

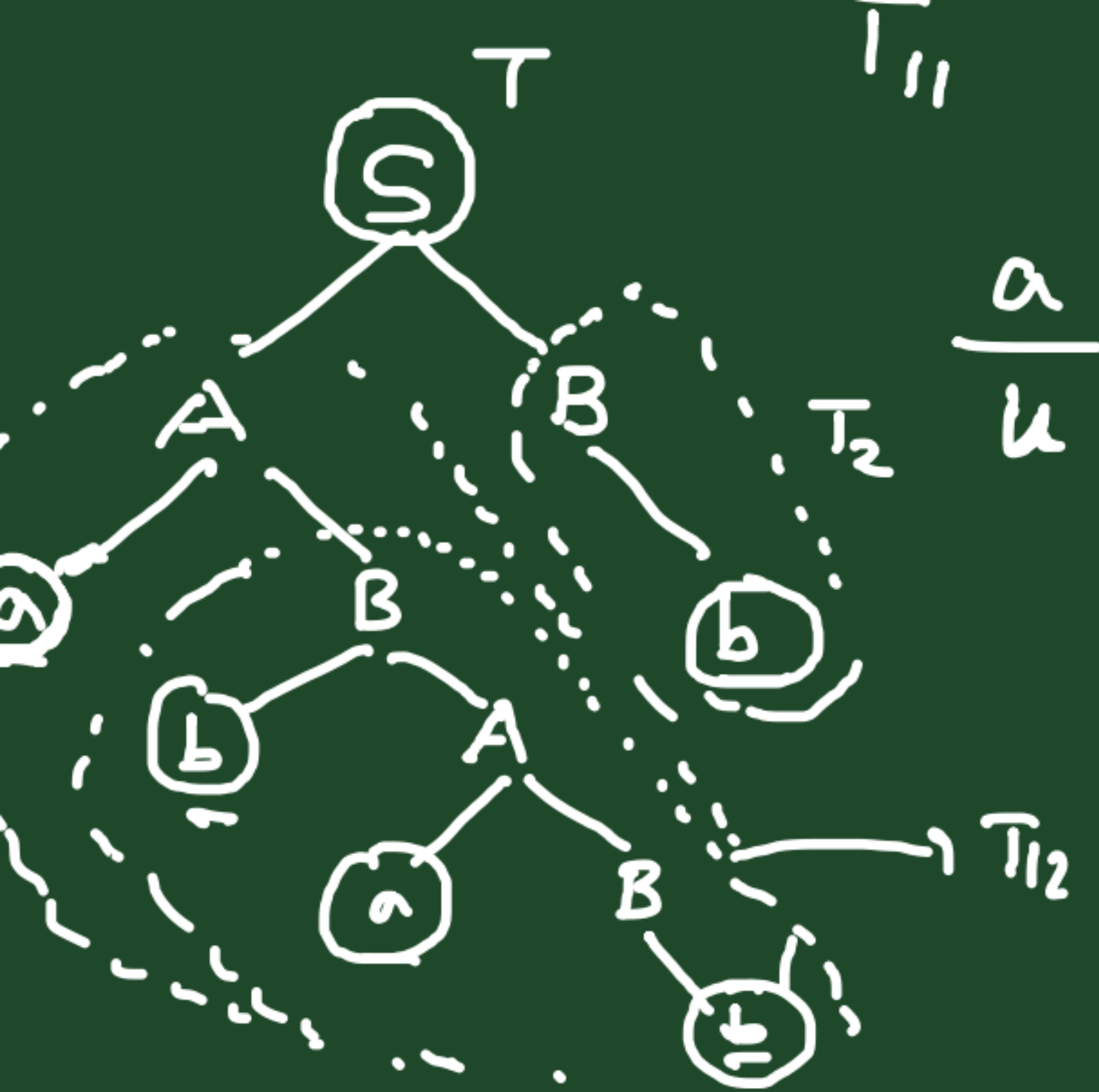
Assumption:

$w \in L(G)$

↓
Pumping Lemma for CFL (Proof of Pumping Lemma)

