ECS - 312

Assessment - I

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Q16

(i) $x + 2y = 1$ and $3x + 6y - 2 = 0$

$a_1 = 1 \quad b_1 = 2 \quad c_1 = 1 \qquad a_2 = 3 \quad b_2 = 6 \quad c_2 = -2$

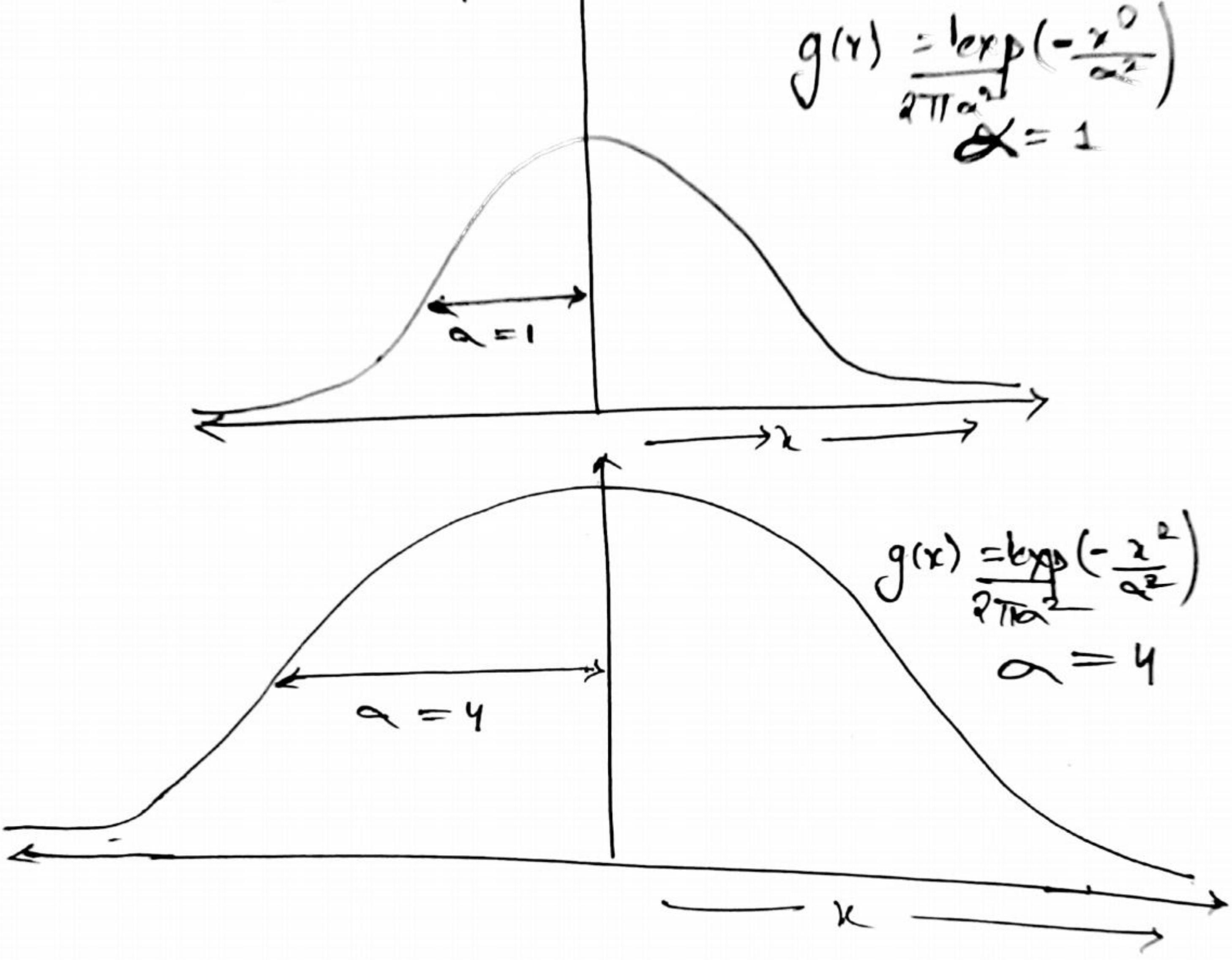
here $\frac{a_1}{a_2} = \frac{1}{3} \quad \frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3} \qquad \frac{c_1}{c_2} = \frac{-1}{2}$

hence $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence two lines are parallel in Euclidean plane

Q1

1D Gaussian f.



