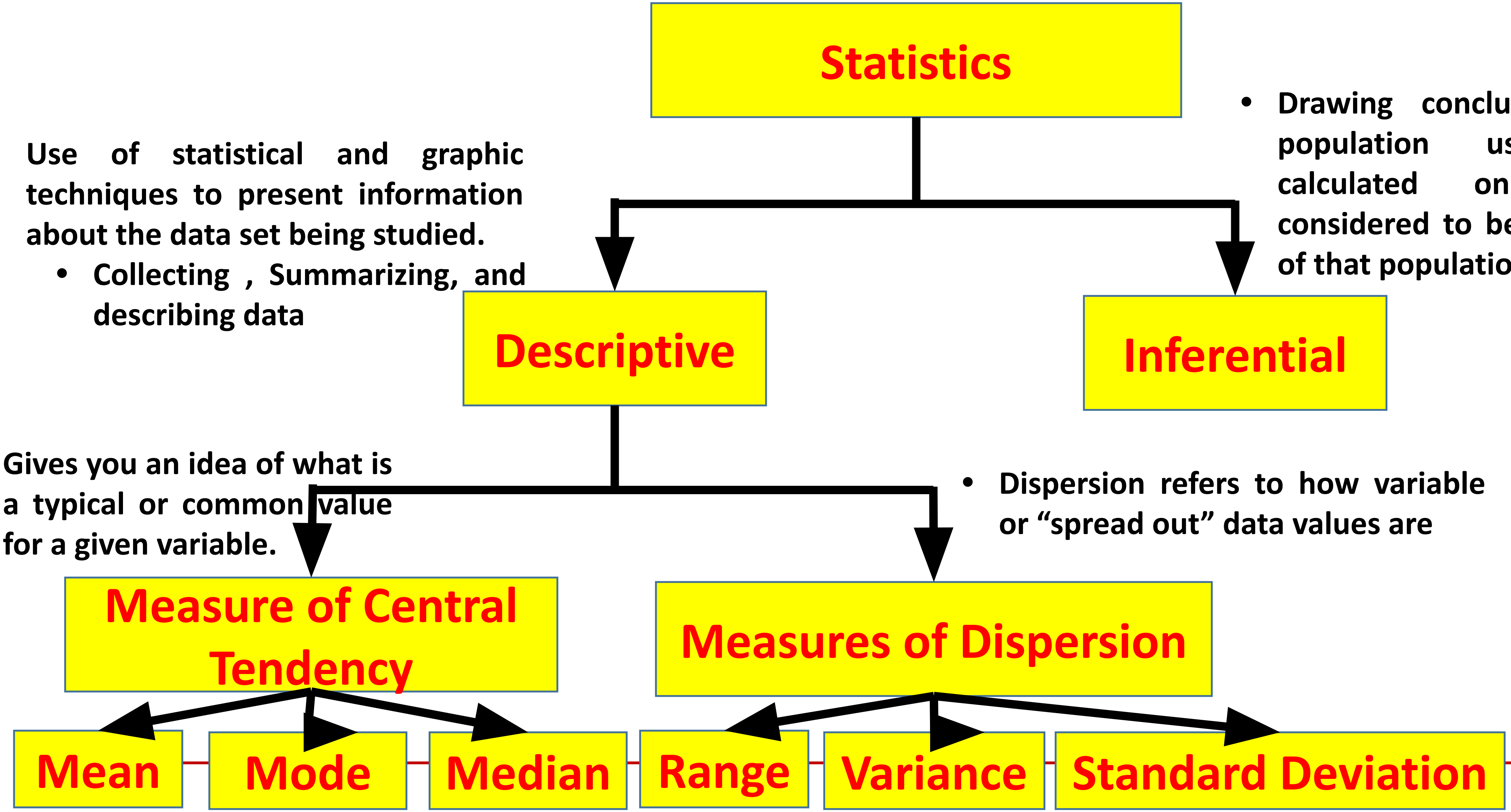
Descriptive Statistics and Data Visualization

Most of the slides are based on the course on Probability and Statistics by Prof. Kevin M. Riordan

Statistics

- **Many studies generate large numbers of data points**
- **How to make sense of all that data?**
  - **Statistics is used to *summarize* the data, to provide a better understanding of overall tendencies within the distributions of scores.**
    - **helps in summarizing the results**
    - **helps us recognize underlying trends and tendencies in the data**
    - **helps in communicating the results to others**

Types of Statistics



Statistics

- Use of statistical and graphic techniques to present information about the data set being studied.
  - Collecting , Summarizing, and describing data

Descriptive

- Drawing conclusions about a population using statistics calculated on a sample considered to be representative of that population.

Inferential

- Gives you an idea of what is a typical or common value for a given variable.

Measure of Central Tendency

- Dispersion refers to how variable or “spread out” data values are

Measures of Dispersion

Mean

Mode

Median

Range

Variance

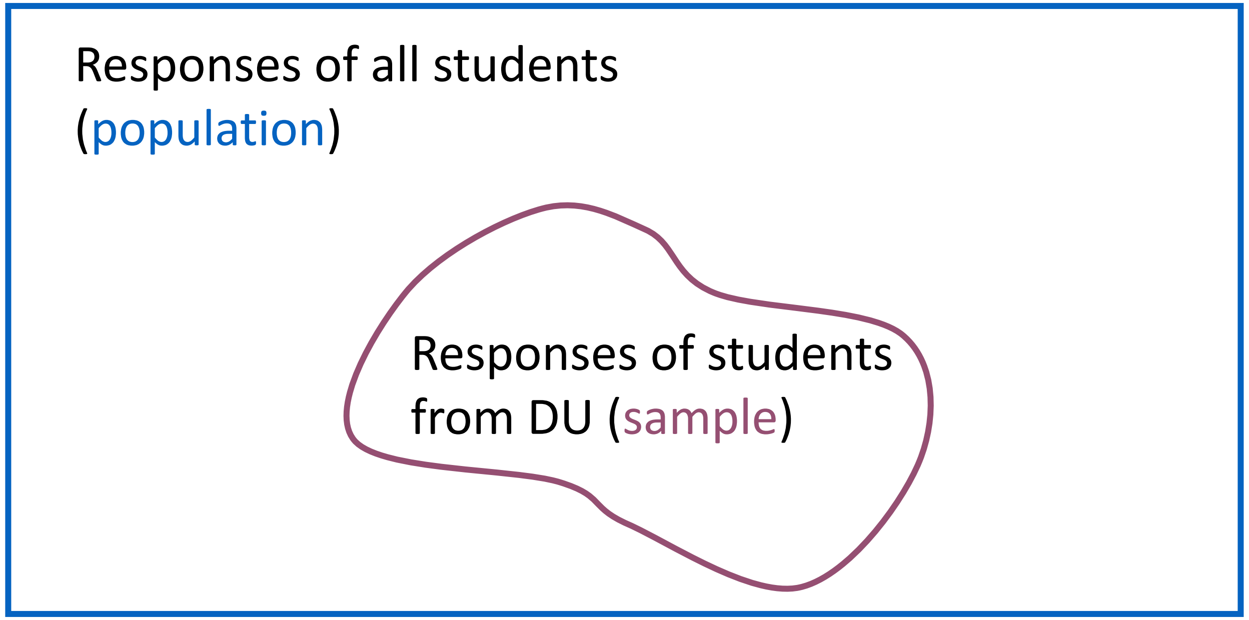
Standard Deviation

Descriptive statistics

- If we wanted to characterize the students in this class, we would find that they are:
  - Young
  - Fit
  - Male
- How young?
- How fit is this class?
- What is the distribution of males and females?
- Goal:
  - To visualize data, understand the patterns, and make quick statements about the system’s behavior
  - To understand relations among variables

Populations & Samples

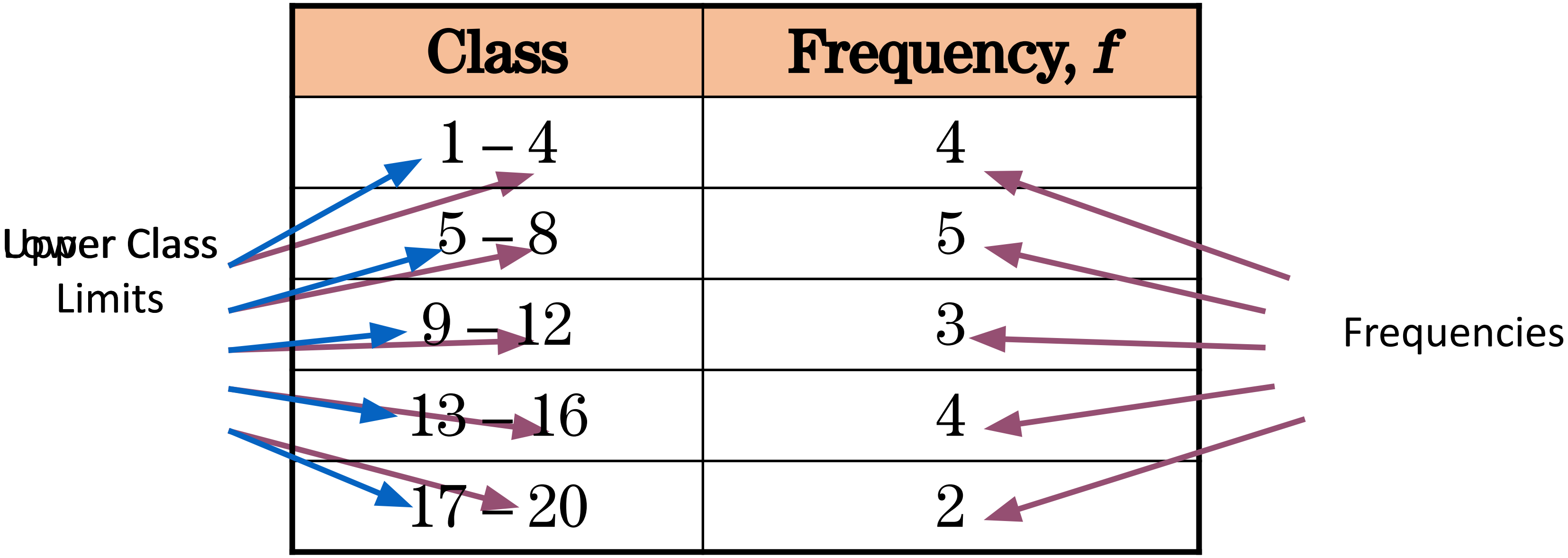
- **In a survey, 250 college students were asked if they study regularly. 35 of the students said yes.**



Responses of all students (population)
Responses of students from DU (sample)

Frequency Distributions

- A **frequency distribution** is a table that shows classes or intervals of data with a count of the number in each class.
- The frequency **f** of a class is the number of data points in the class.

