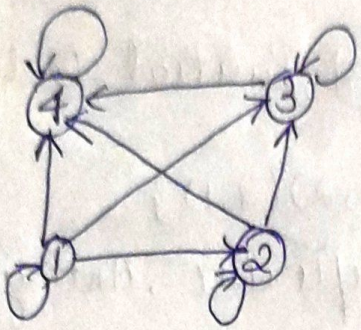
# Hasse diagram / Posets

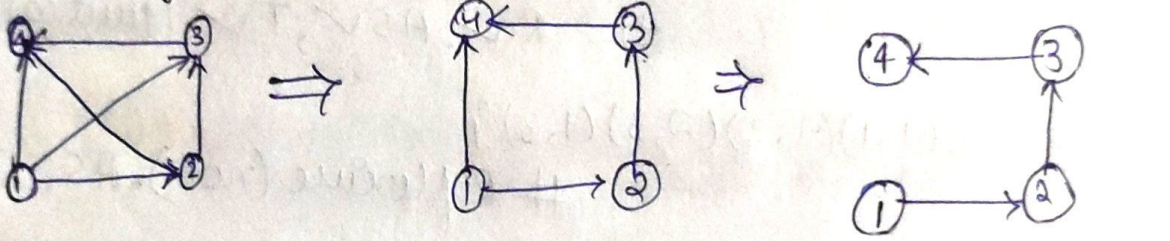
$A = \{1, 2, 3, 4\}$

$R = \{(1,1)(1,2)(1,3)(1,4)(2,2)(2,3)(2,4)(3,3)(3,4)(4,4)\}$



Remove reflexive, antisymmetric, transitive

① Remove reflexive ② Remove AS ③ Remove Transitive



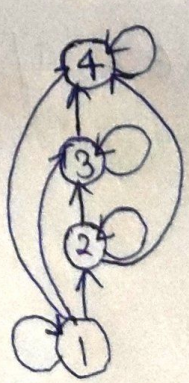
(4) → upper bound

(3) ← least upper bound

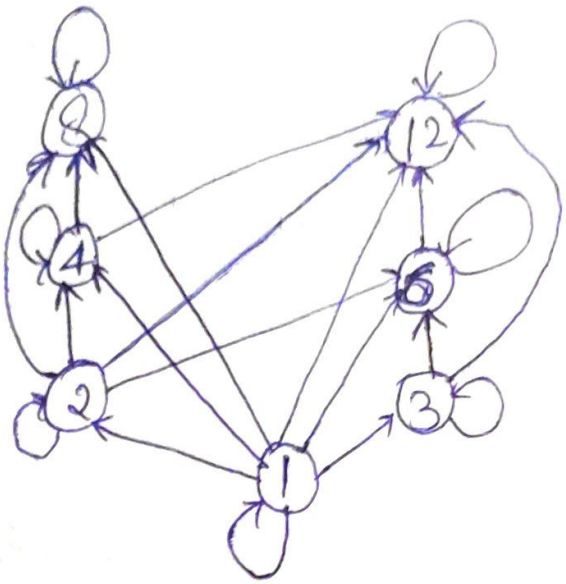
(2) → greatest lower bound

(1) → lower bound

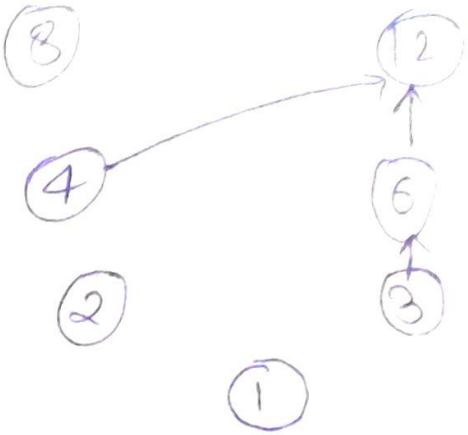
Q Constructing the hasse diagram for $(\{1, 2, 3, 4\}, \leq)$

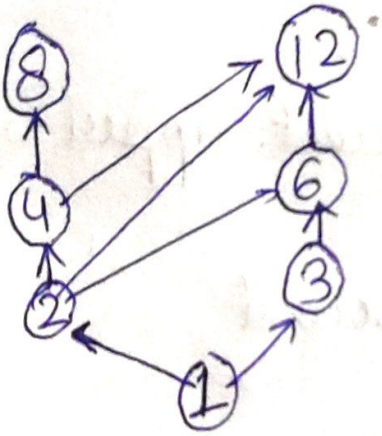


Q Draw a hasse diagram for $\{(a,b) \mid a \text{ divides } b\}$ on $\{1,2,3,4,6,8,12\}$




⇒



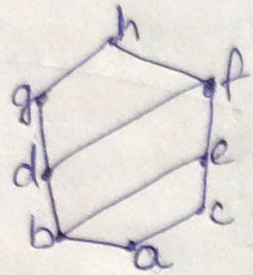



Q Find out lower & upper bounds of subsets $\{a,b,c\}$ $\{j,h\}$ & $\{a,c,d,f\}$ in poset with the hasse diagram in given here.

