18.2 Smoothing Methods

Many manufacturing environments require forecasts for thousands of items weekly or monthly. Thus, in choosing a forecasting technique, simplicity and ease of use are important criteria. The data requirements for the techniques presented in this section are minimal, and the techniques are easy to use and understand.

In this section we discuss three forecasting methods: moving averages, weighted moving averages, and exponential smoothing. The objective of each of these methods is to "smooth out" the random fluctuations caused by the irregular component of the time series, therefore they are referred to as smoothing methods. Smoothing methods are appropriate for a stable time series—that is, one that exhibits no significant trend, cyclical, or seasonal effects—because they adapt well to changes in the level of the time series. However, without modification, they do not work as well when significant trend, cyclical, or seasonal variations are present.

Smoothing methods are easy to use and generally provide a high level of accuracy for short-range forecasts, such as a forecast for the next time period. One of the methods, exponential smoothing, has minimal data requirements and thus is a good method to use when forecasts are required for large numbers of items.

Moving Averages

The **moving averages** method uses the average of the most recent n data values in the time series as the forecast for the next period. Mathematically, the moving average calculation is made as follows.

MOVING AVERAGE

$$\text{Moving average} = \frac{\Sigma(\text{most recent } n \text{ data values})}{n} \tag{18.1}$$

TABLE 18.1
GASOLINE SALES TIME SERIES

Week	Sales (1000s of gallons)
1	17
2	21
3	19
4	23
5	18
6	16
7	20
8	18
9	22
10	20
11	15
12	22

The term *moving* is used because every time a new observation becomes available for the time series, it replaces the oldest observation in equation (18.1) and a new average is computed. As a result, the average will change, or move, as new observations become available.

To illustrate the moving averages method, consider the 12 weeks of data in Table 18.1 and Figure 18.5. These data show the number of gallons of gasoline sold by a gasoline distributor in Bennington, Vermont, over the past 12 weeks. Figure 18.5 indicates that, although random variability is present, the time series appears to be stable over time. Hence, the smoothing methods of this section are applicable.

To use moving averages to forecast gasoline sales, we must first select the number of data values to be included in the moving average. As an example, let us compute forecasts using a three-week moving average. The moving average calculation for the first three weeks of the gasoline sales time series is

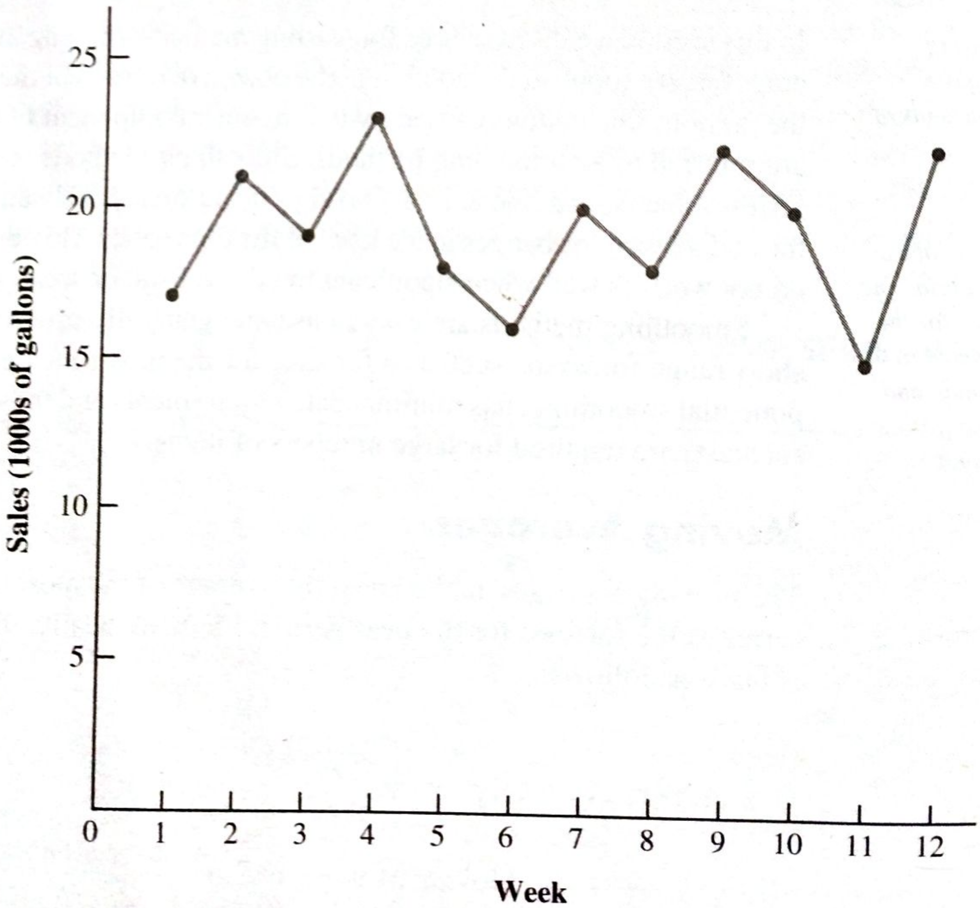
$$\text{Moving average (weeks 1–3)} = \frac{17 + 21 + 19}{3} = 19$$

