OPERATIONS RESEARCH - I

ASSIGNMENT - DECISION THEORY

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(1) Mr Sethi has ₹ 10,000 to invest in one of three options A, B or C. The return on his investment depends on whether the economy experiences inflation, recession or no change at all. The possible returns under each economic condition are given below:

	STATES OF NATURE		
STRATEGY	INFLATION	RECESSION	NO CHANGE
A	2000	1200	1500
B	3000	800	1000
C	2500	1000	1800

What should he decide, using the (i) Maximax (ii) Maximin (iii) Laplace (iv) Minimax regret and (v) Hurwicz ($\alpha = 0.6$) criteria.

SOLUTION:

(i) MAXIMAX CRITERION

	INFLATION	RECESSION	NO CHANGE	MAXIMUM
A	2000	1200	1500	2000
B	3000	800	1000	3000
C	2500	1000	1800	2500

∴ By Maximax criterion, B is the best option

(ii) MAXIMIN CRITERION

	INFLATION	RECESSION	NO CHANGE	MINIMUM
A	2000	1200	1500	1200
B	3000	800	1000	800
C	2500	1000	1800	1000

∴ By Maximin Criterion, A is the best option.

(iii) LAPLACE CRITERION

	INFLATION	RECESSION	NO CHANGE	EXPECTED PAYOFF
A	2000	1200	1500	1566.67
B	3000	800	1000	1600
C	2500	1000	1800	1766.67

∴ By laplace criterion, C is the best option.

(iv) MINIMAX REGRET CRITERION

	INFLATION	RECESSION	NO CHANGE
A	2000	1200	1500
B	3000	800	1000
C	2500	1000	1800
COLUMN MAXIMUM	3000	1200	1800

REGRET TABLE:

	INFLATION	RECESSION	NO CHANGE	MAXIMUM
A	1000	0	300	1000
B	0	400	800	800
C	500	200	0	500

∴ By minimax regret criterion, C is the best option

(v) HURWICZ (α=0.6) CRITERION $W_0 = (\alpha \times MAX) + (1-\alpha) \times MIN$

	INFLATION	RECESSION	NO CHANGE	MAX	MIN	W_0
A	2000	1200	1500	2000	1200	1680
B	3000	800	1000	3000	800	2120
C	2500	1000	1800	2500	1000	1900

∴ By Hurwicz criterion, B is the best option.

(2) Pay offs of three acts X, Y, Z and the states of nature P, Q, R are given below:

PAY-OFFS (₹)

STATES OF NATURE	ACTS X	Y	Z
P	-120	-80	100
Q	200	400	-300
R	260	-260	600

The probabilities of the states of nature are 0.3, 0.3 and 0.4 respectively. Find the best action using:

(i) EMV (ii) EOL criterion and also find EVPI

SOLUTION:

(i) EXPECTED MONETARY VALUE:

STATES OF NATURE	PROB.	ACTS X	Y	Z	EXPECTED VALUE X	Y	Z
P	0.3	-120	-80	100	-36	-24	30
Q	0.3	200	400	-300	60	120	-90
R	0.4	260	-260	600	104	-104	240
				EMV =	128	-8	180

Since Z has the highest EMV, it is the best action.

(ii) EXPECTED OPPORTUNITY LOSS:

STATES OF NATURE	PROB.	ACTS X	Y	Z	COND. OPP. LOSS X	Y	Z	WEIGHTED OPP. LOSS X	Y	Z
P	0.3	-120	-80	100	220	180	0	66	54	0
Q	0.3	200	400	-300	200	0	700	60	0	210
R	0.4	260	-260	600	340	860	0	136	344	0
							EOL =	262	398	210

Since Z has the lowest EOL, it is the best action.

∴ EVPI = 210 ∴ EVPI = min (EOL) = 210

$$EPPI = (100 \times 0.3) + (400 \times 0.3) + (600 \times 0.4)$$

$$= 30 + 120 + 240$$

$$= \underline{\underline{390}}$$

$$EVPI = EPPI - \max(EMV)$$

$$= 390 - 180$$

$$= \underline{\underline{210}} = \min(EOL)$$

(3) Marketing staff of a certain industrial organization has submitted the following payoff table, giving profits in million rupees, concerning a certain proposal depending upon the state of technological advance.