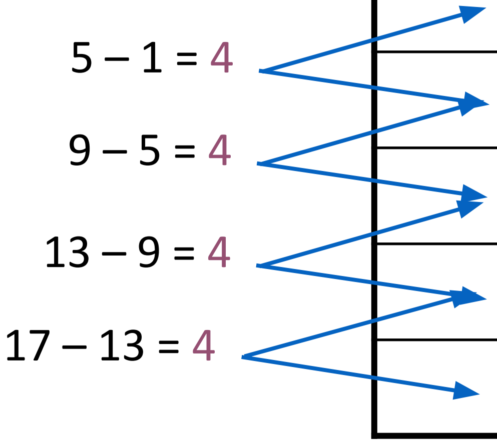


Class	Frequency, *f*
1 – 4	4
5 – 8	5
9 – 12	3
13 – 16	4
17 – 20	2

Frequency Distributions

- The **class width** is the distance between lower (or upper) limits of consecutive classes.

Class	Frequency, f
1 – 4	4
5 – 8	5
9 – 12	3
13 – 16	4
17 – 20	2



5 – 1 = 4

9 – 5 = 4

13 – 9 = 4

17 – 13 = 4

The class width is 4.

- The **range** is the difference between the maximum and minimum data entries.

Constructing a Frequency Distribution

- **Example:**
  - **The following data represents the ages of 30 students in a class. Construct a frequency distribution that has five classes.**

Ages of Students

18	20	21	27	29	20
19	30	32	19	34	19
24	29	18	37	38	22
30	39	32	44	33	46
54	49	18	51	21	21

Constructing a Frequency Distribution

- Number of classes: 5
- The minimum data entry is 18 and maximum entry is 54, so the range is 36.
- Divide the range by the number of classes to find the class width.
  - Class width = $\frac{36}{5} = 7.2$ (Round up to 8)

18	20	21	27	29	20
19	30	32	19	34	19
24	29	18	37	38	22
30	39	32	44	33	46
54	49	18	51	21	21

Constructing a Frequency Distribution

- Lower limit and upper limits of classes will be
  - The lower class limits are 18, 26, 34, 42, and 50.
  - The upper class limits are 25, 33, 41, 49, and 57.

18	20	21	27	29	20
19	30	32	19	34	19
24	29	18	37	38	22
30	39	32	44	33	46
54	49	18	51	21	21

Class	Frequency, *f*
18 – 25	13
26 – 33	8
34 – 41	4
42 – 49	3
50 – 57	2

Relative Frequency

- **The relative frequency of a class is the portion or percentage of the data that falls in that class.**

Class	Frequency, *f*	Relative Frequency
18 – 25	13	0.433
26 – 33	8	0.267
34 – 41	4	0.133
42 – 49	3	0.100
50 – 57	2	0.067

Cumulative Frequency

- **The cumulative frequency of a class is the sum of the frequency for that class and all the previous classes.**

Ages of Students

Class	Frequency, f	Cumulative Frequency
18 – 25	13	13
26 – 33	+ 8	21
34 – 41	+ 4	25
42 – 49	+ 3	28
50 – 57	+ 2	30
	$\sum f = 30$	

Total number of students

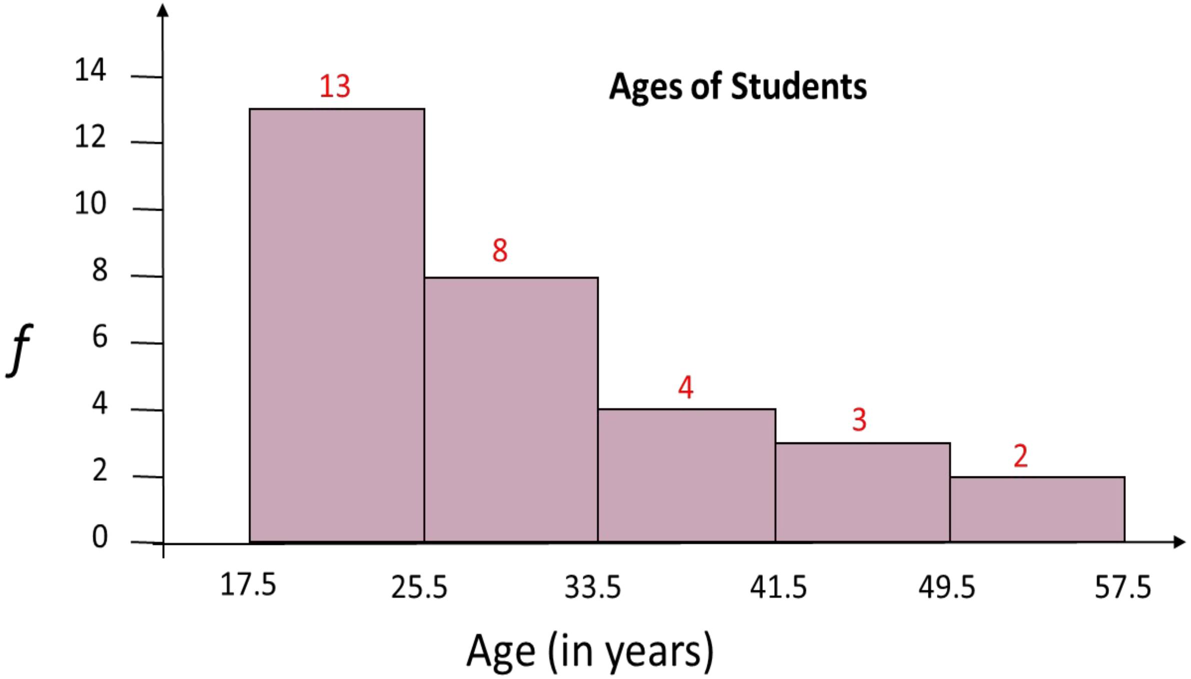
Frequency Histogram

- A frequency histogram is a bar graph that represents the frequency distribution of a data set.
  1. The horizontal scale is quantitative and measures the data values.
  2. The vertical scale measures the frequencies of the classes.
  3. Consecutive bars must touch.
- Class boundaries are the numbers that separate the classes without forming gaps between them.
- The horizontal scale of a histogram can be marked with either the class boundaries or the midpoints.

Frequency Histogram

Ages of Students

Class	Frequency, *f*	Class Boundaries
18 – 25	13	17.5 – 25.5
26 – 33	8	25.5 – 33.5
34 – 41	4	33.5 – 41.5
42 – 49	3	41.5 – 49.5
50 – 57	2	49.5 – 57.5
	$\sum f = 30$	

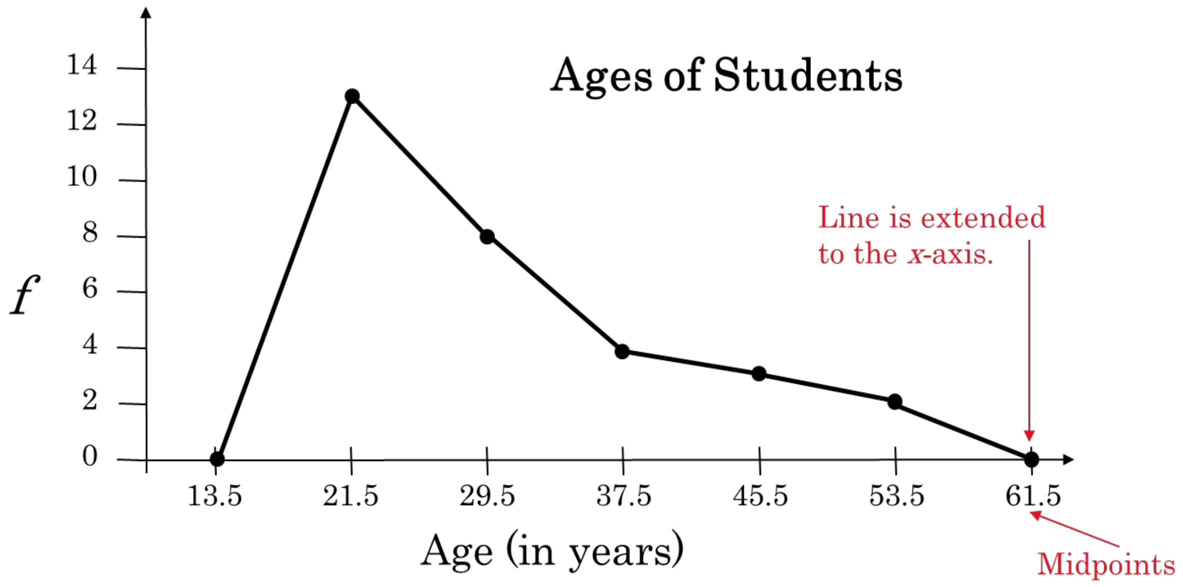


Frequency Polygon

- A **frequency polygon** is a line graph that emphasizes the continuous change in frequencies.

Ages of Students

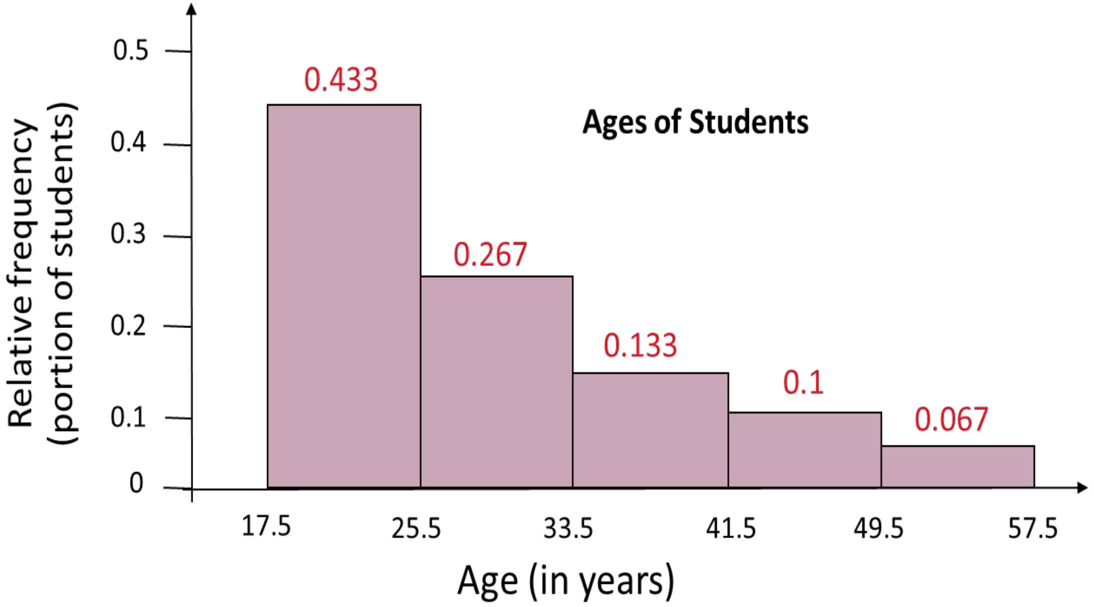
Class	Frequency, f	Mid-Point
18 – 25	13	21.5
26 – 33	8	29.5
34 – 41	4	37.5
42 – 49	3	45.5
50 – 57	2	53.5
	$\sum f = 30$	



Relative Frequency Histogram

- A **relative frequency** histogram has the same shape and the same horizontal scale as the corresponding frequency histogram.