We then use this moving average as the forecast for week 4. Because the actual value observed in week 4 is 23, the forecast error in week 4 is $23 - 19 = 4$. In general, the error associated with any forecast is the difference between the observed value of the time series and the forecast.

The calculation for the second three-week moving average is

$$\text{Moving average (weeks 2–4)} = \frac{21 + 19 + 23}{3} = 21$$

FIGURE 18.5 GASOLINE SALES TIME SERIES



Hence, the forecast for week 5 is 21. The error associated with this forecast is 18 − 21 = −3. Thus, the forecast error may be positive or negative depending on whether the forecast is too low or too high. A complete summary of the three-week moving average calculations for the gasoline sales time series is provided in Table 18.2 and Figure 18.6.

Forecast accuracy is not the only consideration. Sometimes the most accurate method requires data on related time series that are difficult or costly to obtain. Trade-offs are often made between cost and forecast accuracy.

Forecast Accuracy An important consideration in selecting a forecasting method is the accuracy of the forecast. Clearly, we want forecast errors to be small. The last two columns of Table 18.2, which contain the forecast errors and the squared forecast errors, can be used to develop a measure of forecast accuracy.

For the gasoline sales time series, we can use the last column of Table 18.2 to compute the average of the sum of the squared errors. Doing so we obtain

$$\text{Average of the sum of squared errors} = \frac{92}{9} = 10.22$$

This average of the sum of squared errors is commonly referred to as the **mean squared error (MSE)**. The MSE is an often-used measure of the accuracy of a forecasting method and is the one we use in this chapter.

As we indicated previously, to use the moving averages method, we must first select the number of data values to be included in the moving average. Not surprisingly, for a particular time series, moving averages of different lengths will differ in their ability to forecast the time series accurately. One possible approach to choosing the number of values to be included in the moving average is to use trial and error to identify the length that minimizes the MSE. Then, if we are willing to assume that the length that is best for the past will also be best for the future, we would forecast the next value in the time series by using the number of data values that minimized the MSE for the historical time series. Exercise 2 at the end of the section will ask you to consider four-week and five-week moving averages for the gasoline sales data. A comparison of the MSEs will indicate the number of weeks of data you may want to include in the moving average calculation.

