Project

We are interested in finding the hospital with the best mortaility rate, which is equivalent to finding the hospital with the lowest $E(\lambda_i)$. We run a Gibbs-Sampler for the following model:

$$\begin{aligned}
Z_i &\sim Poisson(\lambda_i e_i),\\
\lambda_i &\sim Gamma(\alpha, \beta),\\
\alpha &\sim Exp(a_0), \qquad a_0 = \log(2)/z_0, \qquad z_0 = 0.53,\\
\beta &\sim Gamma(b_0, b_1), \qquad b_0 = 1, \qquad b_1 = 0.65,
\end{aligned}$$

and estimate the mortality rate of hospital i, $E(\lambda_i)$, using

$$\widehat{E(\lambda_i)} = \frac{1}{n}\sum_{j=1}^{n} \lambda_i^{(j)} = \bar{\lambda}_i,$$

where n is the number of observations collected in our sample and $\lambda_i^{(j)}$ denotes the jth observation for λ_i.

Note that under this model, the value of the log of the posterior denisty of $(\boldsymbol{\lambda}, \alpha, \beta)$ at $(\hat{\boldsymbol{\lambda}}, \hat{\alpha}, \hat{\beta}) = (\hat{\lambda}_1, ...\hat{\lambda}_{94}, \hat{\alpha}, \hat{\beta})$ can be calculated in the following manner:

$$\begin{aligned}
f_{(\boldsymbol{\lambda},\alpha,\beta)|\boldsymbol{Z}}(\hat{\boldsymbol{\lambda}}, \hat{\alpha}, \hat{\beta}) &= c\prod_{i=1}^{94} f_{Z_i|\hat{\lambda}_i}(Z_i) f_{\lambda_i|(\hat{\alpha},\hat{\beta})}(\hat{\lambda}_i) f_\alpha(\hat{\alpha}) f_\beta(\hat{\beta})\\
\implies \log f_{(\boldsymbol{\lambda},\alpha,\beta)|\boldsymbol{Z}}(\hat{\boldsymbol{\lambda}}, \hat{\alpha}, \hat{\beta}) &= \log c + \sum_{i=1}^{94} \big(\log f_{Z_i|\hat{\lambda}_i}(Z_i) + \log f_{\lambda_i|(\hat{\alpha},\hat{\beta})}(\hat{\lambda}_i)\\
&\qquad + \log f_\alpha(\hat{\alpha}) + \log f_\beta(\hat{\beta})\big)
\end{aligned}$$

where c is the normalzing constant, which we will ignore when plotting the posterior density.

We initially run a Gibbs-Sampler on the model using $JAGs$ with a thin of 1, a burn-in of 10000 and $n = 50000$ with initial values of $\lambda_i^{(0)} = 1 \forall i$. Figure 1c indicates that the negative log posterior converges slowly to a value around 900, but Figures 1a and 1b indicates that it exhibits a clear sinusoidal pattern. This indicates that the chains generated by this algorithm are most likely getting stuck in regions from which it takes a significant amount of time to escape; in other words, they are *mixing slowly*. The trace plots for α and β in Figure 2a and 2b also seem problematic for the same reason. As expected, the autocorrelation plots for α and β in Figure 2a and 2b reveal extremely high correlation. These plots indicate that a thin of 100 may work better.

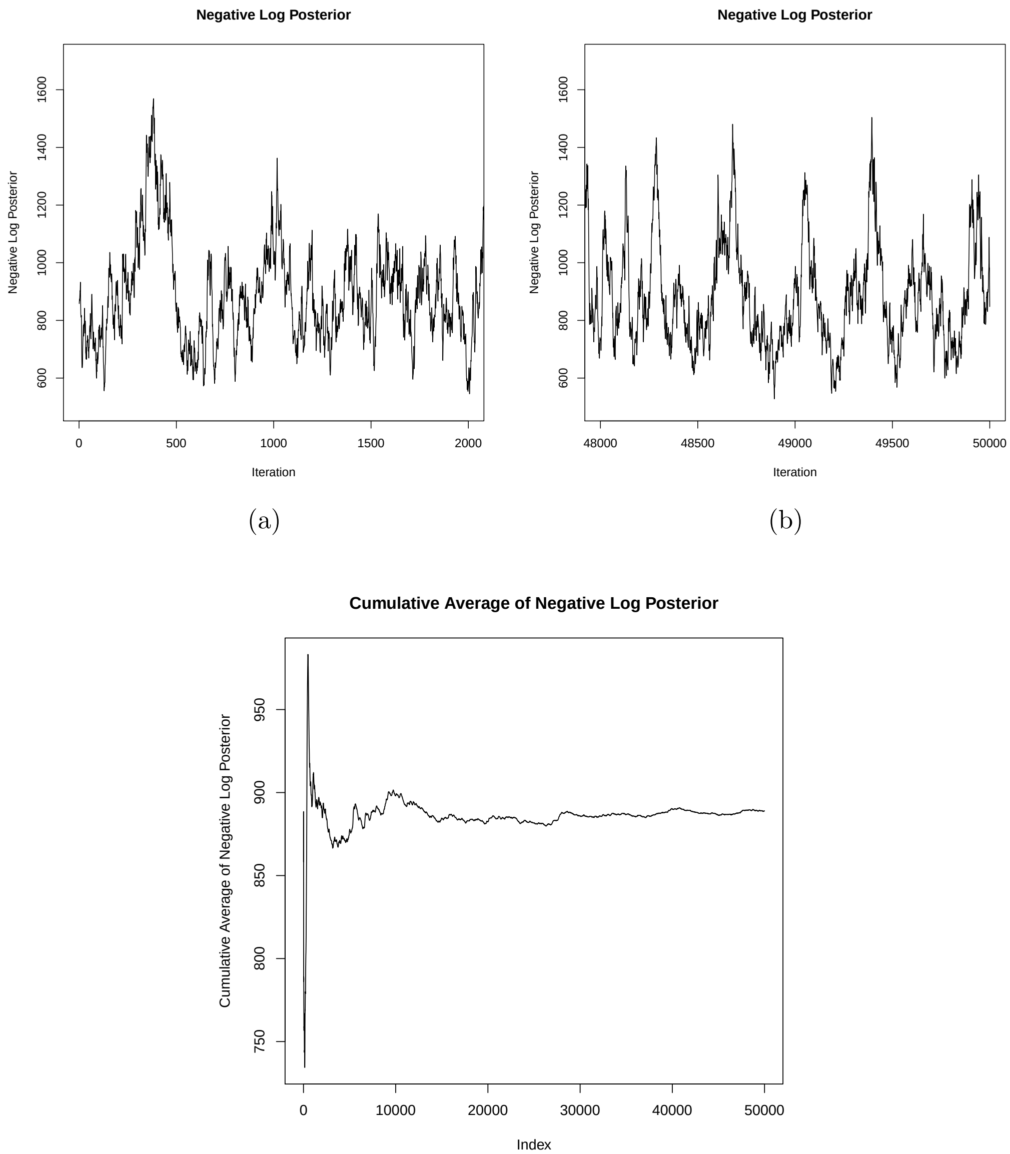
We run another Gibbs-Sampler on the same model again using $JAGs$ with a thin of 100, a burn-in of 10000 and $n = 50000$ this time. Figures 6a, 6b and 6c shows that the negative log posterior for this chain varies evenly around some value between 850 and 900 without getting stuck in any region for too long. These look much better than when we used a thin of 1. Also, as is evident in Figure 7a and 7b, the autocorrelation plots for α and β indicate that the thin of 100 works well. The trace plots for α and β in Figure 7a and 7b also seem much better.

