Homework 4

1. Let F_i denote the Exponential distribution function with paramter λ_i. Also suppose that X_j, which is the time until the deterioration occurs, has an Exponential distribution with parameter $\lambda_i, i = 1, 2$ conditional on being in the interval $A_j = (c_j, d_j)$ for $j = 1, ..., N$ where N is the number of patients. We are interested in running the following model for comparing two treatments:

$$
\begin{aligned}
U_j &\sim Unif(F_i(c), F_i(d)), \\
X_j &= -\frac{1}{\lambda_i}\log(1 - U_j), \\
\lambda_i &\sim Gamma(a, b).
\end{aligned}
$$

Based on our assumption, λ_1 and λ_2 have prior distribution $Gamma(a, b)$. Using the fact that the first quartile is 10 months and the third quartile is 49 months and that $P(X > t) = b/(b + t)^a$ we solve for the following system of equations to find the parameters a and b:

$$b/(b + 10)^a = 0.75 \tag{1}$$

$$b/(b + 49)^a = 0.25 \tag{2}$$

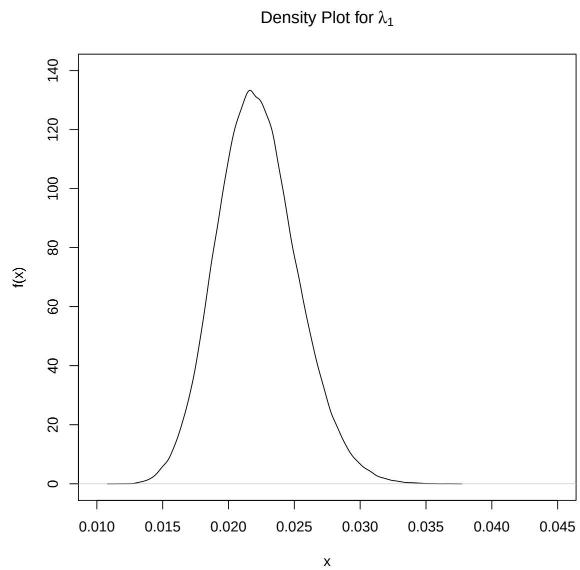
Using the package *nleqslv* and initial values of $a = 2$ and $b = 0.5$ and a tolerance of 0.0001 we get the solutions $a = 33.02833$ and $b = 1143.09157$.

2. We run a gibbs sampler to estimate the distribution of λ_1 and λ_2. The density plots are included in Figures 1a and 1b. We also estimate $P(\lambda_1 > \lambda_2) = P(1/\lambda_1 < 1/\lambda_2)$ and we do this using

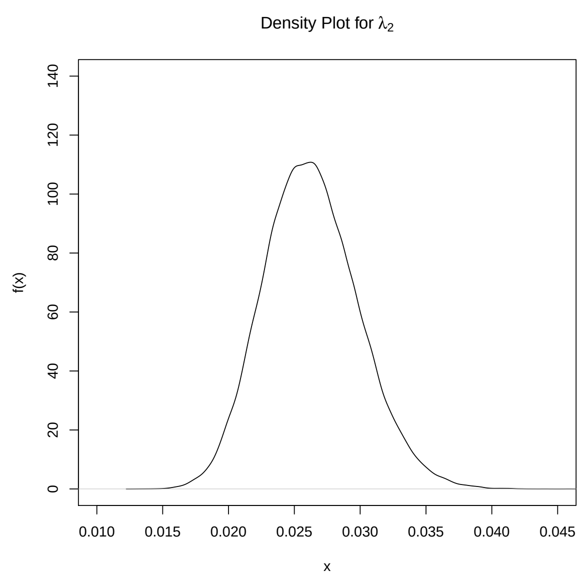
$$\frac{1}{n}\sum_{i=1}^{n} I[\hat{\lambda}_1^i > \hat{\lambda}_2^i] = 0.19092$$

where $\hat{\lambda}_j^i$ denotes the ith observation in our sample for λ_j. This gives evidence that $E[X_2] = 1/\lambda_2$ is most likely greater than $E[X_1]$.

3. The posterior densities of $exp(-36\lambda_j)$ for $j = 1, 2$ calculated by multiplying the $\hat{\lambda}_j^i$'s by -36 and taking exponentials. The densities, estimated using the *density* function in *R*, are presented in Figures 2a and 2b.

