Measurement- The measurement of a given quantity is essentially an act or result of comparision between the quantity (whose magnitude is unknown) and a predefined standard.

Measurand - Unknown variable which is to be measured.

Measured variable - Physical quantity property or condition which is to be measured.

Measurement system performance- Treatment of instrument and measurement System characteristics can be divided into two distinct categories viz:

1- Static characteristics

2- Dynamic characteristics

static characteristics — These are those properties of the system which donot change with time.

Dynamic characteristics - these are those properties of the system which changes with time.

Static cha.

1. Accuracy
2. Sensitivity
3. Reproducibility

} Desirable

4. Drift
5. Static error
6. Dead zone

} Undesirable

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Dynamic characteristics -

1. Speed of response
2. Measuring log
3. fidelity
4. Dynamic error

Accuracy - It is the degree of closeness with which an instrument reading approaches to the true value of the quantity being measured.

* The accuracy may be specified in terms of inaccuracy or limits of error and can be expressed in the following ways.

1> Point accuracy

2> Accuracy as "Percentage of scale Range".

3> Accuracy as "Percentage of True Value".

Point Accuracy - This is the accuracy of the instrument only at one point on its scale. The specification of this accuracy does not give any information about the accuracy at the other points on the scale, or in other words it does not give any information about the general accuracy of the instrument.

Accuracy as "Percentage of Scale Range". -

Let uniform scale from 0°C to 500°C. maximum value is 500°C.

If accuracy is given as 1% of full scale reading.

then % Error = 1 % of full scale reading

$$\text{Error} = \frac{1}{100} \times 500 \quad [\because \text{full scale range is } 500°C]$$

$$\therefore \text{Error} = 5°C$$

$\therefore$ Magnitude of error = 5°C

Let we are measuring 250°C (true value) so measured value will be 250 + 5 = 255°C or 250 − 5 = 245°C [Because error will be fixed]

$$\%\text{error} = \frac{MV - TV}{TV} * 100$$

$$= \frac{5°C}{250°C} * 100$$

$$= \frac{1}{50} \times 100.$$

$$= 2\% \text{ for } 250°C$$

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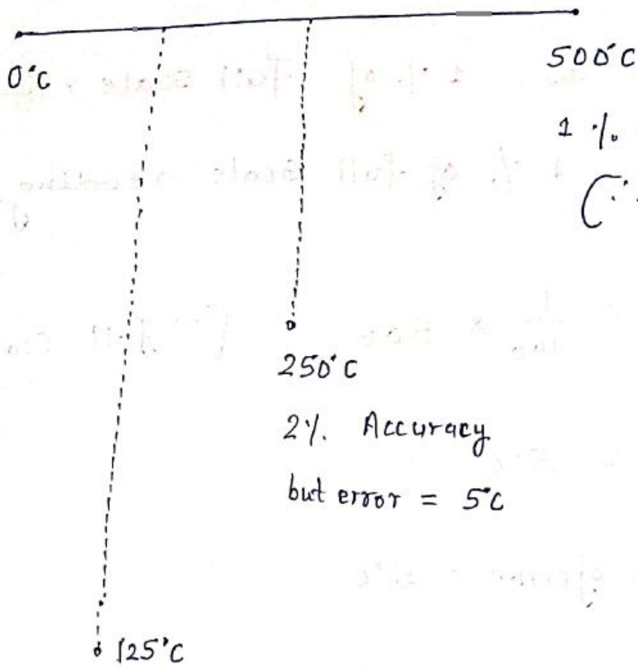


500°C: 1 % Accuracy (∴ error = 5°C)

250°C: 2% Accuracy but error = 5°C

125°C: Accuracy = 2×2 = 4 % but error = 5°C

* Hence for this type of instrument where accuracy is defined as a % of full scale range, the error (Also known as limiting error) is constant but % error will be change accordingly.

500°C	1% Accurate	error = 5°C
250°C	2% Accurate	error = 5°C
125°C	4% Accurate	error = 5°C
62.5°C	8% Accurate	error = 5°C

Accuracy as a % of True Value – The best way to conceive the idea of accuracy is to specify it in terms of the true value of the quantity being measured.

Let scale is from 0°c to 500°C

Let accuracy is given as a 1 % of true value.
So if our true value is 250°c then accuracy is 1 % of 250°c.

$$\text{Error} = 1\ \% \text{ of true value}$$

$$= \frac{1}{100} \times 250\,°C$$

$$= 2.5\,°C$$

if our true value is 500°c

$$\text{then error} = 1\ \% \text{ of the true value}$$

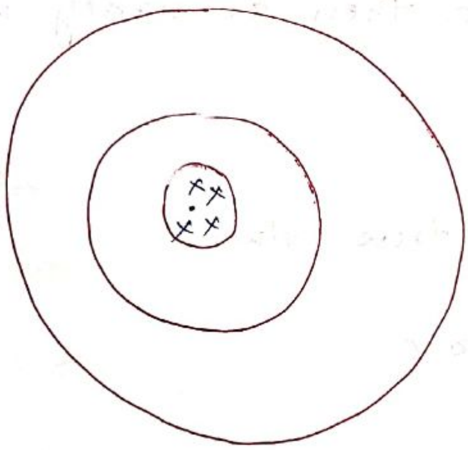
$$= \frac{1}{100} \times 500\,°C$$

$$= 5\,°C$$

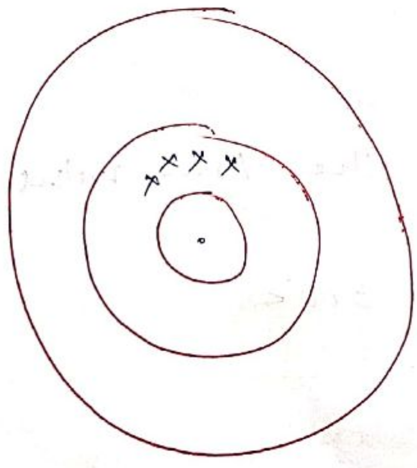
Thus at the starting of the scale our error is low but at the end point of the true value scale, our error is high.

* If reading gets smaller then error will be smaller and if reading gets higher then error will be higher.

Precision – Precision refers to how well measurements agree with each other in multiple tests.



High accurate



High precision

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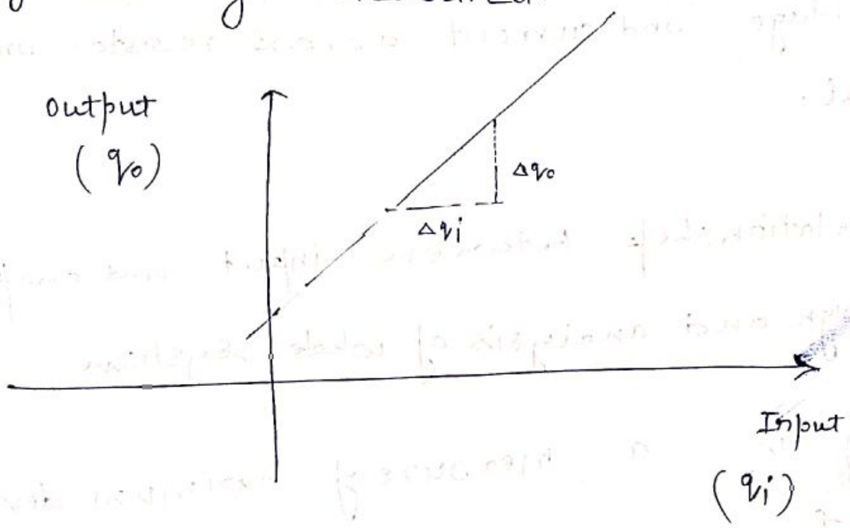
Static sensitivity - The static sensitivity of an instrument or an instrumentation system is the ratio of the magnitude of the output signal or response to the magnitude of the input signal or the quantity being measured.



Static sensitivity at the operating point is defined as:

$$\text{Static sensitivity} = \frac{\text{Infinitesimal change in output}}{\text{Infinitesimal change in input}}$$

$$= \frac{\Delta q_o}{\Delta q_i}$$

* The sensitivity of an instrument should be high.

$$\text{Inverse sensitivity or deflection factor} = \frac{\Delta q_i}{\Delta q_o}$$

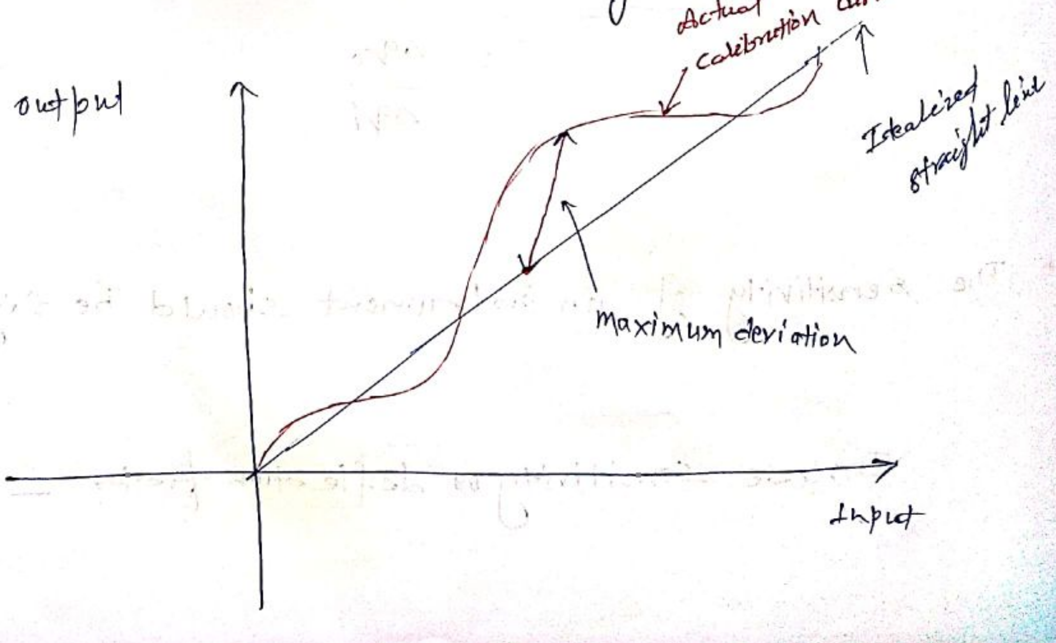
Linearity - Linearity is the property of a mathematical relationship or function which means that it can be graphically represented as a straight line.

Example - voltage and current across resistor are linearly proportional.

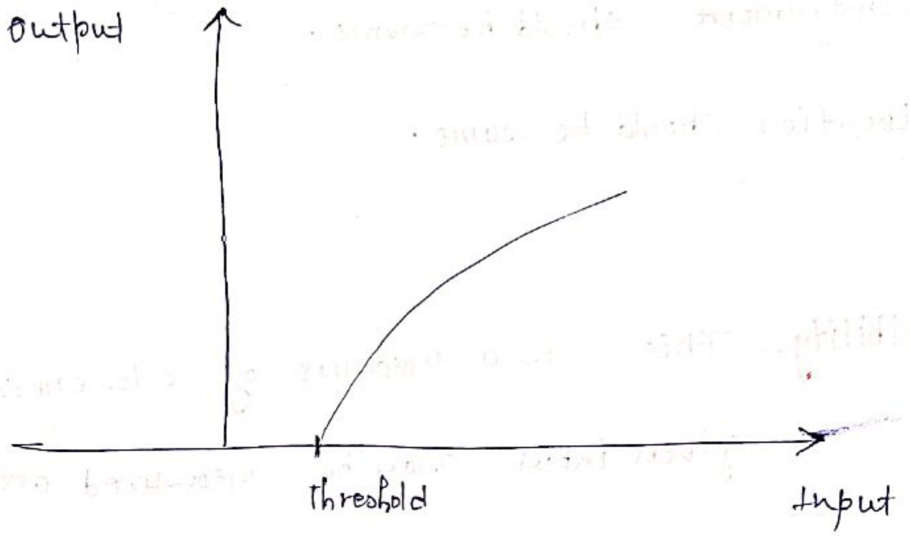
* Linear relationship between input and output simplify the design and analysis of whole system.

* Linearity is a measure of maximum deviation of output from the idealized straight line.

$$\text{Nonlinearity} = \frac{\text{Maximum deviation of output from the idealized straight line}}{\text{Actual reading}} \times 100$$



Threshold – If the input is increased very gradully (slowly) from zero there will be some minimum value below which no output change can be detected. This minimum value is called the threshold of the instrument.



