac{\partial u_4}{\partial x}; \frac{\partial u_1}{\partial z} = -\frac{\partial u_2}{\partial y} = \frac{\partial u_3}{\partial x} = -\frac{\partial u_4}{\partial t}.$$

The infinite-dimensional space of regular four-dimensional functions is a dense subset of the space of 4–vectors with continuously–differentiable components in the G. Further, let ρ_0, c, L – be the characteristic density, velocity, and size of the flow.

Theorem. Equation (1) has a continuum of exact solutions of the form:

$$\begin{aligned}
\rho(t,x,y,z) &= \beta_1 \rho_0 u_1\left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L}\right) + \gamma_1 \int_0^t h(\tau,x,y,z)d\tau \\
\rho U(t,x,y,z) &= \beta_2 \rho_0 c u_2\left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L}\right) + \gamma_2 \int_0^x h(t,\xi,y,z)d\xi \\
\rho V(t,x,y,z) &= \beta_3 \rho_0 c u_3\left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L}\right) + \gamma_3 \int_0^y h(t,x,\eta,z)d\eta \\
\rho W(t,x,y,z) &= \beta_4 \rho_0 c u_4\left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L}\right) + \gamma_4 \int_0^z h(t,x,y,\zeta)d\zeta
\end{aligned} \tag{2}$$

It is easy to see that the solution depends on 19 parameters: four arbitrary functions and 15 scalars.

Keywords: four-dimensional functions,resolving parameters,Cauchy-Riemann conditions,exact solutions

AMS Subject Classification: 76A02,76M40

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ON THE CONTINUUM OF EXACT SOLUTIONS TO THE MASS BALANCE EQUATION

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Let $t \in [0,T]$–the time variable, $\Omega \subset R^3$ is a one-coherent bounded region filled without voids with liquid. If $h(t,x,y,z)$ is a given intensity of mass sources, then the mass balance equation in the region $G = [0,T] \times \Omega \subset R^4$ has the following form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho U)}{\partial x} + \frac{\partial(\rho V)}{\partial y} + \frac{\partial(\rho W)}{\partial z} = h(t,x,y,z). \tag{1}$$

Here: $\rho(t,x,y,z), U, V, W(t,x,y,z)$ – unknown density and velocity components of the fluid.

Definition 1. The resolving parameters of equation (1) are dimensionless scalars (total – 12, of which free – 10) related by the following relations:

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$$\frac{\partial u_1}{\partial t} = \frac{\partial u_2}{\partial x} = \frac{\partial u_3}{\partial y} = \frac{\partial u_4}{\partial z}; \frac{\partial u_1}{\partial x} = -\frac{\partial u_2}{\partial t} = \frac{\partial u_3}{\partial z} = -\frac{\partial u_4}{\partial y};$$
$$\frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial z} = \frac{\partial u_3}{\partial t} = \frac{\partial u_4}{\partial x}; \frac{\partial u_1}{\partial z} = -\frac{\partial u_2}{\partial y} = \frac{\partial u_3}{\partial x} = -\frac{\partial u_4}{\partial t}.$$

The infinite-dimensional space of regular four-dimensional functions is a dense subset of the space of 4–vectors with continuously–differentiable components in the G. Further, let ρ_0, c, L – be the characteristic density, velocity, and size of the flow.

Theorem. Equation (1) has a continuum of exact solutions of the form:

$$\begin{aligned}
\rho(t,x,y,z) &= \beta_1 \rho_0 u_1 \left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L} \right) + \gamma_1 \int_0^t h(\tau,x,y,z)d\tau \\
\rho U(t,x,y,z) &= \beta_2 \rho_0 c u_2 \left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L} \right) + \gamma_2 \int_0^x h(t,\xi,y,z)d\xi \\
\rho V(t,x,y,z) &= \beta_3 \rho_0 c u_3 \left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L} \right) + \gamma_3 \int_0^y h(t,x,\eta,z)d\eta \\
\rho W(t,x,y,z) &= \beta_4 \rho_0 c u_4 \left(\frac{\alpha_1 ct}{L}, \frac{\alpha_2 x}{L}, \frac{\alpha_3 y}{L}, \frac{\alpha_4 z}{L} \right) + \gamma_4 \int_0^z h(t,x,y,\zeta)d\zeta
\end{aligned} \tag{2}$$

It is easy to see that the solution depends on 19 parameters: four arbitrary functions and 15 scalars.

Keywords: four-dimensional functions,resolving parameters,Cauchy-Riemann conditions,exact solutions
AMS Subject Classification: 76A02,76M40

RESULTS OF INVESTIGATION OF SOIL PROPERTIES AND STABILITY CALCULATION OF ALMALUU-BULAK LANDSLIDE

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In the paper [1], the problem is solved under the assumption that the mass of the landslide before and after the descent remains unchanged. To find the mass of the landslide before the exit M_0, it is first necessary to find the most dangerous sliding surface in the unstable slope.

In the paper [2], the landslide pressure E is defined as the difference between the shearing and holding forces. E has the form:

$$F(x, y_2, \dot{y_2}) = [\gamma_s(1-m) + \gamma_b m](y_2 - y_1)[\frac{\dot{y_2} + \tan\varphi}{1 - \dot{y_2}\tan\varphi} - \mu] - c\frac{1 + \dot{y_2}^2}{1 - \dot{y_2}\tan\varphi}, \tag{1}$$

where γ_b, γ_s – respectively, the specific gravity of rainwater and soil, kH/m3; m – soil porosity, units; μ - coefficient of dynamic seismicity [3]; y_1, y_2 – slope and sliding surfaces; φ- is the angle of internal friction of the soil; c - soil cohesion.

We obtain the expression E in an arbitrary section x of the slope in the form of an integral:

$$E = \int_{x_0}^{x} F(x, y_2, \dot{y_2})dx \tag{2}$$

where x_0 is the coordinate corresponding to the starting point of the landslide block.

As a result of determining partial derivatives and performing the necessary transformations, the Euler equation for functional (2) takes the following form:

$$\begin{gathered} 2\dot{y_2}(1+\tan^2\varphi)[[\gamma_s(1-m)+\gamma_b m](y_2-y_1)\tan\varphi - c] - (1-\dot{y_2}\tan\varphi)\{[\gamma_s(1-m)+\gamma_b m] \\ ((1-\dot{y_2}\tan\varphi)(\dot{y_2}+\tan\varphi-\mu(1-\dot{y_2}\tan\varphi)) - (\dot{y_2}-\dot{y_1})(1+\tan^2\varphi))\} = 0 \end{gathered} \tag{3}$$

Since the differential equation (3) is of the second order, two conditions are set for its solution - the position of the starting point and the initial slope of the slip line.

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TRANSLATION-INVARIANT p-ADIC GENERALIZED GIBBS MEASURES FOR THE SOS MODEL WITH FOUR STATES

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Let $\mathbb{Q}$ be the field of rational numbers. The completion of $\mathbb{Q}$ with respect to the p-adic norm defines the p-adic field $\mathbb{Q}_p$ (see [1]).

Let $\Gamma^k(V,L)$ be Cayley tree (see [2]) and $\Phi = \{0,1,2,3\}$. A configuration σ on $A \subset V$ is defined by the function $x \in A \to \sigma(x) \in \Phi$. The set of all configurations on A is denoted by $\Omega_A = \Phi^A$, and $\Omega_V := \Omega$.

A formal p-adic Hamiltonian $H : \Omega \to \mathbb{Q}_p$ of the p-adic SOS model is defined by

$$H(\sigma) = J \sum_{\langle x,y\rangle \in L_n} |\sigma(x) - \sigma(y)|_\infty, \qquad \sigma \in \Omega_{V_n}$$

where $0 < |J|_p < p^{-1/(p-1)}$ for any $\langle x,y\rangle \in L$.

We define a function $z : x \to z_x$, $\forall x \in V \setminus \{x_0\}$, $z_x \in \mathbb{Q}_p$ and consider p-adic probability distribution $\mu_z^{(n)}$ on Ω_{V_n} defined by

$$\mu_{\mathbf{z}}^{(n)}(\sigma) = Z_{n,\mathbf{z}}^{-1} \exp_p\{H_n(\sigma)\} \prod_{x\in W_n} z_{\sigma(x),x}, \tag{1}$$

where $Z_n^{(z)}$ is the normalizing constant.

Theorem 1.[3] *The $p-$adic probability distributions $\mu_{\tilde{z}}^{(n)}(\sigma_n)$, $n = 1,2,...$ in (1) are compatible for p-adic SOS model iff for any $x \in V\backslash\{x^0\}$ the following system of equations holds:*

$$\tilde{z}_{i,x} = \prod_{y\in S(x)} \frac{\sum_{j=0}^{m-1} \theta^{|i-j|_\infty} \tilde{z}_{j,y} + \theta^{m-i}}{\sum_{j=0}^{m-1} \theta^{m-j} \tilde{z}_{j,y} + 1}, \qquad i = 0,1,...,m-1, \tag{2}$$

here $\theta = \exp_p(J)$ and $z_{i,x} = \tilde{z}_{i,x}/\tilde{z}_{m,x}$, $i = 0,1,...,m-1$.

In this case, by the p-adic analogue of the Kolmogorov theorem there exists a unique measure μ_z on the set Ω such that $\mu_z\left(\{\sigma|_{V_n} \equiv \sigma_n\}\right) = \mu_z^{(n)}(\sigma_n)$ for all n and $\sigma_n \in \Omega_{V_n}$ (see [2]).

In this paper we consider translation-invariant p-adic generalized Gibbs measures for the four states SOS model on the Cayley tree of order two and we get the following result:

Theorem. *If $p = 7$, then there exist exactly three translation-invariant p-adic generalized Gibbs measures for the SOS model with four states on the Cayley tree of order two.*

Keywords: Cayley tree, p-adic SOS model, p-adic generalized Gibbs measures, phase transition.
AMS Subject Classification: 46S10, 82B26, 12J12.

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SIMULATION STUDY OF CO2 EOR IN OIL RESERVOIRS OF KAZAKHSTAN

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In this work, the process of increasing oil production by using CO2 in Kazakhstan's oil fields was modeled. First of all, in order to increase CO2 production of oil in Kazakhstan, the important criteria of the earth's crust were collected accordingly. The compiled criteria were presented to the attention of scientific workers in the field of oil production, and an oil field was selected for research. Simulation was implemented in tNavigator, KMGEsim simulators and S3Graf visualizer. As a result of numerical calculations, other results were obtained, such as total oil production, daily production, gas production rate, pressure, water flow rate.The initial production of oil was 48.6976 million st. m3, the total production of oil in the CO2 injection method is 20 mln. SM3 - up to 10 mln. SM3 increased to . That is, according to the obtained results, the total oil product CO2 injection method produced about 2 times greater value than the hydration method. Comparing the obtained results, we can think that it is effective to use CO2 to increase oil production. But the formation pressure decreased to 20 bar in the CO2 injection method, and to 80 bar in the hydration method.The pressure drop may be due to the introduction of CO2 gas only in a dry state. In addition, the total production of water in the CO2 injection method is 25 mln. SM3 - up to 45 mln. increased to SM3. Oil production increased in the CO2 injection method, but formation pressure and water production decreased. That is, other methods of injecting CO2 alternately with water may be more effective than injecting CO2 dry. Therefore, we cannot conclude that this technology is completely effective. After all, it is necessary to compare other important results and to simulate several oil fields in Kazakhstan. If we make sure that the results obtained by modeling in oil fields are positive and beneficial for economic policy, we can come to the conclusion that it is effective to use CO2 to increase oil production in oil fields in Kazakhstan.

Keywords: Enhanced Oil Recovery (EOR), Discrete element method (DEM).

DESIGN OF CENTRIFUGAL PUMP IMPELLER BASED ON HYDRAULIC CALCULATIONS

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This paper is devoted to the design process of centrifugal pump impeller via hydraulic calculations. Emphasis is placed on obtaining optimal impellers geometry and dimensions using empirical formulas, hydraulic laws, and mechanical principles. The centrifugal pump under study has a single inlet, as well as a double curvature at the inlet, which turns into a cylindrical shape at the outlet of the blade shape.

The main goal of the work is to create a systematic approach to the design of an effective impeller that provides the required performance and head of the pump. To achieve this goal, the work explores the fundamental principles of fluid mechanics and pump operation in order to optimize the geometry of the impeller to ensure efficient fluid flow.

Overall, the research results provide a comprehensive understanding of the relationship between impeller design and pump hydraulic efficiency. The resulting geometry and dimensions form the basis for creating an impeller tailored to specific operational requirements. Application of the proposed methodology allows engineers and researchers to design centrifugal pump impellers with improved performance characteristics, higher reliability and reduced energy consumption.

Keywords: centrifugal pump, impeller, blade, hydraulics, hydrodynamic characteristics, efficiency.

AMS Subject Classification: 18B20, 76U99, 97M50.

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NUMERICAL CALCULATION OF THE PARAMETERS OF HEAT CARRIERS IN HELICOID HEAT EXCHANGERS

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The work is devoted to the numerical calculation of heat exchange processes in a helicoid-shaped heat exchanger. In the design of helicoid heat exchangers, profiled tubes and fins of a screw profile are used, leading to an improvement in the heat exchange conditions. Article [1] provides calculations of a heat exchanger for smooth pipes. The paper presents the results of calculations of the oil temperature field depending on various hydrodynamic parameters. Numerical calculation of oil dynamics was performed in the Ansys Fluent software package. Figure 1 shows a quarter of the calculated grid, which shows the thickening of the grid at the pipe boundary, where there are the largest gradients of flow parameters.

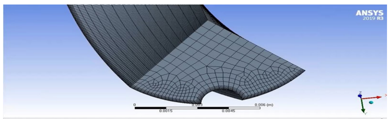

FIGURE 1. Calculation grid for a quarter of a pipe

Figure 2 shows a graph of the dependence of the average mass temperature of oil at the outlet of the pipe on the number of twist N. As can be seen from the figure, with an increase in the number of twists, the temperature of the oil at the outlet increases, that is, there is an intensification of heat exchange between the heat carriers.

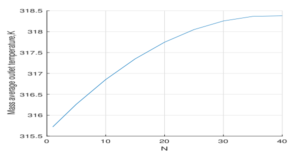

FIGURE 2. Dependence of the average mass temperature at the outlet of the heat exchanger on the twist N.

Keywords: heat exchange, numerical calculation, helicoid heat exchanger, petroleum product

AMS Subject Classification: 76A10

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NUMERICAL SOLUTION OF RELATED PROBLEMS OF MAGNETOELASTICITY

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In the process of deformation of a non-ferromagnetic conductive body, the shape of its surface changes, which leads to a change in the direction of the current, that is, the electromagnetic field of the conductive body changes, eddy currents arise, which, interacting with an external magnetic field, lead to the emergence of forces of electromagnetic origin. These forces change the stress-strain state of the body and the electromagnetic field in it. In this work, a nonlinear two-dimensional model of magneto elasticity of non-ferromagnetic thin shells under the influence of non-stationary electromagnetic forces and mechanical loads is constructed, taking into account electromagnetic anisotropy. Numerical results are obtained and an analysis is made of the electromagnetic effects of the stress-strain state of non-ferromagnetic thin shells, taking into account the electromagnetic anisotropy.

In the mechanics of conjugate fields, an important place is occupied by the study of the motion of a continuous medium, taking into account electromagnetic effects. Analysis of electromagnetic processes is possible only on the basis of a system of equations of electrodynamics, together with material relations. In recent decades, considerable attention in the literature has been paid to the study of the process of deformation of electrically conductive bodies placed in an external alternating magnetic field under the influence of non-stationary force, thermal and electromagnetic loads. Interest in research in this area is associated with the importance of quantitative study and evaluation of the observed effects of the relationship of mechanical, thermal and electromagnetic processes and their practical application in various fields of modern technology in the development of new technologies. When a conductive body moves in a magnetic field or when the magnetic field changes with time, induced currents and the ponderomotive Lorentz forces caused by them arise in the body, which, in turn, is accompanied by deformation of the medium and the appearance of stress waves. The motion of an elastic medium in a magnetic field is described by a joint system of equations of electrodynamics of a slowly moving medium and equations of the dynamic theory of elasticity, taking into account ponderomotive forces. This system of equations is non-linear due to the non-linearity of the relations of the generalized Ohm's law and expressions for ponderomotive forces [1, 2]. Stress-strain states of flexible shells are studied in a nonlinear setting based on a comparison of the results of solutions. The results obtained in the work can be used in the calculations of structural elements in various fields of technology and in the creation of modern technologies for new structural materials subjected to the simultaneous action of non-stationary mechanical and electromagnetic loads, taking into account electromagnetic anisotropy.

