

## Homework 4

**1.** We assume that  $\lambda_1$  and  $\lambda_2$  have prior distribution  $gamma(a, b)$ . Using the fact that the first quartile is 10 months and the third quartile is 49 months and that  $P(X > t) = b/(b+t)^a$  we solve for the following system of equations to find the parameters  $a$  and  $b$ :

$$\begin{aligned} (1) \quad & b/(b+10)^a = 0.75 \\ (2) \quad & b/(b+49)^a = 0.25 \end{aligned}$$

Using the package *nleqslv* and initial values of  $a = 2$  and  $b = 0.5$  and a tolerance of 0.0001 we get the solutions  $a = 33.02833$  and  $b = 1143.09157$ .

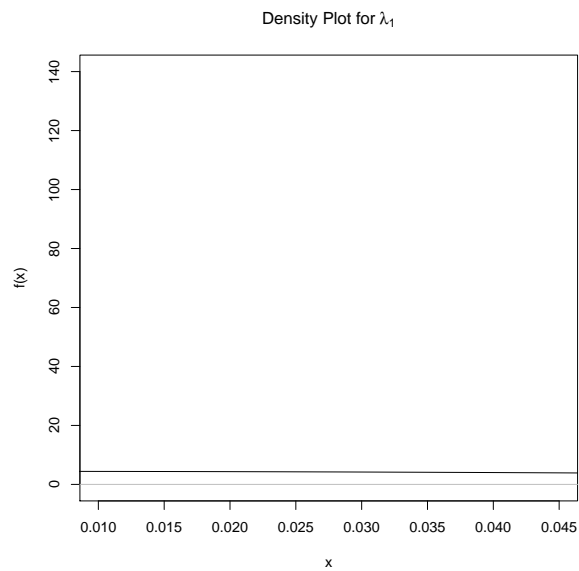
**2.** We run a gibbs sampler to estimate the distribution of  $\lambda_1$  and  $\lambda_2$ . The density plots are included in Figures 1a and 1b. We also estimate  $P(\lambda_1 > \lambda_2) = P(1/\lambda_1 < 1/\lambda_2)$  and we do this using

$$\frac{1}{n} \sum_{i=1}^n I[\hat{\lambda}_1^i > \hat{\lambda}_2^i] = 0.19092$$

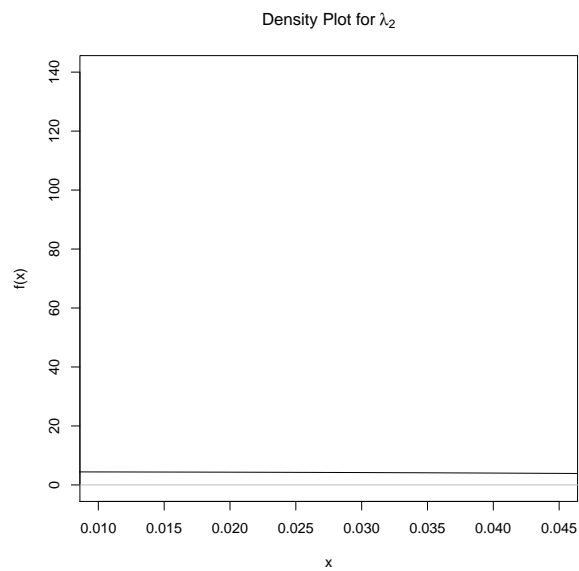
This gives evidence that  $E[X_2] = 1/\lambda_2$  is most likely greater than  $E[X_1]$ .

**3.** The posterior densities of  $exp(-36\lambda_j)$  for  $j = 1, 2$  calculated by multiplying the  $\hat{\lambda}_j$ 's by  $-36$  and taking exponentials. The densities are presented in Figures 2a and 2b.

**4.** To find the posterior density of a new patient under treatment  $j$ , we add create a new vector with the interval equal to  $(0, \infty)$  in the code for the gibbs sampler. The densities for a new patient under each treatment are presented in Figures 3a and 3b for treatments 1 and 2 respectively.

