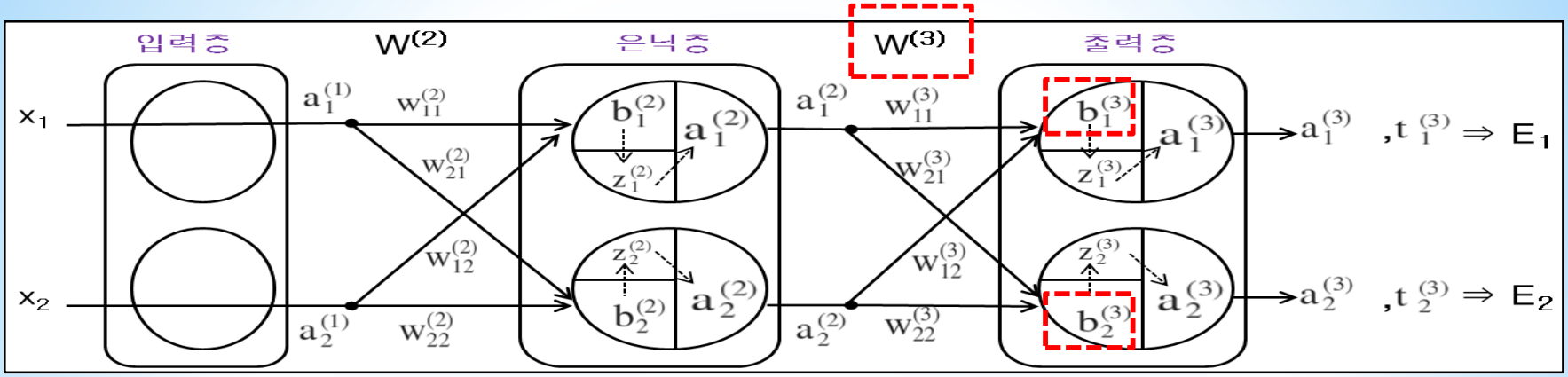
파이썬(Python)으로 구현하는

오차역전파
(Back Propagation)

– 출력층에서의 오차역전파–

출력층 오차역전파 공식 유도 – 출력층 가중치 $W^{(3)}$ / 출력층 바이어스 $b^{(3)}$



$$W^{(3)} = W^{(3)} - \alpha \frac{\partial E}{\partial W^{(3)}}$$

$$b^{(3)} = b^{(3)} - \alpha \frac{\partial E}{\partial b^{(3)}}$$

$$\frac{\partial E}{\partial w_{11}^{(3)}}, \frac{\partial E}{\partial w_{21}^{(3)}}, \frac{\partial E}{\partial w_{12}^{(3)}}, \frac{\partial E}{\partial w_{22}^{(3)}}$$

출력층 가중치 $W^{(3)}$

$$\frac{\partial E}{\partial b_1^{(3)}}, \frac{\partial E}{\partial b_2^{(3)}}$$

출력층 바이어스 $b^{(3)}$

$\partial E/\partial W^{(3)}$, $\partial E/\partial b^{(3)}$ 오차역전파 공식 유도

$$W^{(2)} = W^{(2)} - \alpha \frac{\partial E}{\partial W^{(2)}}$$

$$b^{(2)} = b^{(2)} - \alpha \frac{\partial E}{\partial b^{(2)}}$$

$$\frac{\partial E}{\partial w_{11}^{(2)}}, \frac{\partial E}{\partial w_{21}^{(2)}}, \frac{\partial E}{\partial w_{12}^{(2)}}, \frac{\partial E}{\partial w_{22}^{(2)}}$$

