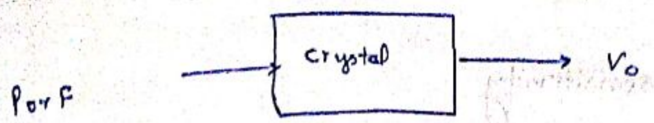
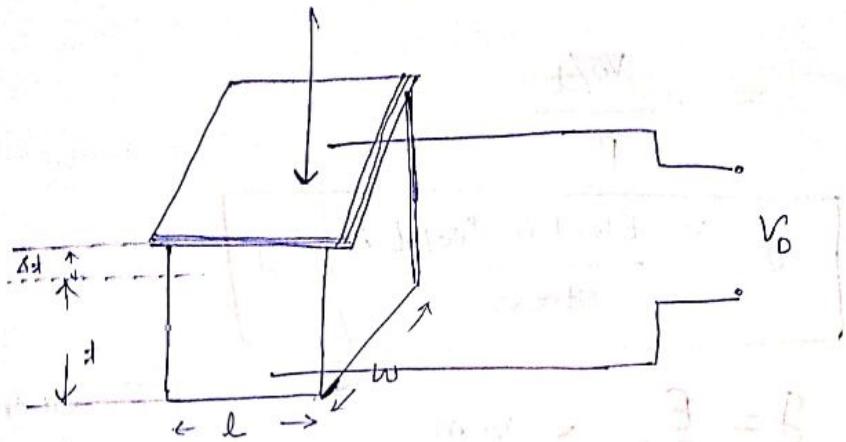
Piezoelectric transducer



Force (F)



charge $\quad Q = K F$

$$K = \frac{Q}{F} = \text{Charge sensitivity} \Rightarrow \frac{\text{Colomb}}{\text{Netwon}}$$

$$V_0 = \frac{Q}{C} = \frac{KF}{C}$$

$$V_0 = \frac{K}{\left(\frac{\epsilon A}{t}\right)} F$$

$$V_0 = \frac{K}{\epsilon} * \frac{F}{A} * t$$

ϵ = dielectric Constant

$\epsilon = \epsilon_0 \epsilon_r$

$$\boxed{V_0 = g \cdot p \cdot t}$$

$$\boxed{g = \frac{K}{\epsilon}}$$ = Voltage sensitivity

$$g = \frac{\text{Volt-meter}}{\text{Newton}}$$

$$g = \frac{V_0}{p \cdot t} = \frac{V_0/t}{p}$$

$$\boxed{g = \frac{\text{Electric field intensity}}{\text{Stress}}}$$

$$g = \frac{\mathcal{E}}{P} \Rightarrow \frac{V\text{-}m}{N}$$

($\mathcal{E}$ = electric field intensity)

P = Pressure or stress

$$g = \frac{K}{\epsilon}$$

$$\therefore \boxed{K = \epsilon \cdot g}$$ Colomb/Newton

Note

$$g = \frac{\text{Electric field intensity}}{\text{Pressure or stress}}$$

$$= \frac{\left(\frac{\text{Volt}}{m}\right)}{\frac{\text{Newton}}{m^2}} = \frac{\text{Volt}}{m} \times \frac{m^2}{\text{Newton}}$$

$$= \frac{\text{Volt} - m}{\text{Newton}}$$

Young modulus :

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta t/t}$$

(t = thickness)

$$Y = \frac{F}{A} * \frac{t}{\Delta t}$$

$$F = Y * \left(\frac{\Delta t}{t}\right) * A$$

$$\text{Sensitivity} = \frac{\text{output}}{\text{Input}} = \frac{V_0}{\Delta t}$$

$$= \frac{g \cdot P \cdot t}{\Delta t}$$

$$= g \frac{F}{A} \frac{t}{\Delta t}$$

$$= g \left[\frac{F}{A} \cdot \frac{t}{\Delta t}\right]$$

$$\boxed{\text{Sensitivity } K = g \cdot Y}$$

unit = $\frac{\text{Volt}}{\text{meter}}$

Note

$$\boxed{K = gY}$$

K = Sensitivity $\left(\frac{V}{m}\right)$

$$\boxed{K = \frac{K_q}{C}}$$

K_q = charge sensitivity $= \frac{Q}{F} = (C/N)$

$V_0 = g P t$

$$\therefore \boxed{g = \frac{V_0}{P \cdot t}}$$

$$\boxed{g = \frac{K}{\epsilon}}$$

g = voltage sensitivity $= \frac{V\text{-}m}{N}$

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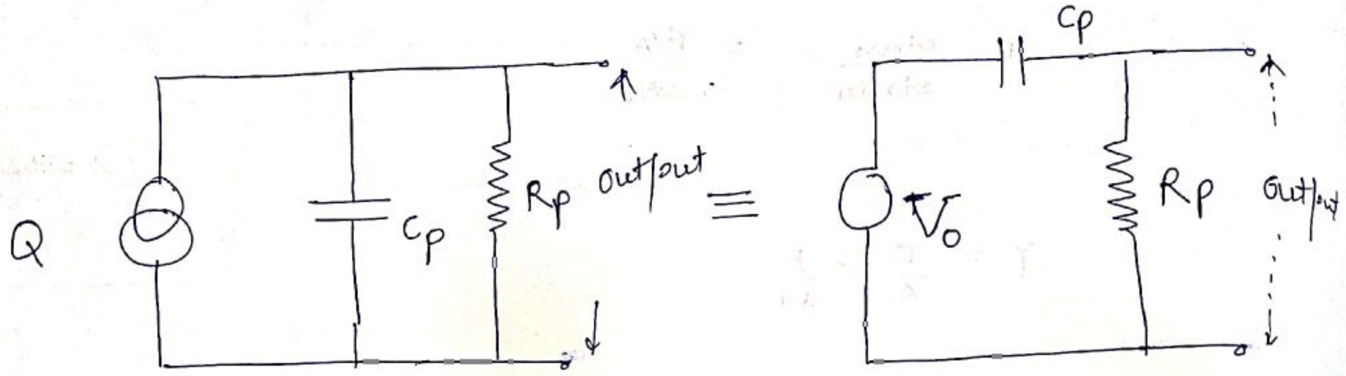
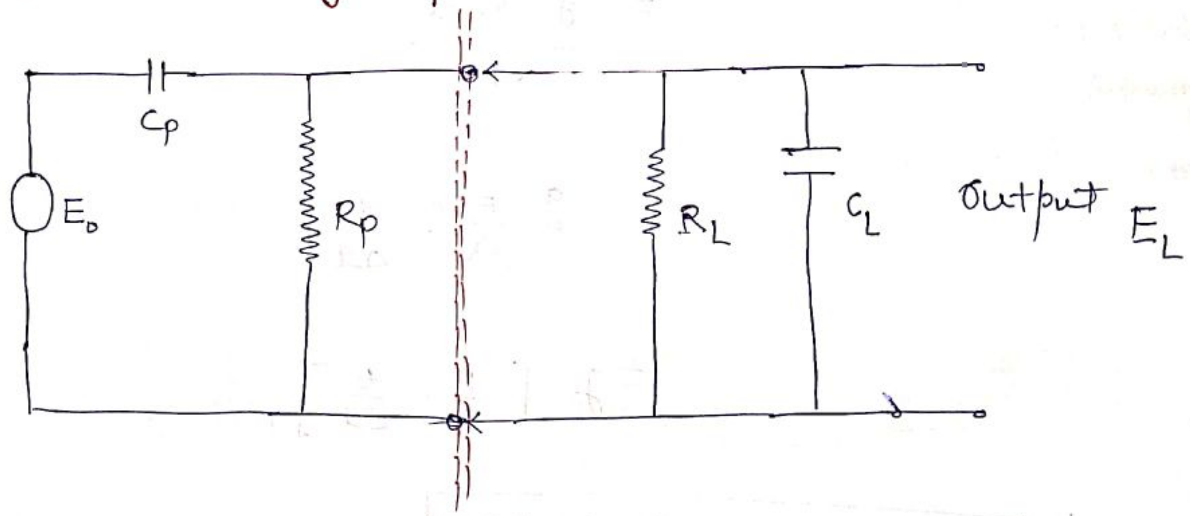
Equivalent circuit of Piezo Electric transducer :



R_p, C_p ⇒ Resistance and capacitance of piezo electric crystal.

