# CS-210 Homework 2

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1.

| A | B | C | A ∪ B | A ∪ B ∪ C | $\overline{A \cup B \cup C}$ | $\overline{A}$ | $\overline{B}$ | $\overline{C}$ | $\overline{A} \cap \overline{B}$ | $\overline{A} \cap \overline{B} \cap \overline{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |

The memberships of both $\overline{A \cup B \cup C}$ and $\overline{A} \cap \overline{B} \cap \overline{C}$ are same so they are equal which proves the De Morgan's Law.

3.

$$\overline{(A \cup C) \cap B} = \overline{B} \cup (\overline{C} \cap \overline{A}).$$

Starting with L.H.S

$\overline{(A \cup C) \cap B}$ ( De Morgan's Law )

$\overline{(A \cup C)} \cup \overline{B}$ ( Commutative Law )

$\overline{B} \cup \overline{(C \cup A)}$ ( De Morgan's Law )

$\overline{B} \cup \overline{C} \cap \overline{A}$ ( L.H.S = R.H.S hence, proved )

5. Suppose that set $|X| = a$ while the sets $|Y| = |Z| = b$. Then $|X \times Y| = ab$ while $|A| = |P(X \times Y)| = 2^{ab}$.

   On the other hand, $|P(X)| = 2^a$ and $|P(Z)| = 2^b$ therefore, $|B| = |P(X) \times P(Z)| = 2^a \times 2^b = 2^{a+b}$.

   Since $2^{ab} \neq 2^{a+b}$, therefore $|A| \neq |B|$. However if $|X| = a = 0$ and $|Y| = |Z| = b = 0$ then it is possible for $|A| = |B|$.

6. $P(\emptyset) = \emptyset$

   $$A = \mathcal{P}(\mathcal{P}(\emptyset)) = \Big\{\emptyset, \{\emptyset\}\Big\}$$

   $$B = \mathcal{P}(A) = \Bigg\{\ \emptyset,\ \Big\{\emptyset\Big\},\ \Big\{\{\emptyset\}\Big\},\ \Big\{\emptyset, \{\emptyset\}\Big\}\ \Bigg\}$$

   $$C = \mathcal{P}(\mathcal{B}) = \Bigg\{\emptyset, \Big\{\emptyset\Big\}, \Big\{\{\emptyset\}\Big\}, \Big\{\{\{\emptyset\}\}\Big\}, \Big\{\{\emptyset, \{\emptyset\}\}\Big\},$$
   $$\Big\{\emptyset, \{\emptyset\}\Big\}, \Big\{\emptyset, \{\{\emptyset\}\}\Big\}, \Big\{\emptyset, \{\emptyset, \{\emptyset\}\}\Big\},$$
   $$\Big\{\{\emptyset\}, \{\{\emptyset\}\}\Big\}, \Big\{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}\Big\},$$
   $$\Big\{\{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\Big\},$$
   $$\Big\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\Big\}, \Big\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\Big\}, \Big\{\{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\Big\}, \Big\{\emptyset,\ \{\{\emptyset\}\},\ \{\emptyset, \{\emptyset\}\}\Big\},$$
   $$\Big\{\emptyset,\ \{\emptyset\},\ \{\{\emptyset\}\},\ \{\emptyset, \{\emptyset\}\}\Big\}\Bigg\}$$

10. (a) $\overline{(A \cap \overline{B})} \cup B = \overline{A} \cup B$

Starting with L.H.S

$\overline{(A \cap \overline{B})} \cup B$ ( De Morgan's Law )

$\overline{A} \cup \overline{(\overline{B})} \cup B$ ( Double Complement Law )

$\overline{A} \cup B \cup B$ ( Idempotent Law )

$\overline{A} \cup B$ ( L.H.S = R.H.S hence, proved )

(b) $A \cup (B \setminus A) = A \cup B$

Starting with L.H.S

$A \cup (B \cap \overline{A})$ ( Distributive Law )

$(A \cup B) \cap (A \cup \overline{A})$ ( Complement Law )

$(A \cup B) \cap U$ ( Identity Law )

$A \cup B$ ( L.H.S = R.H.S hence, proved )

11. $(B \setminus A) \cup (C \setminus A) = (B \cup C) \setminus A$

Starting with L.H.S

$(B \cap \overline{A}) \cup (C \cap \overline{A})$ ( Distributive Law )

$(B \cup C) \cap \overline{A}$

$(B \cup C) \setminus A$ ( L.H.S = R.H.S hence, proved )

15. Suppose that the set $A = \emptyset$, while $C \neq \emptyset$, then by the definition of Cartesian product $A \times C = \emptyset$ so if $A \times C = B \times C$, then $B \times C = \emptyset$ and as we know that $C \neq \emptyset$ then this means $B = \emptyset$. Therefore, $A = B = \emptyset$ proves that $A = B$