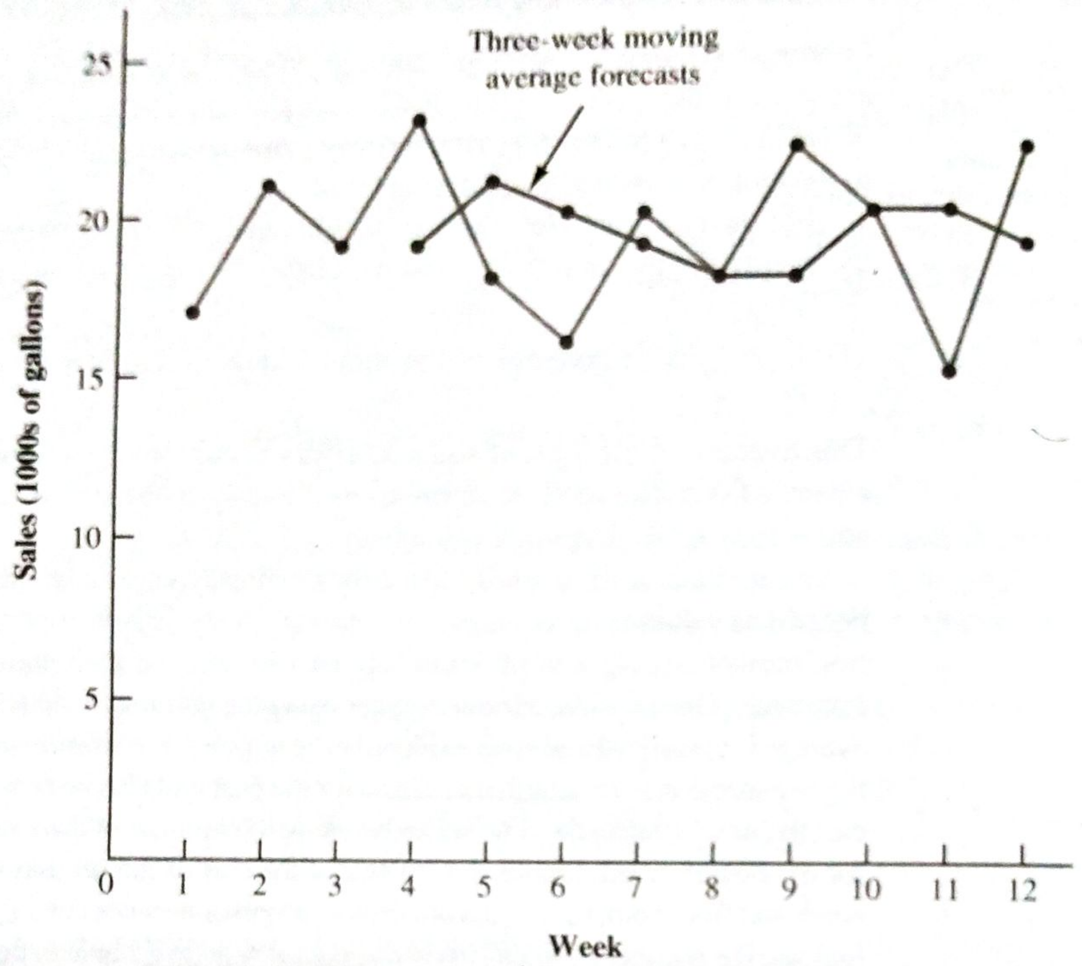
Weighted Moving Averages

In the moving averages method, each observation in the moving average calculation receives the same weight. One variation, known as **weighted moving averages**, involves selecting a different weight for each data value and then computing a weighted average of the most recent n values as

TABLE 18.2 SUMMARY OF THREE-WEEK MOVING AVERAGE CALCULATIONS

Week	Time Series Value	Moving Average Forecast	Forecast Error	Squared Forecast Error
1	17			
2	21			
3	19			
4	23	19	4	16
5	18	21	−3	9
6	16	20	−4	16
7	20	19	1	1
8	18	18	0	0
9	22	18	4	16
10	20	20	0	0
11	15	20	−5	25
12	22	19	3	9
		Totals	0	92

FIGURE 18.6 GASOLINE SALES TIME SERIES AND THREE-WEEK MOVING AVERAGE FORECASTS



the forecast. In most cases, the most recent observation receives the most weight, and the weight decreases for older data values. For example, we can use the gasoline sales time series to illustrate the computation of a weighted three-week moving average, with the most recent observation receiving a weight three times as great as that given the oldest observation, and the next oldest observation receiving a weight twice as great as the oldest. For week 4 the computation is:

$$\text{Forecast for week } 4 = \tfrac{1}{6}(17) + \tfrac{2}{6}(21) + \tfrac{3}{6}(19) = 19.33$$

Note that for the weighted moving average the sum of the weights is equal to 1. Actually the sum of the weights for the simple moving average also equalled 1: Each weight was 1/3. However, recall that the simple or unweighted moving average provided a forecast of 19.

