$$\text{assertion} \overset{T}{} \Rightarrow \text{proposition}^{T/F}$$
$$\not\Leftarrow$$

Page No.	
Date	

In english language we talk about sentence, assertions, orders and questions

Assertions: A statement that says you are strongly believe that something is true

Propositions: A proposition is an ~~assertion~~ | declarative sentence which is either true OR false but not both

eg: 4 is a prime no — prop : F
$2+2=4$
The sun rises in the east — Ass ⇒ Prop — T

eg Not proposition:
$x+y>4$
$x=3$ (depends on other factors i.e x)
Are you ready

* This statement is false: Liar paradox

Propositional variables: A propositional variable denotes an arbitrary proposition with unspecified truth values P, Q, R

Logical Connectives:
P and Q i.e $P \wedge Q$ (Conjunctions)
P OR Q i.e $P \vee Q$ (disjunction)
not P i.e $\neg P$ / $\sim P$ (negation)
P xor Q i.e $P \oplus Q$ (exclusive OR)
$PQ' \vee QP'$

Date

$(\Rightarrow, \rightarrow)$ **Implication: / Conditional operators** Let P and Q be propositions. the conditional statement $P \Rightarrow Q$ is proposition "If P then Q"

The conditional statement $P \Rightarrow Q$ is false when P is true but Q is false and true otherwise

In conditional statement $P \Rightarrow Q$,

P is also called hypothesis (or antecedent or premise)

and

Q is called conclusion (OR consequence)

The following ways to express it:

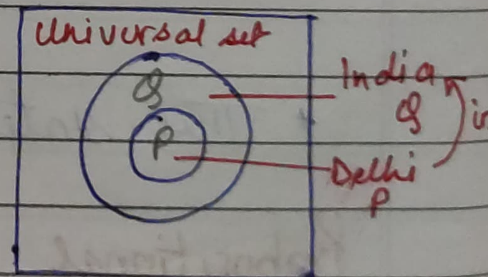
1) If P then Q
2) P implies Q
3) P only if Q
4) P is sufficient for Q
5) A sufficient condition for Q is P
6) A necessary condition for P is Q.

} $P \Rightarrow Q$

Truth table for $P \Rightarrow Q$

	P	Q	$P \Rightarrow Q$
not in Delhi → not in India	0	0	1
not in Delhi → maybe in India (Haryana)	0	1	1
in delhi → not in India	1	0	0
in delhi → in India	1	1	1

eg



$A \Rightarrow B \Rightarrow \neg B \Rightarrow \neg A$

* Converse of $P \Rightarrow Q$ is $Q \Rightarrow P$ not necessarily True
* Contrapositive of $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$ always True
* Inverse of $P \Rightarrow Q$ is $\neg P \Rightarrow \neg Q$ not necessarily True

ex: what are the contrapositive, converse and Inverse of the conditional statement.

S

Page No.
Date

P It is raining
Q home team wins

$P \Rightarrow Q$ If it is raining then the home team wins
$Q \Rightarrow P$ if home team wins then it will rain
$\neg Q \Rightarrow \neg P$ if the home team dsn't win then it will not rain
$\neg P \Rightarrow \neg Q$ if it is not raining then the hometeam will not win

H.W: $P \Rightarrow Q$ If it snows, the trafic moves slowly
Converse: $Q \Rightarrow P$ If the traffic moves slowly then it will snow
Inverse: $\neg P \Rightarrow \neg Q$ It is does not snow, then the traffic moves fast
Contrapositive: $\neg Q \Rightarrow \neg P$ If traffic don't moves slowly then it will not snow

Equivalence: Let P and Q be propositions. The biconditional statement $P \Leftrightarrow Q$ is the proposition "If and only if". The biconditional statement is true when P and Q have same values and is false otherwise. also called bi-implications.

Truth table for $P \Leftrightarrow Q$

P	Q	$P \Leftrightarrow Q$
0	0	1
0	1	0
1	0	0
1	1	1

H.W: Construct the truth table for the compound statement

$(P \vee \neg Q) \Rightarrow (P \wedge Q)$

$(P \vee \neg Q) \Rightarrow (P \wedge Q)$	P	Q	$\neg Q$	$P \vee \neg Q$	$P \wedge Q$
0	0	0	1	1	0
1	0	1	0	0	0
0	1	0	1	1	0
1	1	1	0	1	1

eg: Construct the truth table for $(Q \land \neg P) \Rightarrow P$.

P	Q	$\neg P$	$Q \land \neg P$	$Q \land \neg P \Rightarrow P$
0	0	1	0	1
0	1	1	1	0
1	0	0	0	1
1	1	0	0	1

eg $[(P \land Q) \lor \neg R] \Leftrightarrow P$

P	Q	$P \land Q$	R	$\neg R$	$(P \land Q) \lor \neg R$	$(P \land Q) \lor \neg R \Leftrightarrow P$
0	0	0	0	1	1	0
0	0	0	1	0	0	~~0~~ 1
0	1	0	0	1	1	0
0	1	0	1	0	0	1
1	0	0	0	1	1	1
1	0	0	1	0	0	0
1	1	1	0	1	1	1
1	1	1	1	0	1	1

HW: 1) Show that $\neg(P \lor Q)$ and $(\neg P \land \neg Q)$ are logically equivalent

2) Show that $P \Rightarrow Q$ and $\neg P \lor Q$ are logically equivalent

3) $P \lor (Q \land R)$ and $(P \lor Q) \land (P \lor R)$

(distributive law) over conjuction

(1)

P	Q	$P \lor Q$	$\neg(P \lor Q)$	$\neg P$	$\neg Q$	$\neg P \land \neg Q$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

(2)

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg P \lor Q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

3)

P	Q	R	P∨Q	P∨R	Q∧R	P∨(Q∧R)	(P∨Q)∧(P∨R)
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	0	1	1
1	0	1	1	1	0	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

→ Tautology: It is a formula or assertion that is always true.

eg

P	$\neg P$	$P \vee \neg P$
0	1	1
1	0	1

A contradiction or absurdity is a propositional form which is always false

P	$\neg P$	$P \wedge \neg P$
0	1	0
1	0	0

Contingency: A propositional form which is neither true (under every possible value) nor false (under every possible value)

OR

A propositional which is neither a tautology nor a contradiction

$\neg$ (>) $\wedge$ (>) $\vee$ $P \Rightarrow Q \Leftrightarrow \neg P \vee Q$

Page No. / Date

Logical Identities / Equivalences:

* $P \vee P \Leftrightarrow P$, $P \wedge P \Leftrightarrow P$ } Idempotent rule

* $P \vee Q \Leftrightarrow Q \vee P$, $P \wedge Q \Leftrightarrow Q \wedge P$ } commutative rule/law

* $[(P \vee Q) \vee R] \Leftrightarrow [P \vee (Q \vee R)]$, $[(P \wedge Q) \wedge R] \Leftrightarrow [P \wedge (Q \wedge R)]$ } Associative law

