# CSE 445: Mobile and Wireless Communication

## Lecture 3: Modulation and Multiplexing

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# Modulation



## What is modulation

- **Modulation** is the process of encoding information from a message source in a manner suitable for transmission
- It involves translating a baseband message signal to a bandpass signal at frequencies that are very high compared to the baseband frequency.
- Baseband signal is called *modulating* signal
- Bandpass signal is called *modulated* signal



## Modulation Techniques

- Modulation can be done by varying the
  - Amplitude
  - Phase, or
  - Frequency

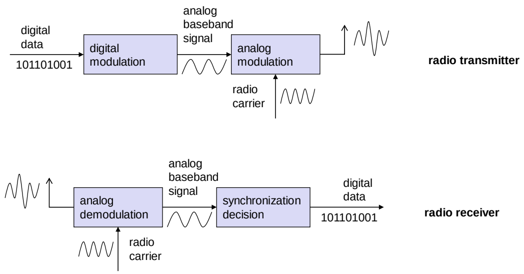
  of a high frequency carrier in accordance with the amplitude of the message signal.
- **Demodulation** is the inverse operation: *extracting the baseband message from the carrier* so that it may be processed at the receiver.

# Modulation

- Digital modulation
  - digital data is translated into an analog signal (baseband)
  - ASK, FSK, PSK - main focus in this chapter
  - differences in spectral efficiency, power efficiency, robustness
- Analog modulation
  - shifts center frequency of baseband signal up to the radio carrier
- Motivation
  - smaller antennas (e.g., $\lambda/4$)
  - Frequency Division Multiplexing
  - medium characteristics
- Basic schemes
  - Amplitude Modulation (AM)
  - Frequency Modulation (FM)
  - Phase Modulation (PM)



# Modulation and demodulation






# Goal of Modulation Techniques

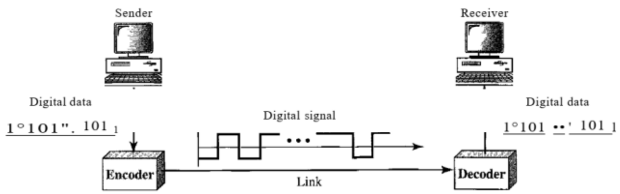
- Modulation is difficult task given the hostile mobile radio channels
  - Small-scale fading and multipath conditions.
- The goal of a modulation scheme is:
  - Transport the message signal through the radio channel with best possible quality
  - Occupy least amount of radio (RF) spectrum.



# Preamble to Digital Modulation

## Line Coding

- Line coding is the process of converting digital data to digital signals.



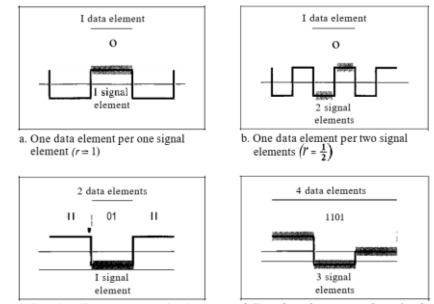



## Signal Element and Data Element

- Our goal is to carry data element.
- In digital communication Signal element Carries Data Element

$$r = \frac{no\ of\ data\ element}{no\ of\ signal\ element}$$

r= no of data elements carried by each signal element




a. One data element per one signal element ($r = 1$)

b. One data element per two signal elements ($r = \frac{1}{2}$)

c. Two data elements per one signal element ($r = 2$)

d. Four data elements per three signal elements ($r = \frac{4}{3}$)



## Data rate and Signal Rate

- The data rate defines the number of data elements (bits) sent in 1s. The unit is bits per second (bps).
- The signal rate is the number of signal elements sent in 1s. The unit is the baud.
- There are several common terminologies used in the literature.
- The data rate is sometimes called the bit rate;
- The signal rate is sometimes called the pulse rate, the modulation rate, or the baud rate.
- We can formulate the relationship between data rate and signal rate as

$$S = C \times N \times \frac{1}{r}$$

- where *N* is the data rate (bps); c is the case factor, which varies for each case; S is the number of signal elements; and *r* is the previously defined factor.