The estimated means and their standard errors, calculated employing the naive method, time series methods (TS) and batch means (BM), using a thin of 1 and a thin of 100 are reported in Table 2 and Table 3, respectively. In calculating the standard errors using the batch means method we used a batch size of $\lfloor n/\sqrt{n} \rfloor$. As expected, the general trend is that variance estimates are smaller when using a thin of 100. With a thin of 1 we also get significantly different values for the standard errors using the different methods, whereas the naive, TS and BM estimates are similar when using a thin of 100. This suggests that the covariance term is almost 0 for the sample collected using a thin of 100, which implies that it is *closer* to an *iid* sample.

The "best" mortality rates are for hospitals 63 and 85 and the "worst" mortality rates are for hospitals 9 and 68. These are presented in Table 1 along with 95% CI's, which were calculated using the TS standard errors from the sample collected with a thin of 100. Trace plots and autocorrelation plots for these λ_i are presented in Figures 4, 5, 9 and 10. Interestingly enough, the λ_i corresponding to these are the hospitals that exhibited the most problematic autocorrelation plots when using a thin of 1. It is also clear from Figure 10 that autocorrelation amongst the λ_i is close of negligible using a thin of 100. Finally we tried multiple starting points using a thin of 100 by setting the initial values of $\lambda_i^{(0)} = 10 \forall i$, $\lambda_i^{(0)} = 100 \forall i$ and $\lambda_i^{(0)}$ as a random sample from the uniform distribution on [1,100] $\forall i$, which still gave us the same results in terms of parameter and error estimates and diagnostic plots.

Best Hospitals:	Mortality Rates	95% CI
85	0.368	(0.366236,0.369764)
63	0.468	(0.465648,0.470352)
Worst Hospitals:	Mortality Rates	95% CI
9	1.512	(1.506512,1.517488)
68	1.642	(1.637688,1.646312)

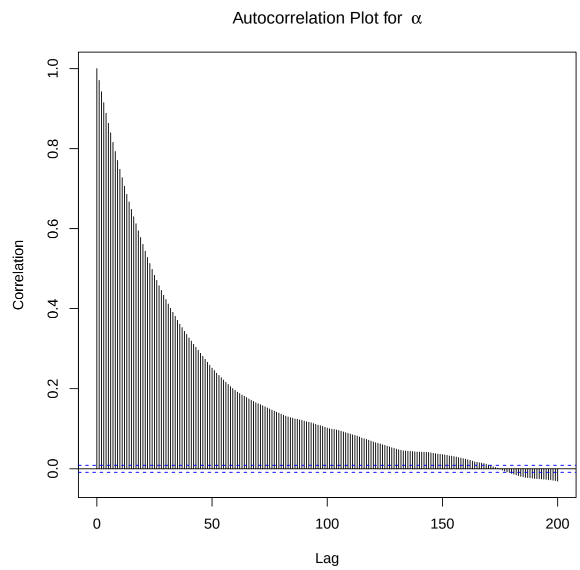
Table 1: Best and Worst Hospitals based on mortality rates using a thin of 100



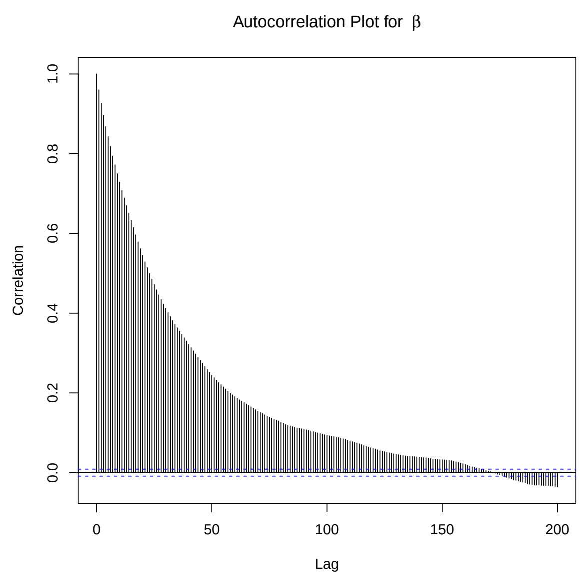
(a) (b)

(c)

Figure 1: Negative Log Posterior with a thin of 1

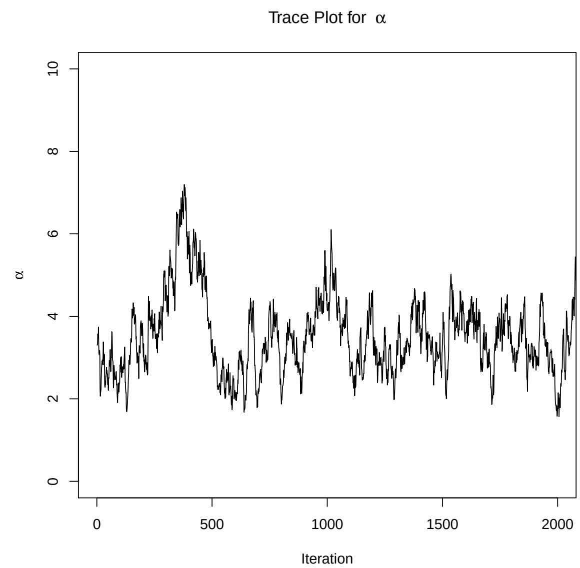


(a)

