Missing Plot technique in RBD

Estimation of one Missing Value in RBD

Let the observation y_{ij} = x (say) in the j^{th} block and receiving the i^{th} treatment be missing, as given in the following table:

					Treatments			
		1	2	…	I	…	t	
Blocks	1	Y_{11}	Y_{21}	…	Y_{i1}	…	Y_{t1}	$y_{.1}$
	2	Y_{12}	Y_{22}	…	Y_{i2}	…	Y_{t2}	$y_{.2}$
	⋮	⋮	⋮	…	⋮	…	⋮	⋮
	J	Y_{j1}	Y_{j2}	…	X	…	y_{jt}	$y_{.j}' + x$
	⋮	⋮	⋮	…	⋮	…	⋮	⋮
	R	Y_{1r}	Y_{2r}	…	y_{ir}	…	y_{tr}	
	Total	$Y_{1.}$	$Y_{2.}$	…	$y_{i.}' + x$	…	$Y_{t.}$	$y_{..}' + x$

where

$y_{i.}'$ is total of known observations getting i^{th} treatment

$y_{.j}'$ is total of known observations in j^{th} block and

Sum of square due to treatment (S.S.Tr) = $\frac{\sum_i y_{i\cdot}{'}}{r} - C.F = \frac{\sum_i \left(y_{i..}{'} + x\right)^2}{r} - C.F$

Sum of square due to

Block (S.S.B) = $\frac{\sum_j y_{.j}{}^2}{t} - C.F = \frac{\left(y_{.j}{'} + x\right)^2}{r} - C.F$

Sum of square due to error = T.S.S –S.S.Tr-S.S.B =

$$(\sum_i \sum_j y_{ij}^2 - C.F) - \left(\frac{\sum_i y_{i.}^2}{r} - C.F\right) - (\frac{\sum_j y_{.j}^2}{t} - C.F)$$

$y_{..}{'}$ is total of all known observations

Correction factor = $\frac{G^2}{tr} = \frac{(G' + x)^2}{tr}$

Total sum of square = $\sum_i \sum_j y_{ij}^2 - \frac{G^2}{tr} = x^2 +$ constant terms independent $of\ x - \frac{(G' + x)^2}{tr}$

Sum of square due to treatment (S.S.Tr) = $\frac{\sum_i y_{i\cdot}{'}}{r} - C.F = \frac{\sum_i \left(y_{i..}{'} + x\right)^2}{r} - C.F$

=

$$x^2 + cons\tan t\, terms\ \ independent\, of\ \ x - \frac{(G'+x)^2}{tr} - \left[\frac{\left(y_{i.}{}' + x\right)^2}{r} - \frac{(G'+x)^2}{tr}\right] - \left[\frac{(y.j'+x)^2}{t} - \frac{(G'+x)^2}{tr}\right]$$

$$= x^2 + cons\tan t\, terms\ independent\ \ of\ x - \frac{(G'+x)^2}{tr} - \frac{\left(y_{i.}{}' + x\right)^2}{r} + \frac{(G'+x)^2}{tr} - \frac{(y.j'+x)^2}{t} + \frac{(G'+x)^2}{tr}$$

$$= x^2 + cons\tan t\, terms\ independent\, of\ x - \frac{\left(y_{i.}{}' + x\right)^2}{r} - \frac{(y.j'+x)^2}{t} + \frac{(G'+x)^2}{tr}$$

Differentiate with respect to x

$$\frac{\partial(S.S.E)}{\partial x}=0$$

$$=2x-2\frac{(yi.'+x)}{r}-\frac{2(y.j'+x)}{t}+2\frac{(G'+x)}{tr}=\frac{0}{2}=0$$

$$x-\frac{\left(y_{i.}{}'+x\right)}{r}-\frac{\left(y_{ij}{}'+x\right)}{t}+\frac{(G'+x)}{tr}=0$$

$$\frac{trx-t(y_{i.}{}'+x)-r\left(y_{.j}{}'+x\right)+(G'+x)}{tr}=0$$

$$trx-t(yi.'+x)-r(y.j'+x)+(G'+x)=0\times tx=0$$

$$trx-ty_{i.}{}'+tx-ry_{.j}{}'+rx+(G'+x)=0$$

$$x(tr-t-r+1)-ty_{i.}{}'-ry_{.j}{}'+G'=0$$

$$x(tr-t-r+1)=ty_{i.}{}'+ry_{.j}{}'-G'$$

$$x((t-1)(r-1))=ty_{i.}{}'+ry_{.j}{}'-G'$$

$$x=\frac{ty_{i.}{}'+ry_{.j}{}'-G'}{(r-1)(t-1)}$$

*Estimation of two missing observations:

Suppose in RBD with t treatments and r Replications, two observations are missing. Let x and y be two missing observations and they belong two different Block and affected different treatment. We assume that x belongs to the jth to the ith treatments and y belong to ith block and mth treatment. Estimate the missing observations x and y.

