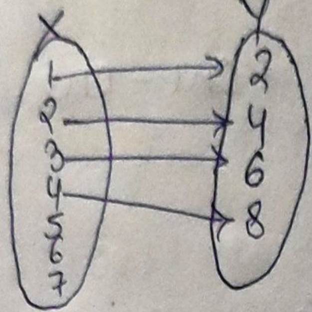
(2) Onto function: A function $f: X \to Y$ is said to onto if every element of Y is image of some element of X i.e codomain = Range.

for $y \in Y$ there is $x \in X$ such that $f(x) = y$

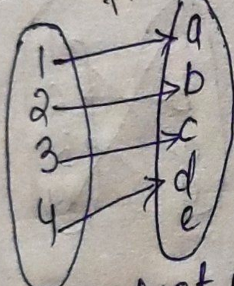


onto

(3) Bijective (Onto + surjective) → one one & onto

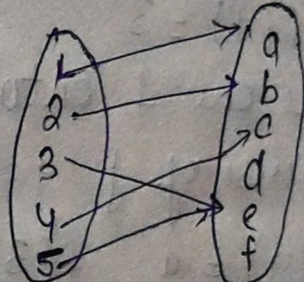
Example →

f1



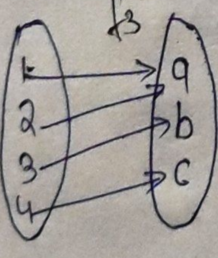
one-one & not onto (into)

(2) f2



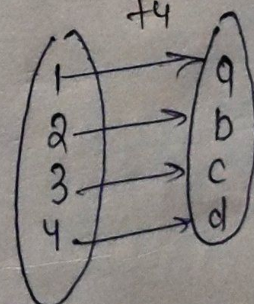
not one-one, into

(3) f3



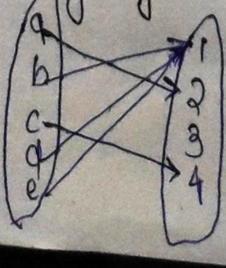
not one-one, onto

(4) f4

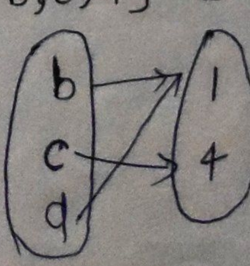


one-one & onto

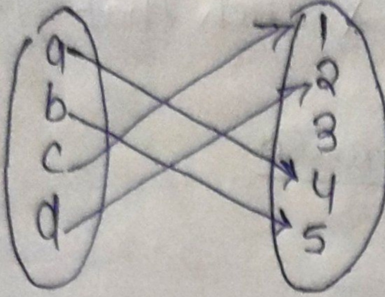
Exa (1) let $A = \{a, b, c, d, e\}$ & $B = \{1, 2, 3, 4\}$ with $f(a) = 2$, $f(b) = 1$, $f(c) = 4$, $f(d) = 1$, $f(e) = 1$.
The image of subset $S = \{b, c, d\}$ is set $f(S) = \{1, 4\}$



⇒



Q Determine whether the function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$ $f(d) = 3$ is one or not



(one - one)

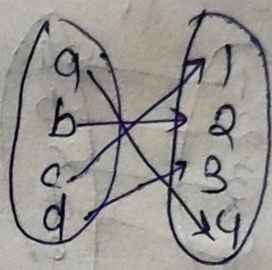
Q Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a) = 4$, $f(b) = 2$ $f(c) = 1$ $f(d) = 3$

→ one one & onto

→ inverse of f

$f^{-1}(4) = a$ $\quad f^{-1}(3) = d$

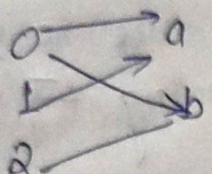
$f^{-1}(2) = b$ $\quad f^{-1}(1) = c$



Relation

A relation is subset of cartesian product ?
Let A & B be the sets, a binary relation from A to B is subset of A×B.