$R_p = 0.10 \times 10^{12}\ \Omega$ (very high)

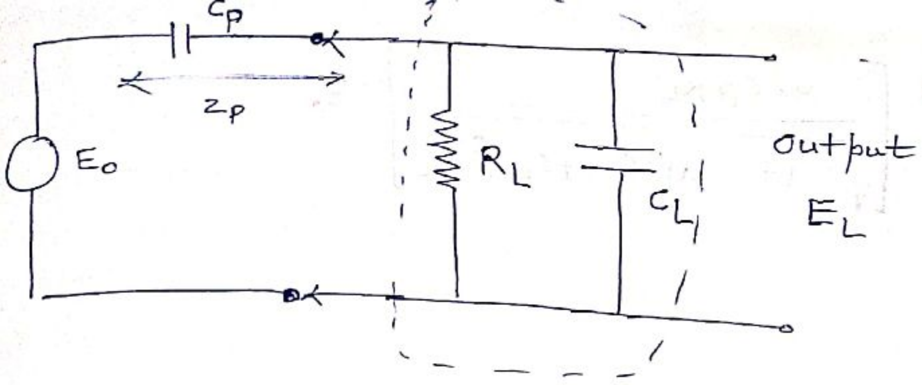
Loading effect and frequency response –



Note :· Leakage resistance of piezoelectric crystal (R_p) is very large ($10^{12}\ \Omega$).

$R_p >> R_L$

So we can neglect the value of R_p.



$$Z_L = R_L \| \frac{1}{j\omega C_L} = \frac{R_L \cdot \frac{1}{j\omega C_L}}{R_L + \frac{1}{j\omega C_L}} = \frac{R_L}{1 + j\omega C_L R_L}$$

$$Z_t = Z_L + Z_p = \frac{R_L}{1 + j\omega C_L R_L} + \frac{1}{j\omega C_p}$$

$$Z_t = \frac{R_L j\omega C_p + 1 + j\omega C_L R_L}{j\omega C_p (1 + j\omega C_L R_L)}$$

$$Z_t = \frac{1 + j\omega R_L (C_p + C_L)}{j\omega C_p (1 + j\omega C_L R_L)}$$

$$E_L = \frac{Z_L}{Z_L + Z_p} * E_o$$

$$= \frac{Z_L}{Z_t} * E_o$$

$$= \frac{R_L}{\cancel{(1 + j\omega C_L R_L)}} * \frac{j\omega C_p [\cancel{1 + j\omega C_L R_L}]}{1 + j\omega R_L (C_p + C_L)} * E_o$$

$$E_L = \frac{j\omega C_p R_L}{1 + j\omega (C_p + C_L) R_L} E_o$$

$$|E_L| = \left[\frac{\omega C_p R_L}{\sqrt{1+\omega^2(C_p+C_L)^2R_L^2}}\right] E_o$$

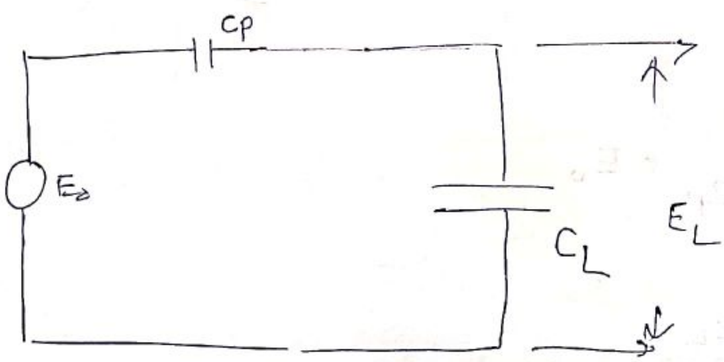
At high frequency:

$$\omega^2(C_p+C_L)^2R_L^2 >> 1$$

$$E_L = \frac{\omega C_p R_L}{\omega(C_p+C_L)R_L} E_o$$

$$\boxed{E_L = \frac{C_p}{C_p+C_L} E_o}$$

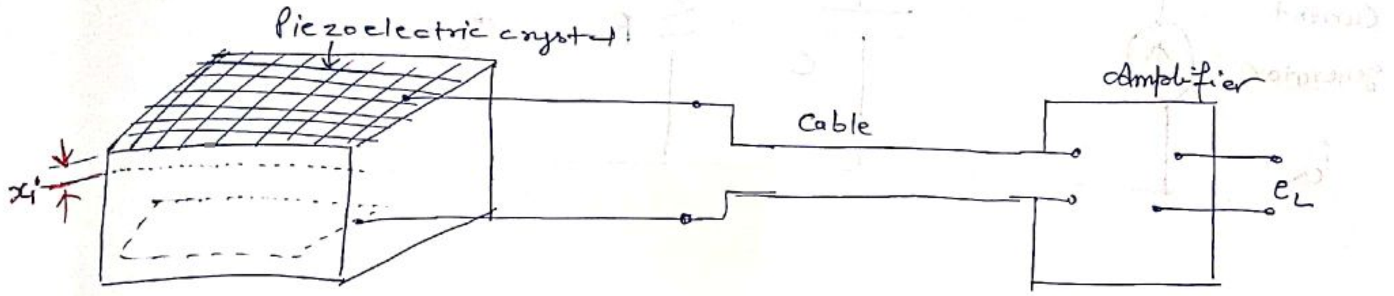
Note At higher frequency: X_L is low So remove R_L.

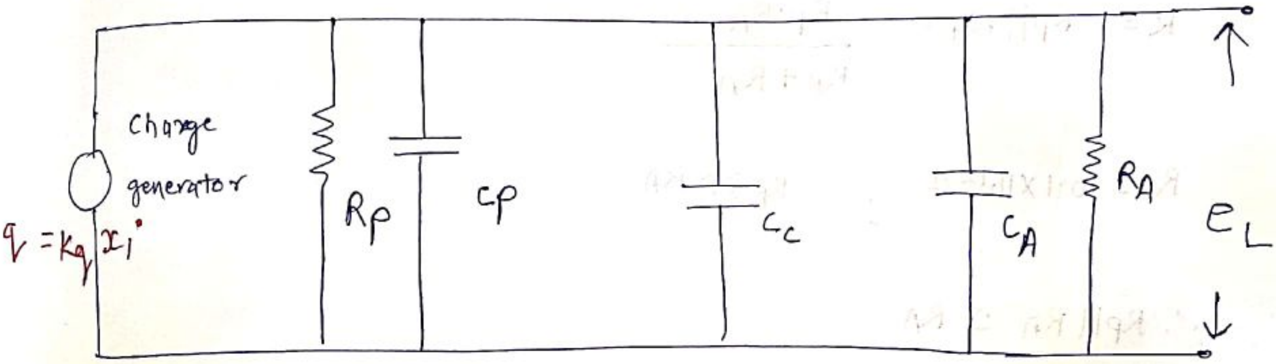


$$E_L = \frac{\frac{1}{j\omega C_L}}{\left(\frac{1}{j\omega C_L}+\frac{1}{j\omega C_p}\right)} E_o$$

$$E_L = \frac{C_p}{C_p+C_L} E_o$$

Frequency response : [Complete setup for measurement of displacement]:





$$q = K_q \; x_i$$

k_q = charge sensitivity = C/meter

q = charge produced

x_i = displacement

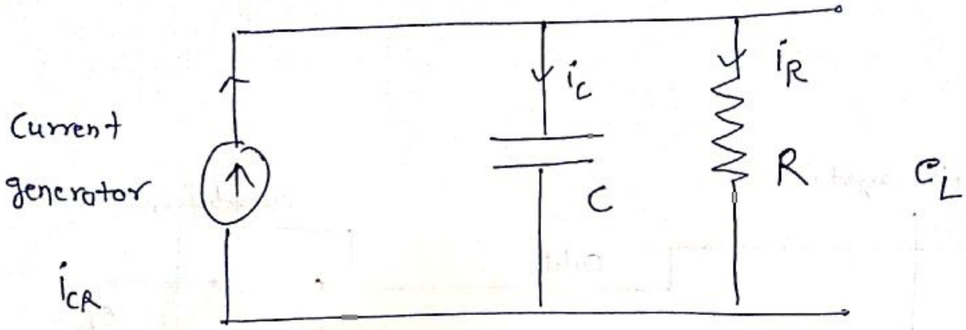
R_p = Leakage resistance of piezoelectric transducer

C_p = Capacitance of piezoelectric transducer

C_c = Capacitance of cable

C_A = Capacitance of amplifier

R_A = Resistance of amplifier.



