OPERATIONS RESEARCH

ASSIGNMENT -1-

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(1) A manufacturer wants to ship 8 loads of his product as shown below. The matrix gives the distance (kms) from the origins to the destinations.

	DESTINATION			
ORIGIN	A	B	C	AVAILABILITY
X	50	30	220	1
Y	90	45	170	3
Z	50	200	50	4
REQUIREMENT	3	3	2	

Shipping costs are ₹10 per load per km.

(a) Find the INITIAL SOLUTION by:

i) NORTH WEST CORNER RULE:

	A	B	C	
X	(1) 50	30	220	~~1~~
Y	(2) 90	(1) 45	170	~~3~~ ~~1~~
Z	50	(2) 200	(2) 50	~~4~~ ~~2~~
	~~2~~ ~~3~~	~~2~~ ~~3~~	~~2~~	8

TOTAL DISTANCE $= (1 \times 50) + (2 \times 90) + (1 \times 45) + (2 \times 200) + (2 \times 50)$

$= 50 + 180 + 45 + 400 + 100$

$= \underline{775}$ kms.

$\therefore$ TOTAL COST $=$ ₹ $(775 \times 10) =$ <u>₹ 7750</u>

(ii) LEAST COST METHOD

	A	B	C	
X	50	(1) 30	220	~~1~~
Y	90	(2) 45	(1) 170	~~3~~ ~~1~~
Z	(3) 50	200	(1) 50	~~4~~ ~~1~~
	~~3~~	~~3~~ ~~1~~	~~2~~ ~~1~~	8

TOTAL DISTANCE

$= (1 \times 30) + (2 \times 45) + (1 \times 170) + (3 \times 50) + (1 \times 50)$

$= 30 + 90 + 170 + 150 + 50$

$= \underline{\underline{490 \text{ kms}}}$

$\therefore$ TOTAL COST = ₹(490 × 10) = ₹4900

(iii) VOGEL'S APPROXIMATION METHOD :

	A	B	C	
X	(1) 50	30	220	~~1~~
Y	90	(3) 45	170	~~3~~
Z	(2) 50	200	(2) 50	~~4~~ ~~2~~
	~~3~~ 1	~~3~~	~~2~~	8

ROW PENALTIES

20	20	20	20
45	45	45	×
0	150	×	×

COLUMN PENALTIES

0	15	120
0	15	×
40	15	×
40	×	×

TOTAL DISTANCE $= (1 \times 50) + (3 \times 45) + (2 \times 50) + (2 \times 50)$

$= 50 + 135 + 100 + 100$

$= \underline{\underline{385 \text{ kms}}}$

$\therefore$ TOTAL COST = ₹(385 × 10) = ₹3850

(b) Also find the optimum shipping schedule to minimize the total transportation cost.

SOLUTION USING <u>VAM</u>:

	A $V_1 = 50$	B $V_2 = 30$	C $V_3 = 50$	
X $U_1 = 0$	$\boxed{1}$ 50	$\boxed{E}$ 30	220	1
Y $U_2 = 15$	90	$\boxed{3}$ 45	170	3
Z $U_3 = 0$	$\boxed{2}$ 50	200	$\boxed{2}$ 50	4
	3	3	2	8

$m = 3,\ n = 3$

$m + n - 1 = 5$

$\neq$ no. of allocated cells

$\Rightarrow$ MATRIX IS <u>DEGENERATE</u>

<u>PENALTIES</u>:

$P_{13} = -170 \quad P_{21} = -25 \quad P_{32} = -170$

$P_{23} = -105$

Since all the values of P_{ij} are ≤ 0, the obtained solution is OPTIMAL.

Therefore,

LEAST DISTANCE $=$ 385 kms

$\Rightarrow$ MINIMUM COST $=$ ₹ 3850

(2) A company has four terminals U, V, W and X. At the start of a particular day, 10, 4, 6 and 5 trailers are available at their terminals. During the previous night 13, 10, 6 and 6 trailers respectively were loaded at plants A, B, C and D. The company dispatcher has come up with the costs between the terminals and plants as follows:

PLANT

TERMINAL	A	B	C	D	
U	20	36	19	28	10
V	40	20	45	20	4
W	75	35	45	50	6
X	30	35	40	25	5
	13	10	6	6	

Find the optimal allocation of loaded terminals from plants to terminals in order to minimize the total transportation cost.

