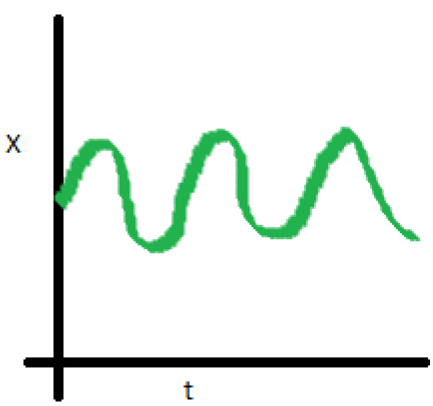
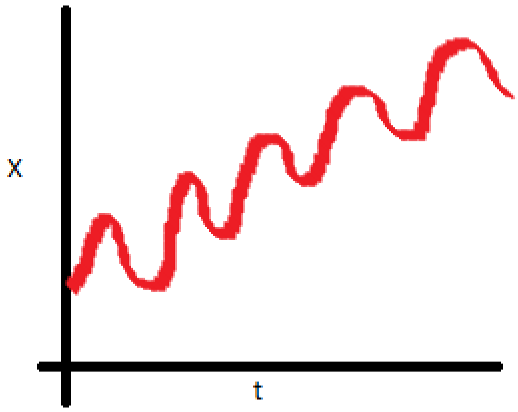
Stationary Series

There are three basic criterion for a series to be classified as stationary series :

1. The mean of the series should not be a function of time rather should be a constant. The image below has the left hand graph satisfying the condition whereas the graph in red has a time dependent mean.

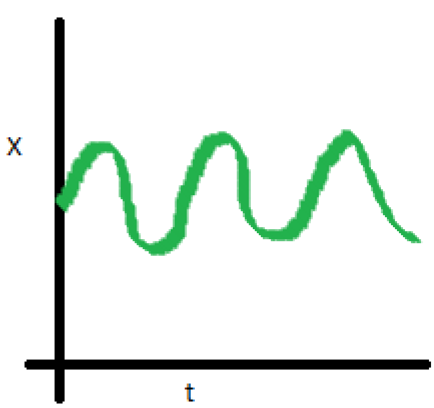


Stationary series

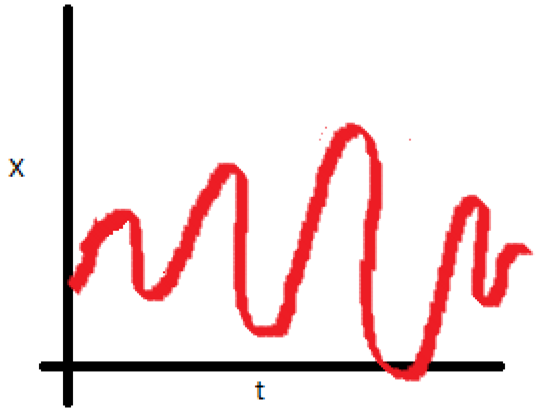


Non-Stationary series

2. The variance of the series should not be a function of time. This property is known as homoscedasticity. Following graph depicts what is and what is not a stationary series. (Notice the varying spread of distribution in the right hand graph)

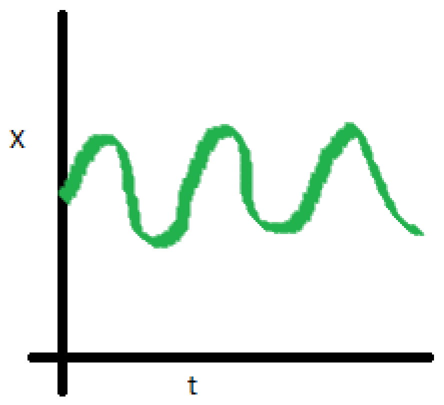


Stationary series

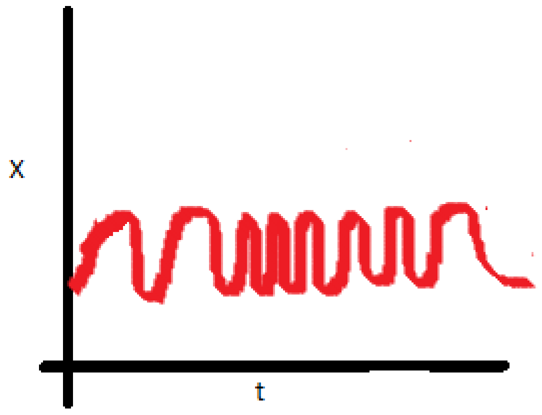


Non-Stationary series

3. The covariance of the i^{th} term and the $(i + m)^{th}$ term should not be a function of time. In the following graph, you will notice the spread becomes closer as the time increases. Hence, the covariance is not constant with time for the 'red series'.



Stationary series



Non-Stationary series

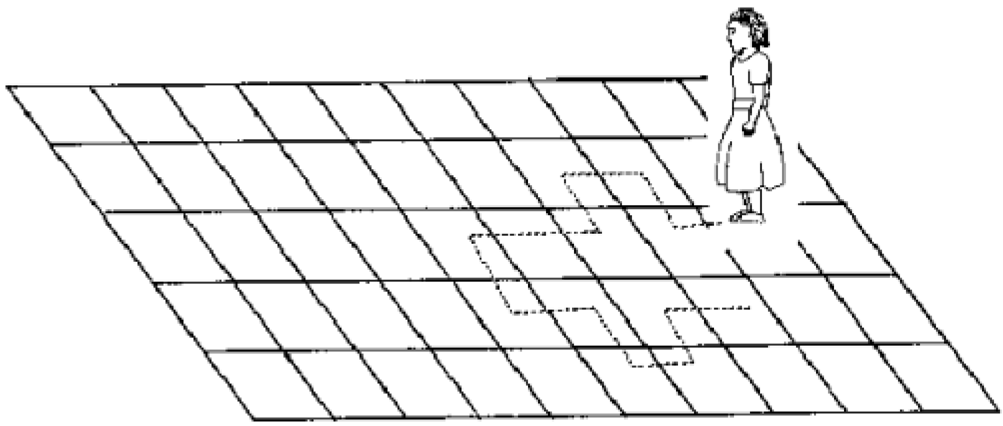
Why do I care about 'stationarity' of a time series?

The reason is that until unless your time series is stationary, you cannot build a time series model. In cases where the stationary criterion are violated, the first requisite becomes to stationarize the time series and then try stochastic models to predict this time series. There are multiple ways of bringing this stationarity. Some of them are Detrending, Differencing etc.

Random Walk

This is the most basic concept of the time series. Let's take an example.

Example: Imagine a girl moving randomly on a giant chess board. In this case, next position of the girl is only dependent on the last position.



(Source: http://scifun.chem.wisc.edu/WOP/RandomWalk.html)

Now imagine, you are sitting in another room and are not able to see the girl. You want to predict the position of the girl with time. How accurate will you be? Of course you will become more and more inaccurate as the position of the girl changes. At t=0 you exactly know where the girl is. Next time, she can only move to 8 squares and hence your probability dips to 1/8 instead of 1 and it keeps on going down. Now let's try to formulate this series :

```
X(t) = X(t-1) + Er(t)
```

where Er(t) is the error at time point t. This is the randomness the girl brings at every point in time.

Now, if we recursively fit in all the Xs, we will finally end up to the following equation :

```
X(t) = X(0) + Sum(Er(1),Er(2),Er(3).....Er(t))
```