Introduction to Algorithm

CS430 Homework 2

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Question -1

Problem-1

Take the following list of functions and arrange them in ascending order of growth rate. That is, if function g(n) immediately follows function f (n) in your list, then it should be the case that f (n) is O(g(n)).

f1(n) = $n^{2.5}$ //Polynomial
f2(n) =√2n //Linear
f3(n) = n + 10 //Linear
f4(n) = 10^n //Exponential
f5(n) = 100^n //Exponential
f6(n) = n^2 log n //polynomial arithmetic

Answer:

Based on the growth rate of above six different functions:

Arrangement of function in Ascending order is: f2, f3, f6, f1, f4, f5

Explanation:

f(n) = O(g(n)) if c and some initial value n_0 are positive when f(n) <= c * g(n) is true.

Linear Function f2(n) and f3(n),

sqrt(2n) < n+10

sqrt(2n) < n

so, f(2) < f(3)

Function polynomial growth functions f1(n) and f6(n)

f6(n) < f1(n) ➔ n*n*log(n)< n * n * sqrt(n) ➔ log(n)< sqrt(n)

f6(n) < f1(n)

If we compare the function f3(n) and f6(n)

F3 is linear function and f6 is quadratic function so,

F3(n) < f6(n)

Partial ordering: f2, f3, f6, f1

For function f1 grow slower than f4 for large value of n

If we take log for both function f1 and f4

Log($n^{2.5}$) < log(10^n) ➔ 2.5 log n < n log 10

f1(n) < f (4)

For exponential function f4(n) and f5(n)

f4(n) < f5(n)

From the equation 1,2 and 3

Function in ascending order: f2, f3, f6, f1, f4,f5

f2(n)= √2n < f3(n)= n + 10 < f6(n)= n^2 log n < f1(n)= $n^{2.5}$ < f4(n)= 10^n < f5(n)= 100^n

Problem-2

Take the following list of functions and arrange them in ascending order of growth rate. That is, if function g(n) immediately follows function f (n) in your list, then it should be the case that f (n) is O(g(n)).

g1(n) = $2^{\sqrt{\log n}}$
g2(n) = 2^n
g4(n) = $n^{4/3}$
g3(n) = n $(\log n)^3$
g5(n) = $n^{\log n}$
g6(n) = 2^{2^n}
g7(n) = 2^{n^2}

Answer:

The Ascending order of Function is: g1, g3, g4, g5, g2, g7, g6

g1 is the smallest growing function compare to other functions

For Exponential function g2 and g7, g2(n) < g7(n)

For Exponential function g6 and g7, g7(n) < g6(n)

For function g2(n) < g7(n) ➔ $2^n < 2^{n^2}$

So, partial ordering of function g1, g2,g7,g6 is : g1 < … g2 < g7 < g6

For polynomial function g3 and g4,

$n (\log n)^3 < n^{4/3}$

$n (\log n)^3 < n * n^{1/3}$ => $(\log n)^9 < n$ = g3(n) < g4(n)

so, g3(n)= $n (\log n)^3$ < g4(n) = $n^{4/3}$

For function g4 and g5

$n^{4/3} < ^{n \log n}$ ➔ 4/3 log n < logn logn ➔ log n < $(\log n)^2$

For function g3 and g5,

g3(n) < g5(n)

$n*(\log n)^3 < n^{\log n}$ ➔ $\log(n(\log n)^3) < \log n . \log n$ ➔ $\log n + \log(\log n)^3 < (\log n)^2$

so, g3(n) < g4(n) < g5(n)

Function g5 grow slower than g2, g6 and g7

So g5(n)= $n^{\log n}$ < g2(n)= 2^n < g7(n) = $2^{n\,2}$ < g6(n) =$2^{2\,n}$

g5 < g2 < g7 < g6

Order sequence is: g3, g4, g5, g2, g7, g6

As g1 is the smallest growing function compare to other functions

So g1 comes before all other functions

g1, g3, g4, g5, g2, g7, g6 if g3(n)= $n (\log n)^3$ < g4(n) = $n^{4/3}$

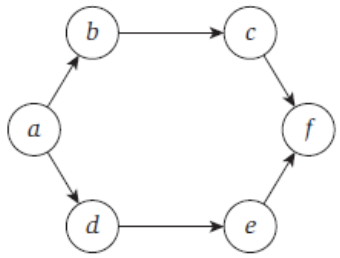
OR

g1, g4, g3, g5, g2, g7, g6 if g4(n)= $n (\log n)^3$ < g3(n) = $n^{4/3}$

Question -2

Consider the directed acyclic graph G in Figure How many topological orderings does it have

Answer:



In-degree ()	
A	0
B	1
C	1
D	1
E	1
F	2

For Simplicity,

a = A , b = B , c = C , d=D ,e= E

Topological ordering:

First node with indegree =0

Output: A

Remove adjacent edge

Indegree of Each node:

B	C	D	E	F
0	1	0	1	2

So next node will be either B or D as their indegree is zero .

Case - I: Next is B

If Select B,

Then indegree of C become zero and next node will be C or D,

Possible topological ordering be: A B C D E

OR A B D C E

OR A B D E C.

Case -II: Next is D

If we choose D,

Then indegree of E also become zero and node will be E or B

Possible topological ordering be: A D E B C

OR A D B C E

OR A D B E C.

