Date: __/__/____ Page no: ____

Shraddha P JAIN

20-PBD-002

CIA-II Assignment

Time Series Assignment

Q.1(a) Explain stationarity of time series distinctly. Is the series

$$y_t = 2 + 0.2 y_{t-1} + 1$$ stationary?

Ans. A time series is defined as stationary if a shift in time doesn't cause a change in the shape of the distribution.

- The basic distribution we are talking about is mean, variance and autocovariance.
- Also, for stationary series, absolute values of roots of characteristic equations should be less than 1, i.e, it should remain within a unit circle.
- First order stationarity implies that the mean of the series does not change with time. Any other moment like variance can change. This is also called weak stationarity.
- Second order stationarity implies that the series has a constant mean, variance and autocovariance that does not change with time. It is also called strong stationarity.

A stationary time series is a series whose properties do not depend on time at which the series is observed. It is a series which does not have trend and seasonality.

To check $Y_t = 2 + 0.2Y_{t-1} + 1$ is stationary, we have to check whether its characteristic roots lie between +1 and −1

$$Y_t = 3 + 0.2Y_{t-1}$$

introducing the Lag operator

$$Y_t = 3 + 0.2LY_t$$

$$Y_t(1-0.2L) = 3$$

It means that $(1-0.2L) = 0$

$$\Rightarrow \quad L^{-1} - 0.2 = 0$$

Let $L^{-1} = \lambda$

$$\Rightarrow \quad \lambda = 0.2$$

$\because$ $|\lambda| < 1$, therefore the series given by $Y_t = 2 + 0.2Y_{t-1} + 1$ is stationary

Q1.(b) Explain exponential smoothing method of forecast. Why do we call it exponential.

Ans The exponential smoothing method is widely used in business decisions. The main importance of this method is that it fundamentally depends upon the actual observations and prediction for that period in the past.

It has the ~~fedal~~ feature that the formula of the forecast can be extended over a period of time.

This method is called exponential as it gives higher importance to the present data and importance decreases as the data moves further in the past. These 'importance' can be called weights, and be mathematically calculated, and its flow of decrement is exponential in nature.

$$F_{t+1} = F_t + \alpha(A_t - F_t) \quad \text{—— (1)}$$

where A_t = actual value at time t.
F_t = forcast at time t.
α = weights

from eq (1), we have

$$F_{t+1} = \cancel{\alpha A_t}\ \alpha A_t + (1-\alpha) F_t \quad \text{— (2)}$$

If $\alpha = 1$, then it says that the actual observation of yesterday is the best prediction for today.

If $\alpha = 0$, it means that forecast of yesterday is the best prediction for today.

Normally $0 < \alpha < 1$.

For α closer to 0, F_t has greater impact on F_{t+1} and α closer to 1 means A_t has greater impact on A_{t+1}.

Repeating the same eqⁿ of F_t.

$$F_t = \alpha A_{t-1} + (1-\alpha) F_{t-1} \quad \text{— (3)}$$

we have.

$$F_{t+1} = \alpha A_t + (1-\alpha) F_t$$
$$= \alpha A_t + (1-\alpha)(\alpha A_{t-1} + (1-\alpha) F_{t-1})$$
$$= \alpha A_t + \alpha(1-\alpha) A_{t-1} + (1-\alpha)^2 F_{t-1} \quad \text{— (4)}$$

The weights assigned decrease exponentially as the terms advance, and because of this, this method is called exponential smoothing.

1 (c) Let $A_t = 24$, $A_{t-1} = 16$, $F_{t-1} = 14$ $\alpha = 0.4$, where the ~~symols~~ symbols are those of exponential smoothing.

Find F_{t+1}.

Ans As we know,

$\cancel{F_{t+1} = F_t}$

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

$$F_t = 14 + 0.4(16-14)$$
$$= 14 + 0.8$$
$$= 14.8.$$

$$F_{t+1} = F_t + \alpha(A_t - F_t).$$
$$= 14.8 + (0.4)(24-14.8)$$
$$= 14.8 + 3.68$$
$$= 18.48$$

$$\therefore \boxed{F_t = 14.8}$$

$$\boxed{F_{t+1} = 18.48}$$

Q. 2(a) Explain AR(2) and AR(1) models. Also explain the error terms

Ans There are two types of AutoRegressive models. These models are planned according to given data and its type. These models are designed for forecasting.

In many cases, the two models can cordinate together and give better results.

AR models are the ones in which the current values are most likely understood to depend on past values.

