2018 Syllabus

BASIC ELECTRICAL AND ELECTRONICS ENGINEERING (Theory)

Course Code	**18ELE13/23**	**Credits**	4
Course type	ES	**CIE Marks**	50 marks
Hours/week: L-T-P	4-0-0	**SEE Marks**	50 marks
Total Hours:	50	**SEE Duration**	3 Hours for 100 marks

Course learning objectives:
To impart an ability to the students to

Demonstrate an understanding of the fundamental concepts of electric system and AC single phase system

Demonstrate an understanding of the fundamental concepts of three phase generators, three phase circuits and applications
Demonstrate an understanding of transformers and induction motors and their applications
Demonstrate an understanding of the Electronic devices and circuits useful in multidisciplinary tasks Such as rectifiers and amplifiers

Demonstrate an understanding of the operational amplifiers and digital circuits and their applications

Unit - I

Typical Electrical System: A typical power system single line diagram, typical domestic wiring layout, protection of electrical system using fuse & MCB, necessity of earthing **2 hrs**

Single-phase A.C. Circuits: Sinusoidal voltage, instantaneous value, average value, root mean square value, form factor and peak factor of sinusoidal varying voltage and current, phasor representation of alternating quantities. analysis of RL, RC and RLC series circuit & RLC series resonant circuit and problems

8 hrs

Self learning topics: Calculation of energy bill for domestic applications

Unit - II

Three phase Synchronous Generators: Principle of operation, types and constructional features, synchronous speed and frequency, expression for emf generated, examples 5 hrs

Three Phase Circuits: Advantages of three phase system, definition of phase sequence, relationship between line and phase values of balanced star and delta connections, power in balanced three-phase circuits, measurements of active and reactive power and power factor by using two-wattmeter method, illustrative examples

5 hrs

Unit – III

Transformer: Principle of operation and construction of single-phase transformer (core and shell type), emf equation, transformation ratio, losses, efficiency, voltage regulation and its significance, illustrative problems on emf equation and efficiency only, applications of transformer (open circuit and short circuit tests, equivalent circuit and phasor diagrams are excluded) **5 hrs**

Three Phase Induction Motor: Concept of rotating magnetic field (no proof), principle of operation, types and constructional features, slip and its significance, applications of squirrel cage and slip ring motors, necessity of a starter, illustrative examples on slip calculations. **5 hrs**

Unit – IV

Semiconductor diode applications: Half wave and Full wave diode rectifiers with and without filter, ripple factor, efficiency and voltage regulation, regulators 7805 & 7905 **5 hrs**

Transistor applications: (Only concepts through circuit diagrams without analysis) Transistor as a switch, RC coupled CE amplifier, power amplifiers class A, class B and class C type, RC phase shift oscillator **5 hrs**

Self learning topics: Nil

Unit – V

Operational Amplifiers: Concept of operational amplifier and integrated circuits, ideal OPAMP Characteristics, inverting, non inverting OPAMP and voltage follower, zero crossing detector (ZCD), addition, subtraction using OPAMP **5 hrs**

Digital Electronic circuits: Boolean algebra and binary number system, logic gates, truth table, half adder and full adder, applications of digital electronics. **5 hrs**

Self learning topics: Nil

Text Books

1. "Basic Electrical Engineering", D. C. Kulshreshtha, TMH publications
2. "Electronic devices and circuit Theory", Robert L. Boylestad, Pearson Education, 9th edition, 2005.

Reference Books

1. "Electrical Technology", E. Hughes International Students 9th Edition, Pearson, 2005.
2. "Basic Electrical Engineering", V. K. Mehta and Rohit Mehta, S. Chand Publications.
3. "Basic Electrical Technology" 3rd Edition, TMH, D.P. Kothari.
4. Electrical Technology B.L. Theraja Latest edition.
5. "Basic Electrical Engineering", Dr. Rajashekharaiah, Prof. B.N.Yoganarasimhan, ,Shiva Book Center Bangalore 2014.
6. "Electronic Devices & Circuits", Dacid A. bell, Oxford niversity press, 5th edition, 2008.