Keywords: deformation, non-ferromagnetic conductive, plate, magnetic field, magneto electisity.

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ON CONSTRUCTION OF A SET OF DYNAMICS EQUATIONS OF A GIVEN STRUCTURE WITH REGARD FOR THE CONSTRAINT STABILIZATION

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Well-known constraint equations restricting the motion of mechanical systems or their analogues

$$g_\alpha(\mathbf{q},t)=0,\quad \alpha=1,\dots,a,$$
$$g_{\beta i}(\mathbf{q},t)\dot q^i+g_\beta(\mathbf{q},t)=0,\quad \beta=a+1,\dots,b,\ \ b\leqslant n,\ \ i=1,\dots,n,$$
$$\mathbf{q}=(q^1,\dots,q^n),\dot q^i=\frac{dq^i}{dt}.$$

can be used for the direct construction of a set of systems of second-order differential equations: $f_i(\ddot{\mathbf{q}},\dot{\mathbf{q}},\mathbf{q},t)=0$, $f_i\equiv\frac{dy_i}{dt}-Y_i(\mathbf{y},\dot{\mathbf{q}},\mathbf{q},t)$, $y_i=\delta_{ij}(\dot q^j-v^j(\mathbf{q},t))$, $Y_i(0,\dot{\mathbf{q}},\mathbf{q},t)$, $i,j=1,\dots,n$. with the subsequent reduction of them to the required structure. The stabilization of the constraints is able to increase the efficiency of the numerical solution combining with methods of solving inverse problems of dynamics [1]. In some cases it is possible to ensure the fulfilment of modified Helmholtz conditions [2], which allows us to construct a Lagrangian and a dissipative function. The solution of the problem of controlling the rolling of a trolley along a plane with a perfectly thin driving wheel in the form of a pointed blade is given [3], the construction of a Lagrangian in the problem of the motion of a point in a potential force field [4], the problem of determining the central force controlling the steady movement of a point along a conical section.

Keywords: equation, dynamics, constraints, stabilization, stability, control.

AMS Subject Classification: 70F17

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SYSTEM OF EQUATIONS OF THE COUPLED DYNAMIC PROBLEMS OF A VISCOELASTIC SHELL LOCATED IN A TEMPERATURE FIELD

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This paper gives a mathematical model and a definition of the dynamic problems for the viscoelastic orthotropic shell, taking into account the interrelation of the temperature and deformation fields.

Let the shell be nonuniformly heated along its thickness and at its median surface to a temperature , $T(x,y,t,z)$ varying with time.

The dependence of temperature T with strain l_x, l_y, l_{xy} and stress components $\sigma_x, \sigma_y, \sigma_{xy}$ in the case of plane stress state has the form

$$\sigma_x = \frac{E_1}{1-\vartheta_1\vartheta_2}\left[(1-R_{11}^*)l_x + \vartheta_2(1-R_{12}^*)l_y - (a_1 + a_2\vartheta_2)(1-R_{13}^*)T\right];$$

$$\sigma_x = \frac{E_2}{1-\vartheta_1\vartheta_2}\left[(1-R_{22}^*)l_y + \vartheta_2(1-R_{21}^*)l_x - (a_1 + a_2\vartheta_2)(1-R_{23}^*)T\right];$$

$$\tau_{xy} = G(1-R_{33}^*)l_{xy}.$$

where a_1, a_2 the coefficients of linear thermal expansion in the direction of the axes Ox, Oy and , respectively; the rest of the notations and the mathematical model obtained are taken as in [1].

Keywords: shell, temperature, stress, strain, mathematical model, equations.

AMS 35M10.

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GRADIENT DEFORMATION CRITERION OF BRITTLE FRACTURE IN SOLID BODIES

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In the article [1], we propose a new fracture criterion based on the assumption that brittle fracture occurs when the modulus of strain gradient reaches to limit value. Analitically solved two planar problems of thermoelasticity related to fractue mechanics. 1. We consider in more detail the fact that the modulus of the temperature gradient, and, consequently, that of the strain, becomes significant in a thin boundary layer where a quick temperature change occurs and on the crack appears on this rectilinear boundary despite the fact that there are no crack-opening stresses. 2. We also present the solution of a new thermoelastic problem of a quick temperature change in a circular area. As it turned out, in the case of a curved boundary (on which a crack appears) of the cooled area E2, where E2 is the Euclidean plane, the fracture criterion becomes complicated and gets a form of a limit value of the gradient modulus of the principal strains sum or just of the principal stresses sum. Failure is determined by the criterion the time of which setting in is less. It is also shown in the article that in the case of a heated circular area in E2 the criterion for brittle fracture of a continuum medium is the limit value of the gradient modulus of the principal strains sum. The results of the preliminary experiment with organic glass samples during a quick temperature change in the area with a rectilinear boundary are presented. During the experiment, there appeared a crack along the boundary of the temperature leap, although the stresses across this boundary are equal to zero, but there is a significant deformation gradient.

Keywords: brittle fracture, gradient deformation criterion of fracture, Cauchy problem, Heaviside functions, regularization of the initial temperature distribution, logarithmic potential of the circle.

AMS Subject Classification: 74H05-Explicit solutions of dynamical problems in solid mechanics.

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FREE VIBRATION OF CROSS-PLY LAMINATED PLATES UNDER CLAMPED BOUNDARY CONDITION: A COMPARATIVE STUDY

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Composite materials are closely attached to civil, aerospace, automobile and aeronautic fields. The use of composite material in engineering field increased drastically due to its mechanical behaviour. Thus, many researchers show more interest to study the mechanical behaviour of these composite materials. Shear deformation theory was introduced by Stavsky [1] for isotropic plates and generalized to laminated anisotropic plates by Yang et al [2].

Consider a rectangular plate with length a, width b and constant thickness h, which made up of even and odd number of layers are given in which the angle orientation fixed at 0° and 90° to analyze the problem. Based on YNS theory, the displacement components are considered as

$$u = u_0(x, y, t) + z\psi_x(x, y, t), \quad v = v_0(x, y, t) + z\psi_y(x, y, t), \quad w = w(x, y, t) \tag{1}$$

where u, v, and w is displacement component in x, y and z directions respectively. u_0 and v_0 are the displacements at middle surface of the plate and ψ_x and ψ_y are shear rotation in middle surface of plate at any point and t is time. The governing equations are obtained from Hamilton's principle as

$$\begin{aligned} &N_{x,x} + N_{xy,y} = I_0\tfrac{\partial^2 u_0}{\partial t^2}, N_{xy,x} + N_{y,y} = I_0\tfrac{\partial^2 v_0}{\partial t^2}, Q_{x,x} + Q_{y,y} = I_0\tfrac{\partial^2 w}{\partial t^2} \\ &M_{x,x} + M_{xy,y} - Q_x = I_1\tfrac{\partial^2 \psi_x}{\partial t^2}, M_{xy,x} + M_{y,y} - Q_y = I_1\tfrac{\partial^2 \psi_x}{\partial t^2} \end{aligned} \tag{2}$$

where N_x, N_y, N_{xy} are total stress resultants, M_x, M_y, M_{xy} are moment resultants, Q_x, Q_y, are transverse shears and I_0, and I_1 are intertia terms. Using YNS theory in Eqn. (2), we obtain the displacement and rotational functions which are assumed in a separable form to get a coupled differential equation. Two different numerical approximations are adopted namely Radial Basis Function method (RBF) and Spline approximation method for the comparative study and the results are presented in term of graphs.

Keywords: Cross-ply; Free vibration; Radial basis function; Splines, Comparative study

AMS Subject Classification: 74H15, 74H45, 74H55

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DEVELOPMENT AND RESEARCH OF AN EXPERIMENTAL MODEL OF A SOLAR ENERGY CONVECTOR

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One of the most pressing issues today is the study and development of alternative solar energy systems. For this purpose, many new types of alternative energy sources are being studied, developed and put into operation in different countries, in different laboratories. One type of these alternative energy sources is concentrated in solar energy systems. In this paper, a model of the concentrated solar system similar to the Stirling system is considered. The efficiency and operability of this assembled unit is studied. Purpose of the work: energy accumulation, receiving heat from the reflected sunlight through the Stirling plate system. Experimental research work consisted of several stages. In the first stages, concentrated solar energy systems were studied and the type of design that best suits our goals was considered. At the second stage, a concentrated solar power plant was assembled. At the third stage, the work of the assembled unit was checked, and experiments were carried out in different weather conditions. At the fourth stage, an analysis of the received information was carried out. During each experiment, the surface of the receiver, sand, water in the tank, and weather temperature indicators were recorded. The experimental study was studied in different weather conditions and at different times of the day, for the purpose of the experiment, a system of manually concentrated solar energy was built, and the possibilities of concentrated solar energy were studied using this device. In the course of the study, the capabilities of the installation were tested at different measurements of solar power. As a result of the study, a system of manually concentrated solar energy was created and the ability to work was tested. The results obtained during the experimental study are presented in tables and plotted in the form of graphs.

Keywords: Solar radiation intensity, Temperature, Thermal conductivity, Thermal capacity, Energy, Reflection coefficient

AMS Subject Classification: 80-05, 80A21, 80M20

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ҚАЗАҚСТАН БОЙЫНША 2023 ЖЫЛДЫҢ АЛҒАШҚЫ ТОҚСАНЫНДАҒЫ СЕЛ ТАСҚЫНДАРЫ МЕН ОЛАРДЫҢ АЛДЫН АЛУ ШАРАЛАРЫ

АЛМАГҮЛ ТИЛЕЙХАН, РЫСБЕК БАЙМАХАН

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Мақалада қарастыруға алдынғы орында қазіргі таңда болуы мүмкін табиғи апаттардың бірі-су тасқыны туралы, Қазақстанның бастапқы төрт обылысының қазіргі жай күйі мен 2023 жылдың басында орын алған сел тасқындарына тоқталып талдау жасалынып, суреттер мен сандық деректер көрсетілген.

Талданатын негізгі мәселелердің қатарында қазіргі таңда табиғи апаттардың бірі- су тасқынын алды алу мақсатында жүргізіліп жатқан іс-шаралар. Қазақстан көлемінде және Оңтістік Қазақстан (ОҚО), Батыс Қазақстан облысы (БҚО), ШҚО және де СҚО бойынша іске асырылып жатқан жұмыстарға тоқталайық.

Наурыз айының басында «Казгидромет» РМК өңделген «NASA» спутниктерінің Жерді қашықтықтан зондтау (ЖҚЗ) суреттері бойынша 2023 жылдың 1-наурызындағы Қазақстанның жағдайна сүйене отырып қардың көлемі 72%-і құрайтынын анықтап, жауын мөлшерін, қардың еруінен болатын су қорын, күзгі топырақ ылғалдылығының ккрсеткіші мен оның қату тереңдігін, өзердердегі мұз көлемін ескере отырып талдау жасап су тасқыны қаупі бар аймақтарға баға берген болатын. Нәтижесінде тәуекелділігі жоғары аймақтар қатарына Алматы мен Жетісу, Қостанай, Солтүстік Қазақстан, Ақмола, Қарағанды, Ақтөбе, батыс Қазақстан, Абай және Шығыс Қазақстан обылыстары кірді. Орташа тәуекелді аймақтар ретінде Ұлытау, Павлодар және Атырау обылыстары, ал тәуекелділігі төмен аймақтар ретінде Жамбыл, Қызылорда, Түркістан және Маңғыстау облыстары саналған болатын.

Әдебиеттер тізімі

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ШАҒЫН ГИДРОЭНЕРГЕТИКАЛЫҚ СУ ЭЛЕКТР СТАНЦИЯСЫНЫҢ ГИДРОТУРБИНАСЫНЫҢ ТИІМДІ ПІШІНІН МОДЕЛЬДЕУ

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Қазіргі уақытта төмен қарқынды ағындар бойымен шағын су электр станцияларын орнату арқылы электр энергиясын өндіру энергия тапшылығы бар аймақтар үшін ыңғайлы және тиімді екені белгілі. Бұл жұмыста шағын СЭС тің түрлерімен танысып, оның ішінде зеріттеу обьектісі ретінде шағын құйынды СЭС таңдалды. Бұл бағытта жүргізілген зеріттеулер негізінде турбина пішіндерінің бірнеше нұсқалары ұсынылған [1,2]. Жұмыстың мақсаты құйынды шағын СЭС турбинасының ұсынылған нұсқаларын салыстыра отырып, оың ішіндегі тиімді пішінін анықтау болып табылады. Турбина пішінінің тиімділігін анықтау мақсатында COMSOL Multiphysics қолданбалы пакетінде сандық эксперимент жүргізіліп, бірнеше геометриясы қарастырылды. Турбинаның шыға берісіндегі жылдамдықтың мәнін табу арқылы энергияны көп беретін шағын СЭС түрі анықталады. Бастапқы жылдамдықтың бірнеше мәнінде (0,5; 1; 1,5; 2; 3; 4; 5 мс) турбина бойындағы қысым, жылдамдық графиктері алынды. Нәтижелері салыстырылып, ойыс түріндегі конус турбина энергияны көбірек беретіні анықталды .

Әдебиеттер тізімі

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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ НАПРЯЖЕННО-ДЕФОРМИРУЕМОГО СОСТОЯНИЯ УПРУГО-ВЯЗКОПЛАСТИЧЕСКОЙ СРЕДЫ С ПОЛОСТЬЮ

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Устойчивость и прочность различных наземных и подземных сооружений зависит от напряженно-деформируемого состояния массива, которое возникает при взрывах и динамических нагрузках. В связи с этим, вопрос о влиянии распространения волн различного характера на устойчивость и прочность массивов является актуальным.

В этой работе рассматривается распространение упруго-вязкопластических волн от динамических нагрузок взрывного характера на упруго-вязкопластической среде с полостью, лежащей на жестком основании. Для решения поставленной задачи создана математическая модель, описывающая напряженно-деформируемое состояние упруго-вязкопластической среды с цилиндрической полостью от воздействия внешних сил.

Для получения системы конечно-разностных уравнений и численного решения использован метод С.К. Годунова . Метод С.К. Годунова наиболее приемлем для численного решения систем уравнений гиперболического типа и для решения широкого круга задач механики сплошной среды.

Полученные численные результаты, которые описывают напряженно-деформируемое состояние массива и вокруг полости, отражены на графиках. Результаты данного исследования найдут свое применение в анализе состояния массивов при влиянии волн от взрывов, различных внешних и внутренних сил.

Ключевые слова: динамическая нагрузка, взрыв, волна, цилиндрическая полость, напряженно-деформированное состояние.

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ЧИСЛЕННЫЕ РЕЗУЛЬТАТЫ ИССЛЕДОВАНИЯ ВЛИЯНИЯ ИСХОДНОГО ЗНАЧЕНИЯ СКОРОСТИ И ТЕМПЕРАТУРЫ ГОРЮЧЕГО НА ПАРАМЕТРЫ ТРЕХМЕРНОГО ФАКЕЛА.

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В настоящее время основная часть используемой энергии вырабатывается при сжигании газов турбулентном потоке. Запасы природного газа ограничены. Поэтому в целях экономного использования горючих газов, совершенствование устроойств для сжигония топлива и увеличение их эффективности необходимо изучение турбулентного горения который представляет болшой практический интерес.

В данной работе приводятся некоторые численные результаты исследования химически реагирующих турбулентных струй состоящий из газовой смеси пропана-бутана истекающихся из сопла прямоугольной формы с конечным отношением длин сторон в спутный (затопленный) поток окислителя. В частности излагается результаты влияния исходного значения скорости и температуры горючего на параметры трехмерного диффузионного факела.

Для описания данного течения используются трехмерные параболизованные системы уравнений Навье-Стокса для многокомпонентного химически реагирующих газовых смесей. Для вычисления коэффициента турбулентной вязкости использовано алгебраическая модель имеющей следующий вид [1] :

$$\mu = \mu_l + \varkappa\rho\left(f^2(y) + f^2(z)\right)\sqrt{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial \omega}{\partial y}\right)^2} \cdot \left(\frac{T}{T_2}\right)^{\alpha}, \tag{1}$$

где μ_l-ламинарная вязкость,$\varkappa, \alpha$-эмпирические константы.

