Homework 7

1. We consider the following hierarchical Bayes model:

$$X_i|\theta_i \sim \mathcal{N}(\theta_i, 1)$$

$$\theta_i|\mu, \tau^2 \sim \mathcal{N}(\mu, \tau^2)$$

$$\mu \sim \mathcal{N}(0, 1000)$$

$$\tau^2 \sim \chi_1^2$$

where μ and τ are considered indenpendent. We run a Gibbs-Sampler using *JAGS* and generate $100,000$ samples using a thin of 1 and a burn-in of 10,000. The posterior densities were fit using the *density* function in *R*. These are presented in Figures 1 and 2.

A comparison of the means under the model presented is shown in Table 1. Its clear that the Efron-Morris is superior, but the James Stein Estimator is worse than the Bayes Estimators we get from the simulations.

Comparison of Estimates:

	Player	True Value	Stein	Efron-Morris	Posterior Means
1	Clemente,Roberto	0.346	0.290	0.334	0.287
2	Robinson,Frank	0.298	0.286	0.313	0.283
3	Howard,Frank	0.276	0.282	0.292	0.279
4	Johnstone,Jay	0.222	0.277	0.277	0.276
5	Berry,Ken	0.273	0.273	0.273	0.272
6	Spencer,Jim	0.270	0.273	0.273	0.272
7	Kessinger,Don	0.265	0.268	0.268	0.268
8	Alvarado,Luis	0.210	0.264	0.264	0.265
9	Santo,Ron	0.269	0.259	0.259	0.260
10	Swaboda,Ron	0.230	0.259	0.259	0.261
11	Petrocelli,Rico	0.264	0.254	0.254	0.256
12	Rodriguez,Ellie	0.226	0.254	0.254	0.256
13	Scott,George	0.303	0.254	0.254	0.256
14	Unser,Del	0.264	0.254	0.254	0.256
15	Williams,Billy	0.330	0.254	0.254	0.256
16	Campaneris,Bert	0.285	0.249	0.249	0.252
17	Munson,Thurman	0.316	0.244	0.244	0.248
18	Alvis,Max	0.200	0.239	0.239	0.244
	Mean Squared Error		0.0007392538	5.760222e-05	0.0005306359