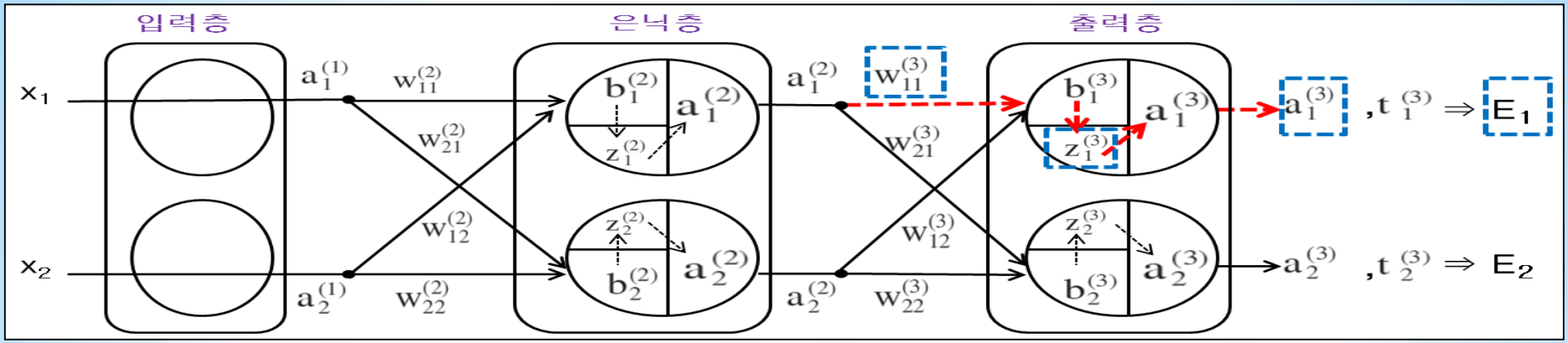
은닉층 가중치 $W^{(2)}$

$$\frac{\partial E}{\partial b_1^{(2)}}, \frac{\partial E}{\partial b_2^{(2)}}$$

은닉층 바이어스 $b^{(2)}$

$\partial E/\partial W^{(2)}$, $\partial E/\partial b^{(2)}$ 오차역전파 공식 유도

출력층 $\frac{\partial E}{\partial w_{11}^{(3)}}$



$$\frac{\partial E}{\partial w_{11}^{(3)}} = \frac{\partial E_1}{\partial w_{11}^{(3)}} + \frac{\partial E_2}{\partial w_{11}^{(3)}} \quad (\partial E_2 \to 0)$$

체인 룰을 이용하여 국소미분으로 분리

$$= \frac{\partial E_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial w_{11}^{(3)}}$$

$$= \frac{\partial \left\{ \frac{1}{2}(t_1^{(3)} - a_1^{(3)})^2 \right\}}{\partial a_1^{(3)}} \times \frac{\partial \mathrm{sigmoid}(z_1^{(3)})}{\partial z_1^{(3)}} \times \frac{\partial (a_1^{(2)} w_{11}^{(3)} + a_2^{(2)} w_{12}^{(3)} + b_1^{(3)})}{\partial w_{11}^{(3)}}$$

$$= (a_1^{(3)} - t_1^{(3)}) \times \mathrm{sigmoid}(z_1^{(3)}) \times (1 - \mathrm{sigmoid}(z_1^{(3)})) \times a_1^{(2)}$$

$$= (a_1^{(3)} - t_1^{(3)}) \times a_1^{(3)} \times (1 - a_1^{(3)}) \times a_1^{(2)}$$

[필요 수식]

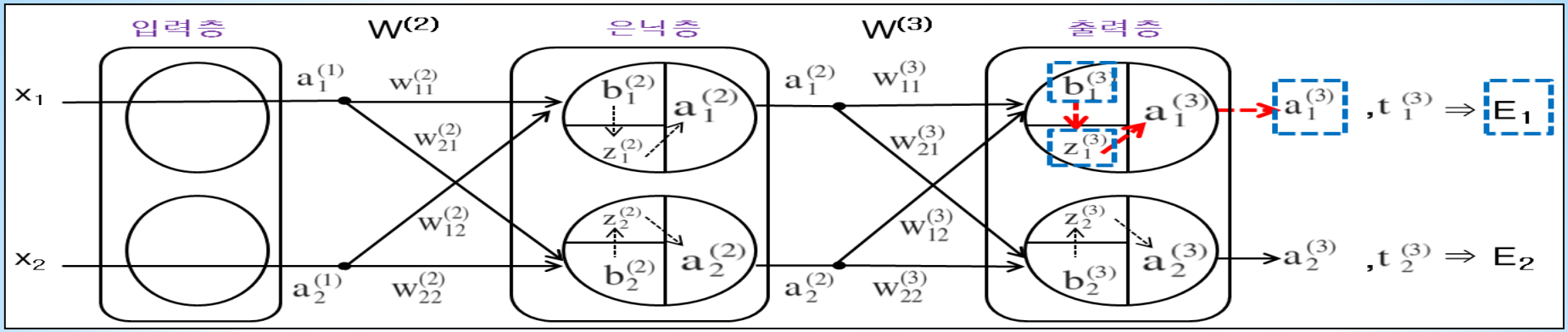
$$E = E_1 + E_2$$

$$E_1 = \frac{1}{2}(t_1^{(3)} - a_1^{(3)})^2$$

$$a_1^{(3)} = \mathrm{sigmoid}(z_1^{(3)})$$

$$z_1^{(3)} = a_1^{(2)} w_{11}^{(3)} + a_2^{(2)} w_{12}^{(3)} + b_1^{(3)}$$

출력층 $\frac{\partial E}{\partial b_1^{(3)}}$



$$\frac{\partial E}{\partial b_1^{(3)}} = \frac{\partial E_1}{\partial b_1^{(3)}} + \cancel{\frac{\partial E_2}{\partial b_1^{(3)}}}^{\,0}$$

