Arden's Theorem

$\Sigma = \{a, b\}$

1) Simplification of RE

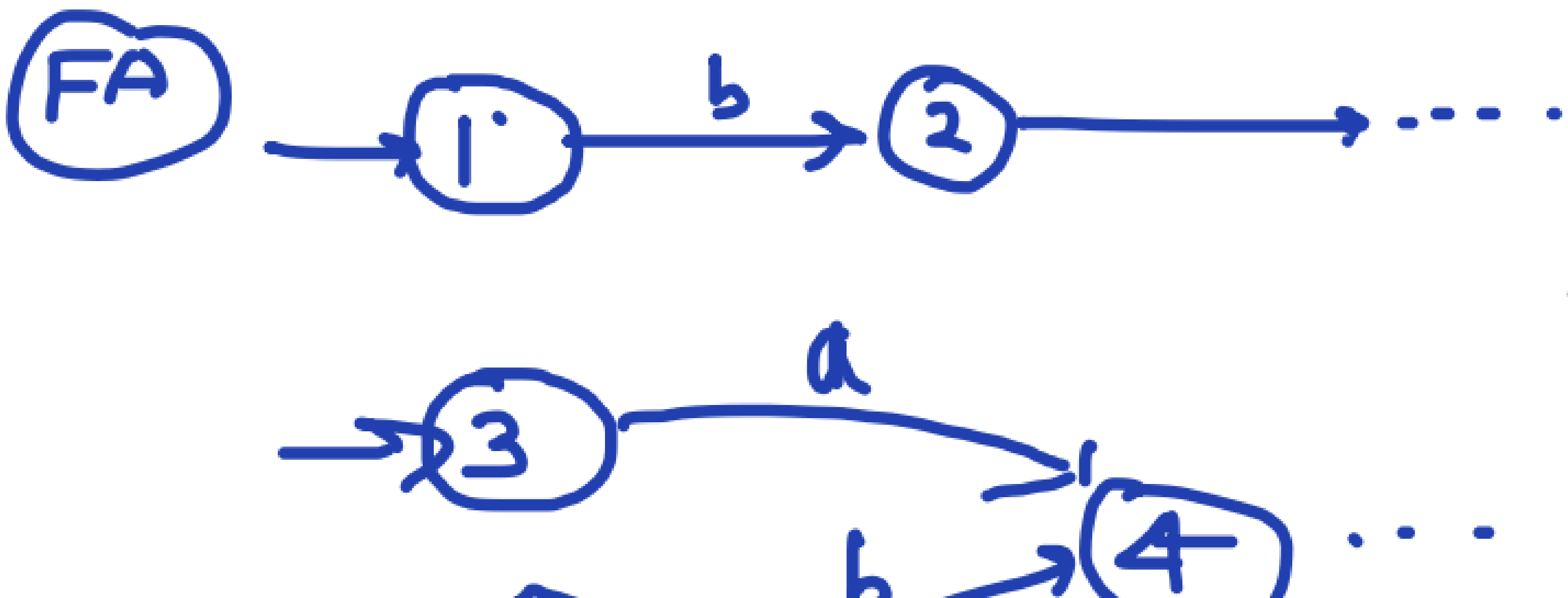
2) FA to RE

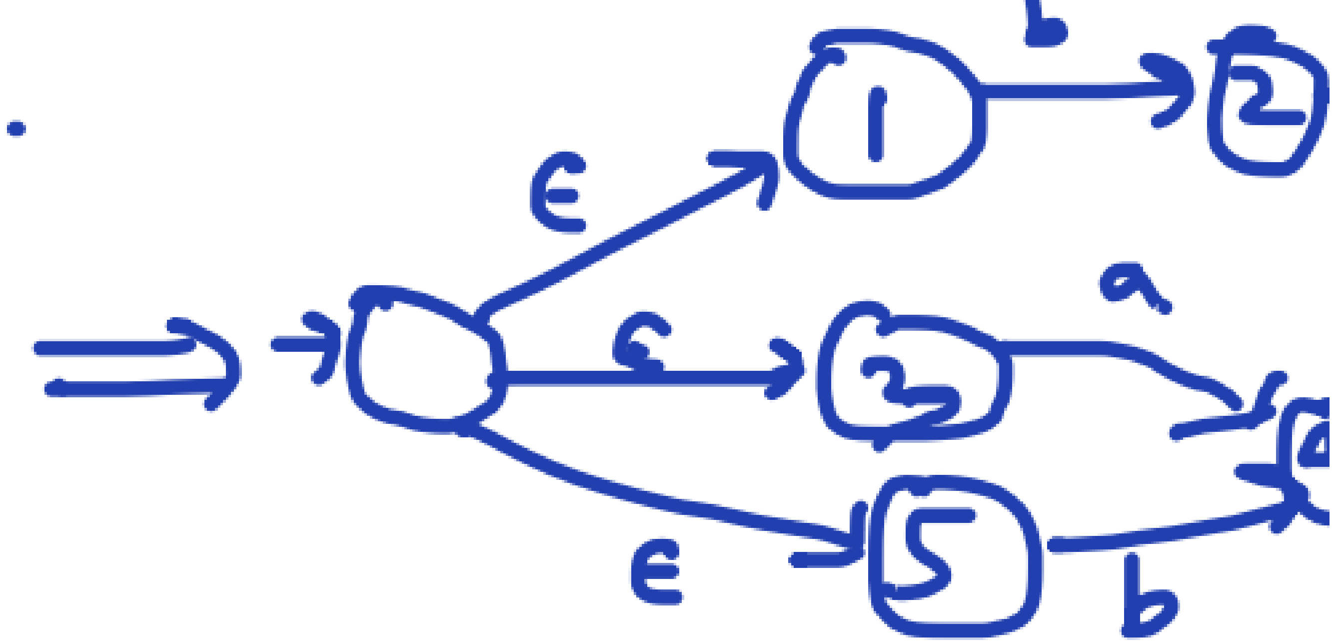
① More than one Initial State

② It allows substring

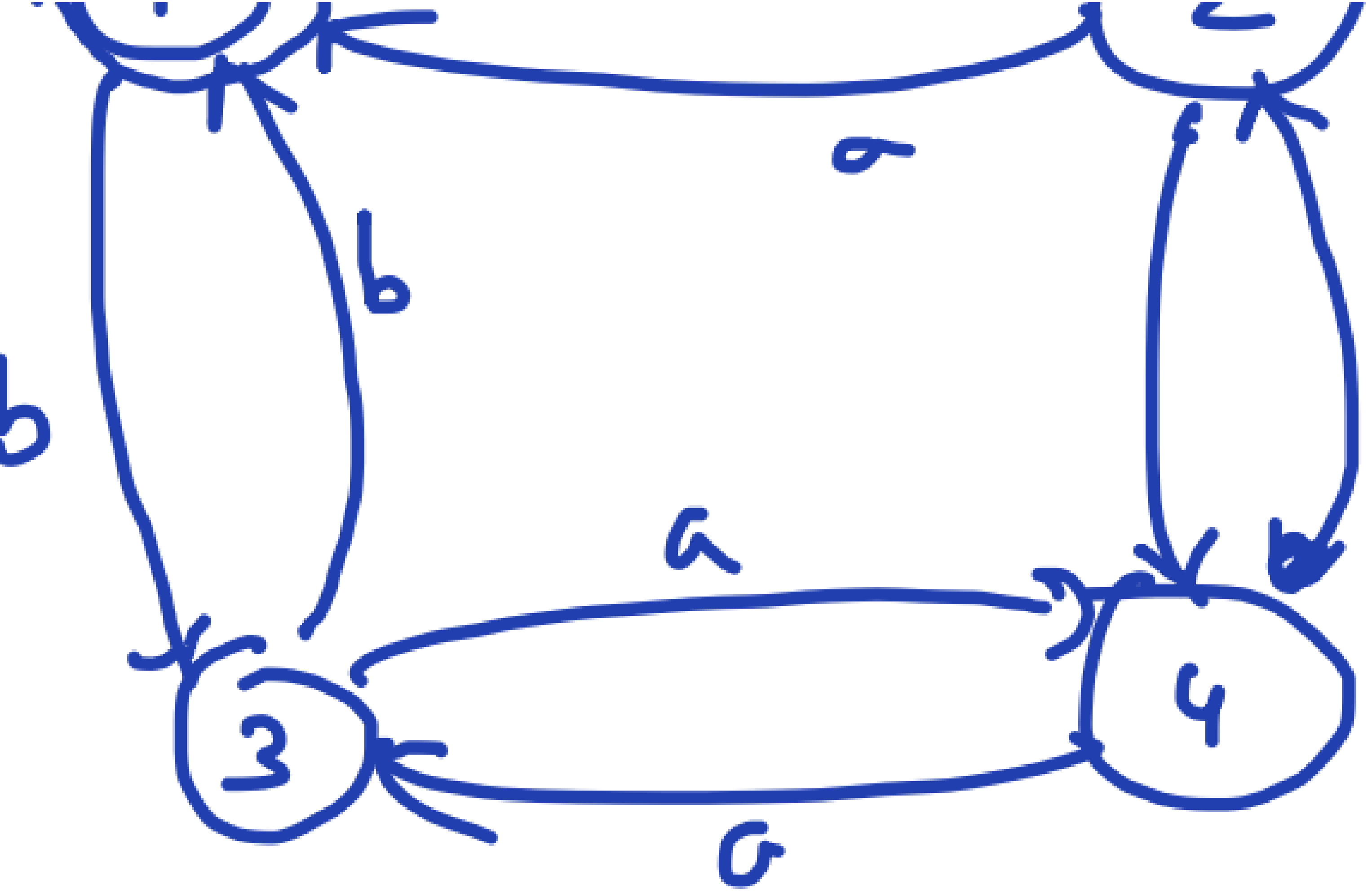
FA to RE

① Unique Non-Reenterable Initial State



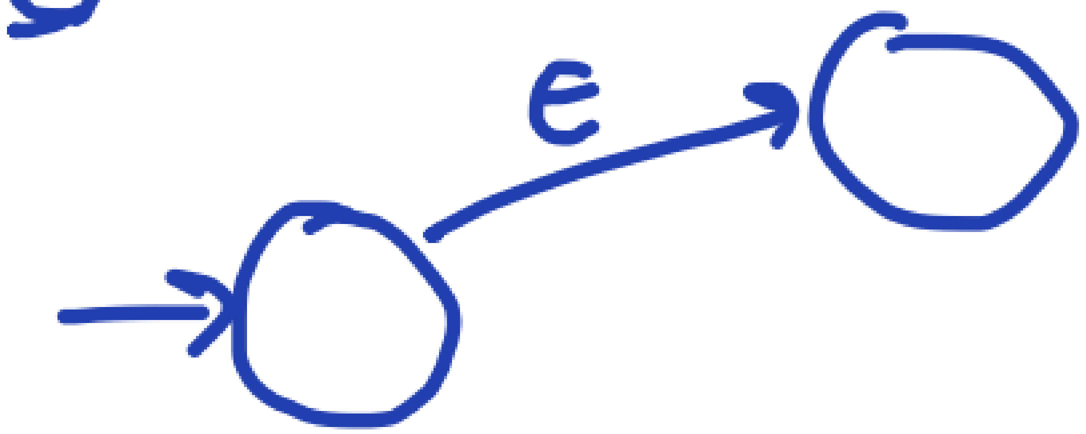


TG