$$C = C_p + C_c + C_A$$

$$R = R_p \| R_A = \frac{R_p \cdot R_A}{R_p + R_A}$$

$$R_p \doteq 0.1 \times 10^{12}\,\Omega \quad ; \quad R_p >> R_A$$

$$\therefore R_p \| R_A \simeq R_A$$

$$R = R_A$$

$$i_{CR} = \frac{dq}{dt} = \frac{d}{dt}(K_q\, x_i) = K_q \frac{dx_i}{dt} \quad \text{—(1)}$$

$$e_L = e_c = \frac{1}{C}\int i_c\, dt = \frac{1}{C}\int (i_{CR} - i_R)\, dt$$

$$\frac{de_L}{dt} = \frac{1}{C}(i_{CR} - i_R)$$

$$C\frac{de_L}{dt} = i_{CR} - i_R$$

$$C\frac{de_L}{dt} = k_q\frac{dx_i}{dt} - \frac{e_L}{R}$$

$$RC\frac{de_L}{dt} + e_L = k_q \cdot R\frac{dx_i}{dt}$$

$$RC\frac{de_L}{dt} + e_L = \frac{k_q}{C} \cdot RC\frac{dx_i}{dt}$$

$$RC\frac{de_L}{dt} + e_L = \left(\frac{K_q}{C}\right)(RC)\frac{dx_i}{dt}$$

$$RC\frac{de_L}{dt} + e_L = K(RC)\frac{dx_i}{dt}$$

$$\tau\frac{de_L}{dt} + e_L = K\tau\frac{dx_i}{dt}$$

$\left[K = \frac{K_q}{C}\right]$ Sensitivity = $\frac{\text{Volt}}{\text{meter}}$

$[\tau = RC]$

taking laplace transform:

$$\tau s\, E_L(s) + E_L(s) = K\tau s\, X_i(s)$$

$$(\tau s + 1)\, E_L(s) = K\tau s\, X_i(s)$$

$$\boxed{\frac{E_L(s)}{X_i(s)} = \frac{K\tau s}{1 + \tau s}} = \frac{KRCs}{1 + RCs}$$

$$\frac{E_L(s)}{X_i(s)} = \frac{K\tau(j\omega)}{1 + \tau \cdot j\omega}$$

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$$\left|\frac{E_L(s)}{X_i(s)}\right| = \frac{K\ \omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$\boxed{M = \left|\frac{E_L(s)}{K\ X_i(s)}\right| = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}} \quad \text{——(1)}$$

$$K = \frac{K_q}{C} = \text{Sensitivity}$$

**

$$K = \frac{q/x_i}{C} = \frac{q}{x_i} * \frac{1}{C}$$

$$K = \frac{CV}{x_i} * \frac{1}{C}$$

$$K = \frac{V}{x_i} = \frac{\text{Volt}}{\text{meter}}$$

Phase shift.

$$\phi = \tan^{-1}\left[\frac{K\tau s}{1+\tau s}\right]$$

$$= \tan^{-1}\left[\frac{K\tau j\omega}{1+\tau j\omega}\right]$$

$$\phi = 90^\circ - \tan^{-1}(\omega\tau)$$

$$M = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{1}{\omega\tau}\right)^2}}$$

Note :
$\omega >> 1$

$\boxed{M = 1}$ and $\phi = 0^\circ$

when $\omega = 0$ (static input) then $M = 0$ output $= 0$.

From equation (1) we see that steady state response of piezo electric crystal to a constant displacement (x_i) is zero. Therefore piezoelectric crystal cannot be used for measurement of static displacement.

* Piezo-electric crystal is mainly used for dynamic measurement. Under static condition, voltage devloped across the piezoelectric crystal is zero.

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Numerical

Q- A quartz piezoelectric crystal having a thickness of 2 mm and voltage sensitivity of 0.055 $\frac{Vm}{N}$ is subjected to a pressure of 1.5 MN/m^2. Calculate the voltage output. If the permittivity of quartz is 40.6×10^{-12} F/m, calculate its charge sensitivity.

Soln:.

$$V_o = g\ p\ t$$

$$V_o = 0.055\ \frac{Vm}{N} \cdot 1.5 \times 10^{6}\ \frac{N}{m^2} \cdot 2 \times 10^{-3}\ m$$

$$V_o = 165V$$

$$g = \frac{K}{\epsilon}$$

$$K = g\,\epsilon = 0.055 \times 40.6 \times 10^{-12}$$

$$K = 2.23 \times 10^{-12}\ C/N$$

Q: Piezoelectric crystal has dimension of 5mm x 5mm x 1.25mm. Force acting on it is 5N. The charge sensitivity of barium titanate is 150 pC/N and its permittivity is 12.5×10^{-9} F/m. If modulus of elasticity of barium titanate is $12 \times 10^{6}\ \frac{N}{m^2}$, Calculate the strain. Also calculate the charge and capacitance.

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Sol^n:

$5mm \times 5mm \times 1.25mm$

$\Rightarrow l \times b \times h$

$\Rightarrow$ length $\times$ Width $\times$ thickness

$A = l \times B$

$A = 5 \times 5 \times 10^{-6} = 25 \times 10^{-6}\ m^2$

$\text{stress} = F/A = P = \frac{5}{25 \times 10^{-6}} = 0.2\ MN/m^2$

$g = \frac{K}{\epsilon} = \frac{K}{\epsilon_0 \epsilon_r} = \frac{150 \times 10^{-12}}{12.5 \times 10^{-9}} = 12 \times 10^{-3} \frac{Vm}{N}$

$V_0 = g P t = (12 \times 10^{-3}) \times (0.2 \times 10^{6}) * (1.25 \times 10^{-3})$

$= 3V$

$Y = \frac{\text{stress}}{\text{strain}}$

$*$ $\text{strain} = \frac{\text{stress}}{Y} = \frac{0.2 \times 10^{6}}{12 \times 10^{6}}$

$= 0.0167$

$Q = K_q F = 150 \times 10^{-12} * 5$

$Q = 750\ pC$

$C_p = \frac{Q}{V_0} = \frac{750 \times 10^{-12}}{3} = 250\ pF$

Q: Piezoelectric transducer has a capacitance of 1000 pF and K_q of 0.4 $\mu C/mm$. Connecting cable has a capacitance of 300 pF, while the oscilloscope used for readout has an input impedance of 1 MΩ parallel with 50 pF.

a) What is the sensitivity (V/mm) of the transducer alone ?

b) What is the high frequency sensitivity $\left(\frac{V}{mm}\right)$ of the entire measuring system ?

c) What is the lowest frequency that can be measured with 5% amplitude error by the entire system ?

d) What value of C must be connected in parallel to extend the range of 5% error down to 10 Hz ?

e) If the C value of part (d) is used what will the system high frequency sensitivity be ?

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$Sol^n \Rightarrow$

$C_p = 1000\,pF$

$K_q = 0.4\,\mu C/mm \quad = 4$

(a) $\quad$ sensitivity $K = \dfrac{K_q}{C_p}$

$$K_{transducer} = \frac{0.4 \times 10^{-6}}{C_p}$$

$$= \frac{0.4 \times 10^{-6}}{1000 \times 10^{-12}}$$

$$= 0.4 \times 10^{3} \quad \frac{V}{mm}$$

$$= 400 \frac{V}{mm}$$

(b)

$$C_{total} = C_p + C_c + C_A$$

$$= 1000 + 300 + 50$$

$$= 1350\,pF$$

high frequency sensitivity of entire system

$$K_{total} = \frac{K_q}{C_{total}} = \frac{0.4 \times 10^{-6}}{1350 \times 10^{-12}} = 296 \times 10^{-6} \times 10^{6}\ V/mm$$

$$= 296 \frac{V}{mm}$$

(c) $R_{eq} = R_{Piezo} \parallel R_{oscilloscope}$

$R_{eq} = R_{oscilloscope}$

$R_{eq} = 1 M\Omega$

$C_{eq} = 1350 pF.$

$\tau = R_{eq} \cdot C_{eq} = 1 \times 10^{6} \times 1350 \times 10^{-12} = 1350 \times 10^{-6}$

$= 1.350 \times 10^{-3}$ second

$$\boxed{M = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}}$$

formula

$$0.95 = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$(0.95)^2 = \frac{(\omega\tau)^2}{1+(\omega\tau)^2}$$

$$1+(\omega\tau)^2 = \frac{(\omega\tau)^2}{(0.95)^2} = 1.108(\omega\tau)^2$$

$$1 = 1.108(\omega\tau)^2 - (\omega\tau)^2$$

$$1 = 0.108(\omega\tau)^2$$

$$\therefore \omega\tau = \sqrt{\frac{1}{0.108}} = \sqrt{9.256}$$

$$\boxed{\omega\tau = 3.04} \text{ ——— (i)}$$

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$$(2\pi f) * (1.35 \times 10^{-3}) = 3.04$$

$$f = \frac{3.04}{2\pi \times 1.35 \times 10^{-3}}$$

$$f = \frac{3.04 \times 10^{3}}{8.478}$$

$$\boxed{f = 358.575 \text{ Hz}}$$

(d)

It is now required that ~~a~~ frequency of 10Hz is to be measured with 5% error.