TECH ADVANCE	DECISION ACCEPT	REJECT
Much	2	3
little	5	2
none	-1	4

The probabilities are 0.2, 0.5 and 0.3 for much, little and none technological advance respectively. Find the optimum decision based on (i) EMV (ii) EOL also find EVPI

SOLUTION:

(i) EMV:

TECH ADVANCE	PROB.	DECISION ACCEPT	REJECT	EXPECTED VALUE ACCEPT	REJECT
Much	0.2	2	3	0.4	0.6
Little	0.5	5	2	2.5	1.0
none	0.3	-1	4	-0.3	1.2
			EMV =	2.6	2.8

Since max (EMV) = 2.8, to reject is the optimum decision

(ii) EOL:

TECH ADVANCE	PROB.	DECISION ACCEPT	REJECT	COND. OPP. LOSS ACCEPT	REJECT	WEIGHTED OPP. LOSS ACCEPT	REJECT
Much	0.2	2	3	1	0	0.2	0
Little	0.5	5	2	0	3	0	1.5
none	0.3	-1	4	5	0	1.5	0
						1.7	1.5

Since min (EOL) = 1.5, to reject is the optimum decision.

Also, EVPI = min (EOL) = 1.5

$$EPPI = (3 \times 0.2) + (5 \times 0.5) + (4 \times 0.3)$$

$$= 0.6 + 2.5 + 1.2$$

$$= \underline{\underline{4.3}}$$

$$EVPI = EPPI - \max(EMV)$$

$$= 4.3 - 2.8$$

$$= \underline{\underline{1.5}} = \min(EOL)$$

(4) A producer of boats has estimated the following distribution of demand for a particular kind of boat:

NO. DEMANDED	0	1	2	3	4	5	6
PROBABILITY	0.14	0.27	0.27	0.18	0.09	0.04	0.01

Each boat costs him ₹ 7000 and he sells them for ₹10,000 each. Any boats left unsold at the end of the season must be disposed off for ₹6000 each. How many boats should be in stock so as to maximize his expected profit.

SOLUTION:

If D denotes the demand and S, the number of boats in ~~supply~~ stock, then the conditional ~~cost~~ profit values are computed as follows:

~~PROFIT~~ FUNCTION } = MARGINAL PROFIT = 10000 - 7000 = 3000
MARGINAL LOSS = 7000 - 6000 = 1000

STATE OF NATURE (DEMAND)	PROB.	CONDITIONAL PROFIT (₹) / COURSE OF ACTION (STOCK)						
		0	1	2	3	4	5	6
0	0.14	0	-1000	-2000	-3000	-4000	-5000	-6000
1	0.27	0	3000	2000	1000	0	-1000	-2000
2	0.27	0	3000	6000	5000	4000	3000	2000
3	0.18	0	3000	6000	9000	8000	7000	6000
4	0.09	0	3000	6000	9000	12000	11000	10000
5	0.04	0	3000	6000	9000	12000	15000	14000
6	0.01	0	3000	6000	9000	12000	15000	18000
EXPECTED PROFIT (EMV):		0	2440	3800	4080	3640	2840	1880

Thus you, the producer should stock 3 boats to maximize profit

(5) A bookstore sells a particular book of tax laws for ₹100 It purchases the book for ₹80 per copy. Since some of the tax laws change every year, the copies unsold at the end of a year become outdated and can be disposed off for ₹30 each. According to Past experience, the annual demand for this book is between 18 and 23 copies
Assuming that the order for this book can be placed only once in a year, the store's manager has to decide how many copies of the book should be purchased.
Create a pay-off matrix and hence find the best course of action using (i) maximax (ii) maximin (iii) Laplace (iv) Minimax regret and (v) Hurwicz ($\alpha = 0.6$) criteria
(II) If the probabilities of occurrence of the various events are respectively 0.05, 0.1, 0.3, 0.4, 0.1 and 0.05, find the best course of action using (i) EMV (ii) EOL methods. Also find EVPI

SOLUTION:

Marginal Profit = 100 - 80 = ₹20

Marginal loss = 80 - 30 = ₹50

(i) MAXIMAX CRITERION

STRATEGIES (COURSES OF ACTION)	STATES OF NATURE (DEMAND)						MAXIMUM
	$E_1 = 18$	$E_2 = 19$	$E_3 = 20$	$E_4 = 21$	$E_5 = 22$	$E_6 = 23$	
$A_1 = 18$	360	360	360	360	360	360	360
$A_2 = 19$	310	380	380	380	380	380	380
$A_3 = 20$	260	330	400	400	400	400	400
$A_4 = 21$	210	280	350	420	420	420	420
$A_5 = 22$	160	230	300	370	440	440	440
$A_6 = 23$	110	180	250	320	390	460	460

∴ A_6 is the best strategy

(ii) MAXIMIN CRITERION

STRATEGIES (COURSES OF ACTION)	STATES OF NATURE (DEMAND)						MINIMUM
	$E_1 = 18$	$E_2 = 19$	$E_3 = 20$	$E_4 = 21$	$E_5 = 22$	$E_6 = 23$	
$A_1 = 18$	360	360	360	360	360	360	360
$A_2 = 19$	310	380	380	380	380	380	310
$A_3 = 20$	260	330	400	400	400	400	260
$A_4 = 21$	210	280	350	420	420	420	210
$A_5 = 22$	160	230	300	370	440	440	160
$A_6 = 23$	110	180	250	320	390	460	110