Задача решена численным методом с привлечением двухслойной десяти точечной неявной конечно-разностной схемы аппроксимации, прогонки и простой итерации.

Для численного интегрирования системы уравнений Навье-Стокса с учетом (1) применена модификация метода SIMPLE (Semi-Implicit Method for Pressure-Linked Equations), т.е. для расчета дисбаланса массы в каждой расчетной точке сетки используется разностное уравнение неразрывности.

Численные исследования проведены при следующих исходных данных горючего и окислителя:

I. Данные окислителя: $u_1 = 0$; $T_1 = 300(400, 500)K$; $(c_1)_1 = 0,232$; $(c_2)_1 = 0$; $(c_3)_1 = 0$; $(c_4)_1 = 0,768$;

II. Данные смеси горючего: $T_2 = 700(900, 1100, 1200)K$; $u_2 = 18,3(30,38,61)$ м/с; $(c_1)_2 = 0$; $(c_2)_2 = 0,12$; $(c_3)_2 = 0$; $(c_4)_2 = 0,88$;

Здесь нижние внешние вторые индексы указывают на принадлежность данного значения к срезу сопла воздуха –1, горючего -2. С точки зрения математического расчета , рассмотрено четырехкомпонентная смесь газов в зоне смешения, состоящую из кислорода O_2- индекс «1», смеси пропана-бутана $(C_3H_8 + C_4H_{10})$-«2», продуктов горения $CO_2 + 9H_2O$- «3», инертного газа N_2- «4».

Предполагается ,что сечение сопло квадратное и давления струи и спутного потока между собой равны и соответствуют атмосферному, а также исходные параметры струи и спутного потока задавались однородными и ступенчатыми значениями.

Приводим некоторые численные результаты. При изменении скорости горючей струи u_2 от 18,3 до 30 м/с линейные размеры факела увеличивается, при увеличивании u_2 до 61 м/с они практически не изменяются. Увеличение исходного значения температуры приводит к медленному убыванию осевой скорости и динамического напора. При малых исходных значениях температуры струи осевое значение температуры медленно растет. Подогрев горючего и окислителя приводит к незначительному удлинению длину факела. При подаче более нагретого горючего граница зоны смещения растет более медленно, однако переход к круглой форме затягивает.

Можно сделать заключение, что при подборе исходного значения скорости и температуры горючей струи и окислителя можно получить желаемую длину, ширину и температуры диффузионного факела.

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РЕЗУЛЬТАТЫ ИССЛЕДОВАНИЙ СВОЙСТВ ГРУНТА И РАСЧЕТА УСТОЙЧИВОСТИ ОПОЛЗНЕВОГО СКЛОНА АЛМАЛУУ-БУЛАК

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В работе [1] задача решается при допущении, что масса оползня до и после схода остается неизменной. Для нахождения массы оползня до схода M_0 сначала необходимо найти в неустойчивом склоне наиболее опасную поверхность скольжения. Склон сложен однородными суглинками, критическая поверхность скольжения неизвестна. Собственный вес склона обусловлен водонасыщенным грунтом в результате инфильтрации атмосферных осадков (дождь, снег).

В работе [2], оползневое давление E определяется, как разность между сдвигающими и удерживающими силами. Выражение, определяющее приращение оползневого давления E, имеет вид:

$$F\left(x, y_2, y_2'\right) = \left[\gamma_{\text{гр}}\left(1-m\right) + \gamma_b m\right]\left(y_2 - y_1\right)\left[\frac{y_2' + \text{tg}\varphi}{1 - y_2'\text{tg}\varphi} - \mu\right] - c\left(1 + \frac{\left(y_2'\right)^2}{1 - y_2'\text{tg}\varphi}\right), \quad (1)$$

где $\gamma_b, \gamma_{\text{гр}}$ – соответственно удельный вес дождевых вод и грунта, кН/м3; m – пористость грунта, д.ед; μ - коэффициент динамической сейсмичности [3]; y_1, y_2 – поверхности склона и скольжения; φ – угол внутреннего трения грунта; c – сцепление грунта.

Получаем выражение E в произвольном сечении x склона в виде интеграла:

$$E = \int_{x_0}^{x} F\left(x, y_2, y_2'\right) dx, \quad (2)$$

где x_0 - координата, соответствующая начальной точке оползневого блока.

В результате определения частных производных и выполнения необходимых преобразований уравнение Эйлера для функционала (2) принимает следующий вид:

$$\begin{gathered} 2y_2''\left(1 + \text{tg}^2\varphi\right)\left[\left(\gamma_{\text{гр}}\left(1-m\right) + \gamma_b m\right)\left(y_2 - y_1\right)\text{tg}\varphi - c\right] - \\ - \left(1 - y_2'\text{tg}\varphi\right)\left\{\left[\gamma_{\text{гр}}\left(1-m\right) + \gamma_b m\right]\left(\left(1 - y_2'\text{tg}\varphi\right)\left(y_2' + \text{tg}\varphi - \mu\left(1 - y_2'\text{tg}\varphi\right)\right) - \right.\right. \\ \left.\left. - \left(y_2' - y_1'\right)\left(1 + \text{tg}^2\varphi\right)\right)\right\} = 0 \end{gathered} \quad (3)$$

Так как дифференциальное уравнение (3) - второго порядка, для его решения задаются два условия – положение начальной точки и начальный наклон линии скольжения.

Были определены водно-физические и физико-механические свойства однородного суглинистого грунта из оползневого слона Алмалуу-Булак (Сузакский район). Они имеют значения: $m = 0,42$; $\varphi = 15^0$; $C = 100$ кН/м^2; $\gamma_{.} = 19$ кН/м^3; $\gamma_b = 10$ кН/м^3; коэффициент динамической сейсмичности $\mu = 0,05$. По данным полевых исследований были построены 3 сечения наиболее характерных участков склона и в качестве примера было взято сечение

1. Была установлена наиболее опасная поверхность скольжения с углом $\alpha = 65^o$. Ей соответствует минимальный коэффициент устойчивости $K_{\text{уст}} = 0,94$ и данное сечение массива является неустойчивым.

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СОСТАВ, СТРОЕНИЯ ГРУНТОВ И ГОРНЫХ ПОРОД МАРСА

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В этом докладе делается попытка обобщить сводку данных о свойствах грунтов планеты Марс. Анализируя данные посадочных аппаратов на основе данных орбитального зондирования, мы показываем их физические и механические свойства о строении и составе грунта Марса. История изучения поверхности Марса спускаемыми автоматическими аппаратами насчитывает 7 успешных американских миссий: посадочные аппараты Викинг 1 и 2 (1976–1982 гг.), марсоход Соджонер с посадочного аппарата Пасфиндер (1997 г.), марсоходы Спирит (2004–2010 гг.) и Опортюнити (с 2004 г.), посадочный аппарат Феникс (2008 г.) и марсоход Кюриосити (с 2012 г.). Все эти аппараты имели ТВ камеры, передававшие изображения, на которых была видна структура поверхности и особенности реголита и коренных пород.

Поверхность Марса покрыта чехлом обломочного материала (реголита), в образовании которого важную роль играли экзогенные процессы: эоловые, локальнофлювиальные, гравитационные, мерзлотные, эоловогляциальные, метеоритная бомбардировка. Мощность реголита колеблется от сотен метров до нескольких километров. Время воздействия указанных процессов на поверхность планеты исчисляется от сотен миллионов до нескольких миллиардов лет. В итоге сформировался осадочный чехол значительной мощности – до нескольких километров в отдельных регионах. Доминирующим процессом, интенсивно действовавшим в начальный период эволюции поверхности Марса (более 4 млрд. лет назад), была метеоритная бомбардировка древней коры планеты. Этот процесс создал сильно кратерированный ландшафт (похожий на поверхность лунных материков) и привел к значительному раздроблению верхних горизонтов литосферы и формированию мощного чехла мегареголита. Мощность такого слоя на Марсе может достигать 2 км [1].

Так как Марс обладал атмосферой и в прошлом, то ветровая деятельность на его поверхности, полностью лишенной растительного покрова (столь характерного для Земли), приводила к «пересортировке» поверхностного материала и дифференцированному переносу его в локальные понижения (кратеры) и обширные депрессии в сотни километров поперечником (ударные бассейны). Такой процесс продолжался всю геологическую историю планеты и сформировал осадочные толщи, мощность которых в отдельных местах может составлять несколько километров [2].

Облака земного типа (из водяных капель на Марсе бывают весьма редко. Атмосфера очень сухая. В самых безводных районах Земли в атмосфере в сотни раз больше водяного пара. В среднем его концентрация в атмосфере Марса близка к 0,05% (по объему), но меняется в десятки раз, во всяком случае в меньшую сторону. В атмосфере содержится азота 1,6% аргона, 0,1—0,4% кислорода, угарный газ (0,06%) и малые количества благородных газов — неон, криптон, ксенон. Но главная составляющая, 95% углекислый газ [3, 4, 5].

Результаты химического анализа поверхностного материала Марса в местах посадки «Викингов-1, -2» показали сходство состава (табл. 1) грунта в районах планеты, удаленных друг от друга на 6000 км. Для образцов грунта были получены следующие отношения химических элементов: Fe/Si = 0,7 ± 0,1 (в земных базальтах 0,7); S/C1 в разных образцах колеблется от 4 до 8; C1/Br – 100. Этот грунт оказался хорошо перемешанной (химически однородной на большой площади) смесью продуктов химического выветривания, главным компонентом которой может быть богатый железом глинистый минерал (или минералы), например нонтронитовый монтмориллонит. Наилучшее приближение к марсианскому грунту дает смесь богатых железом глин, характерных для процессов земного выветривания основных изверженных пород [6].

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Математиканы оқыту әдістемесі және математика тарихы

Methods of teaching mathematics and the history of mathematics

Методика преподавания математики и история математики

THE SYSTEM OF TASKS FOR THE DEVELOPMENT OF CREATIVE ABILITIES OF STUDENTS

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The criteria for creativity in mathematics, to highlight the tasks in the development of students' creative abilities and ways to achieve them are determined.

Purpose of the work. In the late 1950s of the last century, the American psychologist J. Gilford formulated several criteria for creativity:

1. Fluency of thought - the number of ideas that arise in a certain unit of time, the ease of generating ideas.
2. Flexibility of thought - the ability to switch from one idea to another.
3. Originality - the ability to produce ideas that differ from generally accepted stereotypes, the ability to respond to stimuli in a non-standard way.

The analysis represents the following manifestations of the development of students' creative abilities:

Finite set $\rightarrow$ Finite set, Countable set $\rightarrow$ Finite set,

Countable set $\rightarrow$ Countable set, Uncountable set $\rightarrow$ Finite set,

Uncountable set $\rightarrow$ Countable set, Uncountable set $\rightarrow$ Uncountable set,

Countable and bounded set $\rightarrow$ Countable and bounded set Countable and bounded set $\rightarrow$ Countable and unbounded set,

Countable and unbounded set $\rightarrow$ Countable and bounded set,

Countable and unbounded set $\rightarrow$ Countable and unbounded set,

Uncountable and bounded set $\rightarrow$ Countable and bounded set,

Uncountable and bounded set $\rightarrow$ Uncountable and unbounded set,

Uncountable and unbounded set $\rightarrow$ Uncountable and bounded sets,

Uncountable and unbounded set $\rightarrow$ Uncountable and unbounded set.

Finite, countable and uncountable subsets of an interval $(\frac{n}{n+1}, \frac{n+1}{n+2})$.

Sum, difference, product and division functions of given multiple formulas. Construct functions other than given functions in a finite set. Construct an example of an unbounded function in a segment.

Keywords: creativity, finite set, infinite set, countable set, uncountable set.

AMS Subject Classification: 97B10, 97B40, 97B50.

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ON THE HISTORICAL ASPECTS OF THE FORMATION OF COLLECTIVENESS OF MATHEMATICS TEACHING IN SCHOOLS AND PEDAGOGICAL HIGHER EDUCATION INSTITUTIONS

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According to historical data on the territory of modern Kazakhstan in the VIII-XIII centuries there were a number of large cities in which significant successes were achieved in the field of education, science, culture, technology, corresponding to this era. The evidence is the many works of scientists of the IX-XIV centuries, who made a great contribution to the development of science, including philosophy, mathematics, physics, astronomy. Their works were written in Arabic and Persian languages. The greatest scientist in the IX-X centuries, who had a huge impact on the further development of world science and culture, was a native of the Kazakh tribe Abu Nasyr al-Farabi (870-950), known in the world as the Aristotle of the East. His versatile treatises cover almost all areas of science, including mathematics. After the accession of Kazakhstan to the Russian empire in the middle of the XVIII century (1731), the first Russian-Kazakh schools began to open in the country. From the 60s of the XIX century mathematical education in Kazakhstan began to acquire a broader content and entered a new path of development. This was due to the political, economic and cultural changes taking place in the Russian empire. By the end of the XIX century the expansion of the network of Russian-native schools in the Kazakh steppe required the creation on the territory of Kazakhstan of pedagogical educational institutions that trained teachers (Tashkent, Omsk, Orenburg, Omsk). Later teachers' seminaries were opened in the cities of Semipalatinsk, Aktyubinsk, Uralsk, Verny, Akmolinsk. The first textbook on mathematics "Study guide for teaching elementary mathematics"was published in 1914 in two books in Orenburg, the first capital of Kazakhstan. Its author is one of the leaders of the national movement "Alash state and public figure, poet and writer - Myrzhakyp Dulatuly. Undoubtedly, the greatest role in the development of teaching mathematics and the methodology of its teaching in the first decade of Soviet power was played by prominent figures of the national movement "Alash who were repressed and shot in the 30-40s of the last century. They published many textbooks, study and task guides in mathematics in Kazakh language, written in Latin or Arabic script. These first teachers also formed the methodological and mathematical terminology in Kazakh language. During the Soviet period of development of Kazakhstan books for schools, technical institutions and universities in various fields of science, including mathematics, were prepared in Moscow and other scientific-pedagogical centers of Russia in Russian language. At the same time, the number of Kazakh schools began to decline steadily, and since 1930 all educational institutions in Kazakhstan began to use textbooks and study guides by Russian authors, as well as their translation into Kazakh. At the end of the 80s of the last century, the problem of preparing textbooks for schools and pedagogical universities in Kazakh language was again raised, with the main condition being the observance of the principles of forming the continuity of teaching mathematics at all levels. On December 16, 1991, in connection with gaining independence of our country, the problem of mathematical education in Kazakh language was recognized as one of the main directions of the ideological policy of the state and it was reflected in the state programs for the development of education adopted by the Government of the Republic of Kazakhstan and obligatory standards of education. From this period of time the development and implementation of our own curricula, textbooks and study guides in mathematics for secondary schools and pedagogical universities of the republic began, taking into account continuity in teaching mathematics. Along with this the didactics of mathematics began to develop more actively as part of pedagogical science, which made it possible to significantly increase the number of scientific publications and defend many dissertations on the theory and methods of teaching mathematics. It should be especially noted that the subject "Methods of teaching mathematics" has become the main compulsory subject in the preparation of future teachers of mathematics. Its educational and methodological support, textbooks, study guides and additional materials written by methodologists of the country, in particular, teachers of the department of methods of teaching mathematics, physics and informatics of the Kazakh national pedagogical university after Abay contributed to the optimal organization of the educational process at the physics-mathematics faculties of pedagogical universities of Kazakhstan.

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TRAINING A FUTURE TEACHER OF MATHEMATICS: IMPLEMENTATION OF THE EDUCATIONAL MODULE METHODOLOGICAL TRAINING OF A TEACHER OF MATHEMATICS IN THE BASIC CURRICULUM

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In accordance with the state order for the training of teachers of mathematics, the Faculty of Mathematics and Information Technologies of the Osh State University of the Kyrgyz Republic annually enrolls students for the bachelor programs 550200 Physics and Mathematics Education (with training profiles Mathematics , Informatics, Physics).

The design of the main educational program of this direction with the profile of training Mathematics is based on the state educational standard of higher professional education in the program 550200 Physical and mathematical education , qualification: Bachelor [2], the Professional standard of a teacher of a general education organization [4] and the results of research based on the opinions of interested stakeholders. Based on the results of our theoretical and empirical research [1], [6], a basic curriculum was developed on a modular-competence basis, containing twelve educational modules.