$$\omega\tau = 3.04 \qquad \left[\text{from eq}^n \text{ ① of part c}\right]$$

$$\tau = \frac{3.04}{2\pi * 10}$$

$$\tau = \frac{3.04}{62.8} = 48.4076 \times 10^{-3}$$

~~or~~ 48.4×10^{-3} second

$$RC^* = 48.4076 \times 10^{-3}$$

$$C^* = \frac{48.4076 \times 10^{-3}}{1 \times 10^{6}}$$

$$= 48.4076 \times 10^{-9}$$

$$C^* = 48407.6 \times 10^{-12} = 48407.6 \text{ pF}$$

$$\boxed{C_{total} = 48407.6 \text{ pF}}$$

$\therefore$ External capacitance required $= C_e = 48407.6$ pf $- 1350$ pf

$$C_e = 47057.6 \text{ pF}$$

$$\boxed{C_e = 47057.6 \text{ pF}}$$

(e) High frequency sensitivity with external capacitance

tc].

$$K = \frac{K_{qr}}{C_{total}} = \frac{0.4 \times 10^{-6}}{48407.6 \times 10^{-12}} \frac{V}{mm}$$

$$= 8.263 \frac{V}{mm}$$

$$\simeq 8.33 \frac{V}{mm}$$

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Q- A barium titanate piezoelectric transducer is being used to measure pressure varying sinusoidally at a frequency of 50 Hz. Thickness of the crystal is 3mm, diameter 5mm, dielectric constant = 15 nF/m, modulus of elasticity = 120 GPa, Constant g for the crystal = $15 \times 10^{-3} \frac{V-m}{N}$, resistance of the crystal = 1500 GΩ.

It is connected through a cable having a capacitance of 100 pF. The amplifier has an input impedance of 2 MΩ in parallel with a capacitance of 2000 pF. Determine $\left|\frac{E_o}{P}\right|$ and phase lag.

Solⁿ :

$$K_q = KC \quad \text{—— (1)}$$

$$K_q = gYC \quad \text{—— (2)} \qquad \boxed{\because K = gY}$$

Y = Young modulus

$$K_q = gY\left(\frac{\epsilon_o \epsilon_r A}{t}\right) \quad \text{—— (3)}$$

Given :

$g = 15 \times 10^{-3} \frac{V-m}{N}$

$\epsilon_o \epsilon_r$ = dielectric constant = 15 nF/m

A = Area

Y = 120 GPa

t = 3 mm

$$K_q = \frac{(15\times10^{-3})\cdot(120\times10^{9}) * \left[15\times10^{-9} * \frac{\pi}{4}(5\times10^{-3})^2\right]}{3\times10^{-3}}$$

$$K_q = 177 \ \frac{mC}{meter}$$

$$C_{crystal} = \frac{\varepsilon_0 \varepsilon_r A}{t} = \frac{15\times10^{-9}\times\frac{\pi}{4}\times(5\times10^{-3})^2}{3\times10^{-3}}$$

$$= 98.12 \ pF.$$

$C_{amp} = 2000 \, pF$

$C_{able} = 100 \, pF$

$$C = C_{crystal} + C_{Cable} + C_{amp}$$

$$= 98.12 + 100 + 2000 = 2198 \, pF$$

$$R_{eq} = R_{amp} \| R_{crystal} = \frac{R_{amp}\cdot R_{crystal}}{R_{amp}+R_{crystal}} \simeq R_{amp} = 2M\Omega$$

$$\tau = R_{eq}\cdot C_{eq} = 2\times10^{6}\times 2198\times10^{-12} = 4.4 \, ms$$

$$|E_0| = K \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} * |X_i|$$

$$\left|\frac{E_0}{P}\right| = \frac{K_q}{C} \frac{1}{P} \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} \Delta t$$

$$= \frac{K_q}{C} \frac{\Delta t}{\left[Y\frac{\Delta t}{t}\right]} \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$\left|\frac{E_0}{P}\right| = \frac{K_q}{C} \frac{t}{Y} \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$\left|\frac{E_0}{P}\right| = \frac{(177\times10^{-3})}{(2198\times10^{-12})} * \left(\frac{3\times10^{-3}}{120\times10^{9}}\right) * \frac{[2\pi\times50\ (4.4\times10^{-3})]}{\left[\sqrt{1+[2\pi\cdot50\cdot(4.4\times10^{-3})]^2}\right]}$$

$$\left|\frac{E_0}{P}\right| = 1.63\times10^{-6} \quad \frac{\text{Volt}}{(N/m^2)}$$

$$\phi = 90^\circ - \tan^{-1}(\omega\tau) = 90^\circ - \tan^{-1}[2\pi * 50 * 4.4\times10^{-3}]$$

$$\phi = 90^\circ - \tan^{-1}[1.382]$$

$$\phi = 90^\circ - 54.10^\circ$$

$$\boxed{\phi = 35.87^\circ}$$

Capacitive transducer

$$C = \frac{\epsilon A}{d}$$

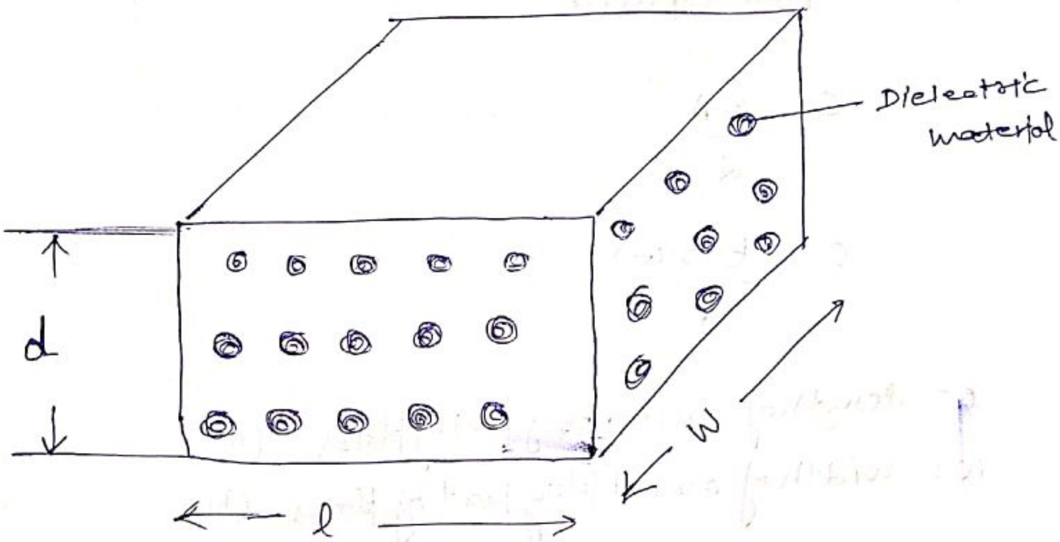
$\epsilon = \epsilon_r \epsilon_0$ (F/m)

ϵ_r = Relative permittivity

ϵ_0 = Permittivity of free space $= 8.85 \times 10^{-12}$ F/m

A = Overlapping area of plates (m^2)

d = Distance between two plates (m)



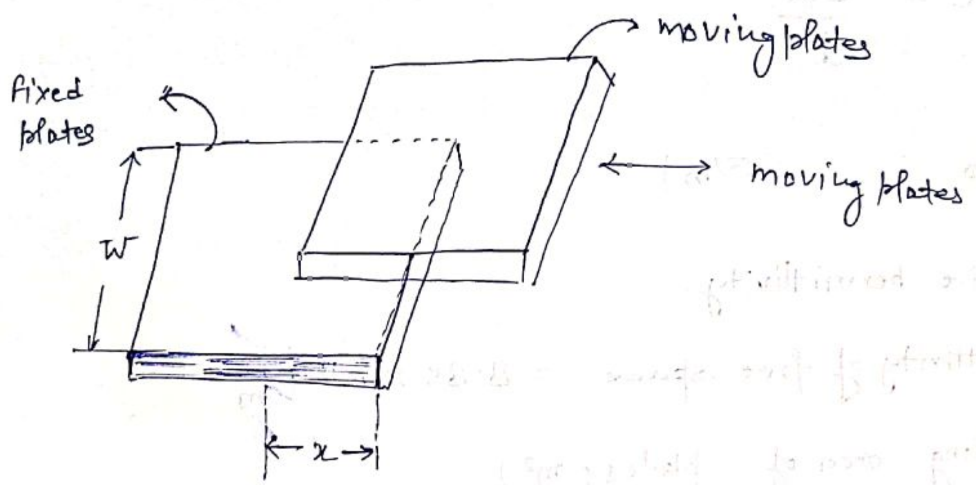
Capacitive transducer works on the principle of change in capacitance which may be caused by:

a) Change in overlapping area

b) Change in the distance between plates

c) Change in dielectric constant.

a) Transducer using change in area of plates -



For a parallel plate capacitor :

$$C = \frac{\epsilon A}{d}$$

$$C = \frac{\epsilon (x w)}{d}$$

x = length of overlapping part of plates (m)
w = width of overlapping part of plates (m)

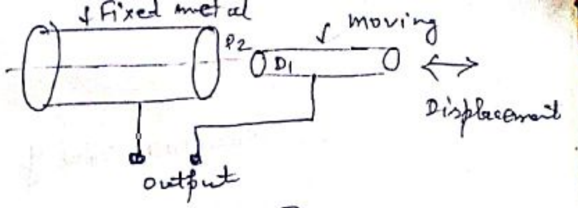
$$\text{sensitivity} = \frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left[\frac{\epsilon x w}{d}\right]$$

$$\boxed{S = \frac{\epsilon w}{d}} \quad F/m$$

* Sensitivity is constant and therefore there is linear relationship beween capacitance and displacement.

