Testing Hypotheses about a Population Variance

Session 17

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Testing Hypotheses about a Population Variance

- In the area of statistical quality control, manufactures try to produce equipments and parts that are consistent in measurement.
- Suppose a company produces industrial wire that is specified to be a particular thickness. Because of the production process, the thickness of the wire will vary slightly from lot to lot and batch to batch.
- By conducting hypotheses tests for the variance of the thickness measurements, the quality control people can monitor for variations in the process.

Chi-square (χ^2)distribution

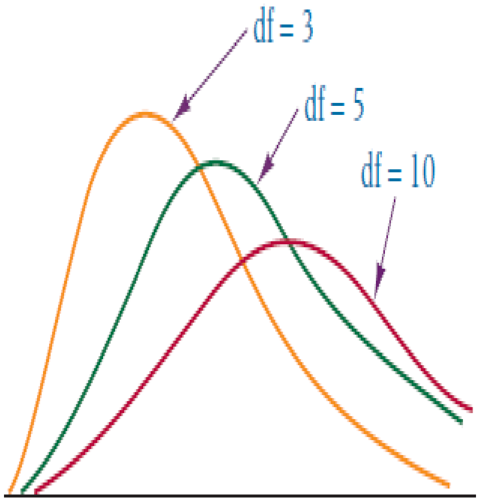
- We know that sample variance can be calculated using the formulae

$$S^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$$

- The relationship of the sample variance to the population variance is captured by the chi-square distribution.
- <u>The ratio of sample variance (S^2), multiplied by (n-1), to the population variance (σ^2)</u> is approximately chi-square distributed, <u>if the population is known to be normally distributed.</u>

Family of chi-square distributions

Three Chi-Square Distributions



Chi-square formula for a single variance

- Like t distribution, the chi-square distribution varies by sample size and contains a degrees of freedom value. The chi-square formula is

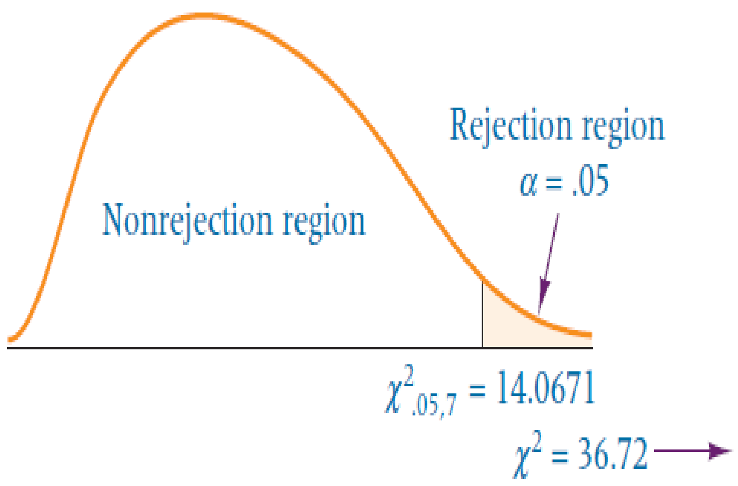
$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

$$df = (n-1)$$

- The chi-square distribution is not symmetrical and its shape will vary according to the degrees of freedom.

Rejection and non-rejection region (right tail test)

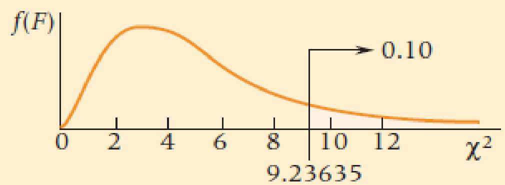
Hypothesis Test Distribution for Pneumatic Tube Example



Chi-square table

The Chi-Square Table

Values of χ^2 for Selected Probabilities



Example: df (Number of degrees of freedom) = 5, the tail above $\chi^2 = 9.23635$ represents 0.10 or 10% of area under the curve.

