

$L = \{a^n b^n : n \geq 1\}$ is not regular.

$$L = \{ab\}$$

$\{ab, aabb, aaabbb, \dots\}$

Step 1: Assume L is regular. Then there is an FA with n states accepting L .

Step 2: Let $w = a^n b^n$. Then $|w| = \underline{2n} > n$ $n \geq 1$
By pumping lemma, we write $w = xyz$ with $|xy| \leq$
and $|y| > 0$.

Step 3 We want to find i so that $xy^i z \notin L$

$|x| \leq n$ (number of states)

$$w = a^n b^n$$
$$|w| = 2n \gg n$$

$$w = xyz$$

$$k \neq 0 \quad \therefore |y| > 0$$

$$\frac{a^{n-k}}{x} \frac{a^k}{y} \frac{b^n}{z}$$

pumped
string

$$a^{n-k} (a^k)^2 b^n$$

$$(i=2)$$

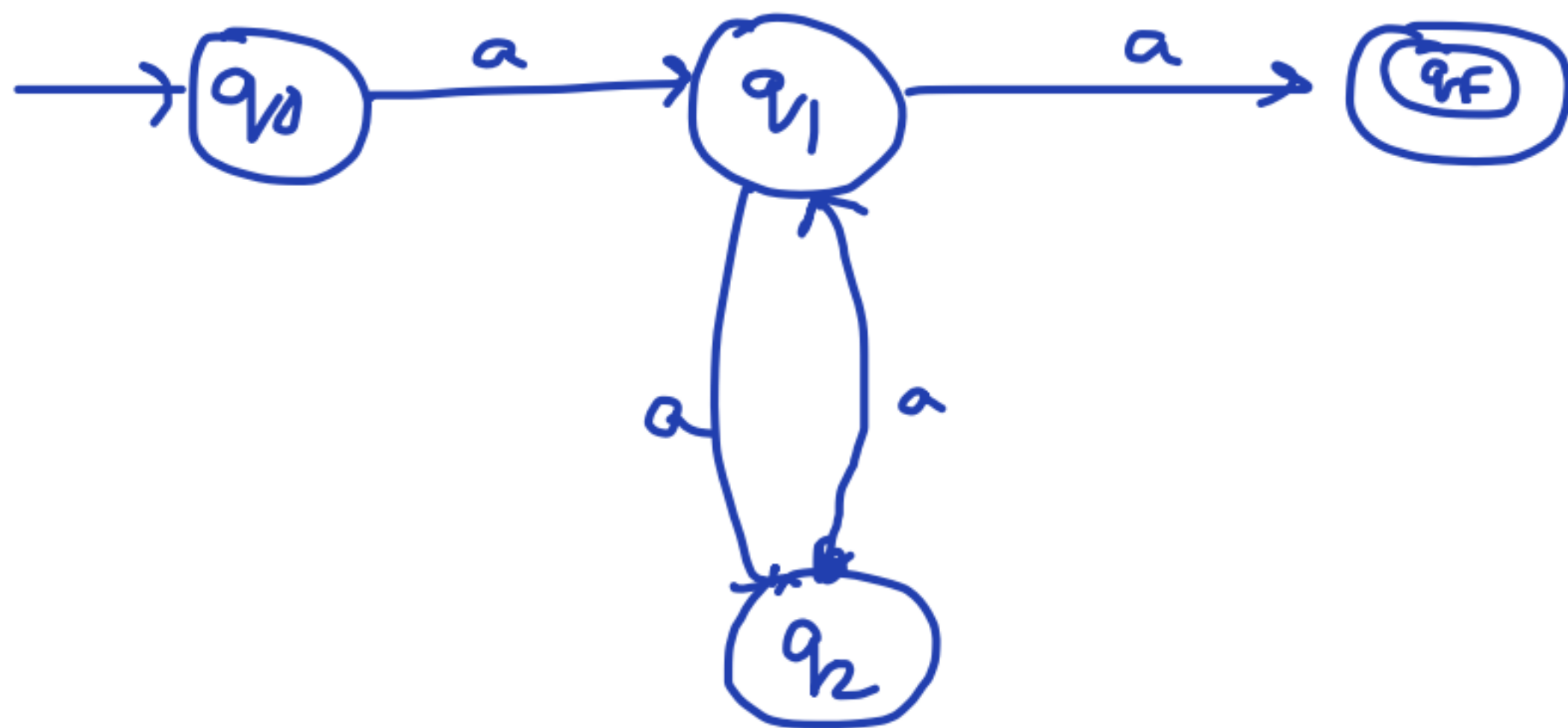
$$\underline{a^{n-k} a^{2k} b^n}$$

$$\underline{a^{n+(k)} b^n} \notin L$$

Contradiction. \therefore language L is non-regular

Δ
 $\{aa, aaaa, aaaaaa, \dots\}$

$$\frac{1}{\sqrt{1-a^2}} (a^2)^i \frac{1}{\sqrt{1-a^2}}, \quad i \geq 0$$



$$\boxed{aa^*bb^*cc^*}$$

$$\underbrace{a^m b^{2n} c^{3p}}$$

$$\underline{aa^* b (bb)^* b ccc (ccc)^*}$$

$$\underline{aa^* bb (bb)^* ccc (ccc)^*}$$

Show that

$L = \{a^p \mid p \text{ is a prime number}\}$ is not regular

Step 1: Assume L is regular. Let n be the number of states in the FA accepting L

Step 2: Let p be a prime number greater than n ($p \geq$

$$|w| = p$$

$$\text{Let } w = a^p$$

By pumping lemma, w can be written

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~~let~~ let $y = a^{(m)}$ for $m \geq 1$ and