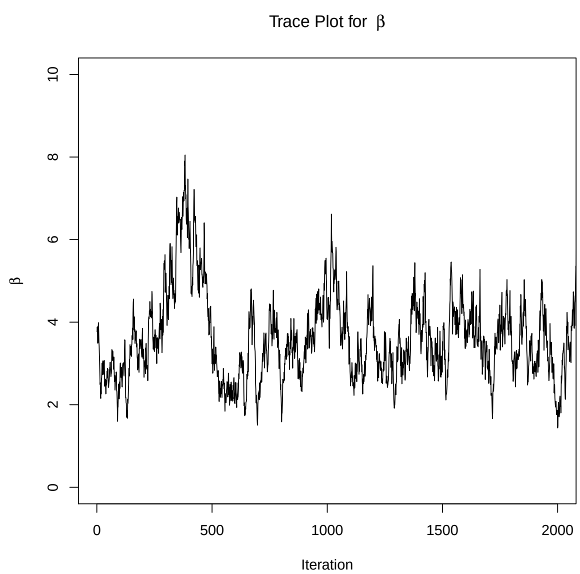


(b)

Figure 2: Auto-Correlation plot for α in Figure 2a and β in Figure 2b with a thin of 1

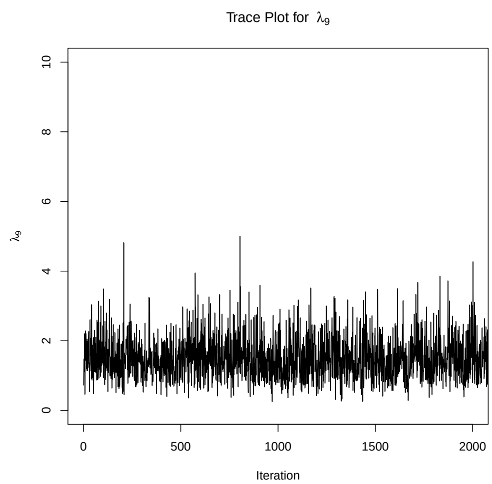


(a)

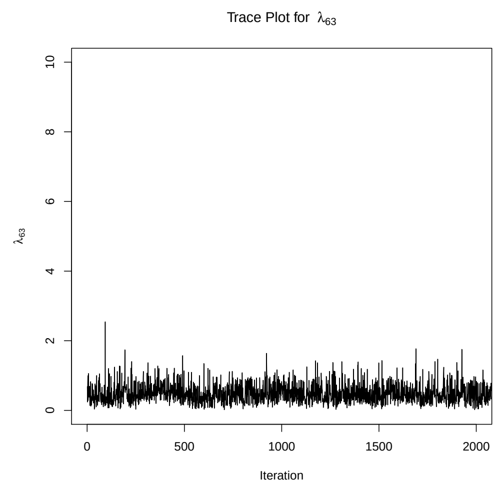


(b)

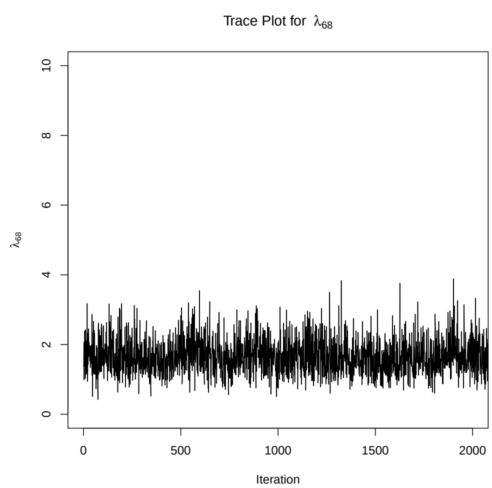
Figure 3: Trace plot for α in Figure 3a and β in Figure 3b with a thin of 1



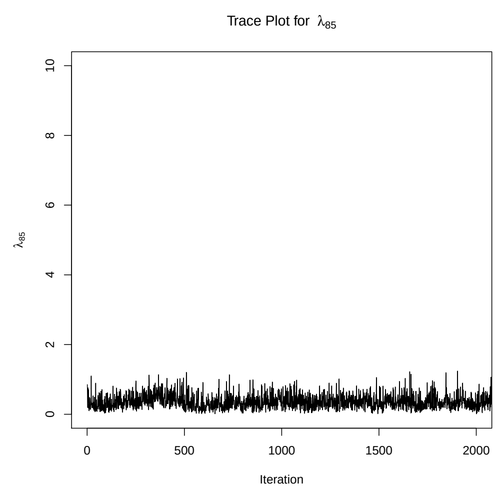
(a)



(b)



(c)



(d)