Table 1: Table comparing the Stein,Efron-Morris and Bayes Estimates for χ_1^2

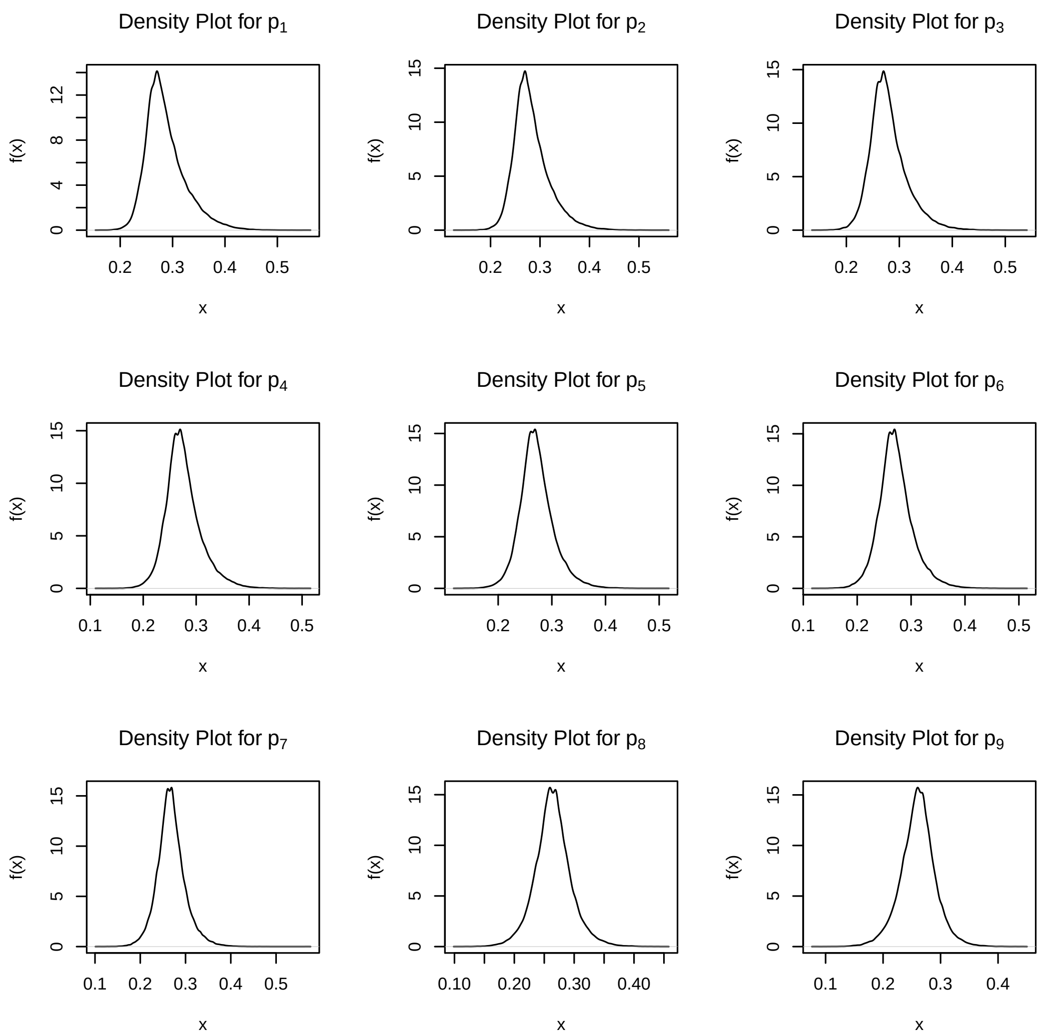

Figure 1: Posterior Density plots for $p_i, i = 1, ..., 9$

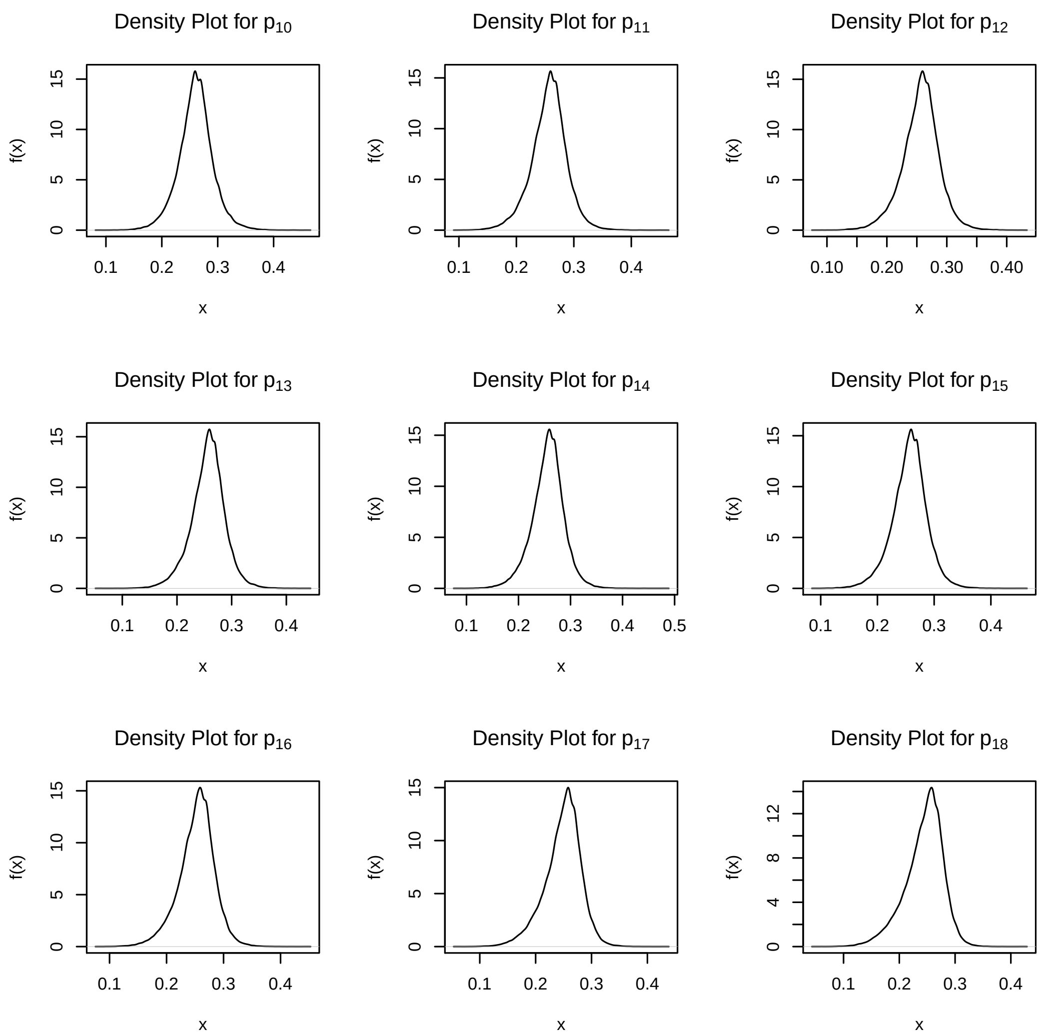

Figure 2: Posterior Density plots for $p_i, i = 10, ..., 18$

The probability that the hitting average of player 2 is greater than player 1 is $P(p_1 < p_2)$. It can be estimated using:

$$\hat{I} = \frac{1}{n}\sum_{i=1}^{n} I_{[p_1^i < p_2^i]} = 0.46747.$$

2. If we consider the following hierarchical Bayes model:

$$X_i|\theta_i \sim \mathcal{N}(\theta_i, 1)$$

$$\theta_i|\mu, \tau^2 \sim \mathcal{N}(\mu, \tau^2)$$

$$\mu \sim \mathcal{N}(0, 1000)$$

$$\tau^2 \sim \chi^2_5$$

then, it seems like the p_i's are shrinking less towards the mean. It is even better than the Stein estimator as Table 2 shows, however the Efron Morris estimator is still clearly better, when compared to the real data.