## Bandwidth of digital signal

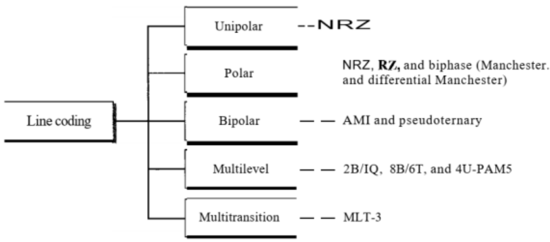
- Although the actual bandwidth of a digital signal is infinite, the effective bandwidth is finite.
- Minimum bandwidth can be given by

$$B_{min} = C \times N \times \frac{1}{r}$$

- The maximum data rate if the bandwidth of the channel is given

$$N_{max} = \frac{1}{C} \times B \times r$$

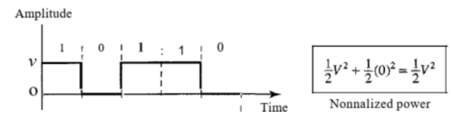
## Line coding schemes






## Line coding schemes

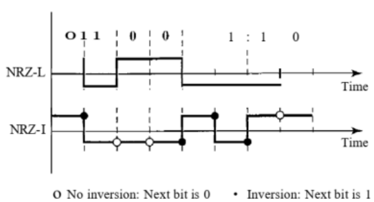
- Unipolar NRZ(Non Return to Zero) scheme






## Line coding schemes

- Polar NRZ L(NRZ Level) and Polar NRZ I (NRZ Invert) scheme






## Line coding schemes

- Unipolar RZ (Return to Zero)

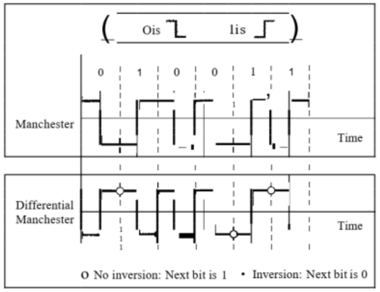
## Line coding schemes

- Bipolar RZ



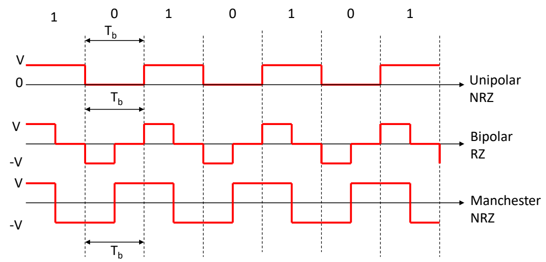
## Line coding schemes

- *Polar bi-phase: Manchester and differential Manchester schemes*






## Line coding example



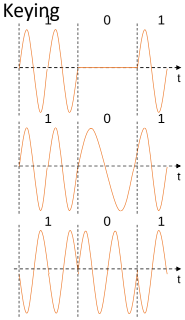



# Digital Modulation

Representing Digital Data using analog signal

## Digital modulation

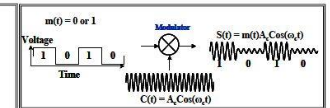
- Modulation of digital signals known as Shift Keying
- Amplitude Shift Keying (ASK):
  - very simple
  - low bandwidth requirements
  - very susceptible to interference
- Frequency Shift Keying (FSK):
  - needs larger bandwidth
- Phase Shift Keying (PSK):
  - more complex
  - robust against interference






## ASK (Review)

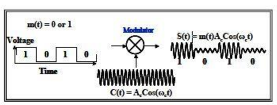
- **Amplitude Shift Keying:** Amplitude changes in discrete levels, in accordance with the input signal.
  - $m(t) = 0$ or $1$ (Input)
  - $C(t) = A_c Cos(\omega_c t)$
  - $S(t) = m(t)\, C(t) = m(t) A_c Cos(\omega_c t)$
- **For $m(t) = 1$:**
  - $S(t) = A_c Cos(\omega_c t)$
- **For $m(t) = 0$:**
  - $S(t) = 0$