체인 룰을 이용하여 국소미분으로 분리

$$= \frac{\partial E_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial b_1^{(3)}}$$

$$= \frac{\partial\left\{\frac{1}{2}(t_1^{(3)}-a_1^{(3)})^2\right\}}{\partial a_1^{(3)}} \times \frac{\partial \mathrm{sigmoid}(z_1^{(3)})}{\partial z_1^{(3)}} \times \frac{\partial(a_1^{(2)}w_{11}^{(3)}+a_2^{(2)}w_{12}^{(3)}+b_1^{(3)})}{\partial b_1^{(3)}}$$

$$= (a_1^{(3)} - t_1^{(3)}) \times \mathrm{sigmoid}(z_1^{(3)}) \times (1-\mathrm{sigmoid}(z_1^{(3)})) \times 1$$

$$= (a_1^{(3)} - t_1^{(3)}) \times a_1^{(3)} \times (1 - a_1^{(3)}) \times 1$$

[필요 수식]

$$E = E_1 + E_2$$

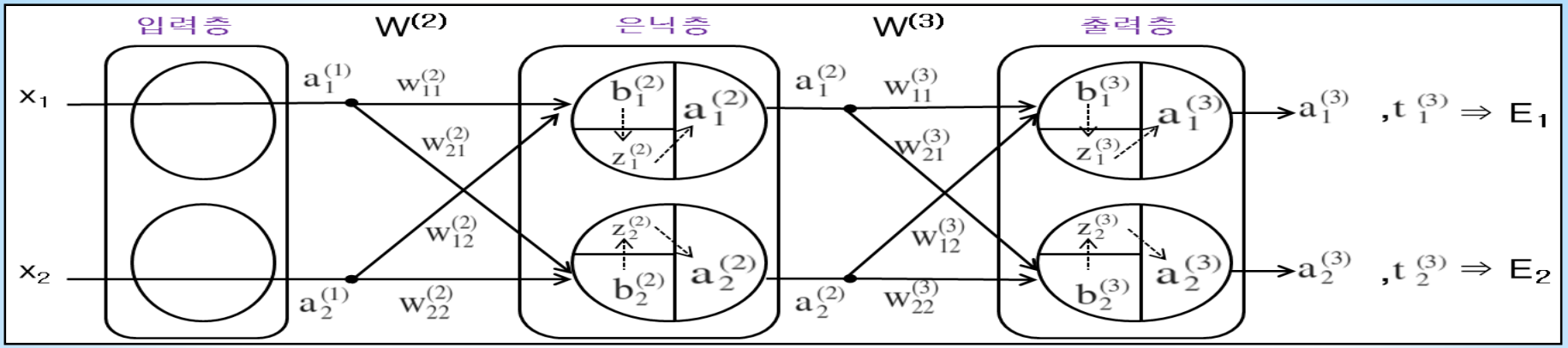
$$E_1 = \frac{1}{2}(t_1^{(3)}-a_1^{(3)})^2$$

$$a_1^{(3)} = \mathrm{sigmoid}(z_1^{(3)})$$

$$z_1^{(3)} = a_1^{(2)}w_{11}^{(3)} + a_2^{(2)}w_{12}^{(3)} + b_1^{(3)}$$

출력층 오차역전파 공식 유도

Self Study



$$\frac{\partial E}{\partial w_{12}^{(3)}} = (a_1^{(3)} - t_1^{(3)}) \times a_1^{(3)} \times (1 - a_1^{(3)}) \times a_2^{(2)}$$

$$\frac{\partial E}{\partial w_{21}^{(3)}} = (a_2^{(3)} - t_2^{(3)}) \times a_2^{(3)} \times (1 - a_2^{(3)}) \times a_1^{(2)}$$

$$\frac{\partial E}{\partial w_{22}^{(3)}} = (a_2^{(3)} - t_2^{(3)}) \times a_2^{(3)} \times (1 - a_2^{(3)}) \times a_2^{(2)}$$

$$\frac{\partial E}{\partial b_2^{(3)}} = (a_2^{(3)} - t_2^{(3)}) \times a_2^{(3)} \times (1 - a_2^{(3)}) \times 1$$