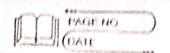
Sufficiency.



DI Let x, x, ... x, be a random sample from $N(H, \sigma^2)$ population. Find the sufficient estimators for H and σ^2 .

Solution: Here $\Theta = (\mu, \sigma^2)$; -ozpizo

We have L = f(x, 0,) f(x, 0) ... f(x, 0)

 $-\frac{n}{n}f(x_i,o_i)=\frac{n}{n}f(x_i)$

(Since x, x, x, x, N (4,0))

 $= \frac{1}{\sigma \int \partial \overline{u}} \left(\frac{\partial u}{\partial x} \right) \frac{1}{\sigma \int \partial \overline{u}} \left(\frac{\partial u}{\partial x} \right) \frac{\partial u}{\partial x} = \frac{1}{\sigma \int \partial \overline{u}} \left(\frac{\partial u}{\partial x} \right) \frac{\partial u}{\partial x}$

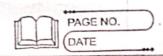
 $= \left(\frac{1}{\sigma \sqrt{3\pi}}\right)^{n} = \left(\frac{x_{1} - \mu}{\sigma}\right)^{2} = \left(\frac{x_{1} - \mu}{\sigma$

 $= \left(\frac{1}{a}\right)^{2} e^{-\frac{1}{a}} \left\{ \frac{2}{12} \left(x_{i} - \mu\right)^{2} \right\}.$

 $= \left(\frac{1}{\sigma \sqrt{\sigma_{ij}}}\right) \cdot e^{\frac{1}{2\sigma^{2}}\left\{i\left(x_{i}^{2} - 2\mu.x_{i}^{2} + \mu^{2}\right)\right\}}$

 $= \left(\frac{1}{\sigma \sqrt{2\pi}}\right) \cdot e^{-\frac{1}{2\sigma^2}\left\{\frac{2}{2}x_1^2 - 2\mu_1\xi x_1 + 2\mu_1^2\right\}}$

= $g_0[t(x)].h(x)$



where
$$g_{e}[t(x)] = \left(\frac{1}{\sqrt{\partial \pi}}\right)^{n} e^{-\frac{1}{2\sigma^{2}}\left\{t_{3}(x) - 2\mu t_{i}(x) + n\mu^{2}\right\}}$$

$$t(x) = \left\{t_{i}(x), t_{3}(x)\right\} = \left\{\xi x_{i}, \xi x_{i}^{2}\right\}$$

and h(x) = 1.

Thus Ex; is sufficient for \u and Ex; is sufficient for or.

a Let x, x, ... x, be a random sample from

 $f(x;0) = 0x^{0-1}; 0<x<1, 0>0.$

Show that to some t,= 11 X; is sufficient

Solution: We have $L = f(x_1, 0) \cdot f(x_2, 0) \cdot \cdots \cdot f(x_n, 0)$.

$$L = \emptyset x_1 \cdot 0 x_2 \cdot 0 x_n$$

 $\frac{n}{-0^n} \frac{n}{11} \frac{0}{2}$



$$L = O' \cdot \prod_{i=1}^{n} x_{i}^{O} \cdot \prod_{i=1}^{l} x_{i}^{O}$$

$$=g(t_{n,0}).h(x_{n},x_{2},...x_{n}).$$

Hence by Factorization Theorem, $t_1 = \overline{U} \times_i$ is sufficient for 0.

Let $x_1, x_2, \dots x_n$ be a random sample of in observations from Poisson population having parameter λ . Find a sufficient statistic for λ ? Q3.

Solution: $P(x=x) = e^{-\lambda} \frac{x}{\lambda} \quad x = 0, 1, 2, \dots$

 $P(x, \lambda) \cdot P(x, \lambda) \cdot \cdot \cdot P(x, \lambda)$ $= e^{\lambda_1 \lambda_2} \cdot e^{\lambda_1 \lambda_2} \cdot e^{\lambda_1 \lambda_2} \cdot e^{\lambda_1 \lambda_2}$

 $= \frac{-n\lambda}{x_1!} \frac{x_1 + x_2 + \cdots + x_n}{x_n!}$

 $= e \qquad \lambda = g(t,\lambda) \cdot h(x,x,x,x_n)$ $= \frac{1}{1} x_i!$ $= \frac{1}{2} x_i!$

	Thus $t_i = Sx_i = n\bar{x}$ is a sufficient Statistic for λ . Statistic for λ . DATE	
4	Let $x_i, x_j = x_i$ be a random sample x_i observations from a distribution x_i x_i $x_j = x_i$ x_i x_i $x_j = x_i$ x_i $x_j = x_i$ x_i $x_j = x_i$ x_i $x_j = x_i$ $x_j = x_j = x_j$	oith
	Find a sufficient statistic for 0?	
	Solution: $L = f(x, 0) \cdot f(x, 0) \cdot f(x, 0)$ $= \frac{1}{10} \cdot e^{-x} \cdot \frac{1}{10} \cdot e^{-x} \cdot \frac{1}{10}$	-x, 0-1
	- 10 To To	- E · Xn
	$= (1)^n - \xi x, \qquad m o-1$	2
	$= (\overline{ro})^n \qquad \stackrel{=}{\underset{i=1}{\overline{ro}}} x_i^n$	
	π̄ χ;	
	$= g(t, o) \cdot h(x_1, x_2, \dots, x_n)$	
	where $g(t_1,0) = \frac{1}{(\overline{lo})}$, $\frac{1}{\overline{l}} \times 0$	
	(10)	

· . If x, is sufficient for 0.