* $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$, $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$ } De' Morgan's law

* $P \Rightarrow (Q \Rightarrow R) \not\Leftrightarrow (P \Rightarrow Q) \Rightarrow R$

* $P \Rightarrow Q \Leftrightarrow (\neg P \vee Q)$

HW
1) $P \Leftrightarrow Q \Leftrightarrow [(P \Rightarrow Q) \wedge (Q \Rightarrow P)]$ — ? equivalence
2) $[(P \wedge Q) \Rightarrow R] \Leftrightarrow [P \Rightarrow (Q \Rightarrow R)]$ — Exportation
3) $[(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q)] \Leftrightarrow \neg P$ — Absurdity
4) $(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$ — Contrapositive

(1)

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)]$	$P \Leftrightarrow Q$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	1	1	1	1

Hence $P \Leftrightarrow Q \Leftrightarrow [(P \Rightarrow Q) \wedge (Q \Rightarrow P)]$

2)

P	Q	R	$Q \Rightarrow R$	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$		$P \Rightarrow (Q \Rightarrow R)$
0	0	0	1	0	1		1
0	0	1	1	0	1		1
0	1	0	0	0	1		1
0	1	1	1	0	1	$\Leftrightarrow$	1
1	0	0	1	0	1		1
1	0	1	1	0	1		1
1	1	0	0	1	0		0
1	1	1	1		1		1

3)

P	Q	$\neg P$	$\neg Q$	$P \Rightarrow Q$	$P \Rightarrow \neg Q$	$(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q)$
0	0	1	1	1	1	1
0	1	1	0	1	1	1
1	0	0	1	0	1	0
1	1	0	0	1	0	0

$\Rightarrow \neg P \Leftrightarrow [(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q)]$

4)

P	Q	$\neg P$	$\neg Q$	$P \Rightarrow Q$		$\neg Q \Rightarrow \neg P$
0	0	1	1	1		1
0	1	1	0	1	$\Leftrightarrow$	1
1	0	0	1	0		0
1	1	0	0	1		1

eg for Tautology: $(P \wedge Q) \Rightarrow (P \vee Q)$

P	Q	$P \wedge Q$	$P \vee Q$	$P \wedge Q \Rightarrow P \vee Q$
0	0	0	0	1
0	1	0	1	1
1	0	0	1	1
1	1	1	1	1

eg: $[(A \Rightarrow B) \vee (A \Rightarrow D)] \Rightarrow (B \vee D)$ $\qquad A \Rightarrow B \Leftrightarrow \neg A \vee B$

$[(\neg A \vee B) \vee (\neg A \vee D)] \Rightarrow (B \vee D)$

$\underbrace{[\neg A \vee (B \vee D)]}_{P} \Rightarrow \underbrace{(B \vee D)}_{Q}$ $\qquad P \Rightarrow Q \Leftrightarrow \neg P \vee Q$

$\neg[\neg A \vee (B \vee D)] \vee (B \vee D)$

$\cancel{\neg A \wedge (B \vee D)}$

$[A \wedge \neg(B \vee D)] \vee (B \vee D)$

$[A \vee (B \vee D)] \wedge [\underbrace{\neg(B \vee D)}_{\neg P} \vee \underbrace{(B \vee D)}_{P}]$ $\qquad \neg P \vee P = 1$ tautology

$[A \vee (B \vee D)] \wedge 1$

$[A \vee B \vee D]$

eg: P: It is snowing
Q: I will go to town
R: I have time

(i) If it is not snowing and I have time, then I will go to the town

$\neg P \wedge R \Rightarrow Q$

ii) I will go to town only if I have time

$Q \Rightarrow R$

iii) It is snowing and I will not go to town

$P \wedge \neg Q$

eg: $Q \Leftrightarrow (R \wedge \neg P)$
I will go to town iff I have time and it is not snowing

$(Q \Rightarrow R) \wedge (R \Rightarrow Q)$ $\qquad (\neg R \vee Q) \wedge (\neg Q \vee R)$
I will go to town iff I have time

eg five persons A, B, C, D, E are in a compartment in a train. A, C, E are men and B, D are women. The train passes through a tunnel and when it emerges. It is found that E is murdered. An enquiry is held, A, B, C, D makes the following statement.

1) A : I am innocent; B was talking to E when train was passing through the tunnel.
2) B : ______ ; I was not talking to E when train was passing through the tunnel.
3) C : ______ ; D committed the murder.
4) D : ______ ; one of the men committed the murder.

Note: four of these eight statements are true and four are false. Assuming that only one person committed the murder. who did it.

Persons	Statement 1	Statement 2	
A	1	1	1 True, 1 False
B	1	1	
C	1	1	makes these two false
D	1	1	

B

3 True - 1 False

Complement of a expression:

$f : A+B \Rightarrow f^c = (\overline{A+B})$
$= \bar{A} \cdot \bar{B}$

A	B	$\bar{A}$	$\bar{B}$	$\overline{A+B}$	$\bar{A} \cdot \bar{B}$
0	0	1	1	0	1
0	1	1	0	1	0
1	0	0	1	1	0
1	1	0	0	1	0

$f = (x+y)\cdot(x\cdot y)$

eg Complement of $f^c = \overline{(x+y)\cdot(x\cdot y)}$

$\Rightarrow (\bar{x}\cdot\bar{y}) + (\bar{x}+\bar{y})$

§ **Logic System** In a positive logic system the higher value/Voltage corresponds to 1, lower value voltage corresponds to 0, In negative logic system, higher value/voltage corr. to '0', lower → '1'

(higher) Positive logic ↑ Voltage corresponds to value '1'

(higher) Negative Logic ↕ Voltage corresponds to value '0'

eg:

"5V" ⇒ Logic '1'
"0V" ⇒ logic '0'

"5V" ⇒ Logic '0'
"0V" ⇒ logic '1'

eg! Logic 0 → −5 V
Logic 1 → 0 V
} positive logic system as 0 > −5

eg Logic 0 → −1.7 V
logic 1 → −0.8 V
} positive L.S

→ Logic 0 → 5.4 V
Logic 1 → −2.7 V
} negative L.S

§ **Duals form**:

+ve & −ve L.S. for AND

A	B	output
L	L	L
L	H	L
H	L	L
H	H	H

(A·B)

+ve L.S for AND

A	B	output
0	0	0
0	1	0
1	0	0
1	1	1

-ve L.S. for AND

A	B	output
1	1	1
1	0	1
0	1	1
0	0	0

FOR OR

A	B	Output
L	L	L
L	H	H
H	L	H
H	H	H

+ve L.S

A	B	output
0	0	0
0	1	1
1	0	1
1	1	1

(1 – high, 0 → low)

-ve L.S. (A+B)

A	B	output
1	1	0
1	0	1
0	1	1
0	0	1

(1 – low, 0 → high)

Hence +ve L.S for AND ≡ -ve L.S for OR
-ve L.S for AND ≡ +ve L.S for OR

* What is dual form: (f^d)

for eg: $f = A \cdot B$ what will be its dual form

$f^d = A + B$ (Changing the +ve L.S to -ve L.S)

OR

$f = A+B$; $f^d = A \cdot B$

eg: $f: a + 0 = a$; $f^d: a \cdot 1 = a$

f: ~~a~~ $a + ab = a$
$(a(1+b)) \rightarrow a(1) = a$

f^d: $a \cdot (a+b) = a$
$a \cdot a + a \cdot b$

* If there is a function $f(a, b, c, \ldots, z, 0, 1, \cdot, +)$
then dual of function $f^d(a, b, c, \ldots, z, 1, 0, +, \cdot)$

we can take dual of expression to convert positive L.S to -ve L.S.

