

Problem Set 3 Deadline: 25th Oct 2020 11:55 pm**1 Instructions**

- Adhere to the deadline. Late submissions are not allowed. You have more than enough time for the homework so it is advised to start early.
- You have to submit the solutions as a .pdf file.
- You may use LaTeX to write the mathematical equations or convert the word files into a pdf.
- The names of the file should be "yourrollnumber_hw1.pdf" e.g. 21100000_hw1.pdf.
- You are allowed to discuss with your peers, but you should not copy statements from each others.
- Follow the basic template you have been given for the homework.
- **You only have to submit solutions for questions 1,2, 6, 7 and 9. The rest are practice questions.**

2 Problems

1. For each of the following functions $f: R \rightarrow R$, determine whether f is invertible, and, if so, determine f^{-1}
 - (a) $f = \{(x, y) | 2x + 3y = 7\}$
 - (b) $f = \{(x, y) | ax + by = c, b \neq 0\}$
 - (c) $f = \{(x, y) | y = x^3\}$
 - (d) $f = \{(x, y) | y = x^4 + x\}$
2. Prove that for a finite set X any **one-to-one** function $f: X \rightarrow X$ is **bijective**.
3. Show that for a function $f: X \rightarrow Y$, f^{-1} exists if and only if f is bijective.
4. How many **one-to-one** functions are there from the set $A = \{a_1, a_2, \dots, a_n\}$ where n is a positive integer, to the set $B = \{b_1, b_2, \dots, b_m\}$, where m is a positive integer?
5. Suppose that f is an invertible function from Y to Z and g is an invertible function from X to Y . Show that the inverse of the composition $f \circ g$ is given by $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.
6. Suppose that f is a function from A to B , where A and B are finite sets with $|A| = |B|$. Show that f is *one-to-one* if and only if it is *onto*.
7. Let $a \in R$ and $b \in R$. Suppose that $a > 0$ and $b > 0$. Give a counter example to show that the following are not valid equalities or show that they are valid.

(a) $\lceil (a+b) \cdot (a-b) \rceil = \lceil (a+b) \rceil \cdot \lceil (a-b) \rceil$

(b) $\lfloor \frac{(a-3)^2}{2} \rfloor = \lfloor \frac{a^2}{2} \rfloor - 3a + 5$

(c) $\lfloor \frac{a-b}{2} \rfloor = \lfloor \frac{a}{2} \rfloor - \lfloor \frac{b}{2} \rfloor$

8. Let $g : \mathbf{N} \longrightarrow \mathbf{N}$ be defined by $g(n) = \max\{0, n - 3\}$ and let $A = \{0, 2, 4, 6\}$.

(a) Evaluate $g(A)$

(b) Evaluate $g^{-1}(A)$

(c) Is g a one-to-one function?

(d) Is g an onto function?

(e) Is g a bijection?

9. With $A = \{x, y, z\}$, let $f, g : A \longrightarrow A$ be given by $f = \{(x, y), (y, z), (z, x)\}$ and $g = \{(x, y), (y, x), (z, z)\}$. Determine each of the following functions. Write your answers as a collection of ordered pairs.

(a) $f \circ g$

(b) $g \circ f$

(c) f^{-1}

(d) g^{-1}