Date
Page

Name: Sradha Kedia
Date: 20th December, 2021
Time: 9:00 to 1:00 pm
Exam Roll num: 20234757053
Name of the programme: MCA
Semester: III
Unique paper code: 223401303
Title of the paper: Automata Theory
Name of the dept: DUCS
Email ID: 200083@cs.du.ac.in
Mobile no: 9899519848
Question no.: 1
Page no.: 4

Ans 1 a) minimizing FA

we start by making equivalence classes of final & non final states.

$$\pi_0 = \{\{F\}, \{I, A, B, C\}\}$$

F is the only final state and I, A, B are identical but for 'b' C goes to F so it has to be separate state.

$$\pi_1 = \{\{F\}, \{I, A, B\}, \{C\}\}$$

now, we consider I, A, B and I, A are identical but B for 'b' goes to C which is another set so we separate out B too.

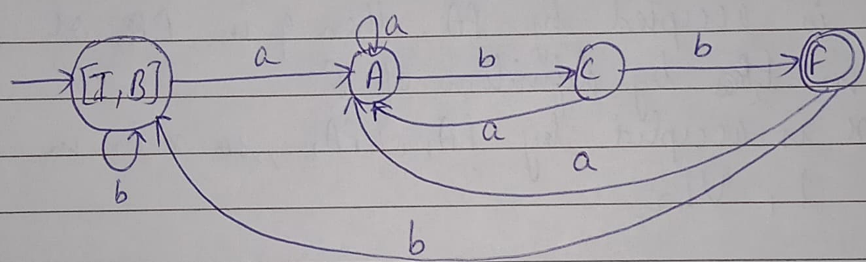
$\pi_2 = \{\{F\}, \{C\}, \{A\}, \{I, B\}\}$

I and B are identical.

so $\pi_3 = \pi_2 = \{\{F\}, \{C\}, \{A\}, \{I, B\}\}$

∴ now, we construct Transition table.

Q \ Σ	a	b
$\{I,B\}$	A	$\{I,B\}$
A	A	C
C	A	F
F*	A	$\{I,B\}$



minimized DFA.

(C) L_1, L_2 are regular languages

to prove $L_1 \cup L_2$ is also regular

So, Yes, the union of a family of regular language necessarily regular.

proof: ∵ L_1 and L_2 are Regular

∴ DFA of L_1, L_2 exist.

$FA_1 = \{Q_1, \Sigma, \delta_1, q_{01}, F_1\}$

$FA_2 = \{Q_2, \Sigma, \delta_2, q_{02}, F_2\}$

we construct DFA, $FA = (Q, \Sigma, \delta, q_0, F)$
s.t.

$Q = Q_1 \times Q_2$
$\delta((x, y), c) = (\delta_1(x, c), \delta_2(y, c))$,
for c in Σ and (x, y) in Q.

$q_0 = (q_{01}, q_{02})$
$F = \{(x, y)$ in $Q \mid x$ in F_1 or y in $F_2\}$

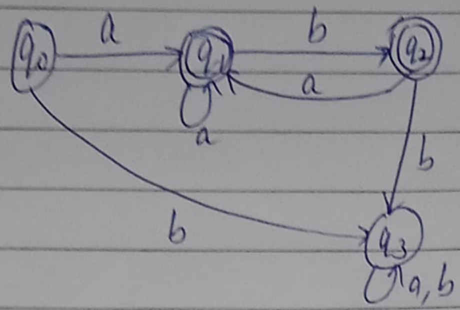
correctness: we recognize $L_1 \cup L_2$. let x be a string whose computation in FA ends in state (q, r)
if x is accepted by FA, then q in F_1 or r is in F_2 by definition of F
$\therefore$ x is accepted by FA_1 or FA_2, so x in in $L_1 \cup L_2$

if x is in $L_1 \cup L_2$, then x is accepted by FA_1 or FA_2. Thus, either q in F_1 or r in F_2
But then (q, r) is in F.
$\therefore$ x is accepted by FA iff x is in $L_1 \cup L_2$

conclusion: There is a DFA, FA that recognises $L_1 \cup L_2$. so, $L_1 \cup L_2$ is regular and the class of RL is closed under union.

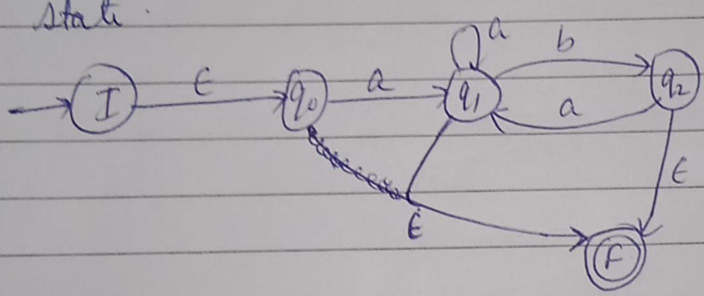
(b) $L = \{ w = \Sigma^*$ & $|w| > 0$: each b is immidiately preceded by 'a'$\}$

L = { a, ab, abab, aba, aaa, aab, ... }

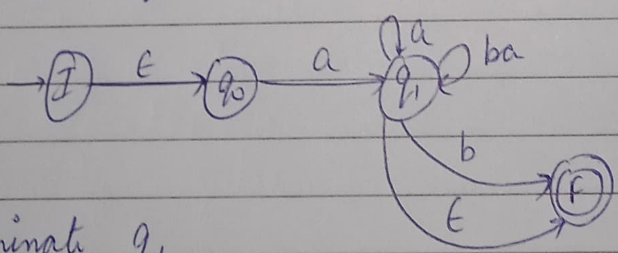


RE conversion:

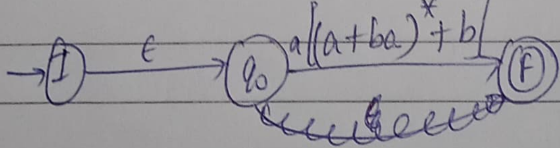
step ① unique non reenterable & unique unleavable final state.



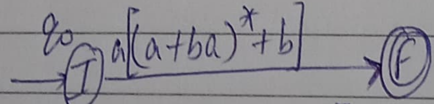
step 2: eliminating q_2, we get



step3: eliminate q_1



step4: eliminate q_0



RE: $a[(a+ba)^* + b]$