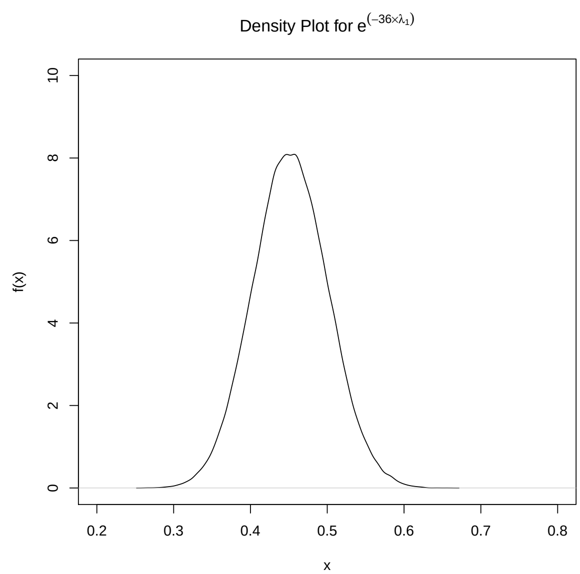


(a)

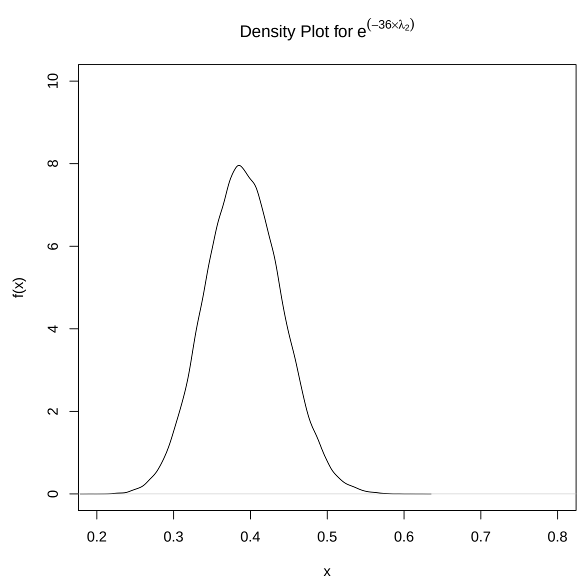


(b)

Figure 1: Density plot for λ_1 and λ_2



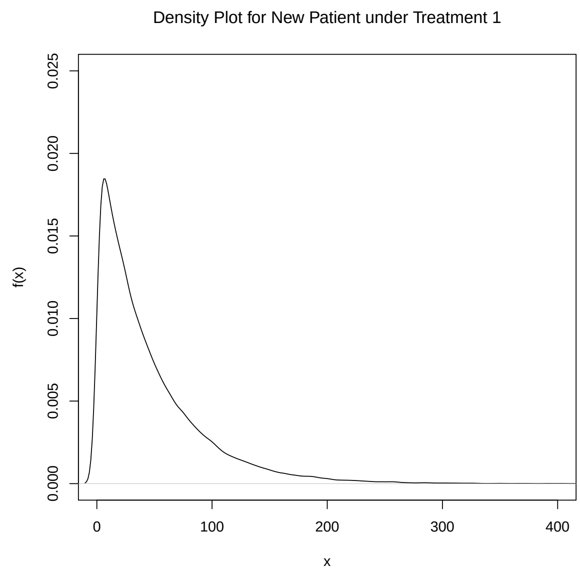
(a)



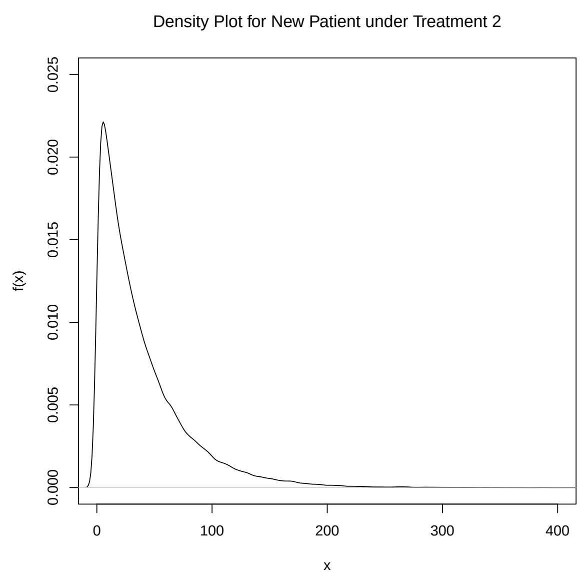
(b)

Figure 2: Density plot for $exp(-36\lambda_1)$ and $exp(-36\lambda_2)$

4. To find the posterior density of a new patient under treatment j, we add create a new vector with the interval equal to $(0, \infty)$ in the code for the gibbs sampler. The densities for a new patient under each treatment are presented in Figures 3a and 3b for treatments 1 and 2 respectively, which was estimated using the *density* function in R.



