Numerical Optimization and Modeling Techniques for Power System Operations and Planning

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Acknowledgments

Outline

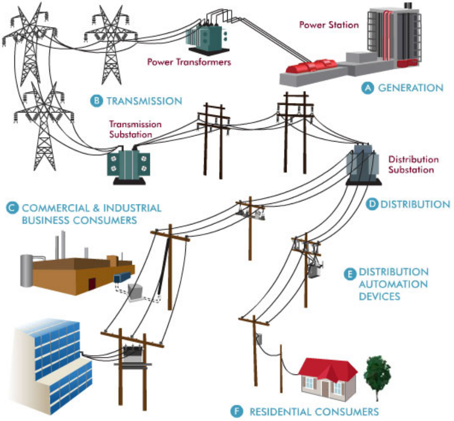
1. Introduction
2. Scenario Analysis
3. Planning and Control
4. Security Assessment
5. Conclusions

Introduction

Introduction
Electric Power Networks

Or electric power grids

- can cover large regions
- have thousands of components
- must be efficient and reliable
- have billions of dollars at stake



https://texaselectricityalliance.wordpress.com/

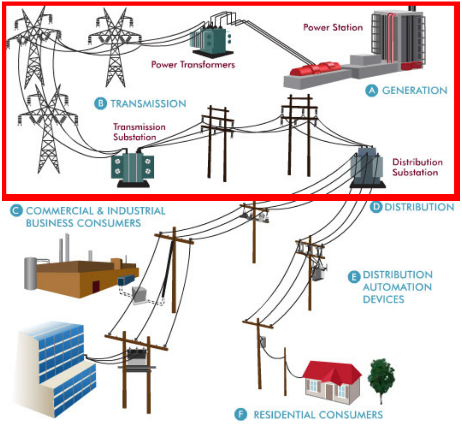
Introduction

Electric Power Networks

Or electric power grids

- can cover large regions
- have thousands of components
- must be efficient and reliable
- have billions of dollars at stake

This work focuses on transmission and generation



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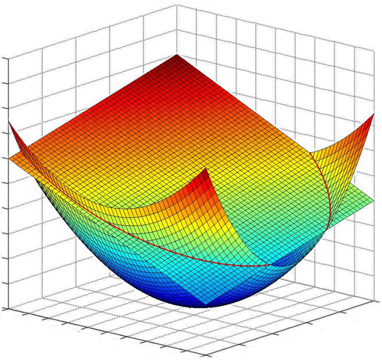
Introduction
Role of Optimization

Optimization plays a crucial role in grid operations and planning


http://www.cs.technion.ac.il/

- optimal power flow (minutes)
- line switching (hours)
- unit commitment (days)
- network expansion (years)

Introduction

Challenges

But algorithms for these problems face many difficult challenges

- non-convex constraints
- discrete variables
- time restrictions
- system stress

http://www.endare.com/

Introduction

Challenges

But algorithms for these problems face many difficult challenges

- non-convex constraints
- discrete variables
- time restrictions
- system stress

This work aims to help deal with each of these

http://www.endare.com/

Introduction

Contributions

More specifically, the contributions are for

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Contributions

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1. scenario analysis
    - obtain system information more reliably

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   - obtain system information more reliably
2. planning and control
   - obtain practical number of system adjustments
   - find better operating points

Introduction

Contributions

More specifically, the contributions are for

1. scenario analysis
   - obtain system information more reliably
2. planning and control
   - obtain practical number of system adjustments
   - find better operating points
3. online security assessment
   - perform faster and more frequent analyses
   - rely less on operator experience

Scenario Analysis

Scenario Analysis
Issues

Relies on solving power flow problems to obtain system state for different operating conditions

http://www.fairfaxcounty.gov/

Scenario Analysis

Issues

Relies on solving power flow problems to obtain system state for different operating conditions