**NOTE:**
- ASK modulator is a Product Modulator

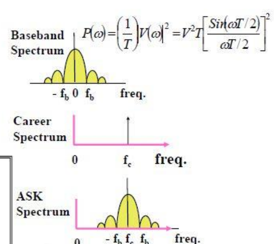


## ASK Power Spectrum: Review



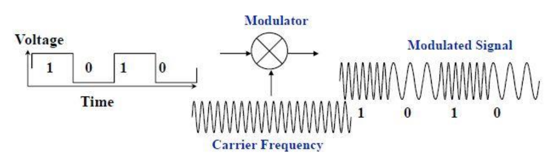

ASK spectrum is the shifted Baseband spectrum:

$$P(\omega) = \left(\frac{1}{T}\right)|V(\omega)|^2 = V^2 T\left[\frac{Sin(\omega_c T/2)}{\omega_c T/2}\right]^2$$



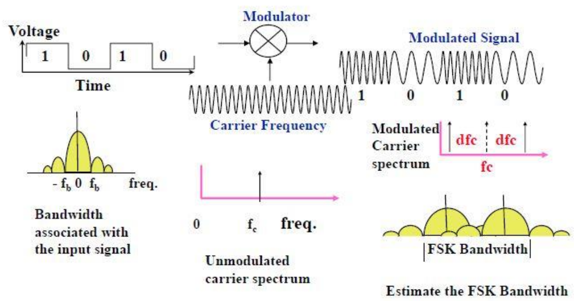



## Frequency Shift Keying: FSK




- **Frequency Shift Keying:** Frequency changes in discrete levels, in accordance with the input signal
  - Input digital signal is the information we want to transmit
  - Carrier is the radio frequency without modulation
  - Output is the modulated carrier, which has two frequencies, coresponding to the binary input signal.

## : FSK Bandwidth



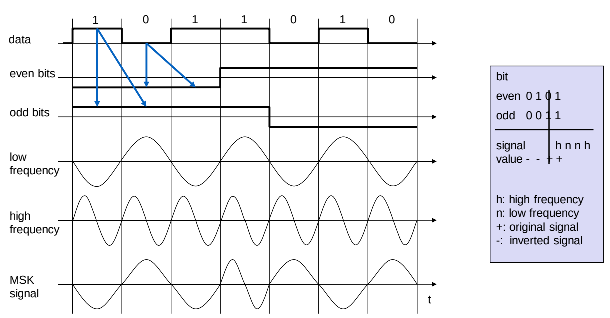



## Advanced Frequency Shift Keying

- Bandwidth needed for FSK depends on the distance between the carrier frequencies
- Special pre-computation avoids sudden phase shifts ➔ MSK (Minimum Shift Keying)
  - bit separated into even and odd bits, the duration of each bit is doubled
  - depending on the bit values (even, odd) the higher or lower frequency, original or inverted is chosen
  - the frequency of one carrier is twice the frequency of the other
  - Equivalent to offset QPSK
- Even higher bandwidth efficiency using a Gaussian low-pass filter ➔ GMSK (Gaussian MSK), used in GSM



## Example of MSK

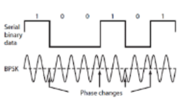


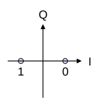

No phase shifts!



## Advanced Phase Shift Keying

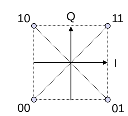
- BPSK (Binary Phase Shift Keying):
  - bit value 0: sine wave
  - bit value 1: inverted sine wave
  - very simple PSK
  - low spectral efficiency
  - robust, used e.g. in satellite systems




- QPSK (Quadrature Phase Shift Keying):
  - 2 bits coded as one symbol
  - symbol determines shift of sine wave
  - needs less bandwidth compared to BPSK
  - more complex

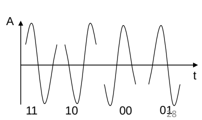
$$s(t) = \begin{cases} A\cos(2\pi f_c t), & \text{binary } 00 \\ A\cos(2\pi f_c t + \frac{\pi}{2}), & \text{binary } 01 \\ A\cos(2\pi f_c t + \pi), & \text{binary } 10 \\ A\cos(2\pi f_c t + \frac{3\pi}{2}), & \text{binary } 11 \end{cases}$$



