

Problem 6

a) $T(1) = 1$

$$T(n) = T(n-1) + T(n-2) + 1 \quad \text{for } n > 1$$

$$T(n) \leq 2T(n-1)$$

$$T(n) < 2^2 T(n-2)$$

$$T(n) < 2^3 T(n-3)$$

$$T(n) < 2^x T(n-x) = 2^x T(0)$$

Hence, $n-x = 0$

$$n = x$$

so, $T(n) = 2^n T(0) = 2^n$

Therefore, $T(n) \in O(2^n)$

b)

```
int memo[10000] = {0} ;
```

```
int fib(int n)
```

```
{
```

```
    if (n == 0 or n==1)
```

```
        return 1;
```

```
    if (memo[n] != 0)
```

```
        return memo[n] ;
```

```
    else
```

```
        memo[n] = fib(n-1) + fib(n-2) ;
```

```
        return memo[n] ;
```

```
}
```