CS-210 Homework 3

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1. (a) $f = \{(x,y)|2x+3y = 7\}$

   $y = \frac{7-2x}{3}$

   For any $y \in$ R, since $\frac{7-3y}{2} \in R$ and $f(\frac{7-3y}{2}) = y$
   so f is onto

   $f(x_1) = f(x_2) \rightarrow \frac{7-2(x_1)}{3} = \frac{7-2(x_2)}{3} \rightarrow x_1 = x_2$
   so f is one-to-one

   Hence, f is a bijection and is invertible

   $f^{-1} = \frac{7-3x}{2}$

   (b) $f = \{(x,y)|ax+by = c, b \neq 0\}$

   $y = \frac{c-ax}{b}$

   For any $y \in$ R, since $\frac{c-by}{a} \in R$ and $f(\frac{c-by}{a}) = y$
   so f is onto

   $f(x_1) = f(x_2) \rightarrow \frac{c-a(x_1)}{b} = \frac{c-a(x_2)}{b} \rightarrow x_1 = x_2$
   so f is one-to-one

   Hence, f is a bijection and is invertible

$f^{-1} = \frac{c-bx}{a}$

(c) $f = \{(x, y)|y = x^3\}$

For any $y \in$ R, since $\sqrt[3]{y} \in R$ and $f(\sqrt[3]{y}) = y$
so f is onto

$f(x_1) = f(x_2) \rightarrow (x_1)^3 = (x_2)^3 \rightarrow x_1 = x_2$
so f is one-to-one

Hence, f is a bijection and is invertible

$f^{-1} = \sqrt[3]{x}$

(d) $f = \{(x, y)|y = x^4 + x\}$

For any $y \in R$ such that $y < -0.47$, the function f does not has a pre-image, which implies that the function is not onto and if not onto and then not a bijective as well. Hence, not invertible.

2. For any one-to-one function to be bijective,it should be an onto function. We must prove that if a function is one-to-one then it must be an onto in order to be a bijective. Now $f : X \to X$, then $|domain| = |co - domain|$, if f is one-to-one then every element in the domain must has a unique image therefore $|range| = |domain|$ and since $|domain| = |co - domain|$, then $|range| = |co - domain|$. This implies that every image has a pre-image image and hence, f is an onto and consequently a bijective as well.

   We must also prove that if f is not one-to-one then it cannot be onto and bijective. Now $f : X \to X$, then $|domain| = |co - domain|$, if f is not one-to-one then every element in domain does not has a unique image therefore, $|range| < |domain|$ and since $|domain| = |co - domain|$, then $|range| < |co - domain|$. This implies that not every image has a pre-image and hence, f is not an onto and consequently not a bijective as well.

6. To show this we must prove that if f is one-to-one then, it must be onto. Let's suppose $|A| = |B| = c$ then $|domain| = |co - domain| = c$, now if f is one-to-one then it will map all c elements in the set A to a unique element in set B. As a result, there are c elements in set B that have a pre-image and hence, $|range| = c$. While on the other hand, we have $|domain| = c$, so by this $|domain| = |range|$ making f an onto function.

   We must also prove that if f is not onto then, it cannot be a one-to-one. Let's suppose $|A| = |B| = c$ so by this $|domain| = |co - domain| = c$. Now if f is not onto then this implies that there are some images which do not have a pre-image and so $|range| < |co - domain|$. As we know that $|domain| = |co - domain| = c$ therefore, it must be the case that $|range| < |domain|$ indicating that f is not one-to-one.

7. (a) $\lceil (a + b) \cdot (a - b) \rceil = \lceil (a + b) \rceil \cdot \lceil (a - b) \rceil$
   Not valid when $a = 2.3$ *and* $b = 2.2$
   $\lceil (2.3 + 2.2) \cdot (2.3 - 2.2) \rceil = \lceil (2.3 + 2.2) \rceil \cdot \lceil (2.3 - 2.2) \rceil$
   $\lceil (4.5) \cdot (0.1) \rceil = \lceil (4.5) \rceil \cdot \lceil (0.1) \rceil$
   $\lceil 0.45 \rceil = (5) \cdot (1)$
   $1 \neq 5$

(b) $\lfloor\frac{(a-3)^2}{2}\rfloor = \lfloor\frac{a^2}{2}\rfloor - 3a + 5$
Not valid when $a = 2.2$
$\lfloor\frac{(2.2-3)^2}{2}\rfloor = \lfloor\frac{2.2^2}{2}\rfloor - 3(2.2) + 5$
$\lfloor 0.16\rfloor = \lfloor 2.42\rfloor - 1.6$
$0 = 2 - 1.6$
$0 \neq 0.4$

(c) $\lfloor\frac{a-b}{2}\rfloor = \lfloor\frac{a}{2}\rfloor - \lfloor\frac{b}{2}\rfloor$
Not valid when $a = 8\ and\ b = 3$
$\lfloor\frac{8-3}{2}\rfloor = \lfloor\frac{8}{2}\rfloor - \lfloor\frac{3}{2}\rfloor$
$\lfloor 2.5\rfloor = \lfloor 4\rfloor - \lfloor 1.5\rfloor$
$2 = 4 - 1$
$2 \neq 3$

9. (a) $fog = \{(x, z), (y, y), (z, x)\}$
   (b) $gof = \{(x, x), (y, z), (z, y)\}$
   (c) $f^{-1} = \{(x, z), (y, x), (z, y)\}$
   (d) $g^{-1} = \{(x, y), (y, x), (z, z)\}$