Finite Automata

Dr. Ankit Rajpal

Assistant Professor

Department of Computer Science

University of Delhi

Alphabets

- An *alphabet* is any finite set of symbols.
- Examples: ASCII, Unicode, {0,1} (*binary alphabet*), {a,b,c}.

Strings

- The set of *strings* over an alphabet Σ is the set of lists, each element of which is a member of Σ.
  - Strings shown with no commas, e.g., abc.
- Σ* denotes this set of strings.
- ϵ stands for the *empty string* (string of length 0).

$\{0,1\}^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, . . . \}$

Languages

- A *language* is a subset of Σ^* for some alphabet Σ.
- Example: The set of strings of 0's and 1's with no two consecutive 1's.
- L = {ϵ, 0, 1, 00, 01, 10, 000, 001, 010, 100, 101, 0000, 0001, 0010, 0100, 0101, 1000, 1001, 1010, . . . }

Deterministic Finite Automata

- A formalism for defining languages, consisting of:
  1. A finite set of *states* (Q, typically).
  2. An *input alphabet* (Σ, typically).
  3. A *transition function* (δ, typically).
  4. A *start state* (q_0, in Q, typically).
  5. A set of *final states* ($F \subseteq Q$, typically).
     - "Final" and "accepting" are synonyms.

The Transition Function

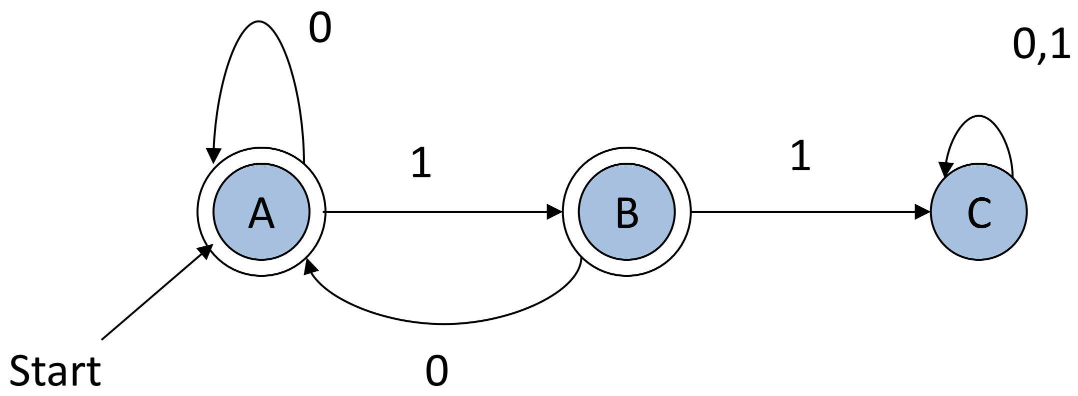
- Takes two arguments: a state and an input symbol.
- $\delta(q, a)$ = the state that the DFA goes to when it is in state *q* and input *a* is received.

Graph Representation of DFA's

- Nodes = states.
- Arcs represent transition function.
  - Arc from state p to state q labeled by all those input symbols that have transitions from p to q.
- Arrow labeled "Start" to the start state.
- Final states indicated by double circles.

Example: Graph of a DFA

Accepts all strings without two consecutive 1's.



Alternative Representation: Transition Table

Final states starred

Arrow for start state

Columns = input symbols

	0	1
* A	A	B
* B	A	C
C	C	C

Rows = states

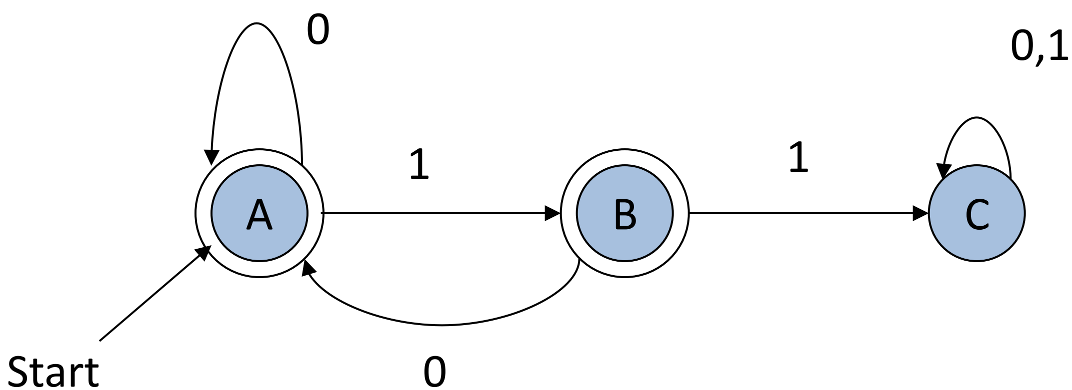
Language of a DFA

- Automata of all kinds define languages.
- If A is an automaton, L(A) is its language.
- For a DFA A, L(A) is the set of strings labeling paths from the start state to a final state.
- Formally: L(A) = the set of strings w such that $\delta(q_0, w)$ is in F.

