Flow Measurement

Units of flow:-

1. Volume flow rate → It is expressed as a volume delivered per unit time. Typical unit is m^3/h or gals/min. It is represented by Q.

2. Flow velocity – It is expressed as the distance the liquid travels in the carrier per unit time. Typical units are m/min or ft/min. It is represented by V.

3. Mass or Weight flow rate – Expressed as mass or weight flowing per unit time. Typical units are kg/h or lb/h. This is related to the volume flow rate by -:

$$m = \rho \cdot Q \quad \text{or} \quad w = \rho \cdot g \cdot Q$$

Q = Volume flow rate

ρ = density

m = mass flow rate

W = Weight flow rate

Unit ⇒

$$m = \rho \cdot Q = \frac{Kg}{\cancel{m^3}} \cdot \frac{\cancel{m^3}}{s} = \frac{Kg}{sec}$$

$$W = \rho \cdot g \cdot Q = \frac{Kg}{\cancel{m^3}} \cdot \frac{m}{s^2} \cdot \frac{\cancel{m^3}}{s} = \frac{Kg-(m/s^2)}{sec} = \frac{N}{sec}$$

Reynold's number :

$$Re = \frac{v d \rho}{\eta}$$

v → velocity of flow (m/s)

d → diameter of pipe throat, or orifice (m)

ρ → density of fluid (Kg/m³)

η → Viscosity (Ns/m²)

Re → Dimensionless quantity

(i) if $Re < 2000$ then flow is streamline or laminar

(ii) if $Re > 2000$ the flow is turbulent.

$$\eta = \frac{\text{Shear stress}}{\left[\frac{\text{Change of velocity}}{\text{Change of distance}}\right]} = \frac{F/A}{\left(\frac{L}{T}\right)/\left(L\right)} = \frac{\left(\frac{F}{A}\right)}{\left(\frac{L}{T}\right) \times \frac{1}{L}}$$

$$= \frac{\text{Force} * \text{time}}{(\text{Length})^2}$$

$$= \frac{\text{Newton} \cdot \text{Second}}{(\text{meter})^2}$$

$$\boxed{\eta = \frac{N\text{-}S}{m^2}}$$

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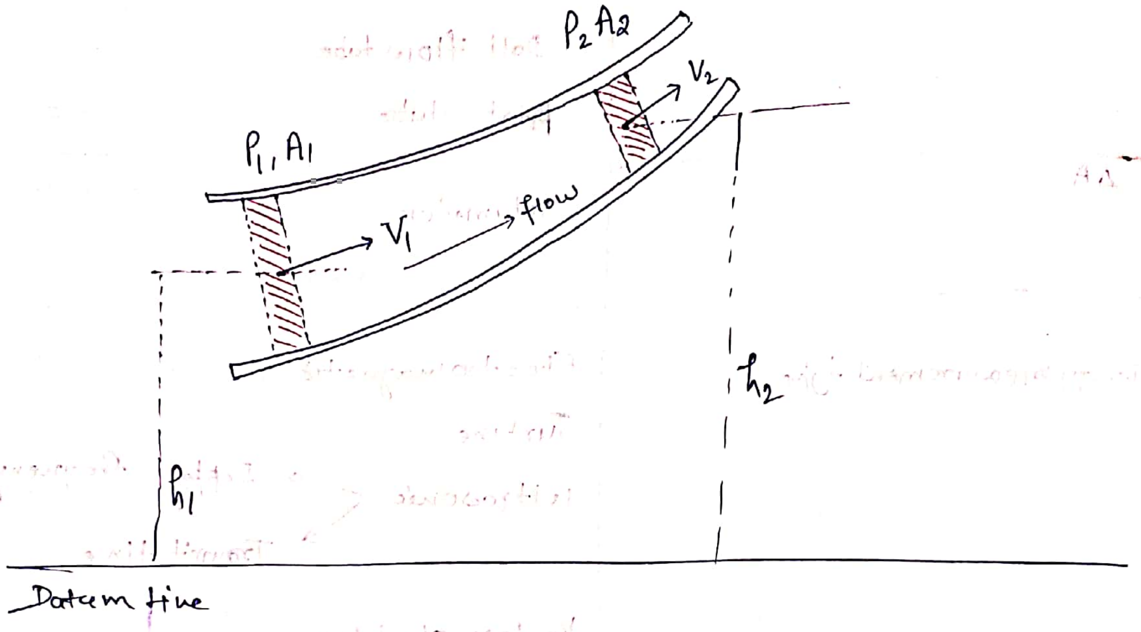
Different types of flow meter

Type	Flow meter
Head type / Differential pressure drop (ΔP)	Orifice Venture Flow nozzle Doll flow tube Pitot tube
ΔA	Rotameter
Velocity measurement type	Electromagnetic Turbine Ultrasonic → Dopper frequency shift Ultrasonic → Transit time Vortex shedding Hot wire → constant current Hot wire → Constant temperature
Mass flow measurement type	Coriolis Thermal Impact
Positive displacement type	Nutating Disc Sliding vane Lobed impeller
Open channel type	Weir Flume

Bernoulli's theorem :-



$$P + \frac{1}{2}\rho V^2 + \rho g h = \text{constant}$$

$$A_1 V_1 = A_2 V_2 = Q$$

Let $\rho \rightarrow$ density of the fluid flowing through the pipe (kg/m^3)

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

if pipe is horizontal then $h_1 = h_2$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$\frac{1}{2}\rho\left(V_2^2 - V_1^2\right) = P_1 - P_2$$

$$V_2^2 - V_1^2 = \frac{2(P_1 - P_2)}{\rho}$$

$$V_2^2 - \left(\frac{A_2 V_2}{A_1}\right)^2 = \frac{2(P_1 - P_2)}{\rho}$$

$$V_2^2 \left[1 - \frac{A_2^2}{A_1^2}\right] = \frac{2(P_1 - P_2)}{\rho}$$

$$V_2^2 \left(\frac{A_1^2 - A_2^2}{A_1^2}\right) = \frac{2(P_1 - P_2)}{\rho}$$

$$V_2^2 = \frac{A_1^2}{(A_1^2 - A_2^2)} \frac{2(P_1 - P_2)}{\rho}$$

$$\boxed{V_2 = \frac{1}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2(P_1 - P_2)}{\rho}}}$$

$$\boxed{Q_t = A_2 V_2 = \frac{A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2(P_1 - P_2)}{\rho}}}$$

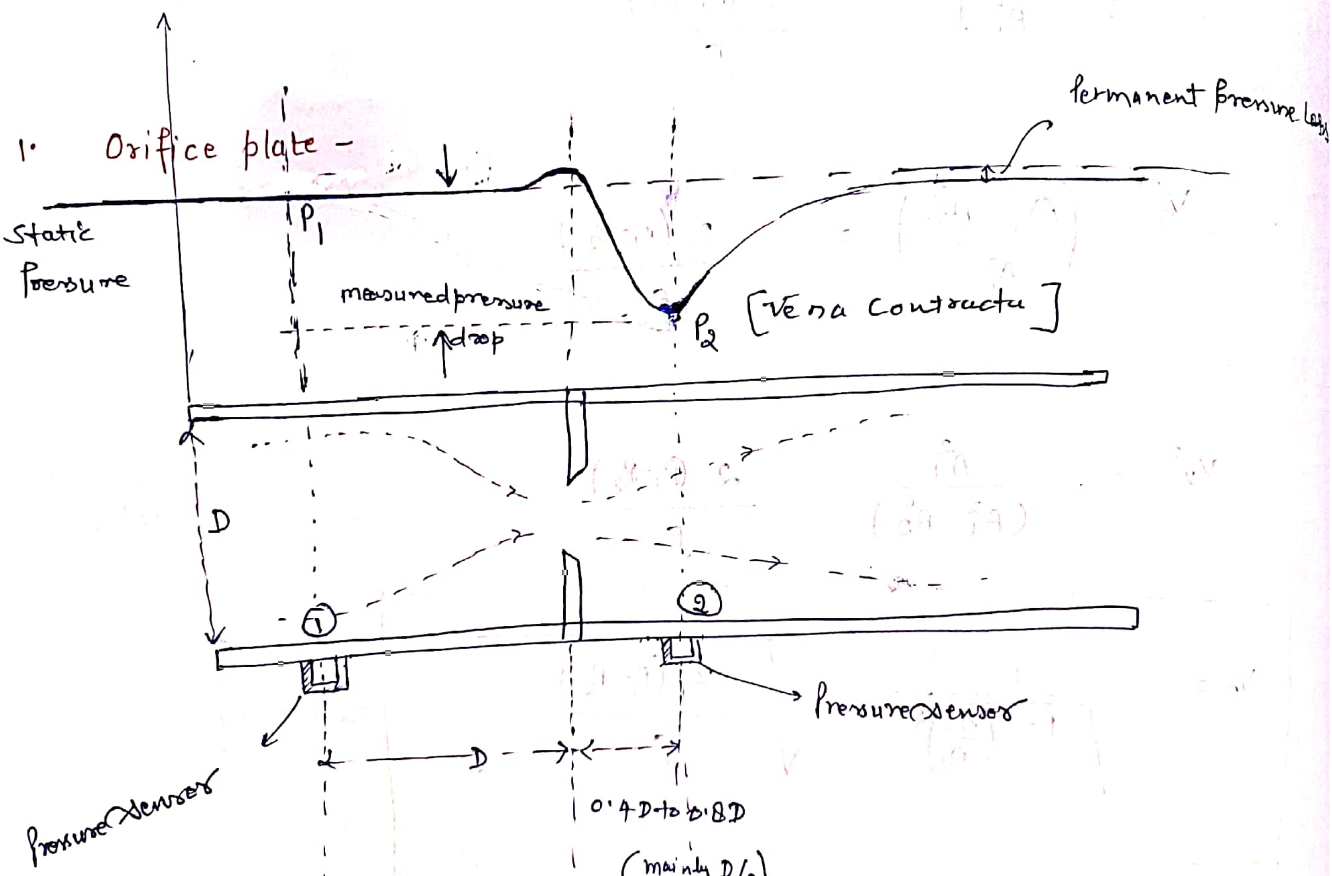
Discharge coefficient $C_d = \frac{\text{Actual flow rate}}{\text{theoretical flow rate}}$

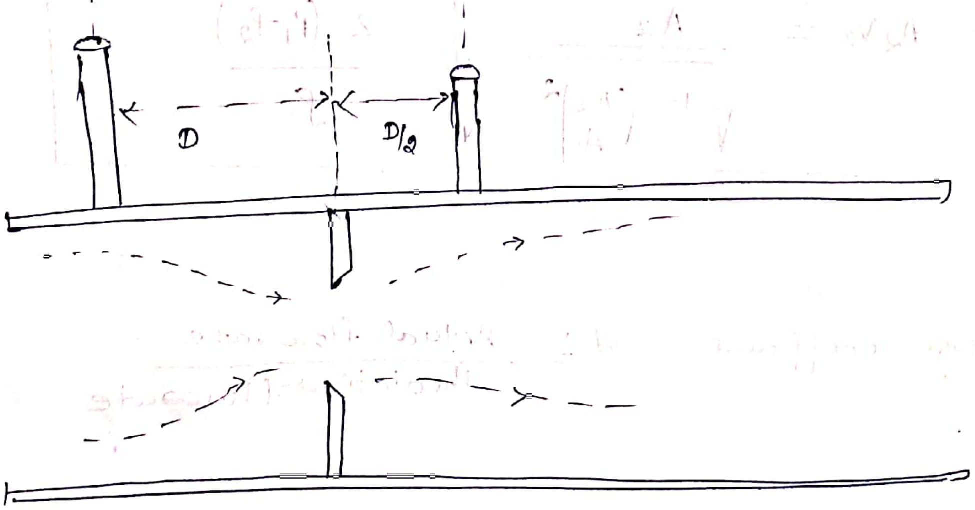
$$C_d = \frac{Q_a}{Q_t}$$

$$Q_a = C_d \cdot Q_t$$

$$\boxed{Q_a = \frac{C_d\ A_2}{\sqrt{1-\left(A_2/A_1\right)^2}} \sqrt{\frac{2\ (P_1-P_2)}{\rho}}}$$

1. Orifice plate -





* Abrupt change in area.
* Easy and cheap to install.

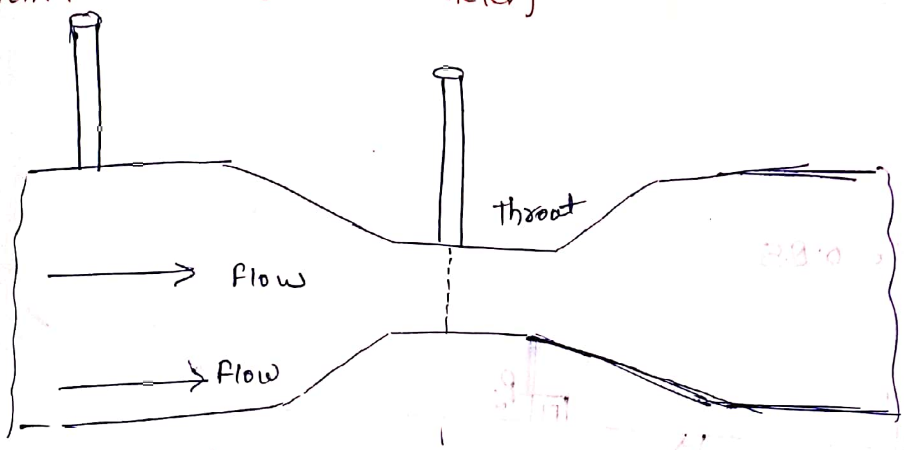
Vena contracta point → The position when fluid velocity is maximum and fluid pressure is minimum, is known as vena contracta point.