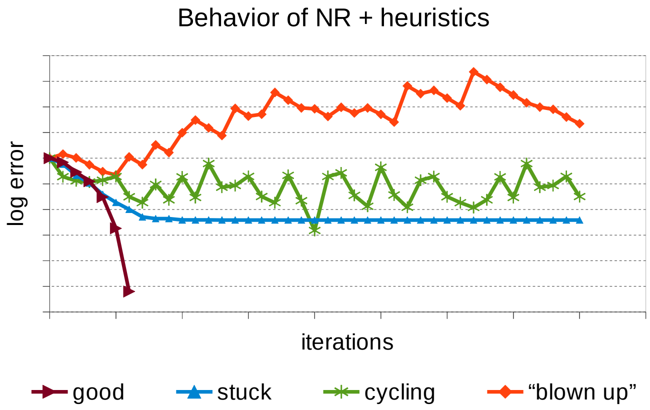
However, widely-used methods

- are based on Newton-Raphson (NR)
- rely heavily on heuristics
- need good initial solution estimate
- cannot handle stressed systems

http://www.fairfaxcounty.gov/

Scenario Analysis

Issues



Scenario Analysis
Power Flow Problem

Find system state (voltages) and unknown generator powers

$$
\begin{array}{ll}
\text{find} & x \\
\text{subject to} & f(x) = 0 \\
& Ax = b \\
& 0 \le Q(x) \perp V(x) \ge 0
\end{array}
$$

Scenario Analysis
Power Flow Problem

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just feasibility

Scenario Analysis

Power Flow Problem

Find system state (voltages) and unknown generator powers

find	x	just feasibility
subject to	$f(x) = 0$	power balance (nonlinear)
	$Ax = b$	
	$0 \leq Q(x) \perp V(x) \geq 0$	

Scenario Analysis

Power Flow Problem

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 & \left.\begin{array}{l} Ax = b \\ 0 \leq Q(x) \perp V(x) \geq 0 \end{array}\right\} & \text{voltage regulation}
\end{array}
$$

Scenario Analysis
Power Flow Problem

Find system state (voltages) and unknown generator powers

find	x	just feasibility
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	$Ax = b$	voltage regulation
	$0 \le Q(x) \perp V(x) \ge 0$	

NR uses "switching heuristics" to handle voltage regulation

Scenario Analysis

New Formulation

Proposed new problem formulation consists of

$$
\begin{aligned}
\text{minimize} \quad & \varphi(x) \\
\text{subject to} \quad & f(x) = 0 \\
& Ax = b \\
& \Phi(x) = 0
\end{aligned}
$$

Scenario Analysis
New Formulation

Proposed new problem formulation consists of

$$
\begin{array}{lll}
\text{minimize} & \varphi(x) & \text{strongly convex} \\
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& \Phi(x) = 0 &
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Scenario Analysis
New Formulation

Proposed new problem formulation consists of

minimize	$\varphi(x)$	strongly convex
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Scenario Analysis

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Proposed new problem formulation consists of

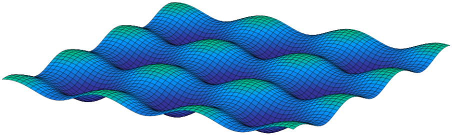
$$\begin{array}{lll} \text{minimize} & \varphi(x) & \text{strongly convex} \\ \text{subject to} & f(x) = 0 & \text{power balance (nonlinear)} \\ & \left.\begin{array}{l} Ax = b \\ \Phi(x) = 0 \end{array}\right\} & \text{voltage regulation (smooth equalities)} \end{array}$$

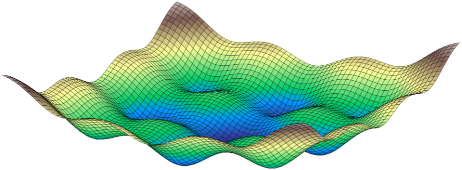
Has properties that help solve the problem