Resolution – If the input is slowly increased from some arbitrary (non-zero) input value, it will again be found that output does not change at all until a certain increment is exceeded. This increment is called resolution of the instrument.

* Resolution is the smallest measurable input change while threshold defines the smallest measurable input.

Repeotability - This is a measure of closeness with which a given input may be measured over and over again and following conditions shold also be satisfied :.

1- Instrument should be same.

2- Location should be same.

Reproducibility - This is a measure of closeness with which a given input may be measured over and over again and following conditions should also be satisfied :-

1- Instrument will be different.

2- Location will be different.

Hysteresis - Hysteresis effect is mainly found in any physical, chemical or electrical phenomenon :

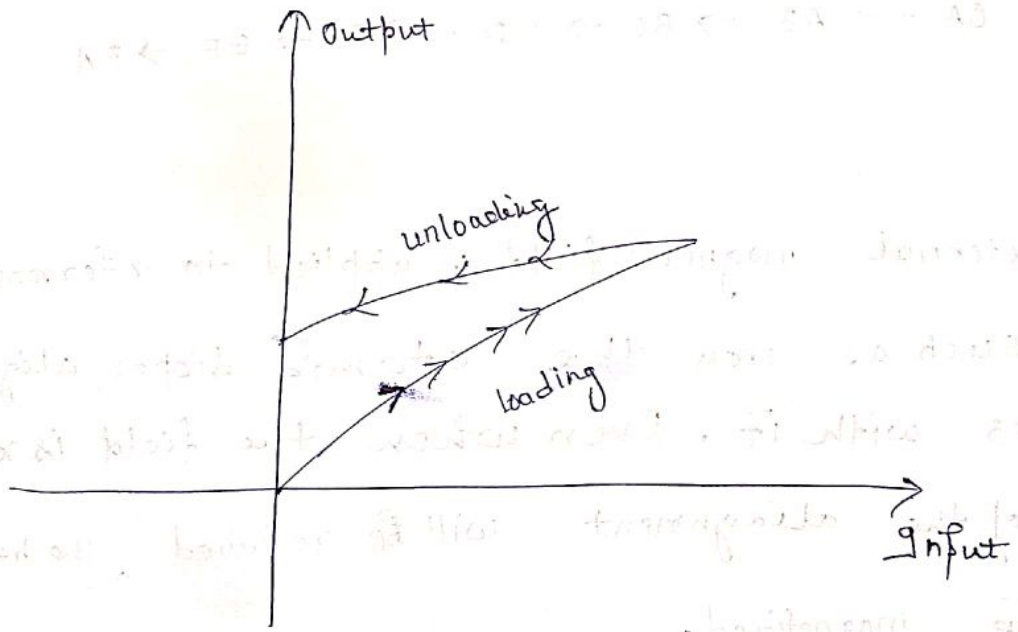
* the term hysteresis meaning deficiency or lagging behind.

* Let input of the instrument increases from zero to full scale (maximum value) then output

will also increase from zero to maximum.

*But when input is returning back from high value to zero the output will not follow the same path as incase of increasing the input. Thus during returning when input goes from high to zero then output goes from high to nonzero value. So output path is different during returning.

* In technical way output does not follow the input is a same way during loading and unloading of input. So this phenomenon is called hysteresis.



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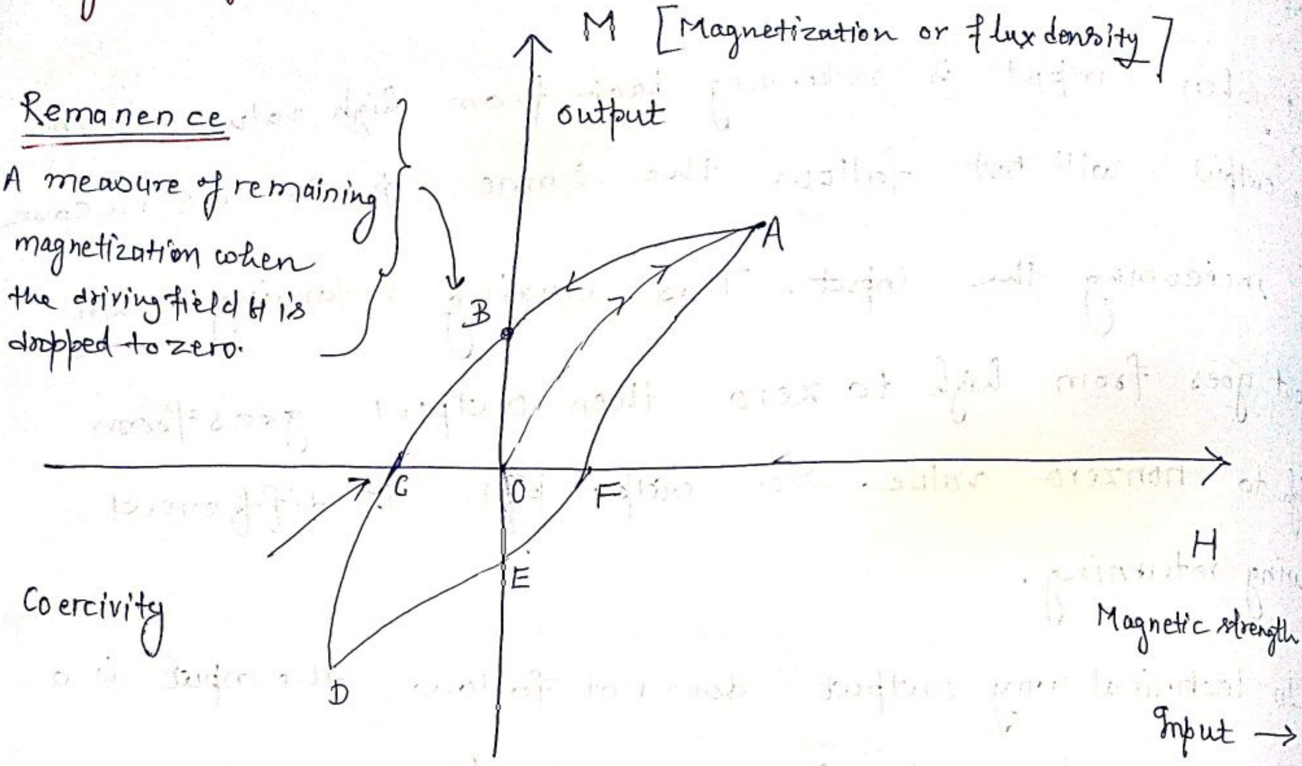
Magnetic hysteresis -



Path: OA ⟶ AB → BC → CD → DE → EF → FA

* When an external magnetic field is applied to a ferromagnetic material such as iron the atomic diopes align themselves with it. Even when the field is removed, the part of the aliagnment will be retained, the material has become magnetized.

Error and their analysis.

True value $\longrightarrow$ Theoritical value or nominal value

or specified value

Measured value $\rightarrow$ It is measured using experiment. It is also called actual value.

Limiting error - (Guarantee Errors) -

* In most instrument the accuracy is guaranteed to be with in a certain percentage of full scale reading.

* Components are guaranteed to be with in a certain percentage of rated value.

* Thus manufacturer has to specify the deviations from the true value (nominal value) of a particular quantity. The limits of the deviations from the specified value are defined as limiting errors or Guarantee errors.

Let True value $= A_s$

Maximum error or limiting error $= \delta A$

Measured value $= A_s \pm \delta A$

$$\boxed{A_a = A_s \pm \delta A}$$

Relative limiting error $= \epsilon_r = \dfrac{\delta A}{A_s}$

$$\epsilon_r = \frac{\epsilon_o}{A_s}$$

$$\epsilon_o = \delta A = \epsilon_r A_s$$

$$A_a = A_s \pm \delta A$$

$$A_a = A_s \pm \epsilon_r A_s$$

$$A_a = A_s [1 \pm \epsilon_r]$$

Let $A_s = 100\Omega$ $\quad \delta A = 10\Omega$

$\therefore$ Relative limiting error $= \epsilon_r = \dfrac{\delta A}{A_s}$

$$\epsilon_r = \pm \frac{10}{100}$$

$$\epsilon_r = \pm 0.1$$

Percentage limiting error $= \% \epsilon_r = 0.1 \times 100$

$$= 10\%$$

$$A_a = A_s [1 \pm \epsilon_r]$$

$$= 100 [1 \pm 0.1]$$

$$= 100 \pm 10 \quad \Omega$$

Note

$$\delta A = A_a - A_s$$

$$\epsilon_r = \frac{\delta A}{A_s}$$

$$= \frac{A_a - A_s}{A_s}$$

$$= \frac{\text{Measured Value} - \text{True Value}}{\text{True Value}}$$

Relative limiting Error $= \epsilon_r = \frac{MV - TV}{TV}$

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Combination of quantities with limiting error:

1- Sum of two quantities -

$$X = x_1 + x_2$$

$$dX = dx_1 + dx_2$$

$$\frac{dX}{X} = \frac{dx_1}{X} + \frac{dx_2}{X}$$

$$\frac{dX}{X} = \frac{x_1}{X} * \frac{dx_1}{x_1} + \frac{x_2}{X} \frac{dx_2}{x_2}$$

$$\boxed{\frac{dX}{X} = \pm \left[\frac{x_1}{X} \frac{dx_1}{x_1} + \frac{x_2}{X} \frac{dx_2}{x_2} \right]}$$

2- Product of two component -

$$X = x_1 * x_2$$

$$\log X = \log_e x_1 + \log_e x_2$$

$$\frac{1}{X} = \frac{1}{x_1} \frac{dx_1}{dX} + \frac{1}{x_2} \frac{dx_2}{dX}$$

$$\boxed{\frac{dX}{X} = \pm \left[\frac{dx_1}{x_1} + \frac{dx_2}{x_2} \right]}$$

Q (1) A 0-150V voltmeter has guaranteed accuracy of 1% of full scale reading. the voltage measured by this instrument is 75V. Calculate the Limiting error in percentage.

Soln:

Accuracy is 1% of full scale reading

$$\text{error} = \frac{1}{100} * 150$$

$$\text{error} = 1.5V$$

Because accuracy is defined for full scale reading so whatever we are measuring the error will be fixed. So when we measure 75V then error will be 1.5V

$$\therefore \text{ Percentage error} = \frac{1.5}{75} \times 100$$

$$= \frac{6}{3} \%$$

$$= 2\%$$

or percentage limiting error = 2%.

Comment – Percentage error increases as the voltage being measured decreases.

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Q-
$R_1 = 37\Omega + 5\%$

$R_2 = 75\Omega + 5\%$

$R_3 = 50\Omega + 5\%$

Determine the magnitude of the resistance and limiting error in ohm and % limiting error of resistance if the resistance is connected in series.

Sol^n

$$R_1 = 37 \pm \frac{5}{100} \times 37 = 37 \pm 1.87\Omega.$$

$$R_2 = 75 \pm \frac{5}{100} \times 75 = 75 \pm 3.75\Omega$$

$$R_3 = 50 \pm \frac{5}{100} \times 50 = 50 \pm 2.50\Omega$$

Limiting value of resultant resistance :.

$$R = 37 + 75 + 50 \pm [1.85 + 3.75 + 2.50]$$

$$= 162 \pm 8.10\Omega$$

∴ Magnitude of resistance $= 162\Omega$

Error in ohm $= \pm 8.10\Omega$

Percentage Limiting error of series combination of resistances

$$= \frac{\pm 8.10}{162} \times 100$$

$$= \pm 5\%$$

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Q- The solution of the unknown resistance for a wheatstone bridge is :

$$R_x = \frac{R_2 R_3}{R_1}$$

$R_1 = 100 \pm 0.5\ \%\ \Omega$

$R_2 = 1000 \pm 0.5\ \%\ \Omega$

$R_3 = 842 \pm 0.5\ \%\ \Omega$

Determine the magnitude of unknown resistance and limiting error in percentage, and in ohm for the unknown resistance R_x.

$$\text{unknown resistance} = R_x = \frac{R_2 R_3}{R_1}$$

$$= \frac{1000 * 842}{100}$$

$$= 8420\,\Omega$$

Relative Limiting error of unknown resistance ⇒

$$\frac{\delta R_x}{R_x} = \pm \left[\frac{\delta R_2}{R_2} + \frac{\delta R_3}{R_3} + \frac{\delta R_1}{R_1}\right]$$

$$= \pm \left[0.5 + 0.5 + 0.5\right] \%$$

$$= \pm 1.5\ \%$$

$\therefore$ Limiting error in ohm $= \dfrac{\pm 1.5}{100} \times 8420 = \pm 126.3\,\Omega$

$$\left.\begin{aligned} 8420 + 126.3 &= 8293.7\,\Omega \\ 8420 - 126.3 &= 8546.3\,\Omega \end{aligned}\right\}\ \text{Guaranteed value of resistance}$$

Q: The resistance of a circuit is found by measuring current flowing and power fed into the circuit. Find the limiting error in the measurement of resistance when the limiting errors in the measurement of power and current are respectively $\pm 1.5\%$ and $\pm 1\%$.