Degrees of Freedom	Area in Upper Tail .995	.99	.975	.95	.9	.1	.05	.025	.01	.005
1	0.0000393	0.0001571	0.0009821	0.0039322	0.0157907	2.7055	3.8415	5.0239	6.6349	7.8794
2	0.010025	0.020100	0.050636	0.102586	0.210721	4.6052	5.9915	7.3778	9.2104	10.5965
3	0.07172	0.11483	0.21579	0.35185	0.58438	6.2514	7.8147	9.3484	11.3449	12.8381
4	0.20698	0.29711	0.48442	0.71072	1.06362	7.7794	9.4877	11.1433	13.2767	14.8602
5	0.41175	0.55430	0.83121	1.14548	1.61031	9.2363	11.0705	12.8325	15.0863	16.7496
6	0.67573	0.87208	1.23734	1.63538	2.20413	10.6446	12.5916	14.4494	16.8119	18.5475
7	0.98925	1.23903	1.68986	2.16735	2.83311	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.34440	1.64651	2.17972	2.73263	3.48954	13.3616	15.5073	17.5345	20.0902	21.9549
9	1.73491	2.08789	2.70039	3.32512	4.16816	14.6837	16.9190	19.0228	21.6660	23.5893
10	2.15585	2.55820	3.24696	3.94030	4.86518	15.9872	18.3070	20.4832	23.2093	25.1881
11	2.60320	3.05350	3.81574	4.57481	5.57779	17.2750	19.6752	21.9200	24.7250	26.7569
12	3.07379	3.57055	4.40378	5.22603	6.30380	18.5493	21.0261	23.3367	26.2170	28.2997

Confidence Interval

CONFIDENCE INTERVAL TO ESTIMATE THE POPULATION VARIANCE (8.6)

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

$$\text{df} = n - 1$$

Example

- The U.S. Bureau of Labor Statistics publishes data on the hourly compensation costs for production workers in manufacturing for various countries. The latest figures published for Greece show that the average hourly wage for a production worker in manufacturing is $16.10. Suppose the business council of Greece wants to know how consistent this figure is. They randomly select 25 production workers in manufacturing from across the country and determine that the standard deviation of hourly wages for such workers is $1.12. Use this information to develop a 95% confidence interval to estimate the population variance for the hourly wages of production workers in manufacturing in Greece. Assume that the hourly wages for production workers across the country in manufacturing are normally distributed.

Solution

- Given standard deviation, S = 1.12, we can obtain the sample variance, S^2= 1.2544 which is the point estimate of the population variance.
- Also given sample size, n=25, the degrees of freedom, n - 1, are 24.
- A 95% confidence interval to estimate population variance is $\frac{(n-1)S^2}{{\chi^2}_{0.025}} \leq \sigma^2 \leq \frac{(n-1)S^2}{{\chi^2}_{0.975}}$.
- From chi-square table, ${\chi^2}_{0.025,24}$ =39.3641 and ${\chi^2}_{0.975}$ =12.40115.
- Substituting and simplifying, we get, 0.7648 $\leq \sigma^2 \leq$2.4276.
- Interpretation :??

Test-4:Test for population variance σ^2
(Assumption : Population is Normally distributed)

	Left tail test	Right tail test	Two tail test
Hypotheses	H_0: $\sigma^2 = \sigma_0^2$ H_1: $\sigma^2 < \sigma_0^2$	H_0: $\sigma^2 = \sigma_0^2$ H_1: $\sigma^2 > \sigma_0^2$	H_0: $\sigma^2 = \sigma_0^2$ H_1: σ^2 # σ_0^2
Test Statistic	$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$	$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$	$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$
Rejection Rule	Reject H_0 if $\chi^2 \leq \chi^2_{(1-\alpha),\, n-1}$	Reject H_0 if $\chi^2 \geq \chi^2_{\alpha, n-1}$	Reject H_0 if $\chi^2 \leq \chi^2_{(1-\alpha/2), n-1}$ or if $\chi^2 \geq \chi^2_{(\alpha/2), n-1}$

Example

- Previous experience shows the variance of a given process to be 14. Researchers are testing to determine whether this value has changed. They gather the following dozen measurements of the process. Use these data and to test the null hypothesis about the variance. Assume the measurements are normally distributed.
- 52, 44, 51, 58, 48, 49, 38, 49, 50, 42, 55, 51

Solution

- H_0: $\sigma^2 = \sigma_0^2$ against H_1: $\sigma^2 \# \sigma_0^2$ where $\sigma_0^2 = 14$
- The sample variance S^2=30.08
- Let $\alpha = 0.01$
- Test statistic $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{11*30.08}{14}$=23.63
- Reject H_0 if $\chi^2 \leq \chi^2_{(1-\alpha/2),n-1}$ or if $\chi^2 \geq \chi^2_{(\alpha/2),n-1}$
- $\chi^2_{0.995,11}$= 2.60320 and $\chi^2_{0.005,11}$= 26.7569
- Since the calculated value of χ^2 does not lie in the rejection region, we do not reject H_0. There is enough statistical evidence to say that shows the variance of a given process to be 14.