SOLUTION:-

Balancing the problem and solving using VAM:

	A	B	C	D	
U	3 20	36	6 19	1 28	~~10~~ ~~4~~ X
V	40	4 20	45	20	~~4~~
W	75	6 35	45	50	~~6~~
X	30	35	40	5 25	~~5~~
Y	10 0	0	0	0	~~10~~
	~~13~~ 3	~~10~~ ~~4~~	~~6~~	~~6~~ 8	35

ROW PENALTIES

1	1	8	8	8
0	0	0	0	x
10	10	15	x	x
5	5	5	5	5
0	x	x	x	x

COLUMN PENALTIES

20	20	19	20
10	15	21	5
10	15	x	5
10	15	x	5
10	x	x	3

$V_1 = 20$ $V_2 = 20$ $V_3 = 19$ $V_4 = 28$

		A	B	C	D	
$U_1 = 0$	U	[3] (+) 20	36	[6] 19	[1] (−) 28	10
$U_2 = 0$	V	40	[4] 20	45	20	4
$U_3 = 15$	W	75	[6] 35	45	50	6
$U_4 = -3$	X	30	35	40	[5] 25	5
$U_5 = -20$	Y	[10] (−) 0	[ε] 0	0	(+) 0	10
		13	10	6	6	35

$m = 5$, $n = 4$

$m + n - 1 = 8$

$\neq$ no. of allocated cells (7)

$\Rightarrow$ Problem is DEGENERATE

<u>PENALTIES:</u>

$P_{12} = -16$ $P_{21} = -20$ $P_{31} = -40$ $P_{41} = -13$ $P_{53} = -1$

$P_{23} = -26$ $P_{33} = -11$ $P_{42} = -18$ $\boxed{P_{54} = 8}$

$P_{24} = 8$ $P_{34} = -7$ $P_{43} = -24$

$V_1 = 20$ $V_2 = 20$ $V_3 = 19$ $V_4 = 20$

		A	B	C	D	
$U_1 = 0$	U	[4] 20	36	[6] 19	28	10
$U_2 = -20$	V	40	[4] 20	45	20	4
$U_3 = 15$	W	75	[6] 35	45	50	6
$U_4 = 5$	X	30	35	40	[5] 25	5
$U_5 = -20$	Y	[9] 0	[ε] 0	0	[1] 0	10
		13	10	6	6	35

$m + n - 1 = 8$

$= 8$

<u>PENALTIES:</u>

$P_{12} = -16$ $P_{21} = -40$ $P_{31} = -40$ $P_{41} = -5$ $P_{53} = -1$

$P_{14} = -8$ $P_{23} = -46$ $P_{33} = -11$ $P_{42} = -10$

$P_{24} = -20$ $P_{34} = -15$ $P_{43} = -16$

Since all the values of P_{ij} are ≤ 0, the obtained solution is OPTIMAL.

MINIMUM TRANSPORTATION COST $= (4 \times 20) + (6 \times 19) + (4 \times 20) + (6 \times 35) + (5 \times 25) + (9 \times 0) + (1 \times 0)$

$= 80 + 114 + 80 + 210 + 125$

$= \underline{609}$

(3) The following table provides all the necessary information on the availability of supply to each warehouse, the requirement of each market, and the unit transportation cost (in ₹) from each warehouse to each market.

WAREHOUSE

	P	Q	R	S	SUPPLY
A	6	3	5	4	22
B	5	9	2	7	15
C	5	7	8	6	8
DEMAND	7	12	17	9	45

Find the optimal schedule and minimum total transport cost.

SOLUTION :-

	P	Q	R	S	
A	6	(12) 3	(2) 5	(8) 4	~~22~~ ~~10~~ 8
B	5	9	(15) 2	7	~~15~~
C	(7) 5	7	8	(1) 6	~~8~~ ✗
	~~7~~	~~12~~	~~17~~ ~~2~~	~~9~~ ✗	45

ROW PENALTIES

1	1	1	2	×
3	3	×	×	×
1	1	1	1	1

COLUMN PENALTIES

0	4	3	2
0	×	3	2
1	×	3	2
1	×	×	2

$V_1 = 3$ $V_2 = 3$ $V_3 = 5$ $V_4 = 4$

		P	Q	R	S	
$U_1 = 0$	A	6	$\boxed{12}$ 3	$\boxed{2}$ 5	$\boxed{8}$ 4	22
$U_2 = -3$	B	5	9	$\boxed{15}$ 2	7	15
$U_3 = 2$	C	$\boxed{7}$ 5	7	8	$\boxed{1}$ 6	8
		7	12	17	9	45

$m = 3$, $n = 4$

$m + n - 1 = 6$

$= 6$

PENALTIES:

$P_{11} = -3 \quad P_{21} = -5 \quad P_{32} = -6$

$P_{22} = -9 \quad P_{33} = -5$

$P_{24} = -6$

Since all the values of P_{ij} are ≤ 0, the given solution is OPTIMAL.

