CS 323 Homework 2

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For all theoretical assignments, you can turn in as either handwriting or typing in. For all programming assignments, please turn in your code along with a solution document. You can discuss questions with others, but the solutions/codes you submit must be entirely your own work, and please mention who you collaborated with in your homework.

Software: MATLAB / C++ /Python/Java (or any language that you are familiar)

Problem 1: Suppose we want to solve

$$\underbrace{\begin{bmatrix} 21 & 32 & 14 & 8 & 6 & 9 & 11 & 3 & 5 \\ 17 & 2 & 8 & 14 & 55 & 23 & 19 & 1 & 6 \\ 41 & 23 & 13 & 5 & 11 & 22 & 26 & 7 & 9 \\ 12 & 11 & 5 & 8 & 3 & 15 & 7 & 25 & 19 \\ 14 & 7 & 3 & 5 & 11 & 23 & 8 & 7 & 9 \\ 2 & 8 & 5 & 7 & 1 & 13 & 23 & 11 & 17 \\ 11 & 7 & 9 & 5 & 3 & 8 & 26 & 13 & 17 \\ 23 & 1 & 5 & 19 & 11 & 7 & 9 & 4 & 16 \\ 31 & 5 & 12 & 7 & 13 & 17 & 24 & 3 & 11 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}}_{\mathbf{X}} = \underbrace{\begin{bmatrix} 2 \\ 5 \\ 7 \\ 1 \\ 6 \\ 9 \\ 4 \\ 8 \\ 3 \end{bmatrix}}_{\mathbf{B}} \tag{1}$$

1. Use 'inv' functions in MATLAB or the like in the programming languages you are using for this homework to get the inverse of $\mathbf{A}$ matrix, and the result $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$. Round $\mathbf{X}$ to 4 decimal places.

2. Program LU decomposition, backward-substitution, and forward substitution, to get the result $\mathbf{X}$. Round $\mathbf{X}$ to 4 decimal places.

3. Considering the $\mathbf{X}$ from Q1 as the accurate results, use L-2 norm to evaluate the relative error $\frac{\|\mathbf{X}-\mathbf{X}_{LU}\|}{\|\mathbf{X}\|}$,where $\mathbf{X}_{LU}$ is the result from Q2. Round the relative error to 4 decimal places.

Problem 2: Suppose we want to find the roots of

$$f(x) = \exp(\cos(x) + \cos(x^2)) + \cos(x) - 1 = 0 \tag{2}$$

where $x \in [0, 2]$.

Set tolerance $\epsilon = 10^{-6}$,

1. Find out the theoretical iterative step for convergence. Write down the specific derivations. (You can directly use/duplicate the derivation process in the Lecture 5-09 slides.)

2. Program the Bisection iterative method to solve this equation; output the initial guess, iterative steps, results at each iterative step, and the converged result. Round the results at each iterative step and the converged result to 4 decimal places.

3. Error plot. Plot X axis: iterative step number k, Y axis: the absolute value of $f(x_k)$, where k represents iterative step. Round $|f(x_k)|$ to 4 decimal places.

Submission should include (a) a solution document (PDF/ WORD/handwritten) showing all results and (b) original codes. Students can find the example codes on Canvas, and can use the example code accordingly. In the solution document, students should include:

1. **Problem 1**
   (a) Q1: $\mathbf{X}$ (Points: 5)
   (b) Q2: Show the specific $\mathbf{L}$ matrix, $\mathbf{U}$ matrix, and the resulting $\mathbf{X}$ (Points: 30)
   Note: For Q2, using functions such as 'lu()', 'inv', matrix division, in MATLAB and the like, will get few credits.
   (c) Q3: show the relative error (Points: 5)

2. **Problem 2**
   (a) Q1: write down the specific derivations to predict the minimal iterative number of a converged result, and the result. (Points: 15)
   (b) Q2: show the initial guess, results x_k at each iterative step, where k represents iterative step, and the converged result. (Points: 30)
   (c) Q3: plot X axis: k (iterative step number), Y axis: $|f(x_k)|$ (the absolute value of residual). (Points: 15)
   Note: To plot, any potential approach is acceptable.

3. **Appendix**
   All codes should also be presented as appendix in the solution document.