Ex → A {0, 1, 2} B = {a, b}



$S = \{(0,a)(0,b)(1,a)(2,b)\}$

There are three types of relations :

1. Reflexive or Irreflexive
2. Symmetric, antisymmetric, symmetric
3. Transitive

① Reflexive Relation → A relation R on a set A is said to be reflexive when
[if $\forall x \in A$, $(x, x) \in R$]

$A = \{1, 2, 3\}$

① $A \times A = (1,1)(2,2)(3,3)(1,2)(1,3)(2,1)(2,3)(3,1)(3,2)$

② $R = \{(1,1)(2,2)(3,3)\}$ ✓

③ $R = \{(1,2)(2,1)(1,1)(2,2)\}$ ✗

④ $R = \{(1,2)(2,3)(1,3)\}$ ✗

② Irreflexive :- A relation R on a set A is said to be irreflexive when
{ if $\forall x \in A$
$(x, x) \notin A$ }

① $R = A \times A$

② $R = \{(1,1)(2,2)(3,3)\}$

③ $R = \{(1,2)\ (2,3)(3,1)\}$

<u>Symmetric</u> <u>relation</u> → A relation R on a set A is said to be symmetric.

If $\forall a,b \in A$

$\left.\begin{array}{c}(a,b) \in R \\ \hline (b,a) \in R\end{array}\right\}$ If then

$A = \{1,2,3\}$

$R = A \times A$

$R = \{(1,1)(2,2)(3,3\}$ ✓

$R = \{(1,2)(2,3)(1,3)\}$ ✗

$R = \phi$ because (a,b) doesnot belong to a relation R so we don't need to check for (b,a) in R

$A \times A =$

	1	2	3
1	11	12	13
2	21	22	23
3	31	32	33

{ Antisymmetric Relation → A relation R on a set A is said to be antisymmetric

$\forall a,b \in A$

If $(a,b) \in R$ then $a = b$
$(b,a) \in R$

allowing only diagonal $(1,1)$ $(2,2)$

$A = \{1,2,3\}$

① $R = \{(1,1)(2,2)(3,3)\}$ ✓

② $R = \{(1,2)\ (2,1)(2,3)(1,1)\}$ ✗

③ $R = \{(1,2)(2,3)(1,3)\ (3,1)(1,1)(3,3)\}$ ✗

④ $R = A \times A$ ✗

⑤ $R = \phi$ ✓

§ Asymmetric Relation → A relation R on a set A is said to be antisymmetric if $\forall a, b \in A$ and $(a,b) \in R, (b,a) \notin R$

diagonals entry not allowed.

① $R = \{(1,2)(2,3)(3,1)\}$ ✓
② $R = \{(1,2)(2,3)(3,2)\}$ ✗
③ $R = \{(1,1)(2,2)(3,3)\}$ ✗
④ $R = \{(1,2)(1,3)(2,3)\}$ ✓
⑤ $R = A \times A$ ✗
⑥ $R = \phi$ ✓

Transitive Relation → A relation R on a set A is said to be transitive if $\forall a, b \in A$

$\left.\begin{matrix}(a,b) \in R \\ (b,c) \in R\end{matrix}\right]$ then $(a,c) \in R$

$A = \{1,2,3\}$

① $A = \{(1,2)(2,1)(1,1)(2,2)\}$ ✓
② $A = \{(1,1)(2,2)(3,3)\}$ ✓
③ $R = \{(1,2)\}$ ✓
④ $R = \{(1,2)(1,3)\}$ ✓
⑤ $R = \phi$ ✓

$R = \{(1,2)(2,3)\}$ ✗

Partial ordering Relation → A relation R on a set A is said to be partial order if it is

(i) Reflexive
(ii) Antisymmetric
(iii) Transitive

$A = \{1, 2, 3\}$

⇒ $R = \{(1,1)(2,2)(3,3)\}$ # Partial ordering.

⇒ $R = \{(1,1)(2,2)(3,3)(1,2)(2,1)\}$

Reflexive ✓, Antisymmetric ✗, transit ✓

⇒ $R = \{(1,1)(2,2)(3,3)(1,3)(2,3)\}$

R ✓, AS ✓, T ✓ (Partial)

⇒ $R = \{(1,1)(1,2)(2,3)(1,3)\}$

Reflexive (not), AS, Transitive

⇒ $R = A \times A$ # Ref ✓, Asym ✗, Transitive ✓

⇒ $R = \phi$ # Ref ✗

Exa → $R_1 = \{(a,b) \mid a, b \in Z, a < b\}$ → Not partial order (because not reflexive)

$R_2 = \{(a,b) \mid a, b \in Z\ a \leq b\}$

→ partial order

$R_3 = \{(a,b) \mid a, b \in Z, a/b \in Z\}$