Q2

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2 = \frac{5 - 2}{3 - 0}(x - 0)$$

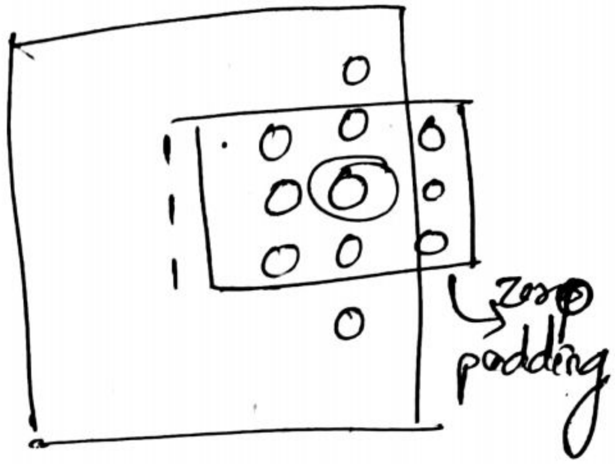
$$y - 2 = x \quad (x = 4$$

$$y = 4$$

Q 11

$$w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

f



(i) correlation

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

filter

⇒ encircle point will ~~with~~ replaced by 0

(ii) Convolution (kernel will remain same as kernel is symmetrical)

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

In both correlation and convolution encircle point will be ~~replaced~~ by $\underline{0}$

Q4 ① correlation

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} \boxed{\begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{matrix}} \end{bmatrix} = \begin{bmatrix} \Sigma \end{bmatrix}$$

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \text{ filter}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \longrightarrow \frac{1}{16} \times (1 \times 4 + 2 \times 1) = 0.375$$

the middle point will become 0.375

② Convolution, for convolution we need to rotate the kernel by 180° C

rotated kernel $w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ (kernel remain same after rotation because it is symmetrical)

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \frac{1}{16} \times (4 \times 1 + 2 \times 1)$$

$$= 0.375$$

the middle point will become 0.375

Q14

No, the histograms of the blurred images will not be equal because, in 1st image when box filter will be applied the image will remained mostly same because, when box (averaging filter) will be applied to half-black region it will remain mostly black

eg

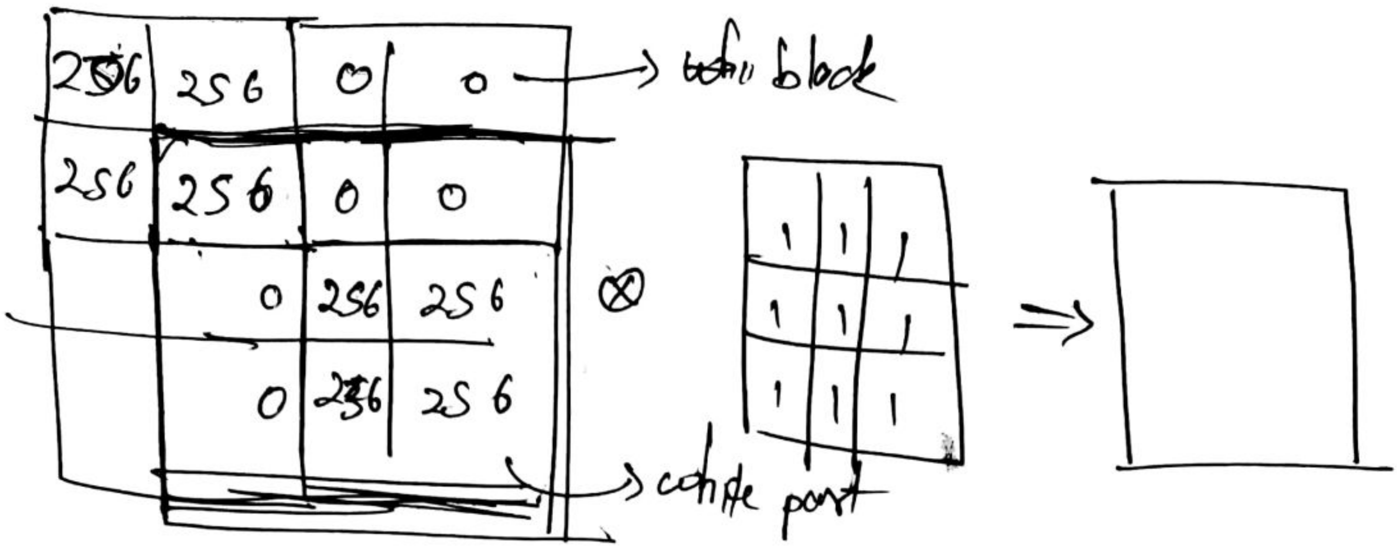
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \otimes \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{\text{box filter}} \times \frac{1}{9} \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

part of black portion

$$\begin{bmatrix} 256 & 256 & 256 \\ 256 & 256 & 256 \\ 256 & 256 & 256 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \times \frac{1}{9} \Rightarrow \begin{bmatrix} 256 & 256 & 256 \\ 256 & 256 & 256 \\ 256 & 256 & 256 \end{bmatrix}$$

From above, we can conclude that frequency of pixels denoting ~~or~~ white and black will not get changed for 1st image

but after box filter is applied to check board pattern



~~the~~ we will get pixels having different value other than 0, and 256,

~~Hen~~ Hence, frequency of 0, and 256 also ~~these~~ other intensities will be different in image 1 and image 2 after applying the filter.

