Problem 7

a) a=8, b=2, c=2

$c < \log_b a$

$2 < \log_2 8$

$2 < 3$

Hence, case 3 of master theorem, $T(n) = O(n^3)$

b) a = 3, b=4, f(n) = nlogn

$\log_b a = \log_4 3 = 0.7925$

$f(n) = n\log n > n^{0.7925}$

Hence, case 1 of master theorem, T(n) = O(nlogn)

c) a = 2, b= 4, c = 0.5

$c = \log_b a$

$0.5 = \log_4 2$

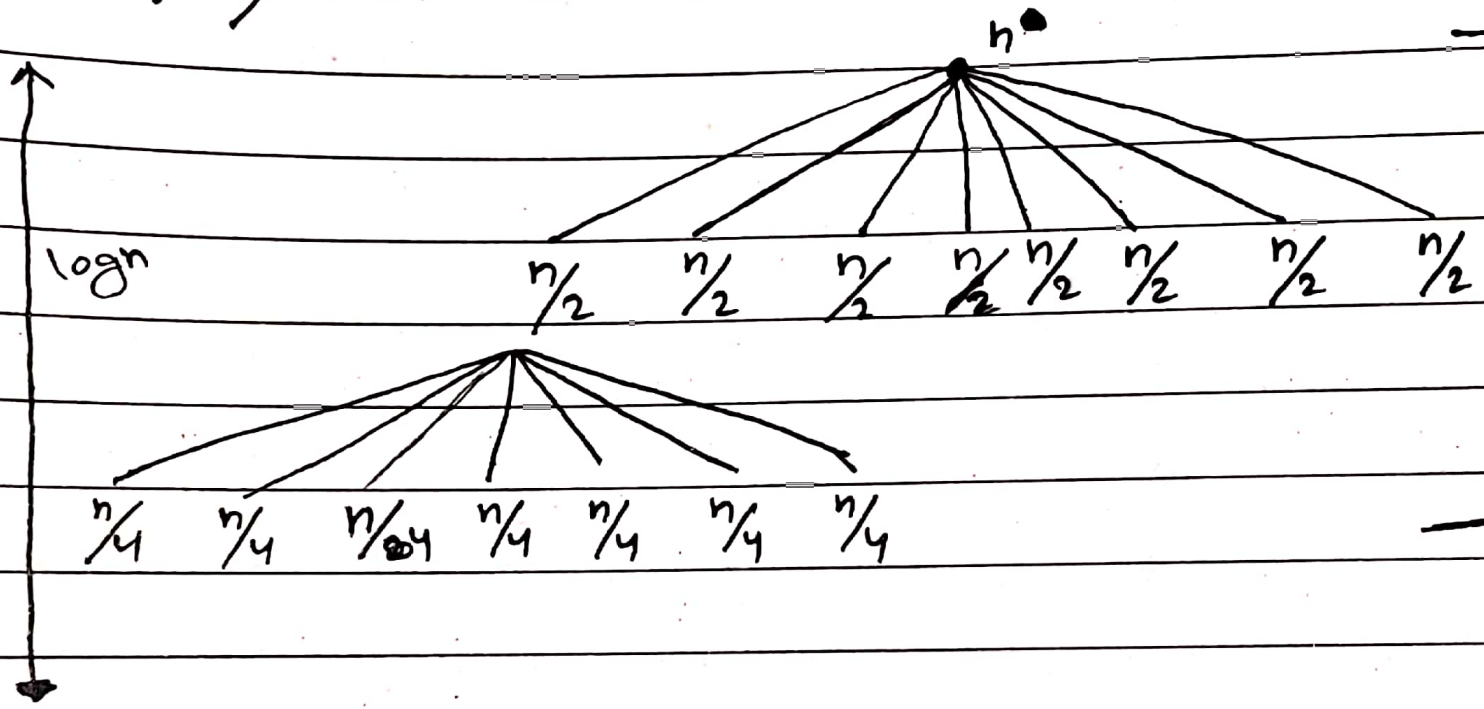
0.5 = 0.5

Hence, case 2 of master theorem, $T(n) = O(\sqrt{n}\ \log n)$

Using Recursion Trees

Date: _____

Q27) a) $8T(n/2) + \Theta(n^2)$



last level → $8^{\log n}(\quad)^2 = 8^{\log n} T(1)$

Total time $= n^2 + 2n^2 + 2^2n^2 + 2^3n^2 + \ldots + 8^{\log n}$

$= n^2 + 2n^2 + 2^2n + 2^3n^2 + \ldots + 2^{3\log n}$

$= \quad // \quad // \quad + \ldots + 2^{\log_2 n^3}$

$= \quad // \quad // \quad // \quad + \ldots + n^3$

Since all the other terms are in n^2 therefore the last term will be n^3 and will be highest so $T(n) = O(n^3)$