∴ A_6 is the best strategy by maximin criterion

(iii) LAPLACE CRITERION

STRATEGIES (COURSES OF ACTION)	STATES OF NATURE (DEMAND) $E_1=18$	$E_2=19$	$E_3=20$	$E_4=21$	$E_5=22$	$E_6=23$	EXPECTED PAYOFF
$A_1=18$	360	360	360	360	360	360	360
$A_2=19$	310	380	380	380	380	380	368.33
$A_3=20$	260	330	400	400	400	400	365
$A_4=21$	210	280	350	420	420	420	350
$A_5=22$	160	230	300	370	440	440	323.33
$A_6=23$	110	180	250	320	390	460	285

∴ By Laplace criterion, A_2 is the best option

(iv) MINIMAX REGRET CRITERION:

Column maximums	360	380	400	420	440	460

REGRET TABLE:

STRATEGIES (COURSES OF ACTION)	$E_1=18$	$E_2=19$	$E_3=20$	$E_4=21$	$E_5=22$	$E_6=23$	MAXIMUM
$A_1=18$	0	20	40	60	80	100	100
$A_2=19$	50	0	20	40	60	80	80
$A_3=20$	100	50	0	20	40	60	100
$A_4=21$	150	100	50	0	20	40	150
$A_5=22$	200	150	100	50	0	20	200
$A_6=23$	250	200	150	100	50	0	250

∴ By minimax regret criterion, A_2 is the best option

(V) HURWICZ ($\alpha = 0.6$) CRITERION $\quad W_0 = (\alpha \times MAX) + (1-\alpha) \times MIN$

STRATEGIES (COURSES OF ACTION)	E_1	E_2	E_3	E_4	E_5	E_6	MAX	MIN	W_0
A_1	360	360	360	360	360	360	360	360	360
A_2	310	380	380	380	380	380	380	310	352
A_3	260	330	400	400	400	400	400	260	344
A_4	210	280	350	420	420	420	420	210	336
A_5	160	230	300	370	440	440	440	160	328
A_6	110	180	250	320	390	460	460	110	320

∴ By Hurwicz criterion, A_1 = 18 books is the best option

(II) (i) EXPECTED MONETARY VALUE :

STATES OF NATURE (DEMAND)	PROB.	COURSES OF ACTION (STRATEGIES) $A_1 = 18$	$A_2 = 19$	$A_3 = 20$	$A_4 = 21$	$A_5 = 22$	$A_6 = 23$
$E_1 = 18$	0.05	360	310	260	210	160	110
$E_2 = 19$	0.10	360	380	330	280	230	180
$E_3 = 20$	0.30	360	380	400	350	300	250
$E_4 = 21$	0.40	360	380	400	420	370	320
$E_5 = 22$	0.10	360	380	400	420	440	390
$E_6 = 23$	0.05	360	380	400	420	440	460
EXPECTED VALUE (EMV) =		360	376.5	386	374.5	335	288.5

∴ By EMV method, $A_3 = 20$ has the highest pay off and hence is the best option.

(iii) EXPECTED OPPORTUNITY LOSS

maximum values of payoff under each state of nature:

$E_1 = 18$	$E_2 = 19$	$E_3 = 20$	$E_4 = 21$	$E_5 = 22$	$E_6 = 23$
360	380	400	420	440	460

STATES OF NATURE (DEMAND)	PROB.	STRATEGIES (COURSES OF ACTION) CONDITIONAL OPPORTUNITY LOSS					
		$A_1 = 18$	$A_2 = 19$	$A_3 = 20$	$A_4 = 21$	$A_5 = 22$	$A_6 = 23$
$E_1 = 18$	0.05	0	50	100	150	200	250
$E_2 = 19$	0.10	20	0	50	100	150	200
$E_3 = 20$	0.30	40	20	0	50	100	150
$E_4 = 21$	0.40	60	40	20	0	50	100
$E_5 = 22$	0.10	80	60	40	20	0	50
$E_6 = 23$	0.05	100	80	60	40	20	0
EOL =		51	34.5	25	36.5	76	122.5

Therefore, using EOL method, $A_3 = 20$ is the best option.

$$\rightarrow \text{EPPI} = (0.05 \times 360) + (0.1 \times 380) + (0.3 \times 400) + (0.4 \times 420) + (0.1 \times 440) + (0.05 \times 460)$$

$$= 18 + 38 + 120 + 168 + 44 + 23 = \underline{411}$$

$$\therefore \text{EVPI} = \text{EPPI} - \max(\text{EMV})$$

$$= 411 - 386 = \underline{25} = \min(\text{EOL})$$