Scenario Analysis

New Formulation

Objective function encourages typical/desirable state properties





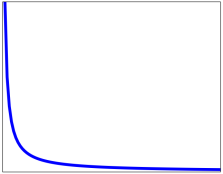
- helps obtain robustness to poor initial points
- helps handle stressed systems ($\frac{df}{dx}$ is near rank-deficient)

Scenario Analysis

New Formulation

Complementarity constraints are replaced with smooth equalities

$$0 \leq a \perp b \geq 0 \qquad\qquad \Phi(a, b) = 0$$



- have better theoretical properties (constraint qualifications)
- suitable for having "elastic" bounds
- helps keep algorithm simple

Scenario Analysis

Solution Approach

Based on Augmented Lagrangian, solves sequence of subproblems

$$\begin{array}{ll} \text{minimize} & \mu\big(\varphi(x) - \lambda^T c(x)\big) + \frac{1}{2}\|c(x)\|_2^2 \quad \left(c = \begin{bmatrix} f \\ \Phi \end{bmatrix}\right) \\ \text{subject to} & Ax = b \end{array}$$

Scenario Analysis
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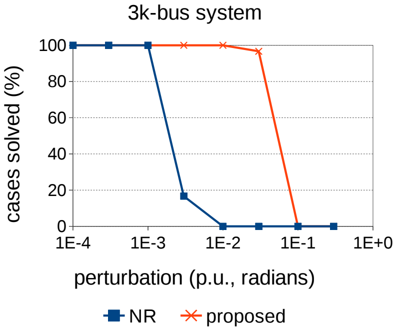
Key features

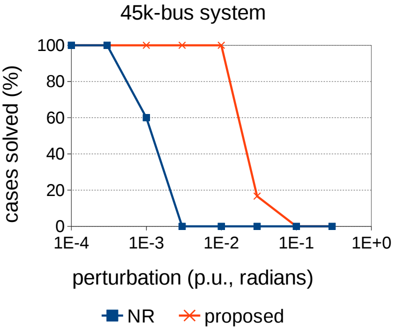
- uses second derivatives to improve convergence
- avoids forming $J^T J$ (J is Jacobian of c)
- allows getting "best" infeasible point easily ("elastic")

Scenario Analysis

Benefits

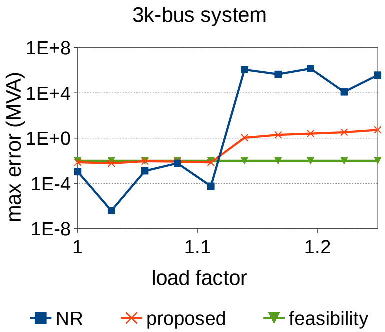
Robustness to poor initial points

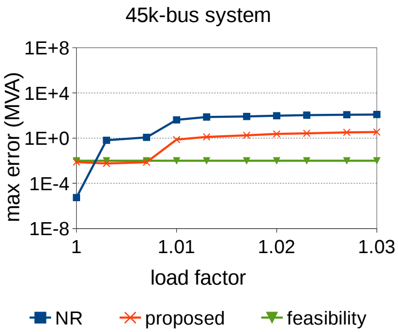




Scenario Analysis
Benefits

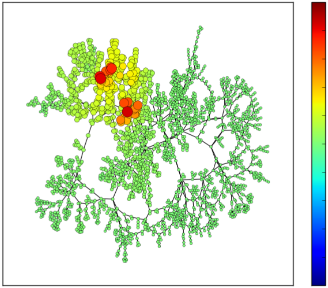
Handling of hard and infeasible cases





Scenario Analysis
Benefits

Lagrange multiplier estimates provide sensitivity information



Help determine how to improve operation (as measured by φ)