The purpose of the article is to develop the content of the educational module Methodological training of a teacher of mathematics , based on the methodology of learning outcomes [3], within the discipline Theory and Methods of Teaching Mathematics . To do so, eight learning outcomes of this discipline [5] have been developed, which should determine its purpose, objectives and content.

The table of correspondence between learning outcomes and the content of sections of the discipline Theory and Methods of Teaching Mathematics indicate the feasibility of applying the methodology of learning outcomes in the methodological training of a future teacher of mathematics.

Keywords: educational program, educational standard, Physical and mathematical education, educational module, learning outcomes, Theory and Methods of Teaching Mathematics.

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CONTINUITY IN TEACHING GEOMETRY IN MIDDLE AND HIGH SCHOOLS

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The efficacy of mathematics instruction in schools and universities within the framework of the continuous mathematical education system relies on a pedagogical approach to the teaching of mathematics.

The objective of this article is to showcase the practical application of geometric theorems across various levels of complexity, encompassing non-standard and olympiad problems, while emphasizing the pivotal role and significance of complex number algebra in their resolution.

The utilization of complex number algebraic methods enriches students' comprehension of mathematics, thereby fostering the development of their conceptual understanding, reasoning abilities, and the cultivation of analytical thinking.

Within the presented article, the employment of complex number algebraic techniques in the solution of geometry problems yields outstanding outcomes ([1] - [5]).

Keywords: complex numbers, centroid, orthocenter, Euler's formula, Mollweide's formula.

AMS Subject Classification: 30-01

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DEVELOPMENT OF STUDENTS LOGIC THINKING USING INTERACTIVE ASSIGNMENTS IN MATHEMATICS

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As a new paradigm of education: teaching students to self-study with the help of Internet technologies is becoming a requirement of the time. The teacher acts as a guide, a consultant who guides the activities of students. Along with high-tech teaching aids, digital technologies are being actively introduced in the modern school.

In our article, we will focus on creating interactive tasks and applying them in the learning process on interactive educational platforms that will help us teach new, effective, creative, and most importantly - accessible and understandable for all teachers and students.

Currently, there are many Internet applications that are used in a variety of ways. Among them are learnigApps, Padlet, Wizer.me which attract users with their insight, simplicity and efficiency, we are talking about online platforms.

They fully comply with the state educational standard and contribute to solving the problems of the education development program to improve the effectiveness of education and digital literacy of students and teachers. The platforms include interactive tasks and are very easy to work with. With the help of online platforms, you can organize various forms of classes: group, pair, problem, search, research, etc.

Interactive tasks are based on the personalization of the educational content space, mobile e-learning, acceptance of the individuality of each child, support for his individual needs and interests, as well as the successful implementation of group activities.

Students complete assignments at home whenever it suits them, and we can send assignments to them via links. Interactive, colorful and engaging activities aim to reinforce the skills acquired in the classroom.

Interactive whiteboard Padlet is very convenient for feedback and exchange of opinions among students. They can also instantly post assigned tasks in a variety of formats. Students really like it, the consolidation of the material takes place in a fun way, the lesson takes place live, students show interest in work, and motivation to study topics increases.

Thus, interactive tasks contribute to the formation and development of students' digital competence. Doing fun interactive activities at home and in the classroom can certainly increase student engagement and spark their interest in learning subjects.

Keywords: interactive problems, competence-based approach, modern information technologies, learning efficiency, online platforms for students' logical thinking.
AMS Subject Classification: 97B50, 97B40.

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EFFECTS, PHENOMENA, BASIC COMPLEXES IN MATHEMATICS

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Academician A.Borubaev ([1] and forthcoming monographs) presented developing of topology as successful implementation of idea of continuity in systems of axioms. In [2] developing of the idea of space was presented. A.Chekeev noted that mathematicians actually use a set of interconnected statements (an axiom is a theorem in another equivalent system of axioms). We propose to call such set as "basic complex" of any branch of mathematics. We [3] drew attention to phenomena as initiators of branches of mathematics. Unresolved and unresolvable problems also used to be initiators.

We list both well-known and found by us effects and their consequences.

1. Effect of infinity. The phenomenon "A part of an infinite object can be equivalent to itself" refuted the intuitively evident Euclid's fifth axiom "the whole is greater than the part". The further development of mathematics was significantly connected with attempts to overcome such paradoxes. At last, P.Cohen (1963) proved that a suitable basic complex is impossible.

2. Effect of continuity. The idea of continuity yields "some tasks for functions have solutions for continuous ones" (K.Weierstrass). Numerous attempts to derive any kind of smoothness from continuity were stopped by K.Weierstrass example (1895). Such hopes were implemented:

3. We distinguished the effect of analyticity: some tasks being non-correct for continuous and even infinitely differentiable functions became correct for analytical ones (K.Alybaev, P.Pankov, T.Imanaliev, L.Askar kyzy).

4. Irgöö effect and numerosity effect. The idea of creating order (cosmos) out of chaos is well-known from ancient times. We found the first mention of concrete implementation of such phenomenon in speech: irgöö means: "random vibration of many balls of different sizes in a round vessel yields migration of the biggest one to the center of their surface." A real process for sufficiently many components (including numerical experiment because computer is a real object) in compact set propelled by outer energy may be self-organized (S.Tagaeva).

5. Synergetic effect for dissipative systems.

6. Effect of higher dimensions. It is known that passing from 1D to 2D (from numbers to vectors), from 2D to 3D yielded new phenomena (strange attractor).

We hope that this list would promote discovering new effects and phenomena in various branches of mathematics and its applications.

Keywords: mathematics, effect, phenomenon, synergetic, numerical experiment, self-organizing
AMS Subject Classification: 00A05

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THE THEORETICAL BASIS AND PRACTICAL IMPLEMENTATION METHODOLOGICAL TRAINING OF MATHEMATICS TEACHERS IN AN INNOVATIVE MANNER

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Methodical science teaching mathematics covers all stages of mathematical education. Lesson methodology has affected the work of mathematics teachers and academics. Hence the importance of methodical preparation of teachers of mathematics and their means of methodic knowledge. Modern mathematics teacher must know the components of the methodology, the methodology of mathematics and learn the methodology of scientific research, learning theory and its applications. This will require the need for special analysis methodology of teaching mathematics. The urgency of this challenge is increased by the ideas of innovation research.

Keywords: Methodology of mathematics, methodology, methodology, teacher training, pedagogical innovation, methodological training of mathematics teachers in an innovative manner.

AMS Subject Classification: 68

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VII World Congress of Turkic World Mathematicians
20-23 September 2023, Turkestan, Kazakhstan

THE VALUE OF INTERPRETATION IN THE DEVELOPMENT OF SPATIAL THINKING

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It is now recognized that education will play a leading role in achieving sustainable development, which is directly called in many UN documents.

The Kyrgyz Republic, like all countries of the world, has committed itself to achieving the Sustainable Development Goals (SDGs) by 2030.

The fourth of the 17 Sustainable Development Goals (SDG-4) focuses on education and is formulated as "Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all" [1].

At the moment, in order to achieve this goal, it is necessary to direct the educational process towards the development of skills of the 21st century, which has led to changes in the State Educational Standard of our republic. The formation of skills for the 21st century lies on the shoulders of teachers, who themselves must meet the requirements of the modern paradigm of education. One of these skills is mathematical literacy, which shows "a person's ability to think mathematically, formulate, apply and interpret mathematics to solve problems in a variety of practical contexts" [2].

In physiology, it is said that the right hemisphere of the human brain responds to intuition, and the left to logic. The interaction of the two hemispheres is necessary to control the functioning of the brain and transfer information from one to the other. The stronger the connection between the two hemispheres of the brain, the higher the person's intellectual development, memory, perception, attention, imagination, speech, thinking. In the learning process, the perception of information and its understanding are, in fact, two different things. Consequently, the geometric interpretation of mathematical concepts and, conversely, the algebraic interpretation of geometric concepts contributes to the formation of concepts and the development of the above mental processes, including spatial thinking, which is necessary for a successful life and work of a person in the areas of his activity.

Keywords: interpretation, spatial thinking, mathematics, students, sustainable development.

AMS Subject Classification: 97

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THE ROLE OF CONSTRUCTIVE LEARNING OF MATHEMATICS IN EDUCATION

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The general task that forms the basis of the educational process is the education of a comprehensively trained and competitive young generation. a comprehensively trained person or a competent person is a person who embodies the highest human qualities, moral values. The upbringing of people with these qualities requires determining what, why and how to teach young people, and it is based not on mechanical memory, but on theoretical (critical) thinking, on increasing the demands for intellectual activity. among the available theories of learning, constructive learning is the most useful, of great educational and vital importance. In terms of its significance, constructive learning is included in a group of theories that are based on the creation of knowledge by the synthesis of new ideas with existing knowledge, that is, the knowledge system does not appear in a passive form, but on the basis of the learner's active life experience. Constructivism considers epistemology as its basis - a system of philosophical thoughts about the nature of knowledge and study.

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EDUCATIONAL CONDITIONS TO DEVELOP COGNITIVE INTEREST OF TECHNICAL COLLEGE STUDENTS IN LEARNING MATHEMATICS

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The objective of the article: theoretical and practical justification of the effectiveness of the use of historical elements of mathematics in educational conditions to develop cognitive interest of students of non-mathematical specialities (technical colleges) who are learning mathematics in the system of secondary vocational education.

Among the many educational conditions that develop cognitive interest in mathematics among students of non-mathematical specialties (in technical colleges) in the system of secondary vocational education, we have identified three, from which the most effective ones, in our opinion, are: rethinking and supplementing the content of the subject of Mathematics in such a way that it includes elements of professional orientation and the history of mathematics; rethinking professionally oriented mathematical problems and design methods in the organization of the process of teaching mathematics; teaching using its historical elements, using interactive methods of teaching mathematics [5, 156 pp.].

The main features of teaching mathematics in college can be called mathematics, the content of which includes historical elements of mathematics, supplemented by professionally oriented tasks. Having studied the state educational standard of basic general education, we will see that the use of historical material at a mathematics lesson allows us to fulfill a number of standard requirements:

1. Shaping a worldview in general, corresponding to the current level of development of science and public practice.

2. Developing a conscious, respectful and benevolent attitude to the culture, history, religion, traditions, values of the peoples of Kyrgyzstan and the world.

3. The ability to independently plan ways to achieve goals, including alternative ways, consciously choose the most effective ways to solve educational and cognitive problems.

4. The ability to create, apply and transform signs and symbols, models and schemes for solving educational and cognitive tasks.

5. The ability to organize educational cooperation and joint activities with teachers and groupmates, individual and group work.

Key words: technical college, cognition, historical element, educational process, secondary vocational education, mathematics, professional tasks.

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DEVELOPMENT OF STUDENTS' RESEARCH SKILLS THROUGH A PROBLEM-ORIENTED MODEL

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Abstract. Effective education integrates cultural values, fosters critical thinking, problem-solving and research skills across disciplines, promoting peer teaching and public speaking.
Keywords: education, critical thinking, problem-solving, research skills, interdisciplinary, peer teaching, public speaking.

The development of education encompasses the provision of cultural and historical values, norms, and traditions to individuals through carefully selected content and forms. The effectiveness of education lies in its focus on the student's personality, necessitating a consistent approach to their educational journey and the subjects and areas of study.

The teaching of mathematics should aim to equip learners with the ability to apply acquired knowledge to making decisions in various real-life situations. Therefore, the effectiveness of each subject is contingent upon its integration into other disciplines and its direct relevance to practical applications.

Diagnostic cards were meticulously developed with the assistance of school psychologists, taking into account these characteristics during the lesson planning process. By incorporating problem-solving questions at each stage of the lesson, students are encouraged to cultivate their ability to express ideas, identify core issues, and pose inquiries. By involving students in setting goals and establishing evaluation criteria, motivation for the lesson is enhanced. Furthermore, students can grasp the significance of adopting an integrated approach to tackling real-world problems.

Critical thinking skills and creativity in students are fostered through systematic utilization across all disciplines, with the aim of honing problem-solving abilities. Group training facilitates the review of actions and the consideration of new problems that arise from the analysis of experiences. I firmly believe that the development of research skills among students holds particular significance in the contemporary world.

The cultivation of students' research skills contributes to the advancement of peer teaching and public speaking abilities. Through the development of research skills, students become adept at teaching one another by employing argumentation techniques to effectively utilize their foundational knowledge. One approach to fostering research skills involves structuring the material. During individual work, certain students are provided with simplified guidelines tailored to their abilities and the outcomes of their psychological diagnosis. It is essential to consistently employ action research aimed at accomplishing the objectives and goals of mathematical education in order to achieve optimal effectiveness.

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OPEN MATHEMATICAL OLYMPIAD FOR UNIVERSITY STUDENTS

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The advent of modern internet technologies has revolutionized traditional methods of mathematical education, creating exciting new possibilities for educators and students alike. One particularly promising area where these technologies can be effectively leveraged is in the organization of mathematical competitions. With contestants able to participate from their own schools or universities, the internet provides a powerful platform for problem-solving and solution submission. Moreover, through the use of basic audio and video connections, participants can easily communicate with organizers and engage in collaborative problem-solving efforts. Overall, the potential impact of internet technologies on mathematics education is truly vast and inspiring.

Keywords: Mathematical Olympiads for university students, Online Olympiads, Internet Technologies, Teaching methods, Comprehensive works on analysis education

AMS Subject Classification: 97I10, 97U50, 97U40, 97D50, 97D30

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DIFFERENTIAL EQUATIONS. SHERLOCK HOLMES AND OTHER APPLICATIONS

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A sultan sent his son to study clairvoyance. He tried his best and succeeded in doing so. Once the sultan took a gemstone ring in his hand and said: "My son, tell me what I have in my hand?" He answered: "It is round, it is a mineral, and it has a hole in the middle." The sultan said: "You named the attributes right, tell me what is this object?" After long deliberation, he answered: "A millstone". The Sultan said: "You established so many exact features with the power of knowledge and learning, but you did not have enough reason to understand that a millstone can not fit in my hand, and I can not take it in my hand." That's how scientists these days "split" a hair in the sciences, but do not know what is important and closest to them [1].

To ensure that our students do not become like this prince, it is necessary to teach them to use mathematics. Since the derivative of the function $y = f(x)$ is the rate of change of y, it is natural that the equations containing derivatives - differential equations are often used to describe the constantly changing reality around us.

Inspiring students, encouraging them to study differential equations, and teaching them to use this tool is a very useful and interesting objectives. In our opinion, word problems can play an important role in addressing this issue. Some of these problems, such as the story of Sherlock Holmes and others, are presented in this paper.

Keywords: differential equations, application of differential equation, a new approach to solving differential equations, mathematics teaching, the story of Sherlock Holmes.

AMS Subject Classification: 34A05, 00A09.

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SOME ASPECTS OF THE PROBLEMS SOLUTION METHODOLOGY IN MECHANICS

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The purpose of the practical component of the Mechanics course for mathematicians is to acquire practical skills in analyzing and solving problems of Statics, Kinematics and Dynamics of the mechanical systems, the ability to shape specific physical problems in an abstract mathematical form, i.e. to construct motion models of mechanical systems and calculate them.

The Physics course for mathematicians at the Kyrgyz-Turkish Manas University constitutes 5 European Credits and includes 48 hours of lectures and 32 hours of laboratory classes in the second semester of the first course. Due to the small time of the course, we limited ourselves to teaching the Mechanics Section. It was selected to train students prior to the Theoretical Mechanics course.

The decline in the schooling level in natural subjects, in physics in particular, creates a serious obstacle for students to master the University Program. In particular, due to the lack of skills and abilities to solve problems in most first-year students, we have to fill in their school gaps on the way. Moreover, the course compactness requires an active methodological search in order to improve performance of practical exercises. Based on the experience with mathematics students, the following approaches are proposed to follow in addressing the issues above.

• The solution of word problems with the ultimate compilation of the motion equation. Based on the data obtained from solving word problems, students should be able to construct the dependence of the distance traveled (coordinates or displacement vector) with time for various motion types, which is a mathematical equation.