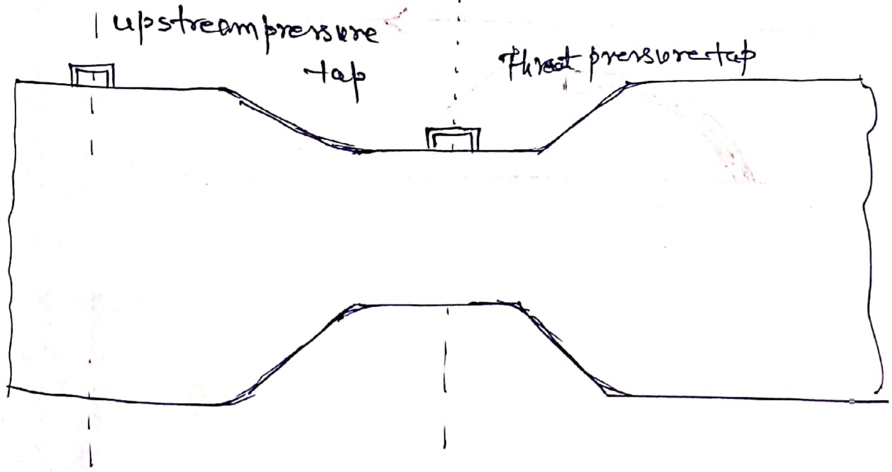
V = maximum

p = minimum

] at vena contracta point

2- Venturi tube - (Venturi meter)

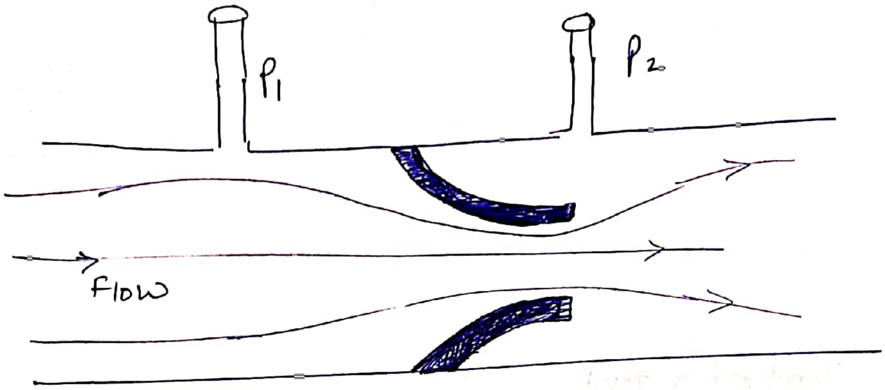




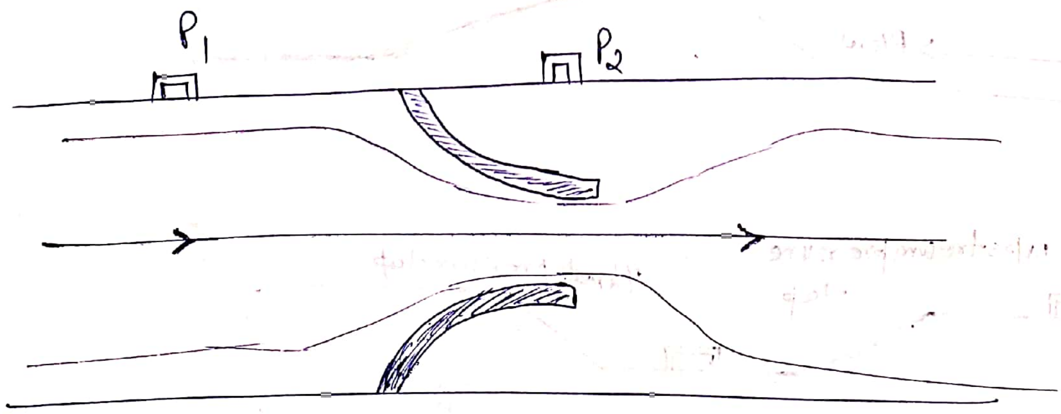
* Smooth change in area
* Occupy greater space and costlier.

* $C_d = 0.95$

3. Flow nozzle -



* $C_d = 0.7$ to 0.85



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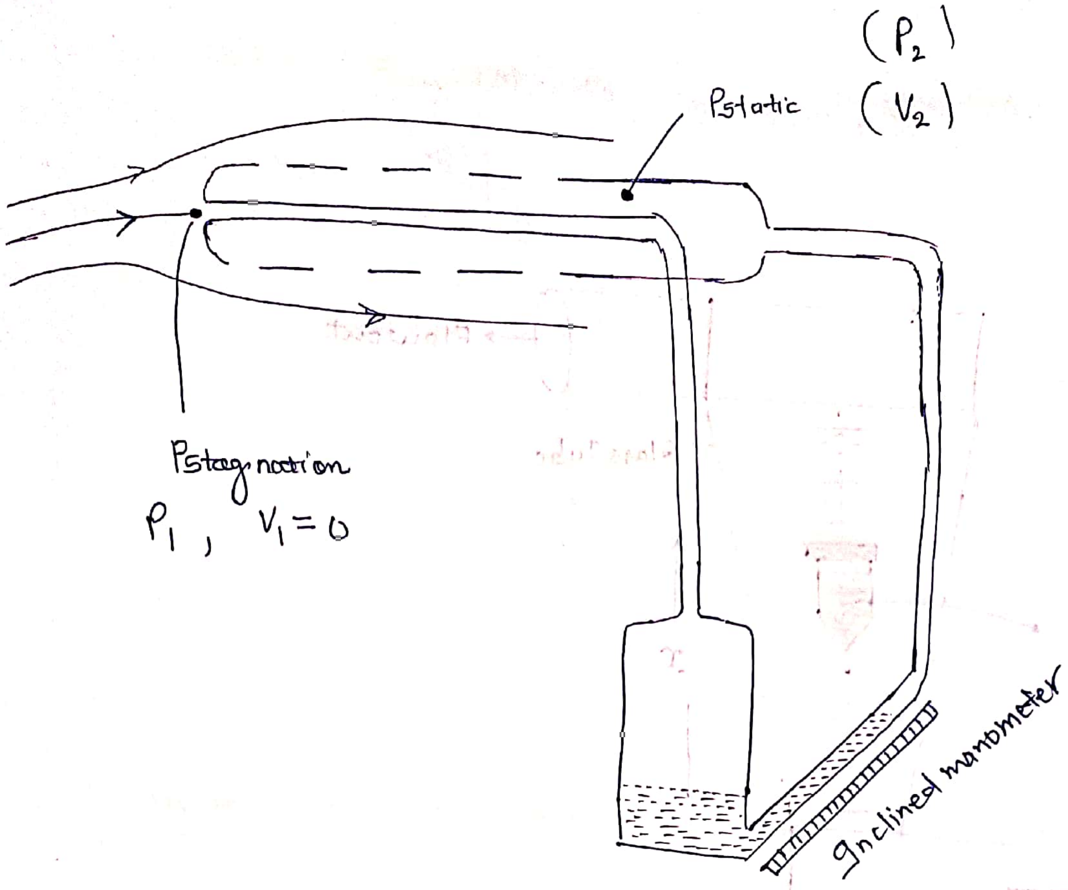
Pitot tube –



$$P_2 + \frac{1}{2}\rho V_2^2 = P_1 + \frac{1}{2}\rho V_1^2$$

$$\left[\begin{array}{l} (h_1 = h_2) \\ \therefore \rho g h_1 = \rho g h_2 \end{array} \right]$$

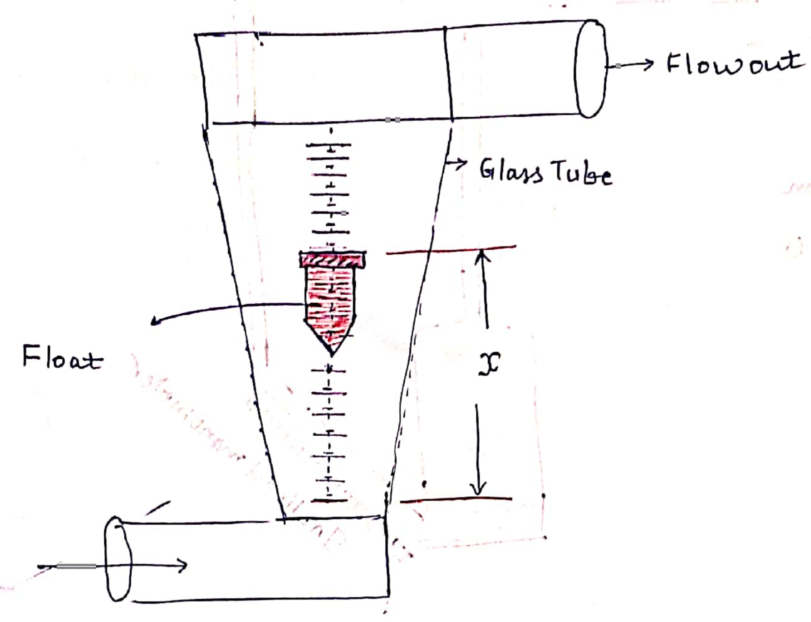
$$P_2 + \frac{1}{2}\rho V_2^2 = P_1 + \frac{1}{2}\rho (0)^2$$

$$P_2 + \frac{1}{2}\rho V_2^2 = P_1$$

$$V_2^2 = \frac{2(P_1 - P_2)}{\rho}$$

$$\boxed{V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}}} = \sqrt{\frac{2\left[P_{stagnation} - P_{static}\right]}{\rho}}$$

Rotameter - Constant pressure drop or variable area flow meter



A_t → Area of tube

A_f → Area of float

V_f = Volume of float

ρ_f = Density of float (kg/m³)

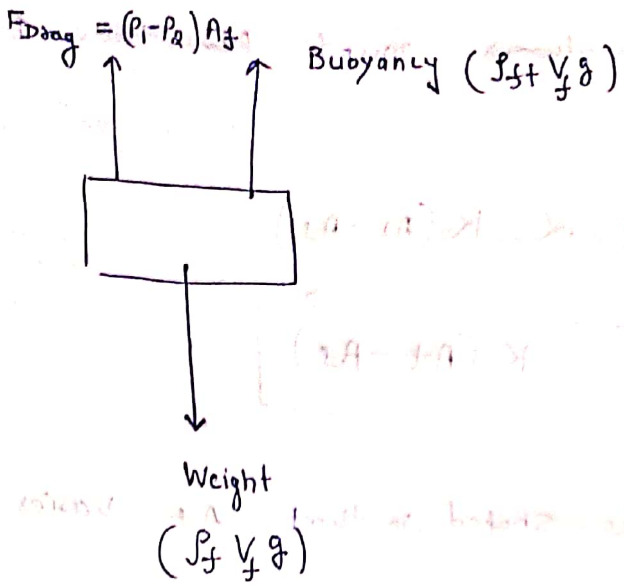
ρ_{ff} = Density of flowing fluid

Q = Volume flow rate (m^3/s)

C_d = Discharge coefficient

P_1 = Pressure on the lower surface of float

P_2 = Pressure on the upper surface of float.



Drag force + Buoyancy force = Weight

$$(P_1 - P_2) A_f + \rho_{ff} V_f g = \rho_f V_f g$$

$$(P_1 - P_2) = \frac{(\rho_f - \rho_{ff}) V_f g}{A_f} \quad \text{--- (1)}$$

$$\therefore Q_a = \frac{C_d (A_t - A_f)}{\sqrt{1 - \left[\frac{(A_t - A_f)}{A_t}\right]^2}} \sqrt{\frac{2}{\rho_{ff}} * \frac{(\rho_f - \rho_{ff}) V_f g}{A_f}}$$

$$Q_a = C_d \frac{(A_t - A_f)}{\sqrt{1 - \left[\frac{(A_t - A_f)}{A_t}\right]^2}} \sqrt{\frac{2 g V_f}{A_f} * \frac{\rho_f - \rho_{ff}}{\rho_{ff}}}$$

If the variation of C_d with float position is slight and if

$\left[\frac{A_t - A_f}{A_t}\right]^2$ is always much less than 1 then

$$Q \propto : (A_t - A_f)$$

$$\boxed{Q = K(A_t - A_f)}$$

* If the tube is shaped so that A_t varies linearly with float position x then :-

$$\boxed{Q = K_1 + K_2 x}$$

It is a linear relationship.

Note:

$$Q = \frac{C_d \; A_t - A_f}{\sqrt{1 - \left(\frac{A_t - A_f}{A_t}\right)^2}} \sqrt{\frac{2 g V_f}{A_f} * \frac{\rho_f - \rho_{ff}}{\rho_{ff}}} \quad \left[\frac{m^3}{s}\right] \quad \text{—(1)}$$

mass flow rate $m = Q \rho_{ff}$ (kg/second)

$$m = \frac{C_d (A_t - A_f)}{\sqrt{1 - \left(\frac{A_t - A_f}{A_t}\right)^2}} \sqrt{\frac{2 g V_f}{A_f} \left[\rho_f - \rho_{ff}\right] * \rho_{ff}} \quad \frac{kg}{second}$$

$$\frac{dm}{d\rho_{ff}} = 0$$

$$\frac{d}{d\rho_{ff}}\left\{ \frac{C_d (A_t - A_f)}{\sqrt{1-\left(\frac{A_t - A_f}{A_t}\right)^2}} * \sqrt{\frac{2 g V_f}{A_f}(\rho_f - \rho_{ff})\rho_{ff}} \right\} = 0$$

$$\left\{ \frac{C_d (A_t - A_f)}{\sqrt{1-\left(\frac{A_t - A_f}{A_t}\right)^2}} * \frac{1}{\sqrt{\frac{2 g V_f}{A_f}(\rho_f - \rho_{ff})\rho_{ff}}} * 2 g V_f \left[\rho_f - 2\rho_{ff}\right] \right\} = 0$$

$$\therefore \rho_f - 2\rho_{ff} = 0$$

$$\boxed{\rho_f = 2\rho_{ff}} \qquad \text{——— (2)}$$

Putting this value in equation —(1)

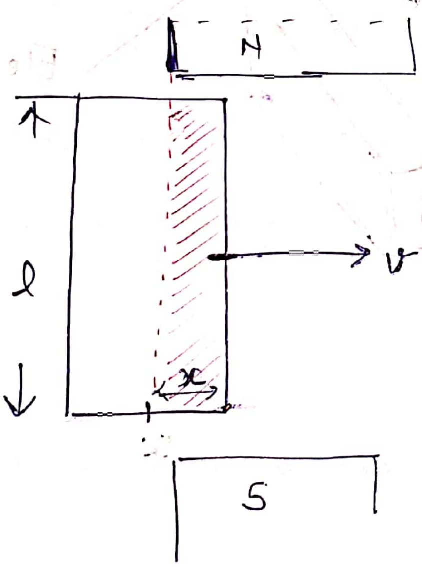
$$Q = \frac{C_d (A_t - A_f)}{\sqrt{1-\left(\frac{A_t - A_f}{A_t}\right)^2}} \sqrt{\frac{2 g V_f}{A_f} \frac{\left[2\rho_{ff} - \rho_{ff}\right]}{\rho_{ff}}} \quad \frac{m^3}{sec}$$

$$Q = \frac{C_d (A_t - A_f)}{\sqrt{1-\left(\frac{A_t - A_f}{A_t}\right)^2}} \sqrt{\frac{2 g V_f}{A_f}} \quad \frac{m^3}{sec}$$

From eqn (1) and (2) : To make the mass flow rate (kg/s) insensitive to changes in the fluid density ρ_{ff}, the float density ρ_f should be equal to twice of the density of flowing fluid.