Sol:

$$P = I^2 R$$

$$R = \frac{P}{I^2} = P I^{-2}$$

$$\therefore \frac{\delta R}{R} = \pm \left[\frac{\delta P}{P} + 2 \frac{\delta I}{I} \right]$$

$$= \pm \left[1.5 + 2 \times 1 \right]$$

$$= \pm 3.5\%$$

Arithmatic mean-

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$= \frac{\Sigma x}{n}$$

Deviation-

$d_1 = x_1 - \bar{x}$

$d_2 = x_2 - \bar{x}$

$d_3 = x_3 - \bar{x}$

$\vdots$

$d_n = x_n - \bar{x}$

Algebric sum of deviations:

$= d_1 + d_2 + d_3 + \cdots + d_n$

$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x})$

$= (x_1 + x_2 + \cdots + x_n) - n\bar{x}$

$= n\bar{x} - n\bar{x}$

$= 0$

* Algebric sum of deviations is zero.

Average deviation -

$$\bar{D} = \frac{|\pm d_1| + |\pm d_2| + |\pm d_3| \cdots + |\pm d_n|}{n}$$

Standard deviation -

(i) (for $n>20$) $$SD = \sigma = \sqrt{\frac{d_1^2 + d_2^2 + \cdots + d_n^2}{n}}$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

(ii) (for $n<20$)

$$SD = S = \sqrt{\frac{d_1^2 + d_2^2 + \cdots + d_n^2}{(n-1)}}$$

$$S = \sqrt{\frac{\Sigma d^2}{(n-1)}}$$

Variance - variance = (standard deviation)^2

$$V = \sigma^2 = \frac{d_1^2 + d_2^2 + \cdots + d_n^2}{n}$$

or

$$V = S^2 = \frac{d_1^2 + d_2^2 + \cdots + d_n^2}{(n-1)}$$

Probable error (PE) $\boxed{PE = 0.6745\sigma}$

Elements of a generalized measurement system -

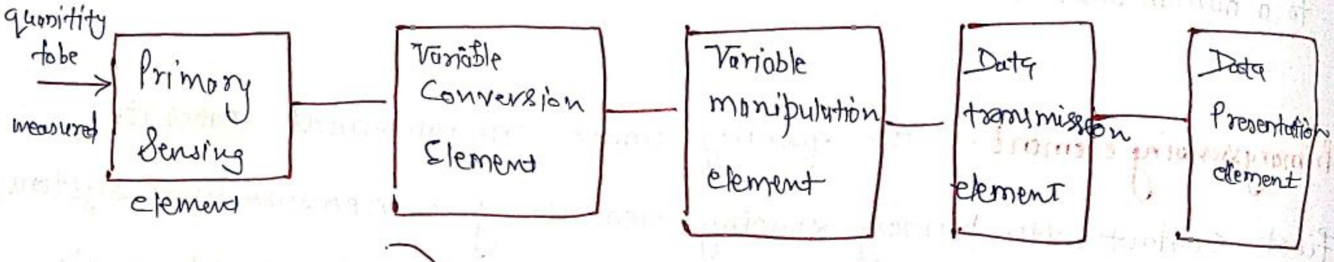
1- Primary sensing element
2. Variable conversion element
3. Data presentation element

* An instrument may be defined as a device or system which is designed to maintain a functional relationship between prescribed properties of physical variables and must include ways and means of communication to a human observer.

Primary sensing element - The quantity under measurement makes its first contact with primary sensing element of a measurement system. the first place is the physical quantity to be measured, in the sensed and detected by an element which gives the output in different analogous form. This output is then converted into an electrical signal by transducer.

Variable conversion element - The output of the primary sensing element may be electrical signal of any form. It may be a voltage, a frequency, or some other electrical parameter. Some times this output is not suited to the system (next). For instrument to perform the desired function, it may be necessary to convert this output to some suitable form while preserving the information content of the original signal.

Variable manipulation element - Manipulation means only a change in numerical value of signal. For example: An electronic amplifier accepts a small voltage signal as input and produces an output signal which is also voltage but of greater magnitude. Thus voltage amplifier acts as a variable manipulation element.



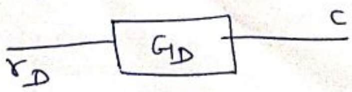
Input - output Configuration of Measuring Instruments and Measurement Systems-

1 - Desired inputs

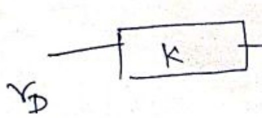
2. Interfacing inputs

3. Modifying inputs

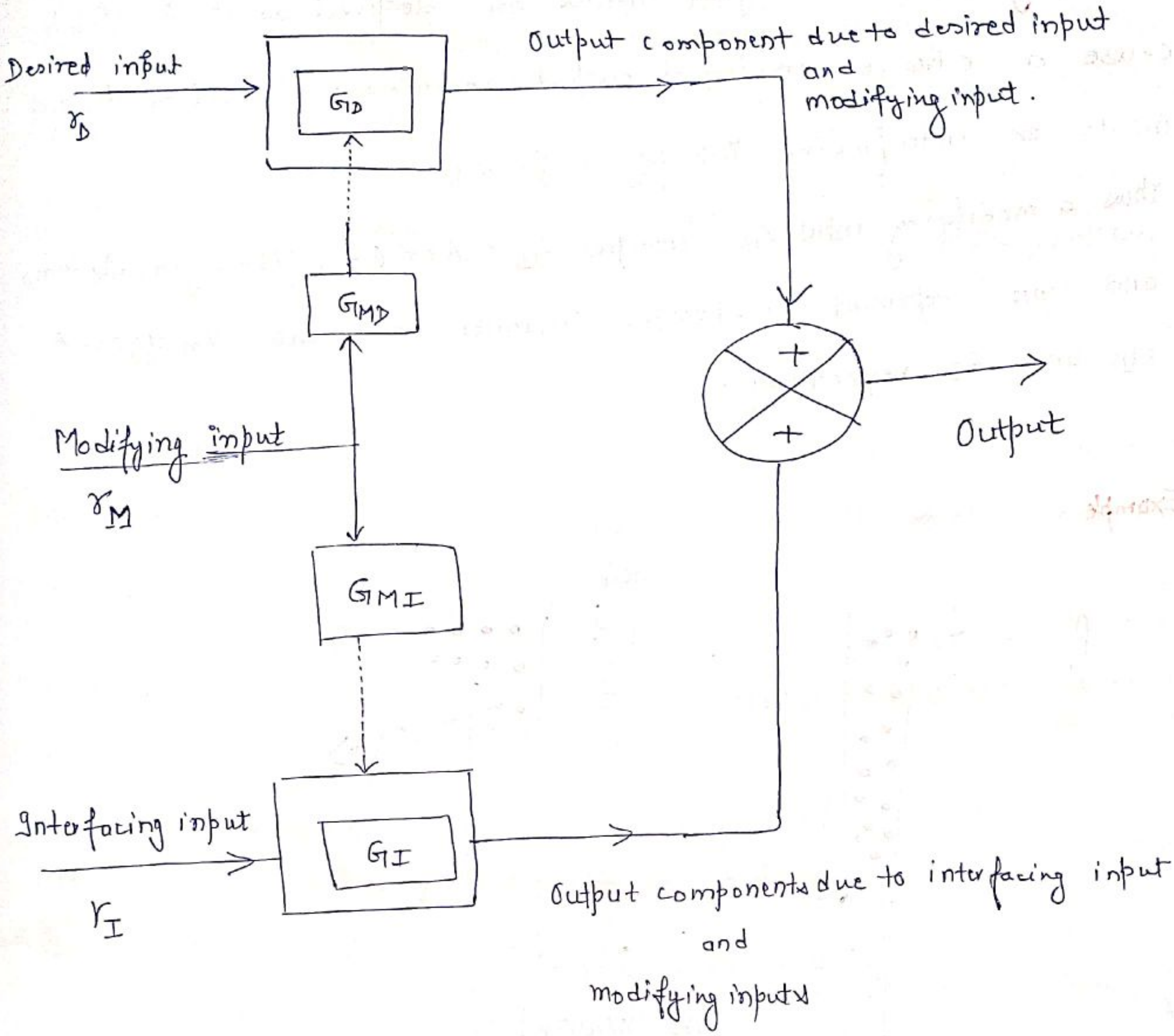
1- Desired inputs - These are the quantities for which the instrument is specifically designed to measure and respond.



$c_D = G_D \cdot r_D$



$c_D = K \cdot r_D$



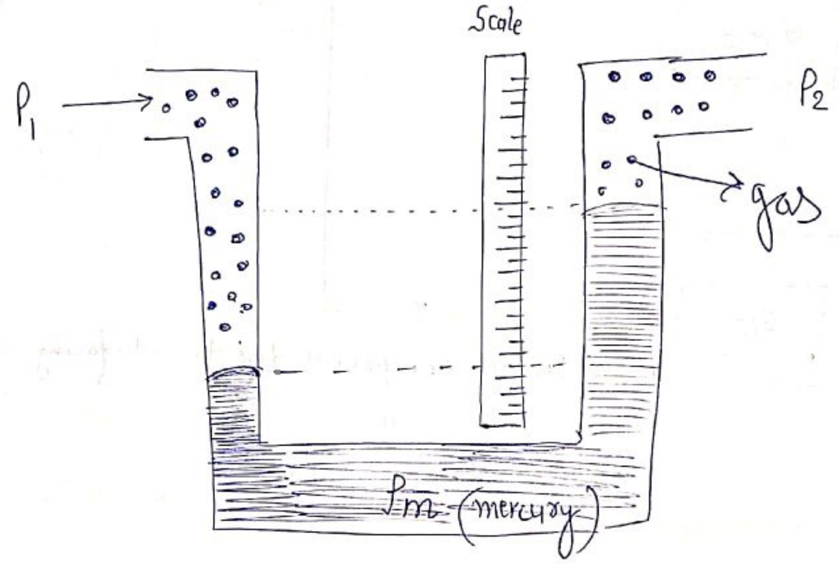
Interfacing inputs– It represent the quantities to which an instrument becom unintentionally sensitive. The instruments are not desired to respond to interfacing inputs but they give an output due to interfacing inputs on account of their principle of working, design and many other factors like the environment in which they are placed.

$$r_I \longrightarrow \boxed{G_I} \longrightarrow c_I = G_I \cdot r_I$$

Modifying inputs - Modifying inputs are defined as inputs which cause a change in input-output relationships for either desired inputs or interfacing inputs or for both.

Thus a modifying input r_M modifies G_D and or G_I. The symbols G_{mD} and G_{mI} represent the specific manner in which r_M affects G_D and G_I respectively.

Example



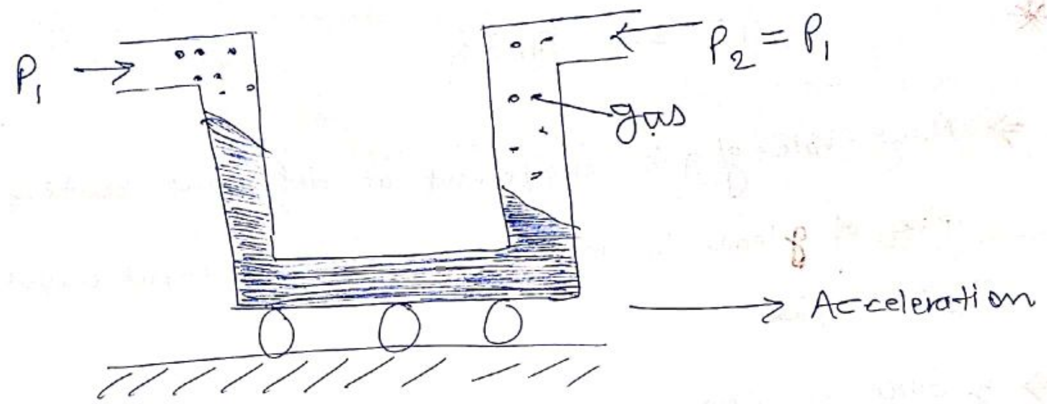
$$P_1 = P_2 + \rho_m g h$$

$$\boxed{P_1 - P_2 = \rho_m g h}$$

(P_1 and P_2 are the desired inputs. h is output)

if $P_1 = P_2$ then $\rho_m g h = 0$ $\therefore h = 0$

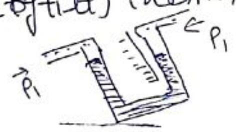
Case - 1 In case (1) even when $P_1 = P_2$ but $h \neq 0$ because of interfacing inputs.