Possible ordering in such a way that for edge b to c, b come before c and for edge d to e, d come before e.

Last node Will be F as all nodes are visited and indegree is zero.

There are six different possible Topological ordering are:

A B C D E , A B D C E , A B D E C , A D E B C , A D B C E , A D B E C.

Question -3

The algorithm described in Section 3.6 for computing a topological ordering of a DAG repeatedly finds a node with no incoming edges and deletes it. This will eventually produce a topological ordering, provided that the input graph really is a DAG.

But suppose that we're given an arbitrary graph that may or may not be a DAG.

Extend the topological ordering algorithm so that, given an input directed graph G, it outputs one of two things:

(a) a topological ordering, thus establishing that G is a DAG; or

(b) a cycle in G, thus establishing that G is not a DAG.

The running time of your algorithm should be O (m + n) for a directed graph with n nodes and m edges.

Answer:

Let S is set of All vertices in a Graph

1. Store each vertex's In-Degree in an array IN

2. Initialize a queue Q1 with all in-degree zero vertices

3. if Q1 is Empty and no of vertices in graph G >1

    Pick a vertex u from set S,

L1: set status of u is visited and store it in Array OUT

    For each Edge (u, v)

        If node v is unvisited

            Goto L1:

        Else

            If v is visited // [already in Array OUT]

            Graphs contain Cycle

            Display Array OUT

5.else

6. While there are vertices in the queue Q1 with indegree = 0

    Dequeue and store a vertex in output Array OUT

    Reduce In-Degree of all vertices adjacent to it by 1

    Enqueue any of these vertices whose In-Degree became zero

4. return output Array OUT

Running time of Algorithm is O(M+N)

Question -4

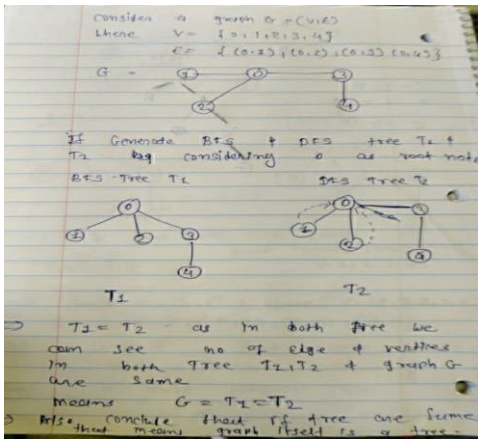
We have a connected graph G = (V, E), and a specific vertex u ∈ V. Suppose we compute a depth-first search tree rooted at u, and obtain a tree T that includes all nodes of G. Suppose we then compute a breadth-first search tree rooted at u, and obtain the same tree T. Prove that G = T. (In other words, if T is both a depth-first search tree and a breadth-first search tree rooted at u, then G cannot contain any edges that do not belong to T.)

Answer:

A Graph is a connected graph, when a path can be found between every pair of different vertices.

Let G= (V, E) Where V= {0,1,2,3,4} AND E= {{1,0), (0,2),(0,3),(3,4)}

Root node = 0



Prove G = T

Proof by Contradiction:

Tree generated by DFS and BFS is same tree T.

Assume

Graph G not equal to Tree T,

Means E1= (v1, v2) is part of Graph G but not belongs to Tree T

If Tree T is DFS tree,

either v1 or v2 must be ancestor of the other or there will be a back edge.

For BFS Tree T, distance between the two nodes v1 and v2 from root u1 can differ by only level 1.

As both BFS and DFS tree are same,

So Either, v1 or v2 should be ancestor of other or both v1 and v2 can differ by almost 1 from common root node.

Implies, E1= (v1, v2) is an edge in Tree T.

Which contradict the statement that (v1, v2) does not belong to tree T.

Question-5

Claim: Let G be a graph on n nodes, where n is an even number. If every node of G has degree at least n/2, then G is connected.

Decide whether you think the claim is true or false and give a proof of either the claim or its negation.

Answer:

Proof by Contradiction

We can **a**ssume Graph is not Connected.

Let node v2 and v1 are two nodes in Graph G.

As each node has degree n/2

Each V1 and v2 are connected with n/2+1 other node

Then, Total Number of nodes in set V = 2*(n/2+1)➔ n+2 nodes

Which contradict with statement that we have only n even nodes in G.

Question-6

Suppose we are given an undirected graph G = (V, E) and we identify two nodes v and w in G.

Give an algorithm that computes the number of shortest v-w paths in G.

Answer:

Graph G= (V, E) where V is number of nodes and E is number of edges.

S = source node

1. Set Discovered[S] = true
2. Discovered[v] = false for all other v
3. Initialize L[0] to consist the single element S
4. Set the layer counter i = 0
5. Length[S]=0 //distance
6. Spath[S]=1 //shortest path
7. While L[i] is not empty

Initialize an empty list L [i + 1]

For each node u ∈ L[i]

Consider each edge (u, v) incident to u

If Discovered[v] = false, then

Set Discovered[v] = true

Add v to the list L [i + 1]

End

If length[v] > length [u]+1

Length[v] =length[u]+1

Spath [v] = Spath [u]

End

Else if length[v]=length[u]+1

Spath[v]= Spath[v]+ Spath [u]

End

End for

8. Increment the layer counter i by one

End while

9. Return spath[D]