Electromagnetic flow meter —

If a conductor of length l, moves with a transverse velocity v across a magnetic field of intensity B, there will be forces on the charge particles of the conductor that will move the positive charges toward one end of the conductor and the negative charges toward other end. Thus a potential gradient is setup along the conductor and there is a voltage difference E between its two ends.



$$\phi = B \cdot A$$

$$\phi = B \cdot [l \cdot x]$$

$l \cdot x$ is area of that part of loop where B is not zero.

$$|Emf| = N \frac{d\phi}{dt} = N \frac{d}{dt}[B\, l\, x]$$

$$= N \cdot B\, l\, \frac{dx}{dt}$$

$$\text{Emf} = N \cdot B\, l\, v$$

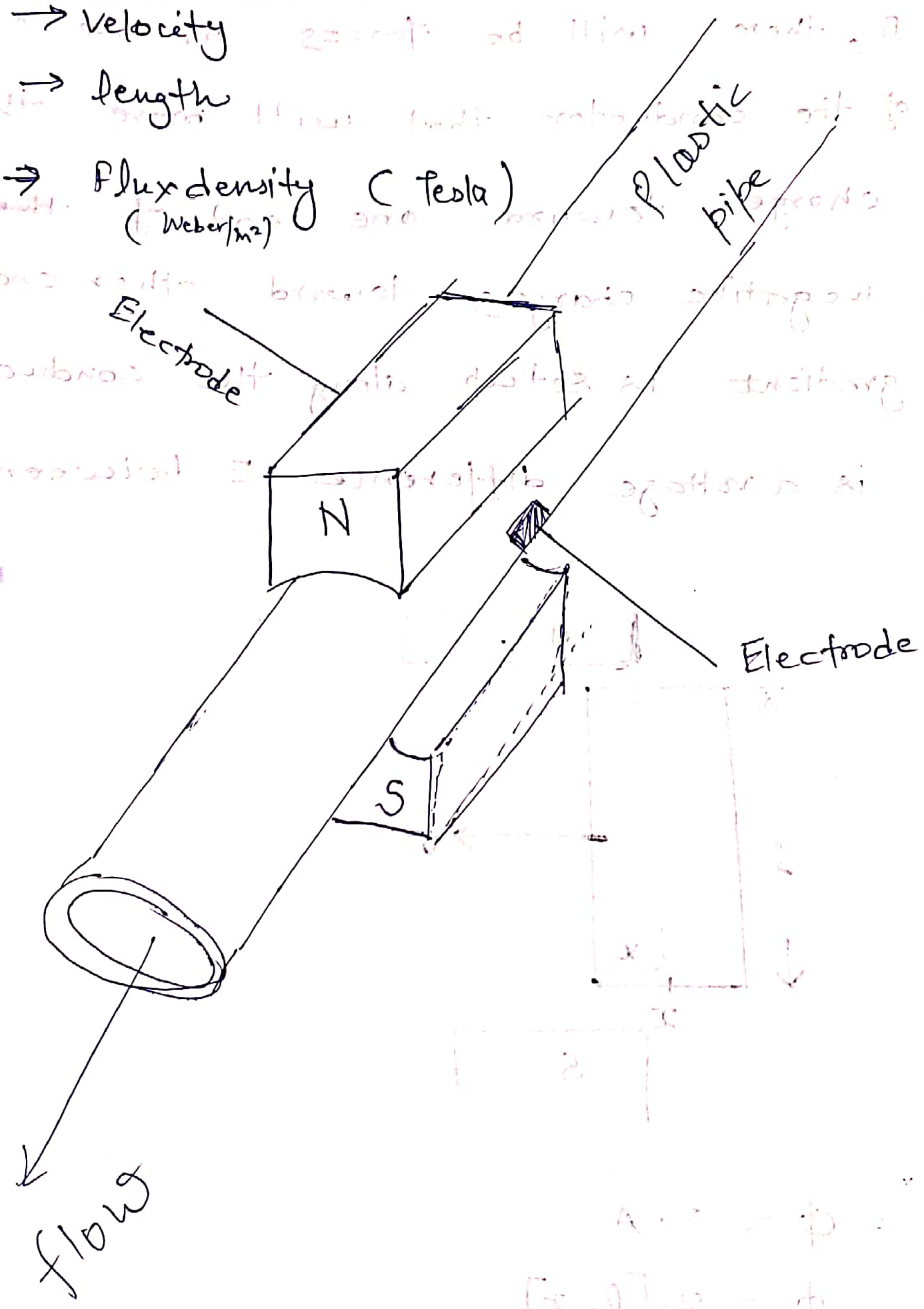
$$\boxed{\text{Emf} = l\, v\, B}$$

Let $N = 1$

$v \rightarrow$ Velocity

$l \rightarrow$ length

$B \Rightarrow$ Flux density (Tesla)
(Weber/m^2)



The schematic diagram of an electromagnetic flow meter is shown above. It consists basically a pair of insulated electrodes buried flush in the opposite sides of non conducting, non magnetic pipe carrying the liquid whose flow is to be measured. The pipe is surrounded by an electromagnet which produces a magnetic field.

This arrangement is analogous to a conductor moving across a magnetic field. Therefore voltage is induced across the electrodes. This voltage is given by:

$$\boxed{Emf = l\,v\,B}$$

(Volt)

$v \rightarrow$ velocity of conductor (flow)

$l \rightarrow$ length of conductor = diameter of pipe

$B \rightarrow$ Flux density (weber/m²)

Note :-

It can be used to measure bidirectional flow.

Disadvantages -

1- Liquid has to be conducting.

2. The pipe should always be full

3. The pipe has to be non-magnetic and non-conducting

meter

of

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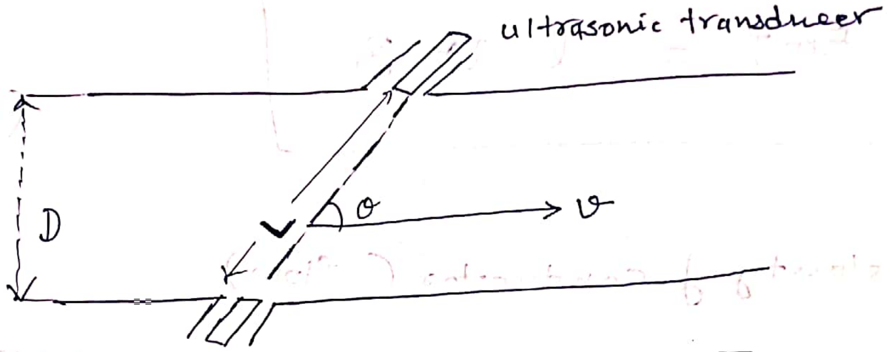
Ultrasonic flow meter —

1. Transit time flowmeter.

2. Doppler frequency shift flowmeter.

1. Transmit time flow meter -



[v → Velocity of fluid
c → Acoustic velocity in the fluid.]

transit time for upstream $t_u = \dfrac{L}{c - V\cos\theta}$

transit time for downstream $t_d = \dfrac{L}{c + V\cos\theta}$

$\Delta t = t_u - t_d$

$= \dfrac{L}{c - V\cos\theta} - \dfrac{L}{c + V\cos\theta}$

$= L\left\{\dfrac{(c + V\cos\theta) - (c - V\cos\theta)}{(c + V\cos\theta)(c - V\cos\theta)}\right\}$

$$= L\left[\frac{2V\cos\theta}{c^2 - V^2\cos^2\theta}\right]$$

$$\boxed{\Delta t = \frac{2Lv\cos\theta}{c^2 - v^2\cos^2\theta}}$$

From above equation we can calculate velocity of flow (v).

Case (ii) If we have t_u and t_d then:

$$v = \frac{L}{2\cos\theta}\left[\frac{1}{t_d} - \frac{1}{t_u}\right]$$

Proof:

$$= \frac{L}{2\cos\theta}\left[\frac{c+V\cos\theta}{L} - \frac{c-V\cos\theta}{L}\right]$$

$$= \frac{1}{2\cos\theta}\left[c+V\cos\theta - c+V\cos\theta\right]$$

$$= \frac{1}{2\cos\theta}\left[2V\cos\theta\right]$$

$$= V$$

This method does not require knowledge of acoustic velocity c.

Speed of acoustic is given by :- [velocity of sound in the fluid = C]

$$c = \frac{L}{2} \frac{(t_u + t_d)}{t_u * t_d}$$

Proof:

$$= \frac{L}{2} \frac{\left[\frac{L}{C - V\cos\theta} + \frac{L}{C + V\cos\theta}\right]}{\left(\frac{L}{C - V\cos\theta}\right) * \left(\frac{L}{C + V\cos\theta}\right)}$$

$$= \frac{\frac{L}{2} * L \left\{ \frac{C + \cancel{V\cos\theta} + C - \cancel{V\cos\theta}}{C^2 - V^2\cos^2\theta} \right\}}{\left\{ \frac{L^2}{C^2 - V^2\cos^2\theta} \right\}}$$

$$= \frac{L}{2} \frac{[2C / C^2 - V^2\cos^2\theta]}{[1 / C^2 - V^2\cos^2\theta]}$$

$$= C$$

Case:

$$t_u = \frac{L}{C - V\cos\theta} \qquad \therefore f_u = \frac{C - V\cos\theta}{L}$$

$$t_d = \frac{L}{C + V\cos\theta} \qquad \therefore f_d = \frac{C + V\cos\theta}{L}$$

$$\Delta f = f_d - f_u$$

$$\therefore \Delta f = \frac{1}{t_d} - \frac{1}{t_u}$$

$$= \left(\frac{c + V\cos\theta}{L}\right) - \left(\frac{c - V\cos\theta}{L}\right)$$

$$\boxed{\Delta f = \frac{2V\cos\theta}{L}}$$

Proof of formula of v and c which is given in case (ii) —— (4)

$$c - V\cos\theta = \frac{L}{t_u} \quad \text{—— (1)}$$

$$c + V\cos\theta = \frac{L}{t_d} \quad \text{—— (2)}$$

$$2c = \frac{L}{t_u} + \frac{L}{t_d} \quad (1) + (2)$$

$$\therefore 2c = L\left[\frac{1}{t_u} + \frac{1}{t_d}\right]$$

$$\boxed{c = \frac{L}{2}\left[\frac{t_u + t_d}{t_u * t_d}\right]}$$

$$c + V\cos\theta = \frac{L}{t_d}$$

$$c - V\cos\theta = \frac{L}{t_u}$$

$$- \quad + \qquad -$$

$$2V\cos\theta = \frac{L}{t_d} - \frac{L}{t_u}$$

$$\therefore \boxed{v = \frac{L}{2\cos\theta}\left[\frac{1}{t_d} - \frac{1}{t_u}\right]}$$

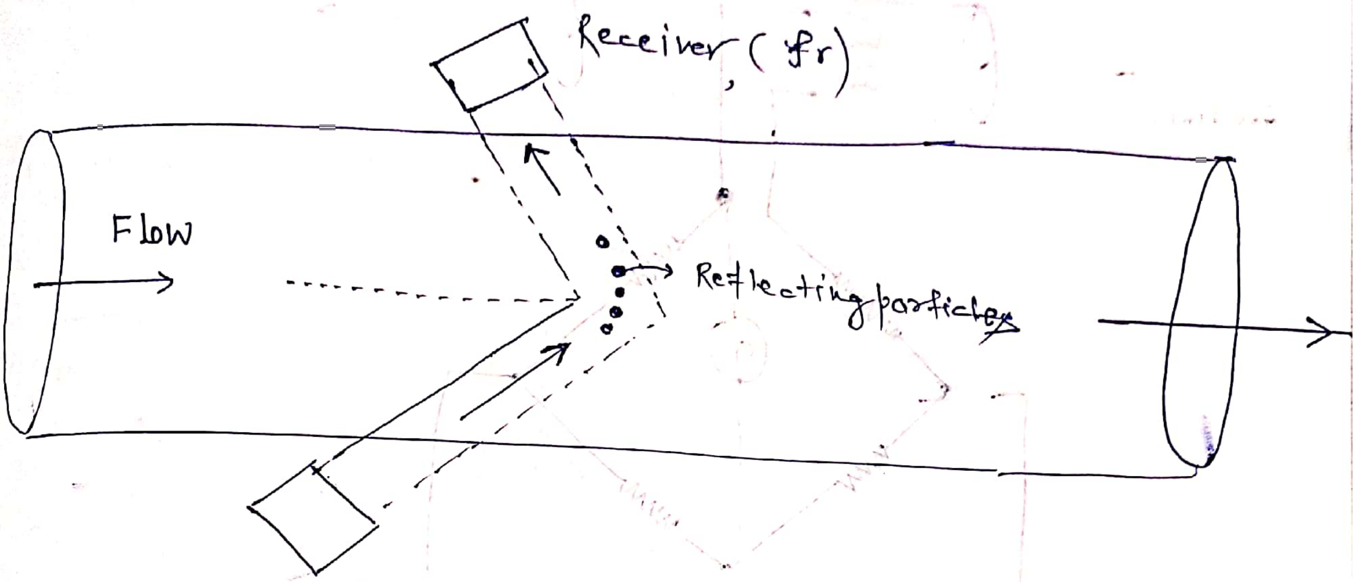
2- Doppler frequency shift flowmeter :-

* Suppose a stationary source of sound is emitting a sound of a certain frequency. To a listener who is moving away from the source of sound, the pitch (frequency) is lower than when he is at rest. Conversely, the pitch will be higher if the listener moves towards the source of sound.