Date

- If any expression is true in its original form then its dual also be true.

eg
$f: a+ab = a$ (original form) $f: a+0 = a$
$f^d: a\cdot(a+b) = a$ (dual form) $f^d: a\cdot 1 = a$

eg
$f: (x+y)\cdot(xy)\cdot(x+y)$
$f^d: (x\cdot y)+(x+y)+(x\cdot y)$

$\Rightarrow$ $f: x(y+0)$ and $f: \bar{x}\cdot 1 + (\bar{y}+z)$

$f^d: x+(y\cdot 1)$, $f^d: (\bar{x}+0)\cdot(\bar{y}\cdot z)$

eg $f = p \vee \neg q$ #

$f^d = \cancel{p \wedge \neg q}$

P	Q	¬P	¬Q	P∨¬Q	P∧¬Q
0	0	1	1	1	0
0	1	1	0	0	0
1	0	0	1	1	0
1	1	0	0	1	0

eg $f = p \wedge (q \wedge (r \wedge T))$

$f^d = p \vee (q \vee (r \vee F))$

eg $f = (x \wedge y) \vee (y \wedge z) \vee (z \wedge x)$

$(x \vee y) \wedge (y \vee z) \wedge (z \vee x)$

$f^d =$

Dual: $f = A + \bar{A} = 1$ $f: A(B+C) = A\cdot B + A\cdot C$

$f^d = A\cdot\bar{A} = 0$ $f^d = A + (B\cdot C) = (A+B)\cdot(A+C)$

why duality?

$f: (\bar{A}\cdot\bar{B})\cdot(C+\bar{D})\cdot(\bar{C}+D)$ primal / original

$f^d: (\bar{A}+\bar{B}) + C\bar{D} + \bar{C}D$ (d^*) Dual

(any of original & dual may/maynot be equal)

Suppose $p^* = d^*$ it is called strong duality
if $p^* \neq d^*$ it is called weak duality

duality gap is $p^* - d^* \geq 0$

if it is zero, then it holds strong duality &
if it is greater than zero, then it holds weak duality

$f: A\cdot B + \bar{C}D + E + 0$

$f^d: (A+B)\cdot(\bar{C}+D)\cdot E\cdot 1$

Normal forms –
1) Conjunctive Normal forms (CNFs)
2) Disjunctive Normal forms (DNFs)
Any well defined formula, we can write either in CNFs OR in DNF's

Date

literal: A variable or negation of variable is called a literal i.e P or $\neg P$

Sum: Disjunction of a literal is called sum i.e
$d_1 \vee d_2 \vee \neg d_3 \vee \cdots \neg d_n$

Conjunction of a literal is called product i.e
$C_1 \wedge C_2 \wedge \cdots \wedge C_n$

A sum of Product is called DNF
eg: $(P_1 \wedge P_2 - P_3) \vee (\neg P_4 \wedge P_5) \vee (\cdots)$

eg $P \wedge (P \Rightarrow Q)$ DNF
as $P \Rightarrow Q \equiv \neg P \vee Q$
$\Rightarrow P \wedge (\neg P \vee Q)$
$\Rightarrow (P \wedge \neg P) \vee (P \wedge Q) \rightarrow$ [Destributive law]
sum of product

Remark: * A product of the variables and their negations in a formula is called an elementary product
eg $P \wedge Q \wedge \neg R$

* Sum of the variables and their negations in a formula is called an elementary sum
eg: $P \vee Q \vee \neg R$

* A necessary and sufficient condition for an elementary product to be identically false is that it contain atleast one pair of factors / or elements in which one is the negation of other

Page No.
Date

eg $P \wedge Q \wedge \neg R \wedge \neg Q \Rightarrow P \wedge \neg R \wedge \underbrace{Q \wedge \neg Q}_{\text{false}} \Rightarrow$ False

→ contradiction

* A necessary and sufficient cond for an elementary sum to be identically true is that is contains atleast one pair of factors / elements in which one is negation of other

eg $P \vee Q \vee \neg R \vee \neg Q \Rightarrow P \vee \neg R \vee \underbrace{Q \vee \neg Q}_{\text{True}} \Rightarrow$ True

→ tautology

Disjunctive Normal form: A formula which is equivalent to a given formula and which consists of a sum of elementary products is called DNF

eg: $\underbrace{(P \wedge Q)}_{\text{product}} \vee (\neg R \wedge \neg Q) \vee (\ldots) \ldots$

eg: $P \wedge (P \Rightarrow Q) \equiv P \wedge (\neg P \vee Q) \equiv (P \wedge \neg P) \vee (P \wedge Q)$

eg: $(\neg P \vee \neg Q) \Rightarrow (P \Leftrightarrow \neg Q)$

$P \Leftrightarrow \neg Q \equiv (P \Rightarrow Q \wedge Q \Rightarrow P)$

$\neg(\neg P \vee \neg Q) \vee ((P \wedge \neg Q) \vee (\neg P \wedge \neg(\neg Q))$

$\equiv (P \vee Q) \wedge (\neg P \vee \neg Q)$

$\neg\neg P \wedge \neg\neg Q \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

$\equiv [(\neg P \wedge Q) \vee (\neg Q \wedge P)]$

$\underbrace{(P \wedge Q)}_{\text{product}} \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

eg: $[P \Rightarrow (Q \wedge R)] \wedge [\neg P \Rightarrow (\neg Q \wedge \neg R)]$

$[\neg P \vee (Q \wedge R)] \wedge [\neg\neg P \vee (\neg Q \wedge \neg R)]$

$[\neg P \vee (Q \wedge R)] \wedge [P \vee (\neg Q \wedge \neg R)]$

$(\cancel{\neg P \wedge P})$ $\neg P$

$\Rightarrow (P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R) \vee (Q \wedge R \wedge \neg Q \wedge \neg R) \vee (P \wedge \neg P)$

