

Problem 9:

a) The outer for loop runs for at most n iterations which is in $O(n)$ while the inner for loop runs for at most n iterations, which is also $O(n)$. Then the summation of array entries in the inner for loop takes at most n steps which is also $O(n)$. The other operations like assigning values and storing them in array takes constant time. Hence over all the algorithm takes at most $O(n) \cdot O(n) \cdot O(n)$ steps which equals to $O(n^3)$ so $f(n) = O(n^3)$

b) Let's consider the first $n/3$ iterations where $i = \frac{n}{3}$

Then the second loop runs for at least $(n - \frac{n}{3}) = \frac{2n}{3}$

The summation takes place from $A[i]$ to $A[j]$ so it takes at least $\frac{2n}{3} - \frac{n}{3} = \frac{n}{3}$

As a result, there are at least $(\frac{n}{3})^3 = \frac{1}{27}n^3$ steps involved in summation.

$$\frac{1}{27}n^3 \leq n^3$$

Hence algorithm is also in $\Omega(n^3)$

c)

For $i=1$ to $n-1$ do

 temp = A[i]

 for $j=i+1$ to n do

 temp = temp + A[j]

 B[i][j] = temp

 EndFor

EndFor

Return B

The outer loop runs for exactly n iterations while the inner for loop takes at most n iterations so total time for the algorithm considering the other operations take constant time is $n \cdot n = n^2$

Hence $f(n) = O(n^2)$