CS 323 Midterm Exam 1-2021

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1. No collaborations.
2. Late submissions will only be accepted one day after the due date. No further late submissions will be accepted. **All late submissions will be taken 30 points over the total**.
3. Submission includes a **PDF** file and a package of all **CODES**. You will get litter or no credit if there is only a 'code' package, or if the answer is presented in the code files.
4. For theoretical problems, you would need to solve it by 'hand'; in other words, the problem is easy and is solvable; and you may need MATLAB and the like as a calculator, but you would definitely not need to program the problem. Show ALL your work. You will get little or No credit for an answer that is not explained. You can take pictures of your handwritten and attached to the solution Pdf file.
5. For Programming problems, follow the instructions carefully, **do not use system function** to simplify coding unless the specific functions are mentioned in the problem descriptions.

Software: MATLAB / C++ / Python/ Java (or any language that you are familiar)

Due date: Rutgers time: March 1 2021 11:59 pm

Problem 1: Theoretical assignment

1. Use Newton's method to solve $x + e^x = 0$ with an accuracy of 3 decimal places. Please show the specific results at each iterative steps until it converges.(Initial guess $x_0 = -1$).
2. Use Secant method to solve $x + e^x = 0$ with an accuracy of 3 decimal places. Please show the specific results at each iterative steps until it converges. (Initial guess $x_0 = -1$, $x_1 = -1.1$).

3. Plot absolute value of residual V.S. iterative steps.

Note:

1. This is a theoretical assignment, you need to solve it by hand. You can use computer to evaluate exponent function if necessary, but you do not need to program everything.

2. Rough definition of converged solution: the results with 3 decimal places doesn't change, and both methods can get converged results within 10 steps in this case.

3. The solution of Problem 1 should include:

   (a) Question 1: write down each iterative step of Newton's method, show the specific evaluation to get x_k. Show x_k and the associated residual at each step. (Points: 20)

   (b) Question 2: write down each iterative step of Secant method, show the specific evaluation to get x_k. Show x_k and the associated residual at each step. (Points: 20)

   (c) Question 3: only output one figure including both secant and Newton's methods. x axis: iterative steps; y axis: absolute value of residual at each step (You have got these values from Questions 1 and 2.) (Points: 5)

Problem 2: Programming assignment Define $\mathbf{A}_{ii} = 40$ and $\mathbf{A}_{i+1,i} = \mathbf{A}_{i-1,i} = -10$ and the other entries in $\mathbf{A}$ are 0. i and j are the slot label at rows and columns, respectively. For example, if $\mathbf{A}$ is a 5×5 matrix, then

$$\mathbf{A} = \begin{bmatrix} 40, & -10, & 0, & 0, & 0 \\ -10, & 40, & -10, & 0, & 0 \\ 0, & -10, & 40, & -10, & 0 \\ 0, & 0, & -10, & 40, & -10 \\ 0, & 0, & 0, & -10, & 40 \end{bmatrix} \tag{1}$$

Following the description, solve $\mathbf{AX} = \mathbf{B}$, where the sizes of $\mathbf{A}$ and $\mathbf{B}$ are 1000×1000 and 1000×1, respectively. All the entries in $\mathbf{B}$ are 1.

1. Use Jacobi, Gauss-Seidel, and SOR method to solve $\mathbf{AX} = \mathbf{B}$. To demonstrate the results $\mathbf{X}$, please do not show the specific $\mathbf{X}$, you only need to show $L1$, L_∞, and $L2$ norms of $\mathbf{X}$.

2. Plot norm-2 of residual vector V.S. iterative steps

Note:

1. You only need to solve the system with $\mathbf{A}_{1000\times1000}$, the $\mathbf{A}_{5\times5}$ is only for demonstration.

2. Since SOR recovers Gauss-Seidel method, you are required to directly program SOR method, and set $\omega = 1$ for Gauss-Seidel simulation, and use $\omega = 1.1$ for SOR simulation.

3. Only use norm of residual $\leq$ Tolerance ($\epsilon = 10^{-10}$) as the stop criteria;

4. The solution of Problem 2 should (only) include:

   (a) Question 1: L_1, L_∞, and L_2 norms of $\mathbf{X}$ solved by Jacobi, Gauss-Seidel, and SOR methods. In total, 9 numbers. (Program $\mathbf{A}_{1000\times1000}$–Points: 12, all norms–Points: 18)

   (b) Question 2: You are required to merge 3 lines into one single plot. Each line represents the residual of one method. (Points: 20)