to
RE



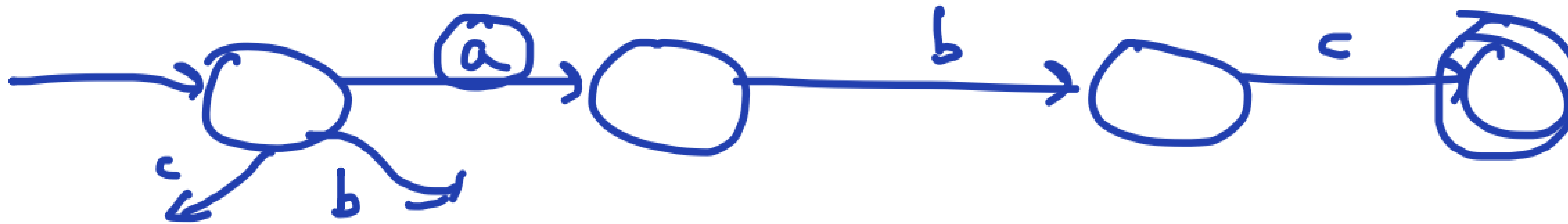
Re-entering Initial state

TG



FA to RE

Uni



TG

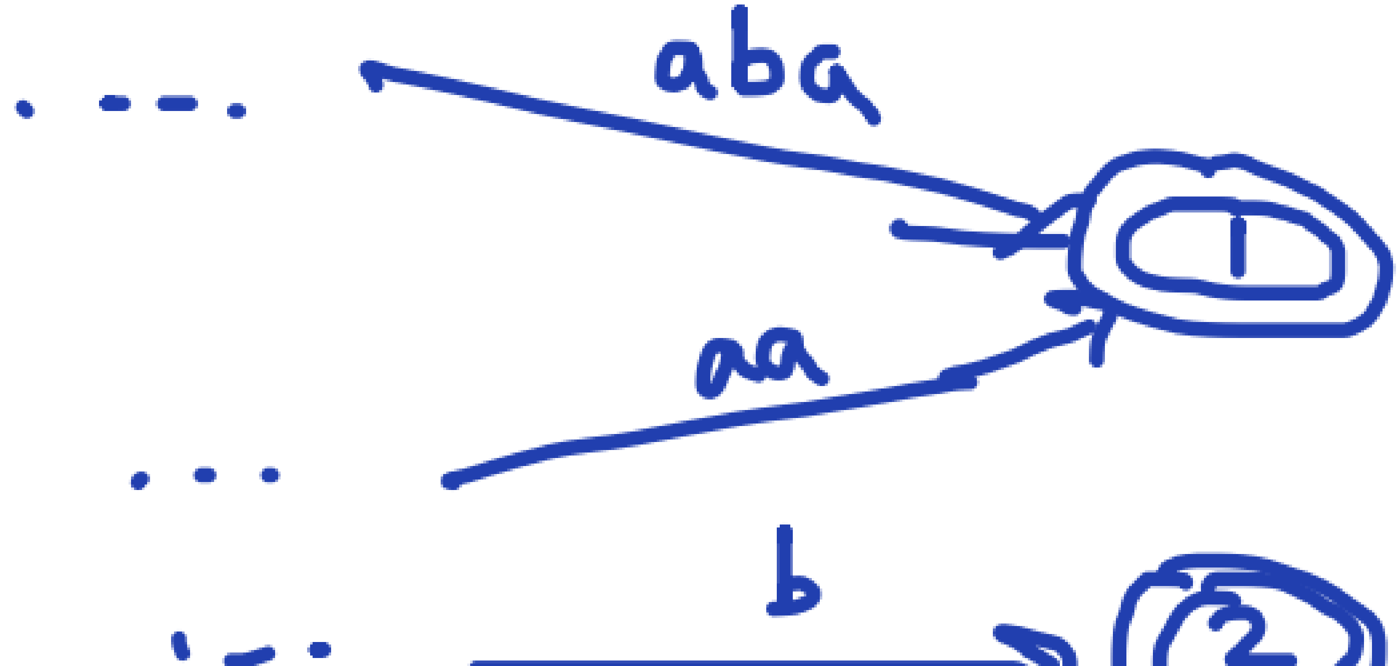
{abc}

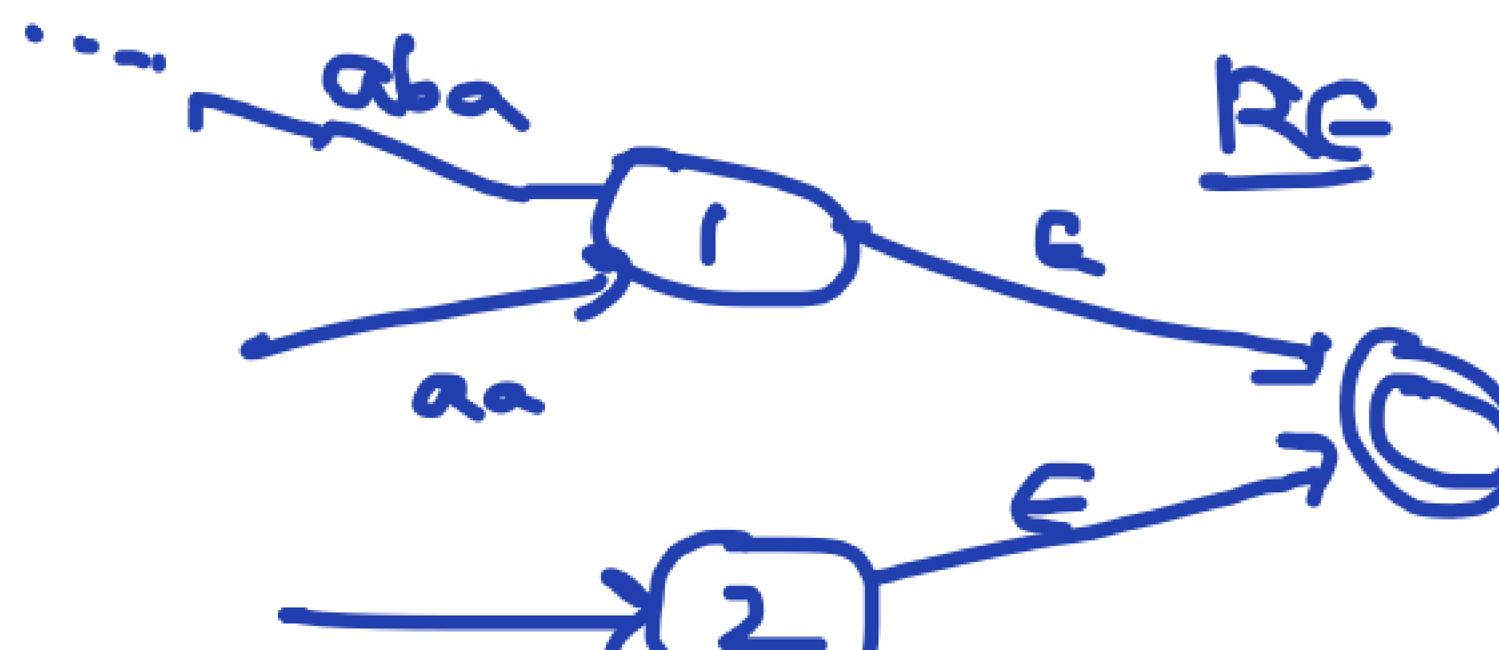
Unique Unleavable final state

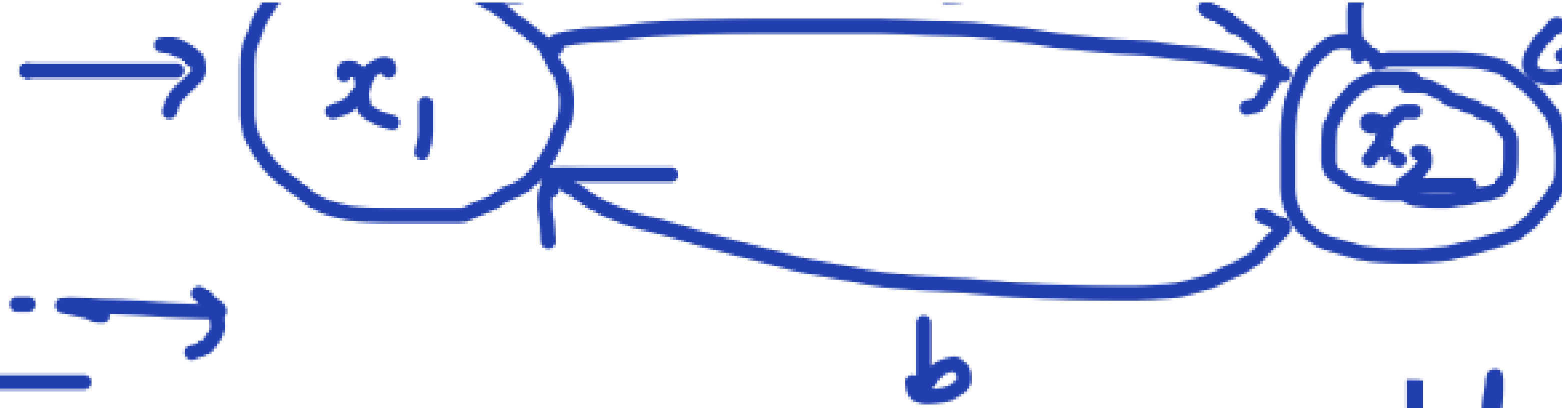
FA

to

RE







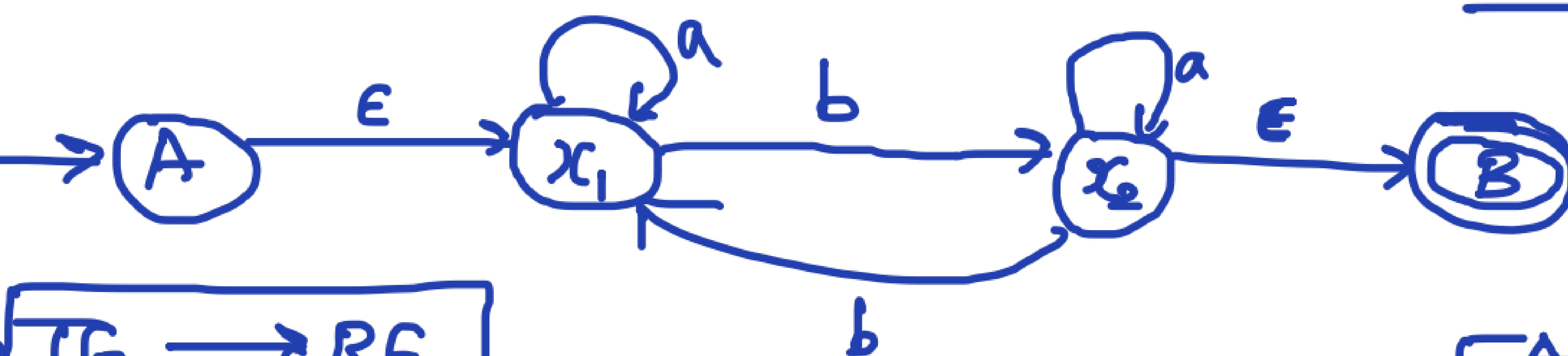
FA →

RE

Initial State