TOTAL COST (MINIMIZED) $= (12 \times 3) + (2 \times 5) + (8 \times 4) + (15 \times 2) + (7 \times 5) + (1 \times 6)$

$= 36 + 10 + 32 + 30 + 35 + 6$

$= \underline{149}$

(4) ABC Limited has three production shops that supply a product to five warehouses. The cost of production varies from shop to shop and cost of transportation from one shop to warehouse also varies. Each shop has a specific production capacity and each warehouse has certain amount of requirement. The costs of transportation are given below:

	WAREHOUSE I	II	III	IV	V	SUPPLY
A	6	4	4	7	5	100
B	5	6	7	4	8	125
C	3	4	6	3	4	175
DEMAND	60	80	85	105	70	

The cost of manufacturing the product at different production shops is:

SHOP	VARIABLE COST	FIXED COST
A	14	7000
B	16	4000
C	15	5000

Find the optimum quantity to be supplied from each shop to different warehouses at the minimum total cost.

TRANSPORTATION COST MATRIX with VARIABLE COST solved for IBFS using VAM.

	I	II	III	IV	V	
A	20	18	(85) 18	21	(15) 19	~~100~~ ~~15~~
B	21	(80) 22	23	20	(45) 24	~~125~~ ~~45~~
C	(60) 18	19	21	(105) 18	(10) 19	~~175~~ ~~70~~ 10
	~~60~~	~~80~~	~~85~~	~~105~~	~~70~~ ~~55~~ ~~10~~	400

ROW PENALTIES

0	1	1	1	-
1	1	1	(2)	-
0	0	1	0	-

COLUMN PENALTIES

2	1	(3)	2	0
2	1	×	(2)	0
(2)	1	×	×	0
×	1	×	×	0
×	×	×	×	(0)

$V_1 = 18$ $V_2 = 17$ $V_3 = 18$ $V_4 = 18$ $V_5 = 19$

		I	II	III	IV	V	
$U_1 = 0$	A	20	18	[85] 18	21	[15] 19	100
$U_2 = 5$	B	21	[80] 22	23	(+) 20	[45] (−) 24	125
$U_3 = 0$	C	[60] 18	19	21	[105] (−) 18	[10] (+) 19	175
		60	80	85	105	70	400

$m = 3$, $n = 5$

$m + n - 1 = 7$

$\underline{\underline{= 7}}$

PENALTIES:

$P_{11} = -2$ $P_{21} = 2$ $P_{32} = -2$

$P_{12} = -1$ $P_{23} = 0$ $P_{33} = -3$

$P_{14} = -3$ $\boxed{P_{24} = 3}$

$V_1 = 18$ $V_2 = 20$ $V_3 = 18$ $V_4 = 18$ $V_5 = 19$

		I	II	III	IV	V	
$U_1 = 0$	A	20	(+) 18	[85] 18	21	[15] (−) 19	100
$U_2 = 2$	B	21	[80] (−) 22	23	[45] (+) 20	24	125
$U_3 = 0$	C	[60] 18	19	21	[60] (−) 18	[55] (+) 19	175
		60	80	85	105	70	400

$m = 3$, $n = 5$

$m + n - 1 = 7$

$\underline{\underline{= 7}}$

PENALTIES:

$P_{11} = -2$ $\quad$ $P_{21} = -1$ $\quad$ $P_{32} = 1$

$\boxed{P_{12} = 2}$ $\quad$ $P_{23} = -3$ $\quad$ $P_{33} = -3$

$P_{14} = -3$ $\quad$ $P_{25} = -3$

$V_1 = 16$ $V_2 = 18$ $V_3 = 18$ $V_4 = 16$ $V_5 = 17$

		I	II	III	IV	V	
$U_1 = 0$	A	20	[15] 18	[85] 18	21	19	100
$U_2 = 4$	B	21	[65] (−) 22	23	[60] (+) 20	24	125
$U_3 = 2$	C	[60] 18	(+) 19	21	[45] (−) 18	[70] 19	175
		60	80	85	105	70	400

$m = 3, n = 5$

$m + n - 1 = 7$

$= 7$

PENALTIES:

$P_{11} = -4$ $\quad$ $P_{21} = -1$ $\quad$ $\boxed{P_{32} = 1}$

$P_{14} = -5$ $\quad$ $P_{23} = -1$ $\quad$ $P_{33} = -1$

$P_{15} = -2$ $\quad$ $P_{25} = -3$

$V_1 = 17$ $V_2 = 18$ $V_3 = 18$ $V_4 = 16$ $V_5 = 18$

		I	II	III	IV	V	
$U_1 = 0$	A	20	[15] 18	[85] 18	21	19	100
$U_2 = 4$	B	21	[20] 22	23	[105] 20	24	125
$U_3 = 1$	C	[60] 18	[45] 19	21	18	[70] 19	175
		60	80	85	105	70	400

