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$T(n) = 4T(n/2) + \lg n \qquad T(n) = O(n^2)$

$T(n) = 1/2\, T(n/2) + 1/n \qquad T(n) =$

$T(n) = aT(n/b) + O(f(n))$

$T(n) = 64T(n/8) - n^2 \lg n \qquad T(n) =$

* Comparision between the growth of functions :

eg: $f_1 = n^2 \qquad f_2 = n^3$

taking lg both sides

$2\log n \qquad 3\log n$

$2 < 3$

$\Rightarrow f_1 < f_2$

OR

$n^2 \qquad n^3$

$1 \qquad n$

$1 \le n \Rightarrow f_1 < f_2$

eg: $n^{\lg n} \qquad n \lg n$

$\log(n \lg n) \to \log n + \log\log n$

taking log on both sides

$\lg n \cdot \lg n \qquad \log n + \lg\lg n$

taking $n = 2^{100}$

$2^{100} \times 2^{100} > 2^{100} + 100$

$2^{100} \log_2 2 \cdot 2^{100} \log_2 2$

$2^{100} \log 2 +$

$\lg(2^{100} \log_2)$

$100 \log_2 2 + \lg \lg_2 2$

eg $2^{\lg n} \qquad n^2$

$n \lg_2 2 \qquad 2\log_2 n$

$n > 2\lg_2 n$ for any large value of n

eg $\sqrt{n} \qquad \lg\lg n$

$\frac{1}{2} \log n^{1/2} \qquad \lg\lg\lg n$

$\to$ taking $n = 2^{2^{2^{100}}} \Rightarrow \frac{1}{2} 2^{2^{100}}$ & 100

$f_1 > f_2$

eg: $f_1 = 2^n$ $f_2 = n^{3/2}$ $f_3 = n\lg n$ $f_4 = n^{\log n}$

$n\log_2 2$ $\frac{3}{2}\log_2 n$ $\log(n) + \log\log n$ $\log n . \log n$

taking $n = 2^{2^{100}}$

as f_1 is exponential fn and the largest

$f_1 > f_4 > f_2 > f_3$ $2^{2^{100}}$ $\frac{3}{2} \times 2^{100}$ $2^{100} \times 100$ $2^{10} \times 2^{100}$

$\boxed{2^n \times n^n \text{ only}}$

	0-99	100-999	1000-
eg $f(n)$	n^3	n^3	n^2
$g(n)$	n	n^3	n^3

$f(n) = O(g(n))$ OR $g(n) = O(f(n))$

$\Rightarrow$ for 0-99 : $g(n) = O f(n)$

for 100-999 $g(n) = O f(n)$ & $f(n) = O(g(n))$

for 1000- $f(n) = O(g(n))$

$\rightarrow$ $f(n) \le c\, g(n)$ where $c = 1$ & $n_0 = 100$

* **MASTER'S Theorem:** The master's method provides a "cookbook" method for solving a recurrance of the form

$T(n) = \cancel{aT(n/b)}$ $aT(n/b) + f(n)$

where $a \geq 1$ and $b > 1$ are constants and $f(n)$ is asymptotically positive fn. The master's theorem requ memorization of three cases.

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$$T(n) = aT(n/b) + \theta(n^k \lg^p n)$$

1) $a > b^k$; then $T(n) = \theta(n^{\lg_b a})$

ii) $a = b^k$; then

a) If $P > -1$ then $T(n) = \theta(n^{\lg_b a} \lg^{P+1} n)$

b) If $P = -1$ then $T(n) = \theta(n^{\lg_b a} \lg\lg n)$

c) If $P < -1$ then $T(n) = \theta(n^{\lg_b a})$

iii) If $a < b^k$

a) If $P \geq 0$ then $T(n) = \theta(n^k \lg^P n)$

b) If $P < 0$ then $T(n) = \theta(n^k)$

eg: $T(n) = 2T(n/2) + n$ $b = 2$ $a = 2$ $k = 1$ $P = 0$

(ii) $a = b^k$ (a) $0 = P > -1$

$\Rightarrow T(n) = \theta(n^{\lg_2 2} \lg^1 n)$

$T(n) = \underline{\underline{\theta(n\lg n)}}$

eg $T(n) = 3T(n/2) + n^2$

$a = 3$, $b = 2$, $k = 2$ $P = 0$

Here

$a < b^k$ i.e $3 < 2^2$ $\Rightarrow$ and $P \geq 0$

$\Rightarrow T(n) = \theta(n^2 \lg^0 n)$

$= \theta(n^2)$

eg $T(n) = T(n/2) + n^2$

$a = 1$, $b = 2$, $k = 2$, $P = 0$

$a < b^k$ $P \geq 0$

$\Rightarrow$ from case (iii) $\theta(n^2)$

eg $T(n) = 2T(n/2) + n\lg n$

$a=2,\ b=2,\ k=1,\ p=1$

$a = b^k \quad p > -1$

$T(n) \Rightarrow \Theta\left(n^{\log_2 2} \log^2 n\right)$

$T(n) \Rightarrow \Theta\left(n \log^2 n\right)$ $\quad \lg^2 n = \lg\lg n$

eg $T(n) = 2T(n/4) + \sqrt{n}$

$a=2,\ b=4,\ k=1/2,\ p=0$

$a = b^k \quad \Rightarrow \Theta\left(n^{\log_4 2} \log n\right)$

$T(n) = \Theta(\sqrt{n} \log n)$

eg $T(n) = 1/2\, T(n/2) + \frac{1}{n}$

Can't apply master's theorem

eg $T(n) = 6T(n/3) + n^2 \lg n$

$a=6,\ b=3,\ k=2,\ p=1$ $\quad \lg_4 2 = 1/2$

$a < b^k \quad p \geq 0$

$\Rightarrow T(n) = \Theta(n^2 \lg n)$