Q17

$$X' = HX$$

$$H = OCBA$$

(i) scale factor s

so, $A = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(ii) rotation by 30° about z-axis

so, $B = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 0 & 0 \\ \sin 30^\circ & \cos 30^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 0 \\ 1/2 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(iii) Shear transformat is n by factr or 2 &
y by fact or 3

So, $C = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

(iv) transform by mom the point in the drcn
d. of [2,1,2].

$$D = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, $H = DCBA$

$$H = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 0 \\ 1/2 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 5\sqrt{3}/2 & -1/2 & 2 & 2 \\ 5/2 & \sqrt{3}/2 & 3 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8)

Given

$$R = \begin{bmatrix} 0.9 & 0.4 & 0.1732 \\ -0.4183 & 0.9043 & 0.0854 \\ -0.1225 & -0.1493 & 0.9812 \end{bmatrix}$$

$$C_w = (-1, -2, -3)^T = \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$$

We know,

extrinsic parameter matrix is given by

$$R[I_3 | -X_0] =$$

$$= \begin{bmatrix} 0.9 & 0.4 & 0.1732 \\ -0.4183 & 0.9043 & 0.0854 \\ -0.1225 & -0.1493 & 0.9812 \end{bmatrix} \left[\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \middle| - \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \right] =$$

$$= \underset{3\times3}{\begin{bmatrix} 0.9 & 0.4 & 0.1732 \\ -0.4183 & 0.9043 & 0.0854 \\ -0.1225 & -0.1493 & 0.9812 \end{bmatrix}} \underset{3\times4}{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}} = \begin{bmatrix} 0.9 & 0.4 & 0.1732 & -2.2196 \\ -0.4183 & 0.9043 & 0.0854 & -1.6465 \\ -0.1225 & -0.1493 & 0.9812 & -2.5225 \end{bmatrix}$$

Q4

$$S*A = \begin{bmatrix} 100 & 900 & 160 & 160 \\ 0 & 110 & 110 & 110 \\ 180 & 300 & 330 & 330 \\ 180 & 180 & 180 & 180 \end{bmatrix}$$

$$(S*A) - B = \begin{bmatrix} 90 & 810 & 144 & 144 \\ 0 & 97 & 93 & 99 \\ 182 & 266 & 299 & 297 \\ 182 & 163 & 161 & 161 \end{bmatrix}$$

$$(((S*A) - B) + P) = \begin{bmatrix} 92 & 812 & 146 & 146 \\ 2 & 112 & 146 & 101 \\ 164 & 268 & 301 & 215 \\ 164 & 185 & 163 & 164 \end{bmatrix}$$

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Assessment 1

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6 Given point = (4, 7)

In homogeneous coordinates, n+1 dimensional vector is used to represent n-dimensional Euclidian point and set dimension n+1 to the value 1.

In Homogeneous coordinates,

$$x = \begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix}$$

where λ is represents the scale factor which is constant

The point (4, 7) is expressed as (4, 7, 1) in homogeneous coordinates.

8. Given lines:

$l: x+y-5=0 \Rightarrow x+y=5$

and, $m: 4x-5y+7=0 \Rightarrow 4x-5y=-7$

Let $x=[x \quad y]^T$ be the point of intersection.

The homogeneous coordinates of line l:

$$l = \begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix}$$

The homogeneous coordinates of line m:

$$m = \begin{bmatrix} 4 \\ -5 \\ 7 \end{bmatrix}$$

Intersection of l and m is expressed in homogeneous coordinates as:

$$x = l \cap m$$

System of linear equations:

$$\begin{bmatrix} l \cdot x \\ m \cdot x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

we need to find the point such that

$l \cdot x = 0$ and $m \cdot x = 0$

Solving the system of linear equations $Ax=b$ using Cramer's rule

$$\begin{bmatrix} 1 & 1 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} +5 \\ -7 \end{bmatrix}$$

$D_1 = \det(A_1) = 5(-5) - 1(-7) = -25+7 = -18$

$D_2 = \det(A_2) = 1(-7) - 5(4) = -7-20 = -27$

$D_3 = \det(A) = 1(-5) - 1(4) = -5-4 = -9$

$x = \frac{D_1}{D_3} = \frac{-18}{-9} = 2$, $y = \frac{D_2}{D_3} = \frac{-27}{-3} = 3$

$X = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ is the point of intersection in homogeneous coordinates

$x = (2, 3)$ is the point of intersection in Euclidian coordinates.

