20-PBD-002

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CIA-II Assignment

Introduction to Econometrics

Q1 (i) Explain the concept of cointegration and error correction mechanism.

Ans. If there is ~~some~~ a linear combination of non-stationary I(1) variables that is stationary (I(0)), then the variables are said to be cointegrated. Cointegration implies that the variables have a similar stochastic trend and never diverge too far from each other.

A way to test whether two variables y_t & x_t are cointegrated is to check whether the residuals $e_t = y_t - \beta_1 - \beta_2 x_t$ are stationary. If the residuals are stationary, then the variables are cointegrated.

The error correction mechanism is a method used in multivariate time series analysis to address the problem of cointegration.

The cointegrated model of Drunkard and her Dog is defined as:

Drunkard: $x_t - x_{t-1} = u_t + c(y_{t-1} - x_{t-1})$

Dog: $y_t - y_{t-1} = w_t + d(x_{t-1} - y_{t-1})$

where u_t, w_t are white noise terms, and second terms of RHS represent error correction terms.

These error correction terms are stationary

Q1. (ii) Specify two time series variables and explain how the above mentioned concepts can be used in analysis.

Ans Consider the two time series variables

(1) Cost of caustic soda ($NaOH$) (that is used ~~a~~ in manufacturing of soap)

(2) Cost of a particular brand of soap (say Lux)

If we consider individually, both these time series variables are stochastic in nature, and may or may not be stationary. If we find both these time series are non-stationary $I(1)$, we can check ~~whether their residu~~ for whether there is cointegration between two time series variables, and if there is, we will need to include the error correction ~~mod~~ mechanism while modelling each of them.

Q. 2. (i) Explain the idea of VAR and define structural, reduced and recursive form equations.

Ans. It can be observed that all variables are in some way interrelated to each other and we do not know which variables are truely exogeneous.

Vector-AutoRegressive Models (VAR) aim to model the interdependencies between variables without imposing arbitary assumptions on the data.

VAR is an extension of auto regressive models, generalised to include the dynamic interrelationship between stationary variables

Considering only 2 variables, and 1 period lag, VAR models are defined as:

1) Structural form:

$$X_t = \eta Z_t + \theta_{1,1} X_{t-1} + \theta_{1,2} Z_{t-1} + u_{1,t}$$
$$Z_t = \gamma X_t + \theta_{2,1} X_{t-1} + \theta_{2,2} Z_{t-1} + u_{2,t}$$

where

X_t, Z_t are endogeneous variables

2) Reduced form:

Since we cannot exstimate the parameters in structural form using OLS, we derive the reduced form:

$$X_t = \beta_{1,1} X_{t-1} + \beta_{1,2} Z_{t-1} + \varepsilon_{1,t}$$
$$Z_t = \beta_{1,2} X_{t-1} + \beta_{2,2} Z_{t-1} + \varepsilon_{2,t}.$$

3) Recursive Form:

Recursive form of VAR contains all the components of reduced form but also allows some ~~function be~~ variables to be functions of other concurrent variables. These short ~~term~~ run relationships allows us to model structural shocks.

Q. 2.ii) Specify a VAR process. Construct Impulse Response Functions for 5 time periods and explain the same.

Ans Consider the time series variables price of wheat, price of rice and price of barley. To analyse these variables, we can use a VAR model

The steps to build a VAR model are:

1. Specify the model
2. Check for stationarity and co-integration
3. Determine the optimal lag.
4. Estimate the parameters of VAR.

Impulse ~~control~~ Response Functions are used to study interactions between variables in VAR model. It is used to describe the evolution of a model's ~~shock~~ variables in reaction to shock in one or more variables.

Example consider the model:

$$X_t = 0.30X_{t-1} + 0.20Y_{t-1} + \varepsilon_{x,t}.$$
$$Y_t = 0.10X_{t-1} + 0.40Y_{t-1} + \varepsilon_{y,t},$$

and assume $X_0 = 0,\ Y_0 = 0$

Considering one time, one unit shock to $\varepsilon_{x,t}$, keeping all $\varepsilon_s = 0$, we have

period (1) : $\hat{X}_1 = 0.30(0) + 0.20(0) + (1) = 1$
$\hat{Y}_1 = 0.10(0) + 0.40(0) + (0) = 0.$

period (2) : $\hat{X}_2 = 0.30(1) + 0.20(0) + (0) = 0.30.$
$\hat{Y}_2 = 0.10(1) + 0.40(0) + (0) = 0.10.$

period (3): $\hat{X}_3 : 0.30(0.30) + 0.20(0.10) + (0) = 0.11.$
$\hat{Y}_3 : 0.10(0.30) + 0.40(0.10) + (0) = 0.07.$

period (4) : $\hat{X}_4 : 0.30(0.11) + 0.20(0.07) + (0) = 0.047$
$\hat{Y}_4 : 0.10(\overset{(0.11)}{\cancel{0.07}}) + 0.40(\overset{(0.07)}{\cancel{0.07}}) + (0) = 0.039.$

period (5) : $\hat{X}_5 : 0.30(0.047) + 0.20(0.039) + 0 = 0.0219$
$\hat{Y}_5 \quad 0.10(0.047) + 0.40(0.039) + 0 = 0.0203.$