Figure 4: Trace Plots for λ_i using a thin of 1

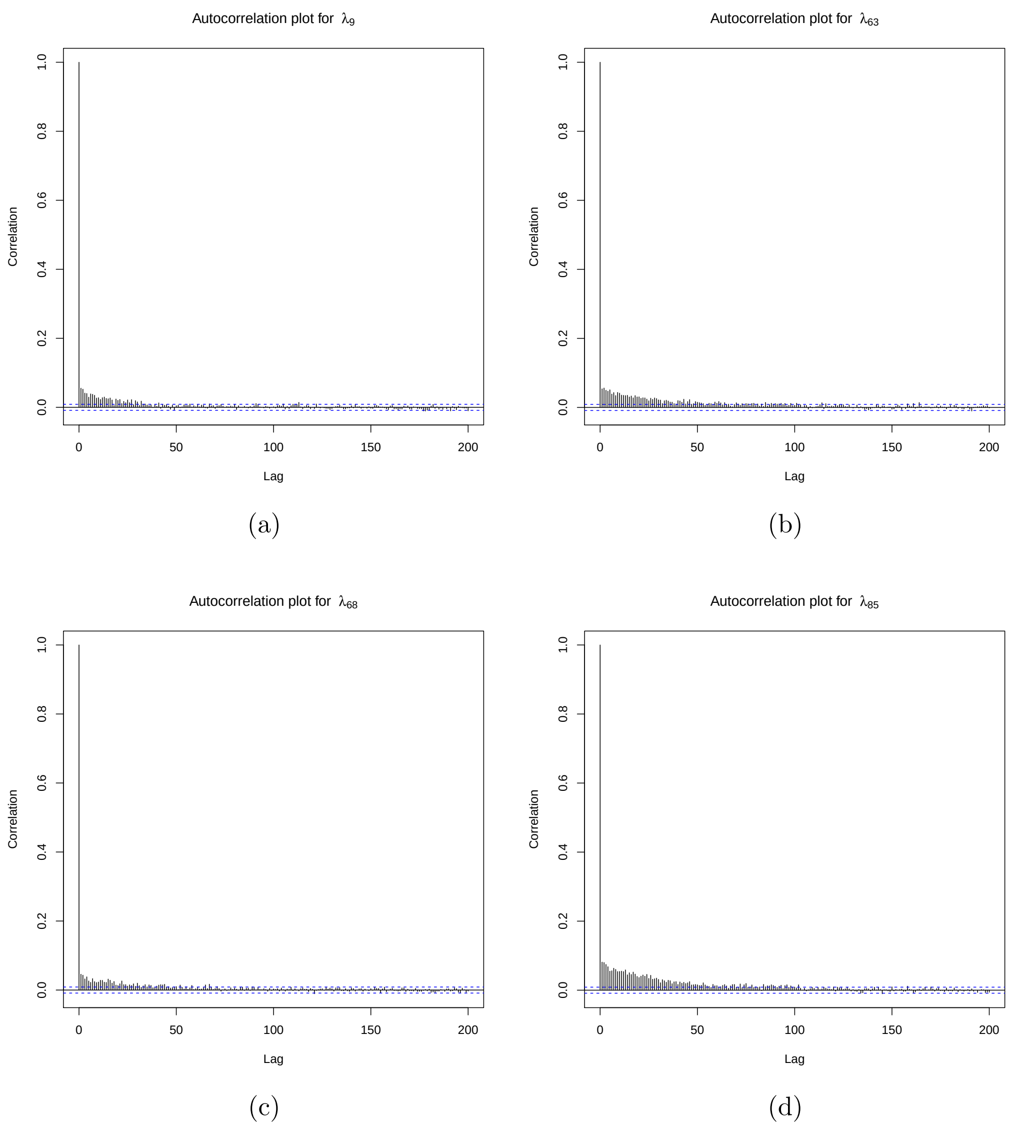
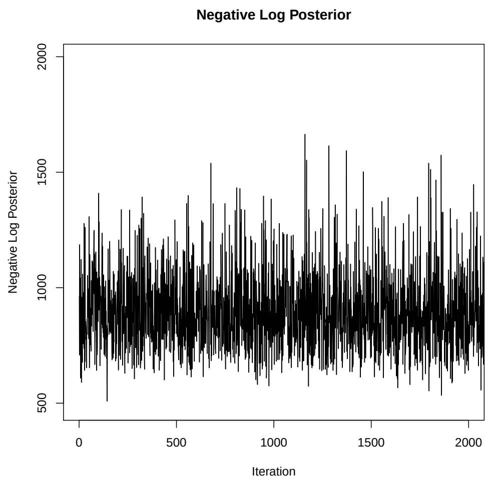
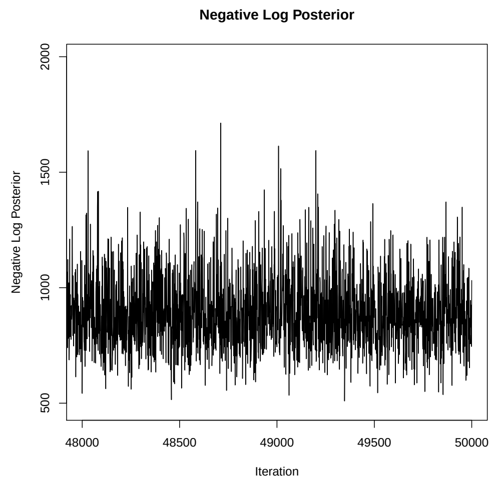


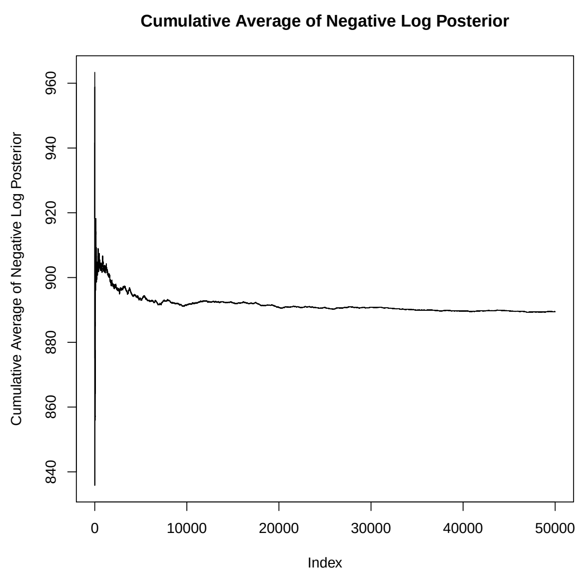
Figure 5: Autocorrelation Plots for λ_i using a thin of 1



(a)

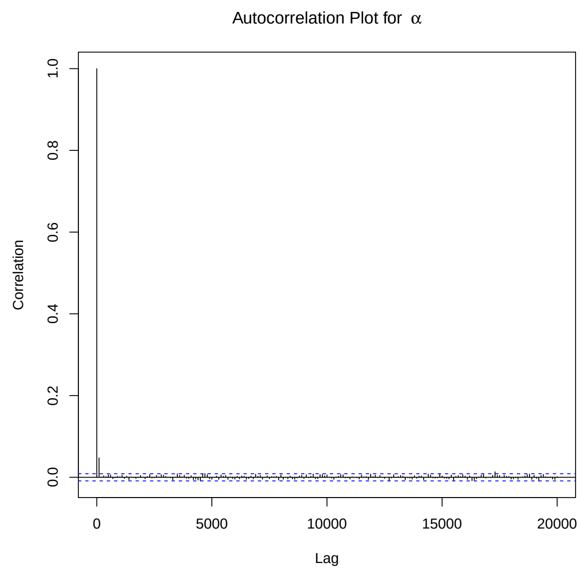


(b)

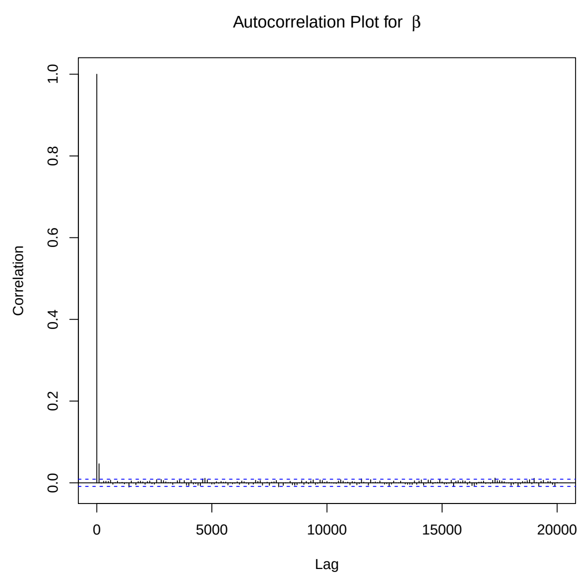


(c)

