CS-210 Homework 1

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1. (a) $\exists x\ P(x)$
   (b) $\exists x\ [S(x) \land P(x)]$
   (c) $\forall x\ [P(x) \to \neg R(x)]$
   (d) $\forall x\ [\neg P(x) \land R(x)]$
   (e) $\exists x\ [P(x) \land R(x)]$
   (f) $\forall x[(P(x) \land T(x)) \to Q(x)]$

4. (a) Statement: "If P is a square, then P is a rectangle" (True)
   (b) Converse: "If P is a rectangle, then P is a square" (False)
   (c) Contrapositive: "If P is not a rectangle, then P is not a square" (True)
   (d) Inverse : "If P is not a square, then P is not a rectangle" (False)
   All statements are not true.

5.
$$\begin{array}{cc}
[(P \lor Q) \land (\neg P \lor R)] \to (Q \lor R) & (\text{ Implication Law }) \\
\neg[(P \lor Q) \land (\neg P \lor R)] \lor (Q \lor R) & (\text{ De Morgan's Law }) \\
(\neg(P \lor Q) \lor \neg(\neg P \lor R)) \lor (Q \lor R) & (\text{ Associate Law }) \\
(\neg(P \lor \neg P) \lor \neg(Q \lor R)) \lor (Q \lor R) & (\text{ Negation Law }) \\
(\neg(T) \lor \neg(Q \lor R)) \lor (Q \lor R) & (\text{ Negation Law }) \\
(F \lor \neg(Q \lor R)) \lor (Q \lor R) & (\text{ Identity Law }) \\
\neg(Q \lor R) \lor (Q \lor R) & (\text{ Negation Law }) \\
T & (\text{ Hence, a tautology })
\end{array}$$

7.

P	Q	R	¬R	(Q ∨ R)	[P → (Q ∨ R)]	(P → Q)	[¬R → (P → Q)]
T	T	T	F	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	T	F	T	T	T

The truth values for $[P \rightarrow (Q \vee R)]$ *and* $[\neg R \rightarrow (P \rightarrow Q)]$ are same , hence the statement is true.

8. (a) $(P \rightarrow R) \vee (Q \rightarrow R) \equiv [(P \wedge Q) \rightarrow R]$

Starting with L.H.S

$(P \rightarrow R) \vee (Q \rightarrow R)$	(Implication Law)
$(\neg P \vee R) \vee (\neg Q \vee R)$	(Associative Law)
$(\neg P \vee \neg Q) \vee (R \vee R)$	(Idempotent Law)
$(\neg P \vee \neg Q) \vee (R)$	(De Morgan's Law)
$\neg(P \wedge Q) \vee (R)$	(Implication Law)
$(P \wedge Q) \rightarrow R$	(L.H.S = R.H.S hence, equivalent)

(b) $(P \wedge (Q \vee R)) \equiv [(P \wedge Q) \vee (P \wedge R)]$

Starting with L.H.S

$(P \wedge (Q \vee R))$	(Distributive Law)
$(P \wedge Q) \vee (P \wedge R)$	(L.H.S = R.H.S hence, equivalent)

(c) $\neg[\neg[(P \vee Q) \wedge R] \vee \neg Q] \equiv (Q \wedge R)$

Starting with L.H.S

$\neg[\neg[(P \vee Q) \wedge R] \vee \neg Q]$	(De Morgan's Law)
$\neg(\neg[(P \vee Q) \wedge R]) \wedge \neg(\neg Q)$	(Double Negation Law)
$((P \vee Q) \wedge R) \wedge Q$	(Commutative Law)
$(R \wedge (P \vee Q)) \wedge Q$	(Distributive Law)
$((R \wedge P) \vee (R \wedge Q))) \wedge Q$	(Commutative Law)
$Q \wedge ((R \wedge P) \vee (R \wedge Q))$	(Distributive Law)
$(Q \wedge R \wedge P) \vee (R \wedge Q \wedge Q)$	(Idempotent Law)
$(Q \wedge R \wedge P) \vee (R \wedge Q)$	(Commutative Law)
$(R \wedge Q) \vee (R \wedge Q \wedge P)$	(Domination Law)
$(Q \wedge R)$	(L.H.S = R.H.S hence, equivalent)