Class	Frequency, *f*	Relative Frequency
18 – 25	13	0.433
26 – 33	8	0.267
34 – 41	4	0.133
42 – 49	3	0.100
50 – 57	2	0.067

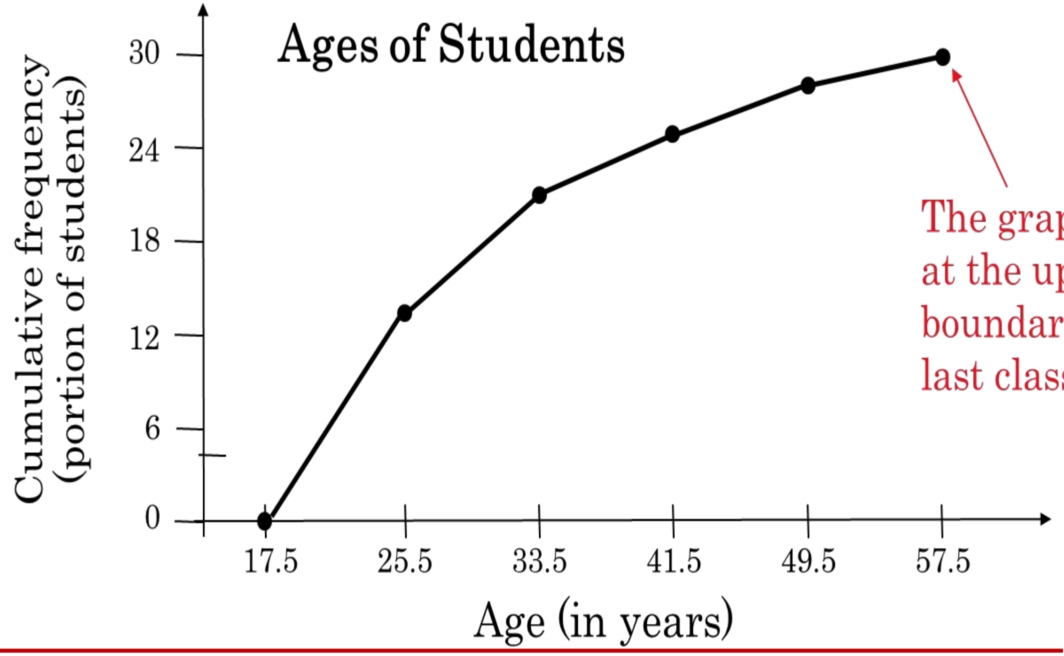


Cumulative Frequency Graph

- A **cumulative frequency graph or ogive**, is a line graph that displays the cumulative frequency of each class at its upper class boundary.

Ages of Students

Class	Frequency, f	Cumulative Frequency
18 – 25	13	13
26 – 33	+ 8	21
34 – 41	+ 4	25
42 – 49	+ 3	28
50 – 57	+ 2	30
	$\sum f = 30$	



The graph ends at the upper boundary of the last class.

Stem-and-Leaf Plot

- In a stem-and-leaf plot, each number is separated into a stem (usually the entry’s leftmost digits) and a leaf (usually the rightmost digit).
- Example:
  - The following data represents the ages of 30 students in a statistics class. Display the data in a stem-and-leaf plot.

Ages of Students

18	20	21	27	29	20
19	30	32	19	34	19
24	29	18	37	38	22
30	39	32	44	33	46
54	49	18	51	21	21

Stem-and-Leaf Plot

Ages of Students

Key: 1|8 = 18

1	8 8 8 9 9 9
2	0 0 1 1 1 2 4 7 9 9
3	0 0 2 2 3 4 7 8 9
4	4 6 9
5	1 4

Most of the values lie between 20 and 39.

This graph allows us to see the shape of the data as well as the actual values.

Stem-and-Leaf Plot

- **Example:**
  - a stem-and-leaf plot that has two lines for each stem.

Key: 1|8 = 18

1	
1	8 8 8 9 9 9
2	0 0 1 1 1 2 4
2	7 9 9
3	0 0 2 2 3 4
3	7 8 9
4	4
4	6 9
5	1 4
5	

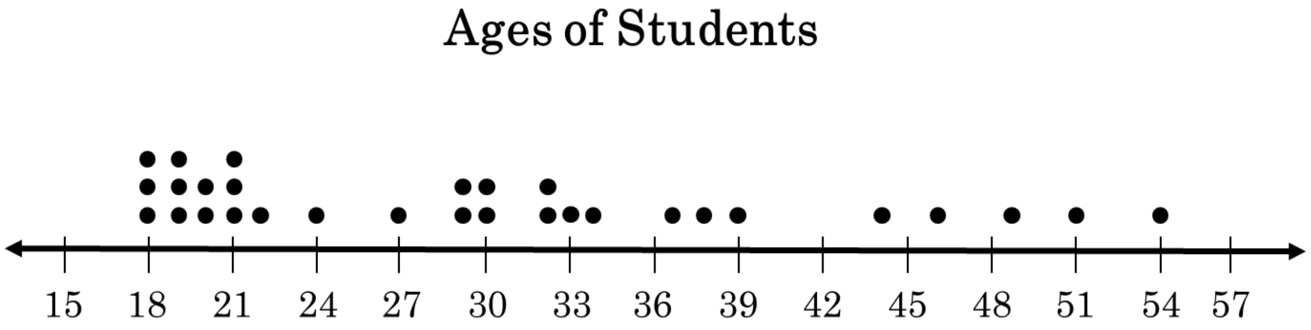
From this graph, we can conclude that more than 50% of the data lie between 20 and 34.

Dot Plot

- In a dot plot, each data entry is plotted, using a point, above a horizontal axis.
- Example:
  - dot plot to display the ages of the 30 students in the class.

Ages of Students

18	20	21	27	29	20
19	30	32	19	34	19
24	29	18	37	38	22
30	39	32	44	33	46
54	49	18	51	21	21

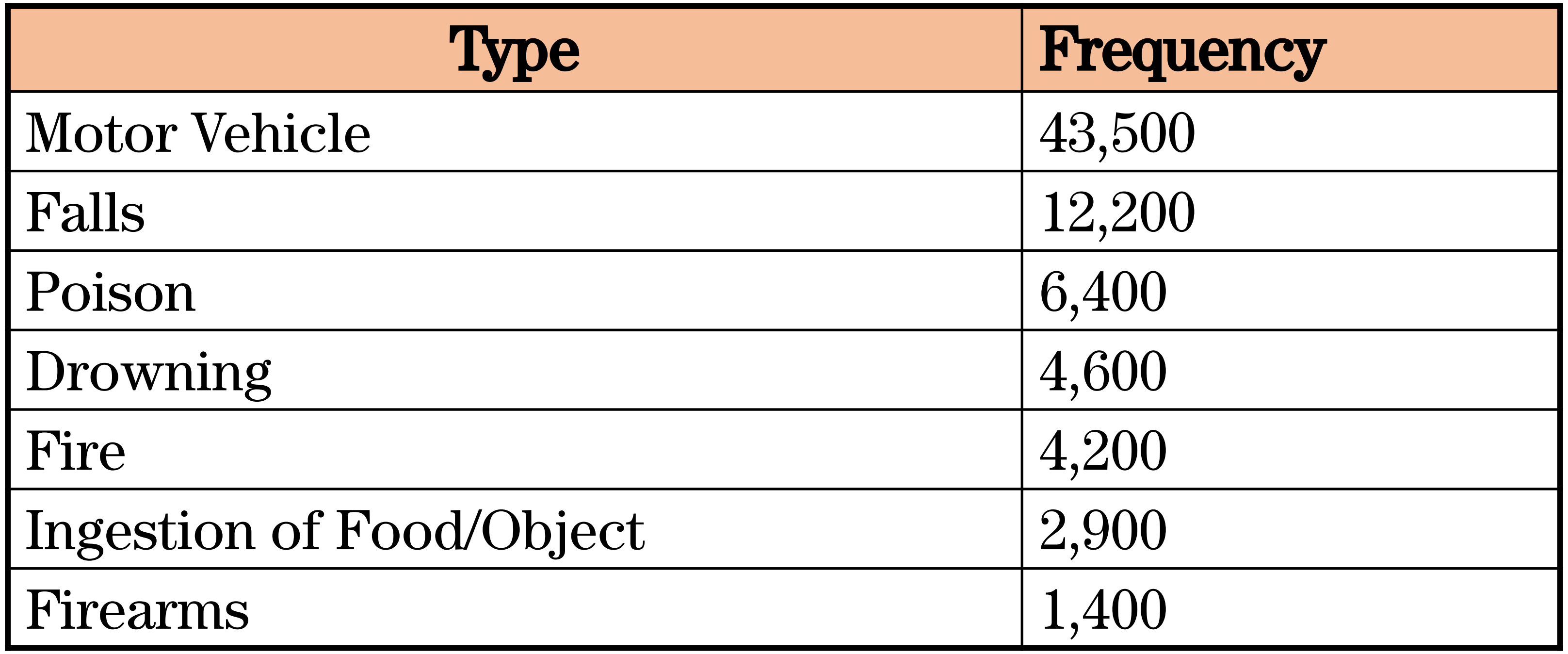


From this graph, we can conclude that most of the values lie between 18 and 32.

Pie Chart

- A pie chart is a circle that is divided into sectors that represent categories. The area of each sector is proportional to the frequency of each category.