odd no. of b's

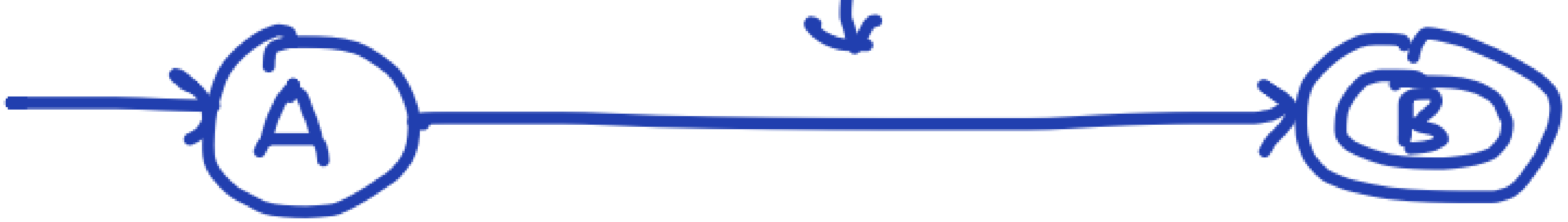
2 Unique unleavable final state

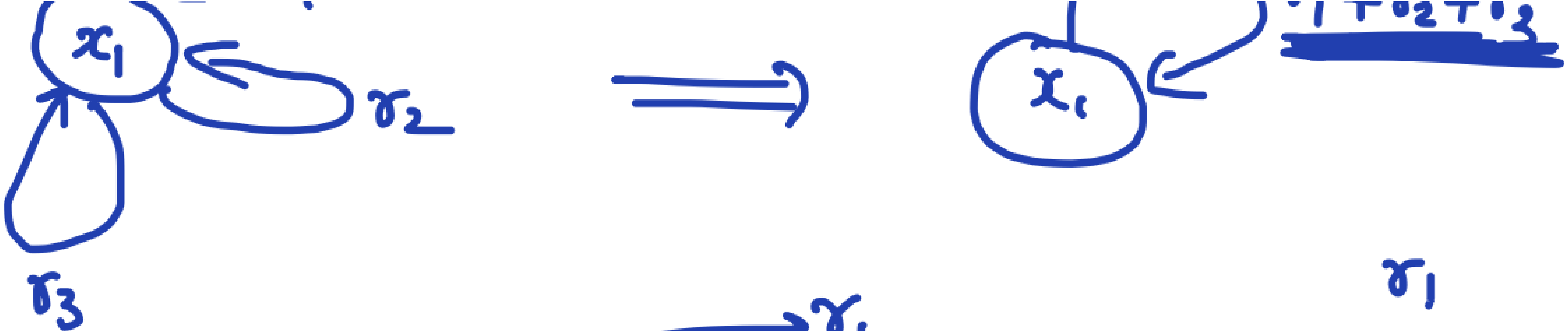


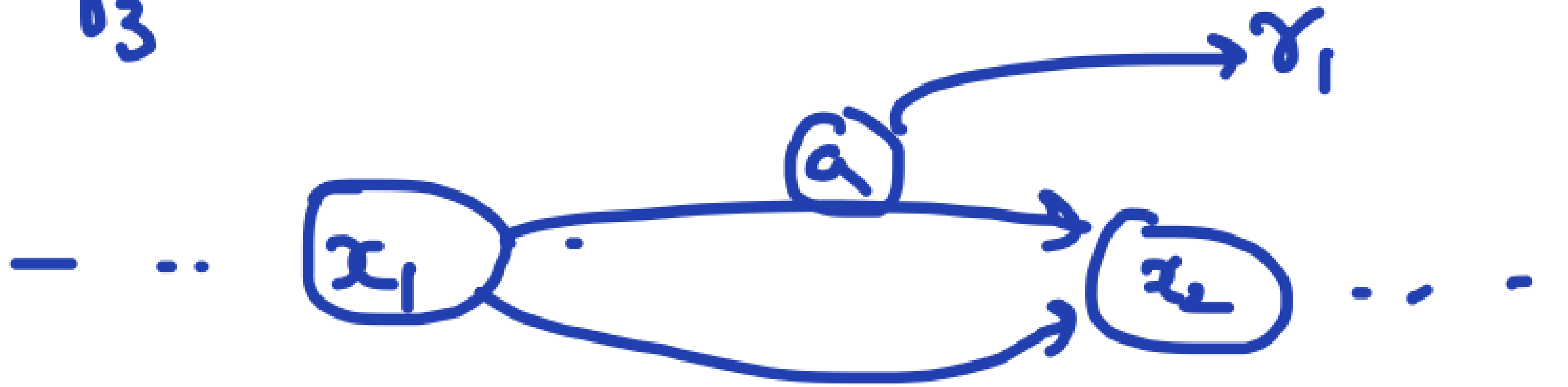
$\boxed{TG \longrightarrow RE}$

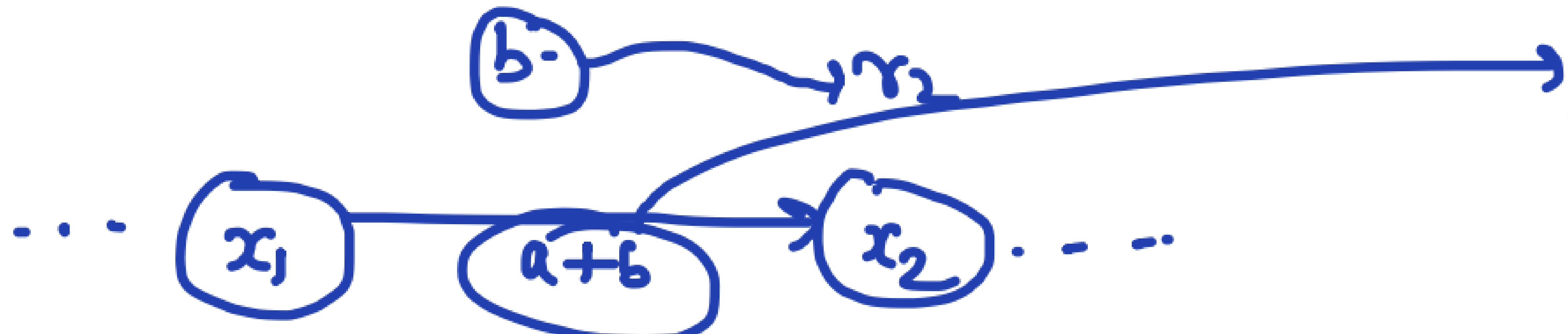
FA to RE

RE









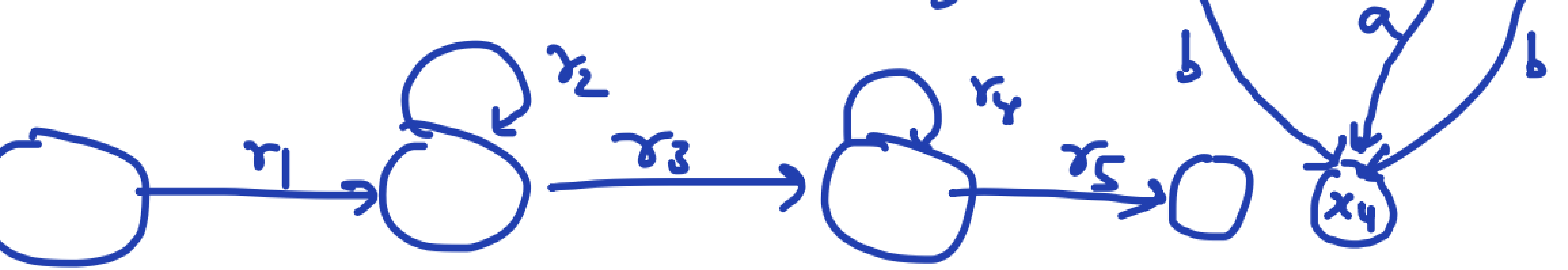
regular expression

$r_1 + r_2$

