

Problem Set 7 Deadline: Nov 30, 2020 11:55 pm**1 Instructions**

- Adhere to the deadline. Late submissions are not allowed. You have more than enough time for the homework so it is advised to start early.
- You have to submit the solutions as a .pdf file.
- You may use LaTeX to write the mathematical equations or convert the word files into a pdf.
- The names of the file should be "yourrollnumber_hw1.pdf" e.g. 21100000_hw1.pdf.
- You are allowed to discuss with your peers, but you should not copy statements from each others.
- Follow the basic template you have been given for the homework.
- **You only have to submit solutions for questions 1, 2, 3. The rest are practice questions.**

2 Problems

1. There are bills of 3 dollars and 7 dollars. Prove that you can form n dollars using these bills when $n \geq 12$. (Hint: Strong Induction)
2. Prove by induction that for each natural number n , each of the following is true.
 - (a) $4^n - 1$ is divisible by 3.
 - (b) $22^n - 1$ is divisible by 3, for integers $n > 0$
3. Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps. Prove your answer by using strong induction
4. Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1, 2^1 = 2, 2^2 = 4$ and so on. [Hint: For the inductive step, separately consider the case where $k + 1$ is even and where it is odd. When it is even, not that $(k + 1)/2$ is an integer.]
5. Prove the Binomial Theorem using mathematical induction.