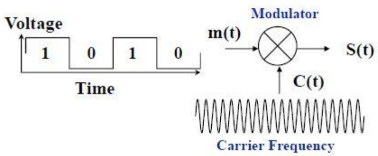

- 8-PSK
- Often also transmission of relative, not absolute phase shift
  - DQPSK - Differential QPSK (IS-136, PHS)

| Dibit | Phase |
|---|---|
| 00 | 0 |
| 01 | 90 |
| 10 | 180 |
| 11 | 270 |

Dibit (2 bits)



## Phase Shift Keying: PSK




- **Phase Shift Keying (PSK)**: Phase of the carrier frequency changes in discrete levels, in accordance with the input signal.
  - Input digital signal is the information we want to transmit
  - Carrier is the radio frequency without modulation



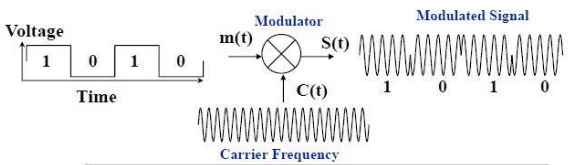
## PSK Types

- Binary Phase Shift Keying: BPSK
- 4-PSK
- 8-PSK
- 16-PSK
- :

**PSK is a solution to high-speed data communication**



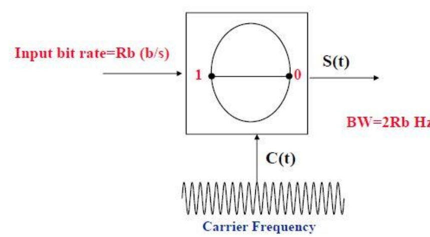
## Binary Phase Shift Keying: BPSK




- **Binary Phase Shift Keying (BPSK)**: Phase of the carrier changes in TWO discrete levels:
  - Phase - 0 deg = 0
  - Phase- 180 deg = 1

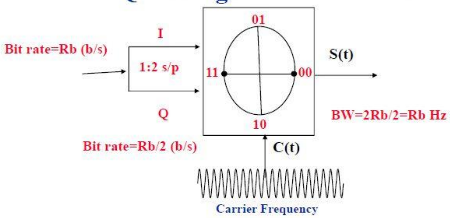


## BPSK Signal Constellation




**Phase Representation:**
- There are 2 phases
- One bit per phase

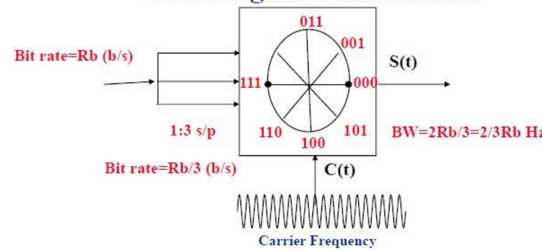
## QPSK Signal Constellation




**Phase Representation:**
- There are 4 phases
- Two bits per phase



## 8PSK Signal Constellation




**Phase Representation:**
- There are 4 phases
- Two bits per phase



## Problem

1. Give a 16 PSK Signal Constellation.
2. If Rb is the input bit rate, estimate the transmission BW.

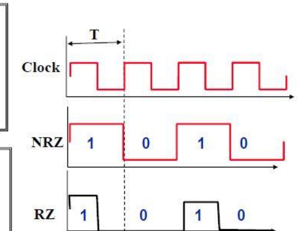


## Problem-1

**GIVEN:**
- A Master Clock
  - Period = T
  - Duty cycle = 50%
- Data-1: Formatted as NRZ
- Data-2: Formatted as RZ

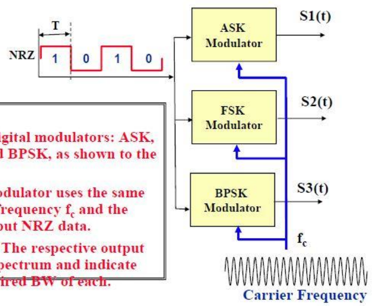
(a) Express and Sketch the power spectrum of the NRZ data.

(b) Express and Sketch the power spectrum of the RZ data.




NRZ: Non Return to Zero
RZ: Return to Zero

## Problem-2



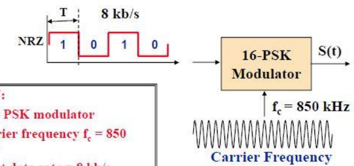

**GIVEN:**

- Three digital modulators: ASK, FSK and BPSK, as shown to the right.
- Each modulator uses the same carrier frequency $f_c$ and the same input NRZ data.

