Sihui Qi
CS 323 Midterm 2

Problem 1
1.
X:4/25/2020 Y:3067
X:5/19/2020 Y:1484
X:5/12/2020 Y:1175
X:6/8/2020 Y:485
X:7/15/2020 Y:313
X:8/7/2020 Y:384
X:9/5/2020 Y:387
X:9/24/2020 Y:618
X:10/16/2020 Y:1219
X:11/17/2020 Y:4492
X:12/22/2020 Y:5107
X:1/27/2021 Y:5001

2.
A:

10x10 double

	1	2	3	4	5	6	7	8	9
1	4	1	0	0	0	0	0	0	0
2	1	4	1	0	0	0	0	0	0
3	0	1	4	1	0	0	0	0	0
4	0	0	1	4	1	0	0	0	0
5	0	0	0	1	4	1	0	0	0
6	0	0	0	0	1	4	1	0	0
7	0	0	0	0	0	1	4	1	0
8	0	0	0	0	0	0	1	4	1
9	0	0	0	0	0	0	0	1	4
10	0	0	0	0	0	0	0	0	1

B:

10x1 double

	1
1	7644
2	-2286
3	3108
4	1458
5	-408
6	1368
7	2220
8	16032
9	-15948
10	-4326

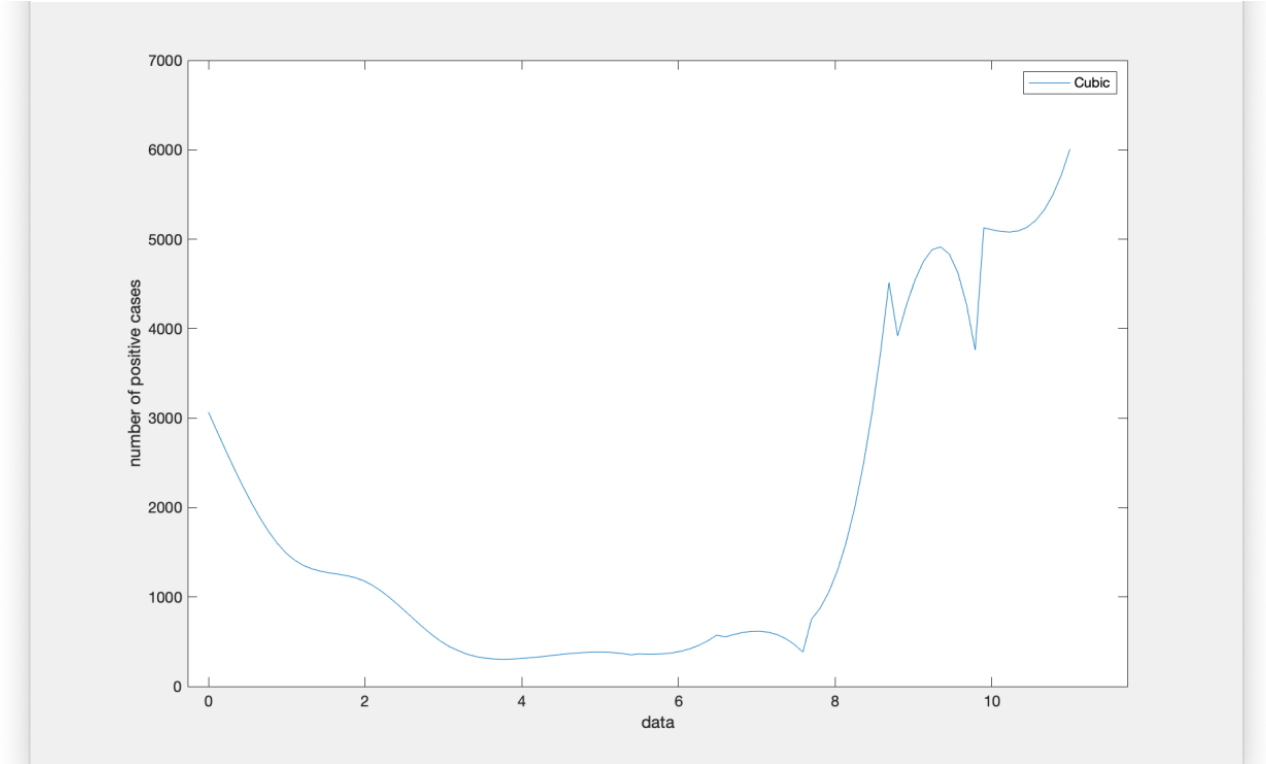
Z:

12x1 double

	1	2
1	0	
2	2.2632e+03	
3	-1.4089e+03	
4	1.0864e+03	
5	171.3493	
6	-313.7858	
7	675.7939	
8	-1.0214e+03	
9	5.6298e+03	
10	-5.4657e+03	
11	284.9177	
12	0	

S0(x)=377.2*x^3 - 1960.0*x + 3067.0
S1(x)=303.0*x - 234.8*(x - 1.0)^3 - 377.2*(x - 2.0)^3 + 803.8
S2(x)=181.1*(x - 2.0)^3 - 1106.0*x + 234.8*(x - 3.0)^3 + 3622.0
S3(x)=28.56*(x - 3.0)^3 - 181.1*(x - 4.0)^3 - 19.49*x + 362.4
S4(x)=151.9*x - 52.3*(x - 4.0)^3 - 28.56*(x - 5.0)^3 - 323.0
S5(x)=112.6*(x - 5.0)^3 - 161.9*x + 52.3*(x - 6.0)^3 + 1246.0
S6(x)=513.9*x - 170.2*(x - 6.0)^3 - 112.6*(x - 7.0)^3 - 2809.0
S7(x)=170.2*(x - 8.0)^3 - 507.5*x + 938.3*(x - 7.0)^3 + 4341.0
S8(x)=5122.0*x - 910.9*(x - 8.0)^3 - 938.3*(x - 9.0)^3 - 40700.0
S9(x)=47.49*(x - 9.0)^3 - 343.4*x + 910.9*(x - 10.0)^3 + 8494.0
S10(x)=5645.0 - 47.49*(x - 11.0)^3 - 58.51*x

Plot:



3.

h=(b-a)/n

=(11-0)/(12-1)

=1

$$\int_{x_0}^{x_{11}} P(x)dx$$

=$$\int_a^b f(x)dx \approx= \frac{h}{2}\left[f_0 + f_n + 2\sum_{i=1}^{n-1} f_i\right]$$

=(1/2)*(3067+5001+2*(1484+1175+485+313+384+387+618+1219+4492+5107))

=19698

Name: Sihui Qi

Problem 2: 1. From the information of question.

$$u' + \lambda u = 10\cos(t/10)$$
$$u' = 10\cos(t/10) - \lambda u \quad | \quad u_0 = 100$$
$$u_{exact} = 10\cos(t/10) - \lambda u + 1$$

Because $h = 0.01$ and $u_{k+1} = u_k + h f(t_k, u_k)$

$$\begin{cases} t_0 = 0 : u_0 = 100 \quad h = 0.01 \\ t_1 = h = 0.01 \quad u_1 = u_0 + h f(t_0, u_0) = 100 + 0.01(10 - 100) = 99.1 \\ t_2 = 2h = 0.02 \quad u_2 = u_1 + h f(t_1, u_1) = 99.1 + 0.01(10\cos(\frac{0.01}{10}) - 99.1) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 98.209 \\ t_3 = 3h = 0.03 \quad u_3 = u_2 + h f(t_2, u_2) = 98.209 + 0.01(10\cos(\frac{0.02}{10}) - 98.209) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 97.3269 \end{cases}$$

Therefore $\begin{cases} t_1 = 0.01 \\ t_2 = 0.02 \\ t_3 = 0.03 \end{cases}$ and $\begin{cases} u_0 = 100 \\ u_1 = 99.1 \\ u_2 = 98.209 \\ u_3 = 97.3269 \end{cases}$

Because $Error = \left| \frac{Numerical - Exact}{Exact} \right|$

by using the exact solution:

$u(0) = 100$

$u(0.01) = 99.1045$

$u(0.02) = 98.2179$

$u(0.03) = 97.3400$

$Error_{u(0)} = \left| \frac{100 - 100}{100} \right| = 0$ $\quad Error_{u(0.01)} = \left| \frac{99.1 - 99.1045}{99.1045} \right| = 0.00004$