In the above figure manometer is placed on a vehicle that is accelerating. Although $P_1 = P_2$ but $h \neq 0$ (height of manometer is not same because of accelerating.)

The scale indicates a reading h. thus acceleration acts as an interfacing input.

* When manometer is not properly aligned with gravity (because of tilt) then $h \neq 0$ (even when $P_1 = P_2$). Angle of tilt θ acts as an interfacing input.



Case ② Modifying inputs for manometer are ambient temperature and gravitational force.

* Changes in ambient temperature change the length of calibrated scale thereby modifying the proportionality factor relating the input $(P_1 - P_2)$ with output h.

* Changes in the ambient temperature change the value of density of mercury ρ_m and therefore modifying the proportionality factor relating the input with output is changed.

* $P_1 - P_2 = \rho_m g h$

⇒ Now value of g is different at different locations. Therefore value of g leads to modification of the input output for the desired inputs.

⇒ g Since g can also be interfacing inputs due to angle of tilt [$\because$ $P_1 - P_2 = \rho_m (g \cos\theta) h$ (θ = angle of tilt)] as explained; there is modification of input-output relationship for interfacing input as well. Thus it is observed that effects of both the desired and interfacing inputs may be modified due to modifying input.

Potentiometer

* Potentiometer is an instrument designed to measure an unknown voltage by comparing it with known voltage. A known voltage may be supplied by a standard cell or any other known voltage reference source.

* Since potentiometer makes use of a balance or null condition, so no current flows and hence no power is consumed in the circuit containing unknown emf when instrument is balanced.

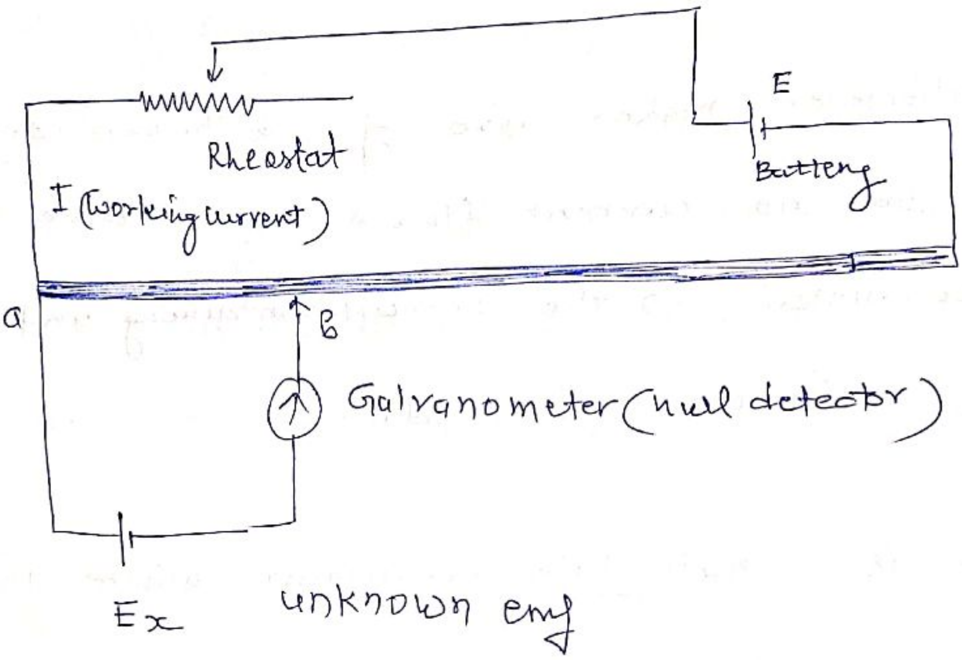
* Potentiometer is a null type instrument where in an unknown emf is measured.

* Note - Null type instruments are very accurate in nature.

Construction & Working - Figure shows the elementary form of D.C. Potentiometer. It is a null type instrument wherein an unknown emf (E_x) is measured. The slide wire of the potentiometer has been calibrated in terms of emf with the help of standard emf source.

* The null detector is the current galvanometer whose deflection is proportional to the

unbalance emf (i.e difference between the emf E_{ab} across the portion ab of the slide wire and the unknown emf E_x.)



* As soon as the two emfs are equal there is no current through the galvanometer and therefore it shows zero deflection thereby indicating null conditions.

$$\therefore \text{Unknown emf } E_x = E_{ab}$$

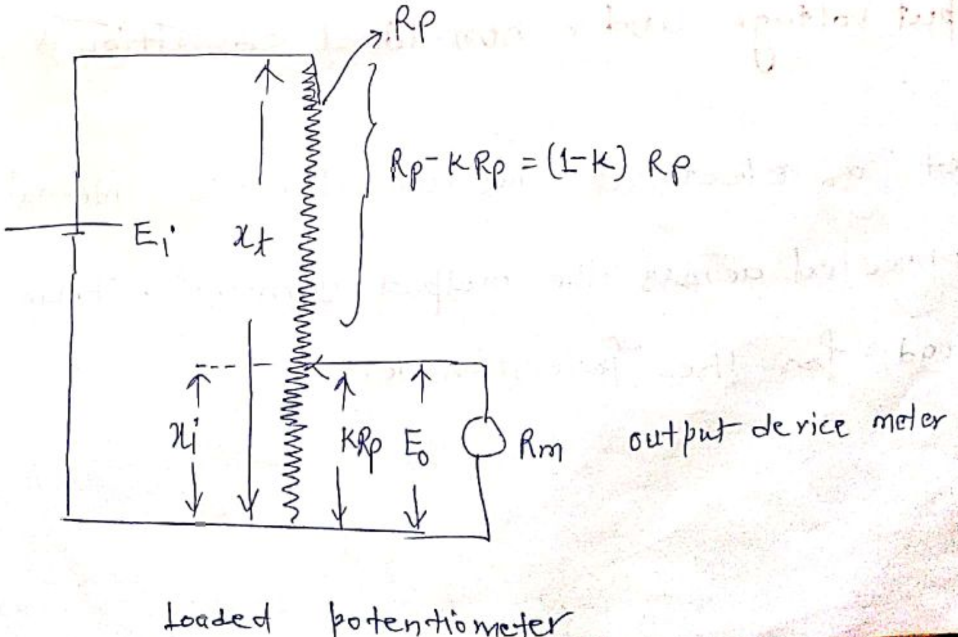
Note: E_{ab} is directly indicated by calibrated scale placed along the slide wire.

Loading Effect :

* In ideal measuring instrument, it will draw no current from the circuit, thereby resulting in the true measurement of electronic parameters.

* Unfortunately, in the real world, all instruments draw current from circuit (or system) as a result of this original signal will be distorted. This distortion may take the form of reduction in magnitude, waveform distortion, phase shift and many times all these undesirable features put together. This makes ideal measurement impossible.

* The incapability of the measuring system (instrument) to faithfully measure, record, or control the input signal (measurand) in undistorted form is called the **loading effect**.



Loaded potentiometer

$E_i \rightarrow$ Input voltage

$E_o \rightarrow$ Output voltage

$x_t \longrightarrow$ Total length of the potentiomer (POT)

$x_i \longrightarrow$ Displacement of wiper from its zero position

$R_p \Rightarrow$ Total resistance of potentiometer

$R_m \Rightarrow$ Resistance of an output device meter

Output voltage under ideal condition ⇒

$$E_o = \frac{\text{Resistance of output}}{\text{Resistance of input}} * \text{input voltage}$$

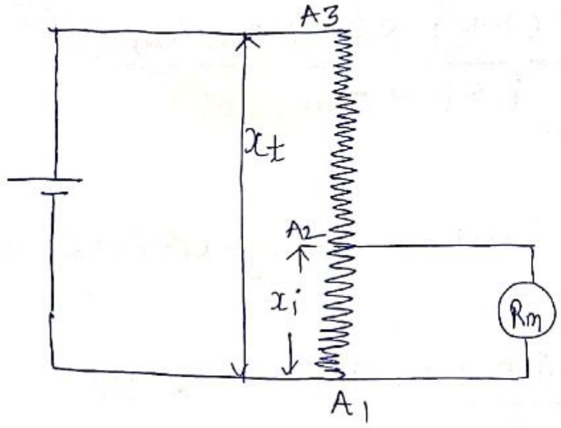
$$E_o = \frac{R_p\left(\frac{x_i}{x_t}\right)}{R_p} \times E_i$$

$$E_o = \frac{x_i}{x_t} E_i$$

$$\boxed{E_o = K E_i} \qquad \left(k = x_i / x_t \right)$$

Output voltage under non-ideal condition ⇒

* Let a electrical instrument (whose internal resistance is R_m) is connected across the output terminal. This instrument form a Load for the Potentiometer.



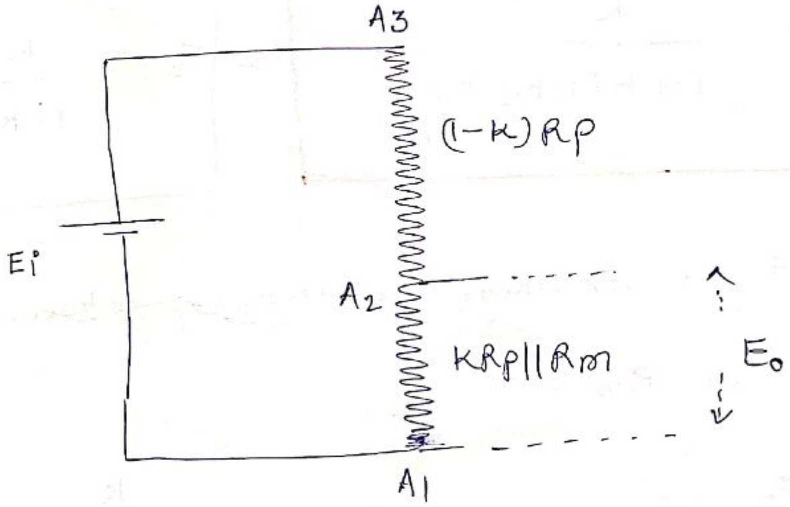
Resistance of distance $x_i = \frac{R_p}{x_t} * x_i$

$$= \frac{x_i}{x_t} R_p$$

$$= K R_p$$

Resistance of remaining portion of potentiometer $= R_p - KR_p$

$$= (1-K) R_p$$



$$KR_p \| R_m \Rightarrow \frac{KR_p \cdot R_m}{KR_p + R_m}$$

Total resistance $= (1-K) R_p + [KR_p \| R_m]$

$$= (1-K) R_p + \frac{KR_p \cdot R_m}{KR_p + R_m}$$

$$= \frac{K(1-K)\,R_p^2 + R_p \cdot R_m}{K R_p + R_m}$$

∴ output voltage will be voltage across the resistance $A_1 A_2$.

$$E_o = \frac{(K R_p \,||\, R_m)}{\text{Total resistance}} * E_i$$

$$E_o = \frac{\dfrac{K R_p R_m}{\cancel{K R_p + R_m}}}{\dfrac{(K(1-K)\,R_p^2 + R_p R_m)}{\cancel{K R_p + R_m}}} * E_i$$

$$E_o = \frac{K R_p R_m}{K(1-K)\,R_p^2 + R_p R_m} E_i$$

$$\boxed{\frac{E_o}{E_i} = \frac{K}{1 + K(1-K)\frac{R_p}{R_m}}} \quad \text{or} \quad \boxed{E_o = \frac{K E_i}{1 + K(1-K)\frac{R_p}{R_m}}}$$

So there exist a non linear relationship between E_o and E_i.

Note :- Let $R_m = \infty$

$$\therefore \frac{E_o}{E_i} = \frac{K}{1 + K(1-K)\frac{R_p}{\infty}} = \frac{K}{1+0} = K$$

$$E_o = K E_i$$

Which is a linear relationship.