Forecast Accuracy To use the weighted moving averages method we must first select the number of data values to be included in the weighted moving average and then choose weights for each of the data values. In general, if we believe that the recent past is a better predictor of the future than the distant past, larger weights should be given to the more recent observations. However, when the time series is highly variable, selecting approximately equal weights for the data values may be best. Note that the only requirement in selecting the weights is that their sum must equal 1. To determine whether one particular combination of number of data values and weights provides a more accurate forecast than another combination, we will continue to use the MSE criterion as the measure of forecast accuracy. That is, if we assume that the combination that is best for the past will also be best for the future, we would use the combination of number of data values and weights that minimized MSE for the historical time series to forecast the next value in the time series.

Exponential smoothing is simple and has few data requirements, which makes it an inexpensive approach for firms that make many forecasts each period.

Exponential Smoothing

Exponential smoothing uses a weighted average of past time series values as the forecast; it is a special case of the weighted moving averages method in which we select only one weight—the weight for the most recent observation. The weights for the other data values are computed automatically and become smaller as the observations move farther into the past. The basic exponential smoothing model follows.

EXPONENTIAL SMOOTHING MODEL

$$F_{t+1} = \alpha Y_t + (1 - \alpha)F_t \tag{18.2}$$

where

$$
\begin{aligned}
F_{t+1} &= \text{forecast of the time series for period } t + 1 \\
Y_t &= \text{actual value of the time series in period } t \\
F_t &= \text{forecast of the time series for period } t \\
\alpha &= \text{smoothing constant } (0 \leq \alpha \leq 1)
\end{aligned}
$$

Equation (18.2) shows that the forecast for period $t + 1$ is a weighted average of the actual value in period t and the forecast for period t; note in particular that the weight given to the actual value in period t is α and that the weight given to the forecast in period t is $1 - \alpha$. We can demonstrate that the exponential smoothing forecast for any period is also a weighted average of *all the previous actual values* for the time series with a time series consisting of three periods of data: Y_1, Y_2, and Y_3. To start the calculations, we let F_1 equal the actual value of the time series in period 1; that is, $F_1 = Y_1$. Hence, the forecast for period 2 is

$F_1 = Y_1$
$F_2 = Y_1$

$$
\begin{aligned}
F_2 &= \alpha Y_1 + (1 - \alpha)F_1 \\
&= \alpha Y_1 + (1 - \alpha)Y_1 \\
&= Y_1
\end{aligned}
$$

Thus, the exponential smoothing forecast for period 2 is equal to the actual value of the time series in period 1.

The forecast for period 3 is

$$F_3 = \alpha Y_2 + (1 - \alpha)F_2 = \alpha Y_2 + (1 - \alpha)Y_1$$

Finally, substituting this expression for F_3 in the expression for F_4, we obtain

$$
\begin{aligned}
F_4 &= \alpha Y_3 + (1 - \alpha)F_3 \\
&= \alpha Y_3 + (1 - \alpha)[\alpha Y_2 + (1 - \alpha)Y_1] \\
&= \alpha Y_3 + \alpha(1 - \alpha)Y_2 + (1 - \alpha)^2 Y_1
\end{aligned}
$$

The term exponential smoothing comes from the exponential nature of the weighting scheme for the historical values.

Hence, F_4 is a weighted average of the first three time series values. The sum of the coefficients, or weights, for Y_1, Y_2, and Y_3 equals one. A similar argument can be made to show that, in general, any forecast F_{t+1} is a weighted average of all the previous time series values.

Despite the fact that exponential smoothing provides a forecast that is a weighted average of all past observations, all past data do not need to be saved to compute the forecast for the next period. In fact, once the **smoothing constant** α is selected, only two pieces of information are needed to compute the forecast. Equation (18.2) shows that with a given α we can compute the forecast for period $t + 1$ simply by knowing the actual and forecast time series values for period t—that is, Y_t and F_t.