* This type of a capacitive transducer is suitable for measurement of linear deisplacement ranging from 1 mm to 10 mm.

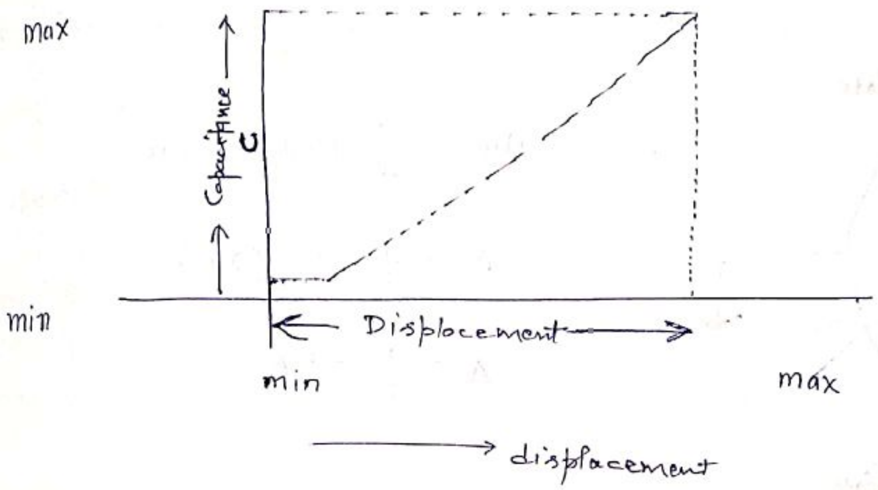
For a cylindrical capacitor ⇒



$$C = \frac{2\pi \epsilon x}{\log_e \left[\frac{D_2}{D_1}\right]} \quad F$$

x = length of overlapping part of cylinders.
D_1 and D_2 are the diameters of electrode

$$\therefore S = \frac{\partial C}{dx} = \frac{2\pi \epsilon}{\log_e \left[\frac{D_2}{D_1}\right]} \quad F/m$$



Capacitance displacement Curve

Measurement of angular displacement ⇒

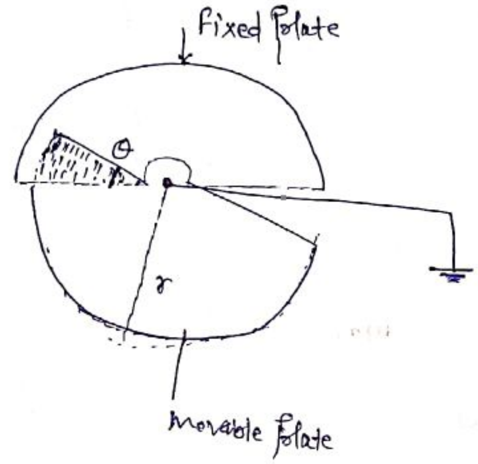
* The principle of change of capacitance with change in area can be employed for measurement of angular displacement.

The figure, shown below, shows a two plate capacitor. One plate is fixed and other is movable. The angular displacement to be measured is applied to movable plate. The angular displacement changes the effective area between the plates and thus changes the capacitance. The capacitance is maximum when the two plates completely overlap each other i.e when $\theta = 180°$.

∴ Maximum value of capacitance $= C_{max} = \frac{\epsilon A}{d} = \frac{\epsilon\left(\frac{\pi r^2}{2}\right)}{d}$

$$= \frac{\pi \epsilon r^2}{2d}$$

[note: $A = \frac{\pi r^2}{2}$ maximum coverd $= 50\%$.]



Area $= \frac{1}{2}$ (radius) * arc

$A = \frac{1}{2} r \cdot (r\theta)$

$A = \frac{1}{2} r^2 \theta$.

Capacitance at angle $\theta = \frac{\epsilon A}{d} = \frac{\epsilon}{d} * \left(\frac{1}{2} r^2 \theta\right)$

$$C = \frac{\epsilon \theta r^2}{2d}$$

θ = angular displacement in radian

$$S = \frac{\partial C}{\partial \theta} = \frac{d}{d\theta}\left[\frac{\epsilon \theta r^2}{2d}\right]$$

$$S = \frac{\partial C}{\partial \theta} = \frac{\epsilon r^2}{2d}$$

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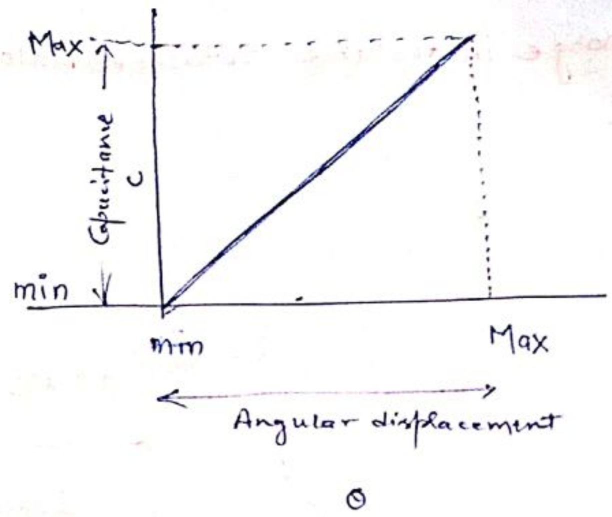
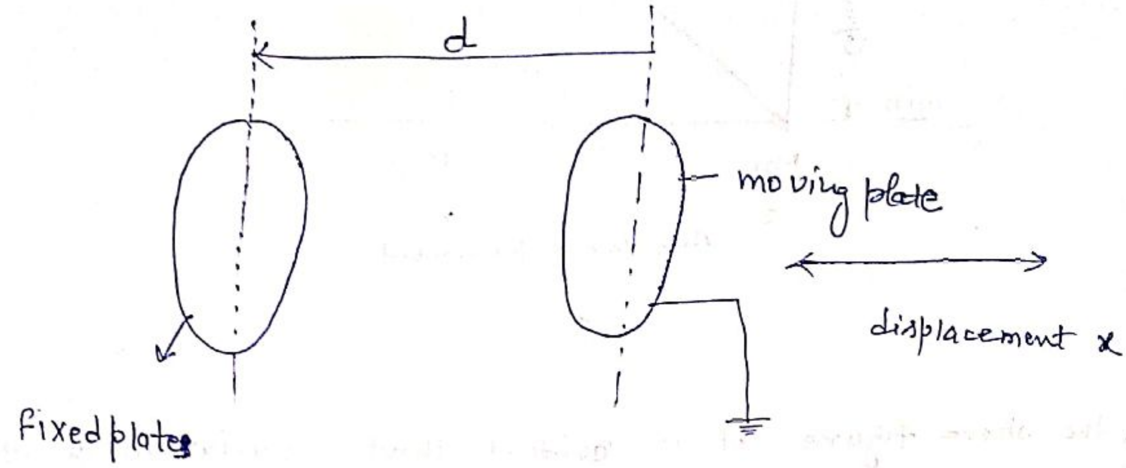


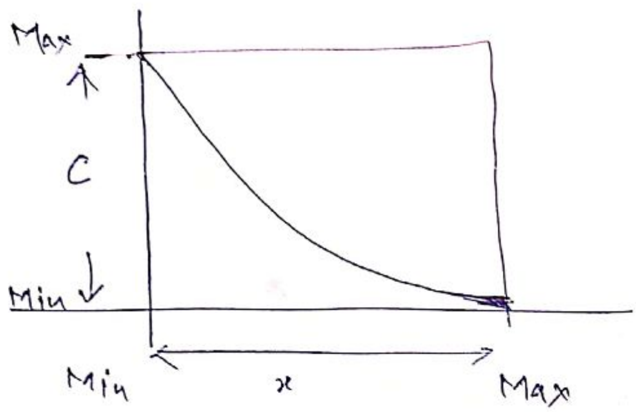
From the above figure it is evident that variation of capacitance with angular displacement (θ) is linear. It should be understood that the above mentioned capacitive transducer can be used for a maximum angular displacement of 180°.