* A similar phenomenon results if the listener is stationary but the source of sound is moving. Known as Doppler effect after the name of its discoverer, the frequency shift observed in this phenomenon is related to the relative velocity of the listener and source.

* This effect is used to measure the flow rate of a fluid, carrying suspended particles. Continuous-wave ultrasonic signal of frequency around 10 MHz is generated by a piezo electric crystal oscillator.

* This signal is scattered by moving suspended particles to produce a signal of changed frequency which is detected by the receiver. It can be shown that the frequency shift is proportional to the velocity of fluid flow, which, in turn, is proportional to the volume ~~flow~~ rate of the fluid.



$$\Delta f = f_{transmitter} - f_{receiver}$$

$$\Delta f = \frac{2 \cdot f_{transmitter} \cos\theta \, V}{C}$$

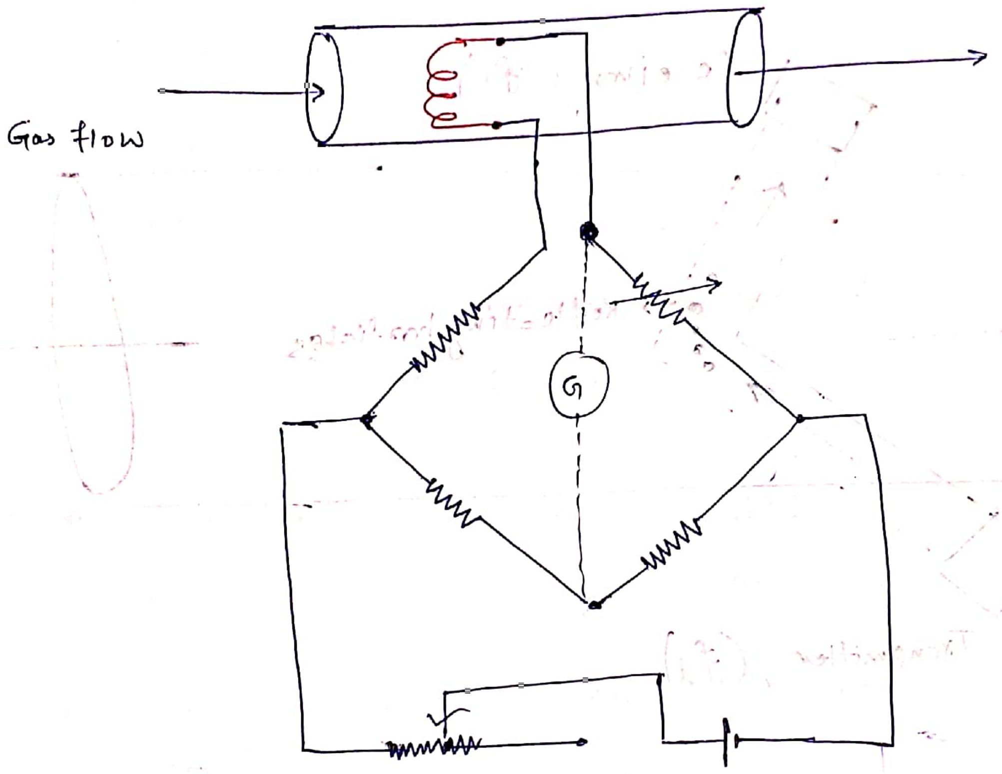
$$\boxed{\Delta f \propto v}$$

Hot wire and Hot film anemometer

1. Constant current type.

2. Constant temperature type.

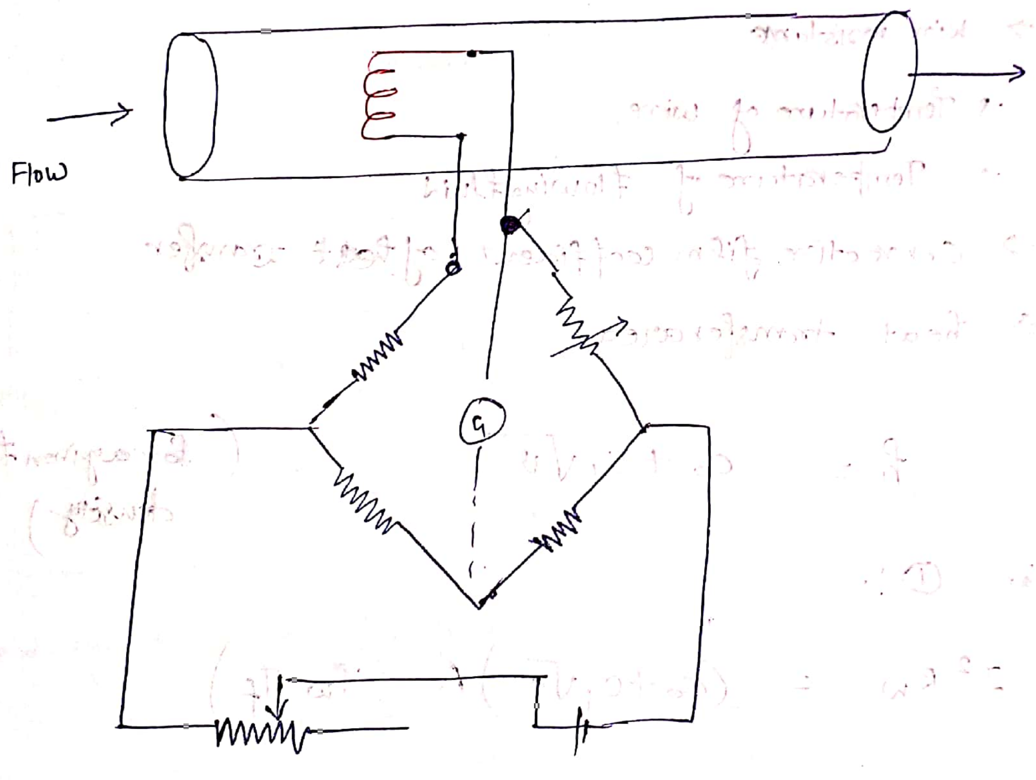
Constant current type -



In the constant current (I) type anemometer, the anemometer is placed in the stream of the fluid whose flow rate needs to be measured. The current of constant magnitude is passed through the wire. The supply voltage of wheatstone bridge is kept constant.

When the wire is kept in the stream of liquid, in that case, the heat is transferred from the wire to the fluid. The heat is directly proportional to the resistance of the wire. If the heat reduces, that means the resistance of the wire also reduces. The wheatstone Bridge measures the variation in resistance which is equal to the flow rate of the fluid (liquid or gas).

Constant temperature method –



In this arrangement, the wire is heated by electric current. The hot wire when placed in the fluid system, the heat is transferred from wire to the fluid. Thus, the temperature of the wire changes which also changes their resistance.

wire.

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* It works on the principle that the temperature of the wire remains constant. The total current required to bring the wire in the initial condition is equal to the flow rate of the gas.

At equilibrium :-

$$\text{Heat generated} = \text{Heat loss}$$

$$I^2 R_W = h A (T_w - T_f) \qquad \text{--- (1)}$$

$I \Rightarrow$ Wire current

$R_W \Rightarrow$ wire resistance

$T_W \Rightarrow$ Temperature of wire

$T_f \Rightarrow$ Temperature of flowing fluid

$h \Rightarrow$ Convective film coefficient of heat transfer

$A \Rightarrow$ heat transfer area

King's Law :

$$h = C_0 + C_1\sqrt{v}$$

(for a given fluid density)

From equation (1) :

$$\therefore I^2 R_W = (C_0 + C_1\sqrt{v}) A (T_w - T_f)$$

$$I^2 = \frac{A (T_w - T_f)(C_0 + C_1\sqrt{v})}{R_W}$$

$$\boxed{I^2 = C_1 + C_2\sqrt{v}}$$

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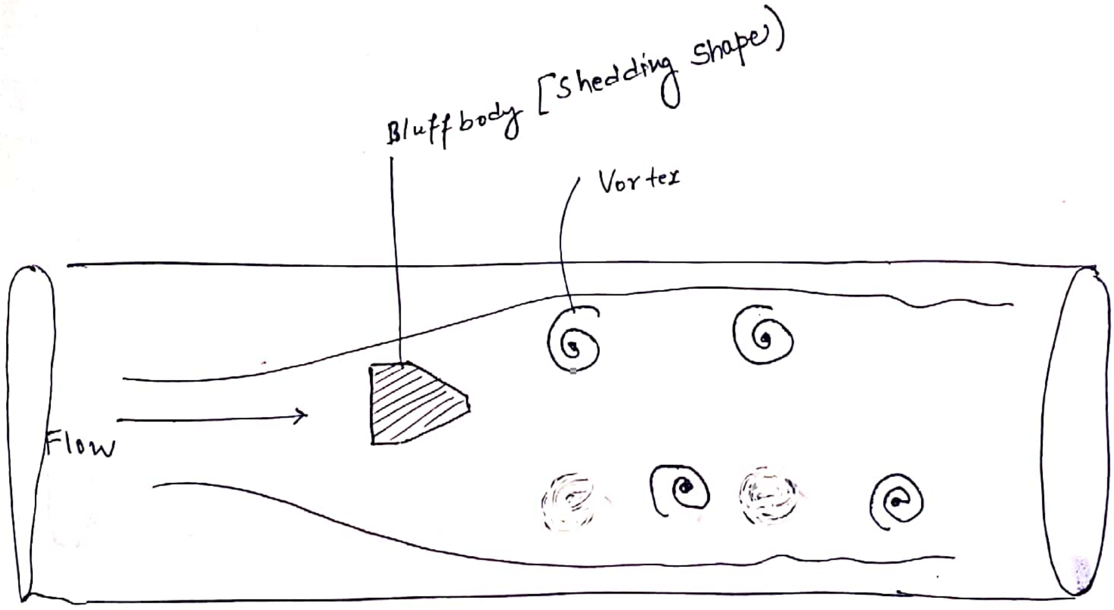
Vortex Shedding flowmeter -



Shedding frequency 'f' is given by :-

$$\boxed{f = \frac{N_{St}\, V}{d}}$$

V = fluid velocity

d = Characteristic dimension of shedding body (Bluff body)

N_{St} = Strouhal number

Drag force flowmeter

$$\text{Drag force } F_d = \frac{C_d A \rho v^2}{2}$$

$C_d \rightarrow$ discharge coefficient

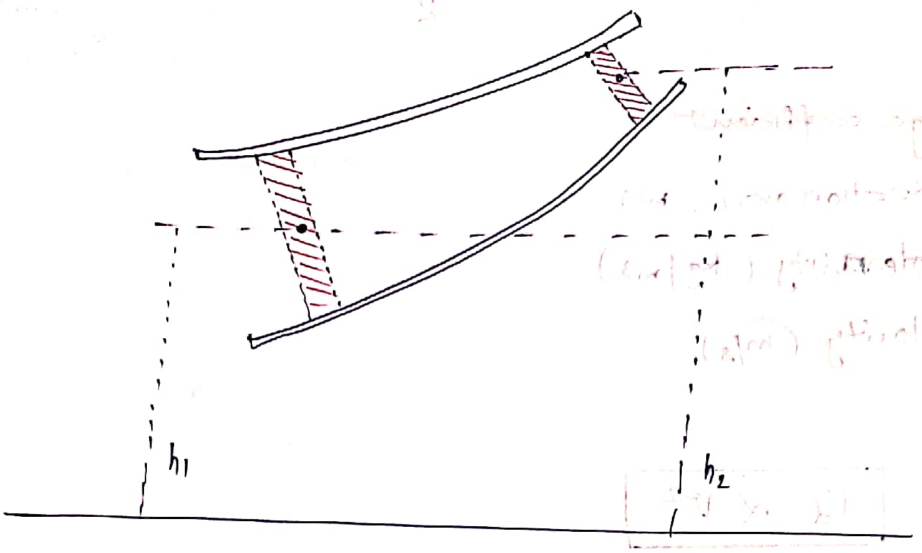
$A \rightarrow$ cross section area (m^2)

$\rho \rightarrow$ fluid ~~dens~~ density (kg/m^3)