To illustrate the exponential smoothing approach to forecasting, consider the gasoline sales time series in Table 18.1 and Figure 18.5. As indicated, the exponential smoothing forecast for period 2 is equal to the actual value of the time series in period 1. Thus, with $Y_1 = 17$, we will set $F_2 = 17$ to start the exponential smoothing computations. Referring to the time series data in Table 18.1, we find an actual time series value in period 2 of $Y_2 = 21$. Thus, period 2 has a forecast error of $21 - 17 = 4$.

Continuing with the exponential smoothing computations using a smoothing constant of $\alpha = .2$, we obtain the following forecast for period 3.

$$F_3 = .2Y_2 + .8F_2 = .2(21) + .8(17) = 17.8$$

Once the actual time series value in period 3, $Y_3 = 19$, is known, we can generate a forecast for period 4 as follows.

$$F_4 = .2Y_3 + .8F_3 = .2(19) + .8(17.8) = 18.04$$

By continuing the exponential smoothing calculations, we can determine the weekly forecast values and the corresponding weekly forecast errors, as shown in Table 18.3. Note that we have not shown an exponential smoothing forecast or the forecast error for period 1 because no forecast was made. For week 12, we have $Y_{12} = 22$ and $F_{12} = 18.48$. Can we use this information to generate a forecast for week 13 before the actual value of week 13 becomes known? Using the exponential smoothing model, we have

$$F_{13} = .2Y_{12} + .8F_{12} = .2(22) + .8(18.48) = 19.18$$

Thus, the exponential smoothing forecast of the amount sold in week 13 is 19.18, or 19,180 gallons of gasoline. With this forecast, the firm can make plans and decisions accordingly. The accuracy of the forecast will not be known until the end of week 13.

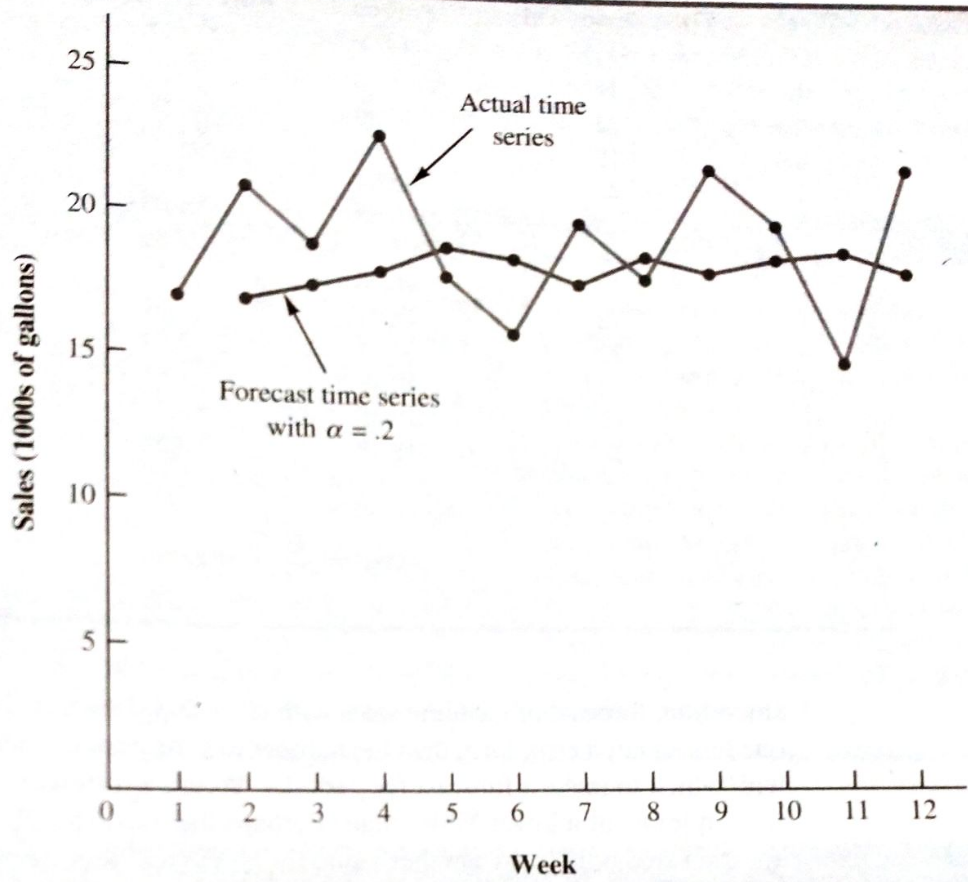
Figure 18.7 is the plot of the actual and forecast time series values. Note in particular how the forecasts "smooth out" the irregular fluctuations in the time series.