$(a,b,c)$ $(j,h)$ $\{a,c,d,f\}$

↓

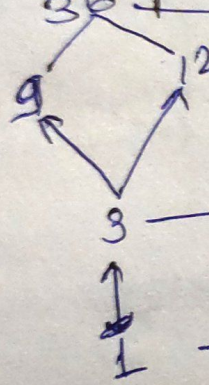
No upper & lower bound




Eg find the greatest lower bound & least upper bound of the sets $\{3,9,12\}$ & $\{1,2,4,5,10\}$ in a poset $(z^+, |)$

$\{3,9,12\}$ ⟶ L.C.M (3,9,12) least upper bound

⇒




3 ⟶ greatest lower bound

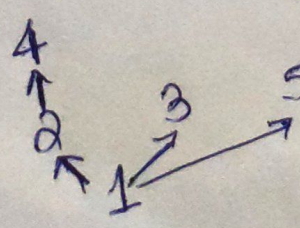
1 ⟶ lower bound

⇒ $\{1,2,4,5,10\}$

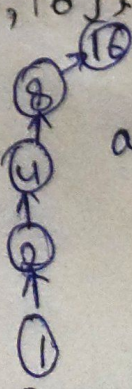
Greatest lower bound = 1

Least upper bound = 20 (L.C.M of 1,2,4,5,10)

Q $(\{1,2,3,4,5\},|)$ & $(\{1,2,4,8,16\},|)$ lattices?



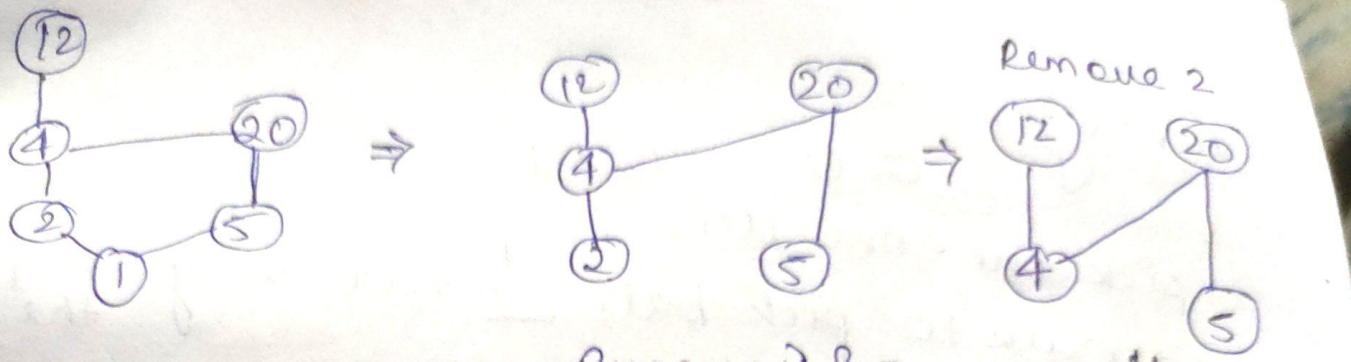

→ it is not lattices we cannot find least upper bound.




after removing all R, AS, T properties this is figure.

it is lattice.

Eg ({1, 2, 4, 5, 12, 20}, 1)

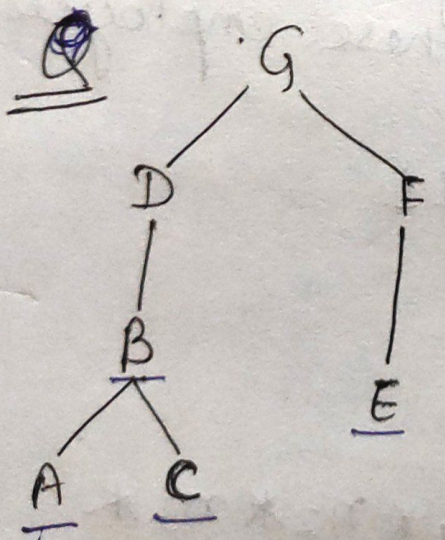



After each step → (topological) sorting

⇒ 1, 2, 5, 4, 12, 20

⇒ 1, 5, 2, 4, 20, 12

Q




→ A C B E D F G (topological sorting)