$V \rightarrow$ fluid velocity (m/s)

$$\therefore \boxed{F_d \propto v^2}$$

Applying Bernoulli's thearem when $h_1 \neq h_2$



$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$\frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 = (P_1 - P_2) + \rho g(h_1 - h_2)$$

$$V_2^2 - V_1^2 = \frac{2(P_1 - P_2)}{\rho} + 2g(h_1 - h_2)$$

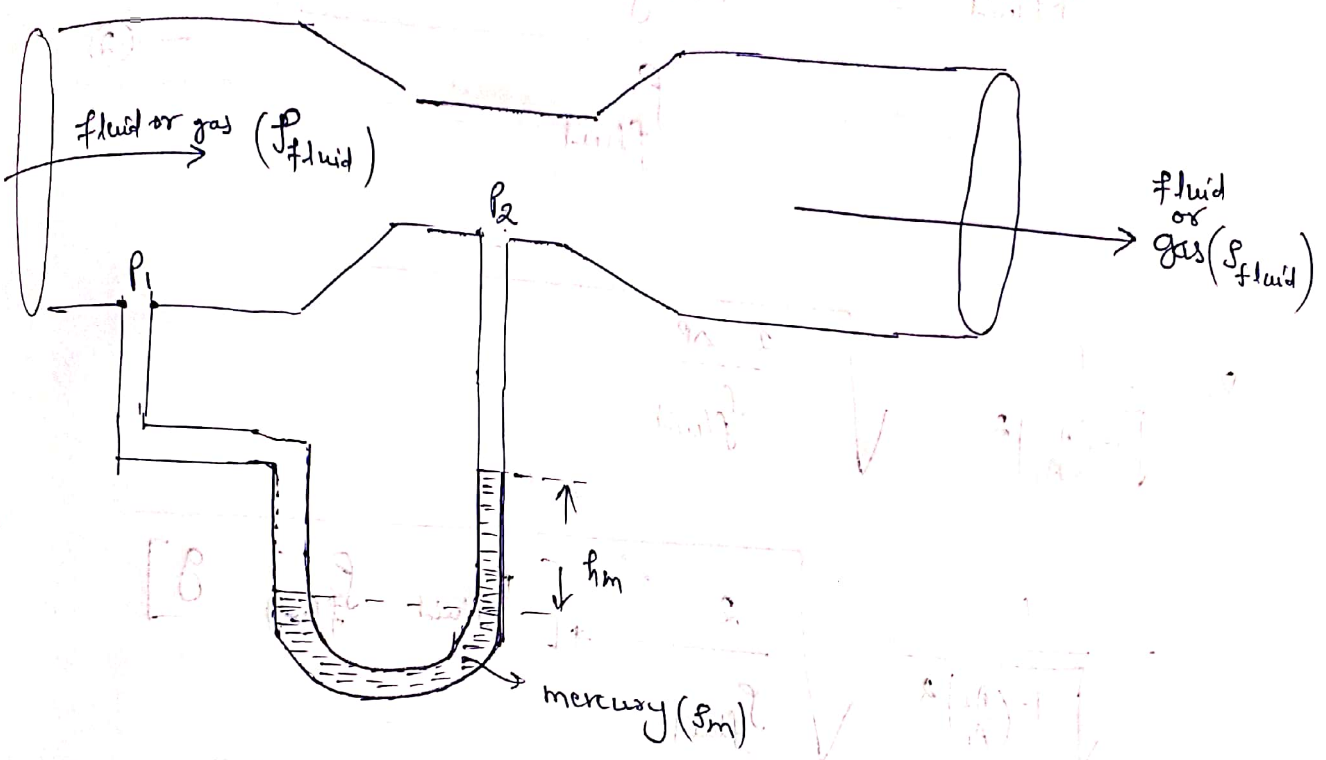
$$V_2^2 - \left(\frac{A_2 V_2}{A_1}\right)^2 = \frac{2(P_1 - P_2)}{\rho} + 2g(h_1 - h_2)$$

$$V_2^2 \left(1 - \frac{A_2^2}{A_1^2}\right) = \frac{2(P_1 - P_2)}{\rho} + 2g(h_1 - h_2)$$

$$V_2 = \frac{1}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2(P_1 - P_2)}{\rho} + 2g(h_1 - h_2)}$$

$$\boxed{V_2 = \frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}\sqrt{\frac{2\Delta P}{\rho} + 2g\Delta h}}$$

Note



$$P_1 - P_2 = \Delta P = h_{mercury}\,\rho_{mercury}\cdot g$$

$$\boxed{\Delta P = h_m \rho_m g}$$

here we have used mercury to measure (P_1-P_2) or ΔP. det we want to measure ΔP using the same fluid which is flowing through the tube. det density of the flowing fluid is ρ_{fluid}. del difference is the colum is h_{fluid}.

$$\Delta P = h_{fluid} \, \rho_{fluid} \, g$$

$$h_{fluid} \, \rho_{fluid} \, g = h_{mercury} \, \rho_{mercury} \, g$$

$$h_{fluid} = \frac{h_{mercury} \cdot \rho_{mercury}}{\rho_{fluid}} \quad \text{——} (2)$$

$$v_2 = \frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2 \, \Delta P}{\rho_{fluid}}}$$

$$= \frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2}{\cancel{\rho_{fluid}}} \times \left[h_{fluid} \cdot \cancel{\rho_{fluid}} \, g \right]}$$

$$\boxed{v_2 = \frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{2 g \, h_{fluid}}}$$

Note

(1) specific mass = Density = $\frac{mass}{Volume}$

$$\rho = \frac{mass}{Volume} \quad \left(\frac{kg}{m3}\right)$$

(2) specific weight = $\frac{Weight}{Volume}$ = $\frac{mass * g}{Volume}$

$$\gamma = \left(\frac{mass}{Volume}\right) \cdot g$$

$$\gamma = \rho \cdot g$$

unit ⇒ $\frac{Newton}{m^3}$

(3)

Specific gravity = Relative density = $\frac{\rho_{substance}}{\rho_{reference}}$

$$\text{Specific gravity} = \frac{\text{Density of Substance}}{\text{Density of reference (maily water)}}$$

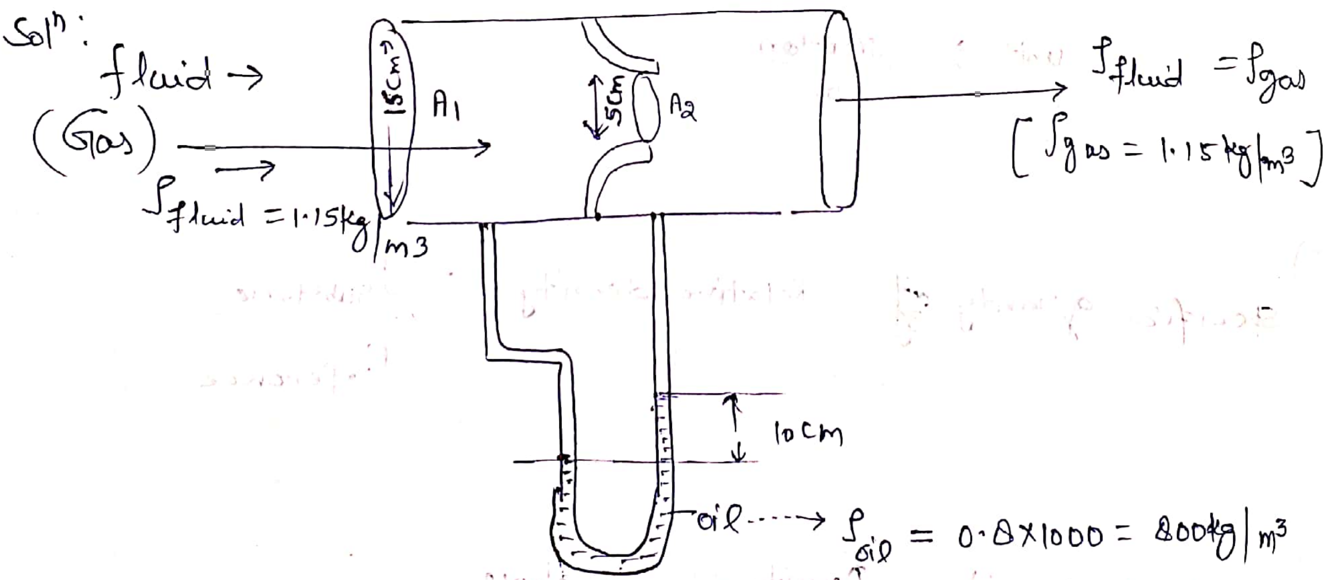
$$= \frac{\rho_{Substance}}{\rho_{H2O}}$$

[Mainly refence is water.]

$\rho_{water} = 1000 \ kg/m^3$

Q: A nozzle is fitted in a horizontal pipe diameter 15 cm carrying a gas of density 1.15 kg/m3 for the purpose of flow measurement. The differential pressure head indicated by an U-tube manometer containing oil of specific gravity 0.8 is 10 cm. If the coefficient of discharge and diameter of nozzle are 0.8 and 5 cm respectively.
Determine the gas flow rate using nozzle flow meter

Soln:



oil → $\rho_{oil} = 0.8 \times 1000 = 800 kg/m^3$

$\rho_{gas} = 1.15 \ kg/m3$ (It flows through the pipe)

$A_1 = \frac{\pi}{4}(0.15)^2 = 0.0176625 \ m^2$

$A_2 = \frac{\pi}{4}(0.05)^2 = 0.0078500 = 0.0019625 \ m^2$

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Density of substance = specific gravity * ρ_{H_2O}

$= 0.8 \times 1000\ kg/m^3$

$$\boxed{\rho_{oil} = 800\ kg/m^3}$$

$\Delta P =$ 10 cm of oil having density of 800 kg/m3

$$\Delta P = h_{oil}\left(\rho_{oil} - \rho_{gas}\right) g$$

$$= 10\times10^{-2} * (800 - 1.15) * 9.8$$

$$= 782.873\ \frac{N}{m^2}$$

$$v_2 = \frac{C_d}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}\sqrt{\frac{2\Delta P}{\rho}}$$

$$\therefore Q = \frac{C_d\ A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}\sqrt{\frac{2\Delta P}{\rho}}$$

$$= \frac{0.8 * (.0019625)}{\sqrt{1-\left[\frac{0.0019625}{0.0176625}\right]^2}} * \sqrt{\frac{2 * 782.873}{1.15}}$$

$$= \frac{0.00157}{\sqrt{1 - .01234567}} * 36.90$$

$$= \frac{0.00157}{\sqrt{0.987654321}} * 36.90$$

$$= \frac{0.00157 * 36.90}{.993807990}$$

$$= \frac{0.057933000}{.993807990}$$

$$= 0.058293957 \ \frac{m^3}{s}$$

$$\boxed{Q = 58293.957 \ \frac{cm^3}{s}}$$

method(2)

$$Q = \frac{C_d A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{2 g h_{fluid}}$$

$$h_{fluid} \rho_{fluid} g = h_{oil} [\rho_{oil} - \rho_{fluid}] g$$

$$h_{fluid} = \frac{h_{oil} [\rho_{oil} - \rho_{fluid}]}{\rho_{fluid}} = \frac{10 \times 10^{-2} * [800 - 1.15]}{1.15}$$

$$= 69.47 \text{ meter}$$

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$$Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{2 g h_{fluid}}$$

$$= \frac{0.8 * 0.0019625}{\sqrt{1-\left(\frac{0.0019625}{0.0176625}\right)^2}} \sqrt{2 * 9.8 * 69.47}$$

$$= 0.00157978 * 36.90$$

$$= 0.05829 3925 \ \frac{m^3}{s}$$

$$\boxed{Q = 58293.92 \ \frac{cm^3}{s}}$$

Note ⇒

$\rho_{oil} = 800 kg/m^3$

$\Delta P = h_{oil}(\rho_{oil} - \rho_{gas}) g = 10 \times 10^{-2} * [800 - 1.15] * 9.8 = 782.873 \frac{N}{m^2}$

$$Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2 \Delta P}{\rho}} = \frac{0.8 * 0.0019625}{\sqrt{1-\left[\frac{5}{15}\right]^4}} * \sqrt{\frac{2 * 782.873}{1.15}}$$

$$= 58293.957 \frac{cm^3}{s}$$

$$h_{fluid} \cdot \rho_{fluid} \cdot g = h_{oil} (\rho_{oil} - \rho_{fluid}) g$$

$$h_{gas} \cdot \rho_{gas} \cdot g = h_{oil} (\rho_{oil} - \rho_{gas}) g$$

$$\therefore h_{gas} = \frac{h_{oil}[\rho_{oil} - \rho_{gas}]}{\rho_{gas}} = \frac{10 \times 10^{-2} [800 - 1.15]}{1.15} = 69.47 m$$

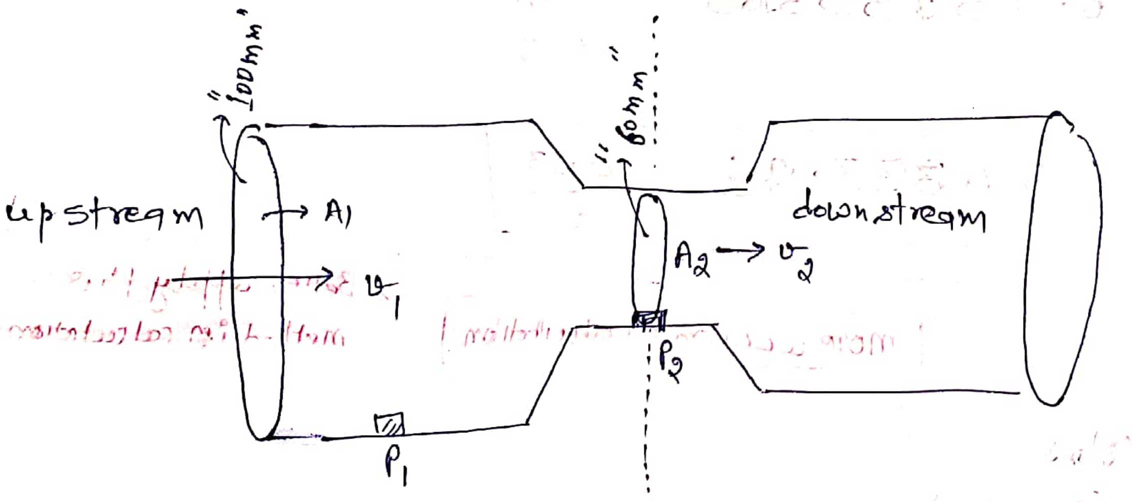
$$\therefore Q = \frac{C_d A_2}{\sqrt{1-(A_2/A_1)^2}} \sqrt{\frac{2 \Delta P}{\rho}} = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2 h_{gas} \rho_{gas} g}{\rho_{gas}}} = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{2 g h_{gas}}$$

$$= 58293.9 \frac{cm^3}{s}$$

Q: A venturi meter tube of throat diameter 60mm is placed in a water pipe of diameter 100mm to measure the volumetric flow rate. Let the volumetric flow rate through the tube is 0.08 m^3/s and the water has a density of 1000 kg/m^3 and viscocity of 10^{-3} N-s/m^2.

(a) Determine the Reynold's number

(b) if the co-efficient of discharge is 0.99, determine the upstream to throat differential pressure.