Comparison of Estimates:

	Player	True Value	Stein	Efron-Morris	Posterior Means
1	Clemente,Roberto	0.346	0.290	0.334	0.328
2	Robinson,Frank	0.298	0.286	0.313	0.318
3	Howard,Frank	0.276	0.282	0.292	0.308
4	Johnstone,Jay	0.222	0.277	0.277	0.297
5	Berry,Ken	0.273	0.273	0.273	0.287
6	Spencer,Jim	0.270	0.273	0.273	0.287
7	Kessinger,Don	0.265	0.268	0.268	0.277
8	Alvarado,Luis	0.210	0.264	0.264	0.266
9	Santo,Ron	0.269	0.259	0.259	0.255
10	Swaboda,Ron	0.230	0.259	0.259	0.255
11	Petrocelli,Rico	0.264	0.254	0.254	0.245
12	Rodriguez,Ellie	0.226	0.254	0.254	0.244
13	Scott,George	0.303	0.254	0.254	0.244
14	Unser,Del	0.264	0.254	0.254	0.245
15	Williams,Billy	0.330	0.254	0.254	0.245
16	Campaneris,Bert	0.285	0.249	0.249	0.234
17	Munson,Thurman	0.316	0.244	0.244	0.222
18	Alvis,Max	0.200	0.239	0.239	0.211
	Mean Squared Error		0.0007392538	5.760222e-05	0.0003418886

Table 2: Table comparing the Stein,Efron-Morris and Bayes Estimates for χ^2_5