# Permutations & Combination

**Counting :-**

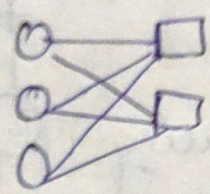
○ ○ ○ □ □

pick one - one item

- 3 ways to pick ball or (+) two ways to pick the box

$3 + 2 = 5$ ways

**Ex** pick one ball and (×) one box



$3 \times 2 = 6$ ways

**Ex** A new company with just two employees, Rahul & ~~Rohit~~ Rohit rents a floor of a building with 12 offices. How many ways are there to assign different offices to these employees.

**Ans**

Rahul → 12
Rohit → 11

$\Rightarrow 12 \times 11$

**Ex**

```
for i1 = 1 to n1
    for i2 = 1 to n2
        !
        for im = 1 to nm
            K = K+1
```

$\Rightarrow n_1 * n_2 * n_3 * \dots * n_m$

Q
```
K = 0
for i1 = 1 to n1 :
    K = K + 1
for i2 = 1 to n2 :
    K = K + 1
!
for in = 1 to nn
    K = K + 1
```

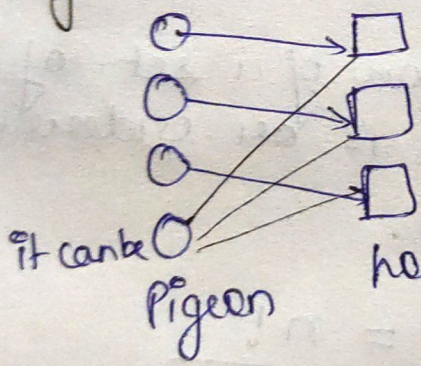
time:
$n_1 + n_2 + n_3 + \dots n_m$

Q How many bit strings of length eight either start with a 1 bit or end with two bits 00.

$1 - - - - - - - = 2^7$
$- - - - - - 0\,0 = 2^6$
$1 - - - - - 0\,0 = 2^5$

$2^7 + 2^6 - 2^5$
$= 128 + 64 - 32$
$= 160$

# Pigeonhole Principle




if k is a positive integer & k+1 or more objects are placed into k boxes, then there is atleast one box containing two or more of objects.

m items
n containers

if $m > n$ there is atleast one container with $\lceil m/n \rceil$ items

Eg Pigeon = 4 ← m
Pigeonholes = 3 ← n

$\lceil m/n \rceil = \lceil 4/3 \rceil = \lceil 1.33 \rceil = 2$

Eg → 10 items
4 containers

$\lceil 10/4 \rceil = \lceil 2.5 \rceil = 3$ Ans

In a group of n peoples, there are two people who have an identical number of friends within the group.

items = n

Each person may have {1, 2, 3 ... n-1}

$$\left\lceil \frac{n}{n-1} \right\rceil = 2$$

Q Among 100 friends there are atleast $\lceil 100/12 \rceil$ = 9 may born in the same month.

Q What is the minimum number of students required in a discrete mathematics class to be sure that atleast six will receive the same grade, if there are five possible grades A, B, C, D, E.

$$\lceil m/5 \rceil = 6$$

$$m = 26$$

Permutations : A permutation of a set of distinct objects is an ordered arrangement of these objects.

$$P(n, r) = {}^nP_r = \frac{n!}{(n-r)!}$$

$n \rightarrow$ first event

$n \rightarrow$ second event

$(m+n)$ $(m*n)$

returns, no. of arrangements of r objects among n objects.

$\Rightarrow n \propto (n-1) * (n-2) * \dots (n-r+1)$ → r objects from n objects

$= n \times (n-1) * (n-2) \text{ ---} * (n-r+1) * \frac{(n-r)(n-r-1) * \text{---} 2 \times 1}{(n-r) * (n-r-1) * \text{---} 2 \times 1}$

$\Rightarrow \frac{n!}{(n-r)!}$

Q How many ways are there to select a first prize winner a second prize winner & a third prize winner from 100 different peoples who have enter contest.

$$^{100}P_3 = \frac{100!}{(100-3)!} = \frac{100 \times 99 \times 98 \times 97!}{97!} = 100 \times 99 \times 98$$

Ans

Q How many permutations of letters ABCDEFGH containing the string ABC?

Ans (ABC)DEFGH = 6!

Q How many poker hands of five card can be dealt from a standard deck of 52 cards? How many ways are there to select 47 cards from the same.

$n = 52$
$r = 5$

$= \frac{52!}{5!\,47!}$ Ans