b> Transduce using change in distance Between plates —



* $C \propto \dfrac{1}{\text{distance Between the plates}}$

* Response of this transducer is not linear. Thus this transducer is useful only for measurement of extremely small displacements

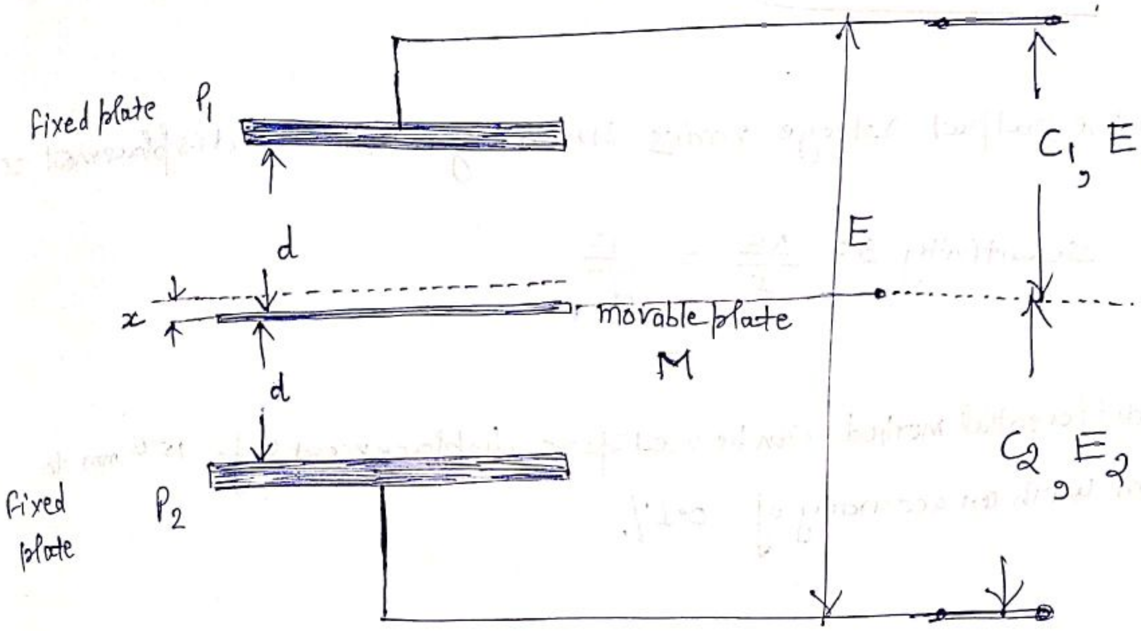


$$C = \frac{\epsilon A}{d}$$

$$S = \frac{\partial C}{\partial d} = \frac{\partial}{\partial d}\left[\frac{\epsilon A}{d}\right]$$

$$S = -\frac{\epsilon A}{d^2}$$

Differential arrangement – A linear characteristic can be achieved by using a differential arrangement for capacitive displacement transducers.



Let the movable plate is moved up due to displacement x.

$$C_1 = \frac{\epsilon A}{d-x} \qquad C_2 = \frac{\epsilon A}{d+x}$$

$$E_1 = \frac{C_2}{C_1+C_2} E = \frac{\left(\frac{\epsilon A}{d+x}\right)}{\left(\frac{\epsilon A}{d-x}\right) + \left(\frac{\epsilon A}{d+x}\right)}$$

$$E_1 = \frac{d-x}{2d} E$$

$$E_2 = \frac{C_1}{C_1+C_2} E = \frac{\left(\frac{\epsilon A}{d-x}\right)}{\left(\frac{\epsilon A}{d-x}\right) + \left(\frac{\epsilon A}{d+x}\right)} E = \frac{d+x}{2d} E$$

$$\therefore \Delta E = E_2 - E_1$$

$$\Delta E = \frac{d+x}{2d}E - \frac{d-x}{2d}E$$

$$\boxed{\Delta E = \frac{x}{d}E}$$

* Therefore the output voltage varies linearly as the displacement

$$\text{sensitivity } S = \frac{\Delta E}{x} = \frac{E}{d}$$

* The differential method can be used for displacements of 10^{-8} mm to 10 mm with an accuracy of 0.1%.

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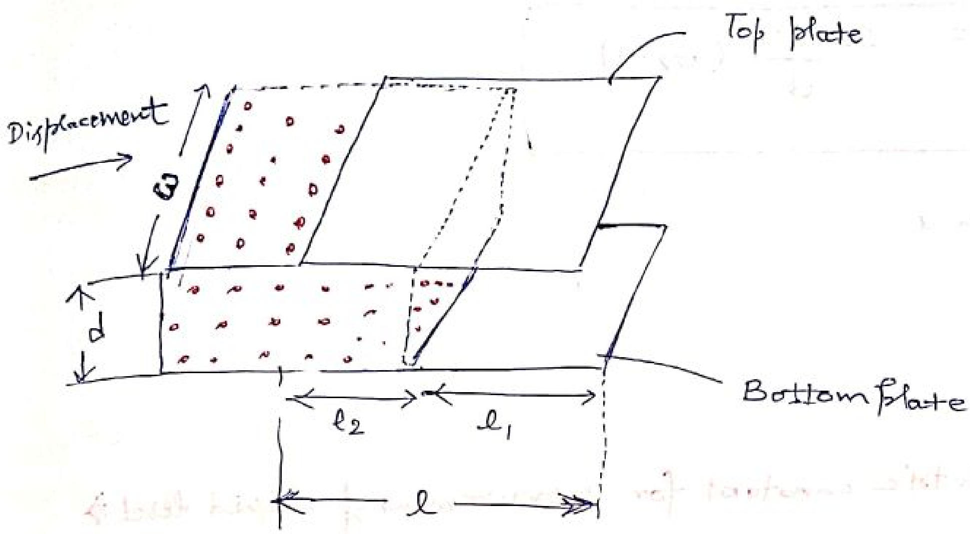
c) variation of dielectric constant for measurement of displacement ⇒



Initial value of capacitance ⇒

$$C_{initial} = \epsilon_0 \frac{wl_1}{d} + \epsilon_0 \epsilon_r \frac{wl_2}{d}$$

$$\boxed{C_{initial} = \frac{\epsilon_0 w}{d}\left[l_1 + \epsilon_r l_2\right]}$$

Let the dielectric be moved through a distance x in the direction indicated. The capacitance changes from c to $c+\Delta c$.

$$\therefore \; c+\Delta c = \epsilon_0 \frac{w}{d}(l_1 - x) + \epsilon_0 \epsilon_r \frac{w}{d}(l_2 + x)$$