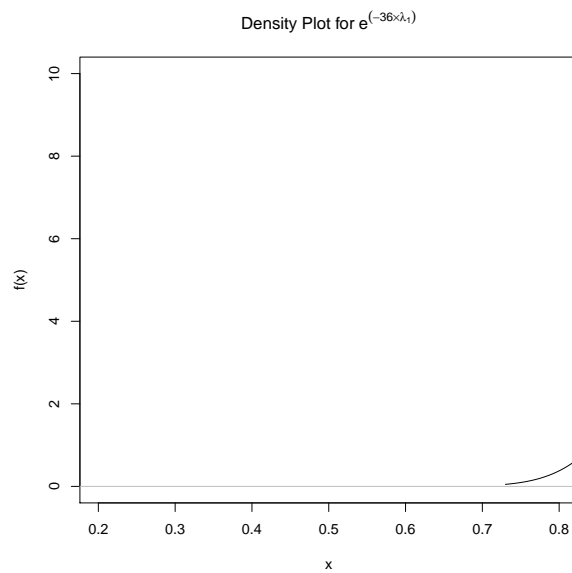


(a)

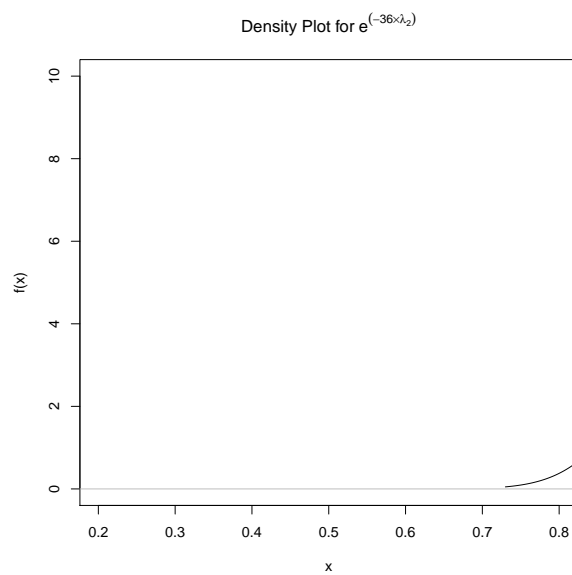


(b)

Figure 1: Density plot for  $\lambda_1$  and  $\lambda_2$

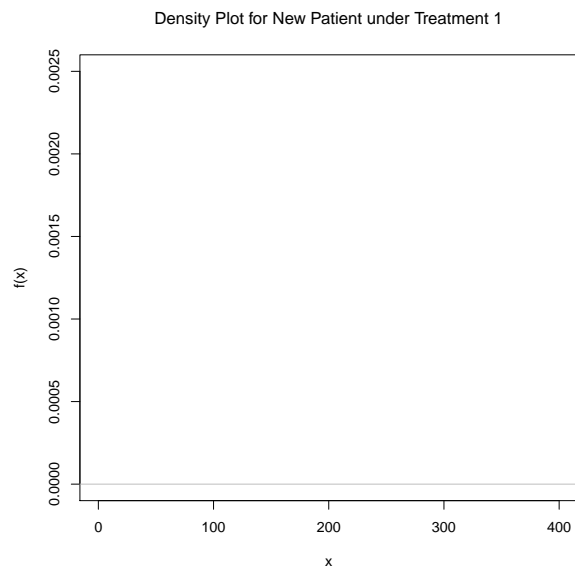


(a)

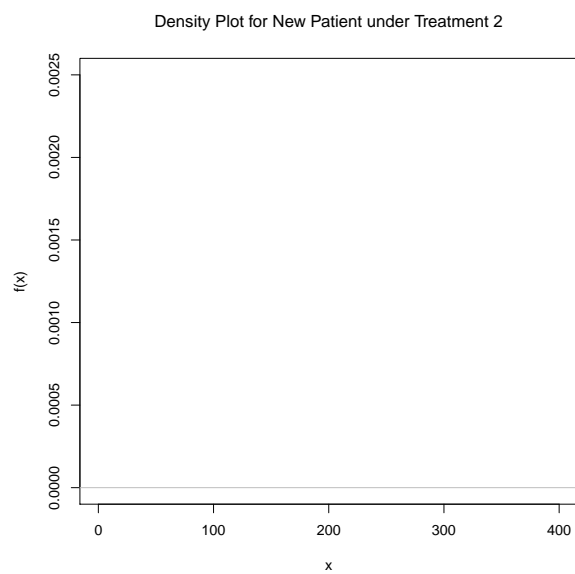


(b)

Figure 2: Density plot for  $\exp(-36\lambda_1)$  and  $\exp(-36\lambda_2)$



(a)