Planning and Control

Planning and Control

Issues

Relies on solving optimal power flow problems to determine system adjustments

http://www.itg.ge/

Planning and Control

Issues

Relies on solving optimal power flow problems to determine system adjustments

However, current methods

- use techniques for handling discrete controls that result in poor-quality solutions
- lack capability of utilizing a practical number of control actions

http://www.itg.ge/

Planning and Control
Optimal Power Flow Problem

Determine "best" system adjustments (generation, controls) to meet power demand

$$
\begin{aligned}
\text{minimize} \quad & \varphi(x) \\
\text{subject to} \quad & f(x) = 0 \\
& l \leq x \leq u \\
& x \in \mathcal{D}
\end{aligned}
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Planning and Control
Optimal Power Flow Problem

Determine "best" system adjustments (generation, controls) to meet power demand

$$
\begin{aligned}
\text{minimize} \quad & \varphi(x) \qquad \text{generation cost, voltage soft limits, etc} \\
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Planning and Control

Optimal Power Flow Problem

Determine “best” system adjustments (generation, controls) to meet power demand

minimize	$\varphi(x)$	generation cost, voltage soft limits, etc
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Optimal Power Flow Problem

Determine "best" system adjustments (generation, controls) to meet power demand

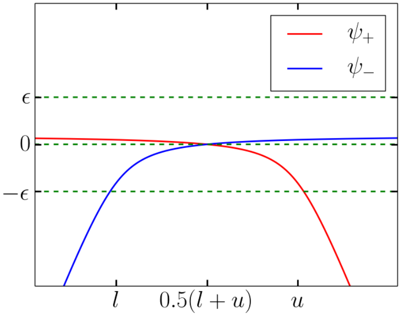
minimize	$\varphi(x)$	generation cost, voltage soft limits, etc
subject to	$f(x) = 0$	power balance (nonlinear)
	$l \leq x \leq u$	hard limits (generators, phase shifters, etc)
	$x \in \mathcal{D}$	control discreteness (taps, shunts)

Planning and Control

Bound Constraints

Replace bounds $l \leq x \leq u$ with equality constraints

$$\psi_+(x) = 0 \quad \text{and} \quad \psi_-(x) = 0$$



Properties

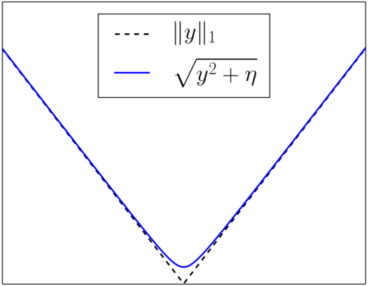
- constraints are satisfied with feasibility tolerance ϵ iff $x \in [l-\delta, u+\delta]$

Planning and Control

Practical Number of Adjustments

Obtained with smooth sparsity-inducing penalty

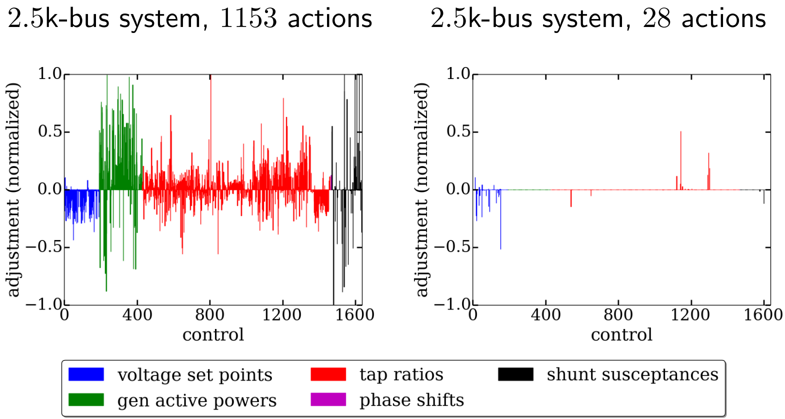
$$P_s(y) = \sum_i \sqrt{y_i^2 + \eta}$$



Properties

- approximates $\|\cdot\|_1$
- strongly convex (for bounded y)

