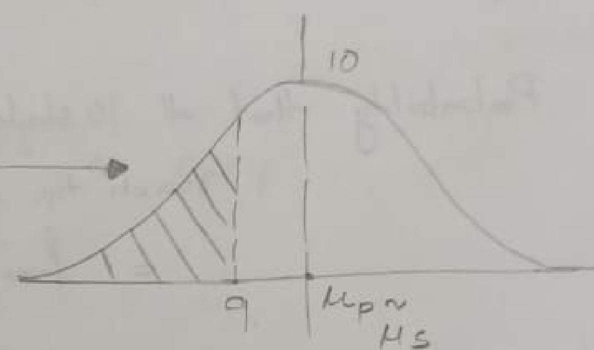
Central Limit Theorem Assignment -

Q Let X be a random variable with $\mu = 10$ and $\sigma = 4$. A sample of size 100 is taken from the population. Find the probability that the sample mean of these 100 observation is less than 9.

$\mu_p = 10$, $\sigma_p = 4$, $N = 100$

$P(\mu_{S100} < 9) = ?$



Calculating, Z score at $x = 9$,

$$Z_s = \frac{x - \mu_s}{\sigma_s}$$

$\mu_p = 10$

$\sigma_s = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{100}}$

$= \cancel{0.04}$

$= 0.4$

$$= \frac{9 - 10}{0.4}$$

$= -2.5$

Area under the curve, at $Z = -2.5$ =

$$P(\mu_{S100} < 9) = 0.0062$$

OPPO Reno2 Z

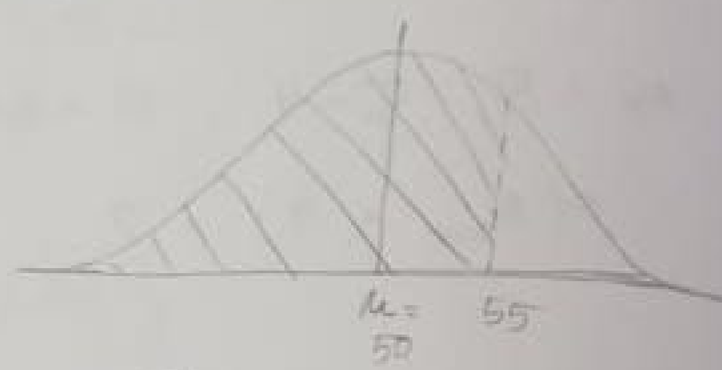
Q2 An elevator can transport a max of [illegible] kg. [illegible] overall student's data, it has shown that the weight of student follow a normal distribution, with a mean of 50 kgs and $\sigma = 15$. What is the probability that 10 students can reach safely to 8th floor?

Maximum the elevator can transport = 550 kg

$\mu_p = 50$ kg

$\sigma_p = 15$

$N = 10$



Probability that all 10 student can safely reach the 8th floor

= probability that the mean of 10 students is less than 55 kg ↳ (550/10)

$$Z = \frac{X - \mu_s}{\sigma_s} = \frac{55-50}{4.7} = 1.06$$

$$\frac{\sigma_p}{\sqrt{n}} = \frac{15}{\sqrt{10}}$$

$= 4.7$

Area under the curve at $Z = 1.06$ = 0.1446

14.46% probability that all 10 student can reach safely.

An officer in the army needs 35 men for a mission. He wants these soldiers to be smart enough to understand the mission.

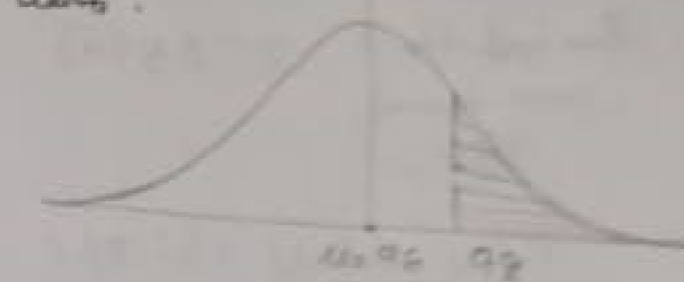
Q4. The average IQ of 35 men must be greater than 98 pts. It is told that the average IQ of a men is 96 with $\sigma = 16$. If the officer is given a random sample of 35 soldiers, what is the probability that he'll get what he wants.