$m = 3, n = 5$

$m + n - 1 = 7$

$= 7$

PENALTIES:

$P_{11} = -3$ $\quad$ $P_{21} = 0$ $\quad$ $P_{33} = -2$

$P_{14} = -5$ $\quad$ $P_{23} = -1$ $\quad$ $P_{34} = -1$

$P_{15} = -1$ $\quad$ $P_{25} = -2$

Since all the values of P_{ij} are ≤ 0, the solution is OPTIMAL

TOTAL TRANSPORTATION COST

$= (15 \times 18) + (85 \times 18) + (20 \times 22) + (105 \times 20) + (60 \times 18) + (45 \times 19) + (70 \times 19)$

$= 270 + 1530 + 440 + 2100 + 1080 + 855 + 1330$

$= 7605$

(5) A transportation problem has the supplies at four sources and requirements at five destinations. The following table shows the cost of shipping one unit from a particular source to a particular destination:

DESTINATION

SOURCE	1	2	3	4	5
1	12	4	9	5	9
2	8	1	6	6	7
3	1	12	4	7	7
4	10	15	6	9	1

Test whether the following pattern has the least possible transportation cost. If not determine the optimal transportation cost.

$x_{11} = 25,\ x_{14} = 30,\ x_{22} = 20,\ x_{23} = 25,\ x_{31} = 15,\ x_{33} = 15,\ x_{43} = 10,\ x_{45} = 40$

TOTAL COST as per the pattern

$= (25 \times 12) + (30 \times 5) + (20 \times 1) + (6 \times 25) + (1 \times 15) + (4 \times 15) + (6 \times 10) + (1 \times 40)$

$= 300 + 150 + 20 + 150 + 15 + 60 + 60 + 40$

$= \underline{\underline{795}} \longrightarrow (1)$

DESTINATION

SOURCE	1	2	3	4	5	
1	12	4	9	5	9	55
2	8	1	6	6	7	45
3	1	12	4	7	7	30
4	10	15	6	9	1	50
	40	20	50	30	40	180

	1	2	3	4	5	
1	12	4	(25) 9	(30) 5	9	~~55~~ 25
2	(10) 8	(20) 1	(15) 6	6	7	~~45~~ ~~25~~ 15
3	(30) 1	12	4	7	7	30
4	10	15	(10) 6	9	(40) 1	~~50~~ 10
	~~40~~ 10	~~20~~	~~50~~ ~~40~~ 25	30	~~40~~	180

ROW PENALTIES

1	1	1	4	3	3
5	5	5	0	2	2
3	x	x	x	x	x
5	5	4	3	4	x

COLUMN PENALTIES

7	3	2	1	6
2	3	0	1	6
2	3	0	1	x
2	x	0	1	x
2	x	0	x	x
4	x	3	x	x

		$V_1 = 11$	$V_2 = 4$	$V_3 = 9$	$V_4 = 5$	$V_5 = 4$	
		1	2	3	4	5	
$U_1 = 0$	1	12	4	$\boxed{25}$ 9	$\boxed{30}$ 5	9	55
$U_2 = -3$	2	$\boxed{10}$ 8	$\boxed{20}$ 1	$\boxed{15}$ 6	6	7	45
$U_3 = -10$	3	$\boxed{30}$ 1	12	4	7	7	30
$U_4 = -3$	4	10	15	$\boxed{10}$ 6	9	$\boxed{40}$ 1	50
		40	20	50	50	40	180

$m = 4,\ n = 5$

$m + n - 1 = 8$

$= 8$

PENALTIES:

$P_{11} = -1$ $\quad P_{12} = 0$ $\quad P_{15} = -5$

$P_{24} = -4$ $\quad P_{25} = -6$

$P_{32} = -18$ $\quad P_{33} = -5$ $\quad P_{34} = -12$ $\quad P_{35} = -13$

$P_{41} = -2$ $\quad P_{42} = -14$ $\quad P_{44} = -7$

Since all the values of P_{ij} are ≤ 0, the solution is OPTIMAL

TOTAL TRANSPORTATION COST

$= (25\times9) + (30\times5) + (10\times8) + (20\times1) + (15\times6) + (30\times1) + (10\times6) + (40\times1)$

$= 225 + 150 + 80 + 20 + 90 + 30 + 60 + 40 = 695$

Therefore from (1), the given allocations to test are NOT optimal.

(6) A company has three plants and four warehouses. The supply and demand in units and the corresponding transportation costs are given. The table below has been taken from the solution procedure of the transportation problem.