10. Given: focal length, $f = 1050$ pixels
Principal point is offset from $(0,0)^T$ of the image plane to location $(+10,-5)^T$

$u_0 = +10$ $v_0 = -5$

Intrinsic parameter matrix $= \begin{bmatrix} f & 0 & u_0 & 0 \\ 0 & f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

In this case,
Intrinsic parameter matrix $= \begin{bmatrix} 1050 & 0 & 10 & 0 \\ 0 & 1050 & -5 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

16. Given: $l: x+2y+1=0$ and $m: 3x+6y-2=0 \Rightarrow x+2y-\frac{2}{3}=0$
Since, $\frac{1}{3} = \frac{2}{6}$

The given two lines are parallel in the Euclidian plane.
In projective geometry, the point of intersection is referred as vanishing point since the parallel lines intersect at a single point in the image plane at point at infinity.
For two parallel lines, their intersection in homogeneous coordinates is

$$l \times m = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ -\frac{2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 2(-\frac{2}{3}) - 2(1) \\ 1(1) - 1(-\frac{2}{3}) \\ 1(2) - 1(2) \end{bmatrix} = \begin{bmatrix} -10/3 \\ 5/3 \\ 0 \end{bmatrix}$$

An infinitely distant point which is the required point of intersection $x_\infty = \begin{bmatrix} -10/3 \\ 5/3 \\ 0 \end{bmatrix}$

3. Camera matrix, P is given. $P = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix}$

Decomposition of matrix M, where $M = KR$

Applying QR decomposition and decomposes a matrix into Q and R

where Q is an orthonormal matrix representing rotational matrix, R and R is an upper triangular matrix representing calibration matrix K.

Let $t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$ be the translation vector.

$$t = K^{-1} h$$

where $h = \begin{bmatrix} m_{14} \\ m_{24} \\ m_{34} \end{bmatrix} = \begin{bmatrix} 0.0067 \\ -0.0878 \\ 1.0000 \end{bmatrix}$

After applying QR decomposition

Orthonormal Rotation matrix, $R = \begin{bmatrix} -0.9672 & -0.2526 & 0.0258 \\ 0.1073 & -0.3146 & 0.9431 \\ -0.2301 & 0.915 & 0.3314 \end{bmatrix}$

Calibration matrix, $K = \begin{bmatrix} -0.7939 & 0.3990 & -0.0576 \\ 0 & 0.4269 & 0.2232 \\ 0 & 0 & -0.8801 \end{bmatrix}$

$$t = \begin{bmatrix} -0.7939 & 3.999 & -0.0576 \\ 0 & 0.4269 & 0.2232 \\ 0 & 0 & -0.8801 \end{bmatrix} \begin{bmatrix} 0.0067 \\ -0.0878 \\ 1 \end{bmatrix}$$

$$t = \begin{bmatrix} 0.269 \\ 0.388 \\ -1.1362 \end{bmatrix}$$

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12. Let filter $= a_1 \exp\left(-\left(\left(\frac{x-a}{c_1}\right)^2 + \left(\frac{y-b}{c_2}\right)^2\right)\right)$

Since, vertical smoothing is greater than horizontal smoothing

Therefore, $c_2 > c_1$

5. i) Coordinate of pixel value (15) is (2,9)

ii) 4-neighbourhood pixels (207):

$N_4(P) = \{106, 233, 137, 179\}$

iii) 8-neighbourhood pixels (191) are

$N_4(P) + N_D(P) = \{168, 193, 156, 166, 97, 84, 174, 252\}$

iv) Green-box coordinate (191) = (8,2)

Red box coordinate (211) = (6,6)

Maroon box coordinate (227) = (11,4)

Distance b/w pixels (191 - 211) $= \sqrt{2^2+4^2}$

$= \sqrt{4+16} = \sqrt{20} = 4.47$

Distance b/w pixels (211 - 227) $= \sqrt{5^2+2^2} = \sqrt{25+4}$

$= \sqrt{29} = 5.38$

Distance b/w pixels (227 - 191) $= \sqrt{3^2+2^2} = \sqrt{9+4}$

$= \cancel{\sqrt{29}}\ \sqrt{13} = 3.6$

Mean distance $= \frac{4.47+5.38+3.6}{3} = 4.48$

vi) As as know pixel > 190 is 1

pixel < 190 is 0

So, blue box (15) is 0

Violet box (207) is 1

Red box (211) is 1

Green box (191) is 1

Orange box (218) is 1

Maroon box (227) is 1

vi) 4X3 neighbourhood of green box (191) is

189	97	165	84
199	168	$\boxed{191}$	193
206	174	155	252

coordinates ~~(7,0)~~ of 189 is (7,0)
97 is (7,1)
165 is (7,2)
84 is (7,3)

coordinates of 199 is (8,0)
168 is (8,1)
193 is (8,3)

coordinates of 206 is (9,0)
174 is (9,1)
155 is (9,2)
252 is (9,3)