① $|w| \leq 2^{n-1}$

②



$$z = \frac{u}{a} (z_1) \frac{y}{b}$$

$$\frac{a}{u} \quad \frac{b}{v} \quad \frac{a}{w} \quad \frac{b}{x} \quad \frac{y}{y}$$

$$z_1 = b$$

z is a yield of T

z_1 is a yield of T_{12}

Pumping

Lemma

Infinite
Language

$u v^k w x^k y$

$\Rightarrow a B B$

$\Rightarrow a b A B$

$\Rightarrow a b a B B$

$\Rightarrow a b a$

Written as $u v w x y$ for some strings

u, v, w, x, y

(ii) $|vx| \geq 1$ or $vx \notin E$ or $|vx| > 0$

(iii) $|vwx| \leq n$

(iv) $u v^k w x^k y \in L$ for all $k \geq 0$

Step 2

$$z = a^n b^n c^n$$

$$|z| = 3n > n$$

We break z into u, v, w, x, y str

$$z = uvwx y$$

Step 3

$$|vwx| \leq n \quad \text{and} \quad |vx| \geq 1$$

& b's than number of c's

Case 2

$$\underline{v = a^i b^j} / \underline{x = b^i c^j}$$

ϵ

✓

↓

$$uv^kwx^k \notin L$$

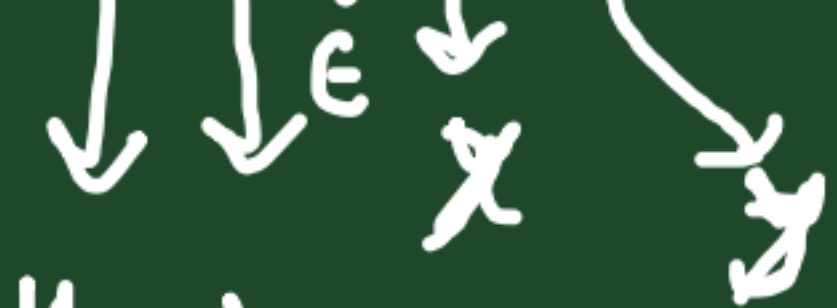
▷ Illegal substring

✓

ϵ

↓

$$a \underline{aa} \underline{abab} \left. \begin{array}{l} bbccc \\ \notin L \end{array} \right\}$$



u v

k=2

$$|vwx| \leq n$$

$$|vx| \geq 1$$

$$\underline{|aab| = 3}$$

Step 2

$$z = a^p$$

$$|z| = p \geq n$$

and we can write

a a a a a a

$$z = uvwx \quad \text{for some string}$$

u, v, w, x, y

$$|vwx| \leq n$$

$$\text{and } |vx| \geq 1$$

$$\underline{|uv^qwxy| = |uwy| + q|x|}$$

$$= q + q\gamma$$

$$\rightarrow \underline{= q(1+\gamma)} \text{ is not a } \text{number}$$

So, $uv^qwxy \notin L$. Assumption was wrong. L is not a CFL.

② Complement

is a CFL and L_2 is a CFL $L_1 \cup L_2$ is a CFL

$$A \rightarrow aA \mid a$$

$$B \rightarrow bB \mid b$$

$$\left. \begin{array}{l} \text{CFL } L_2 \end{array} \right\} \boxed{S \rightarrow \underline{s}, \mid}$$

Rename the NTs of G_1

Rename the NTs of G_2

$$S_1 \rightarrow a s_1 \mid s_1 s_1 \mid A_1 s_1 \mid \epsilon$$

$$A_1 \rightarrow A_1 A_1 \mid b$$

$$S_2 \rightarrow A_2 s_2 \mid s_2 B_2 \mid \epsilon$$

$$A_2 \rightarrow a A_2 \mid a, \quad B_2 \rightarrow b B_2$$

$$S \rightarrow S_1 S_2$$

is a CFL, L^* is a CFL

$$S \rightarrow S_1 S \mid \epsilon$$

$$S \rightarrow a S a \mid b S b \mid a \mid b$$

$$S_1 \rightarrow a S_1 a \mid b S_1 b \mid a \mid b$$