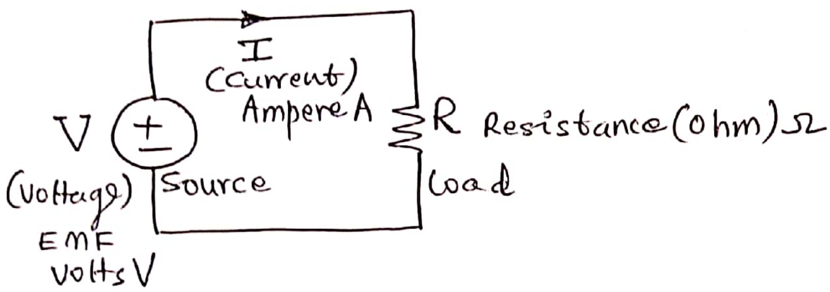
Introduction Topic

Electrical Circuit: It is a closed conducting path consisting of electric source and electric load. Electric Energy sources provide EMF (Electromotive force) to constitute electric current in the circuit.

Types of Electrical Energy Sources | Symbol

1. AC Sources Ex: AC generators, Invertors ⦾~
2. DC Sources Ex: Cell, Battery, DC generator ⊕

A typical Electric Circuit



Ohm's law

$$V = RI$$

OR $I = \frac{V}{R}$

Short circuit:- If resistance (or Impedence Z) is zero then it is called as short circuit. $I = \frac{V}{R} = \frac{V}{0} = \infty$

Hence current is very high ideally infinity. This result in severe fault leading to overheating & fire accident.

Open circuit:- If resistance (or Impedence) is Infinity then it is called as open circuit. $I = \frac{V}{R} = \frac{V}{\infty} = 0$.

Hence current is zero.

Electrical power

In DC circuit Electrical power is $P = VI = I^2R = \frac{V^2}{R}$ Watts

In AC circuit Electrical power $P = VI\cos\phi$ where $\cos\phi$ is called power factor

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Eletrical Energy E = Power × Time = Pt

ie $E = VIt$ Joules (Watt-sec) or Watthour (Wh) or (Killowatthours kWh)

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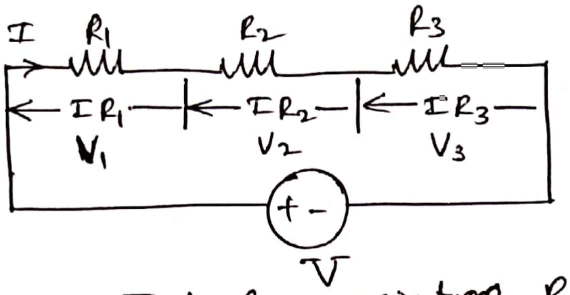
DC Circuits Unit 1

Resistances in series



By KVL, $V - V_1 - V_2 - V_3 = 0$

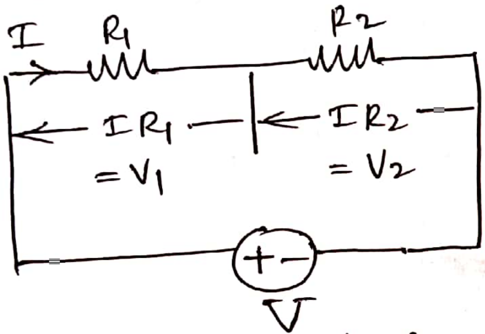
ie $V = V_1 + V_2 + V_3$

ie $V = IR_1 + IR_2 + IR_3$

ie $IR_{eq} = I(R_1 + R_2 + R_3)$

Total resistance $R_{eq} = R_1 + R_2 + R_3$

Voltage Division Rule



$V = V_1 + V_2$

ie $V = IR_1 + IR_2$

ie $IR_{eq} = I(R_1 + R_2)$

i.e $R_{eq} = R_1 + R_2$

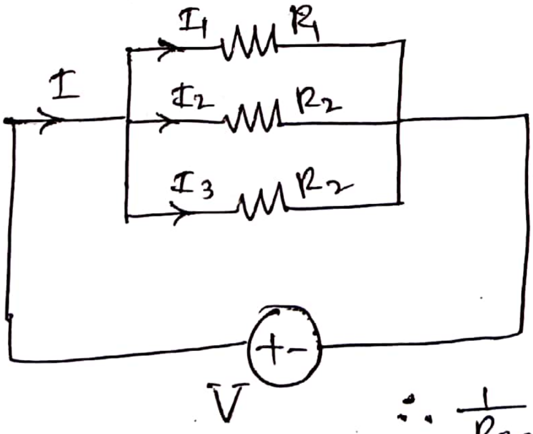
current $I = \frac{V}{R_{eq}} = \frac{V}{R_1 + R_2}$