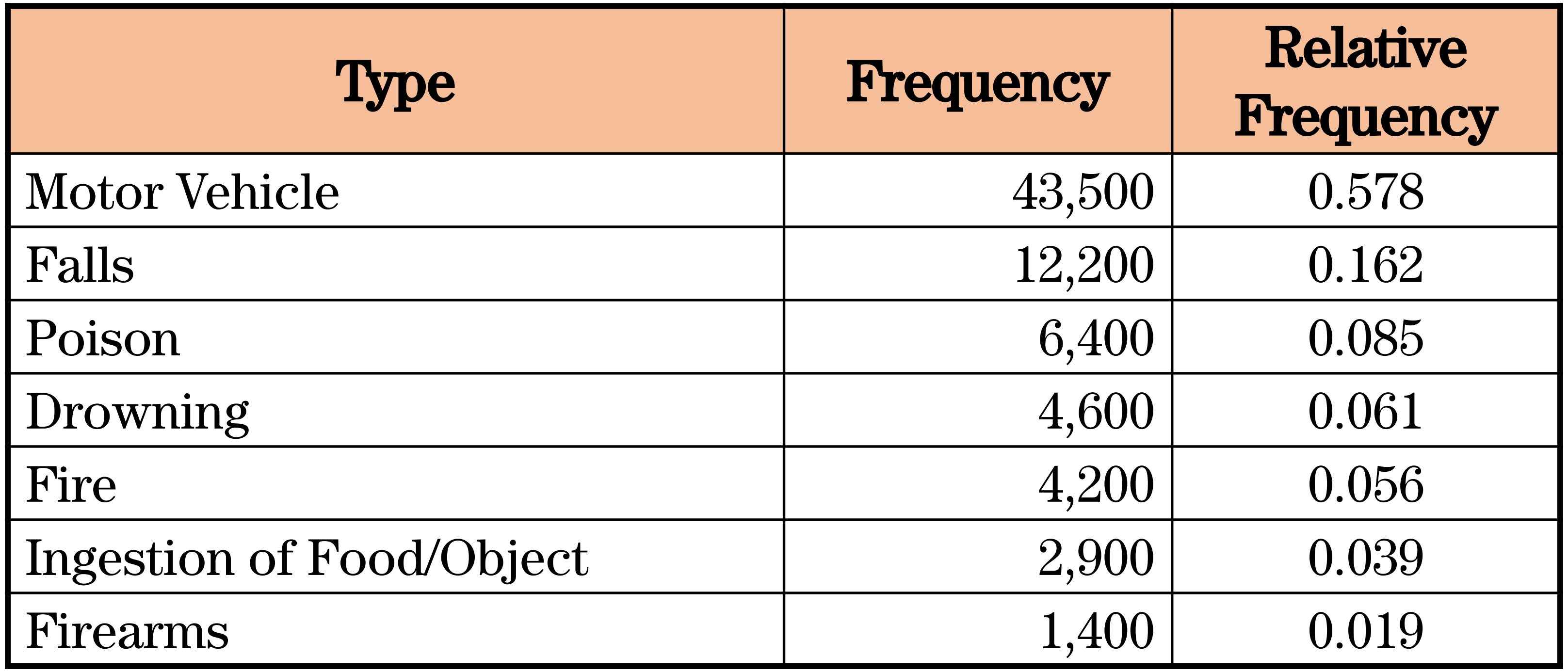
Accidental Deaths in the USA in 2002



Type	Frequency
Motor Vehicle	43,500
Falls	12,200
Poison	6,400
Drowning	4,600
Fire	4,200
Ingestion of Food/Object	2,900
Firearms	1,400

Pie Chart

- To create a pie chart for the data, find the relative frequency (percent) of each category.



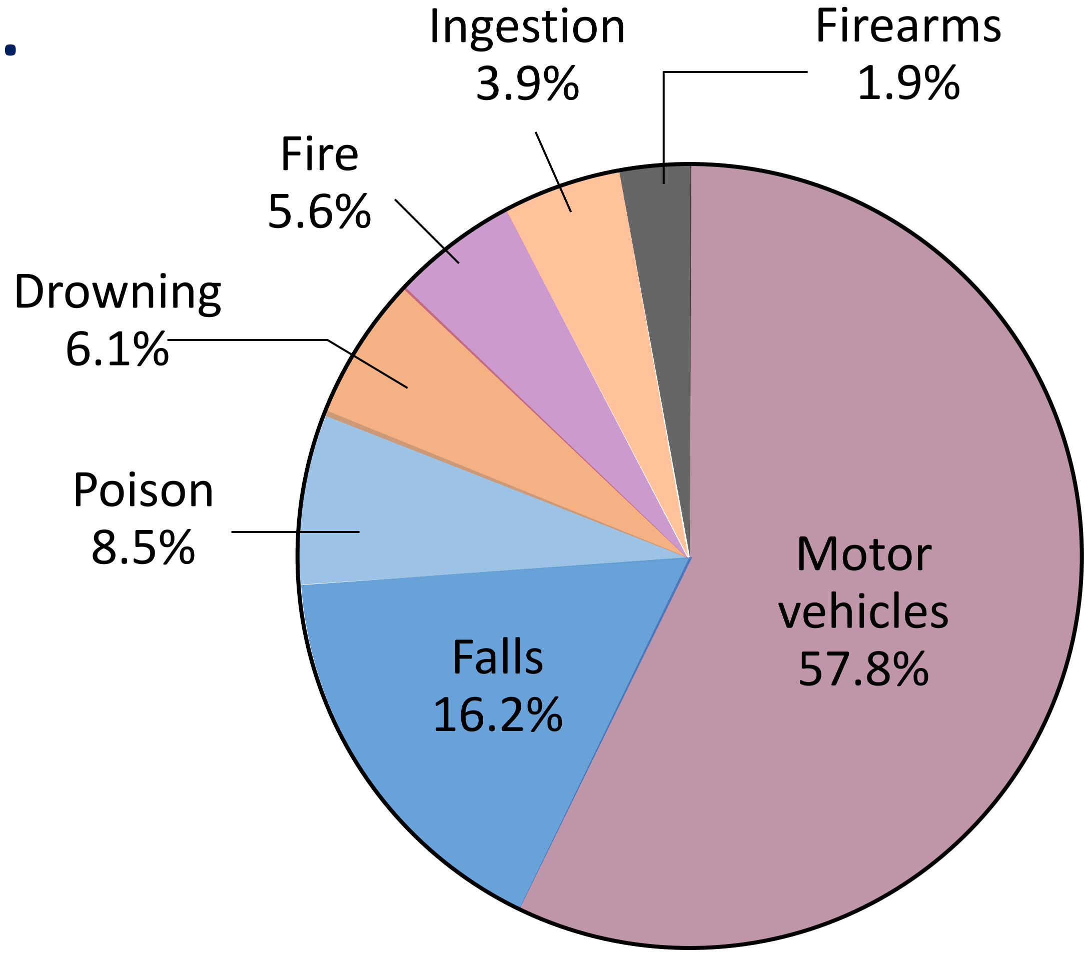
Type	Frequency	Relative Frequency
Motor Vehicle	43,500	0.578
Falls	12,200	0.162
Poison	6,400	0.085
Drowning	4,600	0.061
Fire	4,200	0.056
Ingestion of Food/Object	2,900	0.039
Firearms	1,400	0.019

n = 75,200

Pie Chart

- Next, find the central angle. To find the central angle, multiply the relative frequency by 360°.

Type	Frequency	Relative Frequency	Angle
Motor Vehicle	43,500	0.578	208.2°
Falls	12,200	0.162	58.4°
Poison	6,400	0.085	30.6°
Drowning	4,600	0.061	22.0°
Fire	4,200	0.056	20.1°
Ingestion of Food/Object	2,900	0.039	13.9°
Firearms	1,400	0.019	6.7°

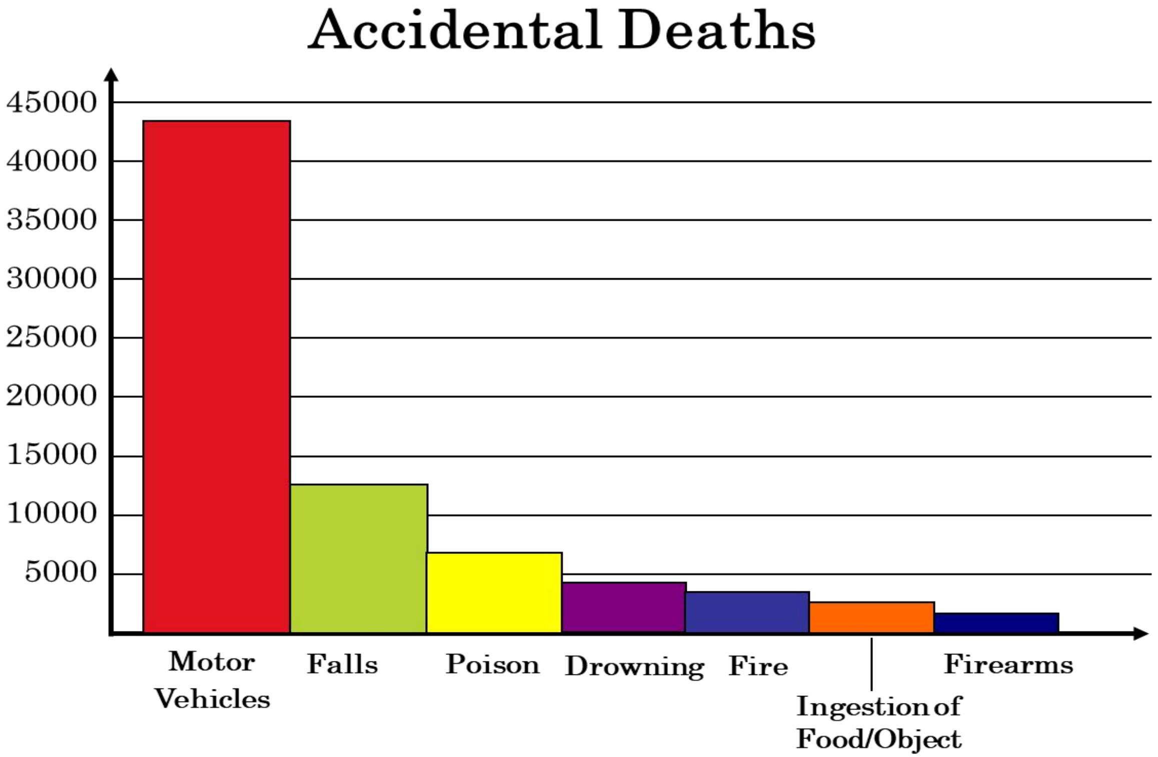


Pareto Chart

- A Pareto chart is a vertical bar graph is which the height of each bar represents the frequency. The bars are placed in order of decreasing height, with the tallest bar to the left.