**SKETCH:** The respective output power spectrum and indicate the required BW of each.



## Problem 3




**GIVEN:**

- A 16 PSK modulator
- Carrier frequency $f_c$ = 850 kHz
- Input data rate = 8 kb/s (NRZ)

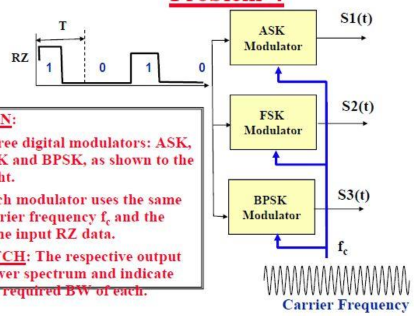
**FIND:**

(a) The bandwidth occupied by S(t)

(b) Sketch the power spectrum of S(t)



## Problem-4



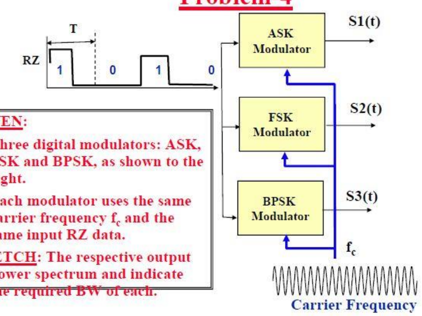

**GIVEN:**

- Three digital modulators: ASK, FSK and BPSK, as shown to the right.
- Each modulator uses the same carrier frequency $f_c$ and the same input RZ data.

**SKETCH:** The respective output power spectrum and indicate the required BW of each.



## Problem-4




**GIVEN:**

- Three digital modulators: ASK, FSK and BPSK, as shown to the right.
- Each modulator uses the same carrier frequency $f_c$ and the same input RZ data.

**SKETCH:** The respective output power spectrum and indicate the required BW of each.

## Problem-5

**GIVEN:**

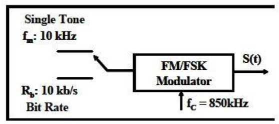
- $f_c$ = 850kHz, $f_m$ = 10kHz, $R_b$ = 10kb/s. When the input is a 10kHz tone, the modulator behaves as an FM modulator with a modulation index β = 1.5 Alternately, when the input is a 10kb/s digital signal, the modulator behaves as an FSK modulator with a frequency deviation Δf = ± 8kHz.

**FIND:**

a ) The FM Bandwidth [use Bessel function & Carson's rule given in the class]

b) The FSK Bandwidth [see class notes]

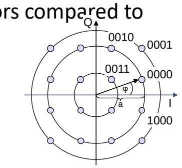
c) FSK Modulation index






## Quadrature Amplitude Modulation

- Quadrature Amplitude Modulation (QAM)
  - combines amplitude and phase modulation
  - it is possible to code n bits using one symbol
  - $2^n$ discrete levels, n=2 identical to QPSK
- Bit error rate increases with n, but less errors compared to comparable PSK schemes
  - Example: 16-QAM (4 bits = 1 symbol)
  - Symbols 0011 and 0001 have the same phase ϕ, but different amplitude a. 0000 and 1000 have different phase, but same amplitude.






# Analog Modulation

Sending Analog data over analog signal



## Frequency versus Amplitude Modulation

- Frequency Modulation (FM)
  - Most popular analog modulation technique
  - Amplitude of the carrier signal is kept constant (constant envelope signal), the frequency of carrier is changed according to the amplitude of the modulating message signal; Hence info is carried in the **phase** or **frequency** of the carrier.
  - Has better noise immunity:
    - atmospheric or impulse noise cause rapid fluctuations in the amplitude of the received signal
  - Performs better in multipath environment
    - Small-scale fading cause amplitude fluctuations as we have seen earlier.
  - Can trade bandwidth occupancy for improved noise performance.
    - Increasing the bandwith occupied increases the SNR ratio.
  - The relationship between received power and quality is non-linear.
    - Rapid increase in quality for an increase in received power.
    - Resistant to co-channel interference (capture effect).