*Layout of RBD:

Treat. / Blocks	**1**	**2**	**…..**	**i**	**…..**	**m**	**….**	**t**	**Total**
1	y_{11}	y_{21}	…..	y_{i1}	……			y_{t1}	$\mathbf{B_1}$
2	y_{12}	y_{22}	…..	y_{i2}	…..			y_{t2}	B_2
….	…		……	…..	…..				….
j				x					$B_j'+x$
….	…..	…..	…..	…		…..	….		….
i						y			$B_i' + y$
….									
r									
Total	T_1	T_2	…..	$T_i'+x$	….	$T_m'+y$	…..	……	$G'+x+y$

Where,

T_i' = Total of the known observations receiving ith treatment

T_m'= Total of the known observations receiving mth treatment

B_j'= Total of the known observations in jth block

B_i'= Total of the known observations in ith block

G'= Total of all the known observations

Correction factor = $\frac{G^2}{tr} = \frac{(G' + x + y)^2}{tr}$

Total sum of square =

$$\sum_i \sum_j y_{ij}^2 - \frac{G^2}{tr} = x^2 + y^2 + cons\tan t\, terms\ independent\, of\ x\, and\ y - \frac{(G' + x + y)^2}{tr}$$

Sum of square due to treatment (S.S.Tr) = $\frac{\sum_i y_i.'}{r} - C.F = \frac{\sum_i \left(y_{i..}' + x + y \right)^2}{r} - C.F$

Sum of square due to

Block (S.S.B) = $\frac{\sum_j y_{.j}{}^2}{t} - C.F = \frac{\left(y_{.j}' + x + y \right)^2}{r} - C.F$

S.S.E=T.S.S – S.S.Tr-S.S.B

$$\left| x^2 + y^2 + cons\tan t\, terms\, indepent\, of\, x\, and\ y - C.F \right|$$

$$= -\left[\left(\frac{T_i' + x}{r} \right)^2 + \left(\frac{T_m' + y}{r} \right)^2 - C.F \right] - \left[\left(\frac{B_j' + x}{t} \right)^2 + \left(\frac{B_i' + y}{t} \right)^2 - C.F \right]$$

$$= \left[\begin{array}{l} x^2 + y^2 + cons\tan t\, terms\, indepent\, of\ x\, and\ y - .C.F - \dfrac{\left(T_i^{'} + x\right)^2}{r} - \dfrac{\left(T_m^{'} + y\right)^2}{r} \\ C.F - \dfrac{\left(B_j^{'} + x\right)^2}{t} - \dfrac{\left(B_i^{'} + y\right)^2}{t} + C.F \end{array} \right]$$

$$= x^2 + y^2 - \frac{\left(T_i^{'} + x\right)^2}{r} - \frac{\left(T_m^{'} + y\right)^2}{r} - \frac{\left(B_j^{'} + x\right)^2}{t} - \frac{\left(B_i^{'} + y\right)^2}{t} + \frac{(G' + x + y)^2}{tr} \quad \ldots(1)$$

Differentiate with respect to x in equation (1)

$$\frac{\partial .S.S.E}{\partial x} = 0$$

$$2x - \frac{2(T_i^{'} + x)}{r} - \frac{2\left(B_j^{'} + x\right)}{t} + \frac{2(G' + x + y)}{tr} = 0$$

$$x - \frac{(T_i^{'} + x)}{r} - \frac{\left(B_j^{'} + x\right)}{t} + \frac{(G' + x + y)}{tr} = \frac{0}{2} = 0$$

$$\frac{xtr - t\left(T_i^{'} + x\right) - r\left(B_j^{'} + x\right) + \left(G' + x + y\right)}{tr} = 0$$

$$xtr - t\left(T_i^{'} + x\right) - r(B_j^{'} + x) + \left(G' + x + y\right) = 0 \times tr = 0$$

$$xtr - tT_i^{'} - tx - rB_j^{'} - rx + G' + x + y = 0$$

$$x(tr - t - r + 1) = tT_i^{'} + rB_j^{'} - G' - y$$

$$x = \frac{tT_i^{'} + rB_j^{'} - G' - y}{(t-1)(r-1)}$$

Differentiate with respect to y in equation (1)

$$\frac{\partial .S.S.E}{\partial y} = 0$$

$$2y - \frac{2(T_m^{'} + y)}{r} - \frac{2\left(B_i^{'} + y\right)}{t} + \frac{2(G' + x + y)}{tr} = 0$$

$$y-\frac{(T_m^{'}+y)}{r}-\frac{\left(B_i^{'}+y\right)}{t}+\frac{(G'+x+y)}{tr}=\frac{0}{2}=0$$

$$\frac{ytr-t\left(T_m^{'}+y\right)-r\left(B_i^{'}+y\right)+\left(G'+x+y\right)}{tr}=0$$

$$ytr-t\left(T_m^{'}+y\right)-r(B_i^{'}+y)+\left(G'+x+y\right)=0\times tr=0$$

$$ytr-tT_m^{'}-ty-rB_i^{'}-ry+G'+x+y=0$$

$$y(tr-t-r+1)=tT_m^{'}+rB_i^{'}-G'-x$$

$$y=\frac{tT_m^{'}+rB_i^{'}-G'-x}{(t-1)(r-1)}$$

* Statistical Analysis of missing plot technique:

Anova is performed in the usual way after substituting the estimated values of the observations. For each missing observation 1 d.f. is subtracted from total and consequently from error d.f. The adjusted treatment ss is obtained by subtracting the adjustment factor, $\frac{\left[y'_{.j}+ty'_{i.}-G'_{..}\right]^2}{\left[t(t-1)(r-1)^2\right]}$ from the treatment SS.

