Problem 4

- According to the definition of big O, a function, f(n) is in O(g(n)) if there exists positive real number c and a real number k such that for all n > k, $0 < f(n) < c \cdot g(n)$

1)

$5n^3 \in O(n^3)$

This is true because it is apparent that for $c \geq 5$, and $n \geq 1$

$5n^3 \leq c \cdot n^3$

2)

$100n^2 \in O(n^4)$

$100n^2 \leq c \cdot n^4$

$$\frac{100}{c} \leq n^2$$

This is true because a quadratic function is always an upper bound over a constant, so this holds true for any $c \geq 0$

3)

$\log n^2 \in O(\log n)$

$2\log n \in O(\log n)$

$2\log n \leq c \cdot \log n$

$2 \leq c$

Hence, this is true because for $c \geq 2$ and $n \geq 1$

4)

$$(n^2 + 7n - 10)^3 \in O(n^6)$$
$$n^2 + 7n - 10 \in O(n^2)$$

This is true because for $c \geq 3$ and $n \geq 1$,
$$n^2 + 7n - 10 \leq \mathrm{c} \cdot n^2$$

5)

$\sqrt{n} \in O((log\ n)^3)$

This is false because, after applying L'hopital rule we get the results below implying that it is incorrect

$$\lim_{n \to \infty} \frac{\sqrt{n}}{(logn)^3} = \infty$$