Problem Set 1 **<u>Deadline: 2nd October 2020 11:55 pm</u>**

1 Instructions

- Adhere to the deadline. Late submissions are not allowed. You have more than enough time for the homework so it is advised to start early.
- You have to submit the solutions as a .pdf file.
- You may use LaTeX to write the mathematical equations or convert the word files into a pdf.
- The names of the file should be "yourrollnumber_hw1.pdf" e.g. 21100000_hw1.pdf.
- You are allowed to discuss with your peers, but you should not copy statements from each others.
- Follow the basic template you have been given for the homework.
- **You only have to submit solutions for questions 1,4,5,7,8,12 and 13. The rest are practice questions.**

2 Problems

1. For the universe of all integers, let $p(x), q(x), r(x), s(x)$ and $t(x)$ be the following statements:

$$\begin{array}{ll} p(x): & x \text{ is even} \\ q(x): & x \text{ is (exactly) divisible by 4} \\ r(x): & x \text{ is (exactly) divisible by 5} \\ s(x): & x > 0 \\ t(x): & x \text{ is a perfect square} \end{array}$$

Write the following statements in symbolic form:

(a) At lease one integer is even.
(b) There exists a positive integer that is even.
(c) If x is even, then x is not divisible by 5.
(d) No even integer is divisible by 5.
(e) There exists an even integer divisible by 5.
(f) If x is even and x is a perfect square, then x is divisible by 4.

2. Consider the universe of all polygons with three or four sides, and define the following open statements for this universe.

$a(x)$:	all interior angles of x are equal
$e(x)$:	x is an equilateral triangle
$h(x)$:	all sides of x are equal
$i(x)$:	x is an isosceles triangle
$p(x)$:	x has an interior angle that exceeds 180
$q(x)$:	x is a quadrilateral
$r(x)$:	x is a rectangle
$s(x)$:	x is a square
$t(x)$:	x is a triangle

Translate each of the following statements into an English sentence, and determine whether the statement is true or false.

a) $\forall x\ [q(x) \oplus t(x)]$
b) $\forall x\ [i(x) \rightarrow e(x)]$
c) $\exists x\ [t(x) \wedge p(x)]$
d) $\forall x\ [(a(x) \wedge t(x)) \leftrightarrow e(x)]$
e) $\exists x\ [q(x) \wedge \neg r(x)]$
f) $\exists x\ [r(x) \wedge \neg s(x)]$
g) $\forall x\ [h(x) \rightarrow e(x)]$
h) $\forall x\ [t(x) \rightarrow \neg p(x)]$
i) $\forall x\ [s(x) \leftrightarrow (a(x) \wedge h(x))]$
j) $\forall x\ [t(x) \rightarrow (a(x) \leftrightarrow h(x))]$

3. Determine whether each of the following conditional statements is true or false:

   (a) If $1 + 1 = 2$, then $2 + 2 = 5$

   (b) If $1 + 1 = 3$, then $2 + 2 = 4$

   (c) If monkeys can fly, then $1 + 1 = 3$

   (d) If $1 + 1 = 2$, then dogs can fly

4. Write the contrapositive, converse and inverse of the statement "If P is a square, then P is a rectangle." Are all the statements true?

5. Show that $[(P \vee Q) \wedge (\neg P \vee R)] \rightarrow (Q \vee R)$ is a tautology using equivalence laws.

6. Use truth table to establish whether the following statement forms a tautology or a contradiction or neither: $((Q \wedge R) \wedge (\neg P \wedge Q)) \wedge \neg Q$

7. Use a Truth table to determine if the statement $[P \rightarrow (Q \vee R)] \equiv [\neg R \rightarrow (P \rightarrow Q)]$ is true.

8. Show the following equivalences using using logical equivalence laws.

   (a) $(P \rightarrow R) \vee (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$

   (b) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$.