$N = 35$

$\mu_p = 96$

$\sigma_p = 16$

$P(\mu_s > 98) = ?$



~~Prob~~ $$Z = \frac{X - \mu_s}{\sigma_s} = \frac{98-96}{2.70} = 0.74$$

$$\frac{\mu_p}{\sqrt{n}} = \frac{16}{\sqrt{35}} = 2.70$$

Area under curve towards left at $Z = 0.74$ = 0.7704

Area under curve toward right = $P(\mu_s > 98) = 1 - 0.7704$

$= 0.22$

$= 22\%$

Q5. Engineers must consider the breadths of male heads when designing motorcycle helmets. Men have head breadths that are normally distributed with a mean of 6.0 inch and a standard deviation of 1 inch.

(a) If a man is randomly selected - find the probability the ~~his~~ head breadth is less than 6.2 inch.

$\mu_p = 6$

$\sigma_p = 1$

$P(\mu_{sam} < 6.2) = ?$

$$Z_{score} = \frac{X - \mu_s}{\sigma_s} = \frac{6.2 - 6}{1} = 0.2$$

$\mu_s \to 6$

$$\sigma_s \to \frac{\sigma_p}{\sqrt{n}} = \frac{1}{\sqrt{1}} = 1$$

Area under the curve towards left $= 0.5793$

$P(\mu_s < 6.2) = 57.93\%$

$N = 1$

(b) for $N = 100$

$$Z_{score} = \frac{X - \mu_s}{\sigma_s} = \frac{6.2 - 6}{0.1} = 2$$

$\mu_s \to 6$

$$\sigma_s \to \frac{\sigma_p}{\sqrt{n}} = \frac{1}{\sqrt{100}} = 0.1$$

Area under the curve toward left of Z $= 0.97$

So, $P(\mu_s < 6.2)$ for 100 sample $= 97\%$

Q6 A production manager for Safeguard Helmet Company plans an initial run of 100 helmets. Seeing the result from part (b), the manager reasons that all helmets should be made for men with head breadths less than 6.2 inch because they would fit but a few men. What is wrong with reasoning?

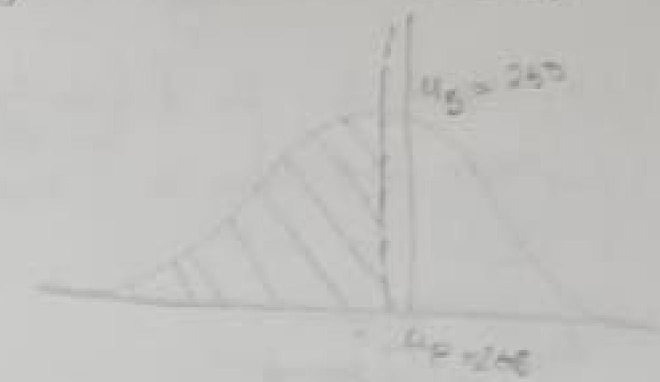
Q7. The length of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days. If 25 women are randomly selected, find the probability that their length of pregnancy have a mean that is less than 260 days.

$\mu_p = 268$

$\sigma_p = 15$

$N = 25$

$P(\mu_s < 260) = ?$



Sampling distribution of sample mean

$$Z_{score} \text{ at } X = 260 = \frac{X - \mu_s}{\sigma_s} = \frac{260 - 268}{3} = -2.66$$

$$\sigma_s \to \frac{\sigma_p}{\sqrt{n}} = \frac{15}{\sqrt{25}} = 3$$

Area under the curve towards left of Z $= 0.0039$

So, $P(\mu_s < 260) = 0.0039$
$= 0.39\%$

Q8. If 25 women are put on a special diet before they become pregnant and they end up having a mean length of pregnancy of less than 260 days, does it appear that the diet has an effect on the length of pregnancy?

Yes.

Since, from the statistics we found, that, there is 0.39% probability of women having a mean length of less than 260. Since, here, the mean is less than 260, so, it mean the diet

Q9. The weight of adult males are normally distributed with a mean of 172 pounds and a standard deviation of 29 pounds

(a) What is the probability that one randomly selected male will weigh more than 190 pounds

(b) What is the probability that 25 randomly selected males will have a mean of more than 190 pound.