WAREHOUSES

PLANT		I	II	III	IV	
	A	5	10	[10] 4	5	10
PLANT	B	[20] 6	8	7	[5] 2	25
	C	[5] 4	[10] 2	[5] 5	7	20
		25	10	15	5	55

Answer the following questions, giving for brief reasons:

		$V_1 = 5$	$V_2 = 3$	$V_3 = 4$	$V_4 = 1$	
		I	II	III	IV	
$U_1 = 0$	A	(+) 5	10	[10] (−) 4	5	10
$U_2 = 1$	B	[20] 6	8	7	[5] 2	25
$U_3 = -1$	C	[5] (−) 4	[10] 2	[5] (+) 5	7	20
		25	10	15	5	55

$m = 3,\ n = 4$

$m + n - 1 = \cancel{6}$

$\cancel{6} = 6$

PENALTIES:

$\boxed{P_{11} = 0}$ $P_{22} = -4$ $P_{34} = -7$

$P_{12} = -7$ $P_{23} = -3$

$P_{14} = -4$

(i) Is this solution feasible? YES, BECAUSE THE SOLUTION IS NOT DEGENERATE AND ALL THE VALUES OF P_{ij} ARE ≤ 0.

(ii) Is this solution optimal? If not find the optimal solution. YES, THIS SOLUTION IS OPTIMAL, BECAUSE ALL THE VALUES OF P_{ij} ARE ≤ 0.

(iii) Does this problem have more than one optimal solution? YES, BECAUSE THE VALUE OF $P_{11} = 0$, INDICATING AN OPTIMAL SOLUTION.

ALTERNATE SOLUTION:

		$V_1 = 5$ I	$V_2 = 1$ II	$V_3 = 4$ III	$V_4 = 1$ IV	
$U_1 = 0$	A	$\boxed{5}$ 5	10	$\boxed{5}$ 4	5	10
$U_2 = 1$	B	$\boxed{20}$ 6	8	7	$\boxed{5}$ 2	25
$U_3 = 1$	C	4	$\boxed{10}$ 2	$\boxed{10}$ 5	7	20
		25	10	15	5	55

(iv) Is this solution degenerate? NO, IT IS NOT. BECAUSE TOTAL NUMBER OF ALLOCATIONS = 6 = m(3) + n(4) − 1

(v) If the cost of route B–III is reduced from ₹7 to ₹6 per unit, what will be the optimum solution?

REWRITING THE MATRIX WITH CHANGES AND SOLVING FOR IBFS USING VAM:

	I	II	III	IV	
A	5	10	(10) 4	5	~~10~~
B	(15) 6	8	(5) 6	(5) 2	~~25~~ ~~20~~ ~~15~~
C	(10) 4	(10) 2	5	7	~~20~~ ~~10~~
	~~25~~ ~~15~~	~~10~~	~~15~~ ~~5~~	~~5~~	55

ROW PENALTIES

1	1	1	×	×
4	4	0	0	0
2	1	1	1	×

COLUMN PENALTIES

1	6	1	3
1	×	1	3
1	×	1	×
2	×	1	×
–	×	–	×

		$V_1 = 4$ I	$V_2 = 2$ II	$V_3 = 4$ III	$V_4 = 0$ IV	
$U_1 = 0$	A	5	10	$\boxed{10}$ 4	5	10
$U_2 = 2$	B	$\boxed{15}$ 6	8	$\boxed{5}$ 6	$\boxed{5}$ 2	25
$U_3 = 0$	C	$\boxed{10}$ 4	$\boxed{10}$ 2	5	7	20
		25	10	15	5	55

$m+n-1 = 6$

$= 6$

PENALTIES:

$P_{11} = -1$ $P_{22} = -4$ $P_{33} = -1$

$P_{12} = -8$ $P_{34} = -7$

$P_{14} = -5$

Since all the values of P_{ij} are ≤ 0, the solution is OPTIMAL

TOTAL TRANSPORTATION COST

$= (10 \times 4) + (15 \times 6) + (5 \times 6) + (5 \times 2) + (10 \times 4) + (10 \times 2)$

$= 40 + 90 + 30 + 10 + 40 + 20$

$= \underline{230}$

(7) A particular product is manufactured in factories A, B, C and D and is sold at centres 1, 2 and 3. The cost in ₹ of product per unit and capacity (in kg) per unit time of each plant is given below:

FACTORY	COST (₹) /UNIT	CAPACITY (kg) /UNIT
A	12	100
B	15	20
C	11	60
D	18	80

The sale price in rupees per unit and the demand in kg per unit are as follows:

SALES CENTER	SALES PRICE (₹/UNIT	DEMAND(kg) /UNIT
1	15	120
2	14	140
3	16	60

Find the optimum sales distribution.