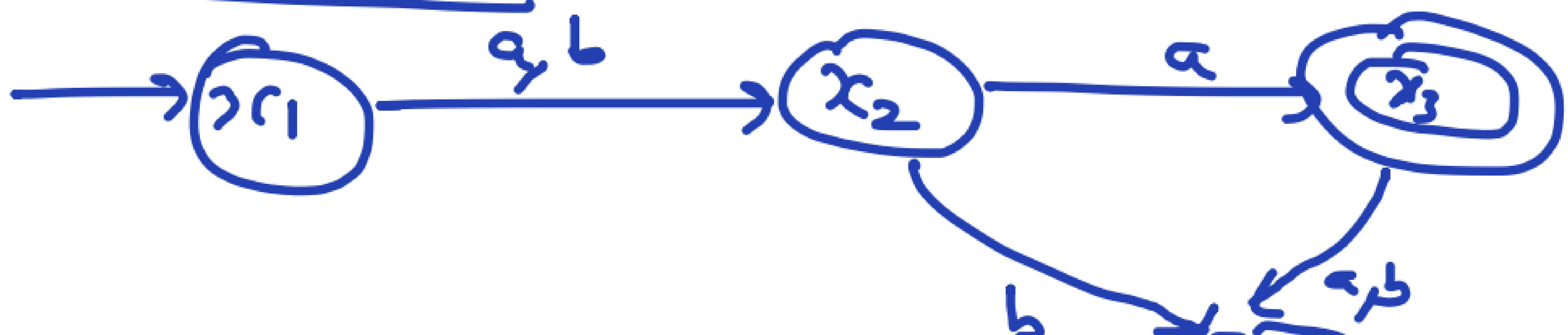
Bypass 2

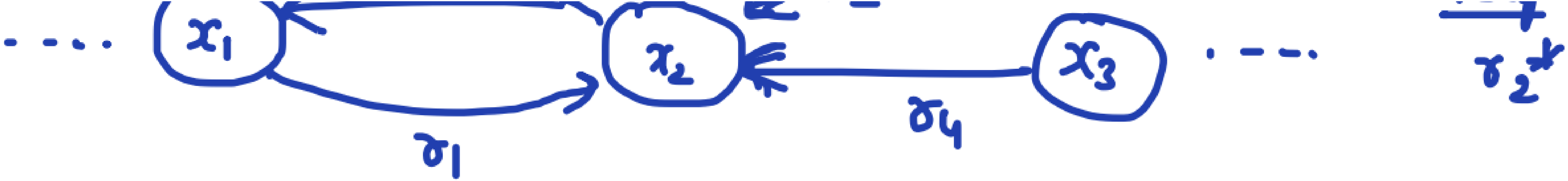
$(a+b)\ a$ $\quad L = \{aa, ba\}$

ϕ

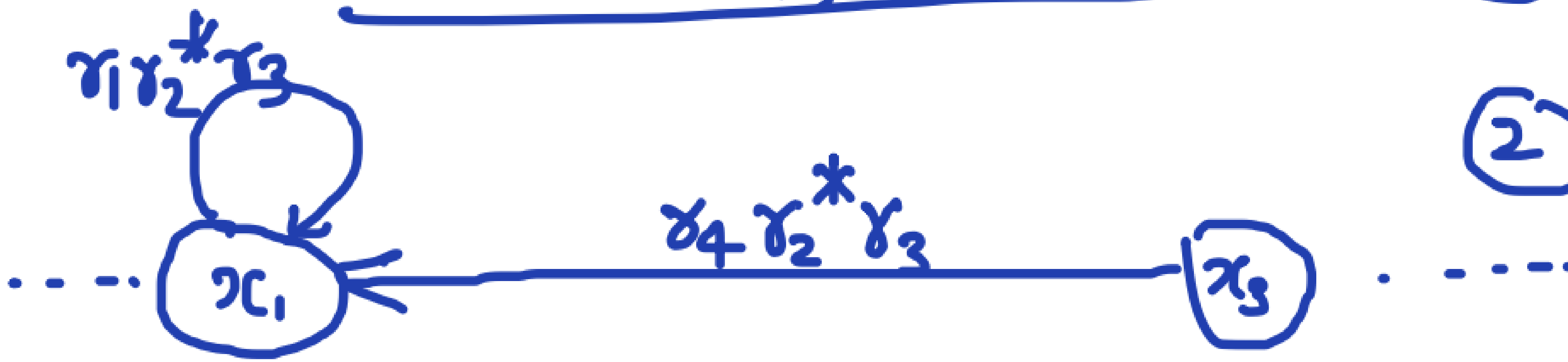


$r_1 r_2^* r_3 r_4 r_5$



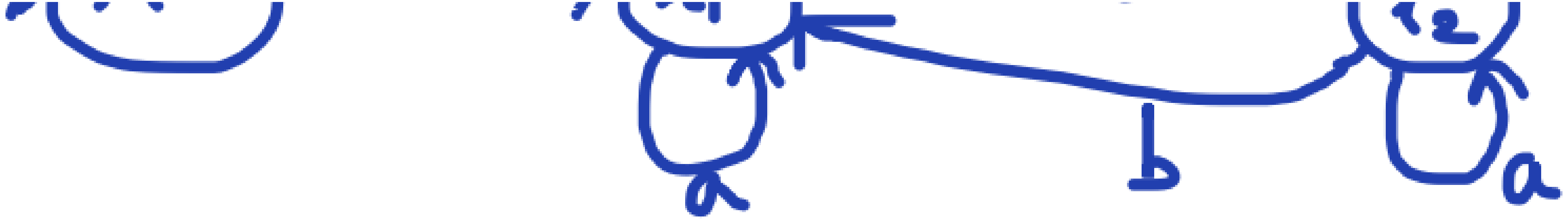


Remove or Bypass x_2



1 path from x
to x_1 via

2 path from x
to x_1 via

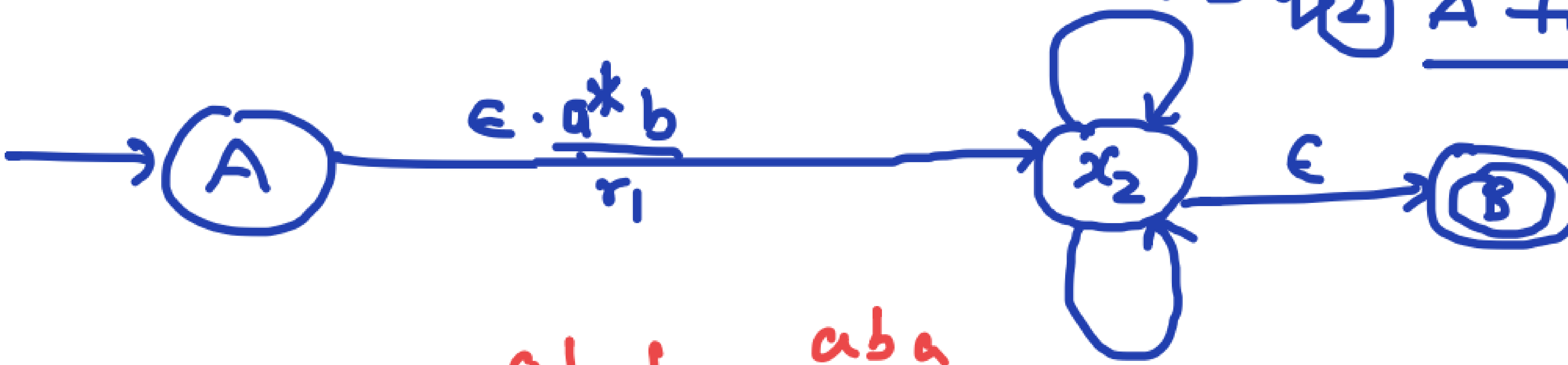