• Test problems for quick assessment and analysis of the level of mastering the theoretical material by students. Test problems are compiled using illustrations, requiring students to independently identify the given values and search for indeterminates to obtain an answer to the test question.

• Involving turnkey package to mainstreaming the intellectual activity of the achievers. The use of software packages is advisable in cases of absence of the analytical solution for the advanced problems.The analysis of the obtained solutions and their dependence on changes in the given parameters is an important point.

Thus, the Mechanics Course is designed to train mathematics students for mastering the disciplines of the general professional cycle, to facilitate understanding the relationship between mathematics and real physical processes and objects.

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FORMATION OF RESEARCH SKILLS OF GIFTED STUDENTS OF THE SECONDARY SCHOOL IN THE PROCESS OF SOLVING MATHEMATICAL PROBLEMS

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In the modern world, science plays an important role in the development of society and the solution of socially significant problems. To become a scientist and contribute to the scientific community, one must have the skills of scientific reasoning. Scientific reasoning skills include the ability to analyze data, formulate hypotheses and conclusions, argue your thoughts, and summarize and interpret research results. These competencies need to be developed already in secondary education, namely among high school students, because the characteristic features of high school students are developed forms of theoretical thinking, the assimilation of scientific knowledge methods that contribute to the formation of not only the need for intellectual activity, but also stimulate the manifestation of research initiative. It is important to note that research activity allows you to realize your own mental capabilities in a variety of educational projects, as well as areas of future professional activity.

This report discusses issues related to the organization of research activities of students, and also assesses the level of readiness of teachers to implement this process.

The main results of the survey conducted among school students and interviews of teachers in the city of Shymkent are described. Based on the analysis of the work experience of teachers in Shymkent, typical difficulties that arise in the formation of research skills among students are highlighted. The study made it possible to understand more deeply what kind of difficulties students face in organizing research activities. Such difficulties may include insufficient knowledge of research methods, lack of skills in working with information resources, as well as insufficient motivation and independence of students.

Especially, the question of the development of research skills with the help of independent solution of various mathematical problems is touched upon.

In conclusion, the necessity of developing a special methodology for organizing research activities in mathematics at school, as well as identifying and supporting gifted students, holding a methodological seminar to increase the level of readiness of teachers to organize research activities of schoolchildren is substantiated.

Keywords: research skills, mathematical problems, survey,independent research.

AMS Subject Classification: 97U30 Teachers' manuals and planning aids (aspects of mathematics education)

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HUMANITARIAN ASPECTS OF TEACHING INVERSE PROBLEMS FOR DIFFERENTIAL EQUATIONS

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Currently, methods and technologies of inverse problems for differential equations are widely used in applied research and allow acquiring scientific information about objects or processes and phenomena occurring not only in places accessible to the researcher, but also in inaccessible places, for example, located deep underground or deep under water. Mastering the theory of inverse problems involves systematic training of such future specialists in higher educational institutions [1, 2, 3]. Here, of course, we mean, first of all, students studying in the fields of applied mathematics. When teaching inverse problems to students, it is important not only to teach mathematical methods of research of applied problems, but also to introduce nature-like and integrative technologies that make it possible to put into practice the humanitarian principles of cognition. It is possible to implement such humanitarian aspects of education on the basis of the development of humanitarian-oriented training sessions. In such classes, the teacher should plan situations when students, based on the results of research on the inverse problem, formulate conclusions not only of an applied nature, but also of a humanitarian nature.

Keywords: inverse problems for differential equations, humanitarian aspects of teaching, pedagogical technologies, student.

AMS Subject Classification: 97M10 Modeling and Interdisciplinarity (Aspects of Mathematical Education).

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ЭФФЕКТИВНЫЙ МЕТОД РЕШЕНИЯ ЗАДАЧ ПО ПЛАНИМЕТРИИ

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В статье раскрывается эффективный метод решения планиметрических задач по геометрии в контексте деятельностного подхода, приводятся соответствующие примеры.

Математическое образование является главным элементом среднего образования. Реформа математического образования, начатая еще в шестидесятые годы , и меняющая до сих пор ежегодно сопровождается введением в школьный курс геометрии современных и рациональных методов решения задач. Достаточно нового качества обучения нетрадиционной для школьного курса геометрия методом решения задач требует разработки соответственного методического обеспечения учебного процесса Под аналитическом методом решения геометрической задачи обычно понимается такой метод ее решения ,в котором в качестве формальной математической модели выступает аналитические соотношения. Мы ограничимся рассмотрением методики обучения нетрадиционным для школьного курса планиметрии векторным методом решения задач в контексте деятельности подхода. Векторный метод решения задач удовлетворяет всем критериям ведущих знаний, однако практика их применения при изучения планиметрии в действующих учебниках и учебных пособиях не соответствует их методической значимости. Мы исходим из необходимости приведения в соответствие методической значимости векторного метода решения планиметрических задач с практикой обучения последним. Векторный метод обогатил геометрической наглядностью алгебру, позволил представить в наглядных геометрических образах течение различных процессов. Одна и та же задача получает различное векторное представление в зависимости от того или иного способа ее решения. Векторный метод эффективен при: а) доказательстве параллельности прямых и отрезков; б) обосновании утверждения о делении отрезка данной точкой в указанном отношении; в) выяснении принадлежности трех точек одной прямой; г) доказательстве перпендикулярности прямых и отрезков; д) доказательстве зависимостей между длинами отрезков; е) нахождении величины угла.

В теории П.Я. Гальперина усвоение знаний рассматривается как процесс, осуществляемый на основе усвоение действий по применению усваиваемых знаний. Формирование новых мыслительных процессов начинается с освоения разработанных обучающим предписаний, представленных в форме развернутых внешних действий. Развернутость действия по решению задачи на элементарные действия или операции делает процесс обучения новым методам решения задач осмысленным, сознательным, т.к. раскрывает объективную логику процесса решения. Включение в процесс решения геометрической задачи аналитическим методом действий по геометрической иллюстрации условия и интерпретации полученного аналитически результат обеспечивает основу для новой идей мыслительной деятельности способствуя формированию исследовательской структуры мышления. Вместе с тем при переходе действий умственный план деятельность ученика по созданию геометрического образа адекватного аналитической модели включает деятельность в свернутые формы рассуждений. Геометрический образ становится "опорным сигналом"и необходимым звеном деятельности по решению задачи аналитическим методом. Применение метода моделирования при разработке методики обучения аналитическим методам решения задач по геометрии представляется по нескольким причинам. В первую очередь эти методы сами являются средством математического моделирования с помощью которых учащиеся получают знаковую модель, адекватную условию задач: преобразование этой модели по соответствующим законам и ее анализ позволяют выполнить требование задачи и перейти от формальной модели к ее геометрический интерпретации. Деятельность которую при этом выполняют учащиеся включают все этапы из которых состоит математическое моделирование. Одновременно выполняется требование задачи и формируется умения и навыки математического моделирования т.е. реализуется цели деятельности – овладения новым методом решения задачи и новым способом рассуждения. Рассмотрим систему задач, способствующих формированию умения делать выводы, получать разнообразные следствия из их условий. Значимость разработанной системы состоит в том, что она позволяет выявлять не только уровень усвоения учащимися теоретического материала по теме «Векторный метод», но и, самое важное, способствует обучению учащихся приемам распознавания геометрических образов на первом этапе.

Использование векторного метода в конкретных ситуациях вызывает определенную умственную деятельность. Для определения содержания за- дач, формирующих умение применять векторы, необходимо выделить действия, адекватные этой деятельности. Анализ показывает, что использование векторного метода в ситуациях а) – е) предполагает владение следующими умениями: 1) переводить геометрический язык на векторный и обратно; 2) выполнять операции над векторами; 3) представлять вектор в виде суммы векторов, разности векторов; 4) представлять вектор в виде про- изведения вектора на число; 5) преобразовывать векторные равенства; 6) переходить от соотношения между векторами к соотношению между их длинами и наоборот; 7) выражать длину вектора через его скалярный квадрат; 8) выражать величину угла между векторами через их скалярное произведение. Благодаря возможности применения общей логической схемы поиска решения задача аналитические методы позволяют разрабатывать предписания для решения классов задач представляющие собой модели деятельности образцы в которых словесно описываются пути поиска решения и реализации требования геометрической задачи аналитическими средствами.

Основная идея деятельностного подхода в интерпретации Г.И Саранцева заключается в целесообразности описания и проектирования деятельности учащихся как системы процессов решения задач. Мы рассматриваем этот подходы к обучения аналитическим методом решения планиметрических задач путем разработки и применения методически целесообразный системы задач. Обучение через задачи в данном случае является единственным путем, идя по которому учащиеся могут овладеть новыми методами решения, оценить их достоинства, познать специфику применения численных методов к изучению свойств геометрических объектов. Успех применения векторного метода в различных ситуациях во многом обусловлен владением специальным словарем, служащим для перевода с языка геометрического на язык векторный и обратно. Этот словарь можно представить следующей таблицей (табл. 1).

Данный словарь включает в себя и ряд опорных задач, являющихся ключом к решению многих более сложных задач с помощью векторов. К ним относятся условия, определяющие принадлежность точки прямой, деление отрезка в данном отношении, принадлежность четырех точек плоскости. Проведенный анализ деятельности применения векторов в различных конкретных ситуациях позволил выделить в ее структуре действия, которые определяют соответствующий ему вид задач, а именно задачи:

- на перевод с геометрического языка на векторный и обратно;
- на выполнение векторных операций;
- на представление вектора в виде суммы (разности) векторов;
- на представление вектора в виде произведения вектора на число;

Таблица 1

Язык геометрии	Язык векторов
$AB \| CD$	$\overrightarrow{AB} = R \cdot \overrightarrow{CD},\ \ R \neq 0$
$AB \perp CD$	$\overrightarrow{AB} \cdot \overrightarrow{CD} = 0$
Точка C принадлежит прямой AB	$\overrightarrow{AB} = R \cdot \overrightarrow{BC}$, либо $\overrightarrow{AC} = R \cdot \overrightarrow{BC}$, либо $\overrightarrow{AC} = R \cdot \overrightarrow{AB}$, либо $\overrightarrow{OC} = p \cdot \overrightarrow{OA} + q\overrightarrow{OB}$, где O – произвольная точка, $p + q = 1$
M – середина отрезка AB	$\overrightarrow{AB} + \overrightarrow{BM} = 0$, либо $\overrightarrow{OM} = \frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB}\right)$, где O – произвольная точка
M_1 – середина отрезка A_1B_1, M_2 – середина отрезка A_2B_2	$\overrightarrow{M_1M_2} = \frac{1}{2}\left(\overrightarrow{A_1A_2} + \overrightarrow{B_1B_2}\right)$
M – центроид ΔABC	$\overrightarrow{OM} = \frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}\right)$, где O – произвольная точка, либо $\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} = 0$
Точка C делит отрезок AB в отношении $AC : CB = m : n$	$\overrightarrow{AC} = \frac{m}{n}\overrightarrow{CB}$, либо $\overrightarrow{OC} = \frac{n}{m+n}\overrightarrow{OA} + \frac{m}{m+n}\overrightarrow{OB}$, где O – произвольная точка
O, A, B, C – точки одной плоскости	$x \cdot \overrightarrow{OA} + y \cdot \overrightarrow{OB} + z \cdot \overrightarrow{OC} = 0$, где x, y, z – действительные числа и $x + y + z = 1$
$ABCD$ – параллелограмм	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$; $\overrightarrow{AB} = \overrightarrow{DC}$; $\overrightarrow{BC} = \overrightarrow{AD}$; $MO = \frac{1}{4}\left(\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} + \overrightarrow{MD}\right)$, где M – произвольная точка, а O – точка пересечения диагоналей параллелограмма $ABCD$
$AB \perp \alpha$, CD и MK – пересекающиеся прямые плоскости α	$\overrightarrow{AB} \cdot \overrightarrow{CD} = 0$ и $\overrightarrow{AB} \cdot \overrightarrow{MK} = 0$

- на переход от соотношения между векторами к соотношению между длинами и обратно;
- на преобразования векторных равенств;
- на нахождение длины вектора и величины угла между ними.

Все виды задач включают в себя основные свойства векторов, большинство задач – предполагает владение умением представлять векторы в виде суммы (разности), переводить геометрический язык на векторный и обратно, владение некоторых действиями с векторами на умственном этапе. Эксперимент показал целесообразность следующих сочетаний уровней формирования умений выполнять операции над векторами и представлять вектор в виде суммы (разности) векторов, произведения вектора на число: 1) формирование взаимно обратных умений осуществляется на материализованном уровне; 2) решение задач на представление вектора в виде комбинации других векторов осуществляется мысленно, а проверка правильности выполнения этого действия – на материальном уровне; 3) формирование взаимно обратных умений осуществляется только в умственном плане.

Таким образом, решение математических задач векторным методом способствует обучению учащихся не только нахождению оптимального векторного решения задачи, но и формируют в их сознании приемы и способы распознавания геометрических образов.

Ключевые слова: задача,метод,деятельностный подход,модель,решение задач.

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ОҚУШЫЛАРДЫ ЕСЕПТЕРДІ ТРИГОНОМЕТРИЯЛЫҚ АЛМАСТЫРУ ТӘСІЛІН ПАЙДАЛАНЫП ШЫҒАРУҒА БАУЛУ

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Мақалада тригонометриялық формулалар мен қасиеттердің математикалық есептерді шешуде қолданылуы ашылады. Мақалада дәлелді шешімі бар сәйкес мысалдар келтірілген.

Жоғарғы сынып оқушыларын математикалық есептерді тригонометриялық алмастыру әдісін пайдаланып шығаруға баулу үшін, ең алдымен олармен синус, косинус, тангенс, котангенс сияқты негізгі тригонометриялық функциялардың қасиеттерін және негізгі тринометриялық теңдеулерді пысықтап, тереңінен қайталап алған жөн. Себебі тригонометриялық алмастыру әдісін қолданғанда берілген теңдеу немесе теңсіздік, ең алдымен қарапайым тригонометриялық функцияға түрленеді. Яғни айнымалы синус, косинус, тангенс, котангенс сияқты негізгі тригонометриялық функциялармен алмастырылады. Кейін тригонометриялық өрнек тригонометриялық функциялардың қасиеттерін және негізгі тригонометриялық формулаларды қолдану арқылы шешіледі. Енді осы әдіске нақты мысалдар келтіру арқылы тоқталып кетейік:

Есеп. Берілген теңдеуді шеш.

$$x+\sqrt{1-x^2}=\sqrt{2}\left(2x^2-1\right). \tag{1}$$

Шешімі: Берілген теңдеудің анықталу облысы $-1<x<1$. Енді x-ті жаңа айнымалымен ауыстырамыз: $x=\cos\omega$, мұндағы $0<\omega<\pi$. Онда бұл теңдеу келесі түрге келеді: $\cos\omega+|\sin\omega|=\sqrt{2}\left(2\cos^2\omega-1\right)$. Мұндағы $0\leq\omega\leq\pi$ болғандықтан, онда $\sin\omega\geq 0$ және $|\sin\omega|=\sin\omega$.

Бұдан шығатыны: $\cos\omega+\sin\omega=\sqrt{2}\cos 2\omega$, $\cos\omega+\sin\omega=\sqrt{2}\left(\cos\omega+\sin\omega\right)\left(\cos\omega-\sin\omega\right)$, $\left(\cos\omega+\sin\omega\right)\left(\sqrt{2}\cos\omega-\sqrt{2}\sin\omega-1\right)=0$.

$\cos\omega+\sin\omega=0$ болсын, онда $\mathrm{tg}\omega=-1$ және $\omega=-\frac{\pi}{4}+\pi n$, мұндағы n-бүтін сан, бірақ $0\leq\omega\leq\pi$ болғандықтан $\omega_1=\frac{3\pi}{4}$.

$\sqrt{2}\cos\omega-\sqrt{2}\sin\omega-1$ болсын, онда $\frac{\sqrt{2}}{2}\sin\omega-\frac{\sqrt{2}}{2}\cos\omega=-\frac{1}{2}$, $\cos\frac{\pi}{4}\sin\omega-\sin\frac{\pi}{4}\cos\omega=-\frac{1}{2}$, $\sin\left(\omega-\frac{\pi}{4}\right)=-\frac{1}{2}$.