Figure 6: Negative Log Posterior with a thin of 100

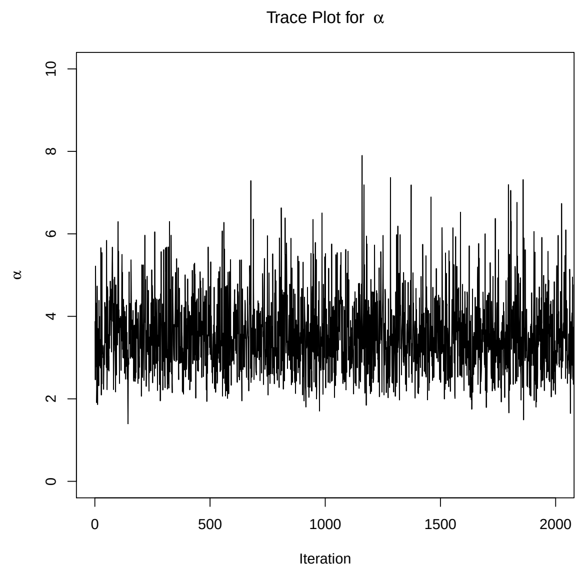


(a)

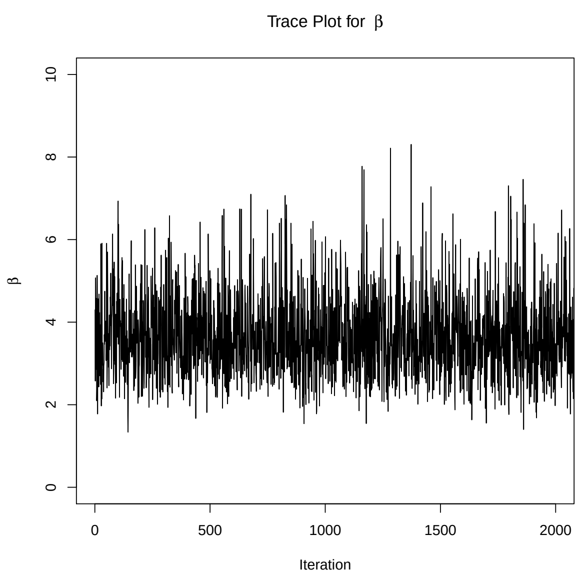


(b)

Figure 7: Auto-Correlation plot for α in Figure 7a and β in Figure 7b with a thin of 100

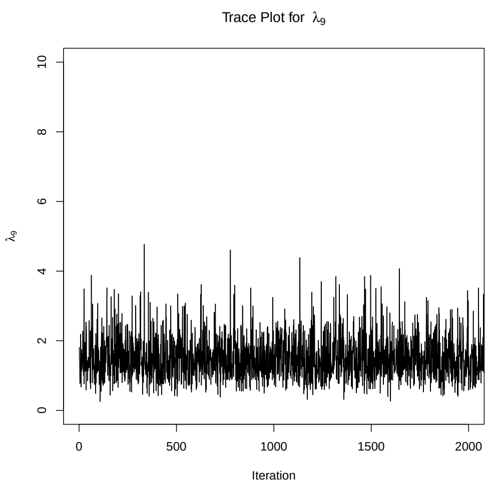


(a)

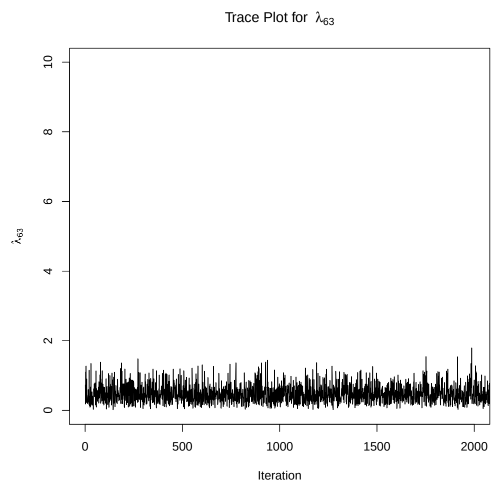


(b)

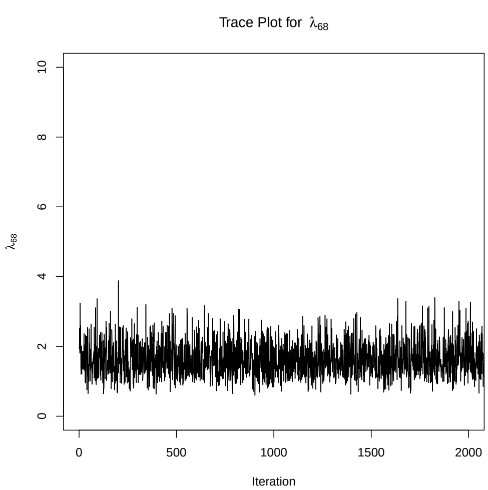
Figure 8: Trace plot for α in Figure 8a and β in Figure 8b with a thin of 100



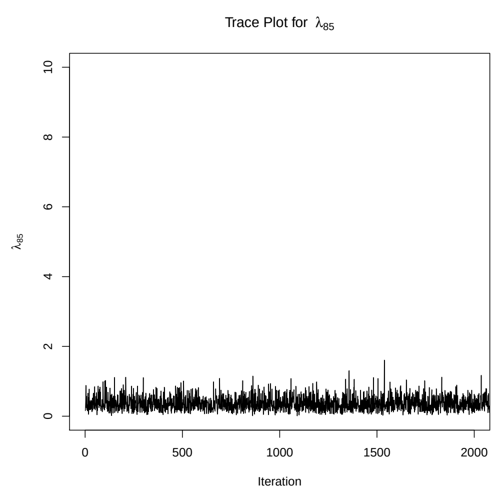
(a)



(b)