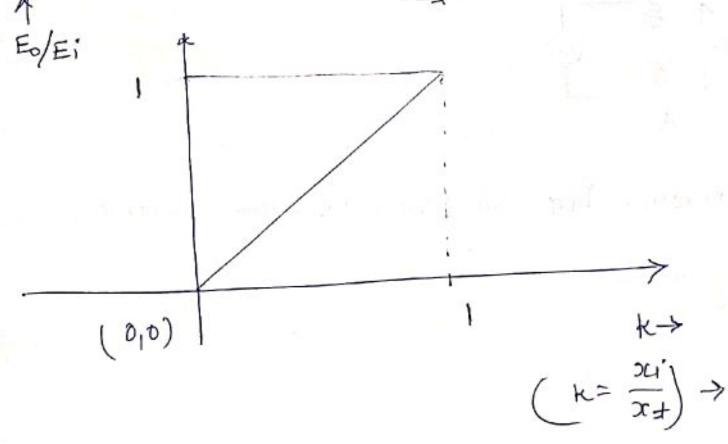
* It means that linearity can be achieved by making output

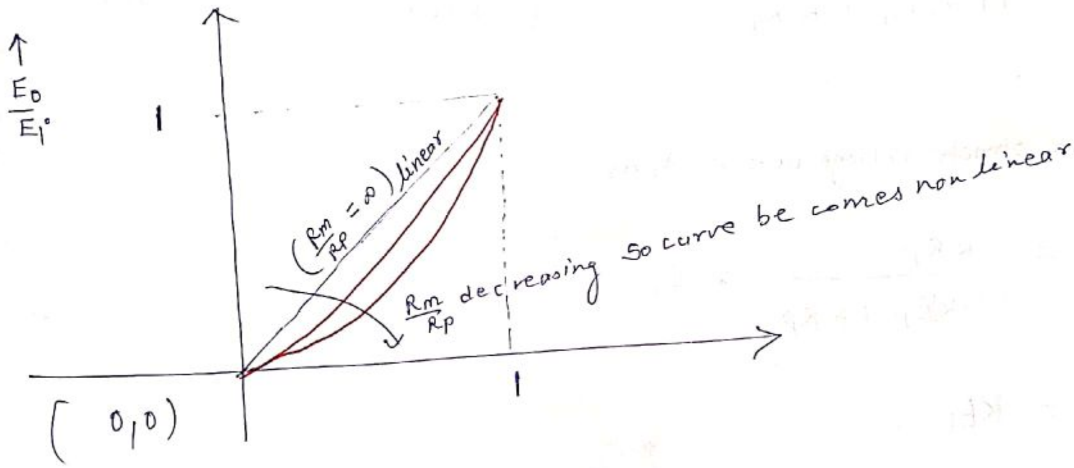
device meter resistance almost infinite ($R_m \to \infty$)

Graph: $E_o = K E_i$

$\frac{E_o}{E_i} = K = \frac{x_i}{x_t}$

A2.





$\frac{x_i}{x_t} = K \longrightarrow$

Conclusion

$\frac{R_m}{R_p} \longrightarrow \infty \Rightarrow$ Linear relationship

$\frac{R_m}{R_p} \longrightarrow$ Finite $\Rightarrow$ Non linear relationship

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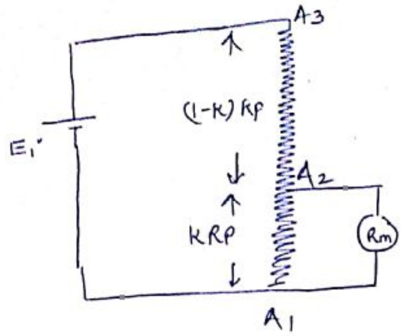
Method - II



We shall apply thevenin's theorem's here. We shall draw the thevenin's equivalent ckt of above figure.

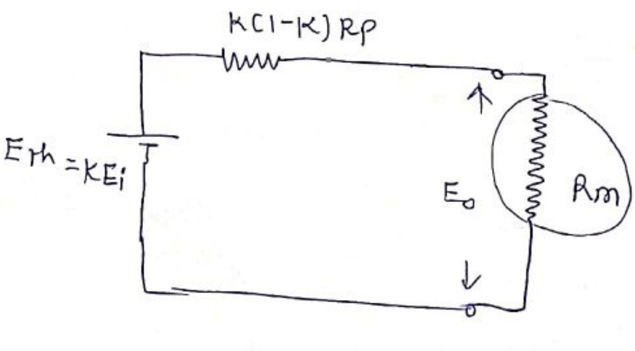
$$R_{th} = (1-K)\, R_p \,||\, K R_p$$

$$= \frac{(1-K) R_p \cdot K R_p}{(1-K) R_p + K R_p} = \frac{K(1-K)\, R_p}{(1-K+K)} = K(1-K) R_p$$

E_{th} = openckt voltage across $A_1 - A_2$

$$E_{th} = \frac{K R_p}{(1-K)R_p + K R_p} * E_i$$

$$E_{th} = K E_i$$



$$E_o = \frac{R_m}{R_m + K(1-K) R_p} * K E_i$$

$$\frac{E_o}{E_i} = \frac{K\, R_m}{R_m + K(1-K)\, R_p} = \frac{K}{\left[1 + K(1-K)\, \frac{R_p}{R_m}\right]}$$

Note ⇒ As the ratio of $\frac{R_m}{R_p}$ decreases, the non-linearity goes on increasing. Thus in order to keep linearity the value of $\frac{R_m}{R_p}$ should be as large as possible. However when we have to measure output voltage with a given meter, the resistance of the potentiometer Rp should be as small as possible.

* Linearity and Sensitivity ⇒

In order to get a high sensitivity the output voltage e_o should be high which in turn requires a high input voltage e_i.

$$\text{Power} = V * I$$

$$P_i = E_i * I_i$$

$$P_i = \frac{E_i^2}{R_p}$$

for (P_i) minimum = $E_i^2 \downarrow$ and $R_p \uparrow$

for low power dissipation ⇒ $E_i \downarrow$ and $R_p \uparrow$

*Also for high sensitivity ⇒ E_i large ⇒ R_p high

* Thus linearity and sensitivity are therefore two conflicting requirements. If Rp is made small the linearity improves but a low value of Rp requires a lower input voltage E_i in order to keep down the power dissipation and low value of E_i results

in a lower value of output voltage E_o resulting in the lower sensitivity.

Thus the choice of potentiometer resistance R_p has to be made considering both the linearity and sensitivity and a compromise between two conflicting requirements has to be struck.

Error ⇒ $\quad$ Error = MV − TV

$$\% \text{ Error} = \frac{MV - TV}{TV}$$

$$\text{Error} = MV - TV$$

$$\text{Error} = \frac{K E_i}{1 + K(1-K)\frac{R_p}{R_m}} - K E_i$$

$$\text{Error} = \frac{-K^2(1-K)\,R_p/R_m}{1 + K(1-K)\frac{R_p}{R_m}}\, E_i$$

$$\boxed{\% \text{ Error} = \frac{-K(1-K)\frac{R_p}{R_m}}{1 + K(1-K)\frac{R_p}{R_m}} \times 100}$$

$R_p/R_m = 1$	$R_p/R_m = 0.1$	$\frac{R_p}{R_m} < 0.1$
$\% \text{ Error} = \frac{K(1-K)}{1+K(1-K)}$ $(\% \text{ Error})_{max} = 12\%$	$(\% \text{ Error})_{max} = 1.5\%$	$(\% \text{ Error})_{max} = 15\left(\frac{R_p}{R_m}\right)$ $(\% \text{ error})_{max}$ will be 1% $(K = 0.67)$

Numerical

Rm

Q.1- The output of a potentiometer is to be read by a 10 kΩ voltmeter (recorder) holding non-linearity to 1%. The family of potentiometer having thermal rating of 5W and resistances ranging from 100Ω to 10kΩ in 100Ω steps are available. Choose from this family the pot (potentiometer) that has the greatest possible sensitivity and meets other requirements. What is the sensitivity of pots are single turn (360°) units.

Solⁿ:- 1% Linearity ⇒

$$1 = 15 \frac{R_p}{R_m}$$

$$R_p = \frac{R_m}{15} = \frac{10{,}000}{15} = 666.7\,\Omega$$

Pots available in this range are 600Ω and 700Ω

[~~100Ω~~, ~~200Ω~~, ~~300Ω~~, ~~400Ω~~, ~~500Ω~~, 600Ω, 700Ω]

When we choose 700Ω - pot ⇒ then nonlinearity gets above 1%.

$$\frac{R_p}{R_m} = \frac{700}{10{,}000} = 0.07$$

So potentiometer resistance may be 100, 200Ω - - - - 600Ω but our requirement is high sensitivity. Therefore we consider highest only.

So we have no alternative but to choose the 600Ω pot.

$$\frac{R_p}{R_m} = \frac{600}{10{,}000} = 0.06$$

$$P = \frac{V^2}{R}$$

maximum excitation voltage ⇒

$$V = \sqrt{P * R}$$

$$= \sqrt{5 * 600}$$

$$= 54.8 \text{ Volt}$$

$$\text{Sensitivity} = \frac{54.8}{360^\circ}$$

$$= 152 \text{ mV/Degree}$$

Soln: Q(2) [Given on next page]:

(a) $R_p = 24 K\Omega$ $\quad x_t = 120 mm \quad x_i = \frac{20+60}{2} = 40 mm$

$R_m = 15 k\Omega \quad \therefore K = \frac{x_i}{x_t} = 40/120 = 1/3$

$$E_o = KE_i = \frac{40}{120} E_i = E_i/3 \quad —①$$

$$E_o = \frac{KE_i}{1+K(1-K)R_p/R_m} = \frac{\frac{40}{120} E_i}{1+\left(\frac{40}{120}\right)\left(1-\frac{40}{120}\right)\frac{24}{15}} = \frac{45}{61} \times \left[\frac{E_i}{3}\right]$$

$$\% \text{ Error} = \frac{MV - TV}{TV} \times 100 = \frac{\frac{45}{61}\left(\frac{E_i}{3}\right) - \left(\frac{E_i}{3}\right)}{\left(\frac{E_i}{3}\right)} \times 100 = -26.22\% \quad \text{Ans}$$

(b) % Error = −3% [It will be negative because here M.V.< T.V.)

$$\% \text{ Error} = -\frac{3}{100} = \frac{-K(1-K)R_p/R_m}{1+K(1-K)\frac{R_p}{R_m}}$$

$$\frac{3}{100} = \frac{\left(\frac{1}{3}\right)\left(1-\frac{1}{3}\right)\frac{24}{R_m}}{1+\left(\frac{1}{3}\right)\left(1-\frac{1}{3}\right)\frac{24}{R_m}}$$

$$\frac{3}{100} = \frac{\left(16/3R_m\right)}{1+\left(16/3R_m\right)} \Rightarrow \frac{3}{100} = \frac{16}{3R_m+16}$$

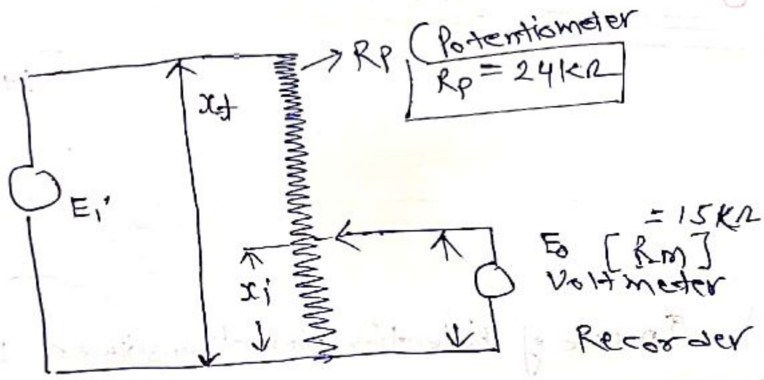
$$\boxed{R_m = 172.44 k\Omega}$$

Q:(2) In a potentiometer transducer, the potentiometer has a total resistance of 24 kΩ for a total wiper travel of 120 mm. During a measurement wiper moves $\boxed{\text{Between 20mm and 60mm}}$ over the potentiometer. $\left[\frac{20+60}{2} = 40mm\right]$

(a) If the voltmeter of 15 kΩ is used to read the output voltage of the transducer, find out the error due to the loading effect at the two measuring points.

(b) If the error due to the loading effect in the above measurement is to be kept within ± 3%, what should be the resistance of the voltmeter?

