Problem 10

Base Case: For any value of x, if y == 0, then return 0. Hence, multiply(x,0) = 0

Inductive Hypothesis: For all integers x , y and there exist a k such that k = |y| then multiply(x, y) = x*y

Induction Step: showing if it holds for (x, y+1) and that multiply(x, y+1) = x(y+1)

multiply(x, y+1) = multiply $\left(cx, c\left\lfloor\frac{y+1}{c}\right\rfloor\right) + x\big((y+1)mod c\big)$

From inductive hypothesis, multiply$\left(cx, c\left\lfloor\frac{y+1}{c}\right\rfloor\right) = cx \cdot \left\lfloor\frac{y+1}{c}\right\rfloor$

$$= cx \cdot \left\lfloor\frac{y+1}{c}\right\rfloor + x\big((y+1)mod c\big)$$

$$= x\left[c \cdot \left\lfloor\frac{y+1}{c}\right\rfloor + \big((y+1)mod c\big)\right]$$

$$= x\left[c \cdot \left\lfloor\frac{y+1}{c}\right\rfloor + \left(c \cdot \left(\frac{y+1}{c} - \left\lfloor\frac{y+1}{c}\right\rfloor\right)\right)\right]$$

$$= x\left[c \cdot \left\lfloor\frac{y+1}{c}\right\rfloor + c \cdot \frac{y+1}{c} - c \cdot \left\lfloor\frac{y+1}{c}\right\rfloor\right]$$

$$= x\left[c \cdot \frac{y+1}{c}\right]$$

$$= x(y+1)$$

Hence, proved that multiply(x,y) = x*y

b) In the worst case, the line 5 will be executed until y is not down to 0 which will take $O(\log_c(y))$ steps