STAT 7934: HW 2

Due in my office on Fr Feb 27 by 5 pm

Group Project:

Consider a classification problem, where the response is binary $y \in \{0, 1\}$ and the predictor variables numerical. .Generate a design matrix X of size 5,000 by 300 (i.e. 5,000 observations and 300 variables). Generate the corresponding response y by selecting an appropriate coefficient vector β and adding a noise term.
Hint: Think what the appropriate criterion (likelihood) function is for this problem.

Consider a model where the true coefficent vector has only 20 non-zero elements and another with 75 non-zero elemensts. Devise a sub-gradient based algorithm (soft-thresholding) to solve the problem, where the Gram matrix $X'X$ is well-conditioned.

Compare the performance of your algorithm for a grid of the tuning parameter λ.

Individual Question:

Consider the problem

$$\min \sum_{i=1}^{m} f_i(x)$$

subject to $x \in P_i, i = 1, \cdots, m$, where $f_i : \mathbb{R}^n \to \mathbb{R}$ and P_i are bounded polyhedral subsets of $\mathbb{R}^n$ with *nonempty* intersection.

Formulate the dual problem.
Do the primal and dual problems have optimal solutions? Is there a duality gap?