Exam Solutions

Problem 1

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problem1:

1. The Newton's method: $g(x) = x - \frac{f(x)}{f'(x)}$

From the question1, the initial guess $x_0 = -1$

$f(x) = x + e^x = 0$

$f'(x) = \frac{d}{dx}(x + e^x)$

$f'(x) = 1 + e^x$

$f(x_0) = -1 + e^{-1} = -1 + \frac{1}{e} = -0.632$

$f'(x_0) = 1 + e^{-1} = 1 + \frac{1}{e} = 1.368$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1 - \frac{-0.632}{1.368} = -1 + 0.462 = -0.538$

$\|error_1\| = -0.538 + e^{-0.538} = 0.046$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -0.538 - \frac{0.046}{1.584} = -0.538 - 0.02904 \approx -0.567$

$\|error_2\| = -0.567 + e^{-0.567} = 0$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -0.567 - \frac{0}{1.767} = -0.567$

$\|error_3\| = -0.567 + e^{-0.567} = 0$

after the first iteration, the second and third iteration have almost the same answer of -0.567, which means step 3 is really close to the real root, thus it converges.

2. The secant's method: $x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} f(x_k)$

From the question2, the initial guess $x_0 = -1$, $x_1 = -1.1$

$f(x_0) = -0.632$

$f(x_1) = -1.1 + \frac{1}{e} = -0.767$

$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1) = -1.1 - \frac{-1.1 - (-1)}{(-0.767) - (-0.632)} \cdot (-0.767)$

$= -1.1 + \frac{0.1}{0.135} \cdot (0.767)$

≈ -0.532

$\|error_2\| = -0.532 + e^{-0.532} = 0.055$

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} \cdot f(x_2) = -0.532 - \frac{-0.532 - (-1.1)}{0.055 - (-0.767)} \cdot 0.055$$

$$= -0.532 - \frac{-0.532 + 1.1}{0.055 + 0.767} \cdot 0.055$$

$$\|error_3\| = -0.57 + e^{-0.57} = 0.004$$

$$= -0.532 - 0.055 \cdot \frac{0.568}{0.822}$$

$$= -0.570$$

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3) = -0.570 - \frac{-0.570 + 0.532}{-0.004 + 0.055} \cdot 0.004$$

$$\|error_4\| = -0.567 + e^{-0.567} = 0$$

$$= -0.57 - \left(-0.004 \cdot \frac{0.038}{0.051}\right)$$

$$= -0.57 + \frac{0.038}{0.051} \cdot 0.004$$

$$= -0.567$$

$$x_5 = x_4 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_4) = -0.567 - \frac{-0.567 + 0.570}{0.0002 + 0.004} \cdot 0.0002$$

$$= -0.567 - \frac{-0.567 + 0.57}{0.0002 + 0.004} \cdot 0.0002$$

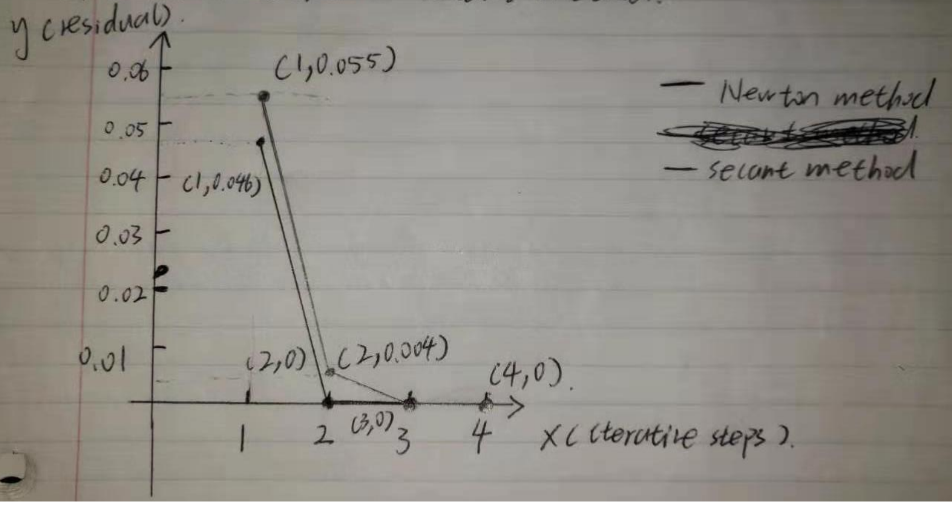
$$= -0.567 - 0.0002 \cdot \frac{0.003}{0.0042}$$

$$\|error_5\| = -0.567 + e^{-0.567} = 0$$

$$= -0.567$$

after the second iteration, the third and fourth iteration almost have the same answer of -0.567, which means the step 4 really close to the value of real roots, thus it converges.

3. Newton's method & secant method:



Problem 2

1. For Jacobi method, it took 38 steps, the L1 norm is 49.9643, the L2 norm is 1.5801, the L∞ norm is 0.500.

A	*1000x1000 double*
B	*1000x1 double*
i	1000
k	38
Nmax	1000
norm_1	49.9634
norm_2	1.5801
norm_inf	0.0500
residual	*1x38 double*
tol	1.0000e-10
X	*1000x1 double*

For Gauss Seidel method, it took 24 steps, the L1 norm is 49.9643, the L2 norm is 1.5801, the L∞ norm is 0.500.

A	*1000x1000 double*
B	*1000x1 double*
i	1000
k	24
Nmax	1000
norm_1	49.9634
norm_2	1.5801
norm_inf	0.0500
residual	*1x24 double*
tol	1.0000e-10
w	1
X	*1000x1 double*

For SOR method, it took 19 steps, the L1 norm is 49.9643, the L2 norm is 1.5801, the L∞ norm is 0.500.

A	*1000x1000 double*
B	*1000x1 double*
i	1000
k	19
Nmax	1000
norm_1	49.9634
norm_2	1.5801
norm_inf	0.0500
residual	*1x19 double*
tol	1.0000e-10
w	1.1000
X	*1000x1 double*

2. The X-axis are the steps, The Y-axis are the residuals corresponding to each step.