(a)



(b)

Figure 3: Density plot for the time to cosmetic deterioration for each treatment

Appendix 1: R code and output

```
model <- function(x) {
    F1 = (x[2]/(x[2]+10))^x[1] - 0.75
    F2 = (x[2]/(x[2]+49))^x[1] - 0.25
    c(F1 = F1, F2 = F2)
}
xstart <- c(2,0.5)
fstart <- model(xstart)
nleqslv(xstart, model, control=list(btol=.0001))

$x
[1]   33.02833 1143.09157

$fvec
[1]  2.361440e-09 -1.034709e-09

$termcd
[1] 1

$message
[1] "Function criterion near zero"

$scalex
[1] 1 1

$nfcnt
[1] 90

$njcnt
[1] 2

$iter
[1] 66

radio = read.table("breast-cancer-data-radiotherapy.txt",
    header=TRUE,na.string = "---")
attach(radio)
```

```
U[is.na(U)]=Inf
lambda1 = 1
sum = sum(L)
a = 33.02833
b = 1143.09157
N = 46
nsim = 100000
lambda1 = rep(0,nsim)
u = rep(0,N)
x = rep(0,N)
unew = rep(0,1)
xnew1 = rep(0,nsim)

for (j in 1:nsim)
    {
        lambda1[j] = rgamma(n=1,a+N,(b+sum))
        lambdatemp= lambda1[j]
        low = pexp(L,rate = lambdatemp)
        up = pexp(U,rate = lambdatemp)
        u = runif(n=46,low,up)
        x = (-1/lambdatemp)*log(1-u)
        unew = runif(n=1,0,1)
        xnew1[j] = (-1/lambdatemp)*log(1-unew)
        sum = sum(x)
    }

radiochem = read.table("breast-cancer-data-radioandchemo.txt",
    header=TRUE,na.string="---")
attach(radiochem)
U[is.na(U)]=Inf
lambdatemp = 1
sum = sum(L)
a = 33.02833
b = 1143.09157
N = 46
nsim = 100000
lambda2 = rep(0,nsim)
u = rep(0,N)
```

```
x = rep(0,N)
unew = rep(0,1)
xnew2 = rep(0,nsim)
for (j in 1:nsim)
    {
        lambda2[j] = rgamma(n=1,a+N,(b+sum))
        lambdatemp= lambda2[j]
        low = pexp(L,rate = lambdatemp)
        up = pexp(U,rate = lambdatemp)
        u = runif(n=46,low,up)
        x = (-1/lambdatemp)*log(1-u)
        unew = runif(n=1,0,1)
        xnew2[j] = (-1/lambdatemp)*log(1-unew)
        sum = sum(x)
    }

success = sum(lambda1>lambda2)
total = nsim
success/total
pdf("lambda1.pdf")
plot(density(lambda1),xlab = "x",ylab = "f(x)",
     main = expression("Density Plot for" ~ lambda[1]),
     ylim=c(0,140),xlim=c(0.01,0.045))
dev.off()
pdf("lambda2.pdf")
plot(density(lambda2),xlab = "x",ylab = "f(x)",
     main = expression("Density Plot for" ~lambda[2]),
     ylim=c(0,140),xlim=c(0.01,0.045))
dev.off()
pdf("explambda1.pdf")
plot(density(exp(-lambda1*36)),xlab = "x",ylab = "f(x)",
     main = expression("Density Plot for" ~ e^(-36%*%lambda[1])),
     ylim=c(0,10),xlim=c(0.2,0.8))
dev.off()
pdf("explambda2.pdf")
plot(density(exp(-lambda2*36)),xlab = "x",ylab = "f(x)",
     main = expression("Density Plot for" ~e^(-36%*%lambda[2])),
     ylim=c(0,10),xlim=c(0.2,0.8))
```

```
dev.off()
pdf("mlambda1.pdf")
plot(density(xnew1),xlab = "x",ylab = "f(x)",
     main = expression("Density Plot for New Patient under Treatment 1"),
     ylim=c(0,0.025),xlim=c(0,400))
dev.off()
pdf("mlambda2.pdf")
plot(density(xnew2),xlab = "x",ylab = "f(x)",
     main = expression("Density Plot for New Patient under Treatment 2"),
     ylim=c(0,0.025),xlim=c(0,400))
dev.off()
```