$\mu_p = 172$ pounds

$\sigma_p = 29$

(a) $N = 1$, $P(\mu_s > 190) = ?$

$$z = \frac{X-\mu_s}{\sigma_s} = \frac{190-172}{29} = 0.62$$

$$\frac{\sigma_p}{\sqrt{N}} = 29$$

Area under the curve towards left $= 0.7324$

Area under the curve toward right $= P(\mu_s > 190)$
$= 1-0.7324$
$= 0.2676$

$= 26.76\%$

(b) Zscore for $(N = 25)$ $= \frac{X-\mu_s}{\sigma_s} = \frac{190-172}{5.8} = 3.10$

$$\frac{\sigma_p}{\sqrt{N}} = \frac{29}{\sqrt{25}} = 5.8$$

Area under the curve towards left of Z $= 0.9990$

Area under the curve towards right of Z $= P(\mu_s > 190) = 1-0.99$
$= 0.01 = 1\%$

(c) An elevator of men's fitness center has a sign that the maximum allowable weight is 4750 pound. If 25 randomly selected men cram into the elevator, what's the probability it will be over the maximum allowable weight.

Probability it will be over-weight of 25 men = Probability that each of these men weight a ~~maximum of~~ 190 pound or more

OR

Probability that the mean of these 25 men is ~~at~~ 190

$\mu_s = \mu_p = 190$ pounds

$\sigma_p = ?$

- The Probability from (b) that 25 men will have a mean weight of 190 pound is 1% So the Probability of the lift being over weight is 1%.

$\sigma_s = \frac{\sigma_P}{\sqrt{n}} = \frac{2.22}{\sqrt{8}} = 0.78$

Zscore for $x = 20$ $= \frac{X - \mu_s}{\sigma_s} = \frac{20 - 21.50}{0.78} = -1.92$

$P(\mu_s < 20) = 0.0274$

Zscore for $x = 23$ $= \frac{23 - 21.50}{0.78} = 1.92$

$P(\mu_s < 23) = 0.9726$

So, $P(20 < \mu_s < 23) = 0.9726 - 0.0274$

$= 0.9452$

$= 94.52\%$

Q14. Suppose the grades in a finite mathematics class are normally distributed with mean of 75 and a standard deviation of 5.

(a) Probability that a randomly selected had a grade of at least 83?

(b) Find the probability that the average grade for 5 randomly selected students was at least 83.

(a) $\mu_P = \mu_S = 75$

$\sigma_P = 5$

$P(\mu > 83) = ?$

for $x = 1$

$\sigma_S = \frac{\sigma_P}{\sqrt{n}} = 5/\sqrt{1} = 5$

Zscore for X as 83 $= \frac{X - \mu_s}{\sigma_s} = \frac{83 - 75}{5}$

$= 1.6$

$P(\mu < 83) = 0.8554$

$P(\mu > 83) = 1 - 0.8554 = 0.1446$

$= 14.46\%$

(b) $n = 5$

$\sigma_S = \frac{\sigma_P}{\sqrt{n}} = \frac{5}{\sqrt{5}} = 2.23$

Zscore for $X = 83$ $= \frac{83 - 75}{2.23} = 3.58$

$P(\mu < 83) = 0.9998$

$P(\mu > 83) = 0.01$

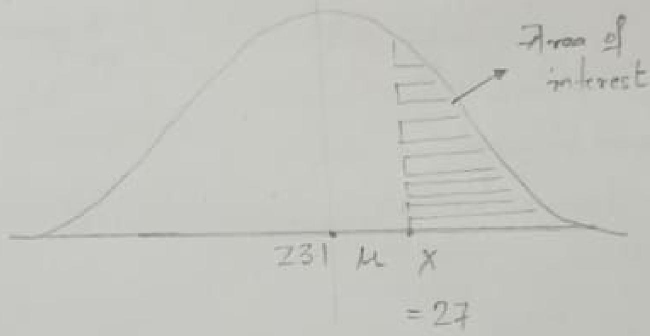
Q11. Suppose the age of a student graduate from Salem State is normally distributed. If the mean age is 23.1 years and the standard deviation is 3.1 years, what is the probability that 6 randomly selected student had a mean age at graduation that was greater than 27 ?

$\mu = 23.1$ years

$\sigma = 3.1$ years

$n = 6$

$X = 27$



Z score of $X = \dfrac{X - \mu_s}{\sigma_s}$ → $\mu_s = \mu = 23.1$

→ Sampling distribution Std $= \sigma/\sqrt{n} = 1.266$

$$= \frac{27 - 23.1}{1.266} = 3.08$$

Area under the cure for $(Z = 3.08)$ (left side) $= 0.9988$

Hence, the probability that selected student had mean age of graduation that was greatr 27 years

$= 1 - 0.9988$

$= 0.0012$