Example: String in a Language

String 101 is in the language of the DFA below.
Start at A.

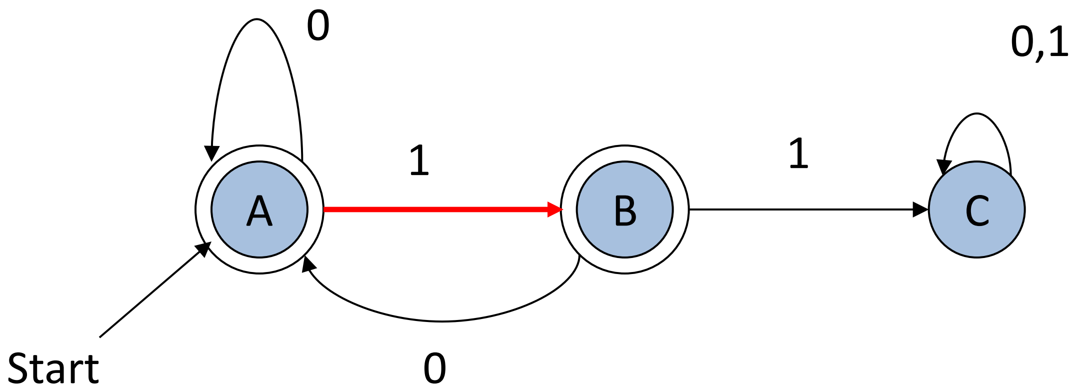
$(0 + 10)^* (1 + \text{epsilon})$



Example: String in a Language

String 101 is in the language of the DFA below.

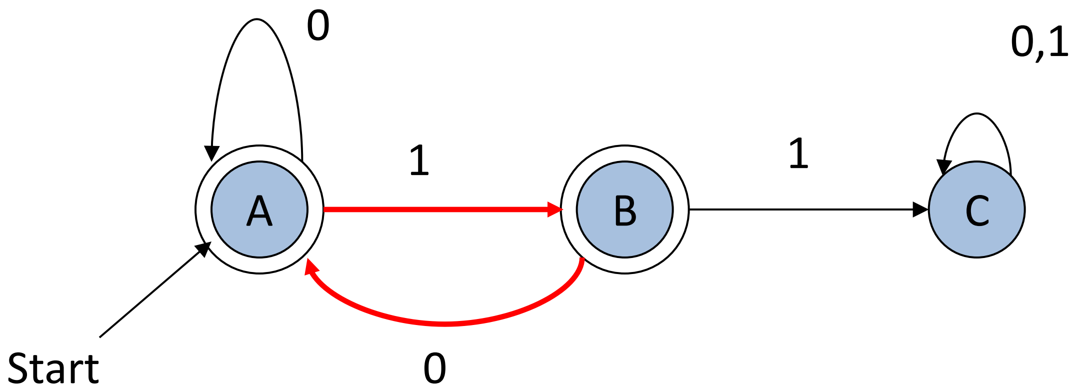
Follow arc labeled 1.



Example: String in a Language

String 101 is in the language of the DFA below.

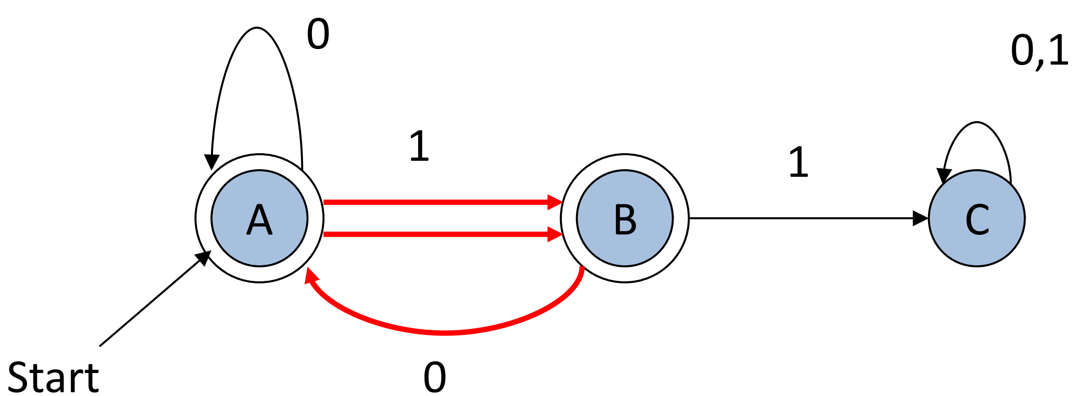
Then arc labeled 0 from current state B.



Example: String in a Language

String 101 is in the language of the DFA below.

Finally arc labeled 1 from current state A. Result is an accepting state, so 101 is in the language.



Example – Concluded

- The language of our example DFA is:

{w | w is in {0,1}* and w does not have two consecutive 1's}

Such that…

These conditions about w are true.

Read a *set former* as "The set of strings w…