

Problem 5

```
CHANGE(n, V, D)
FOR v = 0 TO V
    M[0, v] ← 0
FOR i = 1 TO n
    FOR v = 0 TO V
        IF (Di > v)
            M[i, v] ← M[i - 1, v].
        ELSE
            M[i, v] ← min { M[i - 1, v], 1 + M[i - 1, v - Di] }.
combinations = []
start = V
i = n
WHILE (start != 0 )
    IF M[i, start] == M[i-1, start]
        i = i - 1
    ELSE
        combinations ← Di
        start = start - Di
return combinations
```

Discussing optimality:

Assistance taken from slides

$M[i, v]$ = optimal number of coins with denomination 1,...,i subjected to limit V

Goal = $M[i, V]$

Case 1: item i is not selected then the best of {1,2,3..., i-1} are selected subject to limit V

Case 2 : item i selected then,

$$V = V - D_i$$

number of coins ++

then the best of $\{1, 2, 3, \dots, i-1\}$ are selected subject to limit V

So due to exchange argument, these substructures are optimal, hence, making the whole solution optimal.