Бұдан шығатыны: $\omega-\frac{\pi}{4}=(-1)^{k+1}\frac{\pi}{6}+\pi k$ немесе $\omega=\frac{\pi}{4}+(-1)^{k+1}\frac{\pi}{6}+\pi k$. Мұндағы k -бүтін сан, бірақ $0\leq\omega\leq\pi$ болғандықтан $\omega_2=\frac{\pi}{12}$.

$x=\cos\omega$ болғандықтан $x_1=\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$ және $x_2=\cos\frac{\pi}{12}=-\frac{\sqrt{6}+\sqrt{2}}{4}$.

Жауабы: $x_1=-\frac{\sqrt{2}}{2}$, $x_2=-\frac{\sqrt{6}+\sqrt{2}}{4}$.

Есеп. Теңдеуді шеш және $(0;1)$ аралығында жатқан түбірлерін тап.

$$8x\left(2x^2-1\right)\left(8x^4-8x^2+1\right)=1. \tag{2}$$

Шешімі: Теңдеудің түбірлері $(0;1)$ аралығында жататындықтан $x = \cos\omega$ деп алсақ болады. Мұндағы $0 < \omega < \frac{\pi}{2}$. Онда (2) теңдеуді келесі түрде түрлендіруге болады:

$$8\cos\omega \cdot \left(2\cos^2\omega - 1\right)\left(8\cos^4\omega - 8\cos^2\omega + 1\right) = 1,$$

$$8\cos\omega \cdot \cos 2\omega \cdot \left(-8\cos^2\omega \cdot \sin^2\omega + 1\right) = 1,$$

$$8\cos\omega \cdot \cos 2\omega \cdot \left(1 - 2\sin^2 2\omega\right) = 1,$$

$$8\cos\omega \cdot \cos 2\omega \cdot \cos 4\omega = 1. \tag{3}$$

Әрі қарай (3) теңдеудің екі жағын да $\sin\omega$-ға көбейтеміз (мұндағы, $0 < \omega < \frac{\pi}{2}$) және келесі теңдеуді аламыз:

$$8\sin\omega \cdot \cos\omega \cdot \cos 2\omega \cdot \cos 4\omega = \sin\omega,$$

$$4\sin 2\omega \cdot \cos 2\omega \cdot \cos 4\omega = \sin\omega,$$

$$2\sin 4\omega \cdot \cos 4\omega = \sin\omega, \qquad \sin 8\omega = \sin\omega.$$

Егер $\sin 8\omega = \sin\omega$ болса, онда $8\omega = (-1)^n\omega + \pi n$, мұндағы n - бүтін сан. Екі жағдайды қарастырайық:

1. Егер $n = 2k$ болса, онда $8\omega = \omega + 2\pi k$ немесе $\omega = \frac{2}{7}\pi k$. Мұндағы k - бүтін сан.

$0 < \omega < \frac{\pi}{2}$ болғандықтан, $k = 1$ және $\omega_1 = \frac{2\pi}{7}$.

2. Егер $n = 2k+1$ болса, онда $8\omega = -\omega + \pi(2k+1)$ немесе $\omega = \frac{(2k+1)\cdot\pi}{9}$. Мұндағы k -бүтін сан.

$0 < \omega < \frac{\pi}{2}$ болғандықтан, $k = 0$, $k = 1$ және $\omega_2 = \frac{\pi}{9}$, $\omega_3 = \frac{\pi}{3}$.

Онда (2) теңдеуінің $(0;1)$ аралығында жататын келесі 3 түбірлері бар: $x_1 = \cos\frac{2\pi}{7}$, $x_2 = \cos\frac{\pi}{9}$ және $x_3 = \cos\frac{\pi}{3} = \frac{1}{2}$.

Жауабы: $x_1 = \cos\frac{2\pi}{7}$, $x_2 = \cos\frac{\pi}{9}$ және $x_3 = \cos\frac{\pi}{3} = \frac{1}{2}$.

Есеп. Теңдеудің шешімін табыңдар:

$$8x^3 - 6x - 1 = 0. \tag{4}$$

Шешімі: $x = 0$ бұл теңдеудің шешімі болмайтындықтан, теңдеудің екі жағын да $2x$-ке бөліп жіберейік. Онда:

$$4x^2 = \frac{1}{2x} + 3. \tag{5}$$

Егер $x < -1$ немесе $x > 1$ болса, онда (5) теңдеуінің оң жағы 4-тен кіші, ал сол жағы 4-тен үлкен болады. Бұдан шығатыны (4) теңдеуінің шешімі $-1 \le x \le 1$ аралығында жатады.

$x = \cos\omega$ деп алсақ болады. Мұндағы $0 \le \omega \le \pi$. Онда (4) теңдеуді келесі түрде түрлендіруге болады: $8\cos^3\omega - 6\cos\omega - 1 = 0$, $4\cos^3\omega - 3\cos\omega = \frac{1}{2}$, $\cos 3\omega = \frac{1}{2}$.

$\cos 3\omega = \frac{1}{2}$ теңдеуінің түбірлері $\omega = \frac{\pi}{9}(6n \pm 1)$, мұндағы n - бүтін сан.

$0 \le \omega \le \pi$ болғандықтан, $\omega_1 = \frac{\pi}{9}$, $\omega_2 = \frac{5\pi}{9}$ және $\omega_3 = \frac{7\pi}{9}$. Бұдан $x = \cos\omega$ болғандықтан, $x_1 = \cos\frac{\pi}{9}$, $x_2 = \cos\frac{5\pi}{9}$ және $x_3 = \frac{7\pi}{9}$.

Жауабы: $x_1 = \cos\frac{\pi}{9}$, $x_2 = \cos\frac{5\pi}{9}$ және $x_3 = \frac{7\pi}{9}$.

Есеп. Теңдеудің шешімін табыңдар:

$$\sqrt{1 - x^2} = 4x^3 - 3x. \tag{6}$$

Шешімі: (6) теңдеуінің шешімі $-1 \le x \le 1$ аралығында жатады. Сондықтан $x = \cos\omega$ деп алсақ болады. Мұндағы $0 \le \omega \le \pi$. Онда (6) теңдеуді келесі түрде түрлендіруге болады: $\sqrt{1 - \cos^2\omega} = 4\cos^3\omega - 3\cos\omega$ немесе $|\sin\omega| = \cos 3\omega$. Мұндағы $0 \le \omega \le \pi$ болғандықтан, $\sin\omega \ge 0$ және $\sin\omega = \cos 3\omega$. Бұл теңдеуді $\cos 3\omega = \cos\left(\frac{\pi}{2} - \omega\right)$ теңдеуімен алмастыруға болады. Бұл жерден шығатыны $3\omega = \pm\left(\frac{\pi}{2} - \omega\right) + 2\pi n$, n - бүтін сан. Екі жағдайды қарастырайық:

1. Егер $3\omega = \frac{\pi}{2} - \omega + 2\pi n$, онда $\omega = \frac{\pi}{8} + \frac{\pi n}{2}$. Мұндағы $0 \le \omega \le \pi$ болғандықтан $\omega_1 = \frac{\pi}{8}$ және $\omega_2 = \frac{5\pi}{8}$.

2. Егер $3\omega = -\frac{\pi}{2} + \omega + 2\pi n$, онда $\omega = -\frac{\pi}{4} + \pi n$. Мұндағы $0 \le \omega \le \pi$ болғандықтан $\omega_3 = \frac{3\pi}{4}$.

Бұдан $x = \cos\omega$ болғандықтан (6) теңдеуінің келесі формулалармен есептелетін 3 түбірі бар:

$$x_1 = \cos\omega_1 = \cos\frac{\pi}{8} = \sqrt{\frac{1}{2}\cdot\left(1+\cos\frac{\pi}{4}\right)} = \sqrt{\frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)} = \frac{1}{2}\cdot\sqrt{2+\sqrt{2}},$$

$$x_2 = \cos\omega_2 = \cos\frac{5\pi}{8} = -\sqrt{\frac{1}{2}\cdot\left(1+\cos\frac{5\pi}{4}\right)} = -\sqrt{\frac{1}{2}\left(1-\cos\frac{\pi}{4}\right)} =$$

$$= -\sqrt{\frac{1}{2}\cdot\left(1-\frac{\sqrt{2}}{2}\right)} = -\frac{1}{2}\cdot\sqrt{2-\sqrt{2}},$$

$$x_3 = \cos\omega_3 = \cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}.$$

Жауабы: $x_1 = \frac{1}{2}\cdot\sqrt{2+\sqrt{2}}$, $x_2 = -\frac{1}{2}\cdot\sqrt{2-\sqrt{2}}$ және $x_3 = -\frac{\sqrt{2}}{2}$.

Есеп. Теңсіздікті дәлелде:

$$\left(2x\cdot\sqrt{1-x^2}+2x^2-1\right)^2 \le 2, \qquad \text{мұндағы } -1 \le x \le 1. \tag{7}$$

Дәлелдеу: $-1 \le x \le 1$ болғандықтан $x = \cos\omega$ деп алсақ болады. Мұндағы $0 \le \omega \le \pi$. Онда (7) теңсіздікті келесі түрде түрлендіруге болады:

$$\left(2\cos\omega\cdot|\sin\omega| + 2\cos^2\omega - 1\right)^2 \le 2. \tag{8}$$

Мұндағы $0 \le \omega \le \pi$ болғандықтан, $\sin\omega \ge 0$ және $|\sin\omega| = \sin\omega$ және (8) теңсіздігін келесі түрге түрлендіруге болады: $(\sin 2\omega + \cos 2\omega)^2 \le 2$. Бұл теңсіздіктегі $\sin 2\omega + \cos 2\omega = \sqrt{2}\cdot\sin\left(2\omega+\frac{\pi}{4}\right)$ тең. Ал $\sin^2\left(2\omega+\frac{\pi}{4}\right) \le 1$. Онда бұл жерден $\left(2\cos\omega\cdot|\sin\omega| + 2\cos^2\omega - 1\right)^2 \le 2$ дұрыс екендігі сөзсіз.

Есеп. Теңсіздікті дәлелде:

$$-\frac{1}{2} \le \frac{(x+y)\cdot(1-xy)}{(1+x^2)(1+y^2)} \le \frac{1}{2}. \tag{9}$$

Дәлелдеу: $x = \mathrm{tg}\alpha$ және $y = \mathrm{tg}\beta$ болсын. Мұндағы $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ және $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$. Онда

$$\frac{(x+y)\cdot(1-xy)}{(1+x^2)(1+y^2)} = \frac{(\mathrm{tg}\alpha+\mathrm{tg}\beta)(1-\mathrm{tg}\alpha\cdot\mathrm{tg}\beta)}{(1+\mathrm{tg}^2\alpha)(1+\mathrm{tg}^2\beta)} =$$

$$= \frac{\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}\cdot\frac{\cos(\alpha+\beta)}{\cos\alpha\cos\beta}}{\frac{1}{\cos^2\alpha}\cdot\frac{1}{\cos^2\beta}} = \sin(\alpha+\beta)\cdot\cos(\alpha+\beta) = \frac{1}{2}\sin 2(\alpha+\beta).$$

$-\frac{1}{2} \le \frac{1}{2}\sin 2(\alpha+\beta) \le \frac{1}{2}$ болғандықтан теңсіздік дәлелденді.

Есеп. Теңдеу жүйесін шеш:

$$\begin{cases} x + \sqrt{1-y^2} = 1, \\ y + \sqrt{1-x^2} = \sqrt{3}. \end{cases} \tag{10}$$

Шешімі: (10) теңдеуінде $-1 \le x \le 1$ және $-1 \le y \le 1$ аралығында болғандықтан $x = \cos\varphi$ және $y = \cos\psi$ алмастыруын орындасақ болады. Мұндағы $0 \le \varphi \le \pi$ және $0 \le \psi \le \pi$. Онда $\sqrt{1-x^2} = \sin\varphi$, $\sqrt{1-y^2} = \sin\psi$ және (10) теңдеу келесі теңдеу жүйесіне түрленеді:

$$\begin{cases} \cos\varphi + \sin\psi = 1, \\ \cos\psi + \sin\varphi = \sqrt{3}. \end{cases} \tag{11}$$

(11) теңдеу жүйесінен $\cos^2\varphi = (1-\sin\psi)^2$ және $\sin^2\varphi = \left(\sqrt{3}-\cos\psi\right)^2$. $\sin^2\varphi + \cos^2\varphi = 1$ болғандықтан

$$1 = \left(\sqrt{3}-\cos\psi\right)^2 + (1-\sin\psi)^2 =$$

$$= 3 - 2\sqrt{3}\cos\psi + \cos^2\psi + 1 - 2\sin\psi + \sin^2\psi =$$
$$= 5 - 2\left(\sin\psi + \sqrt{3}\cos\psi\right).$$

Бұдан шығатыны $\sin\psi + \sqrt{3}\cos\psi = 2$ немесе $\sin\left(\psi + \frac{\pi}{3}\right) = 1$. Ал бұл теңдеудің шешімі $\psi = \frac{\pi}{6} + 2\pi n$, мұндағы n - бүтін сан. $0 \leq \psi \leq \pi$ болғандықтан $\psi_1 = \frac{\pi}{6}$ және $y_1 = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

$\cos\psi_1 = \frac{\sqrt{3}}{2}$ және $0 \leq \psi \leq \pi$ болғандықтан $\sin\psi_1 = \frac{1}{2}$ және (11) теңдеуден $\cos\varphi_1 = 1 - \sin\psi_1 = 1 - \frac{1}{2} = \frac{1}{2}$. $x = \cos\varphi$ болғандықтан $x_1 = \cos\varphi_1 = \frac{1}{2}$.

Жауабы: $x_1 = \frac{1}{2}$ және $y_1 = \frac{\sqrt{3}}{2}$.

Есеп. Теңдеу жүйесін шеш:

$$\begin{cases} 4xy\left(2x^2 - 1\right) = 1, \\ x^2 + y^2 = 1. \end{cases} \tag{12}$$

Шешімі: (12) теңдеуінде $x^2+y^2 = 1$ болғандықтан $x = \sin\varphi$ және $y = \cos\varphi$ алмастыруын орындасақ болады. Мұндағы $-\pi \leq \varphi \leq \pi$. Онда (12) теңдеуді келесі түрде жазуға болады: $4\sin\varphi \cdot \cos\varphi \cdot \left(2\sin^2\varphi - 1\right) = 1$, $2\sin 2\varphi \cdot (-\cos 2\varphi) = 1$ немесе $\sin 4\varphi = -1$. Бұл теңдеудің шешімі $\varphi = -\frac{\pi}{8} + \frac{\pi}{2}n = \frac{\pi}{8}(4n - 1)$. Мұндағы n - бүтін сан.

Әдебиеттер тізімі

[1] Папышев А.А. Теоретико-методологические основы обучения учащихся решению математических задач в контексте деятельностного подхода: // Монография. Саранск: Референт, – 2007. – 327 с.

ПРОБЛЕМНЫЕ ВОПРОСЫ МАТЕМАТИКИ В 5 КЛАССЕ

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У многих школьников нашей страны отмечается равнодушие к знаниям, нежелание учиться, низкий уровень развития познавательных интересов, уделенное время уделяется к различным гаджетам.

По этой причине мое мнение, главная задача педагога в этих условиях заключается в поиске более эффективных форм, моделей, способов и условий обучения. Таким образом, на первый план выходит проблема активизации мыслительной деятельности учащихся в процессе обучения переходного момента, так как мы понимаем, что ребенок после начальной школы тяжело адаптируется в начальных моментах. Проблема

Все мы понимаем, что математика является основой для изучения всех предметов любой сферы деятельности. Мы так же понимаем, что математика в 5-6 классе, это основной фундамент в других классах математики. По широте практического применения математическое образование несоизмеримо ни с какими другими видами знаний.

Исходя из этого – ведущая идея в моей педагогической и математической практике – максимально раскрыть перед ребенком спектр приложений математических знаний через активацию мыслительной деятельности учащихся на уроках математики.

Учитывая это в 5-х – 6-х классах очень важно не только дать детям твердые знания начал математики, но и не отпугнуть школьников строгостью царицы наук, увлечь их этим предметом, показать, что за числом стоит предмет или явление. Поэтому я уделяю большое значение этапу сообщения темы, постановки целей и задач урока. Для того, чтобы интерес к урокам не пропал, стараюсь проводить этот этап, используя различные приемы и игры. Рассмотреть самые проблемные места, где может ребенок закрыть данную эпопею. Постараюсь самого ленивого ученка заинтересовать различными методами.