Planning and Control
Practical Number of Adjustments



Planning and Control

Exploration of Discrete-Space

Augmented Lagrangian algorithm that alternates between

- partially solving continuous relaxation (guides search)
- evaluating discrete variable choices in parallel

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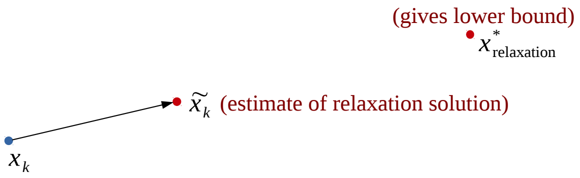

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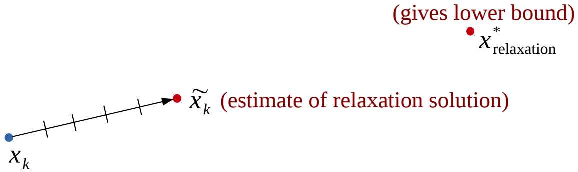

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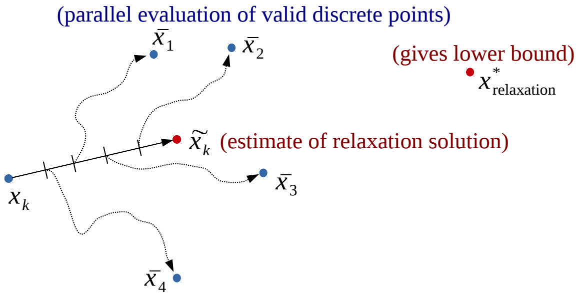

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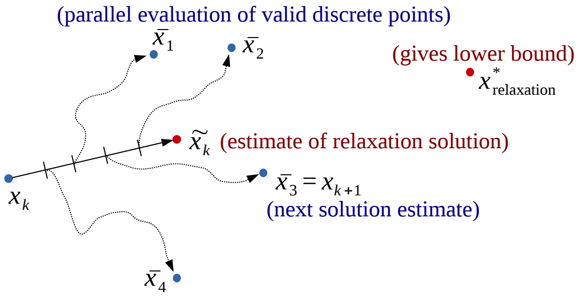

Planning and Control

Exploration of Discrete-Space

Compared time and objective value against those obtained with

- Continuous Relaxation (CR) algorithm (no discrete controls)
- Round and Re-solve (RR) algorithm (current practice)

Twelve 2.5k-bus cases, 8 parallel processes

result	CR	RR	proposed
cases with lowest objective	58%	0%	42%
cases with obj. lower than proposed	58%	8%	—
times slower than RR	0.5	1.0	2.4

Security Assessment

Security Assessment

Issues

Current practices require expensive system simulations and rely on operator experience

http://www.riskmanagementmonitor.com/

Security Assessment

Issues

Current practices require expensive system simulations and rely on operator experience

However, systems are becoming more stressed and unpredictable

- require faster and more frequent security analyses
- require considering more potentially critical quantities

http://www.riskmanagementmonitor.com/

Security Assessment
Critical Operating Boundaries Problem

Consists of three parts

1. determine “nearest” reachable boundaries (thermal, voltage)
2. determine system adjustments to improve security
3. visualize information obtained

Security Assessment
Modeling Approach

Represent system and operating boundaries with

$$Ax = b + Yy$$
$$Cx \geq d$$
$$y^{\min} \leq y \leq y^{\max}$$

Security Assessment
Modeling Approach

Represent system and operating boundaries with

$$Ax = b + Yy \qquad \text{power balance equations}$$
$$Cx \geq d$$
$$y^{\min} \leq y \leq y^{\max}$$

Security Assessment
Modeling Approach

Represent system and operating boundaries with

$Ax = b + Yy$	power balance equations
$Cx \geq d$	branch thermal and bus voltage limits
$y^{\min} \leq y \leq y^{\max}$	