Accidental Deaths in the USA in 2002

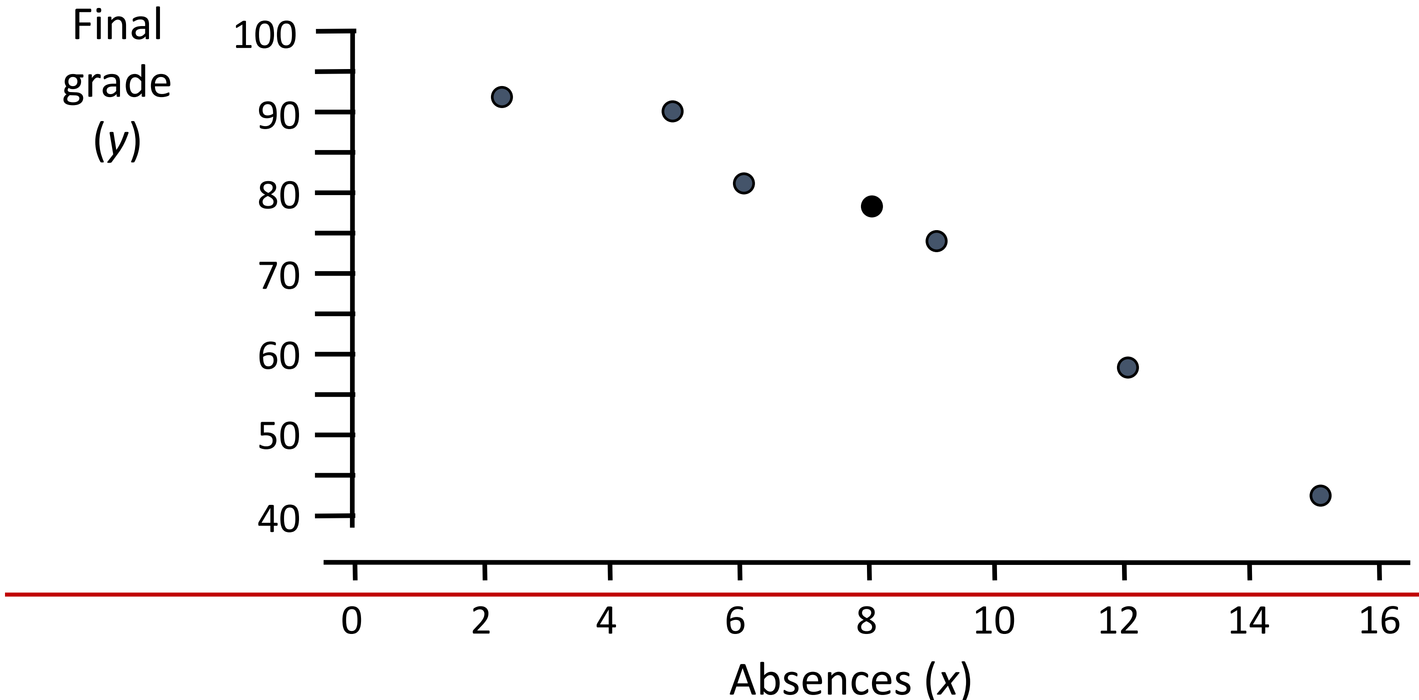
Type	Frequency
Motor Vehicle	43,500
Falls	12,200
Poison	6,400
Drowning	4,600
Fire	4,200
Ingestion of Food/Object	2,900
Firearms	1,400



Scatter Plot

- In a scatter plot, the ordered pairs are graphed as points in a coordinate plane. The scatter plot is used to show the relationship between two quantitative variables.
- The following scatter plot represents the relationship between the number of absences from a class during the semester and the final grade.

Absences x	Grade y
8	78
2	92
5	90
12	58
15	43
9	74
6	81



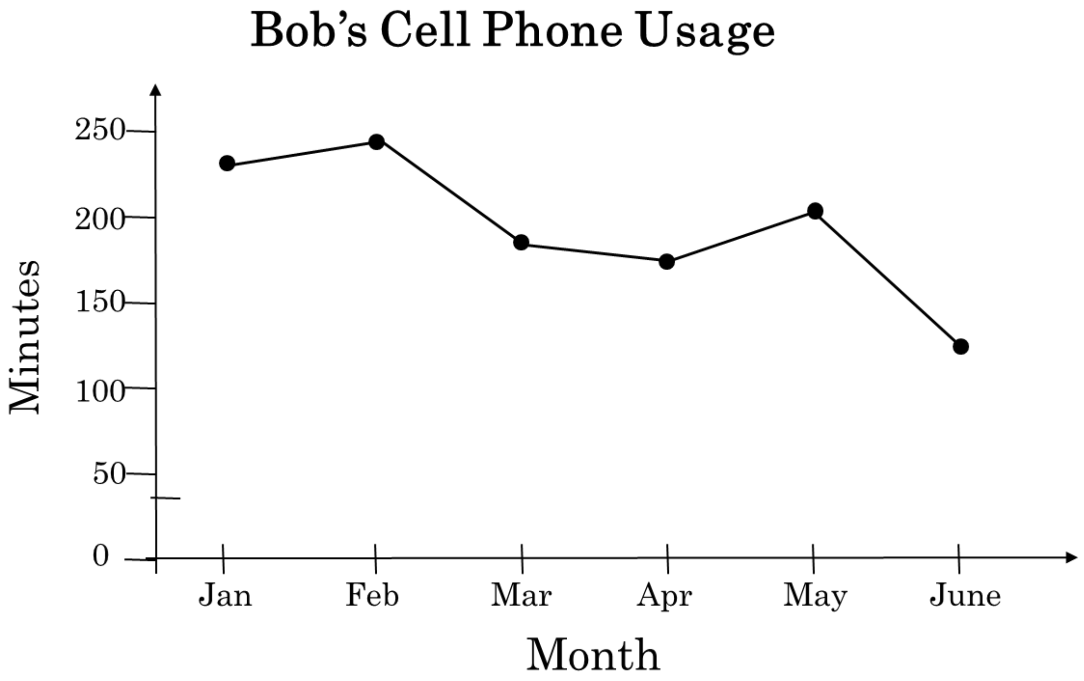
From the scatter plot, you can see that as the number of absences increases, the final grade tends to decrease.

Times Series Chart

- A time series chart is used to graph a time series.
- A data set that is composed of quantitative data entries taken at regular intervals over a period of time is a time series.
- Example:

The following table lists the number of minutes Bob used on his cell phone for the last six months.

Month	Minutes
January	236
February	242
March	188
April	175
May	199
June	135



Measure of Central Tendency

The Mean

- **The mean of a data set is the sum of the data entries divided by the number of entries.**
- **Population mean:** $\boldsymbol{\mu = \frac{1}{N}\sum x}$
- **Sample mean:** $\boldsymbol{\overline{x} = \frac{1}{n}\sum x}$
- **Example**
  - **The following are the ages of all seven employees of a small company:** $\mathbf{53, 32, 61, 57, 39, 44, 57}$
    - $\boldsymbol{\mu = \frac{1}{N}\sum x = \frac{342}{7} = 49\ years}$

The mean age of the employees is 49 years.

The Median

- The median of a data set is the value that lies in the middle of the data when the data set is ordered.
  - If the data set has an odd number of entries, the median is the middle data entry.
  - If the data set has an even number of entries, the median is the mean of the two middle data entries.
- Example
  - The following are the ages of all seven employees of a small company: 53 32 61 57 39 44 57

SORTED DATA 32 39 44 $\boxed{53}$ 57 57 61

The median age of the employees is 53 years.

The Mode

- The mode of a data set is the data entry that occurs with the greatest frequency.
  - If no entry is repeated, the data set has no mode.
  - If two entries occur with the same greatest frequency, each entry is a mode and the data set is called bimodal.
- Example
  - The following are the ages of all seven employees of a small company:

53 32 61 $\boxed{57}$ 39 44 $\boxed{57}$

The mode age of the employees is 57 years.

Outliers

- An outlier is a data point or observation whose value is quite different from the others in the data set being analyzed.
- no absolute agreement about how to define outliers

Comparing the Mean, Median and Mode

- **Example** 53 32 61 57 39 44 57
  - A 29-year-old employee joins the company, and the ages of the employees are now:

    53 32 61 57 39 44 57 29

  - Recalculate the mean, the median, and the mode.
    - Mean = 46.5
    - Median = 48.5
    - Mode = 57

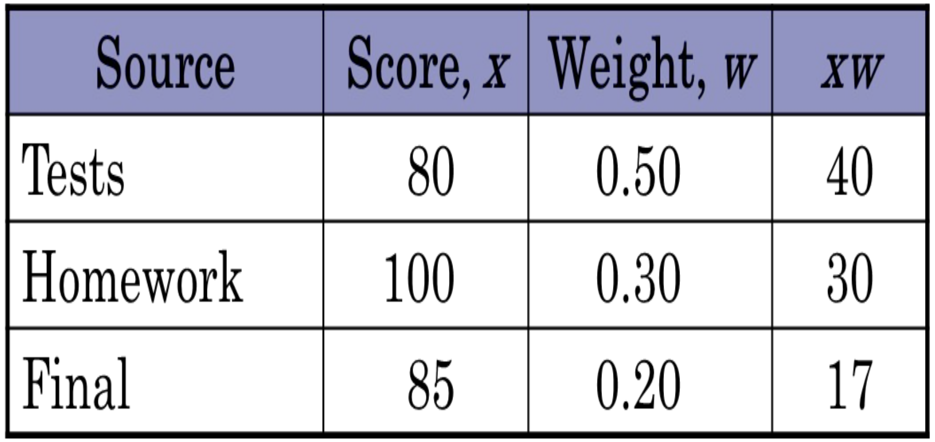
The mean takes every value into account but is affected by the outlier.

Weighted Mean

- A **weighted mean** is the mean of a data set whose entries have varying weights. A weighted mean is given by

$$\bar{x} = \frac{\sum xw}{\sum w}$$

- where **w** is the weight of each entry **x**.
- Example
  - Grades in a statistics class are weighted as follows:
    - Tests are worth 50% of the grade, homework is worth 30% of the grade and the final is worth 20% of the grade. A student receives a total of 80 points on tests, 100 points on homework, and 85 points on his final. What is his current grade?



Source	Score, x	Weight, w	xw
Tests	80	0.50	40
Homework	100	0.30	30
Final	85	0.20	17

$$\bar{x} = \frac{\sum xw}{\sum w} = \frac{87}{100} = 0.87$$

Mean of a Frequency Distribution

- The mean of a frequency distribution for a sample is approximated by

$$\bar{x} = \frac{\sum(x \cdot f\,)}{n} \qquad \text{Note that } n = \sum f$$

- where x and f are the midpoints and frequencies of the classes.