$\boxed{\epsilon \cdot r_1 = r_1}$

Bypass x_1

q(1) x_2 to x_2 via x_1

q(2) A to x_2 via x_1



r_1

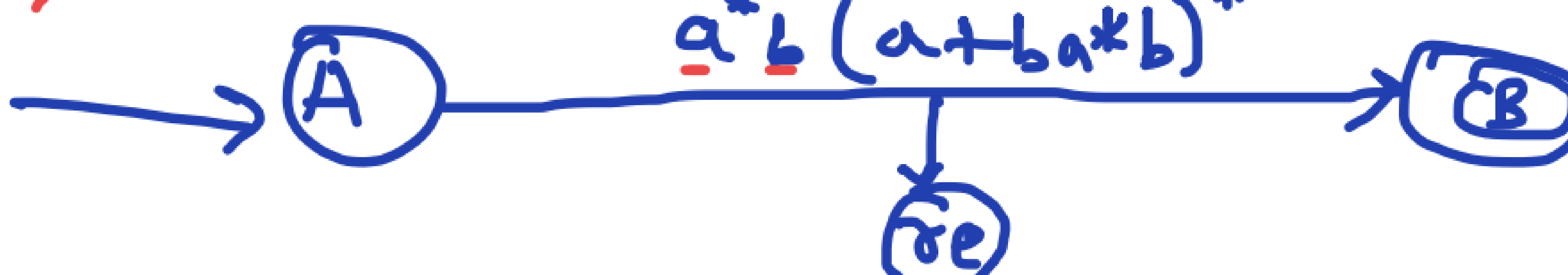
$a+ba$

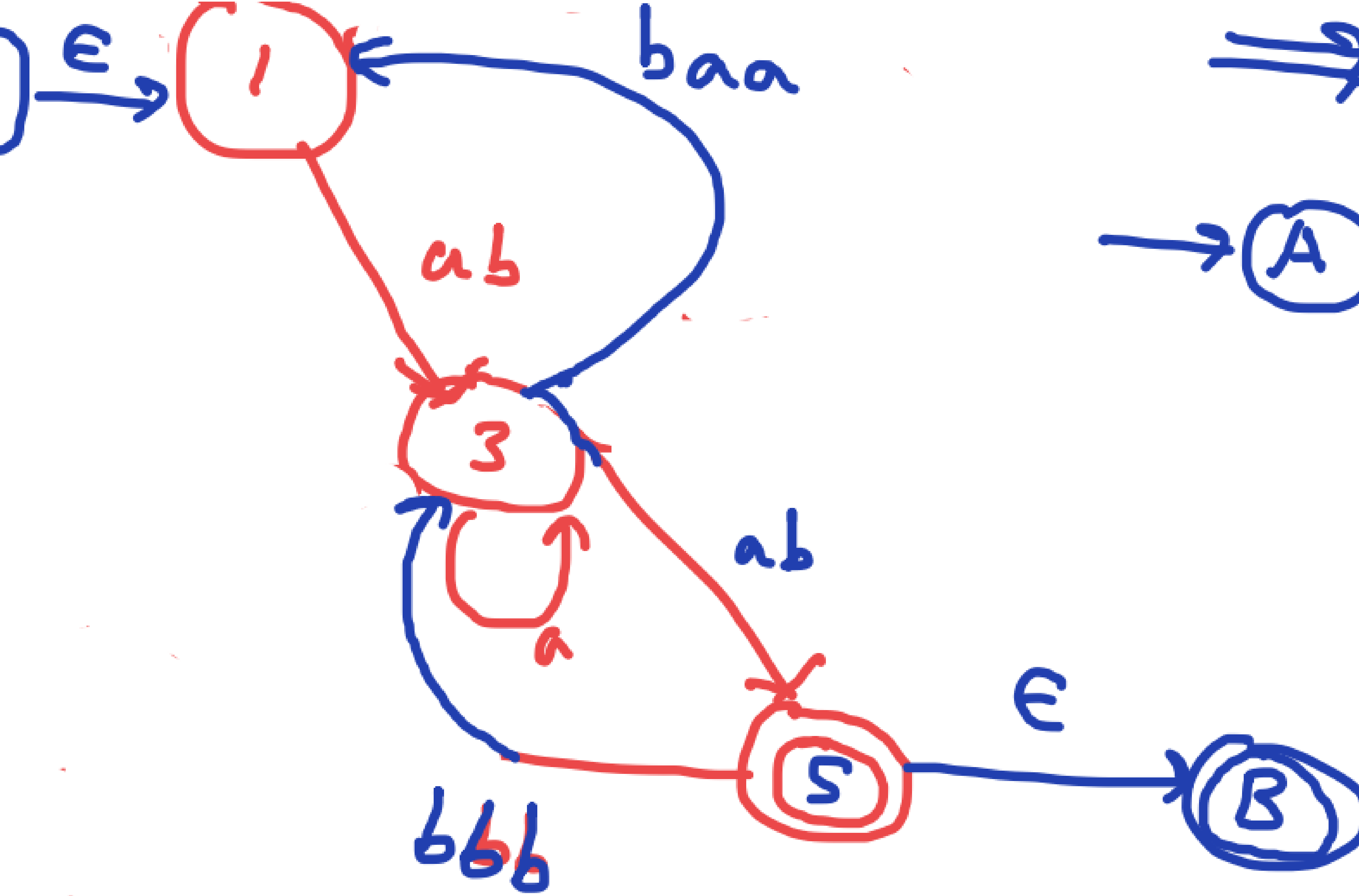
ab, ba

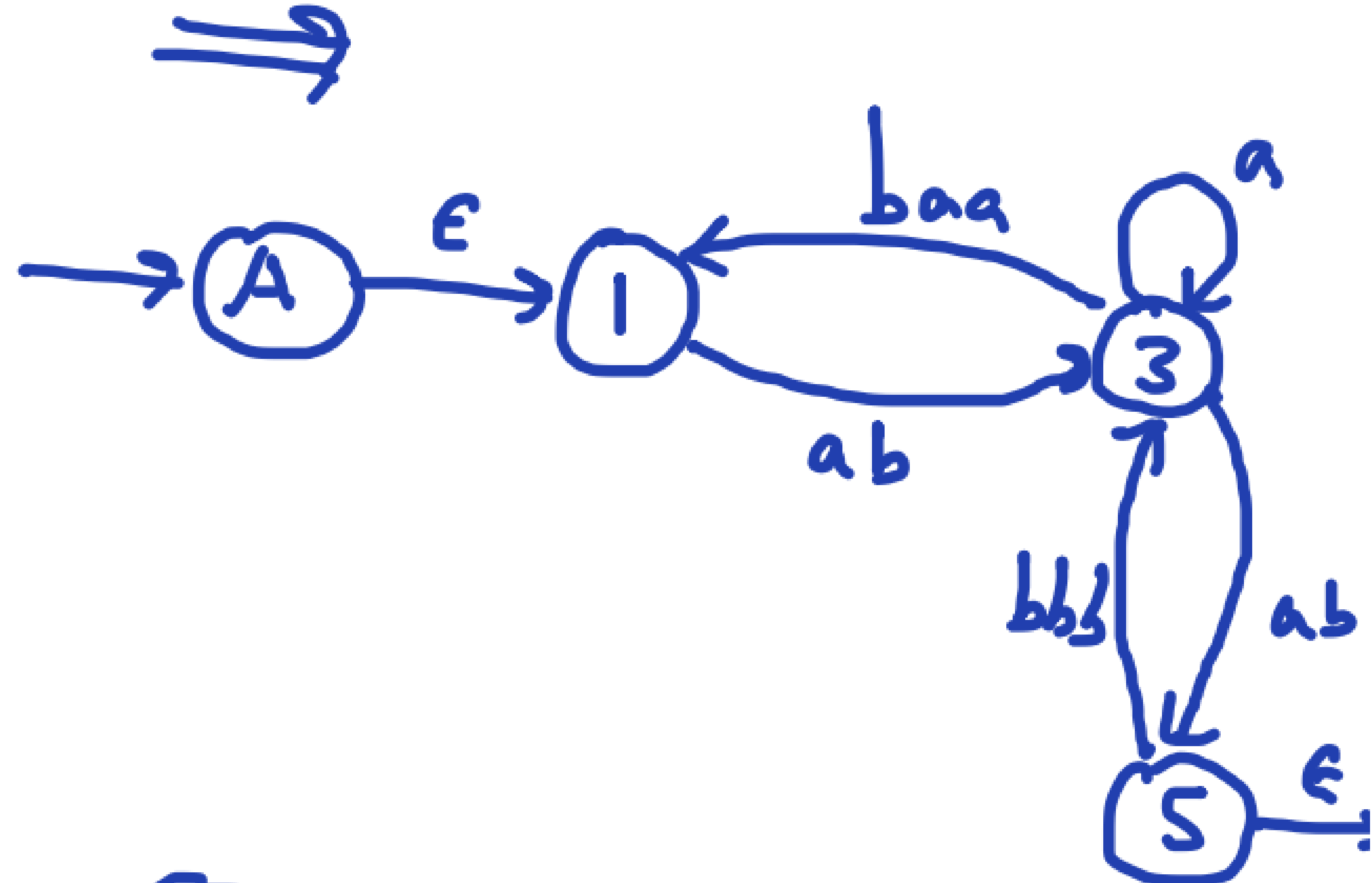
aba
aab
baa bbb

Step2 Bypass x_2

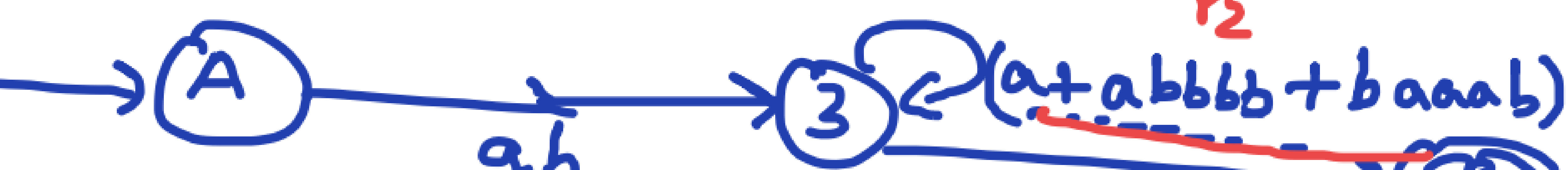
$= \{b, ab, ba,$

