Calculate $P(P_1) = ?$ [find probobity of P_1 calls]

Solution $= P(P_1/A) * P(A) + P(P_1/\bar{A}) * P(\bar{A})$

$= 0.90 * P(A) + 0.05 * P(\bar{A})$

$= 0.90 * \underline{\underline{0.00252}} + 0.05 * \underline{\underline{.9974}}$

$= 0.0521$

$P(A) = P(A/B,E) * P(B \wedge E) +$
$P(A/B,\bar{E}) * P(B \wedge \bar{E}) +$
$P(A/\bar{B},E) * P(\bar{B} \wedge E) +$
$P(A/\bar{B},\bar{E}) * P(\bar{B} \wedge \bar{E})$

$= 0.95 * P(B) * P(E)$
$0.94 * P(B) * P(\bar{E})$
$0.29 * P(\bar{B}) * P(E)$
$0.001 * P(\bar{B}) * P(\bar{E})$

$= 0.95 * 0.001 \times 0.002$
$0.94 * 0.001 * 0.998$
$0.29 * 0.999 * 0.002$
$0.001 * 0.999 * 0.998$

$= 0.00252$

$$P(\bar{A}) = P(\bar{A}/BE) * P(B \wedge E) +$$
$$P(\bar{A}/B\bar{E}) * P(B \wedge \bar{E})$$
$$P(\bar{A}/\bar{B},E) * P(\bar{B} \wedge E)$$
$$P(\bar{A}/\bar{B},\bar{E}) * P(\bar{B} \wedge \bar{E})$$

$$= 0.05 * 0.001 * 0.002$$
$$0.06 * 0.00 * 0.998$$
$$0.71 * .999 * 0.002$$
$$0.999 * .999 * .998$$

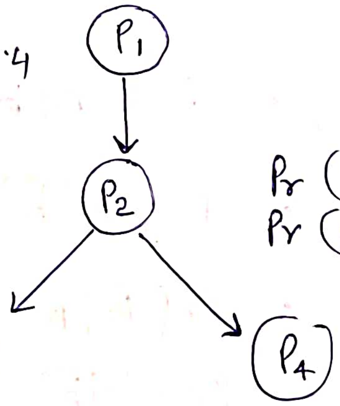
$$= 0.9974$$

Inference (अनुमान लगाना)

Q: Given the network below, calculate marginal and conditional probabilities $Pr(\neg p_3)$, $Pr(p_2 | \neg p_3)$, $Pr(p_1 | p_2, \neg p_3)$ and $Pr(p_1 | \neg p_3, p_4)$. Apply the method of inference by enumeration.

p_1	.4
$\bar{p_1}$	.6

$Pr(p_1) = 0.4$



$Pr(p_2|p_1) = 0.8$
$Pr(p_2|\neg p_1) = 0.5$

	p_2	$\bar{p_2}$
p_1	.8	.2
$\bar{p_1}$	.5	.5

$Pr(p_3|p_2) = 0.2$
$Pr(p_3|\neg p_2) = 0.3$

	p_3	$\bar{p_3}$
p_2	.2	.8
$\bar{p_2}$	.3	.7

$Pr(p_4|p_2) = 0.8$
$Pr(p_4|\neg p_2) = 0.5$

	p_4	$\bar{p_4}$
p_2	.8	.2
$\bar{p_2}$	.5	.5

$Pr(\neg p_3) = ?$

$$= \sum_{P_1, P_2, P_4} Pr(P_1, P_2, \neg p_3, P_4)$$

$$= \sum Pr(P_4|P_2) \cdot Pr(\neg p_3|P_2) \cdot Pr(P_2|P_1) \cdot Pr(P_1)$$

p_4	p_3	p_2	p_1
0	–	0	0
0	–	0	1
0	–	1	0
0	–	1	1
1	–	0	0
1	–	0	1
1	–	1	0
1	–	1	1

$\uparrow$

(Let $\neg p_3 = \overline{p_3}$)

$$= Pr(p_4|p_2) \cdot Pr(\overline{p_3}|p_2) \cdot Pr(p_2|p_1) \cdot Pr(p_1)$$
$$Pr(p_4|p_2) \cdot Pr(\overline{p_3}|p_2) \cdot Pr(p_2|\overline{p_1}) \cdot Pr(\overline{p_1})$$
$$Pr(p_4|\overline{p_2}) \cdot Pr(\overline{p_3}|\overline{p_2}) \cdot Pr(\overline{p_2}|p_1) \cdot Pr(p_1)$$
$$Pr(p_4|\overline{p_2}) \cdot Pr(\overline{p_3}|\overline{p_2}) \cdot Pr(\overline{p_2}|\overline{p_1}) \cdot Pr(\overline{p_1})$$
$$Pr(\overline{p_4}|p_2) \cdot Pr(\overline{p_3}|p_2) \cdot Pr(p_2|p_1) \cdot Pr(p_1)$$
$$Pr(\overline{p_4}|p_2) \cdot Pr(\overline{p_3}|p_2) \cdot Pr(p_2|\overline{p_1}) \cdot Pr(\overline{p_1})$$
$$Pr(\overline{p_4}|\overline{p_2}) \cdot Pr(\overline{p_3}|\overline{p_2}) \cdot Pr(\overline{p_2}|p_1) \cdot Pr(p_1)$$
$$Pr(\overline{p_4}|\overline{p_2}) \cdot Pr(\overline{p_3}|\overline{p_2}) \cdot Pr(\overline{p_2}|\overline{p_1}) \cdot Pr(\overline{p_1})$$