## Frequency versus Amplitude Modulation

- Amplitude Modulation (AM)
  - Changes the amplitude of the carrier signal according to the amplitude of the message signal
  - All info is carried in the amplitude of the carrier
  - There is a linear relationship between the received signal quality and received signal power.
  - AM systems usually occupy less bandwidth then FM systems.
  - AM carrier signal has time-varying envelope.



## Amplitude Modulation

- The amplitude of high-carrier signal is varied according to the instantaneous amplitude of the modulating message signal m(t).


Carrier Signal : $A_c \cos(2\pi f_c t)$

Modulating Message Signal : $m(t)$

The AM Signal : $s_{AM}(t) = A_c[1 + m(t)]\cos(2\pi f_c t)$



## Modulation Index of AM Signal

For a sinusoidal message signal $m(t) = A_m \cos(2\pi f_m t)$

Index is defined as: $k = \frac{A_m}{A_c}$

$S_{AM}(t)$ can also be expressed as:

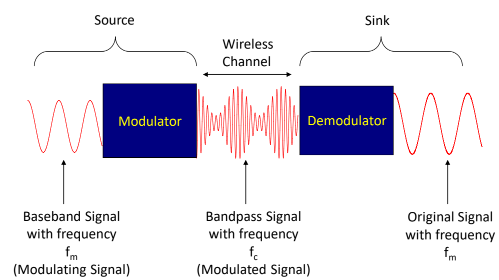
$$s_{AM}(t) = \mathrm{Re}\{g(t)e^{j2\pi f_c t}\}$$

where $g(t) = A_c(1 + m(t))$

g(t) is called the complex envelope of AM signal.

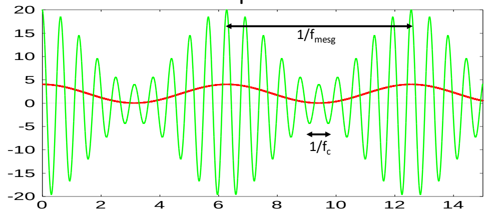


## AM Modulation/Demodulation




$f_c >> f_m$

# AM Modulation - Example




Message signal : $m(t) = 2 + 2\cos(t)$

Carrier signal : $A_c \cos(2\pi f_c t) = 4\cos(10t)$

$s_{AM}(t) = A_c[1 + m(t)]\cos(2\pi f_c t)$

AM Signal : $s_{AM}(t) = 4[1 + 2 + 2\cos(t)]\cos(2\pi f_c t)$

$$f_c = \frac{10}{2\pi} \cong 1.6Hz$$

$$f_{mesg} = \frac{1}{2\pi} = 0.16Hz$$



# AM Modulation - Example

**Example 5.1**

A zero mean sinusoidal message is applied to a transmitter that radiates an AM signal with 10 kW power. Compute the carrier power if the modulation index is 0.6. What percentage of the total power is in the carrier? Calculate the power in each sideband.

**Solution to Example 5.1**

Using equation (5.8) we have

$$P_c = \frac{P_{AM}}{1 + k^2/2} = \frac{10}{1 + 0.6^2/2} = 8.47 \text{ kW}$$

Percentage power in the carrier is

$$\frac{P_c}{P_{AM}} \times 100 = \frac{8.47}{10} \times 100 = 84.7\,\%$$

Power in each sideband is given by

$$\tfrac{1}{2}(P_{AM} - P_c) = 0.5 \times (10 - 8.47) = 0.765 \text{ kW}$$



# Angle Modulation

- Angle of the carrier is varied according to the amplitude of the modulating baseband signal.
- Two classes of angle modulation techniques:
  - Frequency Modulation
    - Instantanoues frequency of the carrier signal is varied linearly with message signal m(t)
  - Phase Modulation
    - The phase θ(t) of the carrier signal is varied linearly with the message signal m(t).