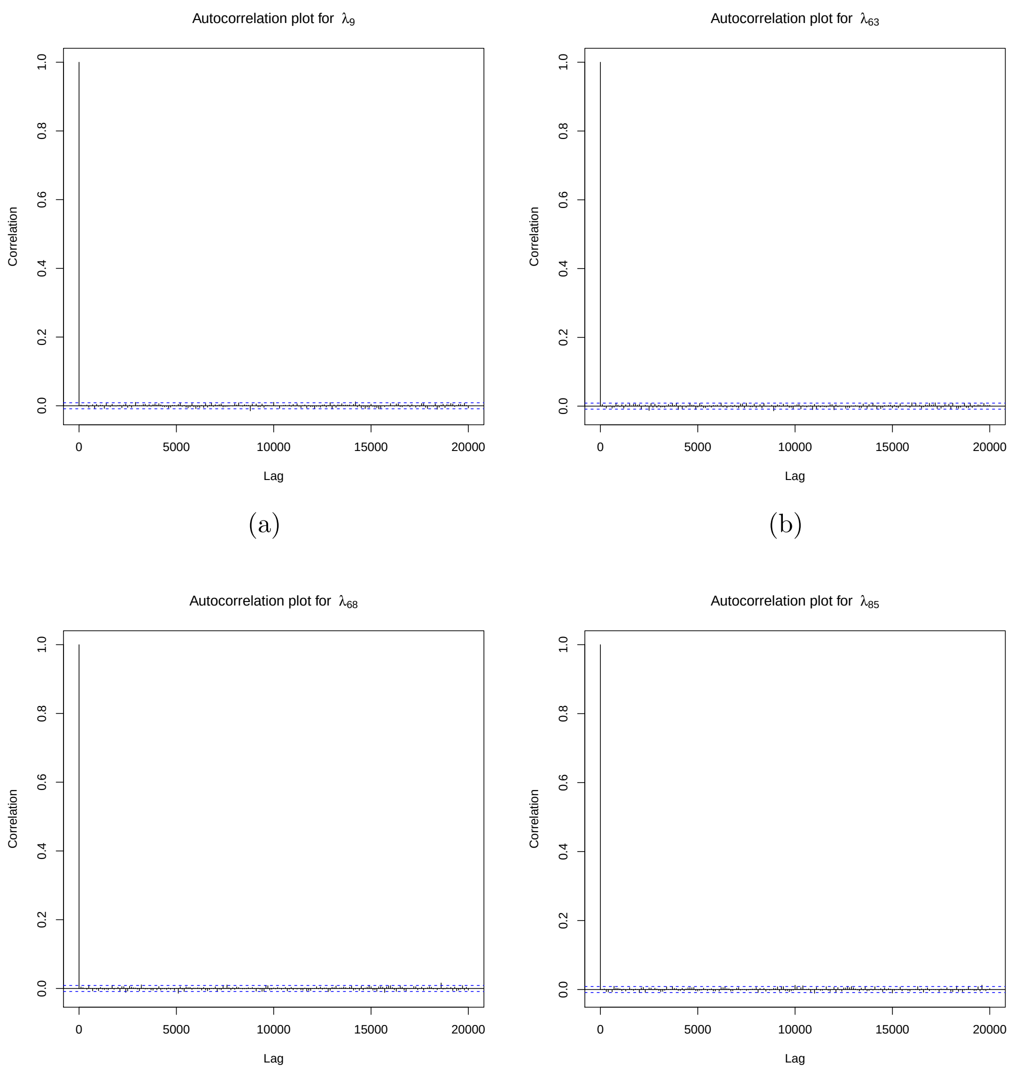


(c)



(d)

Figure 9: Trace Plots for λ_i using a thin of 100



(a) (b) (c) (d)

Figure 10: Autocorrelation Plots for λ_i using a thin of 100

i	Mean	SE	SE(BM)	SE(TS)	i	Mean	SE	SE(BM)	SE(TS)
1	0.843	0.0021	0.0023	0.0022	48	1.031	0.0018	0.0021	0.0019
2	0.829	0.0020	0.0023	0.0022	49	0.587	0.0015	0.0026	0.0024
3	1.304	0.0026	0.0041	0.0036	50	0.605	0.0015	0.0024	0.0023
4	1.047	0.0023	0.0028	0.0024	51	0.851	0.0017	0.0018	0.0017
5	1.000	0.0022	0.0022	0.0023	52	1.325	0.0021	0.0029	0.0026
6	0.717	0.0018	0.0023	0.0020	53	1.257	0.0020	0.0026	0.0024
7	0.762	0.0019	0.0025	0.0022	54	0.752	0.0016	0.0018	0.0017
8	0.899	0.0019	0.0020	0.0020	55	0.729	0.0016	0.0019	0.0019
9	1.511	0.0028	0.0048	0.0048	56	0.872	0.0015	0.0016	0.0015
10	0.751	0.0019	0.0024	0.0021	57	0.717	0.0015	0.0018	0.0017
11	0.801	0.0020	0.0022	0.0021	58	0.706	0.0015	0.0018	0.0018
12	0.832	0.0018	0.0018	0.0019	59	0.978	0.0017	0.0018	0.0018
13	0.785	0.0019	0.0024	0.0022	60	0.627	0.0014	0.0019	0.0018
14	1.176	0.0023	0.0030	0.0027	61	0.867	0.0017	0.0017	0.0017
15	1.381	0.0025	0.0039	0.0038	62	1.371	0.0021	0.0030	0.0026
16	0.726	0.0018	0.0022	0.0020	63	0.467	0.0012	0.0024	0.0024
17	0.704	0.0017	0.0023	0.0022	64	0.720	0.0014	0.0018	0.0015
18	1.218	0.0022	0.0030	0.0027	65	0.881	0.0017	0.0018	0.0018
19	0.623	0.0013	0.0019	0.0018	66	0.695	0.0015	0.0020	0.0018
20	0.939	0.0020	0.0020	0.0021	67	0.693	0.0013	0.0016	0.0015
21	0.935	0.0020	0.0020	0.0021	68	1.643	0.0022	0.0039	0.0037
22	0.933	0.0020	0.0020	0.0021	69	1.322	0.0020	0.0027	0.0024
23	1.481	0.0025	0.0046	0.0044	70	0.570	0.0012	0.0021	0.0019
24	1.276	0.0023	0.0034	0.0032	71	1.263	0.0019	0.0021	0.0021
25	1.195	0.0021	0.0028	0.0024	72	0.977	0.0016	0.0017	0.0016
26	0.813	0.0018	0.0019	0.0019	73	0.572	0.0012	0.0019	0.0018
27	0.716	0.0018	0.0024	0.0021	74	0.848	0.0015	0.0016	0.0015
28	1.110	0.0022	0.0025	0.0023	75	1.033	0.0016	0.0018	0.0016
29	1.052	0.0021	0.0023	0.0022	76	0.596	0.0012	0.0017	0.0016
30	1.438	0.0025	0.0042	0.0039	77	0.747	0.0012	0.0014	0.0013
31	1.392	0.0024	0.0038	0.0035	78	0.725	0.0013	0.0016	0.0014
32	1.287	0.0023	0.0033	0.0030	79	0.934	0.0015	0.0017	0.0016
33	1.063	0.0021	0.0022	0.0021	80	1.026	0.0016	0.0017	0.0016
34	1.285	0.0022	0.0029	0.0027	81	0.633	0.0012	0.0017	0.0014
35	0.878	0.0019	0.0021	0.0020	82	0.987	0.0014	0.0015	0.0015
36	1.220	0.0022	0.0029	0.0026	83	1.322	0.0018	0.0023	0.0020
37	0.705	0.0017	0.0022	0.0020	84	0.570	0.0009	0.0012	0.0011
38	1.475	0.0025	0.0044	0.0039	85	0.367	0.0009	0.0022	0.0021
39	0.851	0.0018	0.0019	0.0019	86	0.982	0.0014	0.0015	0.0015
40	1.049	0.0021	0.0023	0.0021	87	1.253	0.0017	0.0021	0.0019
41	1.216	0.0022	0.0028	0.0026	88	1.080	0.0015	0.0016	0.0015
42	1.353	0.0023	0.0033	0.0032	89	0.657	0.0012	0.0015	0.0013
43	1.409	0.0024	0.0039	0.0036	90	0.609	0.0011	0.0015	0.0013
44	0.901	0.0018	0.0018	0.0018	91	1.092	0.0014	0.0016	0.0014
45	1.031	0.0020	0.0022	0.0021	92	0.795	0.0011	0.0012	0.0011
46	1.243	0.0021	0.0027	0.0024	93	1.374	0.0013	0.0016	0.0014
47	0.919	0.0018	0.0019	0.0019	94	1.300	0.0013	0.0016	0.0014