Sol^n E_i = ______



$R_p = 24k\Omega$

$x_t = 120\,mm$

$\left[\begin{array}{l} x_i \rightarrow 20mm - 60mm \\ \therefore x_i = 40mm \end{array}\right.$ $\left(x_i = \frac{20m + 60mm}{2}\right)$

$R_m = 15k\Omega$

$\boxed{R_m = 15k\Omega}$

Error due to Loading effect = ??

Resistance between 20 mm and 60 mm $\Rightarrow \left(\frac{x_i}{x_t} * R_p = KR_p\right)$

$$= \frac{40\,mm}{120\,mm} * 24\,k\Omega$$

$$= 8\,k\Omega$$

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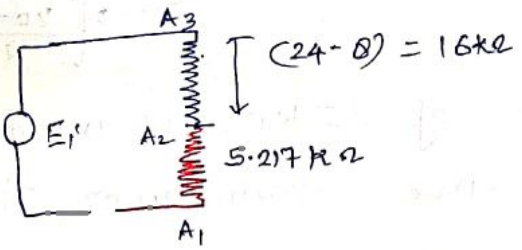
$$(E_0)_{T.V} = \frac{8K\Omega}{24K\Omega} * E_i = \frac{E_i}{3} = 0.3333E_i$$

$R_m = 15K\Omega$ of voltmeter is parallel to $8K\Omega$. So there equivalent resistance

$$= 15 \| 8 = 5.217K\Omega$$



$$(E_0)_{MV} = \frac{5.217K\Omega}{16k\Omega + 5.217K\Omega} * E_i = 0.2459E_i$$

$$\% \text{ Error} = \frac{MV - TV}{TV} \times 100 = \frac{0.2459E_i - 0.3333E_i}{0.3333E_i} \times 100 = -26.22\%$$

Method (ii)

By formula ⇒ % Error

$$= \frac{-K(1-K)\frac{R_P}{R_m}}{1+K(1-K)\frac{R_P}{R_m}} \times 100 = \frac{-\frac{40}{120}\left(1-\frac{40}{120}\right)\frac{24}{15}}{\left[1+\frac{40}{120}\left(1-\frac{40}{120}\right)\frac{24}{15}\right]}$$

$$= \frac{-\frac{1}{3}\left(1-\frac{1}{3}\right)\left(\frac{24}{15}\right)}{1+\frac{1}{3}\left(1-\frac{1}{3}\right)\frac{24}{15}}$$

$$= -26.22\%$$

(b) In case of potentiometer transducer $MV < TV$

So error and therefore % error will always be negative.

$$\therefore -3\% = \frac{-3}{100} = \frac{V_x - 0.3333E_i}{0.3333E_i}$$

$$\Rightarrow -.03 * 0.3333E_i = V_x - 0.3333E_i \Rightarrow V_x = 0.3233E_i$$

$$* \; 0.3233 E_i = \frac{R_{cx}}{16 + R_{cx}} E_i \quad \therefore R_{cx} = 7.6442K\Omega$$

$$* \; R_{cx} = 8K\Omega \| R_{mx}$$

$$\therefore R_{mx} = 172.4K\Omega$$

(Ans)

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Q [3] A potentiometer is used to measure the displacement of a hydraulic drum. The potentiometer is 25 cm long, has a total resistance of 2500 ohms and is operating at 4W with a voltage source. It has linear resistance-displacement characteristics. Determine :

(a) Sensitivity of the potentiometer in Volts/cm (without loading effect.

(b) loading error in the measurement of displacement at actual input displacement of 15 cm, when the potentiometer is connected to a recorder having a resistance of 5000 ohms.

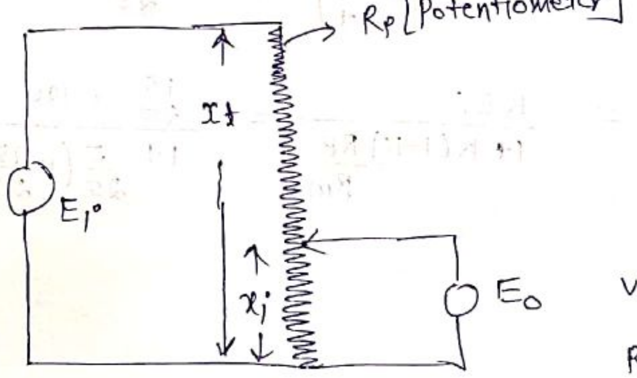
Sol^n —

$R_p = 2500\Omega$

$x_i = 15\,cm$

$x_t = 25\,cm$

Resistance across $x_i = \frac{15}{25} \times 2500$
$= 1500\Omega$



$[K = x_i / x_t]$

$R_m =$ Voltmeter resistance $= 5000\Omega$

$P = I^2 R_p$ $I = \sqrt{P/R_p} = \sqrt{\frac{4}{2500}} = 0.04A$

$E_i = 0.04 * 2500 = 100V$

Sensitivity $= \frac{100\,Volt}{25\,cm} = 4\,Volt/cm$

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(b) $x_i = 15\,cm$

Resistance across $x_i \Rightarrow \frac{15}{25} \times 2500 = 1500\Omega$ (Let this is R_i)

Voltage across $1500\Omega = \frac{1500}{2500} \times 100 = 60$ Volt

Recorder having resistance of 5000Ω has been connected parallel to 1500 ($R_i = 1500\Omega$).

$\therefore$ Combined resistance $= 1500 \| 5000 = \frac{1500 * 5000}{1500 + 5000} = 1153.8$

Hence the total resistance of the circuit is now $= 1153.85 + 1000 = 2153$

Voltage across $R_i = \frac{1153.85}{2153.85} * 100 = 53.57$ Volt

$$\%\text{Error} = \frac{MV - TV}{TV} \times 100 = \frac{53.57 - 60}{60} \times 100 = -10.71\,\%$$

(b) Method (ii) By formula :-

$E_i = 100V$

$\left.\begin{array}{l} x_i = 15\,cm \\ x_t = 25\,cm \end{array}\right\} \therefore K = \frac{x_i}{x_t} = \frac{15}{25}$

$R_p = 2500\,\Omega$, $R_{x_i} = KR_p = \frac{15}{25} \times 2500 =$

$R_m = 5000\,\Omega$

$E_o = KE_i = \left(\frac{x_i}{x_t}\right) E_i = \frac{15}{25} * 100 = 60$ Volt (True value)

$$E_o = \frac{KE_i}{1 + K(1-K)\frac{R_p}{R_m}} = \frac{\frac{15}{25} * 100}{1 + \frac{15}{25}\left(1 - \frac{15}{25}\right) + \frac{2500}{5000}} = \frac{1500}{25 + 15 \times \frac{10}{25} \times \frac{1}{2}}$$

$$E_o = \frac{1500}{28} = 53.57\,V$$

(Measured value)

$$\%\text{Error} = \frac{MV - TV}{TV} \times 100 = \frac{53.57 - 60}{60} \times 100 = -10.71\%$$

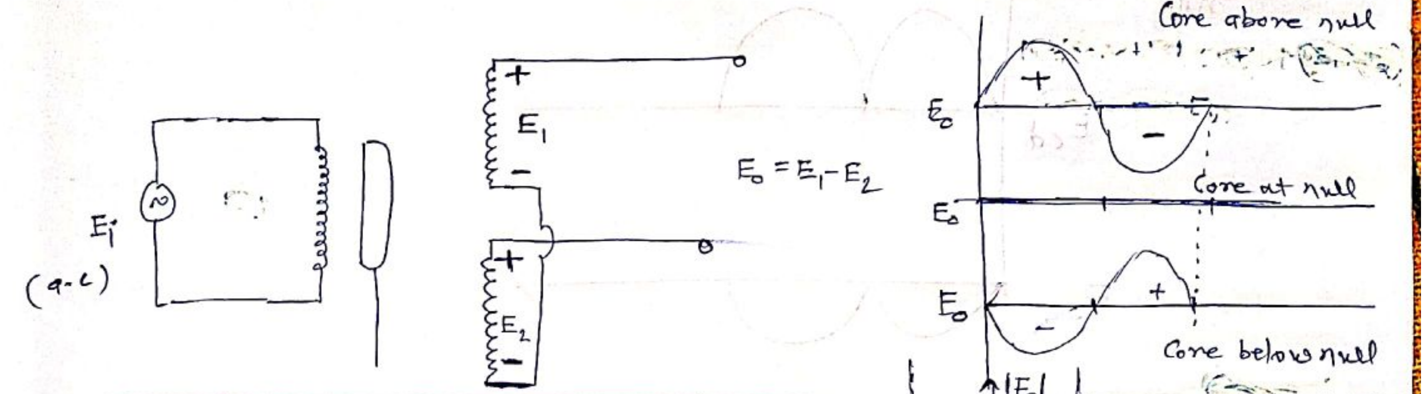
(b) Method (iii) By Direct formula of % Error

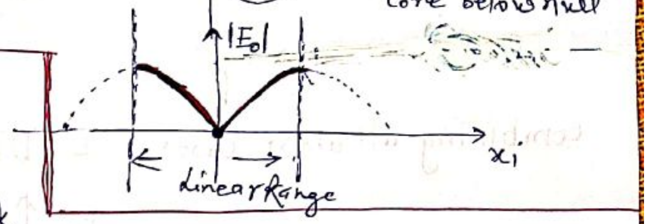
$$\%\text{Error} = \frac{-K(1-K)R_p/R_m}{1 + K(1-K)R_p/R_m} \times 100 = \frac{-\left(\frac{15}{25}\right)\left(1 - \frac{15}{25}\right)\left(\frac{2500}{5000}\right)}{1 + \frac{15}{25}\left(1 - \frac{15}{25}\right)\left(\frac{2500}{5000}\right)}$$

$$= \frac{-\frac{15}{25} * \frac{10}{25} \times \frac{1}{2}}{1 + \frac{15}{25} * \frac{10}{25} \times \frac{1}{2}} \times 100 = \frac{-(3/25)}{(28/25)} \times 100 =$$