$$= \epsilon_0 \frac{w}{d}\left[(l_1 - x) + \epsilon_r (l_2 + x)\right]$$

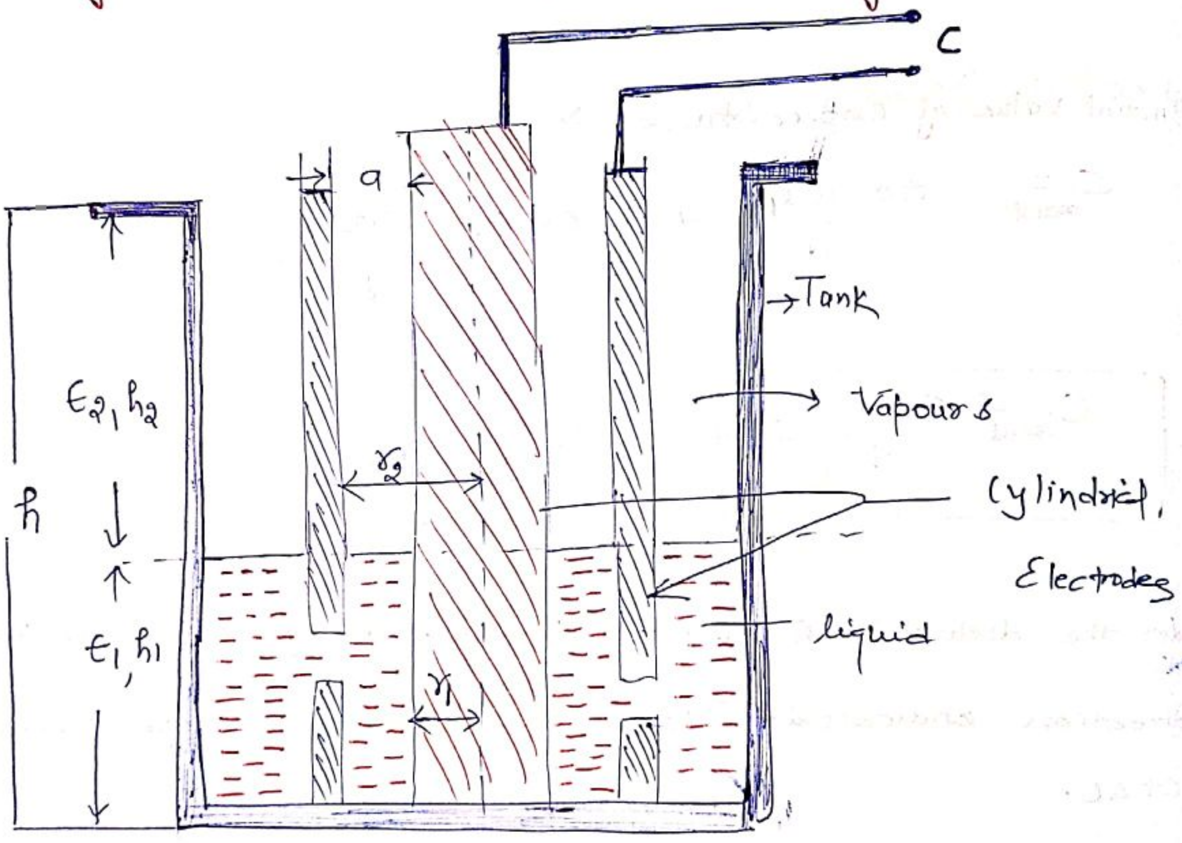
$$= \epsilon_0 \frac{w}{d}\left[l_1 + \epsilon_r l_2\right] + \epsilon_0 \frac{w x}{d}(\epsilon_r - 1)$$

$$C + \Delta C = C_{initial} + \epsilon_0 \frac{w x}{d} (\epsilon_r - 1)$$

$$\boxed{\therefore \Delta C = \epsilon_0 \frac{w x}{d} (\epsilon_r - 1)}$$

$\Delta C \propto x$

* Variation of dielectric constant for measurement of liquid level *



$$C = 2\pi\epsilon_0 \frac{\epsilon_1 h_1 + \epsilon_2 h_2}{\log_e\left(\frac{r_2}{r_1}\right)}$$

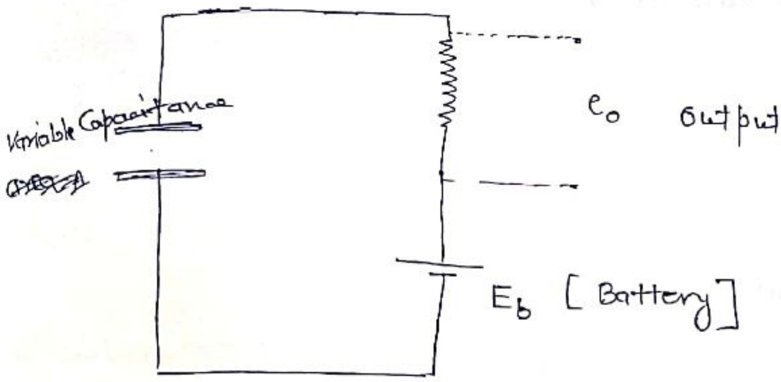
h_1 = height of liquid

h_2 = height of cylinder above liquid.

r_1 = Outside radius of inner cylinder

r_2 = Inside radius of outer cylinder.

Frequency response —



* $C \propto \frac{A}{d}$

Let the distance between plates be x_o when they are stationary under this condition no current flows and output voltage is $e_o = E_B$.

If there is a relative displacement x_i from x_o position, a voltage e_o is produced.

$$\boxed{\frac{E_o(s)}{X_i(s)} = \frac{K\tau s}{1+\tau s}}$$

$$K = \frac{E_B}{x_o} \quad V/m$$

$$\tau = RC$$

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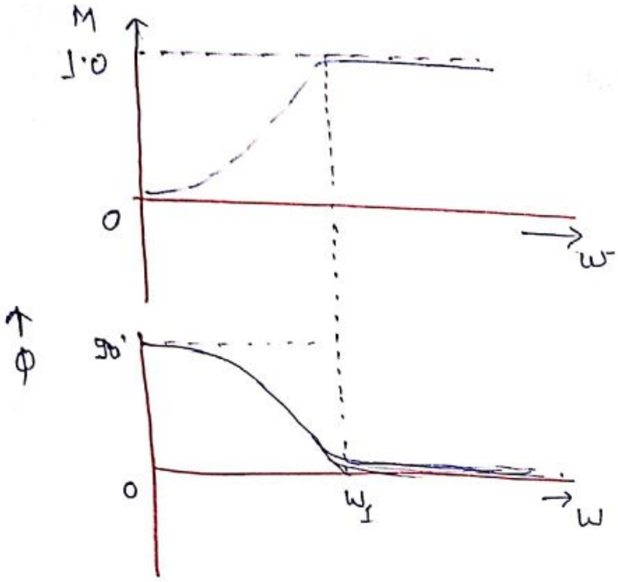
$$\frac{E_o(j\omega)}{X_i(j\omega)} = \frac{j\omega K\tau}{1+j\omega\tau}$$

$$M = \left| \frac{E_o}{K X_i}(j\omega) \right| = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} = \frac{1}{\sqrt{1+\left(\frac{1}{\omega\tau}\right)^2}}$$

$$\text{Phase shift} = \phi = 90^\circ - \tan^{-1}(\omega\tau)$$



* when $w = 0$ $\quad \frac{E_o}{X_i}(j\omega) = 0$

$$E_o = 0$$

* This arrangement should not be used for low frequency application as it will result in inaccuracies. The arrangement should be used for high frequency applications (beyond a frequency ω_1) in order to achieve high degree of accuracy in measurements.

Q(1) Capacitive transducer circuit is used for linear displacement. The transducer is a parallel plate air capacitor wherein the capacitance can be changed by changing the distance between the plates. This transducer is to be used for dynamic measurements. Suppose a flat frequency response with an amplitude ratio within 5% is required down to a frequency range of 20HZ; (a) what is the minimum allowable value of time constant ?

(b) Calculate the phase shift at this frequency.

The area of plates is 300 mm^2 and distance between plates is 0.125 mm. (c) Calculate the value of series resistor R. (d) What is the amplitude ratio at 5HZ with the above time constant ?

(e) Calculate the high frequency voltage sensitivity of the transducer if the battery voltage is 100V.

Soln –

(a) For a flat response within 5%

$$M = 1 - 0.05 = 0.95$$

$$M = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$0.95 = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$(0.95)^2 = \frac{(\omega\tau)^2}{1+(\omega\tau)^2}$$

$$1+(\omega\tau)^2 = \frac{(\omega\tau)^2}{(0.95)^2}$$

$$\frac{(\omega\tau)^2}{(.95)^2} - (\omega\tau)^2 = 1$$

$$(\omega\tau)^2\left[\frac{1-(0.95)^2}{(0.95)^2}\right] = 1$$

$$(\omega\tau)^2\left[\frac{1-0.9025}{0.9025}\right] = 1$$

$$(\omega\tau)^2 * \frac{0.0975}{0.9025} = 1$$

$$\omega\tau = \sqrt{\frac{0.9025}{.0975}}$$

$$\boxed{\omega\tau = 3.04}$$

$$\tau = \frac{3.04}{\omega} = \frac{3.04}{2\pi * 20}$$

$$\tau = \frac{3.04}{125.6}$$

$$\boxed{\tau = 0.024 \text{ second}}$$

Ans

(b) $\phi = 90^\circ - \tan^{-1}(\omega\tau)$

$= 90^\circ - \tan^{-1}[3.04]$

$= 90^\circ - 71.79^\circ$

$$\boxed{\phi = 18.2^\circ}$$

(c) $C = \dfrac{\epsilon A}{d}$

$$C = \frac{(8.85\times10^{-12}) * 300\times10^{-6}}{0.125\times10^{-3}}$$

$C = 21.24\times10^{-12}$ F

$\tau = RC = 0.024$ second

$$R = \frac{\tau}{C} = \frac{0.024}{21.24\times10^{-12}}$$

$$\boxed{R = 1140 M\Omega}$$

(d) $$M = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$M = \frac{(2\pi*5)*(0.024)}{\sqrt{1+(2\pi*5\times0.024)^2}}$$

$$\boxed{M = 0.605}$$

(e)