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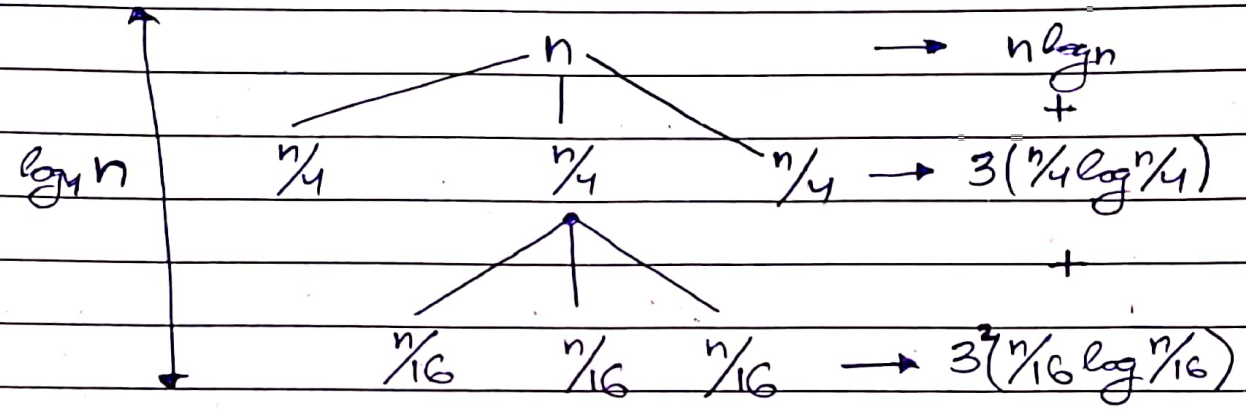
Q7) b) $3T(n/4) + n\log n$

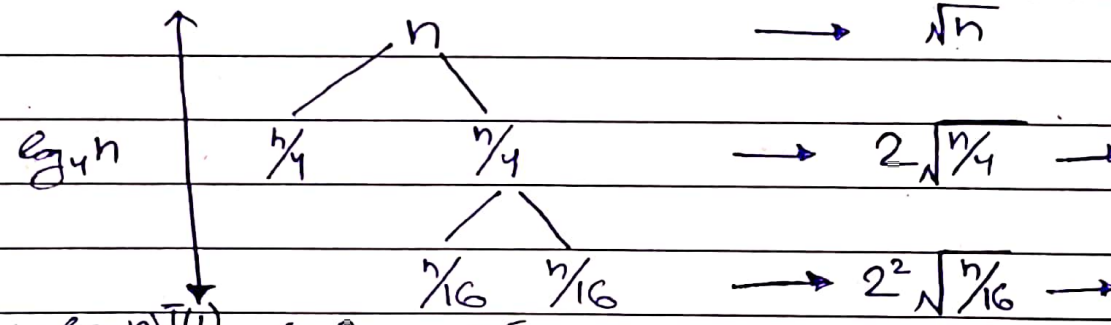


$3^{\log_4 n} T(1) \rightarrow$ last level $\rightarrow n^{\log_4 3}$

Total time = $n\log n + 3(n/4 \log n/4) + \ldots + n^{\log_4 3}$

$n^{\log_4 3} < n^1$ hence highest term in series = $n\log n$ so $T(n) = n\log n$

Q7) c) $2T(n/4) + \sqrt{n}$



last level = $(2^{\log_4 n})T(1) = n^{\log_4 2} = n^{0.5}$

total time = $\sqrt{n} + \sqrt{n} + \ldots \sqrt{n}$

hence total time = $O(\sqrt{n}\log_4 n)$