Mean of a Frequency Distribution

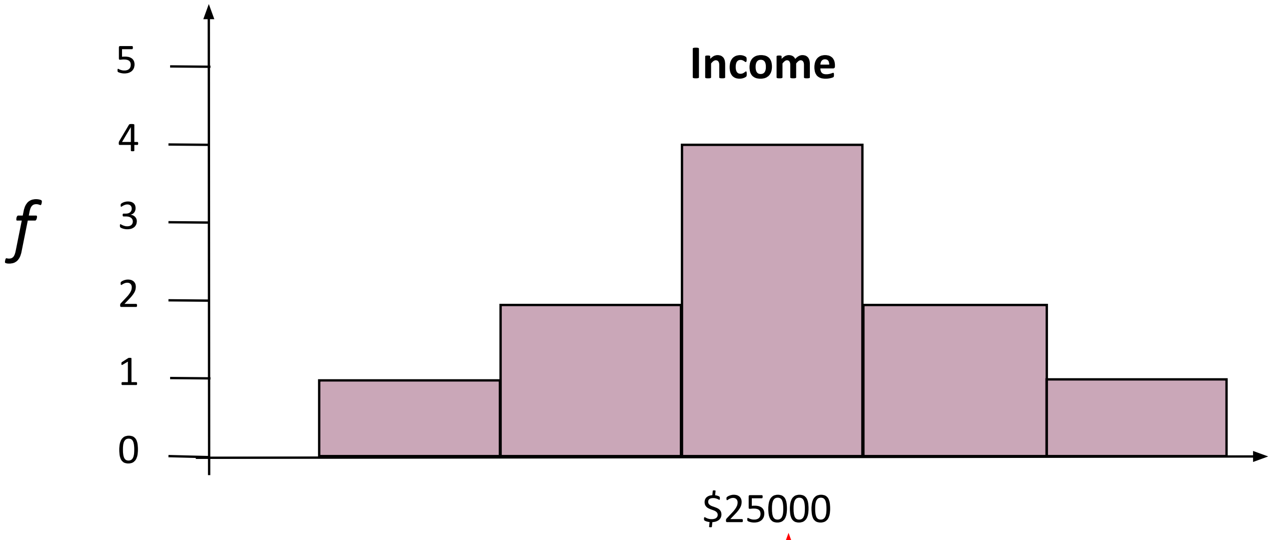
Class	x	f	$(x \cdot f)$
18 – 25	21.5	13	279.5
26 – 33	29.5	8	236.0
34 – 41	37.5	4	150.0
42 – 49	45.5	3	136.5
50 – 57	53.5	2	107.0
		$n = 30$	$\Sigma = 909.0$

$$x = \frac{\Sigma(x \cdot f)}{n} = \frac{909}{30} = 30.3$$

The mean age of the students is 30.3 years.

Shapes of Distributions

- A frequency distribution is **symmetric** when a **vertical line** can be drawn through the **middle** of a graph of the distribution and the **resulting halves** are approximately the **mirror images**.

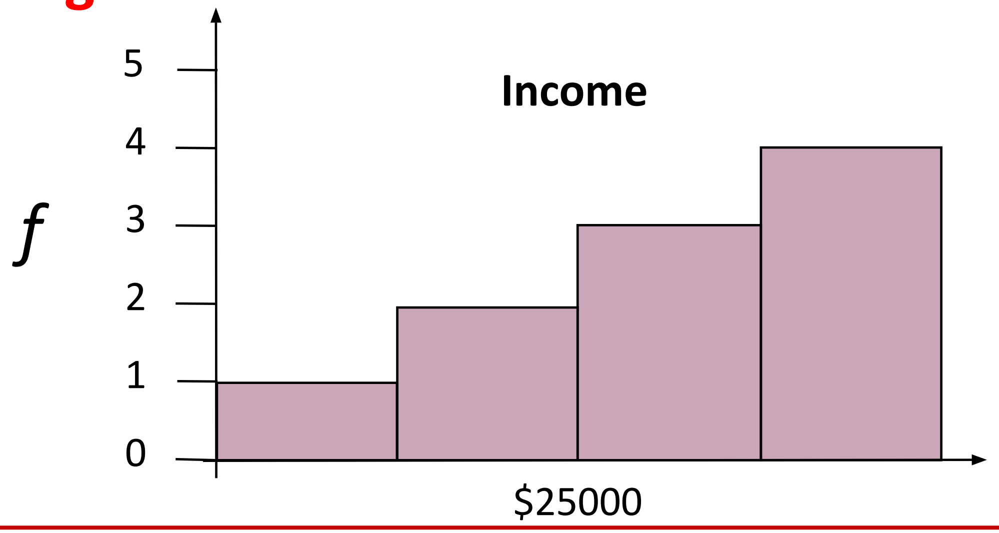


10 Annual Incomes
15,000
20,000
22,000
24,000
25,000
25,000
26,000
28,000
30,000
35,000

mean = median = mode
= $25,000

Shapes of Distributions

- A frequency distribution is skewed if the “tail” of the graph elongates more to one side than to the other.
  - A distribution is skewed left (negatively skewed) if its tail extends to the left.
  - A distribution is skewed right (positively skewed) if its tail extends to the right.



Mean < Median

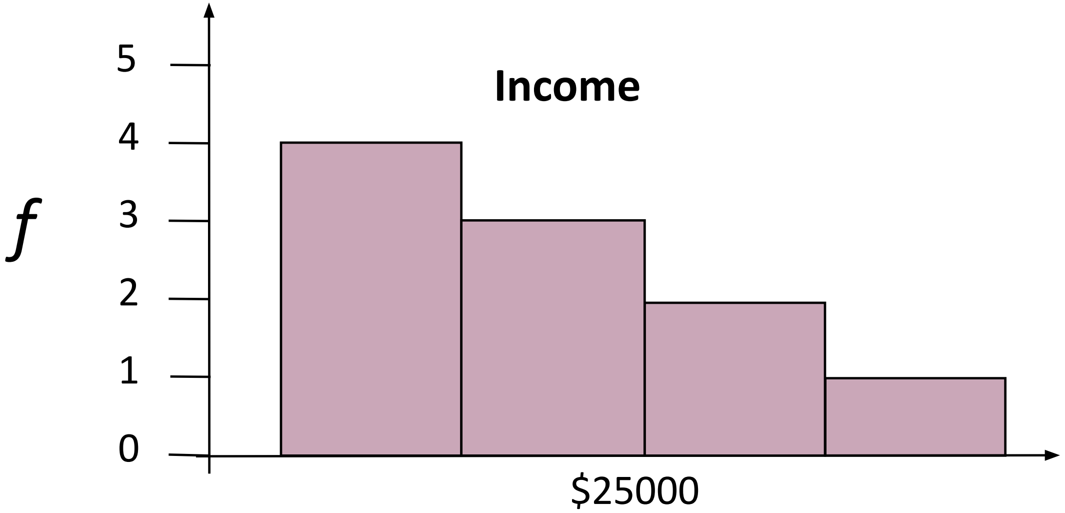
10 Annual Incomes
0
20,000
22,000
24,000
25,000
25,000
26,000
28,000
30,000
35,000

mean = $23,500
median = mode = $25,000

Skewed Right Distribution

10 Annual Incomes
15,000
20,000
22,000
24,000
25,000
25,000
26,000
28,000
30,000
1,000,000

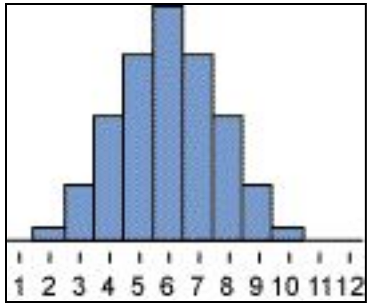
mean = \$121,500
median = mode = \$25,000



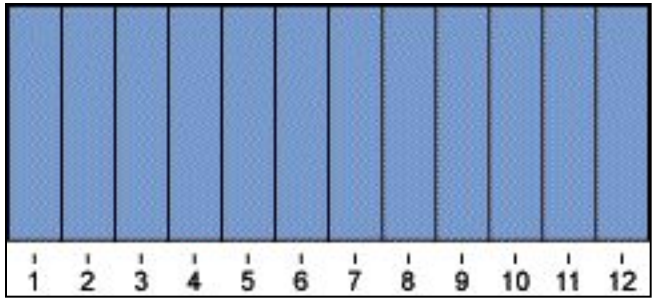
Mean > Median

Summary of Shapes of Distributions

Symmetric

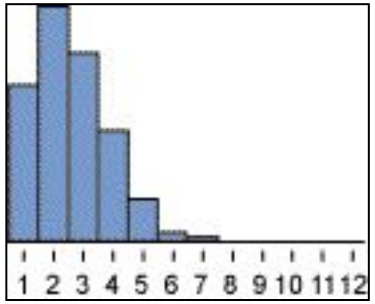


Uniform



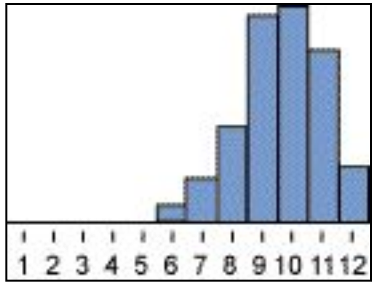
Mean = Median

Skewed right



Mean > Median

Skewed left



Mean < Median

Measures of Dispersion

Range

- The **range** of a data set is the difference between the maximum and minimum date entries in the set.
- Range = (Maximum data entry) – (Minimum data entry)
- Example:
  - The following data are the closing prices for a certain stock on ten successive Fridays.

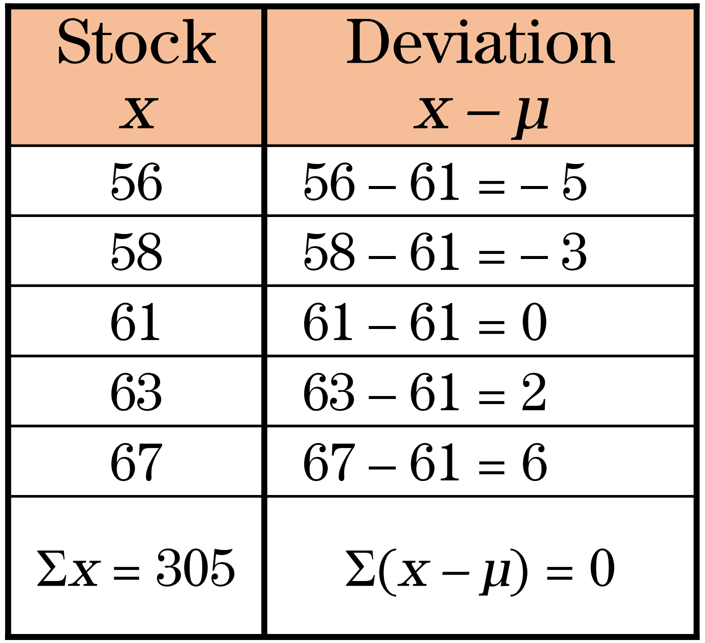
Stock	56	56	57	58	61	63	63	67	67	67

The range is 67 – 56 = 11.

Deviation

- **The deviation of an entry x in a population data set is the difference between the entry and the mean μ of the data set.**
  - **Deviation of x = x – μ**
- **Example:**
  - **The following data are the closing prices for a certain stock on five successive Fridays.**

The mean stock price is
$\mu = 305/5 = 61.$



Stock x	Deviation $x - \mu$
56	56 – 61 = – 5
58	58 – 61 = – 3
61	61 – 61 = 0
63	63 – 61 = 2
67	67 – 61 = 6
$\Sigma x = 305$	$\Sigma(x - \mu) = 0$

Variance and Standard Deviation

- The population variance of a population data set of N entries is

$$\sigma^2 = \frac{\sum(x - \mu)^2}{N}.$$

- The population standard deviation of a population data set of N entries is the square root of the population variance.