Table 2: Estimated Mortality Rates ($\bar{\lambda}_i$) for Hospitals 1 to 94 using a thin of 1

i	Mean	SE	SE(BM)	SE(TS)	i	Mean	SE	SE(BM)	SE(TS)
1	0.844	0.0021	0.0022	0.0021	48	1.034	0.0019	0.0020	0.0019
2	0.832	0.0021	0.0022	0.0021	49	0.587	0.0014	0.0014	0.0014
3	1.300	0.0026	0.0026	0.0026	50	0.603	0.0015	0.0013	0.0015
4	1.046	0.0023	0.0021	0.0023	51	0.850	0.0017	0.0015	0.0016
5	1.002	0.0022	0.0021	0.0022	52	1.323	0.0021	0.0021	0.0021
6	0.716	0.0018	0.0018	0.0018	53	1.264	0.0020	0.0021	0.0020
7	0.760	0.0019	0.0017	0.0019	54	0.750	0.0016	0.0016	0.0016
8	0.895	0.0019	0.0019	0.0019	55	0.725	0.0016	0.0017	0.0016
9	1.512	0.0028	0.0028	0.0028	56	0.870	0.0016	0.0016	0.0016
10	0.751	0.0019	0.0017	0.0019	57	0.719	0.0015	0.0015	0.0015
11	0.804	0.0020	0.0019	0.0020	58	0.708	0.0015	0.0016	0.0015
12	0.834	0.0018	0.0018	0.0018	59	0.976	0.0017	0.0017	0.0017
13	0.784	0.0019	0.0020	0.0019	60	0.628	0.0014	0.0013	0.0013
14	1.175	0.0023	0.0022	0.0024	61	0.865	0.0017	0.0018	0.0017
15	1.383	0.0025	0.0026	0.0025	62	1.373	0.0021	0.0020	0.0021
16	0.730	0.0018	0.0018	0.0018	63	0.468	0.0012	0.0012	0.0012
17	0.706	0.0017	0.0019	0.0017	64	0.720	0.0014	0.0015	0.0014
18	1.221	0.0022	0.0023	0.0022	65	0.883	0.0017	0.0016	0.0017
19	0.623	0.0013	0.0013	0.0013	66	0.693	0.0015	0.0014	0.0015
20	0.939	0.0020	0.0020	0.0020	67	0.695	0.0013	0.0014	0.0013
21	0.934	0.0020	0.0020	0.0020	68	1.642	0.0022	0.0022	0.0022
22	0.934	0.0020	0.0020	0.0020	69	1.321	0.0020	0.0019	0.0020
23	1.481	0.0025	0.0026	0.0025	70	0.570	0.0012	0.0013	0.0012
24	1.274	0.0023	0.0024	0.0023	71	1.267	0.0019	0.0021	0.0019
25	1.198	0.0022	0.0020	0.0022	72	0.977	0.0016	0.0017	0.0016
26	0.817	0.0018	0.0018	0.0018	73	0.571	0.0012	0.0012	0.0012
27	0.716	0.0018	0.0018	0.0018	74	0.854	0.0015	0.0015	0.0015
28	1.110	0.0022	0.0022	0.0022	75	1.033	0.0016	0.0016	0.0016
29	1.053	0.0021	0.0022	0.0021	76	0.595	0.0012	0.0012	0.0012
30	1.439	0.0024	0.0023	0.0024	77	0.747	0.0012	0.0012	0.0012
31	1.390	0.0024	0.0022	0.0024	78	0.724	0.0013	0.0012	0.0013
32	1.288	0.0023	0.0024	0.0023	79	0.931	0.0015	0.0016	0.0015
33	1.062	0.0021	0.0021	0.0021	80	1.024	0.0016	0.0016	0.0016
34	1.283	0.0022	0.0023	0.0021	81	0.633	0.0012	0.0012	0.0012
35	0.878	0.0019	0.0019	0.0019	82	0.984	0.0014	0.0014	0.0014
36	1.219	0.0022	0.0022	0.0022	83	1.319	0.0018	0.0018	0.0018
37	0.705	0.0017	0.0017	0.0017	84	0.571	0.0009	0.0009	0.0009
38	1.476	0.0025	0.0025	0.0025	85	0.368	0.0009	0.0010	0.0009
39	0.852	0.0018	0.0018	0.0018	86	0.985	0.0014	0.0013	0.0014
40	1.049	0.0020	0.0021	0.0020	87	1.254	0.0017	0.0017	0.0017
41	1.216	0.0022	0.0021	0.0022	88	1.082	0.0015	0.0015	0.0015
42	1.352	0.0023	0.0023	0.0023	89	0.657	0.0012	0.0011	0.0011
43	1.407	0.0024	0.0025	0.0024	90	0.611	0.0011	0.0011	0.0011
44	0.906	0.0017	0.0018	0.0017	91	1.092	0.0014	0.0013	0.0014
45	1.030	0.0020	0.0020	0.0020	92	0.798	0.0011	0.0012	0.0011
46	1.246	0.0021	0.0020	0.0021	93	1.373	0.0013	0.0013	0.0013
47	0.918	0.0018	0.0018	0.0018	94	1.304	0.0013	0.0013	0.0013

Table 3: Estimated Mortality Rates ($\bar{\lambda}_i$) for Hospitals 1 to 94 using a thin of 100

Appendix 1: model.txt

```
model {
for (i in 1:N) {
Z[i] ~ dpois(lambda[i]*e[i])
lambda[i] ~ dgamma(alpha,beta)
}
z0 <- 0.53
a0 <- log(2)/z0
alpha ~ dexp(a0)
b0 <- 1
b1 <- 0.65
beta ~ dgamma(b0,b1)
}
```

Appendix 2: script.txt

```
model clear
data clear
model in "model.txt"
data in "data.txt"
compile
inits in "initial_1.txt"
initialize
update 10000
monitor beta, thin(100)
monitor alpha, thin(100)
monitor lambda, thin(100)
monitor Z, thin(100)
update 5000000
coda *
```