$$= 0.8 * 0.8 * 0.8 * 0.4$$
$$+ 0.8 * 0.8 * 0.5 * 0.6$$
$$+ 0.5 * 0.7 * 0.2 * 0.4$$
$$+ 0.5 * 0.7 * 0.5 * 0.6$$
$$+ 0.2 * 0.8 * 0.8 * 0.4$$
$$+ 0.2 * 0.8 * 0.5 * 0.6$$
$$+ 0.5 * 0.7 * 0.2 * 0.4$$
$$+ 0.5 * 0.7 * 0.5 * 0.6$$

$= 0.2048$

0.1920

0.0280

0.1050

0.0512

0.0480

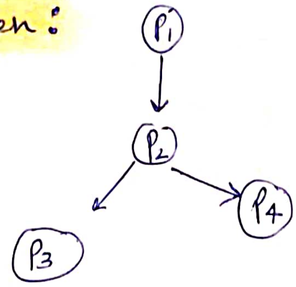
0.0280

0.1050

0.7620

$$\boxed{Pr(\bar{P_3}) = 0.7620}$$

Note : If we are considering only respective parents then:



$Pr(\bar{P_3}) = Pr(\bar{P_3} | P_2) \cdot Pr(P_2) + Pr(\bar{P_3} | \bar{P_2}) \cdot Pr(\bar{P_2})$

$= 0.8 * 0.62 + 0.7 * 0.38$

$= 0.496 + 0.266$

$= 0.762$

$Pr(P_2) = Pr(P_2 | P_1) \cdot Pr(P_1) + Pr(P_2 | \bar{P_1}) \cdot Pr(\bar{P_1})$

$= 0.8 * 0.4 + 0.5 * 0.6$

$= 0.32 + 0.30 = 0.62$ ——— (1)

$Pr(\bar{P_2}) = Pr(\bar{P_2} | P_1) * Pr(P_1) + Pr(\bar{P_2} | \bar{P_1}) \cdot Pr(\bar{P_1})$

$= 0.2 * 0.4 + 0.5 * 0.6$

$= 0.08 + 0.30$

$= 0.38$ ——— (2)

Please see carefully :.

$Pr(\bar{P_3}) = Pr(\bar{P_3} | P_2) \cdot Pr(P_2) + Pr(\bar{P_3} | \bar{P_2}) \cdot Pr(\bar{P_2})$

$= 0.8\,[0.8 * 0.4 + 0.5 * 0.6] + 0.7\,[0.2 * 0.4 + 0.5 * 0.6]$

$= 0.8 * 0.8 \times 0.4 + 0.8 \times 0.5 \times 0.6$
$0.7 \times 0.2 * 0.4 + 0.7 * 0.5 \times 0.6$

$= 0.762$

In this discussion while calculating $Pr(\bar{P_3})$ we have not considered $Pr(P_4)$. But still answer is right.

$Pr(p_2 \mid \bar{p}_3)$ (Calculate)

$$\left[\because P(A \mid B) = \frac{P(A \cap B)}{P(B)} \right.$$

$$Pr(p_2 \mid \bar{p}_3) = \frac{Pr(p_2, \bar{p}_3)}{Pr(\bar{p}_3)}$$

First calculate $Pr(p_2, \bar{p}_3) = \sum_{P_1, P_4} Pr(P_1, p_2, \bar{p}_3, P_4)$

$$= \sum_{P_1 \, P_4} Pr(P_4 \mid p_2) \cdot Pr(\bar{p}_3 \mid p_2) \cdot Pr(p_2 \mid P_1) \cdot Pr(P_1)$$

$$\begin{aligned}
= \; & Pr(p_4 \mid p_2) \cdot Pr(\bar{p}_3 \mid p_2) \cdot Pr(p_2 \mid p_1) \cdot Pr(p_1) \\
+ \; & Pr(p_4 \mid p_2) \cdot Pr(\bar{p}_3 \mid p_2) \cdot Pr(p_2 \mid \bar{p}_1) \cdot Pr(\bar{p}_1) \\
+ \; & Pr(\bar{p}_4 \mid p_2) \cdot Pr(\bar{p}_3 \mid p_2) \cdot Pr(p_2 \mid p_1) , Pr(p_1) \\
& Pr(\bar{p}_4 \mid p_2) \cdot Pr(\bar{p}_3 \mid p_2) \cdot Pr(p_2 \mid \bar{p}_1) \cdot Pr(\bar{p}_1)
\end{aligned}$$