$Error_{u(0.02)} = \left| \frac{98.209 - 98.2179}{98.2179} \right| = 0.00009$

$Error_{u(0.03)} = \left| \frac{97.3269 - 97.34}{97.3400} \right| = 0.00013$

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2.

step 1: $t_1 = h = 0.01$:

$$\begin{cases} K_1 = 0.01 \cdot (10\cos(\frac{0}{10}) - 100) = -0.9 \\ K_2 = 0.01 \cdot (10 \cdot \cos(\frac{0.5 \cdot 0.01}{10}) - (\frac{1}{2} \times (-0.9 + 100)) = -0.8955 \\ K_3 = 0.01 \cdot (10\cos(\frac{0.5 \cdot 0.01}{10}) - (\frac{1}{2} \times (-0.8955 + 100)) = -0.8955 \\ K_4 = 0.01 (10\cos(\frac{0.01}{10}) - (100 - 0.8955) = -0.8910 \end{cases}$$

$u_1 = 100 + \frac{1}{6}(-0.9 - 2(0.8955) - 2(0.8955) - 0.8910)$

$u_1 = 99.1045$.

step 2: $t_2 = 2h = 0.02$:

$$\begin{cases} K_1 = 0.01 \cdot (10\cos(\frac{0.01}{10}) - 99.1045) = -0.8910 \\ K_2 = 0.01 \cdot (10\cos(\frac{0.01 + 0.5 \cdot 0.01}{10}) - (99.1045 + \frac{1}{2}(-0.8911)) \\ \quad = -0.8866 \\ K_3 = 0.01 \cdot (10\cos(\frac{0.01 + 0.5 \cdot 0.01}{10}) - (99.1045 + \frac{1}{2} \cdot \\ \quad = -0.8866 \ (-0.8866)) \\ K_4 = 0.01 \cdot (10\cos \\ \quad (\frac{0.01 + 0.01}{10}) - (99.1045 - 0.8866) \\ \quad = -0.8822 \end{cases}$$

$u_2 = 99.1045 + \frac{1}{6}(-0.8911 - 2(0.8866) - 2(0.8866) - 0.8822)$

$= 98.2179$.

step 3: $t_3 = 3h = 0.03$

$$\begin{cases} K_1 = -0.8821 \\ K_2 = -0.8778 \\ K_3 = -0.8778 \\ K_4 = -0.8734 \end{cases}$$

$u_3 = 98.2179 + \frac{1}{6}(-0.8821 - 2 \times (0.8778) - 2 \times (0.8778) - 0.8734)$

$= 97.3401$

$$Error_{u_1} = \left| \frac{99.1045 - 98.1045}{99.1045} \right| = 0$$

$$Error_{u_2} = \left| \frac{98.2179 - 98.2179}{98.2179} \right| = 0.$$

$$Error_{u_3} = \left| \frac{97.3401 - 97.3401}{97.3401} \right| = 0$$

3. From the Implicit Euler Backward rule.

$(t_1 = h = 0.01)$ $u_{k+1} = u_k + h f(t_k, u_k)$.

$$u_1 = 100 + 0.01\left(-u_1 + 10\cos\left(\frac{0.01}{10}\right)\right)$$

$$u_1 = \frac{10010}{101}$$

$$u_1 \approx 99.1089.$$

$(t_2 = h = 0.02)$ $u_2 = 99.1089 + 0.01\left(-u_2 + 10\cos\left(\frac{0.02}{10}\right)\right)$

$$u_2 = 98.2266$$

$(t_3 = h = 0.03)$ $u_3 = 98.2266 + 0.01\left(-u_3 + 10\cos\left(\frac{0.03}{10}\right)\right)$

$$u_3 = 97.3531$$

$Error_{u_0} = 0$

$$Error_{u_1} = \left| \frac{99.1089 - 99.1045}{99.1045} \right| = 0.00004$$

$$Error_{u_2} = \left| \frac{98.2266 - 98.2179}{98.2179} \right| = 0.00008$$

$$Error_{u_3} = \left| \frac{97.3531 - 97.3401}{97.3401} \right| = 0.00013$$