Appendix 3: R code

```
library(coda)
library(xtable)
data = read.table("ht-data.txt",header=TRUE)
attach(data)
length(e)
```

```
res = read.coda("CODAchain1.txt", "CODAindex.txt")

z0 = 0.53
a0 = log(2)/z0
b0 = 1
b1 = 0.65
logpost = matrix(0,dim(res)[1],1)
for(i in 1:94){
    temp1 <- log(dpois(round(res[,96+i]),res[,2+i]*e[i]))
    temp2 <- log(dgamma(res[,2+i],res[,2],rate = res[,1]))
    temp3 <- log(dexp(res[,2],rate=a0))
    temp4 <- log(dgamma(res[,1],b0,rate = b1))
    logpost = logpost + temp1 + temp2 + temp3 + temp4
}


colnames(res)

pdf("alpha.pdf")
plot(as.numeric(res[,2]),  type= 'l',xlim = c(0,2000),
     ylim = c(0,10),
xlab = "Iteration", ylab = expression(alpha),
     main = expression("Trace Plot for " ~ alpha))
##trace plot
dev.off()


pdf("beta.pdf")
plot(as.numeric(res[,1]),  type= 'l',xlim = c(0,2000),
     ylim = c(0,10),
xlab = "Iteration", ylab = expression(beta),
     main = expression("Trace Plot for " ~ beta))
##trace plot
dev.off()

pdf("tracelmd9.pdf")
plot(as.numeric(res[,2+9]),  type= 'l',xlim = c(0,2000),
     ylim = c(0,10),
```

```
xlab = "Iteration", ylab = expression(lambda[9]),
     main=expression("Trace Plot for " ~ lambda[9]))
##trace plot
dev.off()
pdf("acf9.pdf")
acf(res[,2+9], lag.max = 200, xlab = "Lag",
    ylab = "Correlation",
    main=expression("Autocorrelation plot for " ~ lambda[9]))
## autocorrelation plot
dev.off()

pdf("tracelmd63.pdf")
plot(as.numeric(res[,2+63]),  type= 'l',xlim = c(0,2000),
     ylim = c(0,10),
xlab = "Iteration", ylab = expression(lambda[63]),
     main=expression("Trace Plot for " ~ lambda[63]))
##trace plot
dev.off()
pdf("acf63.pdf")
acf(res[,2+63], lag.max = 200, xlab = "Lag",
    ylab = "Correlation",
    main=expression("Autocorrelation plot for " ~ lambda[63]))
## autocorrelation plot
dev.off()

pdf("tracelmd68.pdf")
plot(as.numeric(res[,2+68]),  type= 'l',xlim = c(0,2000),
     ylim = c(0,10),
xlab = "Iteration", ylab = expression(lambda[68]),
     main=expression("Trace Plot for " ~ lambda[68]))
##trace plot
dev.off()
pdf("acf68.pdf")
acf(res[,2+68], lag.max = 200, xlab = "Lag",
    ylab = "Correlation",
    main=expression("Autocorrelation plot for " ~ lambda[68]))
## autocorrelation plot
dev.off()
```

```
pdf("tracelmd85.pdf")
plot(as.numeric(res[,2+85]),  type= 'l',xlim = c(0,2000),
     ylim = c(0,10),
xlab = "Iteration", ylab = expression(lambda[85]),
     main=expression("Trace Plot for " ~ lambda[85]))
##trace plot
dev.off()
pdf("acf85.pdf")
acf(res[,2+85], lag.max = 200, xlab = "Lag",
    ylab = "Correlation",
    main=expression("Autocorrelation plot for " ~ lambda[85]))
## autocorrelation plot
dev.off()

pdf("logpost.pdf")
plot(as.numeric(-logpost),  type= 'l',xlim = c(0,2000),
xlab = "Iteration", ylab = "Negative Log Posterior" ,
     main = "Negative Log Posterior" )
##plot for log posterior
dev.off()

pdf("cumsum.pdf")
x = cumsum(-logpost)/1:dim(res)[1]
plot(x,  type= 'l',
     ylab = "Cumulative Average of Negative Log Posterior",
     main = "Cumulative Average of Negative Log Posterior")
dev.off()

jpeg("cumsum.jpeg")
x = cumsum(-logpost)/1:dim(res)[1]
plot(x,  type= 'l',
     ylab = "Cumulative Average of Negative Log Posterior")
dev.off()

pdf("acf.pdf")
acf(res[,3], lag.max = 200, xlab = "Lag", ylab = "Correlation",
    main="")
```

```
## autocorrelation plot
dev.off()

pdf("alphaacf.pdf")
acf(res[,2], lag.max = 200, xlab = "Lag", ylab = "Correlation",
    main=expression("Autocorrelation Plot for " ~ alpha))
## autocorrelation plot
dev.off()

pdf("betaacf.pdf")
acf(res[,1], lag.max = 200, xlab = "Lag", ylab = "Correlation",
    main=expression("Autocorrelation Plot for " ~ beta))
## autocorrelation plot
dev.off()

btchsz = floor(dim(res)[1]/sqrt(dim(res)[1]))
nbtch = floor(dim(res)[1]/btchsz)


meanlambda = matrix(0,94,1)
varlambda = matrix(0,94,1)

for(i in 1:94){
meanlambda[i] = mean(res[,2+i])

btchvar = matrix(0,nbtch,1)
for (j in 1:nbtch){
    lb = (j-1)*btchsz
    ub = j*btchsz
btchvar[j] = var(res[lb:ub,2+i])
}

varlambda[i] = (btchsz/(nbtch-1))*sum((btchvar - var(res[,2+i]))^2)

}

res.sum = summary(res[,1:96])
d.res.sum = as.data.frame(res.sum$statistics)
```

```
dim(d.res.sum)

bS=floor(dim(res)[1]/sqrt(dim(res)[1]))

library(xtable)
temp = as.data.frame(cbind(1:47, meanlambda[1:47],
    d.res.sum[(1+2):(47+2),3],
    as.matrix(batchSE(res[,1:96],batchSize=bS)[(1+2):(47+2)]),
    d.res.sum[(1+2):(47+2),4], 48:94, meanlambda[48:94],
    d.res.sum[(48+2):(94+2),3],
    as.matrix(batchSE(res[,1:96],
                      batchSize=bS)[(48+2):(94+2)]),
    d.res.sum[(48+2):(94+2),4]))
colnames(temp) =
    c("i","Mean","SE","SE(BM)","SE(TS)","i",
      "Mean","SE","SE(BM)","SE(TS)")
temptab = xtable(temp,
    caption="Estimated means for $\\lambda_i$",
    label="tab1",digits=c(0,0,3,4,4,4,0,3,4,4,4))
print(temptab,include.rownames=FALSE)
```