

Problem Set 5 Deadline: Nov 15, 2020 11:55 pm**1 Instructions**

- Adhere to the deadline. Late submissions are not allowed. You have more than enough time for the homework so it is advised to start early.
- You have to submit the solutions as a .pdf file.
- You may use LaTeX to write the mathematical equations or convert the word files into a pdf.
- The names of the file should be "yourrollnumber_hw1.pdf" e.g. 21100000_hw1.pdf.
- You are allowed to discuss with your peers, but you should not copy statements from each others.
- Follow the basic template you have been given for the homework.
- **You only have to submit solutions for questions 1, 4, 7, 11, 12. The rest are practice questions.**

2 Problems

1. Prove that if n is an integer, then $n^2 \geq n$ (*Hint: Use Cases*)
2. Use a Proof by cases to show that $|xy| = |x||y|$ where x and y are real numbers.
3. Prove that every positive integer greater than one can be factored as a product of primes.
4. Prove that $\sqrt[3]{2}$ is irrational.
5. Prove that the sum of two rational numbers is rational.
6. Prove that if $2x$ is irrational then x is irrational.
7. Prove that if n^2 is an even number then so is n .
8. Prove that for any integer n , if n is even then $n^2 + n + 5$ is odd.
9. Prove by contrapositive that if $3n + 5$ is even then n is odd.
10. Is it true that for all real numbers x and y , $\lceil x - y \rceil = \lceil x \rceil - \lceil y \rceil$?. Prove your answer.
11. Prove by contradiction that there are no positive integer solutions to: $x^2 + y^2 = 1$
12. Prove that if a is a rational number and b is an irrational number, then $a + b$ is an irrational number.