

Problem 9

Assuming $T(n) = n + 1$ is true

Base Case: $T(0) = 0 + 1 = 1$

Case 1: If the input is even then the $x \bmod 2$ will not be 1 making line 2 false and hence, last line will execute directly, returning $x+1$ directly which is correct.

Case 2: If the input is odd, i.e. $2n+1$ then the algorithm will enter IF statement so,

2 * Increment($\lfloor (2n+1)/2 \rfloor$)

2 * Increment($\lfloor (n+0.5) \rfloor$)

2 * Increment(n)

2 * $(n+1)$

$(2n+1) + 1$

Since, $T(2n+1) = (2n+1)+1$, $T(n) = n+1$ is also true for odd numbers

Time complexity: In worst case scenario, the input could be odd number such that it will be divided all the way to 1, in that case, the 3rd line will execute for $O(\log n)$ time