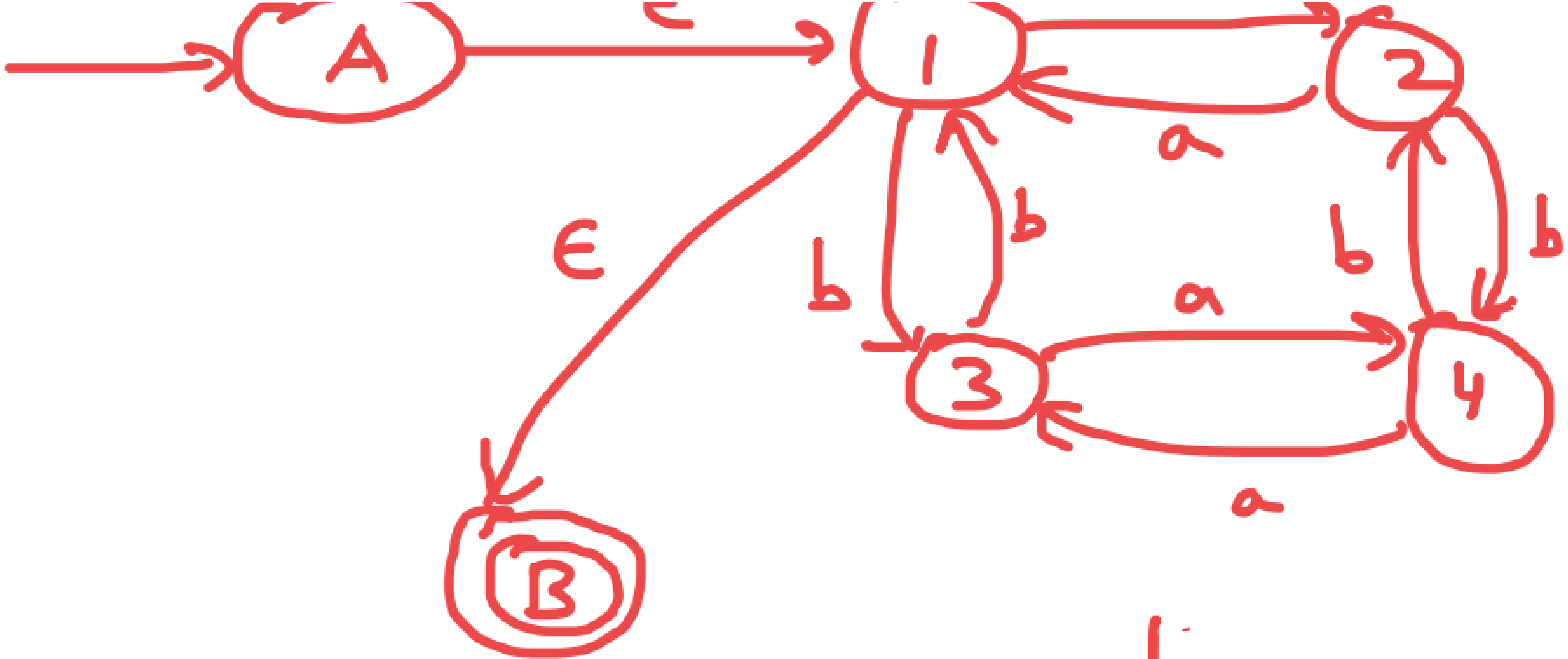






$(a+abbbbb+baaab)^*ab \rightarrow$ RE





$$\left[\underline{aa+bb} + (ab+ba)(aa+bb)^*(ab+ba)\right]$$

2

immediately followed by at least two 1's.

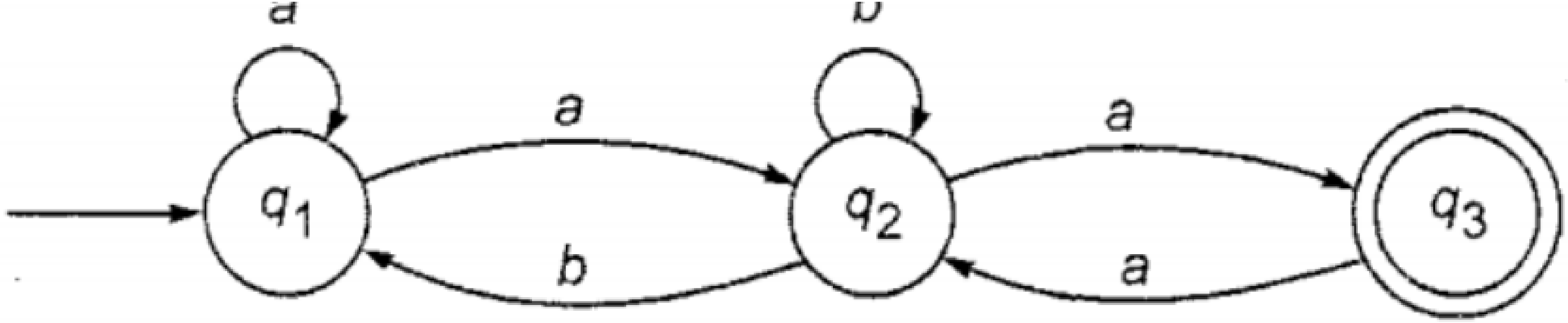
Prove that the regular expression $\mathbf{R} = \Lambda + \mathbf{1^*(011)^*(1^*\ (011)^*)^*}$ describes the same set of strings.

$$R = \epsilon + \underline{1^*(011)^*}(\underline{1^*(011)^*})^*$$

$$\epsilon + RR^* = R^*$$

$$= (\underbrace{1}_{P}{}^*(\underbrace{011}_{Q})^*)^*$$

$$= (1 + 011)^*$$



$= q_1(a + a(b+aa)^* b)$

$R = q_1 , Q = \epsilon$

$, P = a + a(b+$

$q_1 = q_1 a + q_2 b + \epsilon$ — (1)

$q_2 = q_1 a + q_2 b + q_3 a$ — (2)

$q_3 = q_2 a$ — (3)