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Modeling Approach

Represent system and operating boundaries with

$Ax = b + Yy$	power balance equations
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$y^{\min} \leq y \leq y^{\max}$	generator and load power bounds

Security Assessment

Modeling Approach

Represent system and operating boundaries with

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Can express in terms of generator and load powers only

$$Qy \geq t, \quad y^{\min} \leq y \leq y^{\max}$$

measure distances in terms of y changes (generation and load)

Security Assessment
Modeling Approach

Represent system and operating boundaries with

$Ax = b + Yy$	power balance equations
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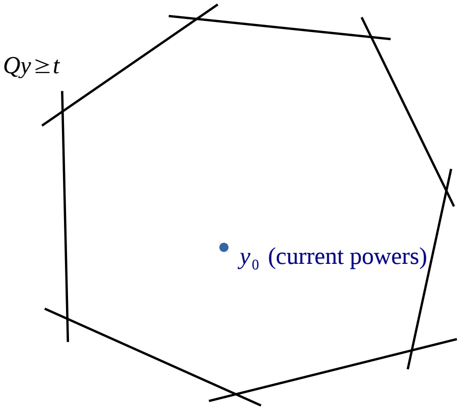
$y^{\min} \leq y \leq y^{\max}$	generator and load power bounds

Can express in terms of generator and load powers only

$$Qy \geq t, \quad y^{\min} \leq y \leq y^{\max}$$ (but Q is dense!)

measure distances in terms of y changes (generation and load)

Security Assessment
Modeling Approach



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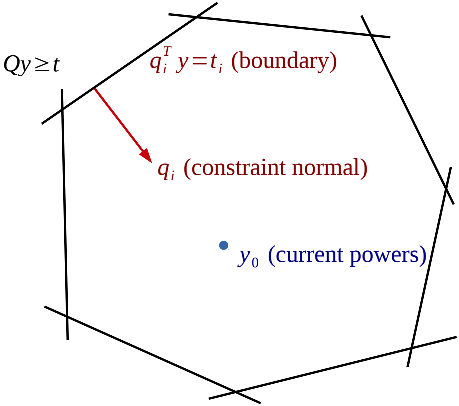
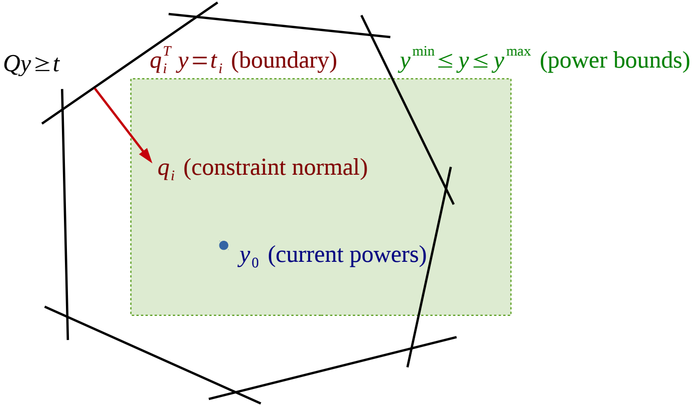


Security Assessment

Modeling Approach



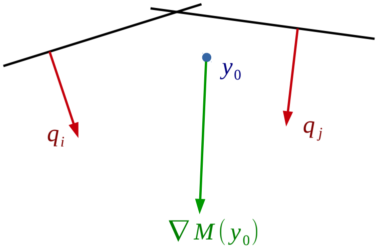
Security Assessment
Modeling Approach

Measure system security with

$$M(y) = \sum_i \log(q_i^T y - t_i)$$

$\nabla M(y)$ gives direction for improving security

- may not be practical
- can use sparse approximation



Security Assessment

Nearest Reachable Boundaries

Identify small set $\mathcal{N}$ of nearest boundaries (say top 200)

- requires comparing boundary distances
- requires knowing $\|q\|_2$ of each row q of Q
- estimate $\|q\|_2$ by multiplying random vectors with Q