Note: DNF is not unique

$7a = 4,5$ $\quad a(1,2,3)$ $\quad (a \cup b) \cap (b \cap a)$

$7b = 1,5$, $\quad b(2,3,4)$

Paga No.
Date

Conjunctive Normal Forms (CNF's): A formula which is equivalent to a given formula and which consists of a product of elementary sums is called a conjunctive normal form of the given formula (wff)

eg: $(P \vee Q) \wedge (P \vee Q \vee \neg R) \wedge (\ldots \vee \ldots \vee) \wedge \ldots$

Any wff of the propositional logic can bring it to CNF

eg:

$A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$

$\neg(P \vee Q) \Leftrightarrow (P \wedge Q)$

$[\neg(P \vee Q) \Rightarrow (P \wedge Q)] \wedge [(P \wedge Q) \Rightarrow \neg(P \vee Q)]$

$[\neg(P \wedge Q) \vee \neg(P \vee Q)] \wedge [(P \vee Q) \vee (P \wedge Q)]$

$[(\neg P \vee \neg Q) \vee (\neg P \wedge \neg Q)] \wedge [(P \vee Q) \vee (P \wedge Q)]$

$(\neg P \vee \neg Q \vee \neg P) \wedge (\neg P \vee \neg Q \vee \neg Q) \wedge (P \vee Q \vee P) \wedge (P \vee Q \vee Q)$

$(\neg P \vee \neg Q) \wedge (\neg P \vee \neg Q) \wedge (P \vee Q) \wedge (P \vee Q)$

$(\neg P \vee \neg Q) \wedge (P \vee Q)$ (product of sums)

eg: $(\neg P \vee Q) \Rightarrow (P \Leftrightarrow \neg Q)$ convert to CNF

$\neg(\neg P \vee Q) \vee (P \Leftrightarrow \neg Q)$

$(P \wedge \neg Q) \vee [(P \Rightarrow \neg Q) \wedge (\neg Q \Rightarrow P)]$

$(P \wedge \neg Q) \vee [(\neg P \vee \neg Q) \wedge (Q \vee P)]$

$(P \vee \neg P \vee \neg Q) \wedge (P \vee Q \vee P) \wedge (\neg Q \vee \neg P \vee \neg Q) \wedge (\neg Q \vee Q \vee P)$

$\underbrace{(P \vee \neg P \vee \neg Q)}_{\text{tautology}} \wedge (P \vee Q) \wedge (\neg P \vee \neg Q) \wedge \underbrace{(\neg Q \vee Q \vee P)}_{\text{tautology}}$

Date

Principal Disjunctive Normal forms (PDNF's)

Let P and Q be two propositional variables

i) $P \wedge Q$
ii) $P \wedge \neg Q$
iii) $\neg P \wedge Q$
iv) $\neg P \vee \neg Q$

} minterms

P	Q
0	0
0	1
1	0
1	1

* these formulas are called minterms

for a given formula, an equivalent formula consisting of disjunctive of minterms only is knows as its principle disjunctive normal form. Such a normal form is also called the sum of products Canonical form.

eg

$P \vee [\neg P \Rightarrow (\neg Q \Rightarrow R)]$

$P \vee [P \vee (Q \vee R)]$

$(P \vee P) \vee (P \vee Q \vee R)]$

$\Rightarrow (P \vee Q \vee R)$

T = tautology

$P \equiv P \wedge T \wedge T$

eg $\boxed{P \equiv P \wedge \overbrace{(Q \vee \neg Q)}^{T} \wedge \overbrace{(R \vee \neg R)}^{T}}$ #

$[(P \wedge Q) \vee (P \wedge \neg Q)] \wedge (R \vee \neg R)$

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (R \wedge \neg Q \wedge \neg R)$

$\boxed{Q \equiv Q \wedge \overbrace{(P \vee \neg P)}^{T} \wedge \overbrace{(R \vee \neg R)}^{T}}$

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (Q \wedge \neg P \wedge R) \vee (Q \wedge \neg P \wedge \neg R)$

$R \equiv R \wedge (P \vee \neg P) \wedge (Q \vee \neg Q)$

$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

$$P \vee Q \vee R = \overset{(7)}{(P \wedge Q \wedge R)} \vee \overset{(5)}{(P \wedge \neg Q \wedge R)} \wedge \overset{(3)}{(\neg P \wedge Q \wedge R)} \wedge \overset{(6)}{(P \wedge Q \wedge \neg R)} \wedge$$
$$\underset{(4)}{(P \wedge \neg Q \wedge \neg R)} \wedge \underset{(2)}{(\neg P \wedge Q \wedge \neg R)} \wedge \underset{(1)}{(\neg P \wedge \neg Q \wedge R)}$$

$\Sigma(1, 2, 3, 4, 5, 6, 7)$ minterms
also called principle disjunctive normal form

* **Principle conjunctive Normal forms: (PCNF's)**
we first define the formula which are called maxterm

for given number of variables, the maxterms consists of disjunctive in which each variable or its negation but not both, appears only once

for a given formula, an equivalent formula consisting of conjunctive of the maxterms only is knows as its principle conjunctive normal forms also called product of sum

i) $P \vee Q$ ii) $P \vee \neg Q$ iii) $\neg P \vee Q$ iv) $\neg P \vee \neg Q$ } maxterms

eg: $P \vee [\neg P \Rightarrow (\neg Q \Rightarrow R)]$
eg $(Q \Rightarrow P) \wedge (\neg P \wedge Q)$
$(\neg Q \vee P) \wedge (\{\neg P \vee (Q \wedge \neg Q)\} \wedge \{Q \vee (P \wedge \neg P)\})$

$(\neg Q \vee P) \wedge [\{(\neg P \vee Q) \wedge (\neg P \vee \neg Q)\} \wedge \{(Q \vee P) \wedge (Q \vee \neg P)\}]$

$\underset{(2)}{(P \vee \neg Q)} \wedge \underset{(1)}{(\neg P \vee Q)} \wedge \underset{(0)}{(\neg P \vee \neg Q)} \wedge \underset{(3)}{(P \vee Q)} \wedge \cancel{(\neg P \vee Q)}$

$\Pi(0, 1, 2, 3)$

eg $P \vee (\neg P \Rightarrow (\neg Q \Rightarrow R))$

$P \vee (P \vee (Q \vee R))$

$\Rightarrow P \vee Q \vee R$

§ Inference theory for propositional logic:

Driving conclusion from evidences

Rule of Inference: It is a template for constructing valid arguments

A set of premises $P_1, P_2 \dots - P_n$. Prove that some conclusion Q in the argument

$(P_1 \wedge P_2 \dots \wedge P_n) \Rightarrow Q$

An argument is valid if the premises logically entail the conclusion

Rule 1: (Modus Ponens)

P True

$P \Rightarrow Q$ True

then Q has to be True

$[P \wedge (P \Rightarrow Q)] \Rightarrow Q$

Tautology

Rule 2: (Modus Tollen)