$$\begin{aligned}
= \; & 0.8 * 0.8 * 0.8 * 0.4 \\
& 0.8 * 0.8 * 0.5 * 0.6 \\
& 0.2 * 0.8 * 0.8 * 0.4 \\
& 0.2 * 0.8 * 0.5 * 0.6
\end{aligned}$$

$$= 0.2048 + 0.1920 + 0.0512 + 0.0480$$

$$= 0.496$$

Rule: shortcut

p_4	p_3	p_2	p_1
0	=	=	0
0	=	=	1
1	=	=	0
1	=	=	1

$$\therefore Pr(p_2 \mid \bar{p}_3) = \frac{Pr(p_2, \bar{p}_3)}{Pr(\bar{p}_3)} = \frac{0.496}{0.762}$$

$$= 0.6509$$

$Pr(p_1 | p_2, \bar{p}_3)$ (Calculate)

$$Pr(p_1 | p_2, \bar{p}_3) = \frac{Pr(p_1, p_2, \bar{p}_3)}{Pr(p_2, \bar{p}_3)}$$

$$Pr(p_1, p_2, \bar{p}_3) = \sum_{P_4} Pr(p_1, p_2, \bar{p}_3, P_4) = \sum_{P_4} Pr(P_4 | p_2) \cdot Pr(\bar{p}_3 | p_2) \cdot Pr(p_2 | p_1) \cdot Pr(p_1)$$

$$= Pr(p_4 | p_2) \cdot Pr(\bar{p}_3 | p_2) \cdot Pr(p_2 | p_1) \cdot Pr(p_1)$$
$$Pr(\bar{p}_4 | p_2) \cdot Pr(\bar{p}_3 | p_2) \cdot Pr(p_2 | p_1) \cdot Pr(p_1)$$

$$= 0.8 * 0.8 * 0.8 * 0.4$$
$$+ 0.2 * 0.8 * 0.8 * 0.4$$

$$= 0.2048 + 0.0512$$
$$= 0.256$$

$$\therefore Pr(p_1 | p_2, \bar{p}_3) = \frac{0.256}{0.496} = 0.5161$$

$$Pr(p_1 \mid \overline{p_3}, p_4) = ??$$

$$Pr(p_1 \mid \overline{p_3}, p_4) = \frac{Pr(p_1, \overline{p_3}, p_4)}{Pr(\overline{p_3}, p_4)}$$

$$Pr(p_1, \overline{p_3}, p_4) = \sum_{P_2} Pr(P_1 \; P_2, \overline{p_3}, p_4)$$

$$= \sum_{P_2} Pr(p_4 \mid P_2) \cdot Pr(\overline{p_3} \mid P_2) \cdot Pr(P_2 \mid p_1) \cdot Pr(p_1)$$

$$= Pr(p_4 \mid p_2) \cdot Pr(\overline{p_3} \mid p_2) \cdot Pr(p_2 \mid p_1) \cdot Pr(p_1)$$
$$+ Pr(p_4 \mid \overline{p_2}) \cdot Pr(\overline{p_3} \mid \overline{p_2}) \cdot Pr(\overline{p_2} \mid p_1) \cdot Pr(p_1)$$

$$= 0.8 * 0.8 * 0.8 * 0.4$$
$$0.5 * 0.7 * 0.2 * 0.4$$

$$= 0.2048 + 0.0280$$
$$= 0.2328$$

$$Pr(\overline{p_3}, p_4) = Pr(p_1, \overline{p_3}, p_4) + Pr(\overline{p_1}, \overline{p_3}, p_4)$$
$$= 0.238 + 0.297$$
$$= .5298$$

$$Pr(\overline{p_1}, \overline{p_3}, p_4) = \sum_{P_2} Pr(\overline{p_1}, P_2, \overline{p_3}, p_4)$$

$$= \sum_{P_2} Pr(p_4 \mid P_2) \cdot Pr(\overline{p_3} \mid P_2) \cdot Pr(P_2 \mid \overline{p_1}) \cdot Pr(\overline{p_1})$$

$$= Pr(p_4 \mid p_2) \cdot Pr(\overline{p_3} \mid p_2) \cdot Pr(p_2 \mid \overline{p_1}) \cdot Pr(\overline{p_1})$$
$$+ Pr(p_4 \mid \overline{p_2}) \cdot Pr(\overline{p_3} \mid \overline{p_2}) \cdot Pr(\overline{p_2} \mid \overline{p_1}) \cdot Pr(\overline{p_1})$$
$$= (0.8 * 0.8 + 0.5 * 0.6) + (0.5 * 0.7 * 0.5 * 0.6) = 0.2970$$

$$\therefore \Pr(p_1 | \bar{p}_3, p_4) = \frac{\Pr(p_1 \bar{p}_3 p_4)}{\Pr(\bar{p}_3, p_4)} = \frac{0.2328}{0.5298}$$

$$= 0.4394$$

Conclusion :-

$\Pr(\bar{p}_3) = 0.762$

$\Pr(p_2 | \bar{p}_3) = 0.6509$

$\Pr(p_1 | p_2, \bar{p}_3) = 0.5161$

$\Pr(p_1 | \bar{p}_3, p_4) = 0.4394$