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Question 6 →

(a) To show: K_6 is non-planar.

So, given vertices is $n=6$

As, we know that;

(i) A K_n complete graph is a non planar iff $n \geq 6$

(ii) If a connected non planar graph G has e edges & v vertices then $3v - e \geq 6$

$$e = \frac{v(v-1)}{2} = \frac{6 \times 5}{2} = 15$$

then, $v = 6$

and $3v - e = 3 \times 6 - 15 = \cancel{12}\ 18 - 15$
$= 3$

& $3v - e = 3 < 6$

i.e. $3v - e \ngeq 6$
$\therefore$ (ii) does not satisfy.

K_6 is non planar by (i) & (ii)

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(b) $a_{n+1} = 3a_n \quad \forall\ n \geq 0,\ a_0 = 2$

$a_n = 3a_{n-1} \quad \forall\ n \geq 1$

then let generating function,

$$G(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$G(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

$$x\,G(x) = \sum_{n=1}^{\infty} a_{n-1} x^n$$

$$a_n - 3a_{n-1} = 0 \quad — \ (1)$$

$$G(x) - 3x\,G(x) = \sum_{n=0}^{\infty} a_n x^n - 3\sum_{n=1}^{\infty} a_{n-1} x^n$$

$$G(x)[1-3x] = a_0 + \sum_{n=0}^{\infty} a_n x^n - 3\sum_{n=1}^{\infty} a_{n-1} x^n$$

$$G(x)(1-3x) = 2 + \sum_{n=1}^{\infty} (0)\,x^n \quad \left(\because \sum_{n=0}^{\infty} (a_n - 3a_{n-1})x^n = 0.x^n\right)$$

By (1)

$$\therefore \boxed{G(x) = \frac{2}{1-3x}}$$

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$\Rightarrow G(x) = \sum_{n=0}^{\infty} 2 \cdot 3^n x^n$

$$\boxed{a_n = 2 \cdot 3^n}$$

(c) Reccurrence $\rightarrow 2T(n/2) + \sqrt{n}$

given, $2T(n/2) + \sqrt{n}$

Master's theorem works for reccurence relation of this form $a(T(n/b)) + f(n)$

where $a \geq 1$ & $b \geq 1$ are constants & $f(n)$ is asymptotically positive function. Master theorems provides three steps –

$T(n) = a(T(n/b)) + \theta(n^k \log^p n)$

case ① $a > b^k$, then $T(n) = \theta(n^{\log_b a})$

② $a = b^k$, case (i) if $p > -1$ then $T(n) = \theta(n^{\log_b a} \log^{p+1} n)$

(ii) if $p = -1$, then $T(n) = 2T(n/2) + \sqrt{n}$

here, $a = 2$, $b = 2$, $k = 1/2$, $p = 0$
from master's theorem, case ① $a > b^k$
$2 > 2^{1/2}$ satisfies

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By condition $a > b^k$

$$T(n) = \theta(n^{\log_b a})$$

putting, $a = 2$, $b = 2$

we get, $T(n) = \theta(n^{\log_2 2})$

$$= \theta(n^1)$$

$$\boxed{T(n) = \theta(n)}$$