$\neg Q$ T

$P \Rightarrow Q$ T

then $\neg P$ has to be true

if Q is false and $P \Rightarrow Q$

$\Rightarrow P$ is false

$\neg Q \wedge (P \Rightarrow Q) \Rightarrow \neg P$

(tautology)

Rule 3: Hypothetical Syllogism:

$P \Rightarrow Q$ T

$Q \Rightarrow R$ T

$P \Rightarrow R$ has to be True

(tautology)

$(P \Rightarrow Q) \wedge (Q \Rightarrow R) \Rightarrow (P \Rightarrow R)$

Date

Rule 4: Disjunctive Syllogisms

$$\begin{array}{ll} P \vee Q & T \\ \neg P & T \\ \hline \therefore Q & T \end{array}$$

tautology
$(P \vee Q) \wedge \neg P \Rightarrow Q$

Rule 5: Addition:

$$\begin{array}{ll} P & T \\ \hline P \vee Q & T \end{array} \qquad \begin{array}{ll} Q & T \\ \hline P \vee Q & T \end{array}$$

Rule 6: Simplification

$$\begin{array}{l} P \wedge Q \\ \hline \therefore P \end{array} \quad \text{OR} \quad \begin{array}{ll} P \wedge Q & T \\ \hline \therefore Q & T \end{array}$$

Rule No 7: Resolution

$$\begin{array}{ll} P \vee Q & T \\ \neg P \vee R & T \\ \hline \therefore Q \vee R & T \end{array}$$

eg: If it rains (R), I will get wet (W) $\equiv$ $R \Rightarrow W$ T

its raining — R T

$\therefore$ I will get wet $\equiv W$ — W T

Ques: If it snow today (A), then I will go for skiing (B)

$A \Rightarrow B$ T

hypothesis: It is snowing $\equiv$ A T

$\therefore$ I will go for skiing $\equiv$ B T

Ques: What is wrong with this famous supposed prove that $1 = 2$

Ques: If $\sqrt{2} > 3/2$, then $(\sqrt{2})^2 > (3/2)^2$. We know that $\sqrt{2} > 3/2$ consequently $(\sqrt{2})^2 = 2 > (3/2)^2 = 9/4$

Page No.
Date

Predicates and Quantifiers:

Limitations of propositional logic.

$P \Rightarrow Q$ — $\underbrace{\text{If it snow today}}_{P}$, $\underbrace{\text{then I will go skiing}}_{Q}$

P

$\therefore Q$ — Modus ponens Rule.

P — everyone enrolled in the university has lived in dormitory

Q — Neha has never lived in dormitory

R — ⇒ Neha is not enrolled in the university.

Predicates: predicates are the statements involving variabeles which are neither true nor false until or unless the value of variables are specified

eg: x is an animal
x is greatest than 3
x is less than 4
$x+y=7$

Consider the statement : 'x is an animal'
here x is a subject & " is an animal" is a predicate

* any statement can be broken into 2 parts
i) Subject ii) Predicate

eg: $\overset{\text{subject}}{X}$ $\underbrace{\text{is greater than 3}}_{G(x)}$. It can be represented by $G(x)$

eg: $G(5)$: 5 is greater than 3 – True
$G(3)$: 3 " " " " → False

Quantifiers: The quantifiers are words that refers to quantities as "some" "all"

eg: I have some apples

Types of Quantifiers:

1) Universal Quantifiers eg:
2) Existential Quantifiers

: Universal Quantifiers:

Let $p(x)$ be a statement $x+1>x$

$P(x) : x+1>x$

$P(1) : 1+1>0$ True ; $P(2): 2+1>2$ True

$\forall x\, P(x)$ will be true where $x \in Z^+$

eg what is the truth value of $\forall x p(x)$. Where $p(x)$ is the statement "$x^2<10$" and domain consists of the +ve integers not exceeding 4.

$T\ 1<10,\ T\ 4<10,\ T\ 9<10,\ 16<10$ ✗ ⇒ False

$\forall x p(x) \Rightarrow P(1) \wedge (P(2) \wedge P(3) \wedge P(4)$

HW: What is the truth value of $\forall x (x^2 \geq x)$. If the domain consists of IR. False $p(x) \geq x$

Z True

: Existential Quantifiers: The EQ of $P(x)$ is the proposition, There exists an element x in the domain such that $p(x)$ OR $\exists x P(x)$

we can say "$\exists x$ s.t $p(x)$" OR "for some $x\, p(x)$"

ex: What is the truth value of $\exists x P(x)$. Where $p(x)$ is the statement "$x^2>10$" and universe of discourse (UoD) of the positive integers not exceeding 4.

Page No.
Date

$x = 1, 2, 3, 4$ } Domain

we know that $P(x) : x^2 > 10$

$P(1) : 1 > 10$ $P(2) : 4 > 10$ $P(3) : 9 > 10$ $P(4) : 16 > 10$

$\exists x P(x) : P(1) \vee P(2) \vee P(3) \vee P(4) \rightarrow$ True

eg Let $p(x)$ be the statement "$x = x^2$". If domain consists of integers. What is the truth value of $\exists x p(x)$

True $P(x) : x = x^2$

$1 = 1^2$ $\exists x P(n)$ $P(1) \vee P(2) \vee \dots \vee P(n) \dots$

Quantifiers with restricted domain: In this, we can limit the domain of quantifiers by modifying the notation bit.

eg: $P(x) : \forall x < 0 \; (x^2 > 0)$ Domain $\mathbb{R}$

True for $x \in (-\infty, 0)$

$P(y) : y^2 = 2$

$\exists y > 0$ True for $y = \sqrt{2}$

HW Determine the truth value of the given statements:

1) $\exists x (x^3 = -1)$ True } Domain $\mathbb{R}$ $\overset{T}{P}(1) \vee \dots P(n)$

2) $\forall x (2x > x)$ False $\,^{F}P(-1) \wedge \dots P(n)$

3) $\exists n (n^2 = 2)$ False } Domain $\mathbb{Z}$

4) $\forall n (n^2 \geq n)$ True $\,^{T}P(1) \vee \overset{T}{P}(2) \vee \dots \,^{T}P(n)$

5) $Q(x) : x + 1 > 2x$ $1 > x$ } Domain $\mathbb{Z}$

find $\exists x Q(x)$, $\forall x Q(n)$, $\forall x \neg Q(x)$, $\exists x \neg Q(x)$

$\exists x Q(x)$ True $Q(-1)^{T} \vee Q(0)^{T} \vee \dots Q(n) \dots \rightarrow$ True

$\forall x Q(x)$ False $\,^{F}Q(1) \wedge Q(n) \rightarrow$ False

$\forall x \neg Q(x)$ False $\,^{F}Q(0) \wedge \dots \rightarrow$ False

$\exists x \neg Q(x)$ True $\,^{T}Q(2) \vee Q(3) \dots \vee Q(n) \rightarrow$ True

* logical equivalence involving predicators and quantifiers

→ two logical statements are said to be equivalent iff they have the same truth value in all possible cases.