$$1 \leq \underline{|y|} \leq n$$

Step 3: ~~let~~ Pump y ($i = p+1$)

$$\underline{|xy^iz|} = |xyz| + |y^{i-1}|$$

$$= p + (i-1)m$$

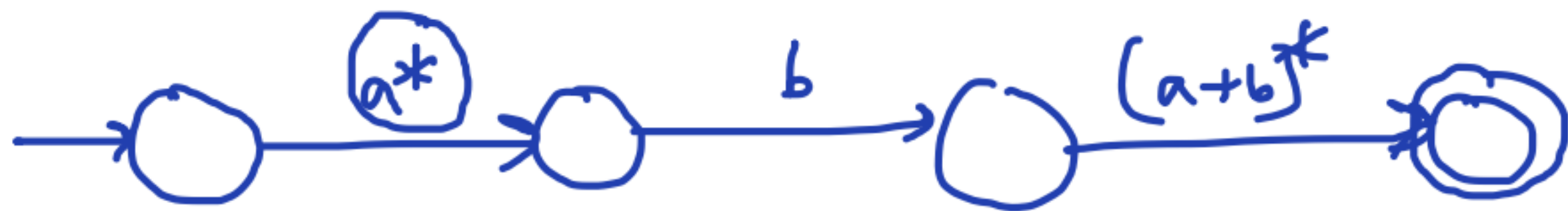
$$= p + pm$$

$$\underline{|xy^iz|} \neq 1$$

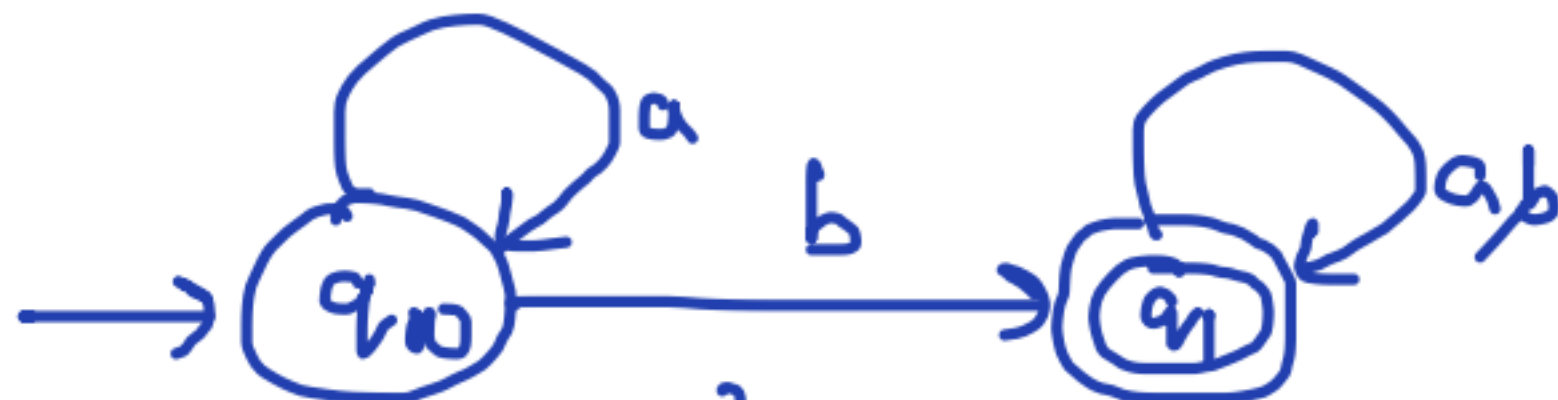
$$= p(1+m) \text{ mod } n$$

hues: L defined by regular expression $\underline{E} = \underline{a^*} \underline{b} \underline{(a+b)^*}$

$$G = (V, T, \underline{S}, P) = \{ \underline{b}, \underline{a}b, \underline{b}a, \underline{b}b, \dots \}$$



NFA
 - edge



all
 edge

FA

$$S \rightarrow \underline{aS}$$

$$S \rightarrow \underline{bS_1}$$

$$S_1 \rightarrow \underline{aS_1}$$

$$S_1 \rightarrow \underline{bS_1}$$

$$S_1 \rightarrow \underline{\epsilon}$$

$$S \rightarrow AbB$$

$$A \rightarrow aA | \epsilon$$

$$B \rightarrow aB | bB | \epsilon$$

$$\underline{aaba}$$

$$G = (\{S, A, B\}$$

$$, \{a, b\}, S, P)$$

$$G = (\{S\}, \{a, b\}, S, P)$$

$$S \rightarrow$$

(5)

To be done

$$\underline{a^*b^*}$$

$$S \Rightarrow a$$

$$\Rightarrow a$$

$$\Rightarrow a$$

$$\Rightarrow a$$

$$S \rightarrow \underline{b}$$

$$S \rightarrow \underline{aS}$$

$$S \rightarrow \underline{Sa}$$

$$S \rightarrow Sb$$