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum(x - \mu)^2}{N}}.$$

Finding the Population Standard Deviation

- **Example:**
  - **The following data are the closing prices for a certain stock on five successive Fridays. The population mean is 61.**

Stock x	Deviation $x - \mu$	Squared $(x - \mu)^2$
56	-5	25
58	-3	9
61	0	0
63	2	4
67	6	36
$\Sigma x = 305$	$\Sigma(x - \mu) = 0$	$\Sigma(x - \mu)^2 = 74$

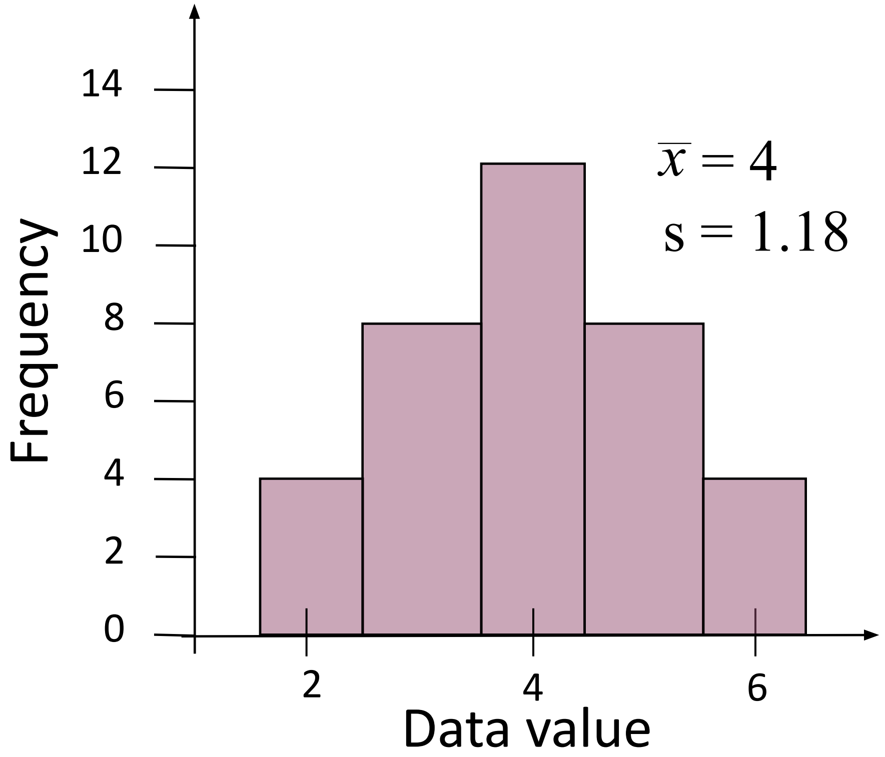
$$\sigma^2 = \frac{\Sigma(\mu x - \quad)^2}{N} = \frac{74}{5} = 14.8$$

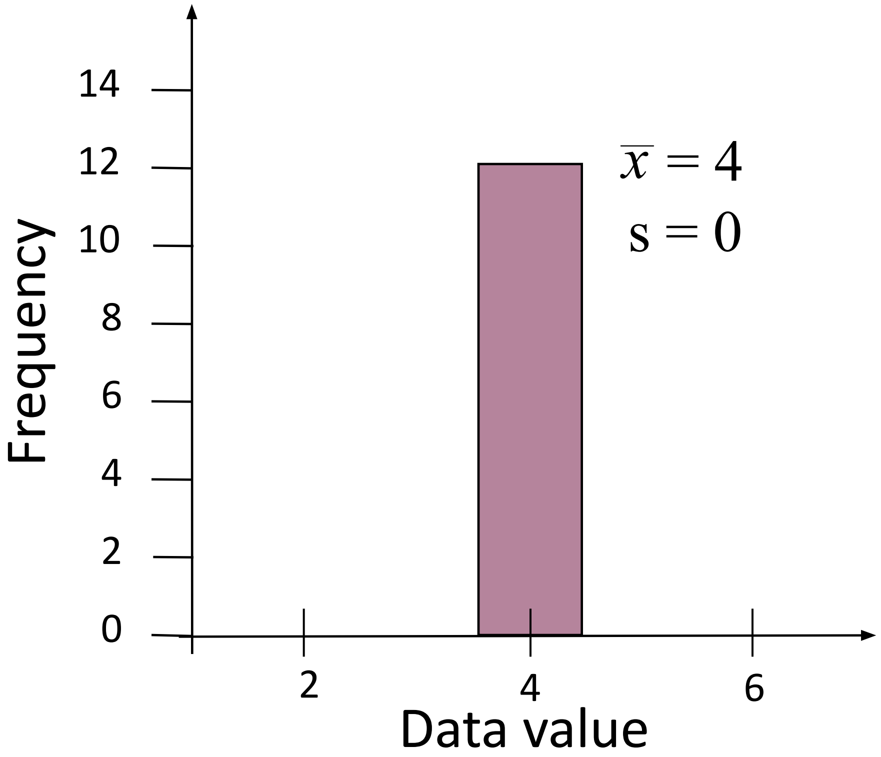
$$\sigma = \sqrt{\frac{\Sigma(\mu x - \quad)^2}{N}} = \sqrt{14.8} \approx 3.85$$

$\sigma \approx \$3.85$

Interpreting Standard Deviation

- standard deviation is a measure of the typical amount an entry deviates from the mean.
- The more the entries are spread out, the greater the standard deviation.

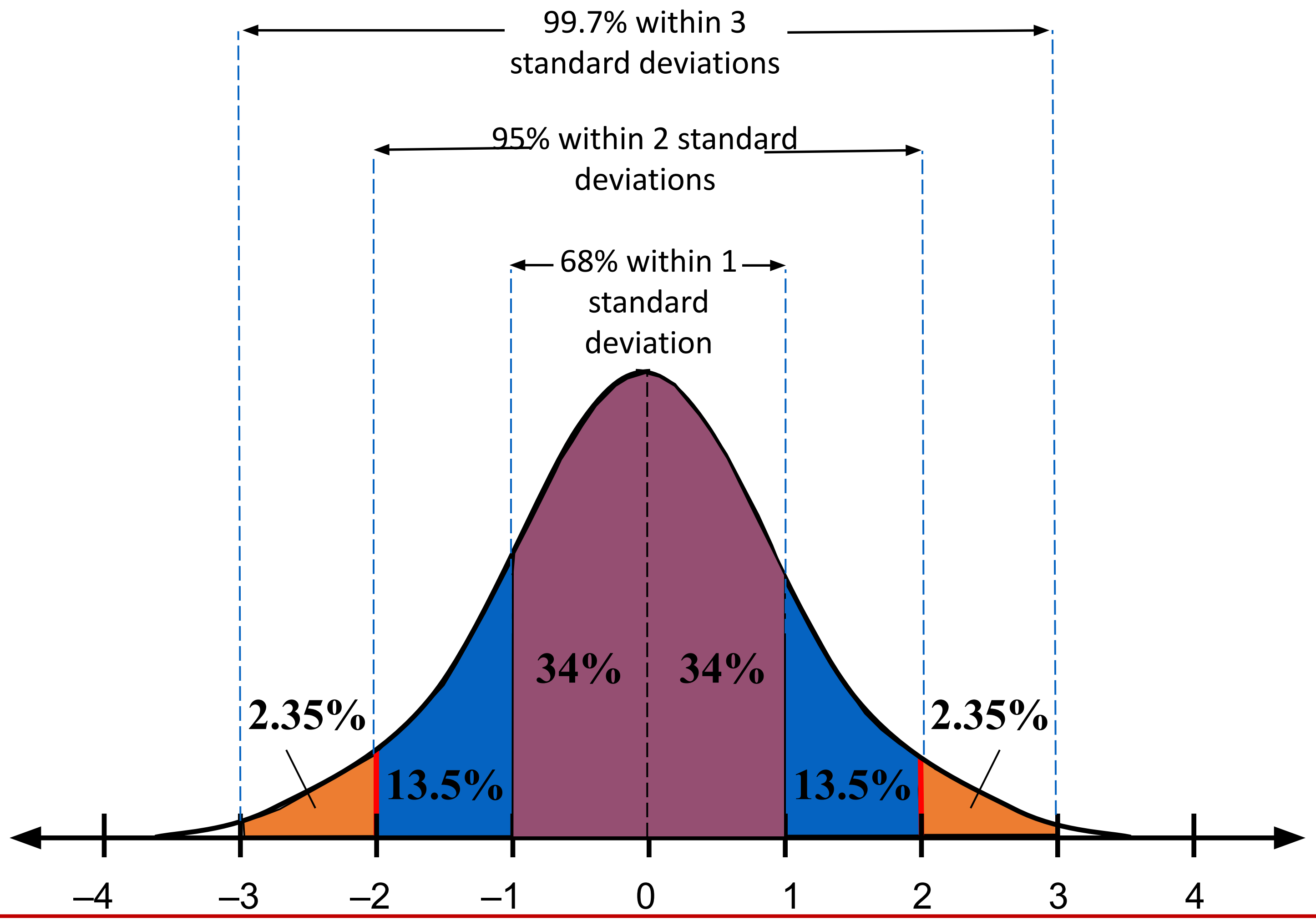




Empirical Rule (68-95-99.7%)

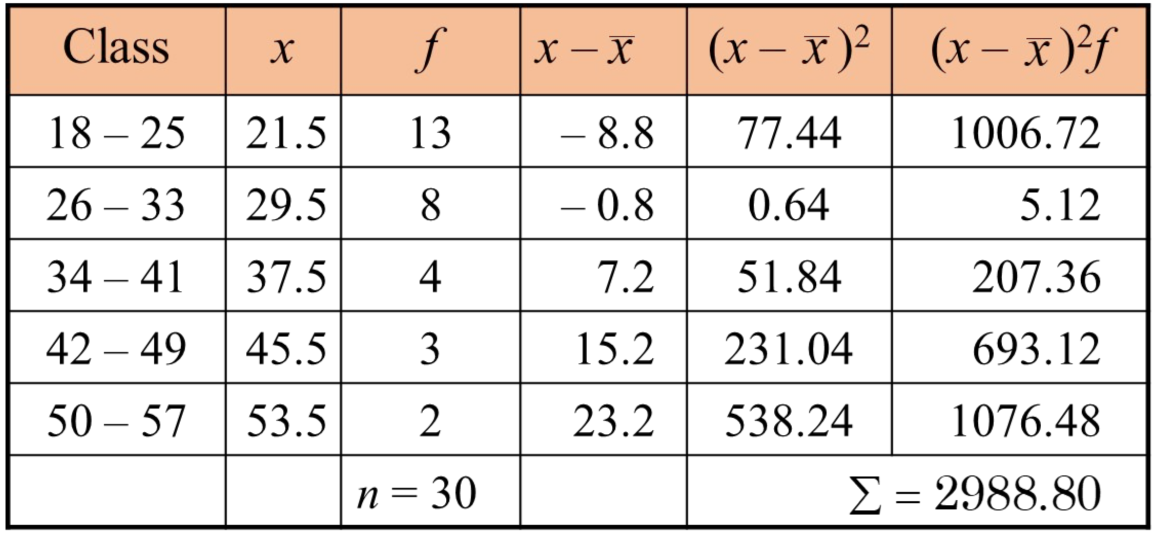
- Empirical Rule
  - For data with a (symmetric) bell-shaped distribution, the standard deviation has the following characteristics.
    1. About 68% of the data lie within one standard deviation of the mean.
    2. About 95% of the data lie within two standard deviations of the mean.
    3. About 99.7% of the data lie within three standard deviation of the mean.

