Problem 2

Proof: T(n) = nlogn

Base Case: T(2) = 2log2

2 = 2log2 hence, satisfied

Induction Hypothesis: Assume T(n) = $nlogn$ is true for $n=2^k$ where k > 1

So, $T(2^k)= 2^k\log2^k$

Induction Step: showing that $n=2^{k+1}$ holds where k > 1 and that,

$T(2^{k+1})= 2^{k+1}\log2^{k+1}$ is true

$$T(2^{k+1}) = 2T\left(\frac{2^{k+1}}{2}\right) + 2^{k+1}$$

$$= 2T(2^k) + 2^{k+1}$$

From Induction hypothesis $T(2^k)= 2^k\log2^k$

$$= 2(2^k\log2^k) + 2^{k+1}$$

$$= 2^{k+1}\log2^k + 2^{k+1}$$

$$= 2^{k+1}(\log2^k + 1)$$

$$= 2^{k+1}(\log2^{k+1})$$

$$So, \qquad T(2^{k+1}) = 2^{k+1}(\log2^{k+1})$$

Hence, proved T(n) = $nlogn$