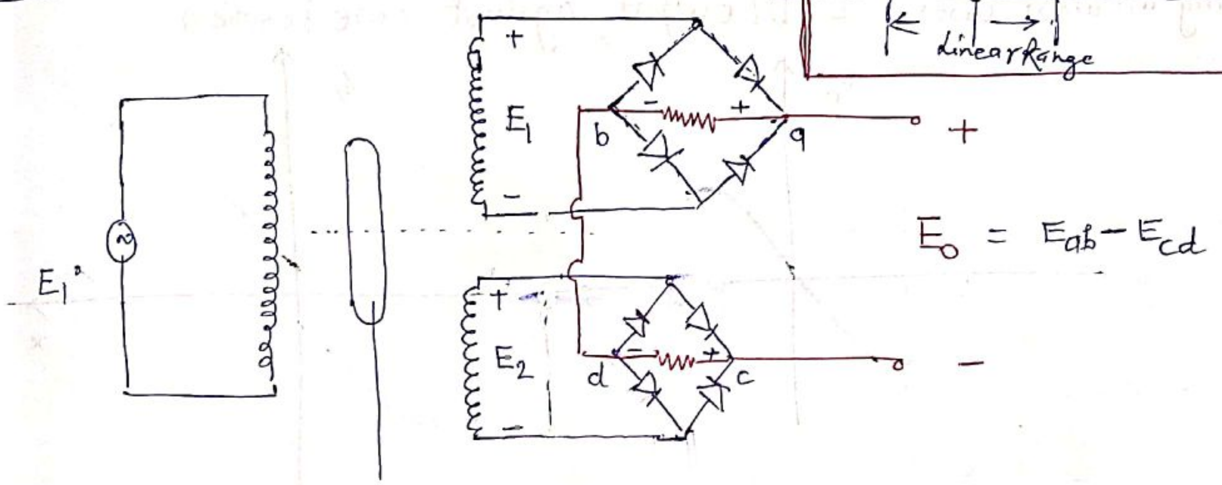
$\boxed{\text{Ans} = -10.71}$

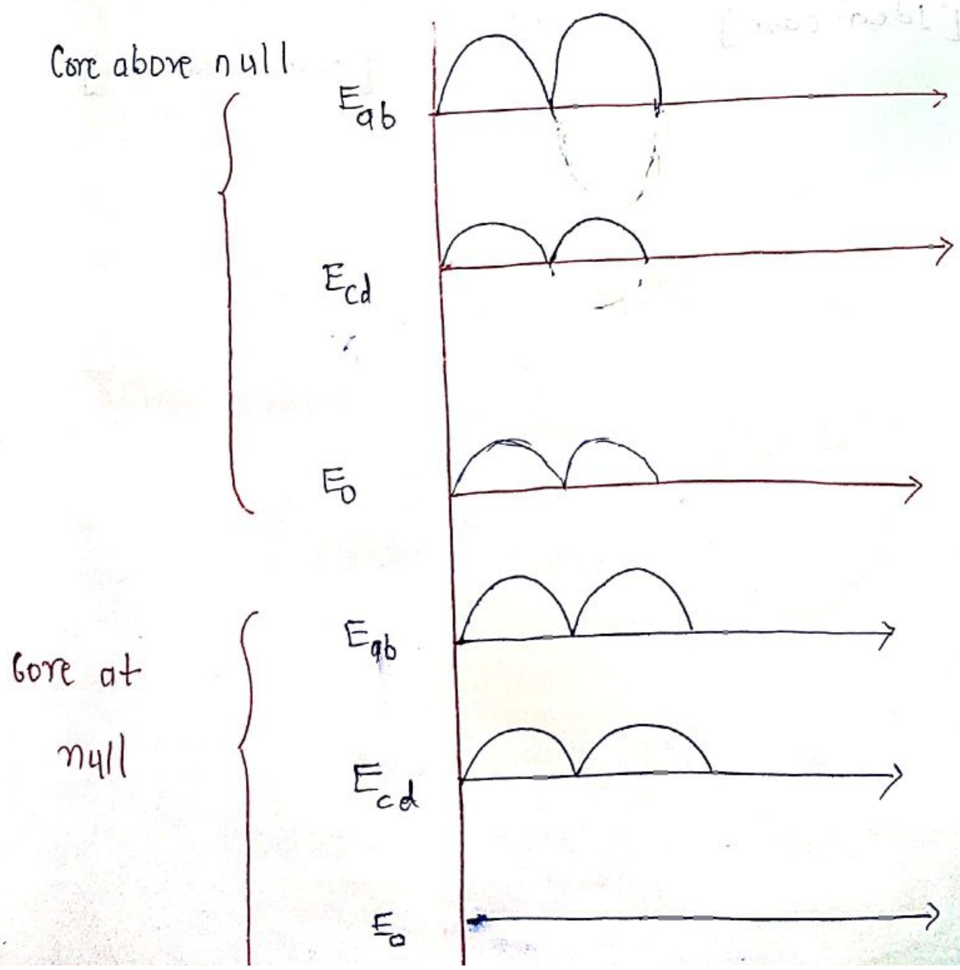
LVDT – [Linear variable differential transformer]



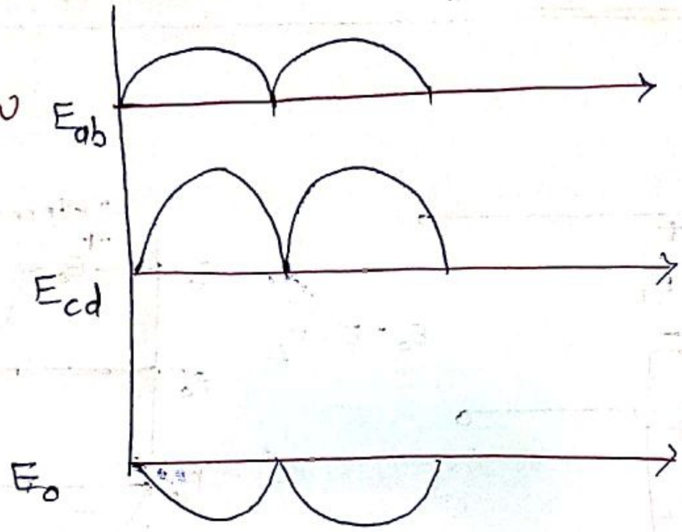


Phase sensitive detection :

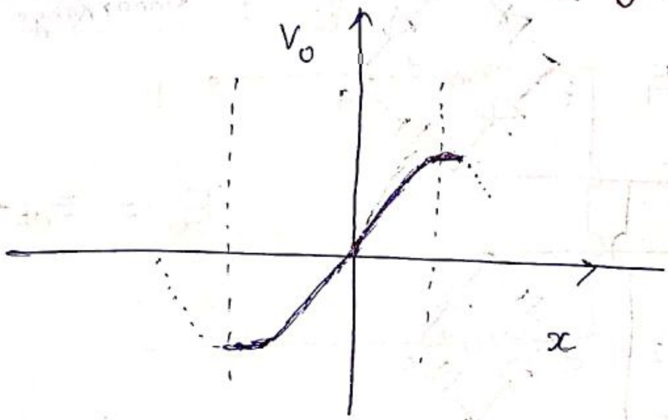




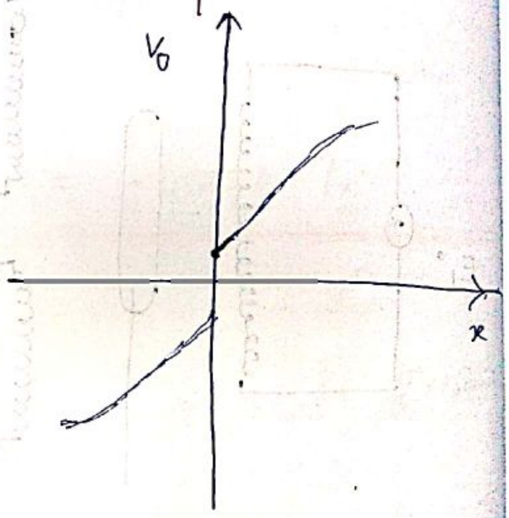
Core below null



Combining all above cases: LVDT output plotted against core position



[ideal case]



[non-ideal case]

Strain Gauge

$$R = \frac{\rho l}{A}$$

change in resistance occurs due to change in l, A and ρ.

* Change in ρ due to strain is called piezo resistive effect.

$$R = \frac{\rho l}{A}$$

$$\log R = \log \rho + \log l - \log A$$

$$\frac{1}{R} dR = \frac{1}{\rho} d\rho + \frac{1}{l} dl - \frac{1}{A} \partial A$$

$$\frac{\partial R/R}{\partial l/l} = 1 + \frac{\partial \rho/\rho}{\partial l/l} - \frac{\frac{1}{A}\partial A}{\partial l/l}$$

$$\frac{\partial R/R}{\partial l/l} = 1 + \frac{\partial \rho/\rho}{\partial l/l} + \frac{\left(-2 dD/D\right)}{\partial l/l}$$

$$G \cdot F = 1 + \frac{d\rho/\rho}{dl/l} + 2\left(\frac{-dD/D}{dl/l}\right)$$

$$GF = 1 + \frac{d\rho/\rho}{\partial l/l} + 2\mu$$

$$\boxed{GF = 1 + 2\mu + \frac{d\rho/\rho}{\partial l/l}}$$

Note :

$$A = \frac{\pi D^2}{4}$$

$$dA = \frac{\pi}{4} 2 D \, dD$$

$$\frac{dA}{A} = \frac{\frac{\pi}{4} \cdot 2D \, dD}{\frac{\pi}{4} D^2}$$

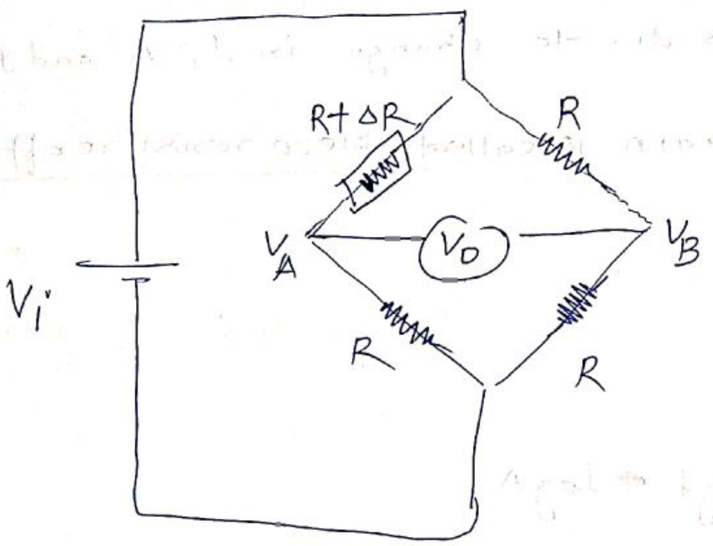
$$= \frac{2 dD}{D}$$

μ = Poisson ratio

$$\mu = -\frac{\partial D/D}{\partial l/l}$$

$$= \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

1 - Quarter bridge



$$V_o = V_A - V_B$$

$$= \frac{R}{R+R+\Delta R} V_i - \frac{R}{R+R} V_i$$

$$= \left(\frac{R}{2R+\Delta R} - \frac{1}{2} \right) V_i$$

$$= \left(\frac{\cancel{2R} - \cancel{2R} - \Delta R}{4R + 2\Delta R} \right) V_i$$

$$= \frac{-\Delta R}{4R+2\Delta R} V_i$$

if $4R >> 2\Delta R$

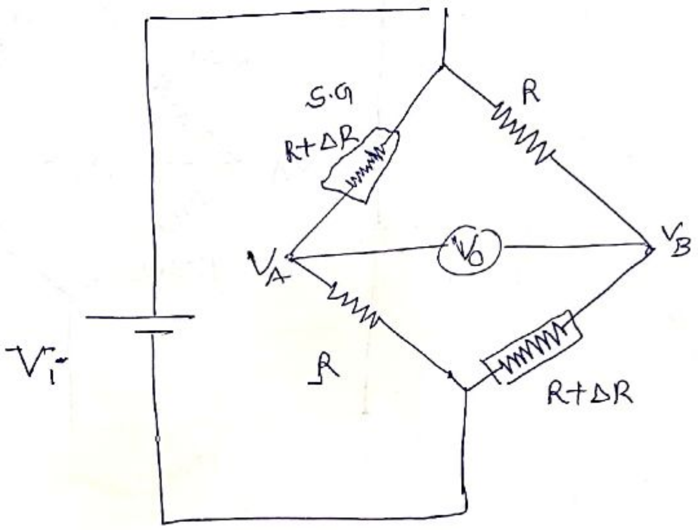
$$V_o = \frac{-\Delta R}{4R} * V_i$$

$$|V_0| = \frac{1}{4} \frac{\Delta R}{R} V_i$$

$$\boxed{|V_0| = \frac{1}{4} \; G.F * strain * V_i}$$ or $$\boxed{V_0 = \frac{1}{4} * G.F. \cdot \epsilon \cdot V_i}$$

[ϵ = strain]

2 - Half bridge -



$$V_0 = V_A - V_B$$

$$= \frac{R}{R+R+\Delta R} V_i - \frac{R+\Delta R}{R+R+\Delta R} V_i$$

$$= \left[\frac{\cancel{R} - \cancel{R} - \Delta R}{2R+\Delta R} \right. V_i$$

$$= \frac{-\Delta R}{2R+\Delta R} V_i$$

$$2R >> \Delta R$$

$$V_0 = -\frac{\Delta R}{2R} V_i$$

$$|V_0| = \frac{1}{2} \frac{\Delta R}{R} V_i$$

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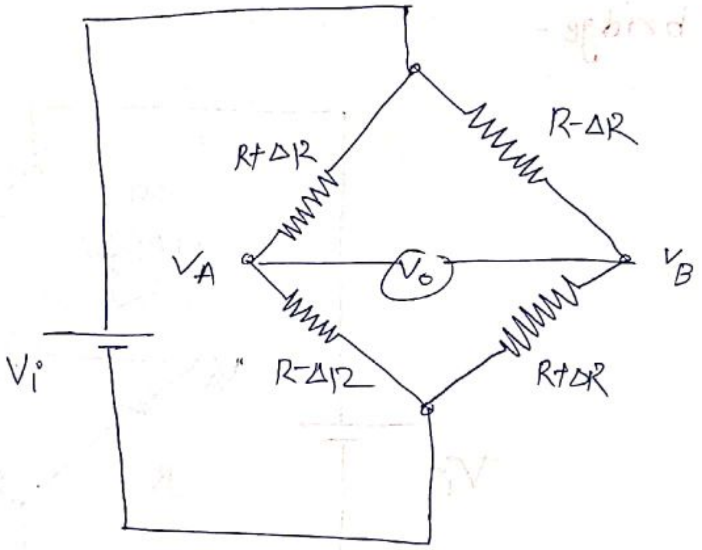
$$\boxed{|V_0| = \frac{1}{2}\ G.F. * \text{stain}\ V_i}$$