Empirical Rule (68-95-99.7%)


99.7% within 3 standard deviations
95% within 2 standard deviations
68% within 1 standard deviation
34%
34%
13.5%
13.5%
2.35%
2.35%
–4
–3
–2
–1
0
1
2
3
4

Standard Deviation for Grouped Data

- Sample standard deviation = $s = \sqrt{\dfrac{\Sigma(x - x)^2 f}{n - 1}}$
- where n = Σf is the number of entries in the data set, and x is the data value or the midpoint of an interval.



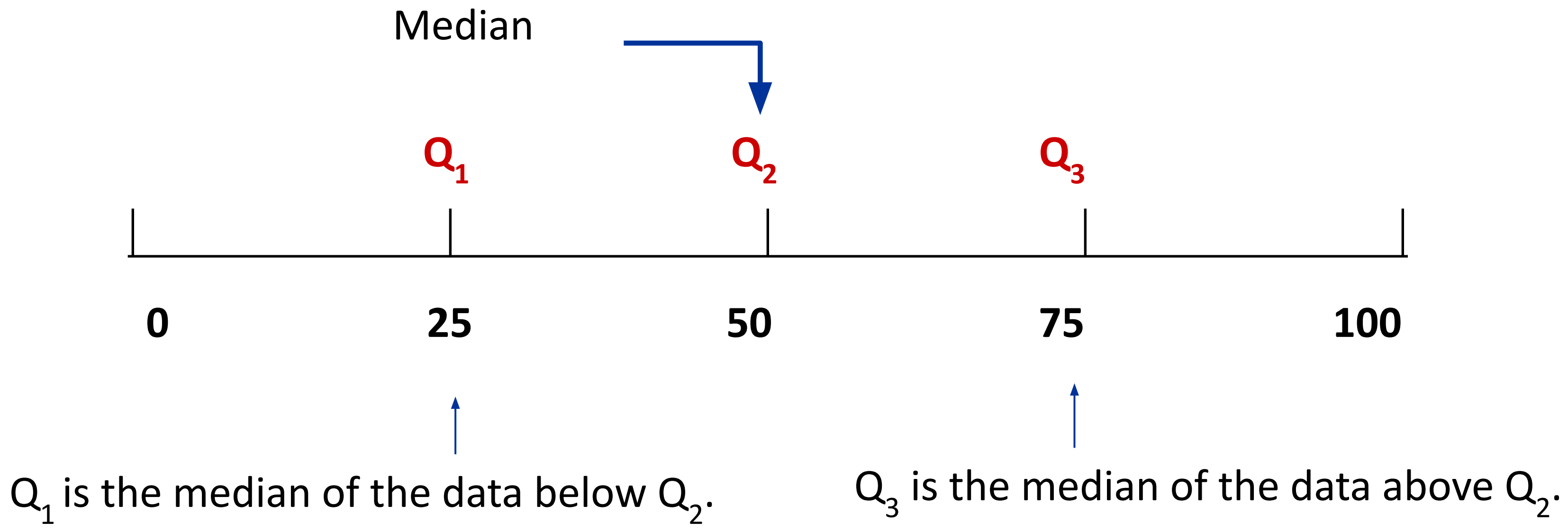
Class	x	f	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
18 – 25	21.5	13	– 8.8	77.44	1006.72
26 – 33	29.5	8	– 0.8	0.64	5.12
34 – 41	37.5	4	7.2	51.84	207.36
42 – 49	45.5	3	15.2	231.04	693.12
50 – 57	53.5	2	23.2	538.24	1076.48
		$n = 30$			$\Sigma = 2988.80$

$$s = \sqrt{\frac{\Sigma(x - x)^2 f}{n - 1}} = \sqrt{\frac{2988.8}{29}} = \sqrt{103.06} = 10.2$$

Measures of Position

Quartiles

- **The three quartiles, Q_1, Q_2, and Q_3, approximately divide an ordered data set into four equal parts.**



Q_1 is the median of the data below Q_2.

Q_3 is the median of the data above Q_2.

Finding Quartiles

- **Example:**
  - **The quiz scores for 15 students are listed below.**

    28 43 48 51 43 30 55 44 48 33 45 37 37 42 38

Order the data.

$$\overbrace{28\quad 30\quad 33\quad \underset{\overset{\uparrow}{Q_1}}{37}\quad 37\quad 38\quad 42}^{\text{Lower half}}\quad \underset{\overset{\uparrow}{Q_2}}{43}\quad \overbrace{43\quad 44\quad 45\quad \underset{\overset{\uparrow}{Q_3}}{48}\quad 48\quad 51\quad 55}^{\text{Upper half}}$$

- **About one fourth of the students scores 37 or less; about one half score 43 or less; and about three fourths score 48 or less.**

Interquartile Range

- **The interquartile range (IQR) of a data set is the difference between the third and first quartiles.**
- **Interquartile range (IQR) = Q3 – Q1.**
- **Example:**
  - **The quartiles for 15 quiz scores are listed below. Find the interquartile range.**

$Q_1 = 37$ $\qquad Q_2 = 43$ $\qquad Q_3 = 48$

$(\text{IQR}) = Q_3 - Q_1$

= 48 – 37

= 11

The quiz scores in the middle portion of the data set vary by at most 11 points.

Box and Whisker Plot

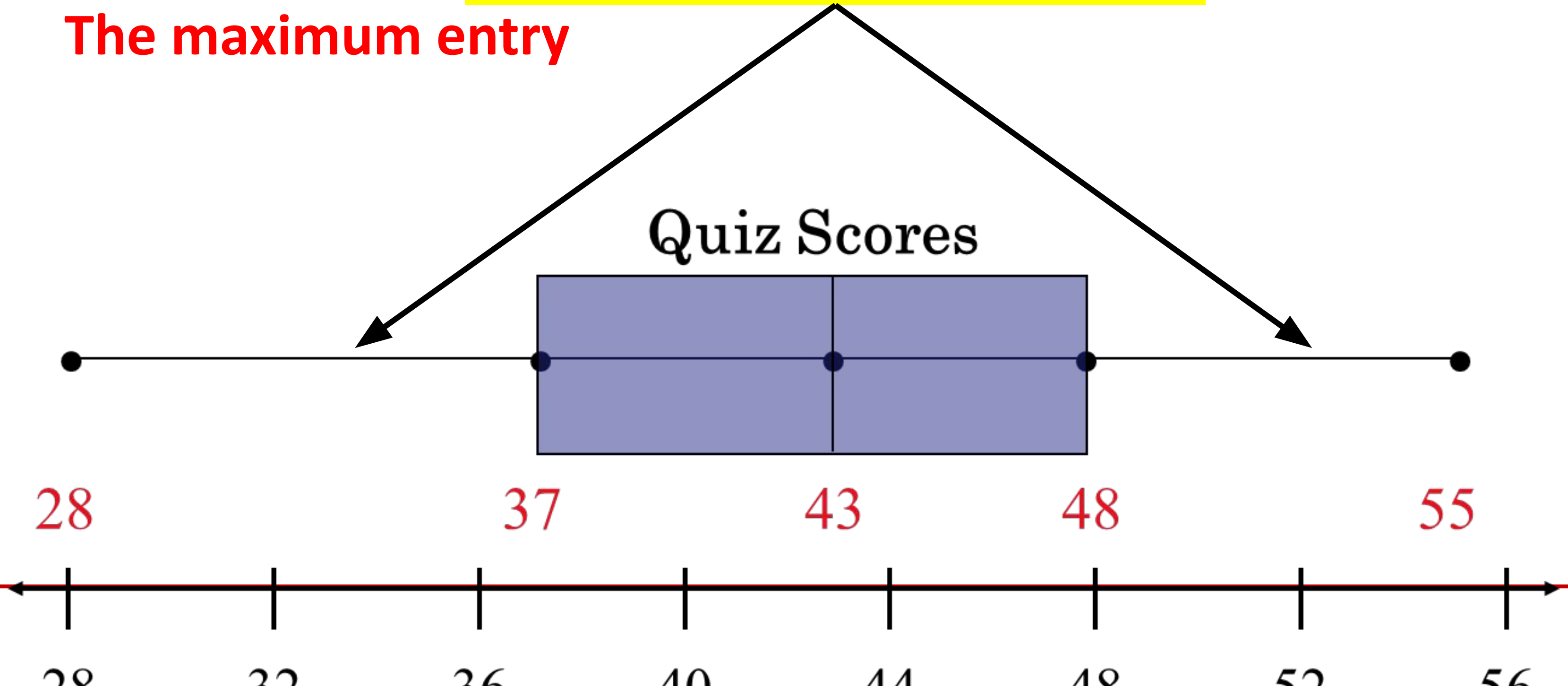
- A box-and-whisker plot is an exploratory data analysis tool that highlights the important features of a data set.
- The five-number summary is used to draw the graph.
  1. The minimum entry
  2. Q1
  3. Q2 (median)
  4. Q3
  5. The maximum entry
- Example:
  - Use the data from the 15 quiz scores to draw a box-and-whisker plot:

    28 30 33 37 37 38 42 43 43 44 45 48 48 51 55

Box and Whisker Plot

- **Five-number summary**
  1. **The minimum entry**
  2. **Q1**
  3. **Q2 (median)**
  4. **Q3**
  5. **The maximum entry**

Whisker: Indicate variability outside the upper and lower quartiles.



Percentiles and Deciles

- **Percentiles** divide an ordered data set into 100 parts. There are 99 percentiles: $P_1, P_2, P_3 \ldots P_{99}$.
  - Percentile = (Number of Values Below “x” / Total Number of Values) × 100
  - Example:
    - The scores for student are 40, 45, 49, 53, 61, 65, 71, 79, 85, 91.
    - percentile for score 71 = (6/10)*100=60
- **Deciles** divide an ordered data set into 10 parts. There are 9 deciles: $D_1, D_2, D_3 \ldots D_9$.

Reference

- Probability and Statistics by Prof. Kevin M. Riordan