PROFIT MATRIX TO MAXIMIZED

	1	2	3	
A	3	2	4	100
B	0	-1	1	20
C	4	3	5	60
D	-3	-4	-2	80
E	0	0	0	60
	120	140	60	320

	1	2	3	
A	(100) 2	3	1	100
B	(20) 5	6	4	~~20~~
C	1	2	(60) 0	~~60~~
D	8	(80) 9	7	~~80~~
E	5	(60) 5	5	~~60~~
	~~120~~ 20	~~140~~ ~~80~~	~~60~~	320

ROW PENALTIES

1	1	×	×	×
1	1	(1)	×	×
(1)	×	×	×	×
1	1	1	.	.
0	0	0	-	-

COLUMN PENALTIES

1	1	1
(3)	2	×
0	1	×
×	(4)	×
×	-	×
×	-	×

$V_1 = 2$ $V_2 = 3$ $V_3 = 1$

		1	2	3	
$U_1 = 0$	A	$\boxed{100}$ 2	$\boxed{\varepsilon}$ 3	$\boxed{\varepsilon}$ 1	$100+\varepsilon$
$U_2 = 3$	B	$\boxed{20}$ 5	6	4	20
$U_3 = -1$	C	$\boxed{\varepsilon}$ 1	2	$\boxed{60}$ 0	$60+\varepsilon$
$U_4 = 6$	D	8	$\boxed{80}$ 9	7	80
$U_5 = 2$	E	5	$\boxed{60}$ 5	5	60
		$120+\varepsilon$	140	$60+\varepsilon$	$320+2\varepsilon$

$m = 5$ $n = 3$

$m+n-1 = 7$

$\neq 5$

PENALTIES:

$P_{13} = 0$ $P_{22} = 0$ $P_{32} = 0$ $P_{41} = 0$ $P_{51} = -1$

$P_{23} = 0$ $P_{43} = 0$ $P_{53} = -2$

Since all the values of P_{ij} are ≤ 0, the current solution is optimal.

∴ TOTAL PROFIT (MAXIMUM):

$(100 + 3) + (20 \times 0) + (60 \times 5) + (80 \times -4) + (60 \times 0)$

$= 300 + 0 + 300 - 320$

$= \underline{\underline{280}}$

---

(8) Find the optimum solution to the following transportation problem to minimize the cost

PRODUCTION COST / UNIT

WEEK	1	2	3	4	SUPPLY
1	10	13	16	19	700
2	M	10	13	16	700
3	M	15	18	21	200
4	M	M	15	18	700
5	M	M	20	23	200
6	M	M	M	15	700
DEMAND	300	700	900	800	~~2700~~ 3200

converting into balanced problem and solving using VAM:

	1	2	3	4	5	
1	300 10	13	400 16	19	0	~~700~~ ~~400~~
2	M	700 10	13	16	0	~~700~~
3	M	15	100 18	100 21	0	~~200~~ 100
4	M	M	400 15	18	300 0	~~700~~ ~~400~~
5	M	M	20	23	200 0	~~200~~
6	M	M	M	700 15	0	~~700~~
	~~300~~	~~700~~	~~900~~ ~~500~~ ~~100~~	~~800~~ 100	500 ~~300~~	3200

ROW PENALTIES

10	13	13	3	3	3	3
10	10	10	3	3	×	×
15	15	15	3	3	3	3
15	15	15	3	3	3	×
20	20	×	×	×	×	×
15	15	15	M-15	×	×	×

COLUMN PENALTIES

M-10	3	2	1	0
×	3	2	1	0
×	3	2	1	0
×	3	2	1	×
×	3	2	2	×
×	×	1	1	×
×	×	2	2	×

	$V_1 = 10$	$V_2 = 11$	$V_3 = 16$	$V_4 = 19$	$V_5 = 1$	
	1	2	3	4	5	
$U_1 = 0$ 1	[300] 10	13	[400] 16	19	0	700
$U_2 = -1$ 2	M	[700] 10	13	16	[ε] 0	700+ε
$U_3 = 2$ 3	M	15	[100] (−) 18	[100] 21	(+) 0	200
$U_4 = -1$ 4	M	M	[400] (+) 15	18	[300] (−) 0	700
$U_5 = -1$ 5	M	M	20	23	[200] 0	200
$U_6 = -4$ 6	M	M	M	[700] 15	0	700
	300	700	900	800	500+ε	$\underline{\underline{3200+\varepsilon}}$

$m = 6,\ n = 5$

$m + n - 1 = 10$

$\neq 9$

$\Rightarrow$ DEGENERATE PROBLEM

PENALTIES:

$P_{12} = -2$ $\quad P_{21} = (-)$ $\quad P_{31} = (-)$ $\quad P_{41} = (-)$ $\quad P_{51} = (-)$ $\quad P_{61} = (-)$

