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Question 3 →

(a) We need to arrange in increasing order of their rate of growth,

(i) $n^{7/4}$ (ii) $n \log n$ (iii) $n^{\log n}$ (iv) $\sqrt{n}$

Taking log in all equation, we get

$= \log n^{7/4}$, $\log(n \log n)$, $\log(n^{\log n})$, $\log(\sqrt{n})$

$= \frac{7}{4} \log n$, $\log n + \log \log n$, $\log n \log n$, $\frac{1}{2} \log n$

computing ① → $\frac{7}{4} \log n = 1.75 \log n$

whereas ② → $\frac{1}{2} \log n = 0.5 \log n$

$\therefore \frac{7}{4} \log n > \frac{1}{2} \log n$

(i) > (iv) —— ①

comparing $n \log n$, $n^{\log n}$

$\log n + \log \log n$ $\quad$ $\log n . \log n$

we can see, clearly,

$\log n + \log \log n > \log n \log n$

$\therefore$ (iii) > (ii) —— II

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comparing (i) $n^{7/4}$ and (ii) $n\log n$

1.75 log n and log n + log log n

if we take $n = 10^{1000}$ (very large)

$= 1.75 \log 10^{1000}$, $\log 10^{1000} + \log \log 10^{1000}$

$= 1.75 \times 10^3 \times 1$, $10^3 + \log 10^3$

$= 1.75 \times 10^3 > 10^3 + 3\log_{10} 10$

$\therefore$ (i) > (ii) —— III

comparing (iii) & (iv)

$n^{\log n}$ $\sqrt{n}$

log log n & 0.5 log n

take any value of $n = 10^{1000}$ (very large)

$\log \log 10^{1000}$, $0.5 \log 10^{1000}$

$= \log 10^3$, 0.5×10^3

$= 3 < 500$

$\therefore$ (iii) < (iv) —— IV

comparing (ii) → n log n & (iv) $\sqrt{n}^{\log n}$

take any ~~log~~ large value $n = 10^{1000}$

$\log n + \log(\log n)$, $\frac{1}{2}\log n$ ⇒ $10^3 + 3 > 0.5 \times 10^3$

$\therefore$ (ii) > (iv) —— V

now comparing (iii) & (i)

$n^{\log n}$, $n^{7/4}$

$n^{7/4} = 1.75 \log n$, $n^{\log n} = \log n \log n$

taking $n = 10^{1000}$

$1.75 \log 10^{1000}$, $(\log 10^{1000})^2$

$= 1.75 \times 1000$, $<$ $(10^3)^2$

$\therefore$ (i) < (iii) — VI

from all equation, we get;

(iv) < (ii) < (i) < (iii)

~~$\sqrt{n} < n^{\log n} < n^{7/4} <$~~

$$\boxed{\sqrt{n} < n\log n < n^{7/4} < n^{\log n}}$$

(c) Head office is in Delhi. Salesmen needs to visit all the branches offices of the company, starting from and returning to its head office

Case 1: No city should be repeated

here, vertices are the cities

$\therefore$ n (no. of vertices) = 6

& starting from Delhi, he returns to Delhi

$\therefore$ closed walk.

and as no cities (vertices) are repeated, hence closed path.

But Banglore is an articulation point i.e Banglore. Hence, it is not possible to find any closed path (Cycle)

Case 2 : If it is allowed to visit city more than once,

if he can repeat vertices. Then its possible to find a walk and return back to Delhi.

It can be like →

Delhi → Banglore → Kolkata → Chandigarh → Ahmdabad → Banglore → Mumbai → Delhi.

(b) To prove, By Mathematical Induction

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n+1)}{30}$$

firstly, take $k=1$, then

$$1^4 = \frac{1(1+1)(2.1+1)(3.1^2+3.1+1)}{30}$$

$$\Rightarrow \quad 1 = \frac{30}{30} = 1 \quad \therefore \text{ true}$$

$\therefore$ it is true for $k=1$

let us assume that it is true for $n=k-1$
we need to prove it for $n=k$.

i.e. $1^4 + 2^4 + 3^4 + \ldots (k-1)^4 + k^4$

$$= \frac{(k-1)(k)(2(k-1)+1)(3(k-1)^2 + 3(k-1) - 1) + k^4}{30}$$

$$= \frac{(k^2-k)(6k^3 - 6k^2 - 2k - 3k^2 + 3k + 1) + 30k^4}{30}$$

$$= \frac{(6k^5 - 6k^4 - 6k^4 - 6k^3 - 2k^3 + 2k^2 - 3k^4 + 3k^3 + 3k^3 - 3k^2 + k^2 - k + 30k^4)}{30}$$

$$= \frac{6k^5 + 15k^4 + 10k^3 - k}{30} \quad \text{— (1)}$$

now, RHS, $\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

$$= \frac{(n^2+n)(2n+1)(3n^2+3n-1)}{30}$$

$$= \frac{6n^5 + 6n^4 - 2n^3 + 9n^4 + 9n^3 - 3n^2 + 3n^3 + 3n^2 - n}{30}$$

$$= \frac{6n^5 + 15n^4 + 10n^3 - n}{30} \quad \text{— (2)}$$

in ① if we put $k=n$

we get $\frac{6n^5+15n^4+10n^3-n}{30}$

$\therefore$ ① = ②

hence we proved for $n=k$, true

$\therefore$ By Mathematical induction,

$$1^4+2^4+3^4+\dots+n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$