Equations

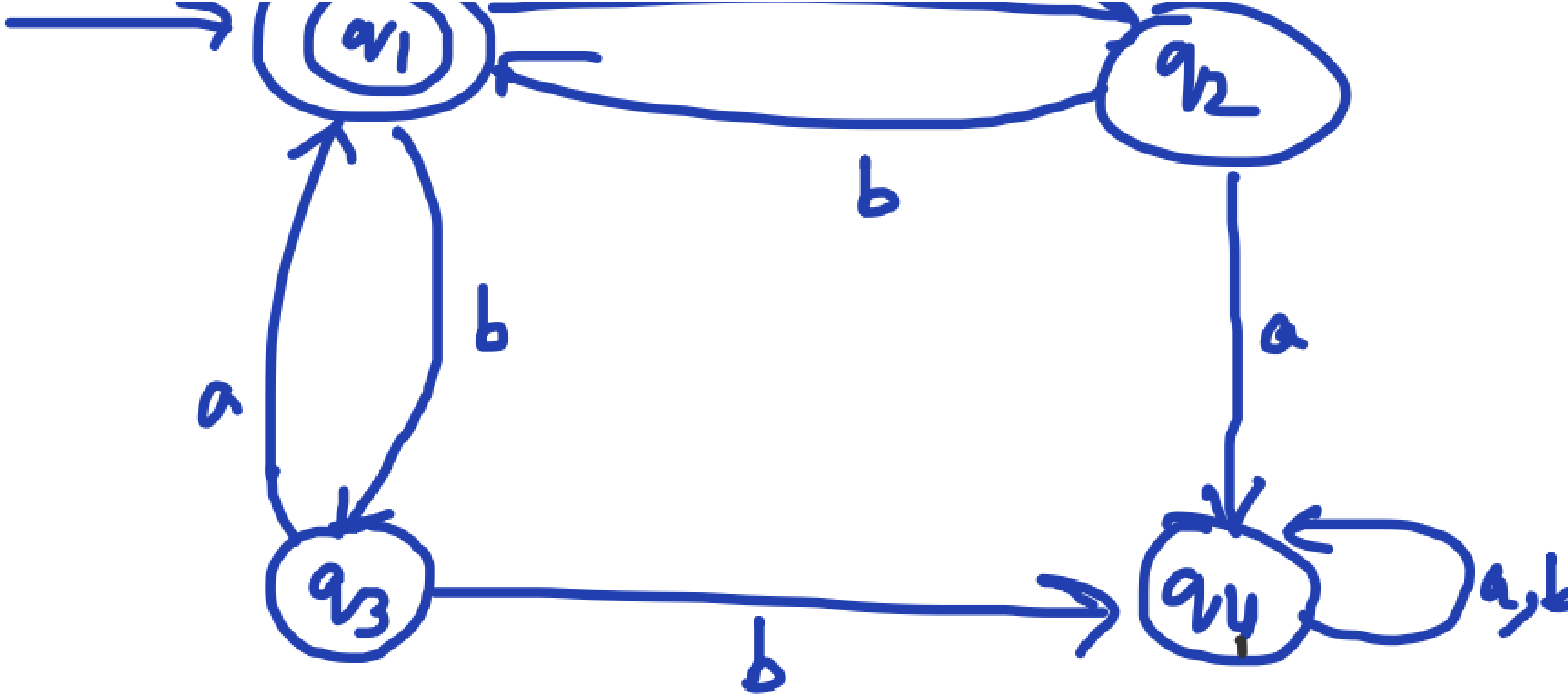
Substituting q_3 in q_2

$q_2 = q_1 a + q_2 b + q_2 a a$

$q_2 = q_1 a + q_2 (b + aa$

$R = Q + RP$

$\Longrightarrow R = QP^*$



FA

$$q_1 = q_2 b + q_3 a + \epsilon$$

$$q_2 = q_1 a$$

$$q_3 = q_1 b$$

$$q_4 = q_2 a + q_3 b + q$$

$$+ q_4$$

$$q_1 = q_1 ab + q_1 ba + \epsilon$$

$$q_1 = q_1 (ab + ba) + \epsilon$$

$$q_1 = (ab + ba)^*$$

$$q_1 = q_1(a + a(b+aa)^*b) + \epsilon$$

$$q_1 = \epsilon\,(a + a\,(b+aa)^*b)^*$$

$$= (a + a\,(b+aa)^*b)^*$$

$$q_1 = (a+a(b+aa)^*b)^*$$

$$q_2 = \underline{(a+a(b+aa)^*b)^*}\; a\,\underline{(b+aa)^*}$$

$$\boxed{q_3 = (a+a(b+aa)^*b)^*\; a\,(b+aa)^*a}$$

Gives FA which accepts all the binary strings $\Sigma = \{0,1\}$ divisible by 3

11

$1 \% 3 = 1$ 01

$2 \% 3 = 2$ 10

$4 \% 3 = 1$ 100

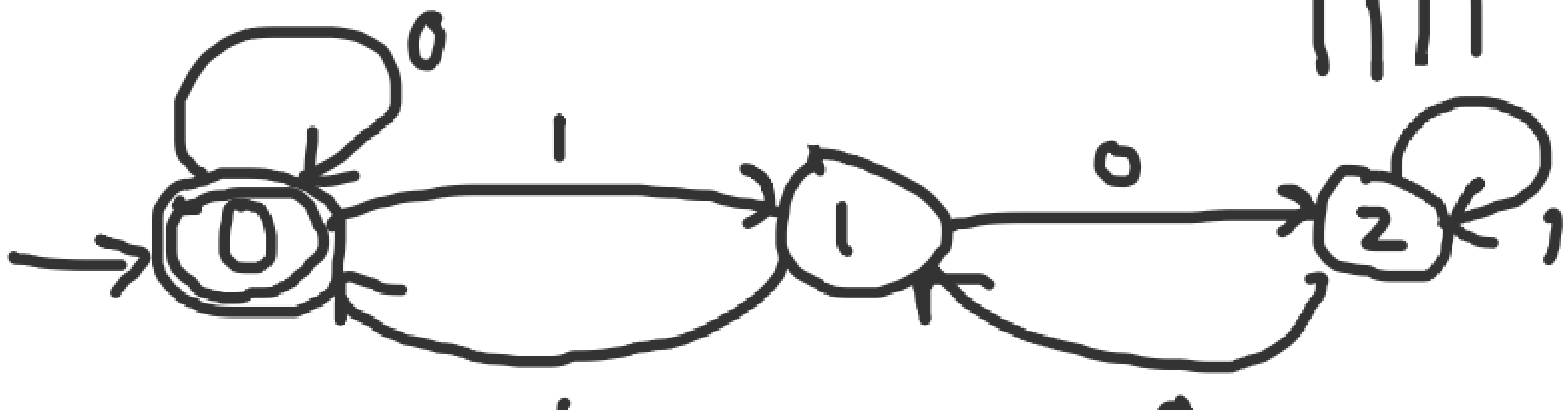
$5 \% 3 = 2$ 101

$7 \% 3 = 1$

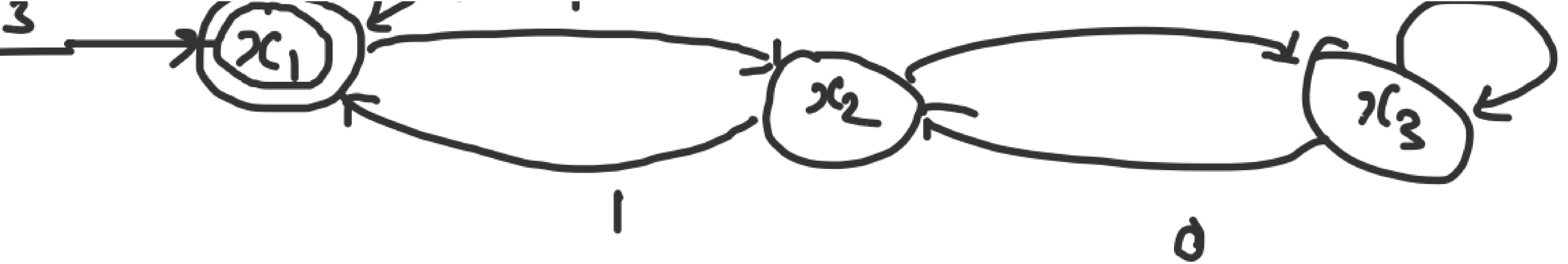
111

0 = 0
11 = 3
110 = 6
1001 = 9
1100 = 12
1111 = 15

Rem



16 8 4 2 1
1 0 1 0 1



divisible by 4

δ \ Σ	0	1
x_1	x_1	x_2
x_2	x_3	x_4
x_3	x_1	x_2
x_4	x_3	x_4

Ans

$\cdot\, x_1$	x_2
$\cdot\, x_3$	x_4
$\cdot\, x_5$	x
$\cdot\, x_7$	x_1
x_2	x_3