assistance taken from the lecture slides

a) COIN-CHANGING-ALGORITHM (x, c1, c2, ..., cn)

SORT coin denominations so that c1 > c2 > > Cn

change = []

WHILE (x > 0)

i = 0

FOR j = 1 to n

IF Cj ≤ x

i = Cj

IF (i = 0)

RETURN "no solution."

ELSE

$$x = x - Ci$$

append i to change

RETURN change

b) No, it is not optimal for any set of denominations. Let's look at a counter example.

Suppose the given denominations = { 1, 10, 21, 34, 70, 100, 350, 1225, 1500 }

While x = 140

Then, according to our greedy algorithm approach,

$$140 - 100 - 34 - 1 - 1 - 1 - 1 - 1 - 1 = 0$$

So, change = 100, 34, 1, 1, 1, 1, 1, 1

However, optimal approach = 140 - 70 - 70 = 0

So optimal change = 70, 70

Hence, it is possible for greedy algorithm to fail on certain denominations.