# Angle Modulation

FREQUENCY MODULATION

$$s_{FM}(t) = A_c \cos(2\pi f_c t + \theta(t)) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(x)dx\right]$$

$k_f$ is the frequency deviation constant (kHz/V)

If modulation signal is a sinusoid of amplitude $A_m$, frequency $f_m$:

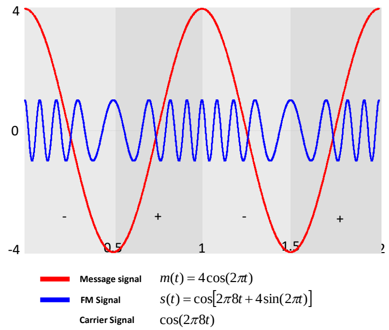
$$s_{FM}(t) = A_c \cos(2\pi f_c t + \frac{k_f A_m}{f_m}\sin(2\pi f_m t))]$$

PHASE MODULATION

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_\theta m(t)]$$

$k_\theta$ is the phase deviation constant

## FM Example:






## FM Index

$$\beta_f = \frac{k_f A_m}{W} = \frac{\Delta f}{W}$$

W: the maximum bandwidth of the modulating signal
$\Delta f$: peak frequency deviation of the transmitter.
$A_m$: peak value of the modulating signal

Example: Given $m(t) = 4\cos(2\pi 4\times10^3 t)$ as the message signal and a frequency deviation constant gain ($k_f$) of 10kHz/V; Compute the peak frequency deviation and modulation index!

Answer: $f_m$=4kHz

$\Delta f$ = 10kHz/V * 4V = 40kHz.
$\beta_f$ = 40kHz / 4kHz = 10



## Spectra and Bandwidth of FM Signals

An FM Signal has 98% of the total transmitted power in a RF bandwidth $B_T$

Carson's Rule

$$B_T = 2(\beta_f + 1) f_m \quad \text{Upper bound}$$
$$B_T = 2\Delta f \quad \text{Lower bound}$$

Example:
Analog AMPS FM system uses modulation index of $B_f$ = 3 and $f_m$ = 4kHz.
Using Carson's Rule: AMPS has 32kHz upper bound and 24kHz lower bound on required channnel bandwidth.



## FM Demodulator

- Convert from the frequency of the carrier signal to the amplitude of the message signal
- FM Detection Techniques
  - Slope Detection
  - Zero-crossing detection
  - Phase-locked discrimination
  - Quadrature detection

## Slope Detector


$$v_1(t) = V_1 \cos[2\pi f_c t + \theta(t)] = V_1 \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(x)dx\right]$$

$$v_2(t) = \frac{dv_1(t)}{dt} = -V_1\left[2\pi f_c + \frac{d\theta}{dt}\right]\sin(2\pi f_c t + \theta(t))$$

$$v_{out}(t) = V_1\left[2\pi f_c + \frac{d}{dt}\theta(t)\right]$$

$$= V_1 2\pi f_c + V_1 2\pi k_f m(t) \longleftarrow$$

Proportional to the original Message Signal



## Important Notes

- For modulation techniques, both analog and digital modulation, I only went through review. I am assuming that you have already studied them in communication engineering.
- I have skipped some details about the modulation and demodulation techniques.
- If you have any confusions refer to the suggested books at the edn of the lecture

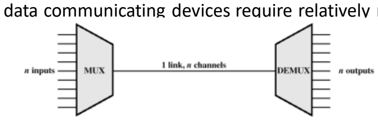


# Multiplexing



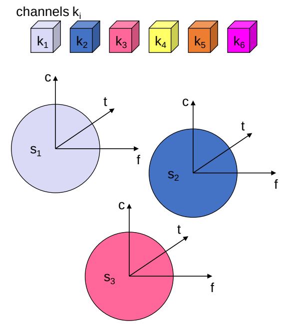
## Multiplexing

- Capacity of transmission medium usually exceeds capacity required for transmission of a single signal
- Multiplexing - carrying multiple signals on a single medium
  - More efficient use of transmission medium
- Reasons:
  - Cost per kbps of transmission facility declines with an increase in the data rate
  - Cost of transmission and receiving equipment declines with increased data rate
  - Most individual data communicating devices require relatively modest data rate support



## Multiplexing Techniques

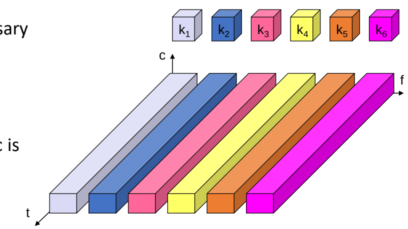
- Multiplexing in 4 dimensions
  - space ($s_i$)
  - time (t)
  - frequency (f)
  - code (c)
- Goal: multiple use of a shared medium
- Important: guard spaces needed!