Logical equivalence is helpful in replacing one expression with other equivalent expression

There are two important equivalences involving quantifiers:

1) $\forall x(P(x) \land Q(x)) \equiv \forall x P(x) \land \forall x Q(x)$

Proof: consider an example

Domain: All the student of delhi university/cs dept

$p(x)$: x has studied discrete mathematics

$Q(x)$: x has scored more than 60% marks in exams

LHS: $\forall x(P(x) \land Q(x))$ → every student of cs dept has studied DS & has scored 60% (more than).

* What happens if we can change $\land$ by $\lor$?

i.e. $\forall x(P(x) \lor Q(x)) \equiv \forall x P(x) \lor \forall x Q(x)$

LHS: $\forall x(P(x) \lor Q(x))$: every student in delhi univ. has studied D.M. or has scored more than 60%

	DM	>60%
A	✓	✗
B	✗	✓

possible

RHS $\forall x P(x) \lor \forall x Q(x)$: Every student in the university has studied D.M OR every student in D.U. has scored more than 60% marks

LHS ≠ RHS

2) * $\exists x(P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$

LHS: $\exists x(P(x) \vee Q(x))$: some students of the D.U. has studied D.M. or has scored more than 60% marks

RHS $\exists x P(x) \vee \exists x Q(x)$: some students of DU has studied D.M. OR some students of DU has scored 60% marks

	DM	>60%
A	✓	✗
B	✗	✗

(True)

Hence $\exists x(P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$

* $\exists x(P(x) \wedge Q(x)) \equiv \exists x P(x) \wedge \exists x Q(x)$

LHS: some students of DU has studied D.M and ~~some the~~ has scored more than 60% marks

	DM	>60%
A	✓	✓
B	✗	✗

RHS: some students of DU has studied D.M and some students has scored more than 60% marks

	DM	>60%
A	✓	✗
B	✗	✓

Negating the quantified expression:
Consider the following statement

" Every student in D.U. has studied the D.M"
Domain: All students of DU

$P(x)$: x has studied D.M.
we know that
"every student in D.U. has studied the D.M"
$\forall x P(x)$

$\Rightarrow$ negation of the statement:
"some students in D.U. has not studied D.M"
$\exists x \neg P(x)$

OR "There are some students in DU who has not studied D.M"

$\Rightarrow \exists x \neg P(x) \equiv \neg \forall x P(x)$

$\forall x P(x) = P_1 \wedge P_2 \wedge \ldots \wedge P_n$

$\exists x P(x) = P_1 \vee P_2 \vee \ldots \vee P_n$

$\neg \forall x P(x) \equiv \neg(P_1 \wedge P_2 \wedge \ldots \wedge P_n)$

$= (\neg P_1 \vee \neg P_2 \ldots \vee \neg P_n$

$= \exists x \neg P(x)$

* **Negating an existential quantification:**

Consider the statement

There is a student of D.U. who has studied D.M.

$\exists x P(x)$

$P(x)$: x has studied D.M.

$\neg(\exists x P(x)) \equiv \forall x (\neg P(x))$

eg: What are the negation of the statements

i) $\forall x (x^2 > x)$ ii) $\exists x (x^2 = 2)$

$\exists x (x^2 \leq x)$ $\forall x (x^2 \neq 2)$

* **Negating the following statement**

$\forall x \exists y [P(x,y) \wedge Q(y)] \rightarrow \exists x \forall y \neg [P(x,y) \wedge Q(y)]$

$\exists x \forall y [\neg P(x,y) \vee \neg Q(y)]$

Page No.
Date

Bound and Free variable :

A variable x is free iff the variable doesn't occur in the scope of quantifiers

$\underbrace{\forall x P(x)}_{\text{bound}}$ $\qquad$ $\underbrace{\forall x P(x)}_{\text{Bound}} \wedge \underbrace{Q(x)}_{\text{Free}}$

A variable x is bound iff the variable occurs in the scope of quantifiers

eg $\underbrace{\exists x (P(x) \wedge Q(x))}_{\text{bound}} \wedge \underbrace{R(x)}_{\text{free}}$

=> all dogs are blue and all cats are grey

$\forall x (D(x) \Rightarrow B(x)) \wedge \forall y (C(y) \Rightarrow G(y))$

$\forall x (D(x) \Rightarrow B(x)) \wedge \forall x (C(x) \Rightarrow G(x))$

Nested Quantifiers : Two quantifiers are nested if one is within the scope of others

eg $\forall x \exists y (x+y=0)$ where $x, y \in \mathbb{R}$ TRUE

eg $\forall x \forall y ((x>0) \wedge (y<0) \Rightarrow xy<0)$ $x, y \in \mathbb{R}$ TRUE

Date

GROWTH of FUNCTIONS :

What is a function : It is a relationship between input and output OR input to output

$y = 2x + 3$

output variable input varible

36^2
4
(144)

Growth : something that has grown or is increasing in size

Why are we learning growth of a function : because we have to compare the algorithms in the functional form

Suppose you are given an algorithm, what should be the good way to measure it :
1 Time Complexity / Space Complexity

* execution time : not a good measure as it is specific to a particular machine

* Number of Steps : not a good measure as the number of steps depends on the programming style / programming language
~~as well as depends on~~

Ideal Solution : Let us assume that we express the running time of a given algorithm as a function of input size 'n' i.e. f(n) and compare these with different functions corresponding to the running time. This kind of comparisons are independent of machine and programming styles.

Page No.
Date

eg for(i = 1; i ≤ n; i = i×2)
Pf('Hello') $\log n = \log 16 = \lg 2^4 = 4 \lg_2^2 = 4$
4 4 times

* Rate of Growth: It is the rate at which the running time increases as a function of input.

eg: Let us assume that

Commonly used rate of growth:

Function	Name	eg
1/c	constant	adding an elt in front of line
lgn	logarithmic	Binary search
n	linear	linear search
nlgn	linear logarithmic	merge sort
n^2	Quadratic	graph algo
n^3	cubic	matrix multiplication
2^n	exponential	Tower of Hanoi

Types of analysis: To analyse the given algorithm, we need to know about with which type of data/input, the algorithm takes less time (performing well) and vice versa.

Input: 4, 3, 5, 1, 2   n = 5
O(1) = best c. O(n) = worst c.