$P_{14} = 0$ $\quad P_{23} = -2$ $\quad P_{32} = -2$ $\quad P_{42} = (-)$ $\quad P_{52} = (-)$ $\quad P_{62} = (-)$

$P_{15} = 1$ $\quad P_{24} = 2$ $\quad \boxed{P_{35} = 3}$ $\quad P_{44} = 0$ $\quad P_{53} = -5$ $\quad P_{63} = (-)$

$P_{54} = -5$ $\quad P_{65} = -3$

	$V_1 = 10$	$V_2 = 11$	$V_3 = 16$	$V_4 = 22$	$V_5 = 1$	
	1	2	3	4	5	
$U_1 = 0$ 1	[300] 10	13	[400] 16	19	0	700
$U_2 = -1$ 2	M	[700] 10	13	16	[ε] 0	700+ε
$U_3 = -1$ 3	M	15	18	[100] (+) 21	[100] (−) 0	200
$U_4 = -1$ 4	M	M	[500] 15	18	[200] 0	700
$U_5 = -1$ 5	M	M	20	23	[200] 0	200
$U_6 = 7$ 6	M	M	M	[700] (−) 15	(+) 0	700
	300	700	900	800	500+ε	$\underline{\underline{3200+\varepsilon}}$

$m = 6,\ n = 5$

$m + n - 1 = 10$

$\underline{\underline{= 10}}$

PENALTIES:

$P_{12} = -2$ $\quad P_{21} = (-)$ $\quad P_{31} = (-)$ $\quad P_{41} = (-)$ $\quad P_{51} = (-)$ $\quad P_{61} = (-)$

$P_{14} = 3$ $\quad P_{23} = 2$ $\quad P_{32} = -5$ $\quad P_{42} = (-)$ $\quad P_{52} = (-)$ $\quad P_{62} = (-)$

$P_{15} = 1$ $\quad P_{24} = 5$ $\quad P_{33} = -3$ $\quad P_{44} = 3$ $\quad P_{53} = -5$ $\quad P_{63} = (-)$

$P_{54} = -2$ $\quad \boxed{P_{65} = 8}$

		$V_1 = 10$	$V_2 = 11$	$V_3 = 16$	$V_4 = 16$	$V_5 = 1$	
		1	2	3	4	5	
$U_1 = 0$	1	[300] 10	13	[400] 16	19	0	700
$U_2 = -1$	2	M	[700] 10	13	16	[ε] 0	700 + ε
$U_3 = 5$	3	M	15	18	[200] (−) 21	(+) 0	200
$U_4 = -1$	4	M	M	[500] 15	18	[200] 0	700
$U_5 = -1$	5	M	M	20	23	[200] 0	200
$U_6 = -1$	6	M	M	M	[600] (+) 15	[100] (−) 0	700
		300	700	900	800	500+ε	3200 + ε

m+n-1
= 10
= 10

PENALTIES:

$P_{12} = \cancel{0} -2$ $P_{21} = (-)$ $P_{31} = (-)$ $P_{41} = (-)$ $P_{51} = (-)$ $P_{61} = (-)$

$P_{14} = -3$ $P_{23} = 2$ $P_{32} = 1$ $P_{42} = (-)$ $P_{52} = (-)$ $P_{62} = (-)$

$P_{15} = 1$ $P_{24} = -1$ $P_{33} = 3$ $P_{44} = -3$ $P_{53} = -5$ $P_{63} = (-)$

$P_{35} = 6$ $P_{54} = -8$

		$V_1 = 10$	$V_2 = 11$	$V_3 = 16$	$V_4 = 22$	$V_5 = 1$	
		1	2	3	4	5	
$U_1 = 0$	1	[300] 10	13	[400] (−) 16	19	(+) 0	700
$U_2 = -1$	2	M	[700] 10	13	16	[ε] 0	700+ε
$U_3 = -1$	3	M	15	18	[100] 21	[100] 0	200
$U_4 = -1$	4	M	M	[500] (+) 15	18	[200] (−) 0	700
$U_5 = -1$	5	M	M	20	23	[200] 0	200
$U_6 = 7$	6	M	M	M	[700] 15	0	700
		300	700	900	800	500+ε	3200+ε

m+n-1
= 10
= 10

PENALTIES:

$P_{12} = -2$ $P_{21} = (-)$ $P_{31} = (-)$ $P_{41} = (-)$ $P_{51} = (-)$ $P_{61} = (-)$

$P_{14} = 3$ $P_{23} = 2$ $P_{32} = -5$ $P_{42} = (-)$ $P_{52} = (-)$ $P_{62} = (-)$

$\boxed{P_{15} = 0}$ $P_{24} = 5$ $P_{33} = -3$ $P_{44} = 3$ $P_{53} = -5$ $P_{63} = (-)$