Forecast Accuracy In the preceding exponential smoothing calculations, we used a smoothing constant of $\alpha = .2$. Although any value of α between 0 and 1 is acceptable, some

TABLE 18.3 SUMMARY OF THE EXPONENTIAL SMOOTHING FORECASTS AND FORECAST ERRORS FOR GASOLINE SALES WITH SMOOTHING CONSTANT $\alpha = .2$

Week (t)	Time Series Value (Y_t)	Exponential Smoothing Forecast (F_t)	Forecast Error ($Y_t - F_t$)
1	17		
2	21	17.00	4.00
3	19	17.80	1.20
4	23	18.04	4.96
5	18	19.03	−1.03
6	16	18.83	−2.83
7	20	18.26	1.74
8	18	18.61	−.61
9	22	18.49	3.51
10	20	19.19	.81
11	15	19.35	−4.35
12	22	18.48	3.52

FIGURE 18.7 ACTUAL AND FORECAST GASOLINE SALES TIME SERIES WITH SMOOTHING CONSTANT $\alpha = .2$



values will yield better forecasts than others. Insight into choosing a good value for α can be obtained by rewriting the basic exponential smoothing model as follows.

$$F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$$
$$F_{t+1} = \alpha Y_t + F_t - \alpha F_t$$
$$\underbrace{F_{t+1}}_{} = \underbrace{F_t}_{\text{Forecast in period } t} + \alpha\underbrace{(Y_t - F_t)}_{\text{Forecast error in period } t} \tag{18.3}$$

Thus, the new forecast F_{t+1} is equal to the previous forecast F_t plus an adjustment, which is α times the most recent forecast error, $Y_t - F_t$. That is, the forecast in period $t + 1$ is obtained by adjusting the forecast in period t by a fraction of the forecast error. If the time series contains substantial random variability, a small value of the smoothing constant is preferred. The reason for this choice is that, because much of the forecast error is due to random variability, we do not want to overreact and adjust the forecasts too quickly. For a time series with relatively little random variability, larger values of the smoothing constant provide the advantage of quickly adjusting the forecasts when forecasting errors occur and thus allowing the forecasts to react faster to changing conditions.

The criterion we will use to determine a desirable value for the smoothing constant α is the same as the criterion we proposed for determining the number of periods of data to include in the moving averages calculation. That is, we choose the value of α that minimizes the mean squared error (MSE). A summary of the MSE calculations for the exponential

TABLE 18.4 MSE COMPUTATIONS FOR FORECASTING GASOLINE SALES WITH $\alpha = .2$

Week (t)	Time Series Value (Y_t)	Forecast (F_t)	Forecast Error ($Y_t - F_t$)	Squared Forecast Error $(Y_t - F_t)^2$
1	17			
2	21	17.00	4.00	16.00
3	19	17.80	1.20	1.44
4	23	18.04	4.96	24.60
5	18	19.03	−1.03	1.06
6	16	18.83	−2.83	8.01
7	20	18.26	1.74	3.03
8	18	18.61	−.61	.37
9	22	18.49	3.51	12.32
10	20	19.19	.81	.66
11	15	19.35	−4.35	18.92
12	22	18.48	3.52	12.39
			Total	98.80

$$\text{MSE} = \frac{98.80}{11} = 8.98$$

smoothing forecast of gasoline sales with $\alpha = .2$ is shown in Table 18.4. Note that there is one less squared error term than the number of time periods, because we had no past values with which to make a forecast for period 1. Would a different value of α provide better results in terms of a lower MSE value? Perhaps the most straightforward way to answer this question is simply to try another value for α. We will then compare its mean squared error with the MSE value of 8.98 obtained by using a smoothing constant of $\alpha = .2$.