~~Relevante~~ Case:
best case ≤ average case ≤ worst case

Date

Asymptotic Notation: The asymptotic notations that will permit us to bound the function of "f" for suitably large value of "n". The time and space measured in terms of asymptotic notations only. It ignores the multiplicative and additive constants and concerns only with rate of growth

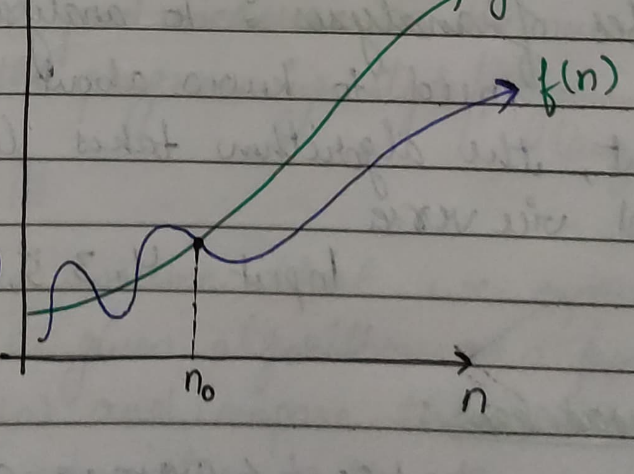
$$f(n) = n^4 + 10n^2 + 100n + 500 \qquad n = 10000$$
$$O(n^4)$$

Worst Case / "O" big O notation
Best Case / "Ω" big Ω notation
Average Case / "θ" big θ notation
small o ("o") notation
small omega ("ω") notation

Worst Case / big O ("O") notation: Let $f(n)$ and $g(n)$ be the functions that map positive integer to positive real numbers. We denote $f(n)$ is $O(g(n))$ or $f(n) \in O(g(n))$

$$O(g(n)) = \{ f(n) : \text{there exists constants } c > 0,\ n_0 \geq 1 \text{ s.t. } 0 \leq f(n) \leq c \cdot g(n)\ \forall\ n \geq n_0 \}$$

$f(n) \leq c \cdot g(n)$
$f(n) = O(g(n))$



Here the function $g(n)$ is upper bound of function $f(n)$

eg

$f(n) = 4n + 3$

$0 \leq f(n) \leq c \cdot g(n)$

$4n + 3 \leq c \cdot n$

$4 \times 3 + 3 \leq 5 \cdot 3 \qquad c = 5, n_0 = 3$

Page No.
Date

eg $f(n) = n^3 + 20n + 1$

c, n_0, $f(n)$

22, 1, n^3

eg Show that $\sum_{i=1}^{n} \lg i$ is $O(n\lg n)$

$$\sum_{i=1}^{n} \lg i = \lg 1 + \lg 2 + \cdots \lg n$$

$$= \lg(1*2*3 \cdots *n)$$

$$= \lg(n!)$$

$$f(n) = \sum_{i=1}^{n} \lg i = \lg(n!)$$

$$0 \le f(n) \le c.g(n)$$

Since we know $n! \le n^n$

$$\Rightarrow \lg(n!) \le \lg(n^n)$$

$$0 \le f(n) \le c.g(n)$$

$$0 \le \lg(n^n) \le cg(n) \text{ i.e } n\lg n \le c(n\lg n)$$

$\underline{n\lg n \le cn\lg n}$ $c \ge 1$, $n_0 = 1$

HW If $f(n) = a_m n^m + a_{m-1} n^{m-1} + \cdots + a_2 n^2 + a_1 n + a_0$
then prove that

$$f(n) = O(n^m)$$

$f(n) = \frac{1}{2} n^2 - 3n$ is $O(n^2)$
find the value of c, $g(n)$ and n_0

$c = 1$, $n = 1$

we know that $f(n) = O(g(n))$

$f(n) = O(n^2)$ | $c > 0$ $n_0 \geq 1$

$f(n) \leq cn^2$

$\frac{1}{2}n^2 - 3n \leq cn^2$

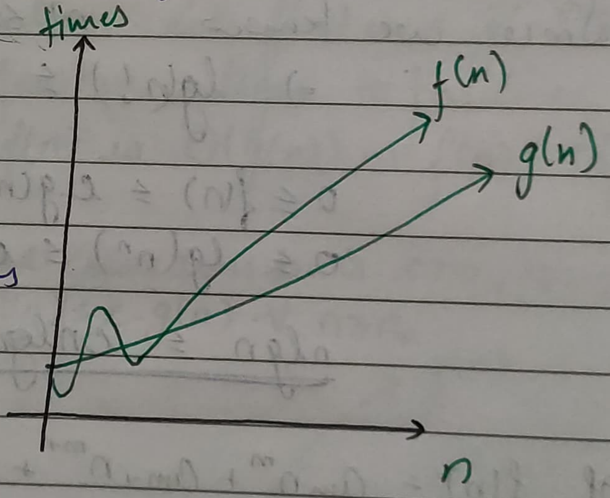
$\frac{1}{2} - \frac{3}{n} \leq c$

for $n = 7$ $c \geq 0$ $\Rightarrow$ $\boxed{c \geq 1/14 \ , \ n_0 = 7}$

§ **Big Omega (Ω) notation / Best case:** Let $f(n)$ and $g(n)$ be functions that map positive integers to the real no.
We denote $f(n)$ is $\Omega(g(n))$ or $f(n) = \Omega(g(n))$

$$\Omega(g(n)) = \left\{ \begin{array}{l} f(n) : \exists \text{ constants } c > 0 \ \& \ n_0 \geq 1 \text{ s.t.} \\ 0 \leq c \cdot g(n) \leq f(n) \end{array} \right.$$

- here the fn $g(n)$ is an asymptotic lower bound for the function $f(n)$
- Include all the functions whose rate of growth is same as or higher than that of $g(n)$



eg: $f(n) = 16$ find the value of c, $g(n)$ & n_0

$f(n) = \Omega(g(n))$

$f(n) \geq c \cdot g(n)$ $c = 13, 12, 15, 14$, $n_0 = 1$

$16 \geq 17 \cdot 1$

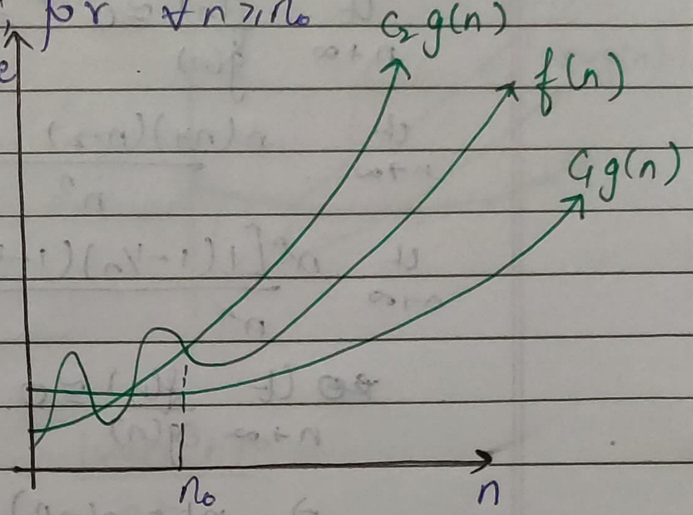
$f(n) = 3n + 5$ $c, g(n), n_0$

$3n + 5 \geq c \cdot n$

$f(n) = \frac{1}{2}n^2 - 3n \qquad g(n) = n^2$

Theta (θ) Notation / Average Case: Let $f(n)$ and $g(n)$ be fn that map positive constants c_1, c_2 and n_0 s.t.

