Problem Set 4 Deadline: 4th Nov, 2020 11:55 pm

1 Instructions

- Adhere to the deadline. Late submissions are not allowed. You have more than enough time for the homework so it is advised to start early.
- You have to submit the solutions as a .pdf file.
- You may use LaTeX to write the mathematical equations or convert the word files into a pdf.
- The names of the file should be "yourrollnumber_hw1.pdf" e.g. 21100000_hw1.pdf.
- You are allowed to discuss with your peers, but you should not copy statements from each others.
- Follow the basic template you have been given for the homework.
- **You only have to submit solutions for questions 2, 3, 6, 7, 8. The rest are practice questions.**

2 Problems

1. What is the term a_8 of the sequence a_n if a_n equals:
   (a) 2^{n-1}
   (b) 7
   (c) $1 + (-1)^n$
   (d) $-(-2)^n$
2. Evaluate the following sums. When needed assume that n is a large positive even integer.
   (a) $\sum_{j=0}^{8}(2^{j+1} - 2^j)$
   (b) $\sum_{i=1}^{5} i^2 + \sum_{i=1}^{5} 7$
   (c) $\sum_{i=1}^{4}(i^2 + i)$
   (d) $\sum_{k=1}^{4} k^2 + \sum_{k=1}^{4} k$
   (e) $\sum_{i=0}^{4}(3i^2 + 2i)$
   (f) $3\sum_{k=0}^{4} k^2 + 2\sum_{k=0}^{4} k$

(g) $\sum_{k=111}^{3000} k$

(h) $\sum_{k=-n}^{n} k$

3. Find at least three different sequences beginning with the terms 1, 2, 4 whose terms are generated with a simple formula or rule. Assuming your formula is correct, determine the next 5 terms of each.

4. For each of these lists of integers, provide a a simple rule or numerical formula that generates the terms of an integer sequences that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

   (a) $3, 6, 11, 18, 27, 38, 51, 66, 83, 102, \ldots$

   (b) $7, 11, 15, 19, 23, 27, 31, 35, 39, 43, \ldots$

   (c) $1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, \ldots$

   (d) $0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, \ldots$

   (e) $1, 3, 15, 105, 945, 10395, 135135, 2027025, 34459425, \ldots$

   (f) $1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, \ldots$

   (g) $2, 4, 16, 256, 65536, 4294967296, \ldots$

5. Compute the sums of these geometric progressions.

   (a) $\sum_{k=0}^{9} 2^k$

   (b) $\sum_{k=0}^{15} 3 * (-2)^k$

   (c) $\sum_{j=0}^{12} 3 * 2^j$

6. Let a_n be the nth term of the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, \ldots$, constructed by including the integer k exactly k times. Show that

$$a_n = \lfloor \frac{1}{2} + \sqrt{2n + \frac{1}{4}} \rfloor$$

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7. Let $|r| < 1$ be a real number. Evaluate

$$\sum_{i=0}^{\infty} ir^i$$

8. Find a formula for $\sum_{k=0}^{m} \lfloor \sqrt{k} \rfloor$ where m is a positive integer.