(b)

Figure 3: Density plot for the time to cosmetic deterioration for each treatment

**Appendix 1: R code and output**

```
model <- function(x) {  
  F1 = (x[2]/(x[2]+10))^x[1] - 0.75  
  F2 = (x[2]/(x[2]+49))^x[1] - 0.25  
  c(F1 = F1, F2 = F2)  
}  
xstart <- c(2,0.5)  
fstart <- model(xstart)  
nleqslv(xstart, model, control=list(btol=.0001))  
  
$x  
[1] 33.02833 1143.09157  
  
$fvec  
[1] 2.361440e-09 -1.034709e-09  
  
$termcd  
[1] 1  
  
$message  
[1] "Function criterion near zero"  
  
$scalex  
[1] 1 1  
  
$nfcnt  
[1] 90  
  
$njcnt  
[1] 2  
  
$iter  
[1] 66  
  
radio = read.table("breast-cancer-data-radiotherapy.txt",  
header=TRUE,na.string = "---")  
attach(radio)
```

```
U[is.na(U)]=Inf
lambda1 = 1
sum = sum(L)
a = 33.02833
b = 1143.09157
N = 46
nsim = 100000
lambda1 = rep(0,nsim)
u = rep(0,N)
x = rep(0,N)
unew = rep(0,1)
xnew1 = rep(0,nsim)
for (j in 1:nsim)
{
  lambda1[j] = rgamma(n=1,a+N,(b+sum))
  lambdatemp= lambda1[j]
  low = pexp(L,rate = lambdatemp)
  up = pexp(U,rate = lambdatemp)
  u = runif(n=46,low,up)
  x = (-1/lambdatemp)*log(1-u)
  unew = runif(n=1,0,1)
  xnew1[j] = (-1/lambdatemp)*log(1-unew)
  sum = sum(x)
}
radiochem = read.table("breast-cancer-data-radioandchemo.txt",
header=TRUE,na.string="---")
attach(radiochem)
U[is.na(U)]=Inf
lambdatemp = 1
sum = sum(L)
a = 33.02833
b = 1143.09157
N = 46
nsim = 100000
lambda2 = rep(0,nsim)
u = rep(0,N)
x = rep(0,N)
unew = rep(0,1)
```

```

xnew2 = rep(0,nsim)
for (j in 1:nsim)
{
  lambda2[j] = rgamma(n=1,a=N,(b+sum))
  lambdatemp= lambda2[j]
  low = pexp(L,rate = lambdatemp)
  up = pexp(U,rate = lambdatemp)
  u = runif(n=46,low,up)
  x = (-1/lambdatemp)*log(1-u)
  unew = runif(n=1,0,1)
  xnew2[j] = (-1/lambdatemp)*log(1-unew)
  sum = sum(x)
}
success = sum(lambda1>lambda2)
total = nsim
success/total

pdf("lambda1.pdf")
plot(density(lambda1),xlab = "x",ylab = "f(x)",
main = expression("Density Plot for" ~ lambda[1]),ylim=c(0,140),
xlim=c(0.01,0.045))
dev.off()
pdf("lambda2.pdf")
plot(density(lambda2),xlab = "x",ylab = "f(x)",
main = expression("Density Plot for" ~ lambda[2]),ylim=c(0,140),
xlim=c(0.01,0.045))
dev.off()
pdf("explambda1.pdf")
plot(density(exp(-lambda1*36)),xlab = "x",ylab = "f(x)",
main = expression("Density Plot for" ~ e^(-36*%lambda[1])),
ylim=c(0,10),xlim=c(0.2,0.6))
dev.off()
pdf("explambda2.pdf")
plot(density(exp(-lambda2*36)),xlab = "x",ylab = "f(x)",
main = expression("Density Plot for"
~e^(-36*%lambda[2])),ylim=c(0,10),
xlim=c(0.2,0.6))
dev.off()

```

```
pdf("mlambda1.pdf")
plot(density(xnew1),xlab = "x",ylab = "f(x)",
main = expression("Density Plot for New Patient under Treatment 1"),
xlim=c(0,400))
dev.off()
pdf("mlambda2.pdf")
plot(density(xnew2),xlab = "x",ylab = "f(x)",
main = expression("Density Plot for New Patient under Treatment 2"),
xlim=c(0,400))
dev.off()
```