$P_{54} = -2$ $P_{65} = 8$

$V_1 = 10$ $V_2 = 13$ $V_3 = 16$ $V_4 = 21$ $V_5 = 0$

		1	2	3	4	5	
$U_1 = 0$	1	300 | 10	13	200 | 16	19	200 | 0	700
$U_2 = 0$	2	M	700 | 10	13	(+) 16	ε (−) 0	700+ε
$U_3 = 0$	3	M	15	18	100 (−) 21	100 (+) 0	200
$U_4 = -1$	4	M	M	700 | 15	18	0	700
$U_5 = 0$	5	M	M	20	23	200 | 0	200
$U_6 = 6$	6	M	M	M	700 | 15	0	700
		300	700	900	800	500+ε	3200+ε

m+n-1
= 10
= 10

PENALTIES:

$P_{12} = 0$ $P_{14} = -2$

$P_{21} = (-)$ $P_{23} = 3$ $\boxed{P_{24} = 5}$

$P_{31} = (-)$ $P_{32} = -2$ $P_{33} = -2$

$P_{41} = (-)$ $P_{42} = (-)$ $P_{44} = 2$ $P_{45} = -1$

$P_{51} = (-)$ $P_{52} = (-)$ $P_{53} = -4$ $P_{54} = -2$

$P_{61} = (-)$ $P_{62} = (-)$ $P_{63} = (-)$ $P_{65} = 6$

$V_1 = 10$ $V_2 = 15$ $V_3 = 16$ $V_4 = 21$ $V_5 = 0$

		1	2	3	4	5	
$U_1 = 0$	1	300 | 10	13	200 (+) 16	19	200 (−) 0	700
$U_2 = -5$	2	M	700 | 10	13	ε | 16	0	700+ε
$U_3 = 0$	3	M	15	18	100−ε (−) 21	100+ε (+) 0	200
$U_4 = -1$	4	M	M	700 (−) 15	(+) 18	0	700
$U_5 = 0$	5	M	M	20	23	200 | 0	200
$U_6 = -6$	6	M	M	M	700 | 15	0	700
		300	700	900	800	500+ε	

m+n-1
= 10
= 10

PENALTIES:

$P_{12} = 2$ $P_{14} = 2$

$P_{21} = (-)$ $P_{23} = -2$ $P_{25} = -5$

$P_{31} = (-)$ $P_{32} = 0$ $P_{33} = -2$

$P_{41} = (-)$ $P_{42} = (-)$ $\boxed{P_{44} = 2}$ $P_{45} = -1$

$P_{51} = (-)$ $P_{52} = (-)$ $P_{53} = -4$ $P_{54} = -2$

$P_{61} = (-)$ $P_{62} = (-)$ $P_{63} = (-)$ $P_{65} = -6$

		$V_1 = 10$ (1)	$V_2 = 13$ (2)	$V_3 = 16$ (3)	$V_4 = 19$ (4)	$V_5 = 0$ (5)	
$U_1 = 0$	1	[300] 10	13	[300-ε] 16	19	[100+ε] 0	700
$U_2 = -3$	2	M	[700] 10	13	[ε] 16	0	700+ε
$U_3 = 0$	3	M	15	18	21	[200] 0	200
$U_4 = -1$	4	M	M	[600+ε] 15	[100-ε] 18	0	700
$U_5 = 0$	5	M	M	20	23	[200] 0	200
$U_6 = -4$	6	M	M	M	[700] 15	0	700
		300	700	900	800	500+ε	

$m+n-1 = 10$

$= 10$

PENALTIES:

$P_{12} = 0$ $\quad P_{21} = (-)$ $\quad P_{31} = (-)$ $\quad P_{41} = (-)$ $\quad P_{51} = (-)$ $\quad P_{61} = (-)$

$P_{14} = 0$ $\quad P_{23} = 0$ $\quad P_{32} = -2$ $\quad P_{42} = (-)$ $\quad P_{52} = (-)$ $\quad P_{62} = (-)$

$P_{25} = -3$ $\quad P_{33} = -2$ $\quad P_{45} = -1$ $\quad P_{53} = -4$ $\quad P_{63} = (-)$

$P_{34} = -2$ $\quad P_{55} = -4$ $\quad P_{65} = -4$

Since all the values of P_{ij} are ≤ 0, the solution is OPTIMAL.

TOTAL TRANSPORTATION COST

$= (300 \times 10) + (300 \times 16) + (100 \times 0) + (700 \times 10) + (200 \times 0) + (600 \times 15) + (100 \times 18) + (200 \times 0) + (700 \times 15)$

$= 3000 + 4800 + 0 + 7000 + 0 + 9000 + 1800 + 0 + 10500$

$= 36100$