(load

[3] Full bridge strain gauge -



$$V_0 = V_A - V_B$$

$$= \frac{R-\Delta R}{(R-\cancel{\Delta R}) + (R+\cancel{\Delta R})} V_i - \frac{R+\Delta R}{(R+\cancel{\Delta R}) + (R-\cancel{\Delta R})} V_i$$

$$= \frac{R-\Delta R}{2R} V_i - \frac{R+\Delta R}{2R} V_i$$

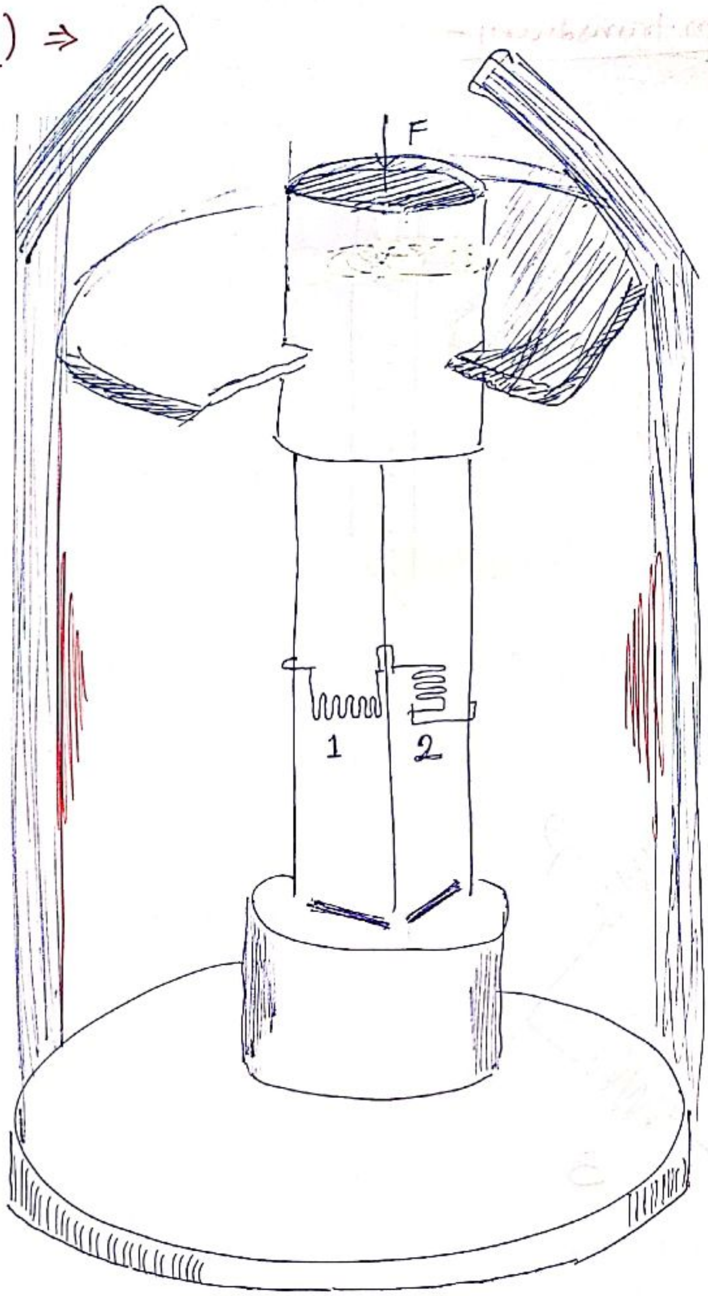
$$= \left[\frac{\cancel{R} - \Delta R - \cancel{R} - \Delta R}{2R}\right] V_i$$

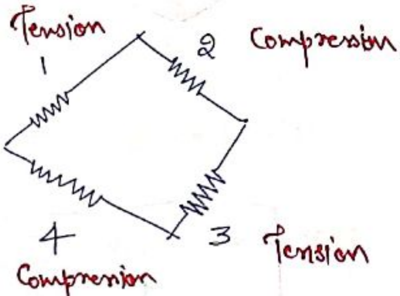
$$= \frac{-2\Delta R}{2R} V_i$$

$$V_0 = \frac{\Delta R}{R} V_i$$

$$\boxed{V_0 = 1 * GF * \text{strain} * V_i}$$

(load cell) ⇒



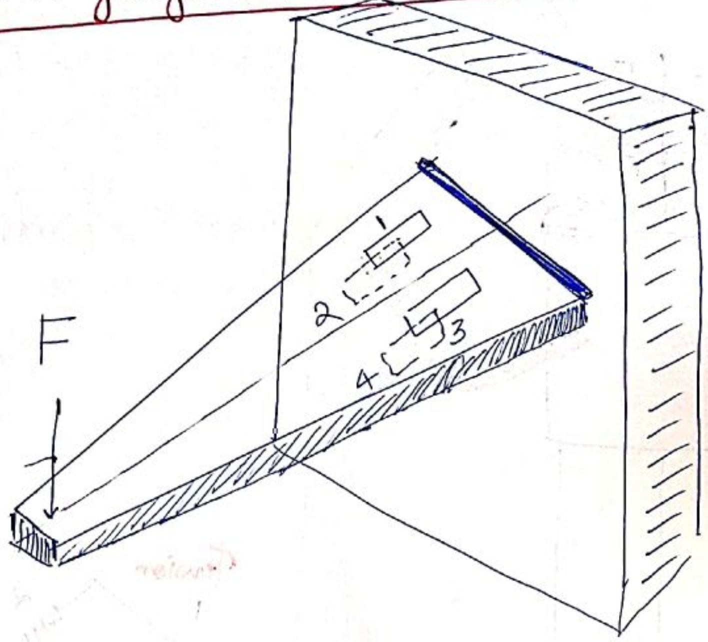


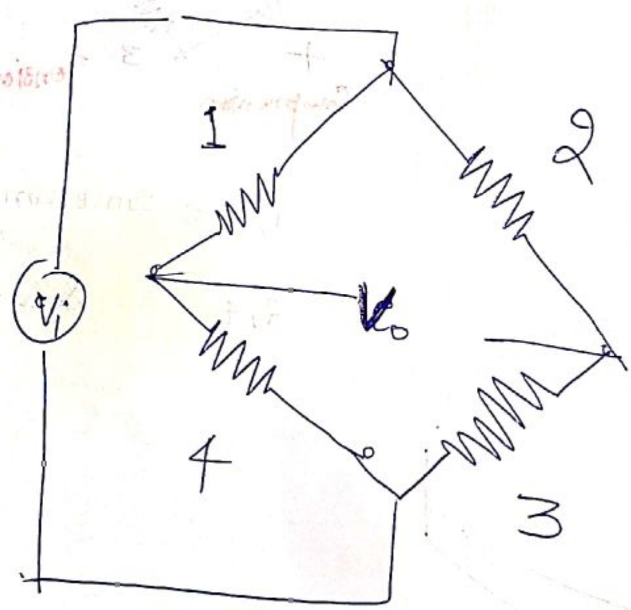
1, 3 ⇒ Same variation

2, 4 ⇒ Same variation

Strain Gauge Load cell

Strain gauge beam transducer -





Numericals.

Q. Consider a wheatstone bridge circuit having all resistances equal to 120 Ω. if each strain gage cannot sustain a power dissipation of more than 0.25W. What is the maximum value of input excitation? if this system gages (4 No.) is used for measuring strains on a tensile specimen (mounted for maximum sensitivity and temperature compensation) what is the output voltage per unit of strain (assume gauge factor = 2)

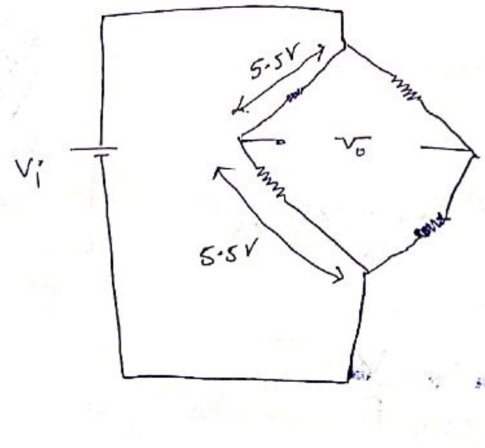
Sol^n ⇒ for each strain gage ⇒

$P = 0.25 W$

$V_i = ?$

$V_0 = ?$

$R = 120 \Omega$



$P = \frac{V^2}{R}$

$V_1 = \sqrt{P \cdot R}$

$= \sqrt{0.25 \times 120}$

$= \sqrt{30}$

$= 5.5$

$V_i = 5.47 + 5.47$

~~$V_i = 11 Volt$~~

$V_i = 10.94 V$

$V_o = \frac{1}{1} \frac{\Delta R}{R} V_i$

$= \frac{1}{1} \cdot G.F * \epsilon V_i$

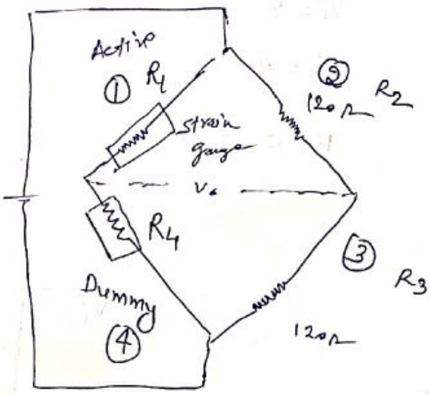
$= \frac{1}{1} * 2 * strain * 10.94 = 21.88 Volt * strain$ $\therefore \frac{V_o}{strain} = 21.88V$

Q: In a wheatstone bridge - leg 1 is an active strain gage of advace alloy and 120Ω resistor, leg 4 is a similar dummy gage and legs 2 and 3 are fixed 120Ω resistors. The maximum gage current is to be 0.030 A.

a) What is the maximum permissible dc Bridge excitation voltage? [use this value for part (b)]

(b) If the active gage is on a steel member, what is the bridge output voltage per 7 MPa of stress?

$Y = 200\,GPa$ $G.F. = 2$



Sol^n:

$I = 0.030\,A$

$I = \frac{V}{R}$

$= \frac{V_1'}{120+120} = 0.30$

$V_1' = .30 * 240$

$V_1 = 7.2\,V$

$R_1 \cdot R_3 = R_2 R_4$

$\frac{R_1}{R_4} = \frac{R_2}{R_3}$

$\frac{R_1 + \Delta R_1}{R_4 + \Delta R_4} = \frac{R_2}{R_3}$

ΔR_1 and ΔR_4 should Same and dummy placed on adjacent to strain gauge with temperature

$$\boxed{V_o = \frac{1}{4} \quad \frac{\Delta R}{R} \quad V_i}$$

$$V_o = \frac{1}{4} \quad G.F \cdot \epsilon \cdot V_i$$

$$= \frac{1}{4} * 2 * \left[\frac{stress}{Y}\right] V_i$$

$$= \frac{1}{4} * 2 * \frac{7 \times 10^6}{200 \times 10^9} * 7.2$$

$$\boxed{V_o = 0.126 \text{ mV}}$$

* variation in straingauges is same in dummy and in active strain gauge due to temperature. Therefore overall effect is zero.

Q: A tensile loded specimen experiences a uniaxial stress of 100 MPa. if two strain gages are mounted, one in the axial direction and other in the transverse direction and connected to the adjacent arms of a Wheat stone's bridge, determine the bridge constant of the measurement system. If the input voltage is 10V and the change in voltage due to loading is 250 microvolts, estimate the average gage factor of the gages. Assume the specimen to be steel with $E = 200$ GPa ; Poisson's ratio $= 0.3$

Sol^n :.

Stress $P = 100$ MPa

Determine bridge constant = ?

$V_i = 10V$

$V_o = 250 \times 10^{-6}$

Average gage factor = ?

$E = 200 \times 10^9$

$\mu = 0.3$

$$V_o = \frac{1}{2} \; GF \cdot \epsilon \, V_i$$

$$V_o = \frac{1}{2} \; GF * \left[\frac{\text{Stress}}{E}\right] V_i$$

$$250 \times 10^{-6} = \frac{1}{2} \cdot G.F. * \frac{100 \times 10^6}{200 \times 10^9} * 10$$

$$G.F. = \frac{250 \times 10^{-6} \times 2 \times 2 \times 10^2}{1}$$

$$G.F. = 1000 \times 10^{-4} = 0.1 .$$