If the treatment show significant effect, then the S.E. of the difference between two treatment means is:

i) $\sqrt{\frac{2s_E^2}{r}}$,if none of the treatments contains the missing value; and

ii) $\sqrt{s_E^2\left(\frac{2}{r}+\frac{t}{r(r-1)(t-1)}\right)}$, if one of the treatments corresponds to missing observation.

We subtract 1 d.f for each estimated value from the total ss and error ss.

***Example: Estimation of one missing observation:**

Suppose that the value for treatment 2 is missing in replication III. The data will then be as presented in the table below.

		Replication			
Treatment	I	II	III	IV	**Total**
1	22.9	25.9	39.1	33.9	**121.8**
2	29.5	30.4	x	29.6	**89.5**
3	28.8	24.4	32.1	28.6	**113.9**
4	47.0	40.9	42.8	32.1	**162.8**
5	28.9	20.4	21.1	31.8	**102.2**
Total	**157.1**	**142.0**	**135.1**	**156.0**	**590.2**

We know that the missing observation can be estimated using the following formula:

$$x=\frac{ty_{i.}'+ry_{.j}'-G'}{(r-1)(t-1)}$$

……………….[1]

where

$y_{i.}'$ is total of known observations getting ith treatment

$y_{.j}'$ is total of known observations in jth block and

G' = total of all the known observations

Here, i = 2, j = 3

$r = 4$, $t = 5$, $y_{i.}' = \mathbf{89.5}$, $y_{.j}' = \mathbf{135.1}$, $G' = 590.2$

Thus, substituting in equation [1], we get

= 4(135.1) + 5(89.5) – 590.2/(3)(4)

= 397.7/12

= 33.1

After substituting the estimated missing value, we get

Treatment 2 total = 89.5 + 33.1 = 122.6,

Replication 3 total = 135.1 + 33.1 = 168.2, and

The grand total = 590.2 + 33.1 = 623.3

Treatment SS = ¼ [(121.8)² + (122.6)² + (113.9)² + (162.8)² + 102.2)²] – (623.3)²/20

= 19946.9725 – 19425.1445

= 521.8280

Now, Adj. SSt = SSt – A.F. = $\frac{1}{r}\sum_{i=1}^{t} T_{i.}^2 - C.F.$ - A.F

= 521.8280 – A.F.

Where, A.F. = $\dfrac{\left[y'_{.j} + ty'_{i.} - G'_{..}\right]^2}{\left[t(t-1)(r-1)^2\right]}$

= [135.1 – 4(33.1)]²/(5)(4)

= 7.29/20

= 0.3645

Hence, Adj. SSt = 521.8280 – 0.3645

= 521. 3645

Now,

Total SS. = Total S.S. = $\sum_{i=1}^{t}\sum_{j=1}^{r} y_{ij}^{2} - \frac{G^{2}}{rt}$ = 938.6160

SSB = $\frac{1}{t}\sum_{j=1}^{r} T_{.j}^{2} - C.F$

SSE= Total SS – Adj. SSt – SSB = 347. 9475

*ANOVA table:

Source of variation	df	SS	MS	F
Replication	3	69.1855	23.0618	1
Treatment	4	521.4635	130.3659	4.117
Error	11	347.9475	31.6316	
Total	18	938.9610		

Now, F_{tab} = F(11, 3) = 0.114112 < F_r cal.

Thus, replicates (blocks) are not homogeneous.

F_{tab}= F(4,11) = 3.3566 < $F_{cal.}$ at 5% level of significance.

Hence, we conclude that treatments are not homogeneous.

For this we compute the critical difference (C.D.) which is given by,

C.D.(treatment means) = $t_{(r-1)(t-1)}$ for $(\alpha/2)\%*\sqrt{\frac{2MSSE}{r}}$

C.D = $\sqrt{s_E^2\left(\frac{2}{r}+\frac{t}{r(r-1)(t-1)}\right)}$ if one of the treatments corresponds to missing observation.

= 4.3705

Treatments	Treatments Total	Means	CD
1	**121.8**	**30.45**	**10.31**
2	**122.6**	**30.65**	**4.3705***
3	**113.9**	**28.475**	**10.31**
4	**162.8**	**40.7**	**10.31**
5	**102.2**	**25.55**	**10.31**

* Since the observation is missing with respect to treatment 2.

On comparing absolute difference between any two treatment means with CD we conclude that whether the corresponding pair is homogeneous or not.

(Note: Students are requested to compare the absolute mean difference and write your conclusion)