Security Assessment
Nearest Reachable Boundaries

Identify small set $\mathcal{N}$ of nearest boundaries (say top 200)

- requires comparing boundary distances
- requires knowing $\|q\|_2$ of each row q of Q
- estimate $\|q\|_2$ by multiplying random vectors with Q

Keep ones that can be reached within power bounds

- requires constructing rows of Q and solving trivial LPs
- practical since $\mathcal{N}$ is small

Security Assessment
Nearest Reachable Boundaries

Identify 500 nearest boundaries

2.5k-bus system, 10k boundaries

algorithm	time (s)	% top 500
full-Q	13.92	100
proposed	0.08	95.6

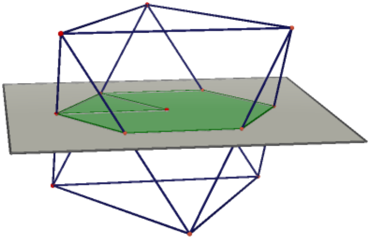
45k-bus system, 200k boundaries

algorithm	time (s)	% top 500
full-Q	4836.34	100
proposed	1.28	95.1

Security Assessment
Visualization

Strategy

- determine suitable plane
- reconstruct boundaries on plane



http://gauss.math.nthu.edu.tw/

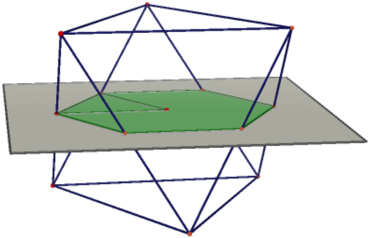
Security Assessment
Visualization

Strategy

- determine suitable plane
- reconstruct boundaries on plane

Choice of plane

- one that best preserves critical boundary distances
- requires finding top 2 singular vectors of a matrix



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Security Assessment
Visualization

More specifically

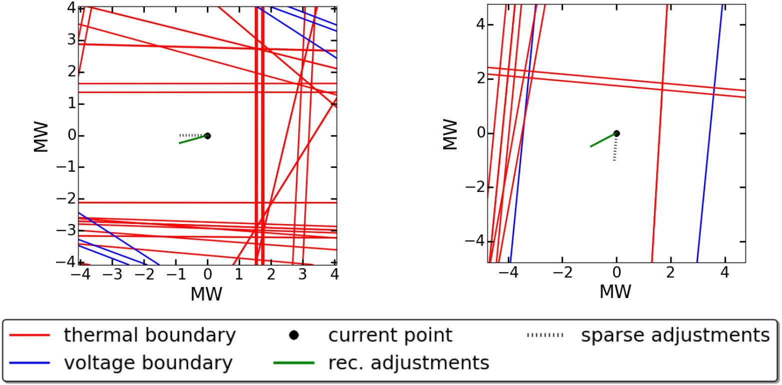
proposed plane is spanned by columns of matrix that solves

$$\begin{aligned} \underset{U \in \mathbb{R}^{m\times 2}}{\text{maximize}} \quad & \sum_{i \in \bar{\mathcal{N}}} \beta_i^2 \|U^T q_i\|_2^2 \\ & U^T U = I \end{aligned}$$

$$\left(\begin{array}{l} \bar{\mathcal{N}} \text{ is set of nearest reachable constraints} \\ \beta_i \text{ are weights inversely proportional to boundary distance} \end{array} \right)$$

Security Assessment
Visualization

For two 2.5k-bus systems



Conclusions

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In this work, we have addressed issues in

1. scenario analysis
2. planning and control
3. security assessment

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Contributions

- robust and informative power flow method tested on real and large networks

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Contributions

- practical number of control actions
- distributed approach for handling discrete variables and getting higher-quality solutions

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Contributions

- fast constraint filtering techniques
- visualization strategies for enhancing security awareness

Questions

Thank you for your attention

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