P_1 = upstream pressure

P_2 = Downstream pressure (throat pressure)

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi}{4}\left[100 \times 10^{-3}\right]^2 = 7.85 \times 10^{-3} \; m^2$$

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi}{4}\left(60 \times 10^{-3}\right)^2 = 2.8260 \times 10^{-3} \; m^2$$

$$Q = A_2 V_2$$

$$[V_2]_{throat} = \frac{Q}{[A_2]_{throat}}$$

$$= \frac{0.08 \ m^3/s}{2.8260 \times 10^{-3} \ m^2}$$

$$= 28.309 \ m/s$$

(a) Reynold's number $Re = \frac{v \, d \, \rho}{\eta}$

$$Re = \frac{[V_2]_{throat} \, [d_2]_{throat} \, \rho_{fluid}}{\eta}$$

$$= \frac{28.309 * (60 \times 10^{-3}) * 1000}{10^{-3}}$$

$$= 1.69 \times 10^{6}$$

m^2

m^2

[b] $\Delta P = P_1 - P_2$

= upstream to throat differential pressure.

$$Q = A_2 V_2 = \frac{Cd * A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

$$Q = Cd * \frac{A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

$$\sqrt{\frac{2\Delta P}{\rho}} = \frac{Q\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}{Cd * A_2}$$

$$\Delta P = \frac{Q^2}{C_d^2} * \frac{\left[1-\left(\frac{A_2}{A_1}\right)^2\right]}{A_2^2} \left[\frac{\rho}{2}\right]$$

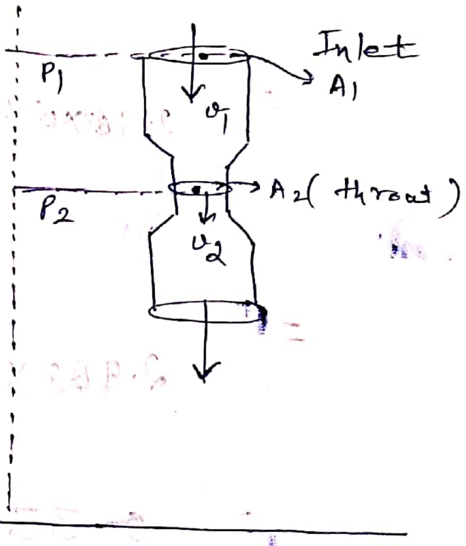
$$= \frac{(\cdot 08)^2}{(\cdot 99)^2} * \frac{\left[1-\left(\frac{2\cdot 8260\times 10^{-3}}{7\cdot 05\times 10^{-3}}\right)^2\right]}{\left[2\cdot 8260\times 10^{-3}\right]^2} * \frac{10^3}{2}$$

$$= [0\cdot 006530] * \frac{0\cdot 8704}{7\cdot 9862 * 10^{-6}} * \frac{(10^3)}{2}$$

$$= \frac{0\cdot 006530 \times \cdot 8704 * 0\cdot 5}{7\cdot 9862} \times 10^9 = 359\cdot 901 \times 10^3 \ N/m^2$$

Q: A venturimeter is used to measure the volume flow rate of an oil having a density of 850 kg/m^3. It is fitted in a vertical pipe line with oil flowing downwards. Its diameter at the inlet and throat are 0.3 m and 0.2 m respectively. The pressure at the inlet and throat are measured by pressure transducers are are found to be $1.8 \times 10^5 \frac{N}{m^2}$ and $1.4 \times 10^5 \frac{N}{m^2}$ respectively. The difference in height between the inlet and throat is 0.5 m. The discharge coefficient of the venturi meter tube is 0.95. Determine the volume flow rate of the oil.

Soln ⇒



$\rho_{fluid} = 85\ kg/m^3$

$d_1 = 0.3\ m$

$d_2 = 0.2\ m$

$\Delta P = P_1 - P_2$

$= 1.8 \times 10^5 - 1.4 \times 10^5$

$= 0.4 \times 10^5\ N/m^2$

$C_d = 0.95$

$\Delta h = 0.5$ meter

Datum line

$\therefore A_1 = \frac{\pi d_1^2}{4} = \frac{\pi}{4}(0.3)^2 = 7.065 \times 10^{-2}\ m^2$

$\therefore A_2 = \frac{\pi d_2^2}{4} = \frac{\pi}{4}(0.2)^2 = 3.14 \times 10^{-2}\ m^2$

here height of inlet and throat is not same it means

$h_{inlet} \neq h_{throat}$.

So formula for this case will be →

$Q = A_2 V_2$ →

$$Q_{actual} = C_d \, A_2 \frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho} + 2g\Delta h}$$

$$= 0.95 * (3.14 \times 10^{-2}) \frac{1}{\sqrt{1-\left[\frac{3.14 \times 10^{-2}}{7.065 \times 10^{-2}}\right]^2}} \sqrt{\left[\frac{2\cdot(0.4 \times 10^5)}{850}\right] + [2 * 9.81 * 0.5]}$$

$$= 2.983 \times 10^{-2} * 1.11631 * \sqrt{94.11765 + 9.81}$$

$$= 2.983 \times 10^{-2} \; 1.11631 * 10.1945$$

$$= 0.33947 \; m^3/\text{second}$$

Q: A Submarine moves horizontally in the sea and has its axis much below the surface of the sea water. A pitot tube properly placed just in front of the submarine is connected to a differential pressure gauge. The pressure difference between pitot pressure and static pressure was found to be 20 KN/m2. Find the speed of the submarine if the density of sea water is 1026 Kg/m3.

Soln :

$$\Delta P = P_{stagnation} - P_{static}$$

$$\Delta P = 20 \ KN/m^2$$

This pressure head is due to the velocity of fluid with respect to submarine i.e. due to the velocity of the submarine.

$$v = \sqrt{\frac{2\Delta P}{\rho}}$$

$$= \sqrt{\frac{2 \times 20 \times 1000}{1026}}$$

$$= \sqrt{38.98}$$

$$\boxed{v = 6.24 \ m/s}$$

Q: Calculate the generated emf by the magnetic flowmeter if the ordinary tap water flows at the rate of 400 litres per minutes through its tube of 8 cm diameter. Assume the maximum flux density of the alternating magnetic field established in the flowmeter to be 400 lines per cm^2

$1 \text{ Litre} = 1000 \text{ cm}^3 = 10^{-3} \text{ m}^3$

Soln:

$Q = 400 \text{ litre / minute}$

$Q = \frac{400}{60} \text{ litre/s}$

$Q = 6.667 \text{ litre/s}$

$= 6.667 \times 10^{-3} \text{ m}^3/\text{sec}$

$\because Q = AV$

$v = \frac{Q}{A}$

$= \frac{6.667 \times 10^{-3}}{\frac{\pi}{4}(0.08)^2}$

$= 1.327 \frac{m}{\text{second}}$

l = diameter of pipe $= 0.08$ m

$B = 400 \times 10^{-4}$ tesla

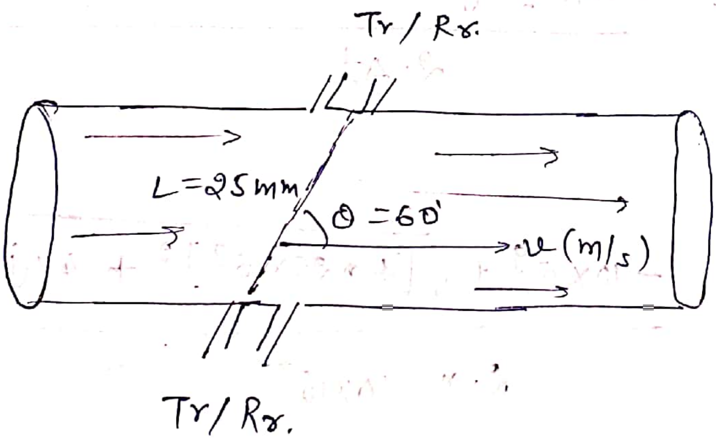
$Emf = l \cdot v \cdot B$

$= [0.08] * [1.327] * 400 \times 10^{-4}$

$= 4.24 \times 10^{-3}$

$= 4.24$ mV

Q: In an ultrasonic flowmeter, transducers are seperated by a distance $L = 25mm$ with the line joining the transducers making an angle of $60°$ with the direction of flow. The transit time difference between upstream and downstream measurements is 10 ns with the sound velocity in the medium being 1000 m/s. Assume that the size of transducers is very small as compared to the diameter of pipe, Calculate the flow velocity.



Given →

$L = 25 \times 10^{-3} m$

$\theta = 60°$

$\Delta t = 10 \times 10^{-9}$ sec.

$C = 1000$ m/s

$v = ?$

$$\Delta t = \frac{2 L v \cos\theta}{c^2 - v^2 \cos^2\theta} \quad \text{——— (1)}$$

$$\Delta t = \frac{2 L v \cos 60°}{c^2 - v^2 \cos^2 60°}$$

$$\Delta t = \frac{2L\, v/2}{c^2 - v^2/4} \qquad \left(\cos 60° = \frac{1}{2}\right)$$

$$c^2 \Delta t - \frac{v^2}{4} \cdot \Delta t = Lv$$

$$\frac{v^2}{4} \cdot \Delta t + Lv - c^2 \Delta t = 0$$

$$(\Delta t) v^2 + (4L) v - 4c^2 \Delta t = 0 \quad \text{——— (2)}$$

$$ax^2 + bx + c = 0 \quad \text{——— (2)}$$

$$x = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$$

$$= \frac{-\Delta t \pm \sqrt{(4L)^2 + 4 \Delta t * (4c^2 \Delta t)}}{2 \cdot \Delta t}$$

$$= \frac{-10 \times 10^{-9} \pm \sqrt{[4 * 25 \times 10^{-3}]^2 + 4 \times (10 \times 10^{-9})^2 * 4 * (10^3)^2}}{2 * 10 \times 10^{-9}}$$

$$= \frac{\quad}{\quad} \ m/s$$

Method-(II)

From eqn (2):

$$v^2 + \frac{4L}{\Delta t} v - \frac{4c^2 \Delta t}{\Delta t} = 0$$

$$\boxed{v^2 + \left(\frac{4L}{\Delta t}\right) v - 4c^2 = 0}$$

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$$v^2 + \left[\frac{4\times 25\times 10^{-3}}{10\times 10^{-9}}\right] v - 4\times(10^3)^2 = 0$$

$$\boxed{v^2 + 10^7 v - 4\times 10^6 = 0}$$

$$v = \frac{-10^7 \pm \sqrt{(10^7)^2 + 4\times 1 \times 4\times 10^6}}{2}$$

$$= \frac{-10^7 \pm \sqrt{10^{14} + 16\times 10^6}}{2}$$

$$= \frac{-10^7 \pm \sqrt{1.00000016 \times 10^{14}}}{2}$$

$$= \frac{-10^7 \pm 1.00000008\times 10^7}{2}$$

$$= \frac{-10^7 + 1.00000008\times 10^7}{2}$$

$$= \frac{8\times 10^{-1}}{2}$$

$$= 4\times 10^{-1}$$

$$= 0.4 \ m/s$$

Method III

$$\Delta t = \frac{2Lv\cos\theta}{c^2 - v^2\cos^2\theta}$$

$\because$ $c^2 >> v^2\cos^2\theta$ so we can neglect the term $v^2\cos^2\theta$.

$$\therefore \Delta t = \frac{2Lv\cos\theta}{c^2}$$

$$\therefore v = \frac{\Delta t \cdot c^2}{2L\cos\theta}$$

$$= \frac{10\times10^{-9} \times [10^3]^2}{2\times 25\times 10^{-3} \times \cos 60^\circ}$$

$$= \frac{10^{-2}}{25\times10^{-3}}$$

$$= \frac{10}{25}$$

$$\boxed{v = 0.4 \ m/s}$$

Q: A hari

(a) D

(B)

Soln

(b

Q: A Pitot tube is used for measurement of velocity of water having a density of 1000 kg/m3.

(a). Determine the velocity of flow at the heat of the pitot tube if it produces a differential pressure of 10 KN/m2 between its two outlets.

(b) The same diffenential pressure is obtained in air at an altitude where the density of air is 0.65 kg/m3. Determine the velocity of air flow.

Soln —

(a) $$v = \sqrt{\frac{2\Delta P}{\rho}}$$

$$= \sqrt{\frac{2*10*1000}{1000}} = \sqrt{20} = 4.47 \text{ m/s}$$

(b) $$\text{Velocity of air} = \sqrt{\frac{2\Delta P}{\rho}}$$

$$= \sqrt{\frac{2*10\times10^3}{0.65}}$$

$$= 175.41 \text{ m/s}$$

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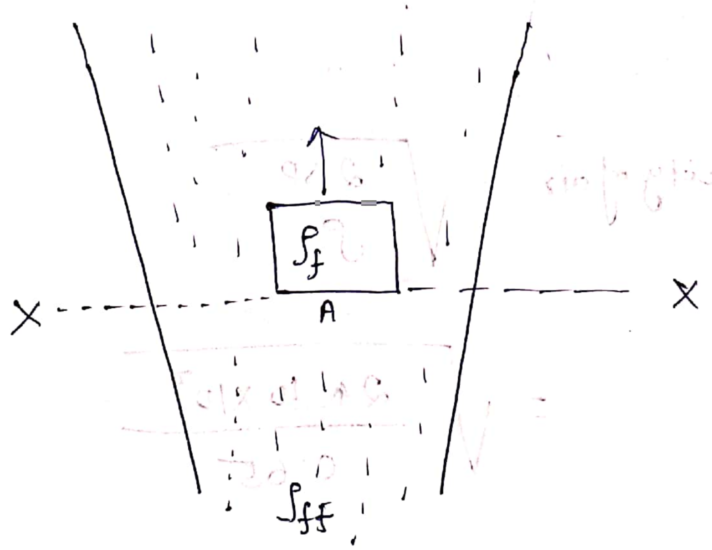
Q: The figure shows the float of a rotameter stationary at level XX for a certain flow rate of water. The specific gravity of the float is 2, mass 10^{-2} kg and base area 1.0×10^{-2} m^2.

Neglect the effect due to viscosity, (9)

a) Calculate the drag force.

(b) Calculate the pressure drop across the float

(c) if the velocity of water before the float is $V_1 = 0.1$ m/s, then using Bernoulli's equation find the velocity just after the section XX.



