

# Pseudorandom Permutations (aka block ciphers)

Conceptual: PRF vs PRP

PRF: like having a huge, randomly initialized key → value store (associative array)

PRP: same, but no value chosen twice  
( $\text{range}(T) = \text{set of "values"}$ )

"Example"  
cryptogram

$A \rightarrow F$   
 $B \rightarrow Q$   
 $C \rightarrow R$   
⋮

permutation  
of  $\{A, \dots, Z\}$

better cryptogram

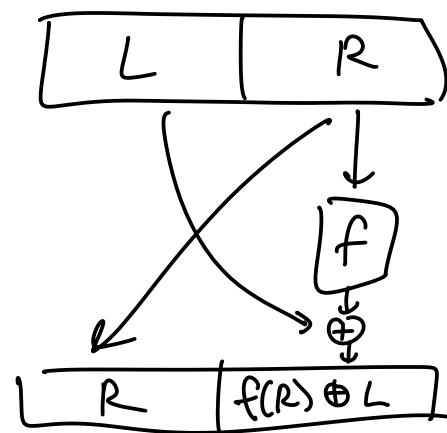
$AA \rightarrow QR$   
 $AB \rightarrow FW$   
 $AC \rightarrow AM$

permutation  
over  
 $\{AA, \dots, ZZ\}$

even better  
permutation

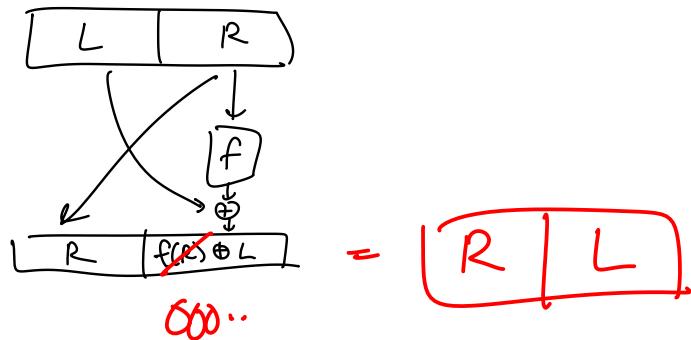
$\{0,1\}^{128} \rightarrow \{0,1\}^{128}$

Feistel ciphers:

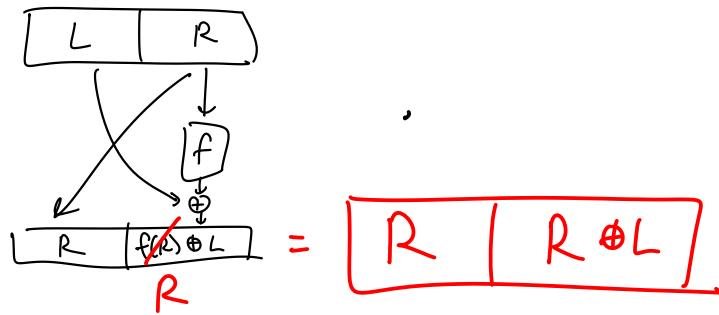


What happens when round function is "trivial"?

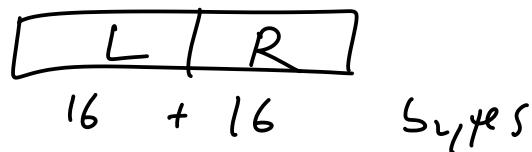
- $f(x) = 000 \dots 00$



- $f(x) = X$



Example feistel



## Attack 1 / 2-round Feistel:

1 round:  $(L, R) \mapsto (R, f(R) \oplus L)$

want to distinguish

QUERY(LR):

return  $(R, f(k, R) \oplus L)$

QUERY(LR):

random stuff

Distinguisher:

$(A, B) = \text{QUERY}(0^n 0^n)$

return  $A \stackrel{?}{=} 0^n$

In presence of left (l.b.,  
 $\Pr[\text{output 1}] = 1$

In presence of right (r.b.,  
 $\Pr[\text{output 1}] = \frac{1}{2^n}$

## 2-round Feistel:

$(L, R) \longrightarrow (R, f(R) \oplus L)$

$\longrightarrow (f(R) \oplus L, f(f(R) \oplus L) \oplus R)$

want to distinguish

QUERY(LR):

return  $(f(R) \oplus L, f(f(R) \oplus L) \oplus R)$

QUERY(LR):

random stuff

Obs:  $f(R) \oplus L$  looks weak with respect to  $L$

Idea: Call twice

$$(L, R) \rightarrow (f(R) \oplus L, f(f(R) \oplus L) \oplus R)$$

$$(L', R') \rightarrow (f(R') \oplus L', f(f(R') \oplus L') \oplus R')$$

If  $R = R'$ :

$\underbrace{\text{same } f(R) \text{ term}}$   $\Rightarrow$  XOR these 2 things  
to get  $L \oplus L'$

Calling prog:

pick arbitrary  $L, R, L'$

$(A, B) = \text{QUERY}(L, R)$

$(C, D) = \text{QUERY}(L', R)$

return  $A \oplus C \stackrel{?}{=} L \oplus L'$

In left lib:  
always outputs 1

In right lib:  
at time of 2<sup>nd</sup> query  
 $L, L', A$  fixed,  
 $C$  to be chosen  
randomly