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МЕКТЕП МАТЕМАТИКАСЫНДАҒЫ ЭЛЛЕКТИВТІ КУРС МАҢЫЗДЫЛЫҒЫ

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Мақалада жалпы білім беретін мектепте бейіналды даярлық және бейіндік оқытуды іске асыру мәселелері қамтылады. Элективті курс бағдарламасын әзірлеу алгоритмінің сипаттамасы және онымен бірге жүретін құжаттар жиынтығы берілген. Мектеп математикасында білім беру саласына қатысты курстың мазмұнын таңдау міндеттері көрсетілген.

Кілттік сөздер: элективті курс, элективті курсты әзірлеу алгоритмі, бейіналды даярлық, бейіндік оқыту, курстың міндеттері.

ЫҚТИМАЛДЫҚТАР ТЕОРИЯСЫ МЕН МАТЕМАТИКАЛЫҚ СТАТИСТИКА КУРСЫН ОҚЫТУДА АҚПАРАТТЫҚ ТЕХНОЛОГИЯ ЖЕТІСТІКТЕРІНІҢ ҚОЛДАНЫЛУ ЕРЕКШЕЛІКТЕРІ

БАГДАГУЛЬ ҚАДЫРБАЕВА[1], КЕНЖЕБЕК ЖАНТЛЕУОВ[2]

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Мақалада болашақ мамандарды «Ықтималдықтар теориясы мен математикалық статистика» курсын оқытуда ақпараттық технология жетістіктерін қолдану мәселелері баяндалған.

Әлемді әбігерге салған пандемия барлық мемлекеттердегі сияқты Қазақстандағы білім салаларында да оқытудың қашықтан ұйымдастырылуын және ақпараттық технология мүмкіндіктерін жоғары оқу орындары оқытушылардың да жедел меңгеріп, қолдануларын талап етті. Осы талап Абай атындағы Қазақ Ұлттық педагогикалық университетінде де басты назарда болып, өте жылдам қарқынмен оқыту үдерісінің ұйымдастырылуында нәтижелі жүзеге асырылды.

Университетімізде аталған бағытта қашықтан оқыту технологиясын ендірудің дидактикалық және әлеуметтік-психологиялық мәселелері зерттеліп, оқытушылардың озық тәжірибелері таратылуда, дегенмен мәселенің шешімін табудың ең тиімді жолы қазіргі замандағы ақпараттық технология мүмкіндіктерінің жоғары оқу орындарындағы оқыту үдерісінде оқытушылардың пәндерді оқытуды ұйымдастыруда қолдану кәсібилігіне байланысты десек қателеспейтініміз анық.

Оқытушының кәсібилігі дегенде, аталған мәселе бойынша, жазбаша онлайн емтихан сұрақтарын накты қоя алуы және жауаптарының «ашық кітап» түріндегі бағалау критерийлерін анықтап бере алуында дер едім.

«Қазақстан білім беру ұйымдары үшін қашықтан оқыту жүйесі мен автоматтандырылған прокторинг» сервисінің өнімдерінің жазбаша емтихан «ашық кітап» түрі бойынша берілген жауаптардың антиплагиат көрсеткіштері бойынша тексерілуі оқу үдерісінде жаратылыстану бағыты, оның ішінде математика мамандығы бойынша білім алып жатқан студенттерге «Ықтималдықтар теориясы және математикалық статистика» курсын оқытуда айтарлықтай қиындық тудырып отырғаны ақиқат. Оған себеп, студенттер емтиханда жауап бергенде сұрақтар бойынша теорема, тұжырымдарды дәлелдеу қажеттілік тудырмайды, өйткені дәлелдеулер көшірме болып саналады. Сондықтан бұл оқыту үдерісін ұйымдастырудағы білімді бағалау мәселесінде ақпараттық технология мүмкіндіктерін қолданудың ерекшеліктерін, өзекті тұстарын анықтауды қажет ететіні ақиқат.

Тәжірибемізде білімді бағалау алғашында тест және zoom, т.б. платформаларда онлайн жауап берудің форматында, одан кейін «Қазақстан білім беру ұйымдары үшін қашықтан оқыту жүйесі мен автоматтандырылған прокторинг» сервисінің өнімдерінің жазбаша емтихан «ашық кітап» түрі бойынша жүзеге асырылды. Ол үшін алдымен тақырыпты зерттеу бағытынды есептердің алатын рөлін анықтап алу қажет болды.

Математика, психология және педагогика ғылымдары салаларында «есеп» терминіне байланысты пәннің ерекшелігіне қарай әртүрлі көзқарастар қалыптасқан, есеп ұғымын ғалымдар «қандайда бір танымдық нәтижеге қол жеткізудегі ойлау әрекетінің жемісі ретінде де түсіндіреді. Сондықтан есепті шығару үдерісін адам мүмкіндіктерінің іске асырылуы деп қарап, оның дамуының көрсеткіші деп те талдайды. Ал адам мүмкіндіктерінің бірі ретінде оқушының өзіндік іс-әрекетін алуымызға болады.

Ғалымдардың (Караев Ж.М, Мұхамбетжанова С.Т [1,2] және т.б.) айтқан пікірлеріне сүйене отырып, біз оқушының өзіндік іс-әрекеті деп олардың есеп шығару барысында өздігінен жоспарлау, бағдарлау, тексеру және қорытынды жасау сияқты шығармашылық ойлау қабілетін талап ететін әрекеттер тізбегін түсінеміз.

Осы айтқан тұжырымға сүйеніп ғалым Б.А.Қадырбаева оқушылардың өзіндік іс-әрекетін арттыру төмендегі мәселелер негізінде анықтады:

-өтілген материал мен өтілетін материалдың арасындағы байланысты тағайындайтын әр түрлі жаттығулар, есептер қарастыру арқылы оқушы білімін толықтыру;

-оқушының өзіндік жұмысын ұйымдастыру мақсатында олардың оқыту үдерісіндегі белсенділігін арттыру;

-оқушыға берілетін теориялық білімді күнделікті тұрмыспен, халықтық педагогикамен байланыстыра оқыту арқылы өздігінен білім алып, білім деңгейлерін кеңейту дағдыларын қалыптастыру;

-деңгейлік дамыта оқыту есептерін шығарту арқылы оқушының шығармашылық қабілетін дамытуды қамтамасыз ету[3].

Оқушылардың сыныпта өзіндік жұмыс жасау түрлерін Л.П.Аристова бес бөлікке бөлді:

1) мұғалімнің толық ашып түсіндірмеген мәселелеріне оқушылардың зер сала оқуы;

2) мұғалімнің түсіндіруінде қарастырылмаған мәселелерді үйге өздігінен оқып-үйренуге тапсыру;

3) бұрын меңгерген білімдерін жаңа алған біліммен логикалық байланыстырып талқылау арқылы ұғыну.

4) жаңа алған білімдерді пысықтау;

5) мұғалімнің түсіндіруінен және басқадай білім көздерінен алынған білімдерді пысықтау [4;42-68б].

Аталған мәселелерді негізге ала отырып, И.И.Малкин оқушылардың сабақтағы өздігінен орындайтын жұмыстарын дидактикалық жүйелеуді жүзеге асырды[5].

Ғалымдардың аталған пікірлерін ескере отырып, Б.А.Қадырбаева оқушылар өзіндік жұмыс жасаудағы әрекеттерідегі қолданылатын тәсілдерді негізгі мәселелерді бөліп алу тәсілі, салыстыру және бақылау тәсілдері түсіндіру тәсілі, жалпылау тәсілі деп бөліп қарастырды[3].

Бірінші «негізгі мәселелерді бөліп алу тәсілі» бойынша оқушылар төмендегідей ақыл-ой операцияларын жүзеге асырады:

анализ және синтез, салыстыру мен нақтылау, абстракциялау және жалпылау.

Негізгі мәселені бөліп алып қарастыруда төмендегі көрсетілген әрекеттер тізбектей орындалады;

а)мәселенің не туралы екенін анықтау;

ә) мәселеге қатысты негізгі ұғымдар мен заңдылықтарды іріктеу;

б) қарастырылып отырған материалдар ішінен тірек мәселелерін бөліп алу;

в) қысқаша тұжырымдауда негізгі тірек мәселелерді белгілеп алу;

г) бөлініп алынған тірек мәселелерінен негізгісін жазу.

Екінші «Салыстыру және бақылау тәсілдерінің» негізгі мақсаты - оқушыға деңгейлік дамытуға арналған есептерді шығаруда тапсырмалардың орындалу тәсілдерін бақылай отырып салыстыруға үйрету.

Бұл жағдайда келесі мәселелер ескерілуі қажет:

а) біртекті обьектілерді салыстыру;

ә) берілген обьектілерді белгілі бір қасиеттеріне байланысты ажырата отрып салыстыру;

Әрине, обьектінің түрі, белгісі, категориясына қатысты салыстырғанға қарағанда фактілерді салыстыру әлдеқайда жеңіл.

Салыстыруды оқу үдерісінің барлық кезеңдеріне, мәселен, жаңа материалды қабылдау барысында, оны жүйелеу мен кеңейтуде, т.б. жағдайларда қолдануға болады. Бақылау және салыстыру тәсілдері бойынша оқушы берілген материалдың өткен материалдармен ұқсастығын, ерекшелігін анықтайды. Нәтижесінде оқушы негізгі мәселені есіне сақтап қалады.

Үшінші «түсіндіру тәсілі» бойынша құбылыстарды түсіндіру барысында оқушы төмендегі мәселелерді үйренуі керек:

а) берілген есептерді шығарудың алгоритмдерін түсіну және олардың байланысы мен ерекшелігін ажырата білу;

ә) берілген есепті шығаруға қажетті анықтама, ереже, заңдылықтарды анықтай білуге үйрену;

б) ережелер мен заңдылықтарды нақтылап, есепті шығаруға қолдану.

Төртінші «жалпылау тәсілі» арқылы құбылыс, үдерістердің заңдылықтары, белгілері, қасиеттерінің ашылуы мүмкін болады.

Жалпылау үдерісі даралау үдерісімен тығыз байланысты және қарастырылатын обьектінің қасиеттері ішінен кейбір қасиетін, негізгісі бөлініп алынады.

«Математикадан білім беру стандартынан асатын деңгейлік дамыта оқытудың әдістемелік жүйесін (мақсат, мазмұн, әдіс, құрал, түр) құру оқушылардың өзіндік іс-әрекетін жетілдіруге бағытталады және оқушылардың өзіндік жұмыс жасау қабілетін жетілдірудің құралы - деңгейлік дамыта оқыту есептері мен осы есептерді шығару қызметі болуы тиіс»,- деген тұжырым жасалды [3].

Есептердің мазмұны жөнінде айтқанда біріншіден, не абстрактілі математикалық сипатына, не нақты тұрмыстық, өндірістік, қызығушылық сипатына сәйкес оның мазмұнын, екіншіден, есептердің көмегімен меңгерілетін теориялық материалдарды (түсініктер, қасиеттер, формулалар, ережелер және т.б,) түсінеміз.

Белгілі психолог А.Н.Леонтьевтің [6] теориясы бойынша оқушы үшін маңызды болып көрінген мәселелер ғана олардың белсенділіктерін арттырады, өзі үшін жаңа болып көрінетін ақиқаттарды өз бетінше ізденуіне көмектеседі, оқуға деген ынтасын арттырып, ойлау және елестету қабілеттерін жетілдіреді.

Олай болса, оқушыларға аталғандай өз бетінше ізденуіне келтіретіндей жағдайды жасауға мұғалімдер жауапты болатыны ақиқат.

Белгілі шамалар немесе берілген салдарлардың себептері ізделінетін *кері есептердің*, яғни белгісіз шамалар себеп-салдар байланыстар арқылы емес, оларға «қарсы» шамаларды анықтауды мақсат ететін есептердің мұғалімнің оқушыларға өз бетінше ізденуіне келтіретіндей жағдайды жасауға қызмет атқара алатынын тәжірибеміз көрсеткенін айта кеткім келеді.

Мысалы, кездейсоқ шаманың үлестірім тығыздығы берілсін:

$$f\left(x\right)=\left\{\begin{array}{c} 0,\quad x\leq 0 \\ \frac{2}{\pi}cos^2x,\ -\frac{\pi}{2} \quad < x\leq\frac{\pi}{2} \\ 0,\ x>\frac{\pi}{2} \end{array}\right.$$

Кездейсоқ шаманың $\left(0;\frac{\pi}{4}\right)$ аралығында X кездейсоқ шама
үш тәуелсіз сынақтың екеуінде пайда болу ықтималдығын табу керек болсын[7;92б].
Шешуі: мұнда белгілі, тақырыпта меңгеріліп отырған формулалар арқылы: $P\left(0<\mathrm{X}<\frac{\pi}{4}\right)=\frac{2}{\pi}\int_0^{\frac{\pi}{4}}cos^2xdx=\frac{\pi+2}{4\pi}$, $P_3\left(2\right)=C_3^2\left(\frac{\pi+2}{4\pi}\right)^2\frac{3\pi-2}{4\pi}$ табылады.

Берілгендері немесе белгілі шамалары бойынша кездейсоқ шаманың ықтималдығы ізделінетін осы есеп *тура есеп болады*, яғни белгісіз себепті байланыстар үлестірім тығыздығы арқылы ізделінеді.

Енді, осы мысалдың есептері болатын басқа жағдайларында кездейсоқ шаманың үлестірім функциясы беріліп, оның тығыздығы арқылы анықталу керек болса, онда ол есеп те тура есеп болып саналады, өйткені белгісіз себепті байланыстар үлестірім функциясы арқылы ізделінеді.

Кері есептер қарастыру жағдайларында белгісіз шамалар себеп-салдар байланыстар арқылы емес, оларға «қарсы» белгілі болу керек шамаларды, яғни үлестірім тығыздығын анықтайтын мәліметтерді анықтауды мақсат етеді. Онда туындайтын қиындық үлестірім тығыздығы анықталатын аралықтағы шарттар үлестірім функциясынан тәуелді және төменгі шекарада ол нөлге, жоғарғы шекарада бір шамасына ұмтылуы қажет[8].

Қорытындылай келе, «Ықтималдықтар теориясы мен математикалық статистика» курсының тақырыптарын меңгергенде, оқушылардың өзіндік жұмыс жасау қабілетін жетілдіруге бағытталған оқытуды ұйымдастыра алу үшін, студенттердің біліктіліктері жетілген, кері есептерді талдай білуле алатын, оны шығару жолдарын біліктілікпен орындай білетін болулары қажет және бұл студенттердің емтиханда жауап бергендерінде көшірме болатын сұрақтар бойынша теорема, тұжырымдарды дәлелдеулер арқылы емес, кері есептер талдау арқылы білім, білік, дағдыларының қай деңгейде екендіктерінің көрсеткіштері болатынын көрсетеді. Олай болса, оқыту үдерісін ұйымдастырудағы білімді бағалау мәселесінде ақпараттық технология мүмкіндіктерін қолданудың ерекшеліктерін, өзекті тұстарын анықтауда аталғандай оқыту мәселелердің шешімі болатынын көрсетеді.

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[8] Қадырбаева Б.А.,Жантлеуов К.К.Үзіліссіз кездейсоқ шамалардың үлестірім тығыздығы тақырыбын оқытуда кері есептерді қолданудың ерекшеліктері «Жаратылыстанудың кері және қисынды емес есептері» атты халықаралық ғылыми конференция материалдары. Абай ат ҚазҰПУ,РҒА СБ, Математика институтының Халықаралық математикалық орталығы 11-12 сәуір, 2023 ж. Алматы қ., 36 б.

ЦИФРЛЫҚ БІЛІМ БЕРУ ЖАҒДАЙЫНДА БОЛАШАҚ ИНФОРМАТИКА МҰҒАЛІМДЕРІН ДАЙЫНДАУДА ДИСКРЕТТІ МАТЕМАТИКА ЭЛЕМЕНТТЕРІН ОҚЫТУДЫҢ ӘДІС-ТӘСІЛДЕРІ

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Дискретті математика элементтерін цифрлық білім беру мүмкіндіктерін қолдана отырып оқыту қазіргі білім беру процесінде алға қойылған басты мәселенің бірі болуда. Аталған мәселені шешу барысында оқытудың әдістемесін әзірлеу қажеттілігі туындап отыр. Осыған орай, мақалада болашақ информатика мұғалімдерін дайындауда дискретті математика элементтерін оқытуда цифрлық ресурстарды тиімді түрде қолдана отырып, оқытудың әдіс-тәсілдері баяндалады. Зерттеудің негізгі мақсаты, цифрлық білім беру жағдайында болашақ информатика мұғалімдеріне дискретті математика элементтерін тиімді цифрлық ресурстар мен әдіс-тәсілдерді таңдап, оқытуды ұйымдастыру болып табылады.