## Frequency multiplex

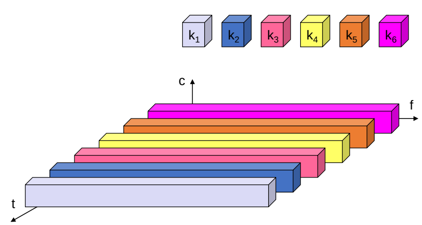
- Separation of the whole spectrum into smaller frequency bands
- A channel gets a certain band of the spectrum for the whole time
- Advantages
  - no dynamic coordination necessary
  - works also for analog signals
- Disadvantages
  - waste of bandwidth if the traffic is distributed unevenly
  - inflexible






## Time multiplex

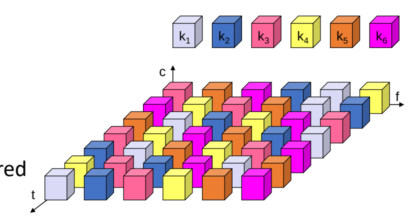
- A channel gets the whole spectrum for a certain amount of time
- Advantages
  - only one carrier in the medium at any time
  - throughput high even for many users
- Disadvantages
  - precise synchronization necessary





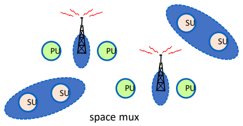

## Time and frequency multiplex

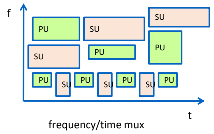
- Combination of both methods
- A channel gets a certain frequency band for a certain amount of time
- Example: GSM, Bluetooth
- Advantages
  - better protection against tapping
  - protection against frequency selective interference
- but: precise coordination required



## Cognitive Radio

- Typically in the form of a spectrum sensing CR
  - Detect unused spectrum and share with others avoiding interference
  - Choose automatically best available spectrum (intelligent form of time/frequency/space multiplexing)
- Distinguish
  - Primary Users (PU): users assigned to a specific spectrum by e.g. regulation
  - Secondary Users (SU): users with a CR to use unused spectrum
- Examples
  - Reuse of (regionally) unused analog TV spectrum (aka white space)
  - Temporary reuse of unused spectrum e.g. of pagers, amateur radio etc.

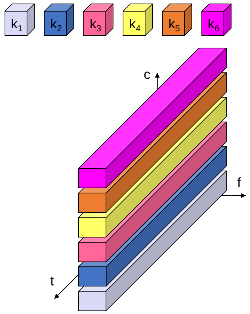


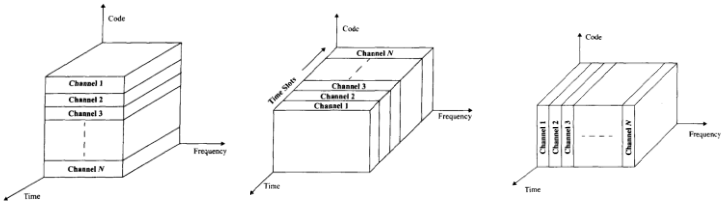



## Code multiplex

- Each channel has a unique code
- All channels use the same spectrum at the same time
- Advantages
  - bandwidth efficient
  - no coordination and synchronization necessary
  - good protection against interference and tapping
- Disadvantages
  - varying user data rates
  - more complex signal regeneration
- Implemented using spread spectrum technology






3 types of multiplexing (Review)



## Recommended Text for this lecture

- Mobile Communications by Jochen Schiller
  - Chapter 2: 2.5,2.6,
- Wireless Communications Principle and Practice by Theodore S. Rappaport
  - Chapter 5
- Data Communications and Networking by Behrouz A. Forouzan
  - Chapter 4

## Disclaimer

- This Presentation contains some edited version of slides provided by Jochen Schiller writer of the book Mobile Communications.
- This Presentation contains some edited version of slides provided by Professor Ibrahim Korpeoglu.
- There is also some screenshots from different books.
- Some online images are also used.

*I have tried to cite any source. But if any citation is missed, kindly contact me to add your citation.



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