Let density of water = 1000 kg/m^3

Sol^n: specific gravity $= \dfrac{\rho_{substance}}{[\rho_{H_2O}]_{Reference}}$

$\rho_{substance} = \text{specific gravity} * \rho_{H_2O}$

$\rho_{float} = 2 * 1000 \; kg/m^3$

$$\boxed{\rho_{float} = 2000 \; kg/m^3}$$

(4) Drag force + Buoyancy force = Weight

$F_{drag} = F_{weight} - F_{buoyancy}$

$= V_f \rho_f g - V_f \rho_{ff} g$

$F_{drag} = (\rho_f - \rho_{ff}) V_f \cdot g$ ——— (1)

Volume of float = $\dfrac{\text{Mass of float}}{\text{Density of float}}$

$V_f = \dfrac{m}{\rho_f}$

$F_{drag} = [\rho_f - \rho_{ff}] \left(\dfrac{m}{\rho_f}\right) g$

$= \left[\dfrac{\rho_f - \rho_{ff}}{\rho_f}\right] m \cdot g$ ——— (2)

$= \left[1 - \dfrac{\rho_{ff}}{\rho_f}\right] m \cdot g = \left[1 - \dfrac{1000}{2000}\right] * [10^{-2}] * 9.81$

$= 4.90 \times 10^{-2}$ N

F_{drag} = $\Delta P \cdot$ ~~Base~~ area of float

$$F_{drag} = \Delta P * A_f$$

$$\Delta P = \frac{F_{drag}}{A_f}$$

$$= \left(\frac{\rho_f - \rho_{ff}}{\rho_f}\right) \frac{m \cdot g}{A_f} \qquad \text{—— (3)}$$

as given in ques:
$S_g = 2$
$[\rho_{ff} = \rho_{water}]$
$[\rho_f = 2\rho_{water}]$

$$= \frac{mg}{A_f} * \left[1 - \frac{\rho_{ff}}{\rho_f}\right]$$

$$= \frac{m \cdot g}{A_f}\left[1 - \frac{1000}{2000}\right]$$

$$= \frac{[10^{-2}] * [9.81]}{1.0 \times 10^{-2}} * \left[1 - \frac{1}{2}\right]$$

$$P_1 - P_2 = \boxed{\Delta P = 4.9 \ N/m^2}$$

[C]
Note: The difference in height of the top and bottom of the float is

$$h_1 - h_2 = \frac{\text{Volume of float}}{\text{Area of float}} = \frac{V_f}{A_f} = \frac{m/\rho_f}{A_f}$$

$$(h_1 - h_2) = \frac{m}{\rho_f A_f} \qquad \text{—— (4)}$$

[C]

$$P_1 + \frac{1}{2}\rho_{ff} V_1^2 + \rho_{ff} g h_1 = P_2 + \frac{1}{2}\rho_{ff} V_2^2 + \rho_{ff} g h_2$$

$$\frac{1}{2}\rho_{ff} V_2^2 = (P_1 - P_2) + \rho_{ff} g (h_1 - h_2) + \frac{1}{2}\rho_{ff} V_1^2$$

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_{ff}} + 2 \cdot g \cdot (h_1 - h_2) + V_1^2}$$

$$= \sqrt{\frac{\left[\frac{2mg}{A_f}\frac{(\rho_f - \rho_{ff})}{\rho_f}\right]}{\rho_{ff}} + 2g \cdot \frac{m}{\rho_f A_f} + V_1^2}$$

Put $P_1 - P_2$ and $h_1 - h_2$ from eq^n (3) and (4)

$$= \sqrt{\frac{2mg}{A_f \rho_{ff}} - \cancel{\frac{2mg}{A_f \rho_f}} + \cancel{\frac{2mg}{A_f \rho_f}} + V_1^2}$$

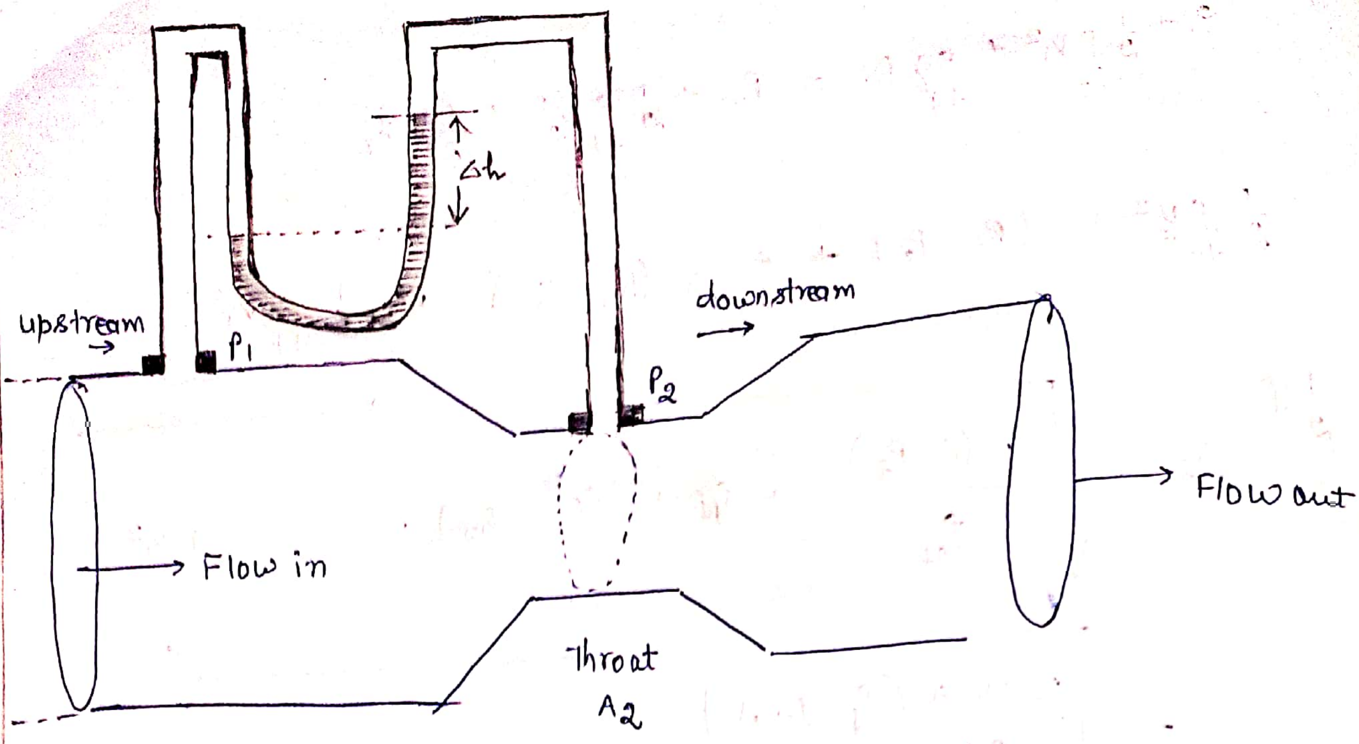
$$= \sqrt{\frac{2mg}{A_f \rho_{ff}} + V_1^2}$$

$$= \sqrt{\frac{2 \times 10^{-2} \times 9.81}{1.0 \times 10^{-2} * 1000} + (0.1)^2}$$

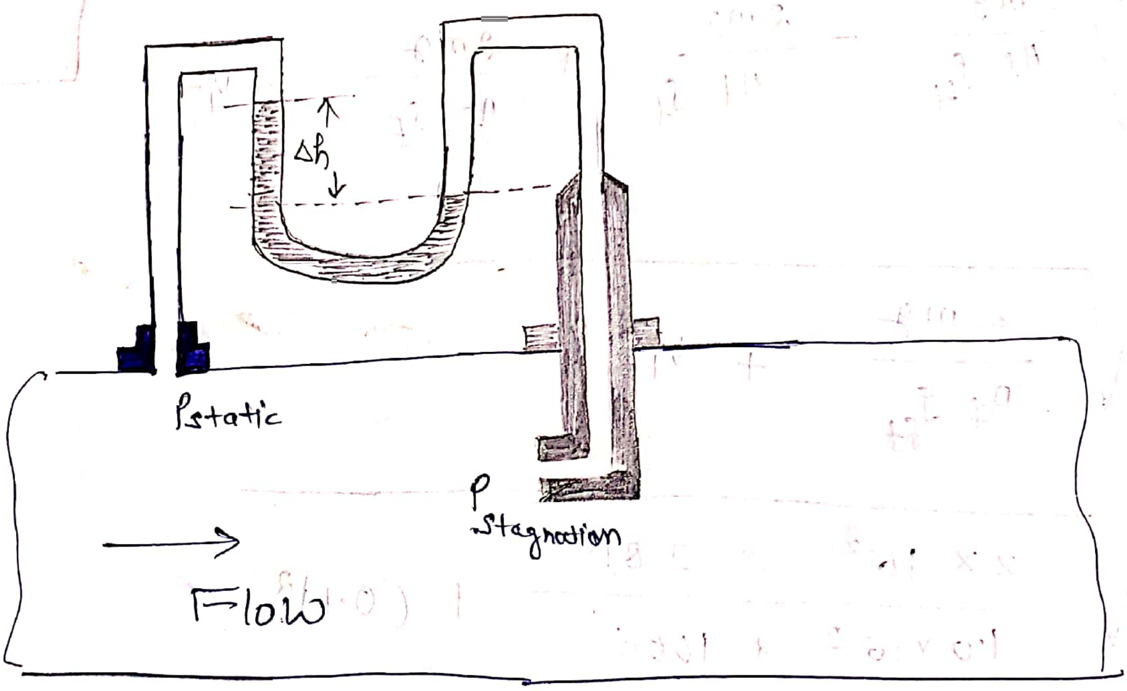
$$= \sqrt{(19.62 \times 10^{-3}) + 0.01}$$

$$= \sqrt{0.02962} = 0.17 \text{ m/s}$$

Note



Venturimeter



Pitot tube

Q: A venturi meter is used to measure flow rate of water that is 4 kg/second. Venturimeter throat diameter is 3.5 cm. Pipe diameter is 7 cm. Find the maximum range of differential pressure ΔP.

Let $C_d = 0.95$; $\rho_{water} = 1000\ kg/m^3$

Sol^n:

Volume flow rate : Q (m^3/s)

$$Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

mass flow rate = 4 kg/second

$$m = \rho \cdot Q$$

$$\therefore Q = \frac{m}{\rho}$$

$$Q = \frac{4\ kg/second}{1000\ kg/m^3}$$

$$\boxed{Q = 4 \times 10^{-3}\ m^3/s}$$

$$Q = \frac{cd\ A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

$$\Delta P = \frac{Q\sqrt{1-\left[\frac{\pi d_2^2/4}{\pi d_1^2/4}\right]^2}}{cd\left[\pi d_2^2/4\right]} \times \frac{\rho}{2} = \frac{Q\sqrt{1-\left(\frac{d_2}{d_1}\right)^4}}{cd\left[\frac{\pi d_2^2}{4}\right]} \quad \frac{\rho}{2}$$

$$= \frac{4\times10^{-3}\sqrt{1-\left[\frac{3.5\times10^{-2}}{7\times10^{-2}}\right]^4}}{0.95 * \frac{\pi}{4}(3.5\times10^{-2})^2} * \frac{1000}{2}$$

$$= \frac{4\times10^{-3} * (0.9682) * 500}{0.95 * 9.61625\times10^{-4}} = 2.11966\times10^{3}$$

=

$$\boxed{\Delta P = 2.11\ \frac{KN}{M^2}}$$

Answer

Q: A pitot tube is placed in aeroplane. U tube manometer manometer connected to it shows deflection of 10 cm of water.

Find :

a) Air velocity if density of air $= 1.2\ kg/m^3$

$\rho_{water} = 1000\ kg/m^3$

Soln

$$v = \sqrt{\frac{2\Delta P}{\rho}}$$

$$= \sqrt{\frac{2\Delta P}{\rho_{air}}}$$

→ Because air is flowing through pitot tube

ΔP = 20 cm of water

$$\Delta P = gh\left[\rho_{water} - \rho_{air}\right]$$

$$= 9.8 \times [10\times10^{-2}] * [1000 - 1.2]$$

$$= 978.824\ N/m^2$$

$$v = \sqrt{\frac{2\Delta P}{\rho_{air}}}$$

$$v = \sqrt{\frac{2 * 978.924}{1.2}}$$

$$\boxed{v = 40.39 \ \frac{m}{s}}$$

Q: A pitot tube is placed in front of submarine horizontally. U tube manometer filled with mercury shows deflection of 15 cm of mercury between stagnation and static. Find the speed of submarine.

Given:
Density of sea water = 1026 kg/m3
Density of mercury = 13600 kg/m3

Soln:

$$\Delta P = P_{stagnation} - P_{static}$$

$$\Delta P = 15 \text{ cm of mercury}$$

water is flowing througth the pitot tube.

$$\Delta P = g h [\rho_{mercury} - \rho_{water}]$$

$$= 9.81 * [15 \times 10^{-2}] * [13600 - 1026]$$

$$\Delta P = 18.48 \ \frac{KN}{m^2}$$

$$v = \sqrt{\frac{2\Delta P}{\rho_{water}}} = \sqrt{\frac{2 \times 18.48 \times 1000}{1026}} = 6 \ m/s$$

Let if it is asked to convert 15 cm of mercury column to water column then procedure is given below:-

$$h_w \rho_w g = h_m [\rho_m - \rho_w] g$$

$$h_w \rho_w = h_m [\rho_m - \rho_w]$$

$$h_w * [1026] = [15 \times 10^{-2}] [13600 - 1026]$$

$$\boxed{h_w = 1.8 \text{ meter}}$$

Thus if water is used in place of mercury to measure the pressure difference then it will reach up to 1.8 meter.

Q: In a variable area flow meter (rotameter), the volumetric flow rate of fluid is Q_1 then float stands at a height of 10 mm from inlet; and when volumetric flow rate of fluid is Q_2 then float moves a height of 20 mm from the inlet. Find the ratio of $\frac{Q_1}{Q_2}$.