(d) $(P \vee Q \vee R) \wedge (P \vee T \vee \neg Q) \wedge (P \vee \neg T \vee R) \equiv P \vee [R \wedge (T \vee \neg Q)]$

Starting from L.H.S

$(P \vee Q \vee R) \wedge (P \vee T \vee \neg Q) \wedge (P \vee \neg T \vee R)$	(Distributive Law)
$P \vee [(Q \vee R) \wedge (T \vee \neg Q) \wedge (\neg T \vee R)]$	(Associative Law)
$P \vee [(R \vee Q) \wedge (R \vee \neg T) \wedge (T \vee \neg Q)]$	(Distributive Law)
$P \vee [R \vee (Q \wedge F) \wedge (T \vee \neg Q)]$	(Identity Law)
$P \vee [R \vee (F) \wedge (T \vee \neg Q)]$	(Identity Law)
$P \vee [R \wedge (T \vee \neg Q)]$	(L.H.S = R.H.S)

12. (a) $F\ when\ x = 1\ and\ y = -1$
$x\ is\ not\ equal\ to\ y\ because\ -1 \neq 1.$
$(x = y)\ is\ (F)\ hence,\ \neg(x = y)\ is\ (T).$
$Similarly,\ x^2\ is\ equal\ to\ y^2\ because\ (-1)^2 = 1^2$
$(x^2 = y^2)\ is\ (T)\ hence\ \neg(x^2 = y^2)\ is\ (F)$
Hence, the whole is expression is going to be (F)

(b) $F\ when\ x = 7\ and\ y = 1.9129$
If we take x = 7 then for (x = y^3) to be True, y = $\sqrt[3]{7}$.
However since $\sqrt[3]{7}$ is not an integer this statement is (F).

(c) $F\ when\ x = 2\ and\ y = 0.5$
If we take x = 2 then ($\frac{1}{2}$ = y), since y must be an integer and can't be a fraction therefore it can never have a value that can equal $\frac{1}{2}$.

(d) T

(e) $F\ when\ x = 2\ and\ y = 3$
(xy = y) is (F) when x = 2 and y = 3.

(f) $F\ when\ x = 0\ and\ y = 0$
($x^3 \neq y^2$) is (F) when x = 0 and y = 0.

13. (a) $\neg\forall x\ \forall y\ P(x,y)$
$\exists x\ \exists y\ \neg p(x,y)$

(b) $\neg\forall x\ \exists y\ (P(x,y) \vee Q(x,y))$
$\exists x\ \forall y\ \neg(P(x,y) \vee Q(x,y))$ (De Morgan's Law)
$\exists x\ \forall y\ (\neg P(x.y) \wedge \neg Q(x,y))$

(c) $\neg\forall x\ (\forall y\ P(x,y) \wedge \exists y\ Q(x,y))$
$\exists x\ \neg(\forall y\ P(x,y) \wedge \exists y\ Q(x,y))$ (De Morgan's Law)
$\exists x\ (\neg\forall\ P(x,y) \vee\ \neg\exists y\ Q(x,y)$ (De Morgan's Law)
$\exists x\ (\exists y\ \neg P(x,y) \vee\ \forall y\ \neg Q(x,y)$

(d) $\neg(\exists x\ \exists y\ \neg P(x,y) \wedge \forall x\ \forall y\ Q(x,y))$ (De Morgan's Law)
$\neg(\exists x\ \exists y\ \neg P(x,y)) \vee \neg(\forall x\ \forall y\ Q(x,y))$
$\neg\exists x\ \exists y\ \neg P(x,y) \vee \neg\forall x\ \forall y\ Q(x,y)$
$\forall x\ \forall y\ P(x,y) \vee \exists x\ \exists y\ \neg Q(x,y)$