Қойылған мақсатқа жету барысында болашақ информатика мұғалімдеріне дискретті математика элементтерін оқытудың моделі анықталды. Анықталған модель негізінде оқытудың мақсаты мен мазмұны анықталды. Заманауи оқытуға мүмкіндік беретін цифрлық оқыту құралдары жасалынды және интернет сервистердің мүмкіндіктері кеңінен қолданыла отырып, оқыту жүйесіне енгізілді, нәтижесі практика жүзінде педагогикалық эксперименттен өткізілді. Зерттеу барысында қол жеткізген, болашақ информатика мұғалімдеріне арналған «Информатикадағы дискретті математиканың элементтері» курсының оқу бағдарламасы негізгі үш модульді қамтиды: компьютер архитектурасының логикалық негіздері; алгоритмдеу және программалаудың математикалық негіздері; компьютерлік желілердегі графтар теориясының элементтері.

Курсты оқыту барысында болашақ информатика мұғалімдерінде төмендегідей құзіреттіліктер қалыптасады:

- заманауи математикалық құралдарды, іргелі тұжырымдамалар мен жүйелік әдіснамаларды, ақпараттық технологияларды кәсіби тұрғыдан қолдану, кәсіби қызметте заманауи аспаптық және есептеу құралдарын пайдалану;
- базалық математикалық білімді меңгеру, ғылыми-техникалық есептер мен қолданбалы есептерді шешу үшін тиімді қолдану;
- негізгі дискретті математика элементтерін информатика бойынша кәсіби пәндер мазмұнында практика жүзінде қолдану, дұрыс шешімдер қабылдауға, өнімді қызмет процесінде туындайтын міндеттерді шығармашылықпен тиімді шеше білу;
- цифрлық білім беру ресурстарын пайдалана отырып, өз жоспарларын жауапкершілікпен іске асыру, өзінің практикалық кәсіби қызметінде дискретті математикамен байланысты мәселелерді шешу.

Бұл зерттеу информатика саласындағы болашақ мамандардың сапалы іргелі дайындығын қамтамасыз ету үшін дискретті математика элементтерін зерттеу қажеттілігін растай отырып, оны оқытудың әдістемесін жасауға және оны білім беру құралы ретінде қарастыруға мүмкіндік берді.

Әрбір оқытушы өзінің ақпараттық құзіреттілігі мен дискретті математика элементтері бойынша сауаттылығын біріктіріп, әдістемелік шеберлігімен цифрлық білім беру жағдайында оқытудың белсенді тиімді әдістерін қолдана отырып, сапалы оқу процесін қамтамасыз ете алады.

Кілттік сөздер: дискретті математика, информатика, интеграция, кәсіби құзыреттілік, әдістеме, граф, сындарлы оқыту.

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МЕТОД МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ ПРИ ИЗУЧЕНИИ ПРОИЗВОДНОЙ

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Практико-ориентированные задачи решаемые с помощью метода математического моделирования используются как очень эффективное средство усвоения учащимися понятий, методов, вообще математических теорий, как наиболее действенное средство развития мышления учащихся, как универсальное привитие учащимся умений и навыков практического применения математики.

Большинство исследователей и авторы учебников для средней школы рассматривают трехэтапное математическое моделирование[1]:

1) Построение математической модели;

2) Исследование построенной модели, т.е. решение полученной математической задачи с помощью математических инструментов;

3) Интерпретация полученного результата.

Авторы предлагают введение понятия производной через рассмотрение задач, приводящих к производной, которые решаются как прикладные, с явным выделением этапов решения прикладной задачи[2]. Сама же производная появляется как математическая модель, описывающая определенное свойство (характеристику) рассматриваемой реальной ситуации. Привлечение нескольких практических задач из различных дисциплин формирует у учащихся представление о большой общности построенной модели и ее возможностях. Такой подход к введению понятия производной позволяет уже на начальном этапе изучения формировать у учащихся представления о ней как о математической модели тех реальных ситуаций, которые рассматривались, способствует отработке умений абстрагирования, перевода с языка рассматриваемого явления на язык математики. Таким образом, в процесс введения математического понятия вносятся черты, специфические для прикладной деятельности.

Ключевые слова: развитие мышления, практическое применение, математическое моделирование, отработка умений, прикладная деятельность.

Предметная классификация AMS: 97.

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СУЩНОСТЬ И СТРУКТУРА МАТЕМАТИЧЕСКОЙ КОМПЕТЕНТНОСТИ БУДУЩИХ ПЕДАГОГОВ ПО ЕСТЕСТВЕННОНАУЧНЫМ ПРЕДМЕТАМ

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На протяжении многих лет проблема качественного образования была и остается одной из приоритетных в педагогической науке и практике. Сегодня очень остро стоит вопрос о профессиональной подготовке учителя. Качество образования будущего учителя и уровень сформированности его профессиональной компетентности являются социальными критериями состояния и результативности процесса образования. Будущий учитель должен соответствовать потребностям современного общества в формировании и развитии профессионально-личностной компетентности специалиста [1].

Безусловно, уровень профессиональной компетентности будущих учителей по естественнонаучным предметам в большей степени зависит от качества математической подготовки. Поскольку у математики огромная междисциплинарная функция, то значительную основу подготовки будущих учителей по естественнонаучным предметам составляют математические дисциплины и курсы. Для того, чтобы будущий учитель по естественнонаучным предметам был способен применить математические методы, активно участвовать в их использовании и внедрении, он должен иметь качественную подготовку по математическим дисциплинам [2].

Не вызывает сомнений, что развитие математических компетенций нуждается в сравнительной оценке. Отметим, что факторы влияют на развитие личности при определенных условиях. Влияние факторов на развитие математических компетенций мы можем оценить на основе показателей и критериев. Для уточнения сущности математических компетенций были определены показатели оценки уровня сформированности математических компетенций будущих учителей по естественнонаучным предметам [3].

Ключевые слова: математическая компетентность, компетентностный подход, междисциплинарный подход, математика, естественнонаучные предметы.

Предметная классификация AMS: 97.

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ТЕХНОЛОГИЯ ТРЕХМЕРНОЙ МЕТОДИЧЕСКОЙ СИСТЕМЫ ОБУЧЕНИЯ – ЭФФЕКТИВНЫЙ МЕХАНИЗМ ГАРАНТИРОВАННОГО ОБЕСПЕЧЕНИЯ КАЧЕСТВА ОБРАЗОВАНИЯ

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В статье раскрывается сущность авторской педагогической технологии, основанной на трехмерной методической системе обучения, которая служит ядром деятельностно-развивающей дидактики. Актуальность технологизации образовательного процесса обусловлена тем ее свойством, что она обеспечивает гарантированное достижение учебных успехов каждого обучаемого. Использование на практике технологии трехмерной методической системы обучения (ТТМСО) способствует повышению качества обучения и эффективности всей системы образования.

Предлагается новое понятие «Дидактическая матрица», которая является научно-обоснованной платформой рассматриваемой технологии и проектом деятельностно-развивающего образовательного процесса с критериальной системой оценивания. Дидактическая матрица включает модифицированную таксономию целей Б.Блума, которая завершается уровнем «создание». Этого требует трансформация образовательного процесса на основе STEM подхода, направленного на формирование инженерно-технологических навыков у обучаемых в контексте требований Общество 4.0 с Индустрией 4.0.

Дидактическая матрица интегрирует все компоненты современной теории обучения и является основой проектирования образовательного процесса дидактики XXI века.

В статье предлагается новая модель педагогической квалиметрии, которая позволяет не только критериально (объективно) оценивать качество обучения, но и стимулирует обучаемого к успеху. Новая модель педквалиметрии способствует превращению самой стрессовой части учебного процесса, то есть, его контрольно-оценочного этапа, в увлекательную интеллектуальную игру-соревнование. Здесь ученики соревнуются в интеллектуальном марафоне, пробегая вверх по лестнице дидактической матрицы, получая при этом поощрение за преодоление очередной ступени данной лестницы.

Многолетние отечественные и зарубежные опыты по применению ТТМСО на практике показали, что она гарантирует 100%-е усвоение учебного материала на уровнях «знание» и «понимание» всеми учениками и обеспечивает динамичный рост показателей уровней «применение» и «создание».

ТРАНСФОРМАЦИЯ СИСТЕМЫ ОБРАЗОВАНИЯ НА ОСНОВЕ STEM ПОДХОДА КАК УСЛОВИЕ ПОДГОТОВКИ КОНКУРЕНТОСПОСОБНОГО ЧЕЛОВЕЧЕСКОГО КАПИТАЛА В СОВРЕМЕННОМ МИРЕ

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В данной статье рассмотрены вопросы трансформации системы среднего образования в контексте запросов «общества будущего» – Общества 4.0 с Индустрией 4.0. Обоснована важная роль STEM образования в реализации гуманистической парадигмы развития образования, личностно-деятельностного и компетентностного подходов в обучении, а также их модификации в соответствии с требованиями процесса цифровизации и инженерно-технологического образования. Показана актуальность введения элементов инженерно-технологического образования в школы в условиях индустриально-цифровой эры развития человечества.

Обоснованы дидактические условия реализации инженерно-технологического образования в школе. Показано значение осуществления проектной и учебно-исследовательской деятельности обучаемых в формировании STEM компетенции. Анализированы дидактические особенности STEM обучения.

Раскрыта сущность непрерывного содержания STEM образования. Основу содержания STEM-образования в начальной школе составляет содержание предмета «Технология». В основной школе – содержание «Технологии», «Робототехники» и «Информатики», а также интегрированное содержание предметов ЕМЦ.

Показано, что в основе STEM-подхода лежат четыре принципа: проектная форма организации образовательного процесса, прикладной характер учебных задач, межпредметный характер обучения, охват дисциплин, которые являются ключевыми для подготовки инженера или специалиста по прикладным научным исследованиям: предметы естественнонаучного цикла (физика, химия, биология, география), математики, современные технологии и инженерные дисциплины.

Обоснована важная роль интерактивных методов и технологии трехмерной методической системы обучения в реализации STEM обучения. Проанализированы психолого-педагогические основы организации предпрофильной подготовки и профильного обучения в контексте требований STEM подхода. Показано, что «Атлас новых профессий» является главным навигатором профориентационной работы в школе.

Реализация STEM образования предполагает сетевого взаимодействия: школа – дополнительное образование – ТиПО – Вуз – производство. В «технологическом профиле» школы учащимся предлагаются сложные задания в виде проектов, направленных на решение реальных производственных задач. Показано стратегическое значение трансформации системы образования на основе STEM подхода в подготовке конкурентоспособного человеческого капитала, который является главным условием создания цифровой экономики в контексте требований четвертой промышленной революции.

ПЕДАГОГИЧЕСКИЕ ОСОБЕННОСТИ РАЗВИТИЯ ГИБКИХ НАВЫКОВ В ПРОЦЕССЕ ПОДГОТОВКИ БУДУЩИХ МАТЕМАТИКОВ

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Современный многомерно трансформирующийся мир ставит человека перед вызовами новой реальности. При этом в последнее время всё большую значимость приобретают универсальные социокультурные компетенци„ связанные с формированием так называемых гибких, или мягких навыков (soft skills). На Всемирном экономическом форуме определены десять ведущих soft skills как комплекс неспециализированных, важных для карьеры надпрофессиональных навыков, которые отвечают за успешное участие в рабочем процессе, высокую производительность и являются сквозными, то есть не связаны с конкретной предметной областью. Вариантом так называемой Давосской десятки является также модель «Four Cs» (collaboration, communication, creativity, critical thinking). Математика, являясь инструментом познания мира, анализа и представления объективной реальности, играет в образовании особо важную роль. Потенциал математики как учебной (школьной, университетской) дисциплины может быть задействован для формирования таких гибких навыков, как системное и критическое мышление (целостное восприятие и абстрагирование, анализ и синтез, сравнение, обобщение, классификация и систематизация, способность задавать вопросы, аргументировать и оценивать достоверность и новизну идеи), креативность (способность генерировать идеи и решения) и т.п. Не имеют явного отношения к математической науке, но необходимы для организации работы в команде при разработке и практической реализации научно-исследовательских и прикладных проектов в данной области коммуникация, координация и кооперация (способность выражать и интерпретировать мысли в устной и письменной форме, эффективно взаимодействовать с другими людьми), а также эмоциональный интеллект (способность понимать свои эмоции и осознавать их причины, распознавать и разделять эмоции другого человека, управлять эмоциями). В докладе обсуждаются особенности математического мышления и соответствующей картины мира, специфика профессиональной деятельности, влияющие на развитие гибких навыков в процессе подготовки будущих математиков в университете. Представлены результаты анкетирования студентов-математиков высших учебных заведений Казахстана и России (Международный Казахско-Турецкий университет имени Ходжи Ахмеда Ясави, Российский университет дружбы народов имени Патриса Лумумбы и др.). Выявлены сходства и различия в предпочтениях студентов-математиков относительно степени и способов развития тех или иных гибких навыков. Обсуждается взаимосвязь полученных результатов с этнокультурными и методическими особенностями подготовки математиков в указанных странах, восходящих к европейской (Пифагор, Евклид, И. Ньютон, Р. Декарт, Л. Эйлер, Г. В. Лейбниц, Д. Гильберт, Н. И. Лобачевский, А. Н. Колмогоров и др.) и восточной (Омар

Хайям, Аль-Бируни, Аль-Хорезми, Аль-Фараби, М. Улугбек, У. М. Султангазин и др.) традициям.

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Ключевые слова: математика, soft skills, подготовка студентов, Казахстан, Россия.

Предметная классификация AMS: 97-06 (Математическое образование).

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НЕКОТОРЫЕ ЗАМЕЧАНИЯ ОТНОСИТЕЛЬНО ТЕПЛОВОГО ПОТОКА

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В статье приводится прогноз о том, каким путем великий французский ученый Ф. Фурье пришел к открытию закона теплового потока. Конечно, нашу теорию нельзя назвать математически точной. Однако можно предположить, что именно эти идеи привели великого ученого к великому открытию.

О ТЕЗАУРУСНОМ ПОДХОДЕ К ОБУЧЕНИЮ МАТЕМАТИКЕ

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Стремительное развитие информационного общества требует повышения качества образования, особенно математического. На сегодняшний день существует несколько подходов к повышению качества образования, но они не решают задачи полноценного использования факторов, от которых зависит повышение качества образования в современных условиях.

Одним из путей совершенствования математического образования в условиях информационного общества, является использование информационного подхода к обучению математике. С этой точки зрения обучение интерпретируется как информационный процесс, как расширение тезауруса личности обучающегося при включении в него новой информации.

Изучение различных трактовок понятия тезауруса в педагогике [1, 2] показывает использование понятий "предметного тезауруса", "учебного тезауруса", "личностного тезауруса". .

Помимо термина «личностный тезаурус», в научной литературе также используется термин «лексикон». Лексикон в переводе с древнегреческого буквально означает «словарная книга». Лексикон – это набор слов, которыми владеет человек, его словарный запас.

Под учебным тезаурусом предмета математика мы понимаем совокупность данных, состоящих из понятий, устойчивых словосочетаний, приёмов учебной и общематематической деятельности, основных задач и приёмов решения этих задач [3].

Лексикон учащегося, относящийся к учебно-познавательной математической деятельности, называется его математическим лексиконом. Он состоит из математических знаний, умений, навыков, опыта, способности, ценностей и математической культуры учащегося, который является результатом непрерывного математического образования или самостоятельного обучения [3].

Обоснованы принципы тезаурусного подхода, описаны основные этапы технологии формирования математического лексикона студента, цели, состав и формы деятельности, а также приёмы достижения результатов каждого этапа.

Ключевые слова: учебный тезаурус, учебная информация, математический лексикон студента.

Предметная классификация AMS: 97B99.

Әдебиеттер тізімі

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