Sol^n:

$$Q \propto (A_t - A_f)$$

$$Q = K_1 + K_2 x$$

$$Q \propto x$$

$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

$$\frac{Q_1}{Q_2} = \frac{70mm}{20mm}$$

$$\frac{Q_1}{Q_2} = \frac{7}{2}$$

$$\boxed{\frac{Q_1}{Q_2} = 3.5}$$

Q: A rotameter is calibrated for measure the flowrate of liquid of density 1000kg/m3 with a scale ranges from 1 to 100 litre/minute. It is used to measure the flow of another material having density of 1.25 kg/m3 with flow ranges between 20 to 2,0.00 l/minute. Find the density of new float if original float density is 2000kg/m3.

Soln:

[1]

$\rho_{ff_1} = 1000\ kg/m^3$

$Q_1 \rightarrow 1$ to 100 litre/min

$\rho_{f_1} \rightarrow 2000\ kg/m^3$

[2]

$\rho_{ff_2} = 1.25\ kg/m^3$

$Q_2 = 20$ to $2,000$ l/minute

$\rho_{f_2} = ?$

$$Q = \frac{C_d\ (A_t - A_f)}{\sqrt{1 - \left(\frac{A_t - A_f}{A_t}\right)^2}} * \sqrt{\frac{2g V_f}{A_f} * \frac{\rho_f - \rho_{ff}}{\rho_{ff}}}$$

$$Q \propto \sqrt{\frac{\rho_f - \rho_{ff}}{\rho_{ff}}}$$

$$\frac{Q_1}{Q_2} = \frac{\sqrt{\frac{2000 - 1000}{1000}}}{\sqrt{\frac{\rho_{f_2} - 1.25}{1.25}}}$$

$$\frac{100\ l/min}{2000\ l/min} = \frac{\sqrt{\frac{1000}{1000}}}{\sqrt{\frac{\rho_f - 1.25}{1.25}}}$$

$$\frac{1}{20} = \sqrt{\frac{1.25}{\rho_{f_2} - 1.25}}$$

$$\rho_{f_2} - 1.25 = 400 \times 1.25$$

$$\rho_{f_2} = 500 + 1.25$$

$$\boxed{\rho_{f_2} = 501.25 \quad kg/m^3}$$

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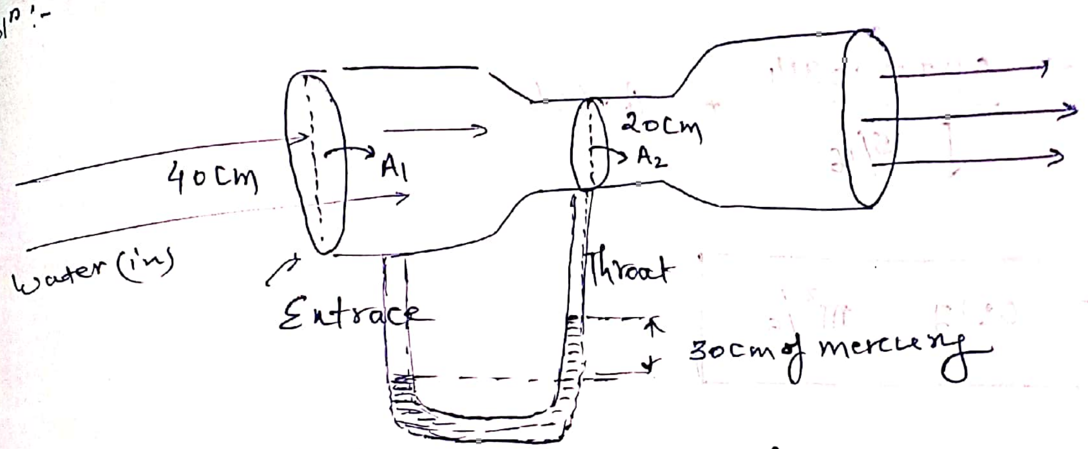
Q: A 40cm X 20cm Venturi meter is used to measure flow rate of water in pipe line. U-tube mercury manometer connected between entrance and throat indicates deflection of 30cm [mercury column]. Find the rate of flow of water.

$Cd = 0.97$.

$\rho_{mercury} = 13600$ kg/m3

$\rho_{water} = 1000$ kg/m3

Soln:-



$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi [40\times10^{-2}]^2}{4} = 0.1256\ m^2$$

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi [20\times10^{-2}]^2}{4} = 0.0314\ m^2$$

$\Delta P =$ 30 cm of mercury

$$\Delta P = g h [\rho_m - \rho_w]$$

$$= 9.8 \times [30\times10^{-2}] * [13600 - 1000]$$

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$$= 37.04 \ \frac{KN}{m^2}$$

$$\hat{Q} = \frac{C_d \quad A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

$$= \frac{0.97 * (0.0314)}{\sqrt{1 - \left[\frac{\left\{\frac{\pi}{4}(20\times10^{-2})^2\right\}}{\left\{\frac{\pi}{4}(40\times10^{-2})^2\right\}}\right]^2}} \sqrt{\frac{2 \times 37.04 \times 1000}{1000}}$$

$$= \frac{0.97 \times .0314}{\sqrt{15/16}} * 8.606$$

$$\boxed{Q = 0.27 \quad m^3/s}$$

Note Find the pressure difference in terms of water column.

$$h_w \rho_w g = h_m [\rho_m - \rho_w] g$$

$$h_w = \frac{h_m [\rho_m - \rho_w]}{\rho_w}$$

$$= \frac{[30\times10^{-2}][13600 - 1000]}{1000}$$

$$\boxed{h_w = 3.78 \text{ meter}}$$

$$Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

$$= \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} * \sqrt{\frac{2\Delta P}{\rho_{water}}} \quad \text{—— (1)}$$

[∵ flowing fluid is water so $\rho = \rho_{water}$]

If ΔP is measured using the same fluid which is flowing through the pipe; here water is flowing through the pipe and ΔP is measured interms of same fluid (water) then

$$\Delta P = h_w \rho_w g$$

Put this value in equation (1)

$$Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} * \sqrt{\frac{2[h_w \rho_w g]}{\rho_w}}$$

$$\boxed{Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{2 g h_w}}$$

$h_w = 3.78$ meter

$$\therefore Q = \frac{0.97 * \frac{\pi}{4} * (20\times10^{-2})^2}{\sqrt{1-\left[\frac{\pi/4\,(20\times10^{-2})^2}{\frac{\pi}{4}\,(40\times10^{-2})^2}\right]^2}} * \sqrt{2\times9.8\times3.78}$$

$$= \frac{0.97 * .0314}{\sqrt{15/16}} * \sqrt{74.088}$$

$$\boxed{Q = 0.27 \ m^3/s}$$

Note

$$Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho_f}}$$

let $\rho = \rho_f$

$$Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{d_2}{d_1}\right)^4}} \sqrt{\frac{2\Delta P}{\rho_f}}$$

But :

$$\boxed{\Delta P = h_m(\rho_m - \rho_f)g}$$

$$Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\{h_m(\rho_m-\rho_f]g\}}{\rho_f}}$$

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$$\boxed{Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{2g\left(\frac{\rho_m}{\rho_f}-1\right)h_m}}$$

Note ⇒ $\rho = \rho_f$ = Density of fluid which is flowing through the pipe.

$$A_2 = \frac{\pi}{4} d_2^2$$

$$A_1 = \frac{\pi}{4} d_1^2$$

$$\frac{A_2}{A_1} = \frac{\frac{\pi}{4} d_2^2}{\frac{\pi}{4} d_1^2} = \frac{d_2^2}{d_1^2}$$

$$\left(\frac{A_2}{A_1}\right)^2 = \left(\frac{d_2^2}{d_1^2}\right)^2 = \left(\frac{d_2}{d_1}\right)^4$$

Q: A pitot tube is used to measure the velocity of an air stream at 20°C and 0.1 MPa. If the velocity is 10 m/s, what is the dynamic pressure in newton/m^2.

Sol^n air stream is at 20°C and 0.1 MPa

$$PV = nRT$$

$$P = \rho R^* T \qquad T = 20+273$$

$$\rho = \frac{P}{R^* T} = \frac{0.1 \times 10^6}{287 * (293)}$$

$$\boxed{\rho_{air} = 1.189 \ kg/m^3}$$

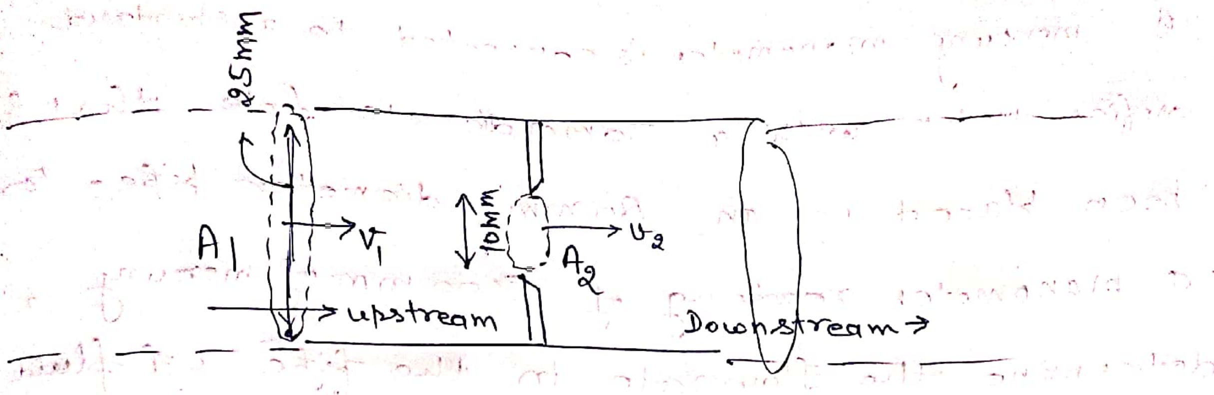
$$v_{air} = \sqrt{\frac{2\Delta P}{\rho_{air}}} \Rightarrow \therefore \Delta P = \frac{1}{2} \rho_{air} v^2$$

$$= \frac{1}{2} [1.189] [10]^2$$

$$= 59.45 \ \frac{N}{m^2}$$

Q: An orifice is to be used to indicate the flow rate of water in a 25 mm diameter line. The orifice diameter is 10 mm. What pressure reading in mm of mercury will be experienced on the orifice for a line flow velocity [upstream velocity] of 5 m/s ?

$$\left[\begin{array}{l} \rho_{water} = 1000 \ kg/m^3 \\ \rho_{mercury} = 13600 \ kg/m^3 \end{array}\right]$$



$v_1 \rightarrow$ upstream velocity

$v_2 \rightarrow$ Down stream velocity

$$Q = A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi d_1^2/4}{\pi d_2^2/4} v_1 = \left(\frac{d_1}{d_2}\right)^2 v_1$$

$$= \left[\frac{25\times10^{-3}}{10\times10^{-3}}\right]^2 \times 5$$

$$v_2 = 31.25 \text{ m/s}$$

$$\because v_2 = \frac{Cd}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

$$31.25 = \frac{1}{\sqrt{1-\left(\frac{d_2}{d_1}\right)^4}} \sqrt{\frac{2\Delta P}{\rho}}$$

$$31.25 * \sqrt{1-\left(\frac{10}{25}\right)^4} = \sqrt{2\Delta P/\rho}$$

$$\boxed{\Delta P = 475781.25 \text{ N/m}^2}$$

$$\Delta P = h_{mercury}\left[\rho_{mercury} - \rho_{water}\right] g$$

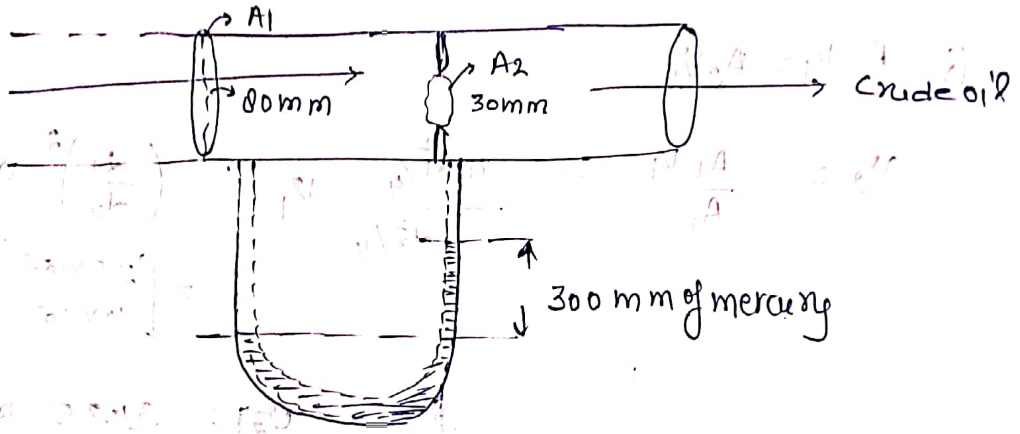
$$h_m = \frac{\Delta P}{[\rho_m - \rho_w] g} = \frac{475781.25}{[13600-1000]\times 9.8} = 3.85 \text{ m}$$

(Ans)

Q: A mercury manometer is connected to a standard orifice meter with a 30mm diameter hole that has been placed in an 80 mm diameter pipe. For a manometer reading of 300 mm of mercury, determine the flow rate in the pipe if fluid is crude oil at 20°C

$\rho_m = 13600 kg/m^3$ $\rho_{crude\ oil} = 870 kg/m^3$ at 20°C

Soln:



ΔP = 300 mm of mercury

$$\Delta P = h_m [\rho_m - \rho_{crude oil}]\, g$$

$$= 300 \times 10^{-3} * [13600 - 870] * 9.8$$

$$= 37426.2\ N/m^2$$

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi}{4} * [80 \times 10^{-3}]^2 = 5.024 \times 10^{-3}\ m^2$$

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi}{4} * [30 \times 10^{-3}]^2 = 7.065 \times 10^{-4}\ m^2$$

$$Q = \frac{C_d A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

$$= \frac{1 * 7.065 \times 10^{-4}}{\sqrt{1-\left(\frac{30}{80}\right)^4}} \sqrt{\frac{2. \times 37426.2}{870}}$$

$$Q = 0.0066 \ m^3/s$$