$0 \le c_1 \cdot g(n) \le f(n) \le c_2\, g(n)$; for $\forall n \ge n_0$



eg: $f(n) = 10n^3 + 5n^2 + 17$

find: $c_1, c_2, g(n)$ & n_0

$n_0 = 1, \quad g(n) = n^3, \quad c_1 = 10, \quad c_2 = 32$

eg: Show that $\frac{1}{2}n^2 - 3n$ is $\theta(n^2)$

$c_1 =$

$c_2 =$

$g(n) = n^2$

$n_0 = 7$

* Little (o) notation: Let $f(n)$ and $g(n)$ be functions that map +ve integers to positive real numbers. we define $f(n) = o(g(n))$ s.t. $f(n) < c \cdot g(n)$ $\forall$ $n > n_0$ & $c > 0$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \qquad f(n) = o(g(n))$$

eg $f(n) = 2n \qquad \lim_{n \to 0} \frac{2}{n} = \frac{1}{\infty} = 0$ true

$g(n) = n^2$

* Write a program to Implement Binary Search

* Write a program to Implement the following rules

i) Identity laws $\quad P \wedge T \equiv P$ and $P \vee F \equiv P$

2) Domination laws $\quad P \vee T \equiv T$ and $P \wedge F \equiv F$

3) Double negation laws $\quad \neg(\neg P) \equiv P$

Date

Prove that $n! = o(n)$

$f(n) = n!$ $\quad g(n) = n^n$

$f(n) = o(g(n))$

$f(n) < c \cdot g(n)$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} \cong 0 = \lim_{n \to \infty} \frac{n!}{n^n}$

$\lim_{n \to \infty} \frac{n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1}{n^n}$

$\lim_{n \to \infty} \frac{\cancel{n^n}[1(1-1/n)(1-2/n) \cdots (3/n)(2/n)(1/n)]}{\cancel{n^n}}$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} \cong 0$

$\Rightarrow n! = o(n^n)$

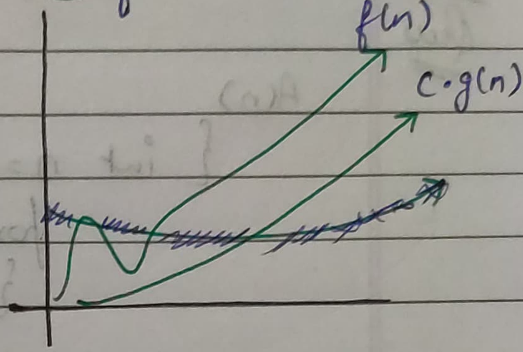
Hw Prove that $n^k = o(2^n)$ $\quad f(n) = o(g(n)) \Rightarrow \lim_{n \to \infty} \frac{f(n)}{g(n)} \cong 0$

$\lim_{n \to \infty} \frac{n^k}{2^n}$

Page No.
Date

Little omega (ω) notation : For a given functions $f(n)$ and $g(n)$ we genote

$$\omega(g(n)) = \{ f(n) : \text{for all +ve integer } n_0 > 0 \text{ s.t. } 0 \le c \cdot g(n) < f(n) \ \forall\ n \ge n_0$$



$$\lim_{n \to \infty} \frac{f(n)}{g(n)} \cong \infty$$

Ques

$f(n) = 100n^3 + 70$

$g(n) = n^2$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} \cong \infty \Rightarrow \frac{100n^3 + 70}{n^2}$$

$$\lim_{n \to \infty} 100n + 70/n^2 \to \infty$$

Hence $f(n) = \omega(g(n))$

eg

$f(n) = n!$

$g(n) = \omega(2^n)$

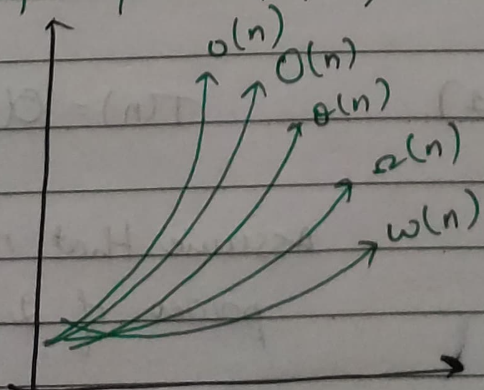
Prove that $f(n) = \omega(g(n))$

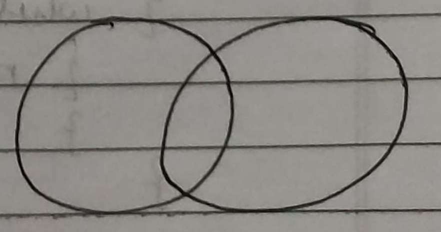
$$\lim_{n \to \infty} \frac{f(n)}{g(n)} \cong \infty \Rightarrow \lim_{n \to \infty} \frac{n!}{2^n}$$

$$\frac{a_{n+1}}{a_n} \quad \frac{(n+1)(\cancel{n!})}{\cancel{2}\cdot 2 \cdot \cancel{n!}} \quad \frac{n+1}{2} \to \infty \text{ as } n \to \infty$$

$$= \frac{n^n (1 - 1/n)(1 - 2/n) \cdots (2/n)(1/n)}{2^n}$$

Relationship b/w O, o, Θ, Ω, ω notations :





Page No. | Date

Best case ≤ Average case ≤ worst case

Ques Compare the order of growth of $\lg_2 n$ and $n!$

Ques Find the Θ bound for $f(n) = \frac{n^2}{2} - \frac{n}{2}$

Ques

```
A(n)
{ int n = 2^2^k;
    for(i=1, i<=n, i++)
    { j=2
      while (i<=n)
      { j=j^2
          pf ("Hi");
      }
    }
}
```

$T(n) = O(?)$

eg

```
A(n)
{ int i, j, n ;
   for (i=1, i<=n, i++)
   { for (j=1, j<=n, j=j+i)
     {    pf("Hello");
     }
   }
}
```

$T(n) = O(?)$

eg

```
A(n)
{ while (n>1)
   { n = n/2
   }
}
```

$T(n) = O(?)$

Assume that n should be power of 2

* $T(n) = 2T(n/2) + n$ $\qquad T(n) = O(n \lg n)$ using masters theorem

Date

$T(n) = 4T(n/2) + \lg n \qquad T(n) = O(n^2)$

$T(n) = 1/2\ T(n/2) + 1/n \qquad T(n) =$

\# $T(n) = aT(n/b) + \Theta(f(n))$

$T(n) = 64T(n/8) - n^2 \lg n \qquad T(n) =$