The exponential smoothing results with $\alpha = .3$ are shown in Table 18.5. With MSE = 9.35, we see that for the current data set, a smoothing constant of $\alpha = .3$ results in less fore-

TABLE 18.5 MSE COMPUTATIONS FOR FORECASTING GASOLINE SALES WITH $\alpha = .3$

Week (t)	Time Series Value (Y_t)	Forecast (F_t)	Forecast Error ($Y_t - F_t$)	Squared Forecast Error $(Y_t - F_t)^2$
1	17			
2	21	17.00	4.00	16.00
3	19	18.20	.80	.64
4	23	18.44	4.56	20.79
5	18	19.81	−1.81	3.28
6	16	19.27	−3.27	10.69
7	20	18.29	1.71	2.92
8	18	18.80	−.80	.64
9	22	18.56	3.44	11.83
10	20	19.59	.41	.17
11	15	19.71	−4.71	22.18
12	22	18.30	3.70	13.69
			Total	102.83

$$\text{MSE} = \frac{102.83}{11} = 9.35$$

cast accuracy than a smoothing constant of $\alpha = .2$. Thus, we would be inclined to prefer the original smoothing constant of $\alpha = .2$. Using a trial-and-error calculation with other values of α, we can find a "good" value for the smoothing constant. This value can be used in the exponential smoothing model to provide forecasts for the future. At a later date, after new time series observations are obtained, we analyze the newly collected time series data to determine whether the smoothing constant should be revised to provide better forecasting results.

NOTES AND COMMENTS

1. Another measure of forecast accuracy is the *mean absolute deviation* (MAD). This measure is simply the average of the absolute values of all the forecast errors. Using the errors given in Table 18.2, we obtain

$$\text{MAD} = \frac{4 + 3 + 4 + 1 + 0 + 4 + 0 + 5 + 3}{9} = 2.67$$

One major difference between MSE and MAD is that the MSE measure is influenced much more by large forecast errors than by small errors (because for the MSE measure the errors are squared). The selection of the best measure of forecasting accuracy is not a simple matter. Indeed, forecasting experts often disagree as to which measure should be used. We use the MSE measure in this chapter.

2. Spreadsheet packages are an effective aid in choosing a good value of α for exponential smoothing and selecting weights for the weighted moving averages method. With the time series data and the forecasting formulas in the spreadsheets, you can experiment with different values of α (or moving average weights) and choose the value(s) of α providing the smallest MSE or MAD.

Exercises

Methods

SELF test

1. Consider the following time series data.

Week	1	2	3	4	5	6
Value	8	13	15	17	16	9

   a. Develop a three-week moving average for this time series. What is the forecast for week 7?
   b. Compute the MSE for the three-week moving average.
   c. Use $\alpha = .2$ to compute the exponential smoothing values for the time series. What is the forecast for week 7?
   d. Compare the three-week moving average forecast with the exponential smoothing forecast using $\alpha = .2$. Which appears to provide the better forecast?
   e. Use a smoothing constant of .4 to compute the exponential smoothing values. Does a smoothing constant of .2 or .4 appear to provide the better forecast? Explain.

2. Refer to the gasoline sales time series data in Table 18.1.
   a. Compute four-week and five-week moving averages for the time series.
   b. Compute the MSE for the four-week and five-week moving average forecasts.
   c. What appears to be the best number of weeks of past data to use in the moving average computation? Remember that the MSE for the three-week moving average is 10.22.

3. Refer again to the gasoline sales time series data in Table 18.1.
   a. Using a weight of 1/2 for the most recent observation, 1/3 for the second most recent, and 1/6 for third most recent, compute a three-week weighted moving average for the time series.