High frequency sensitivity of the transducer

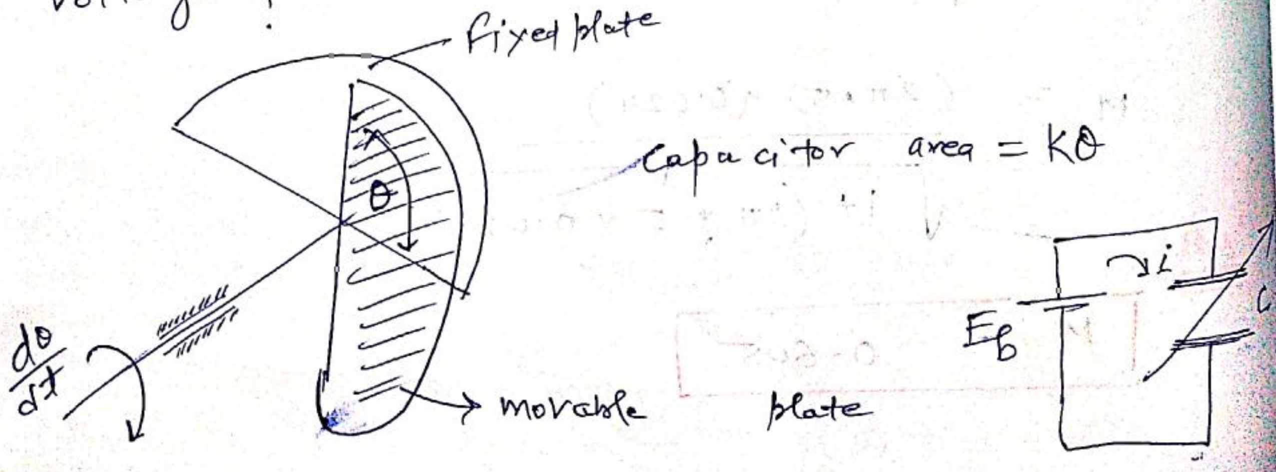
$$K = \frac{E_B}{x}$$

$$K = \frac{100\,V}{0.125 \times 10^{-3}}$$

$$K = 800 \times 10^{3}\ V/m$$

$$\boxed{K = 800\ kV/m}$$

Q:(2) In the variable-capacitance velocity pick-up shown in figure, prove that current i is directly proportional to the angular velocity $\frac{d\theta}{dt}$. Since voltage signals are more readily (easily) manipulated, how might the current signal be transduced to a proportional voltage?



Sol^n:

$$i = \frac{dq}{dt}$$

$$= \frac{d}{dt}(E_B C)$$

$$= E_B \frac{dC}{dt}$$

$$= E_B \frac{d}{dt}\left(\frac{\epsilon A}{d}\right)$$

$$= E_B \frac{\epsilon}{d} \frac{dA}{dt}$$

$$= E_B \frac{\epsilon}{d} \frac{d}{dt}(K\theta)$$

$$= E_B \frac{\epsilon K}{d} \frac{d\theta}{dt}$$

$$\boxed{i = K' \frac{d\theta}{dt}}$$

E_B : Battery voltage E_B is Constant

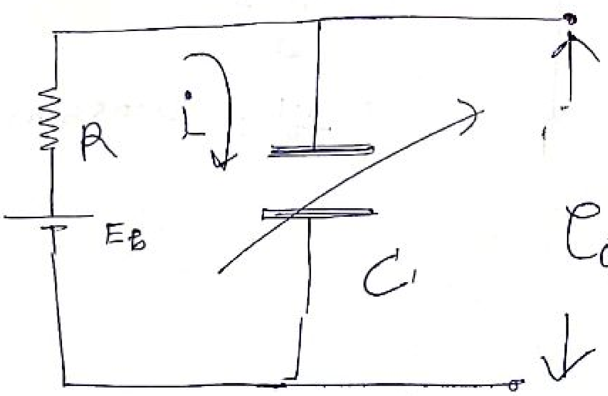
$C \rightarrow$ Value of Capacitance is variable and depends upon angular displacement

* The overlapping area of plates is proportional to angular displacement θ.

* $A = K\theta$

Hence i is proportional to $\frac{d\theta}{dt}$ or the angular velocity.

(b) Circuit for current to voltage signal conversion ⇒

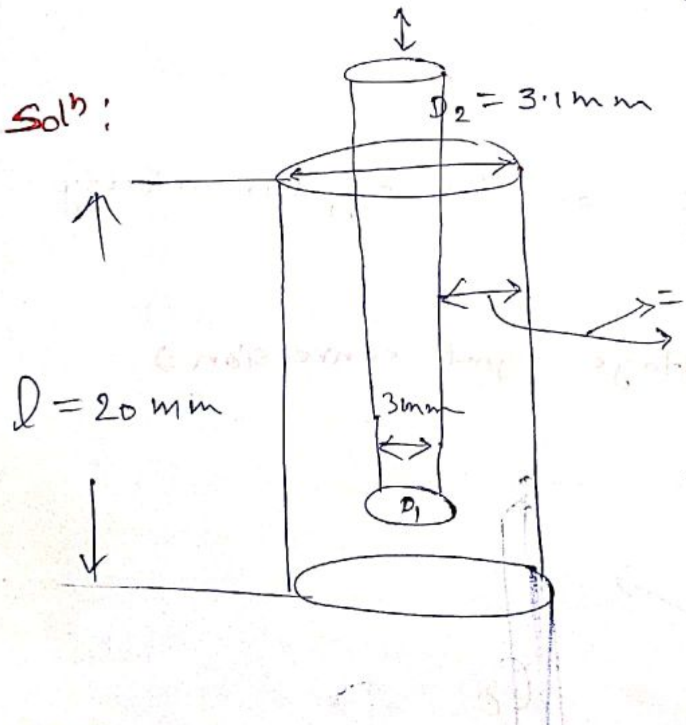


Q(3) A capacitive transducer is made up of two concentric cylindrical electrodes. The outer diameter of the inner cylindrical electrode is 3mm and the dielectric medium is air. The inner diameter of the outer electrode is 3·1 mm. Calculate the dielectric stress when a voltage of 100V is applied across the electrodes.

(b) Is it within safe limits?

(c) length of electrodes is 20mm. Calculate the change in capacitance if the inner electrode is moved through a distance of 2mm. The breakdown strength of air is 3KV/mm.

Sol^n:



$D_1 = 3\,mm$

$D_2 = 3.1\,mm$

$= (r_2 - r_1)$

$\frac{3.1}{2} - \frac{3}{2}$

$= \frac{3.1 - 3}{2}$

$= \frac{0.1}{2} = 0.05\,mm$

Length of air gap between two electrodes $= (3.1-3)/2$

$= 0.1/2$ mm

$= 0.05$ mm

$\therefore$ Dielectric stress $= \frac{100}{0.05} = \frac{10000}{5}$

$= 2$ KV/mm

The breakdown strength of air is 3KV/mm and hence the dielectric is safe.

$$C = \frac{2\pi \epsilon l}{\log_e\left(\frac{D_2}{D_1}\right)}$$

$$= \frac{2 \times 3.14 \times (8.85 \times 10^{-12}) * (20 \times 10^{-3})}{\log_e\left(\frac{3.1}{3}\right)}$$

$= 33.9$ pF

Now inner electrode is shifted by 2mm.

$l = 20 - 2 = 18$ mm

$$c = \frac{2 \times 3.14 \times (8.85 \times 10^{-12}) * 18 \times 10^{-3}}{\log_e\left(\frac{3.1}{3}\right)}$$

$= 30.5$ pF

$$\Delta C = 33.9 - 30.5 \text{ pF}$$
$$= 3.4 \text{ pF}$$

Derivation of formula of sensitivity of capacitance →

$$\text{Sensitivity} = \frac{\Delta V}{\Delta x}$$

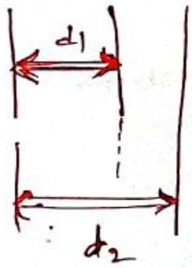
$$V_2 - V_1 = \frac{q}{C_2} - \frac{q}{C_1}$$

$$= q\left[\frac{1}{C_2} - \frac{1}{C_1}\right]$$

$$= CV\left[\frac{d_2}{\epsilon A} - \frac{d_1}{\epsilon A}\right]$$

$$= \left(\frac{\epsilon A}{d_1}\right)V\left[\frac{d_2}{\epsilon A} - \frac{d_1}{\epsilon A}\right]$$

$$\left[\begin{array}{l} q = CV \\ q = \left(\frac{\epsilon A}{d_1}\right)V \end{array}\right]$$



$$V_2 - V_1 = V\left[\frac{d_2 - d_1}{d_1}\right]$$

$$\boxed{\frac{V_2 - V_1}{d_2 - d_1} = \frac{V}{d_1}}$$

$$\therefore \boxed{\frac{\Delta V}{\Delta x} = \frac{E_{battery}}{x_0}}$$