Golden Section Search Method

$$\frac{a}{b} = \frac{a+b}{a} = 1.618$$

Proof (1)

$$x_1 - L = R - x_2$$

$$\boxed{x_1 + x_2 = R + L}$$

Proof (2) Apply $\frac{a}{b} = \frac{a+b}{a} = 1.618$

$\because \begin{cases} a = x_2 - L \\ b = R - x_2 \end{cases}$

$$\frac{x_2 - L}{R - x_2} = \frac{R-L}{x_2 - L} = 1.618$$

$$\frac{R-L}{x_2-L} = 1.618$$

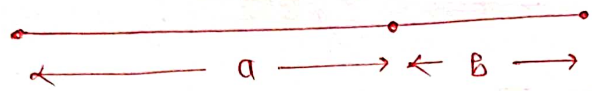
$$\frac{x_2-L}{R-L} = \frac{1}{1.618}$$

$$\frac{x_2-L}{R-L} = 0.618$$

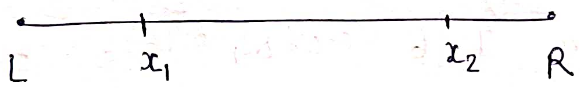
$$x_2 - L = 0.618(R-L)$$

$$\boxed{x_2 = L + 0.618(R-L)}$$

Golden section search method



$$\frac{a}{b} = \frac{(a+b)}{a} = 1.618$$



$$x_2 = L + 0.618\,(R-L) \qquad \text{——— (1)}$$

$$x_1 + x_2 = L + R \qquad \therefore\ x_1 = L + R - x_2 \qquad \text{——— (2)}$$

Step

(1) * First calculate x_1 and x_2 from equation (1) and (2).

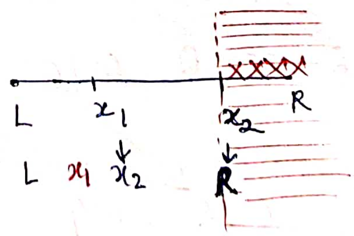
(2) * If $f(x_1) < f(x_2)$

$R \leftarrow x_2$

$x_2 \leftarrow x_1$

Preserve L

$x_1 = L + R - x_2$



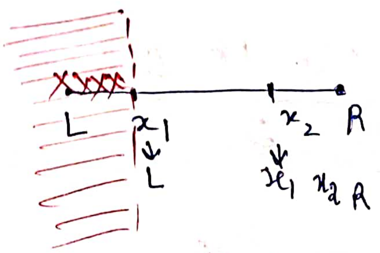
* If $f(x_1) > f(x_2)$

$L \leftarrow x_1$

$x_1 \leftarrow x_2$

Preserve R

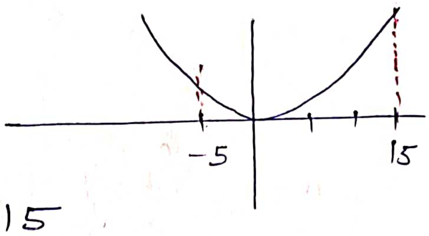
$x_2 = L + R - x_1$



* Now repeat the step (2) for next iterations.

[1] Minimize $f(x) = x^2$ over $[-5, 15]$ using golden section search method. (Take $n = 7$)

Sol^n $f(x) = x^2$



$[-5 \quad 15]$

$L = -5 \qquad\qquad R = 15$

$x_2 = L + 0.618(R-L) = -5 + 0.618\,[15+5] = 7.36$

$x_1 = L + R - x_2 = -5 + 15 - 7.36 = 2.64$

K	L	x_1	x_2	R	$f(x_1)$	$f(x_2)$	L/R	Comment
1	-5	2.64	7.36	15	6.9696	54.17	L	$f(x_1) < f(x_2)$
2	-5	-0.28 $(L+R-x_2)$	2.64	7.36	0.0784	6.9696	L	$f(x_1) < f(x_2)$
3	-5	-2.08 $(L+R-x_2)$	-0.28	2.64	4.3264	0.0784	R	$f(x_1) > f(x_2)$
4	-2.08	-0.28	0.84 $(L+R-x_1)$	2.64	0.0784	0.7056	L	$f(x_1) < f(x_2)$
5	-2.08	-0.96	-0.28	0.84	0.9216	0.0784	R	$f(x_1) > f(x_2)$
6	-0.96	-0.28	0.16	0.84	0.0784	0.0256	R	$f(x_1) > f(x_2)$
7	$\boxed{-0.28}$	0.16	0.4	$\boxed{0.84}$	0.0256	0.16		

$$x^* = \frac{(-0.28) + (0.84)}{2} = \frac{0.56}{2} = 0.28$$

$$f(0.28) = (0.28)^2 = 0.0784$$

Ans