CS 323 Midterm Exam 2-2021

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1. No collaborations.
2. Late submissions will only be accepted one day after the due date. No further late submissions will be accepted. **All late submissions will be taken 30 points over the total**.
3. Submission includes a single **PDF** file and a package of all **CODES**. You will get litter or no credit if there is only a 'code' package, or if the answer is presented in the code files.
4. For theoretical problems, you are required to solve it by 'hand'; in other words, the problem is easy and is solvable; and you may need MATLAB and the like as a calculator, but you would definitely not need to program the problem. Show ALL your work. You will get little or No credit for an answer that is not explained. You can take pictures of your handwritten and attached to the solution Pdf file.
5. For Programming problems, follow the instructions carefully, **do not use system function** to simplify coding unless the specific functions are mentioned in the problem descriptions.

Software: MATLAB / C++ / Python/ Java (or any language that you are familiar)

Due date: Rutgers time: April 23 2021 11:59 pm

Problem 1:

Based on the source link:
https://coronavirus.jhu.edu/data/new-cases-50-states/new-jersey
(The data is taken from the above link and is only used for the purpose of teaching.)

1. Extract 12 sampling points from Mar 2020 to the present. Type in these data in your Solution PDF. (Hints: the data would be x : date vs. y : number of positive cases. You can choose these data by hand, no coding is required.) (20 points)
2. Use Cubic Spline to perform interpolation functions and plot the results. Round all numbers/coefficients in cubic spline functions to 4 significant digits and show these functions and the plot in your solution PDF. Do not show intermediate terms, for example, fractions are required to be further evaluated to decimal numbers. (You can use the lecture code directly. Note that, if you use 'spline' function in Matlab or the like, there would be no credits for this question.) (20 points)
3. Use Trapezoid rule to find the following integral:

$$\int_{x_0}^{x_{11}} P(x)dx$$

where x_0 and x_11 denote the first and the last data points, respectively. Show details in your solution PDF. (Hints: We have got 12 points (x, y) from Question 1, so there is no need of $P(x)$), (20 points)

Problem 2: Theoretical assignment

We first consider the following single degree of freedom (SDOF) problem:

$$\begin{aligned} \dot{u} + \lambda u &= 10\cos(t/10) \\ u_0 &= 100 \end{aligned} \tag{1}$$

where the exact solution is given by:

$$u(t) = \left(u_0 - \frac{1000\lambda}{1+100\lambda^2}\right)\exp^{-\lambda t} + \frac{100[10\lambda\cos(t/10) + \sin(t/10)]}{1+100\lambda^2} \tag{2}$$

and the numerical error is defined as:

$$Error = \left|\frac{Numerical - Exact}{Exact}\right| \tag{3}$$

Define the time step length: $h = 0.01$, and

1. Use Explicit Euler Forward scheme to get u at t_1, t_2 and t_3 and the associated errors (20 points)
2. Use RK 4 to solve this problem to get u at t_1, t_2 and t_3 and the associated errors (20 points)
3. Use Implicit Euler Backward to to get u at t_1, t_2 and t_3 and the associated errors (20 points)