These past values may be from last (previous) year to say 'p' years. This 'p' may be 1, 2, .. or any number, but we use such a p that gives ~~most~~ most accurate predictions

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots \quad -(1)$$
$$(1-x)^{-1} = 1 + x + x^2 + \cdots \quad -(2)$$

are the standard results that we will use.

the operator L is lag operator.

$L(Y_t) = Y_{t-1}$

$L^2(Y_t) = Y_{t-2}$.

In general $L^k(Y_t) = Y_{t-k}$. $\quad -(3)$

AR(1) model is model with one unit back in time (i.e, $p=1$)
It is given as:

$$AR(1) \Rightarrow Y_t = \mu + \phi Y_{t-1} + \varepsilon_t$$

If we wish to use 2 observations of past 2 units, we need AR(2) model.
This is given by

$$AR(2) \Rightarrow Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$$

ε_t is the error term, which is completely random, but is assumed to be $N(0, \sigma^2)$.

ε_t represents the actual error of today.

Q.2(b) Explain MA models and invertible property of MA(1) model.

Ans Moving Average model is used when the current value depends on only error terms in the past data. We know error terms follow 'white noise'

When MA model depends on 'q' of its ~~past~~ past value errors, the model is denoted by MA(q).
It is given by:

$$Y_t = \beta_0 + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t \quad (1)$$

ε_t is the error likely to occur. It follows white noise, and $E(\varepsilon_t) = 0$, $V(\varepsilon_t) = 1$.

$$Y_t = \beta_0 + \theta U_{t-1} + \theta_2 U_{t-2} \ldots \cancel{\theta} + U_t \quad - (2)$$

θ are the parameters, estimated by maximum likelihood. we can use linear regression model because of white noise nature of error terms

$$MA(1) \Rightarrow Y_t = \mu + \theta_1 U_{t-1} + U_t$$

$$MA(2) \Rightarrow Y_t = \mu + \theta_1 U_{t-1} + \theta_2 U_{t-2} + U_t$$

Invertible property of MA(1):
We can mathematically show that MA(1) model is equivalent to ~~AR(2)~~ AR(∞) model. This is the invertible property of MA(1) model.

This can be shown as:

$$MA(1): \quad Y_t = A + BU_{t-1} + U_t \quad - (1)$$

Introducing the lag operator:

$$Y_t = A + BL(U_t) + U_t$$

$$Y_t = A + (1+BL)U_t \quad - (2)$$

(2) - Can be written as

$$U_t = \frac{Y - A}{1+BL}$$

$$= (Y-A)(1-BL)^{-1}$$

$\Rightarrow (Y_t - A)(1 - BL + (BL)^2 - (BL)^3 + \dots) = U_t$.

$\Rightarrow [1 - BL + (BL)^2 - \dots] Y_t - A[1 - BL + (BL)^2 + \dots] = U_t$.

Applying the L operator, we have.

$(Y_t - BY_{t-1} + B^2 Y_{t-2} - B^3 Y_{t-3} + \dots) - A(1 - B + B^2 \dots) = U_t$.

$\Rightarrow (Y_t - BY_{t-1} + B^2 Y_{t-2} \dots) - A(1-B)^{-1} = U_t$ = today's error. — ③

③ is the form of AR(∞).

Hence MA(1) is equivalent to AR(∞) model.

Q2 (c) Let general forecasting formula be:

$$\hat{Y}_t = 10 + 0.6\, Y_{t-1}.$$

You have $Y_{t-2} = 8$, $Y_{t-1} = 16$, $Y_t = 20$.

find errors U_{t-1} and U_t.

Ans We have:

$$\hat{Y}_t = 10 + 0.6\, Y_{t-1} \quad \text{— ①}$$

Now,

We know that

$$U_t = Y_t - \hat{Y}_t \quad \text{②}$$

So,

$$U_{t-1} = Y_{t-1} - \hat{Y}_{t-1}$$

$$U_{t-1} = Y_{t-1} - (10 + 0.6\,Y_{t-2})$$

$$U_{t-1} = 16 - (10 + (0.6)(8))$$

$$U_{t-1} = 16 - (10 + 4.8)$$

$$U_{t-1} = 16 - 14.8$$

$$\boxed{U_{t-1} = 1.2}$$

and

$$U_t = Y_t - \hat{Y}_t$$

$$= Y_t - (10 + 0.6\,Y_{t-1})$$

$$= 20 - (10 + 0.6(16))$$

$$= 20 - (10 + 9.6)$$

$$= 20 - 19.6$$

$$\boxed{U_t = 0.4}$$