Hypotheses Testing about the Difference in Two Means : Independent Samples and Population Variances Unknown and unequal

Session 21

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- In this case, we test hypotheses about the difference in two population means when the population variances are unknown and un equal.
- If the population variances are not known, the z methodology is NOT appropriate.
- An assumption underlying this technique is that the measurement or characteristic being studied is normally distributed for both populations.
- Also we consider both samples are independent

t formula to test the difference in means assuming σ_1^2 and σ_2^2 are unknown and not equal

$$t = \frac{(\overline{x_1}) - (\overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$df = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

- Because this formula requires a more complex degrees-of-freedom component, it may be unattractive to some users. Many statistical computer software packages offer the user a choice of the “pooled” formula or the “unpooled” formula. The “pooled” formula in the computer packages is formula including S_p, in which equal population variances are assumed.

Test-6b:Test about difference in population means (Population variances are unknown and unequal, Characteristic being studied is Normally distributed, Independent samples)

	Left tail test	Right tail test	Two tail test
Hypotheses	$H_0:\mu_1-\mu_2= 0$ $H_1:\mu_1-\mu_2 < 0$	$H_0:\mu_1-\mu_2= 0$ $H_1:\mu_1-\mu_2 > 0$	$H_0:\mu_1-\mu_2= 0$ $H_1:\mu_1-\mu_2 \neq 0$
Test statistic	$t = \dfrac{(\overline{x_1}) - (\overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ $\quad df = \dfrac{\left[\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right]^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$		
Rejection Rule	Reject H_0 if $t \leq -t_{\alpha,df}$	Reject H_0 if $t \geq t_{\alpha,df}$	Reject H_0 if $\lvert t\rvert \geq t_{\frac{\alpha}{2},df}$

- Example : A researcher is interested in finding the average duration of marriage based on the educational qualifications of couples. Two groups were considered for the study. Group 1 consisted of couples with no Bachelor’s degree (both partners) and Group 2 consisted of couple who both have Bachelor’s degree or higher. The data in the following table shows average duration of marriage in years. At α=0.05, test whether the average duration of marriage is more for couples with no Bachelor’s degree as compared to couples with Bachelor’s degree.

Group	Sample size	Duration of marriage in years	Standard Deviation estimated from sample
Couples with no Bachelor’s degree	120	10.1 years	2.4 years
Couples with Bachelor’s degree or higher	100	9.5 years	3.1 years

Solution

- Assumptions : Duration of marriage (in years) data are normally distributed and the population variances are unequal, but unknown. Samples are independent.
- Given $\alpha = 0.05$
- $\bar{x}_1$= 10.1, $\bar{x}_2$= 9.5, s_1= 2.4 and s_2= 3.1, n_1= 120, n_2= 100

Assuming population 1: Couples with no Bachelor's degree and population 2: Couples with Bachelor's degree or higher.

- The null and alternative hypothesis are

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_1: \mu_1 - \mu_2 > 0$$

Solution(contd…)

- The test statistic is $$t = \frac{(\overline{x_1}) - (\overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$
- Calculated t =1.5805
- $df = \frac{\left[\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right]^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$=184
- Now for $\alpha = 0.05$, $t_{\alpha,df} = t_{0.05,184}$= 1.653.
- Rejection rule is reject H_0 if $t \geq t_{\alpha,df}$
- Since 1.5805 < 1.653, we do not reject the null hypothesis . Hence there is no statistical evidence to conclude that the average duration of marriage is more for couples with no Bachelor's degree as compared to couples with Bachelor's degree.