   (c) $\neg[\neg[(P \vee Q) \wedge R] \vee \neg Q] \equiv Q \wedge R$

   (d) $(P \vee Q \vee R) \wedge (P \vee T \vee \neg Q) \wedge (P \vee \neg T \vee R) \equiv P \vee [R \wedge (T \vee \neg Q)]$

9. Consider the propositional function $M(x, y) =$"x has sent an email to y", and $T(x, y) =$"x has called y".The predicate variables x, y take values in the UoD D={students in class }. Express the following statements using symbolic logic:

   (a) There are atleast two students in class such that one student has sent other an email and the second student has called the first student.

   (b) There are some students in class who have emailed everyone.

10. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

    (a) $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$

    (b) $\forall x \exists y (x + y = 1)$

    (c) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$

    (d) $\forall x \forall y \forall z ((x = y) \rightarrow (x + z = y + z))$

11. Let $W(x, y)$ mean that student x has visited website y, where the UoD for x consists of all students in your school and the UoD for y consists of all websites. Express each of these statements by a simple English sentence.

    (a) $\exists y (W(John, y) \wedge W(Cindy, y))$

    (b) $\exists y \forall z (y \neq David) \wedge (W(David, z) \rightarrow W(y, z))$

    (c) $\exists x \exists y \forall z ((x \neq y) \wedge (W(x, z) \iff W(y, z)))$

12. Let the universe of discourse for both variables be the set of integers. Determine whether the following universally quantified statements are **true** or **false**. If **false** give one counter-example for each part.

    (a) $\forall\ x\ \forall\ y\ (\neg(x = y) \rightarrow \neg(x^2 = y^2))$

    (b) $\forall\ x\ \exists\ y\ (x = y^3)$

    (c) $\forall\ x\ \exists\ y\ (1/x = y)$

    (d) $\forall\ x\ \exists\ y\ (y^3 < 100 + x)$

    (e) $\forall\ x\ \forall\ y\ (xy = y)$

    (f) $\forall\ x\ \forall\ y\ (x^3 \neq y^2)$

13. Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

    (a) $\neg \forall x \forall y P(x, y)$

    (b) $\neg \forall x \exists y (P(x, y) \vee Q(x, y))$

    (c) $\neg \forall x (\forall y P(x, y) \wedge \exists y Q(x, y))$

    (d) $\neg (\exists x \exists y \neg P(x, y) \wedge \forall x \forall y Q(x, y))$

14. Suppose a software system has 9 components $\{A, B, C, D, E, F, G, H, I\}$. Each component has either exactly one of the two types of bugs (Bug 1 and Bug 2) or has no bug (is clean). We want to identify which components have Bug 1 or Bug 2 or is clean. The software testing team has summarized its findings as follows.

    (a) Let $P(x)$ be the predicate that component x has Bug 1, let $Q(x)$ be the predicate that component x has Bug 2 and let $R(x)$ be the predicate that component x is clean. Translate each of the below findings in terms of the predicates $P(x)$, $Q(x)$ and $R(x)$.

        i. E and H do not have the same bug.

        ii. If G has Bug 1 then all components have Bug 1.

        iii. If E has Bug 1 then H has Bug 1 too.

        iv. If C has Bug 1 then D and F do not have Bug 1.

        v. If either E or H has Bug 1 then I does not have Bug 2.

        vi. At least 4 components have Bug 1.

        vii. If A has either bug then all components have Bug 2.

        viii. A and F are not in the same category.

        ix. B has Bug 2.

        x. At least one of C and G have the same bug as B.

        xi. Exactly 2 components have Bug 2.

        xii. If I has bug 2 then at least one of D, F and A have Bug 2 too.

        xiii. If E or G have bug 2 then all components have either Bug 1 or Bug 2.

    (b) Determine, using the above findings, which components have Bug 1, which ones have Bug 2 and which are clean.

        Hint: First determine which components have Bug 1 *then determine which ones have Bug* 2 *and then list the clean ones.*

15. Express the following propositions into predicate logic. Make up predicates as you need. State what each predicate means. Also state the universe of discourse for that predicate.

    (a) "Anyone who completes all homework assignments will pass this course."

    (b) "Not everyone likes to do the homework."

    (c) "There is one class that all of my friends have taken."

    (d) "No student failed Logic, but at least one student failed History."