$\therefore$ $V_1 = IR_1 = \frac{V}{R_1+R_2} R_1$ & $V_2 = I R_2 = \frac{V}{R_1+R_2} R_2$

Knowing total voltage V, R_1, R_2 the voltages across R_1 & R_2 can be determined using voltage division Rule. This is used in many circuit analysis problems.

—x—x—x—x—x

Resistances in Parallel



Voltage across the branches in parallel are equal.

$V = I_1 R_1 = I_2 R_2 = I_3 R_3$

Also, $I = I_1 + I_2 + I_3$

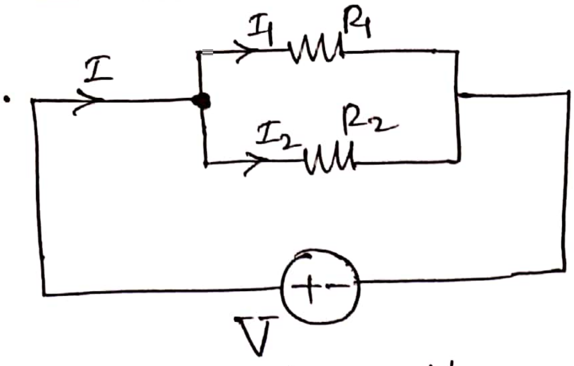
ie $\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$\therefore$ Total equivalent resistance $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

R_1, R_2, R_3 connected in parallel are symbolically written as $R_1 \| R_2 \| R_3$

Current Division Rule



$$I = I_1 + I_2$$

ie $\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$

$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

or $R_{eq} = \frac{R_1 R_2}{(R_1+R_2)}$ ie $R_1 \| R_2$

where $I = V/R_{eq} = \frac{V}{\left(\frac{R_1R_2}{R_1+R_2}\right)}$ and hence $V = I R_{eq} = \cancel{I\left(\frac{R_1R_2}{R_1+R_2}\right)}$

Currents in the branches indivisually are given by

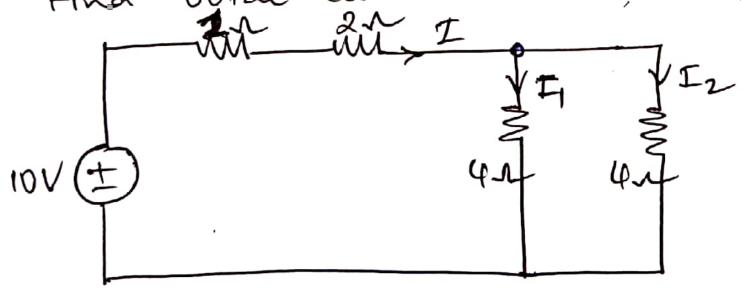
$$\boxed{I_1 = \frac{V}{R_1} = \frac{I}{R_1}\frac{R_1R_2}{(R_1+R_2)} = I\frac{R_2}{R_1+R_2}}$$

$$\boxed{I_2 = \frac{V}{R_2} = \frac{I}{R_2}\frac{R_1R_2}{(R_1+R_2)} = I\frac{R_1}{(R_1+R_2)}}$$

Knowing the total current I and branch resistances R_1 & R_2 currents in the individual branches can be determined. Current Division Rule is useful in many circuit analysis cases.

Problems

Ex1. Find total current I and I_1 and I_2 & power in 4Ω.



Sol^n equivalent resistance $R_{eq} = (4\|4) + 2 + 1$

ie $R_{eq} = \frac{4\times4}{4+4} + 2 + 1 = 2 + 2 + 1 = 5\Omega$

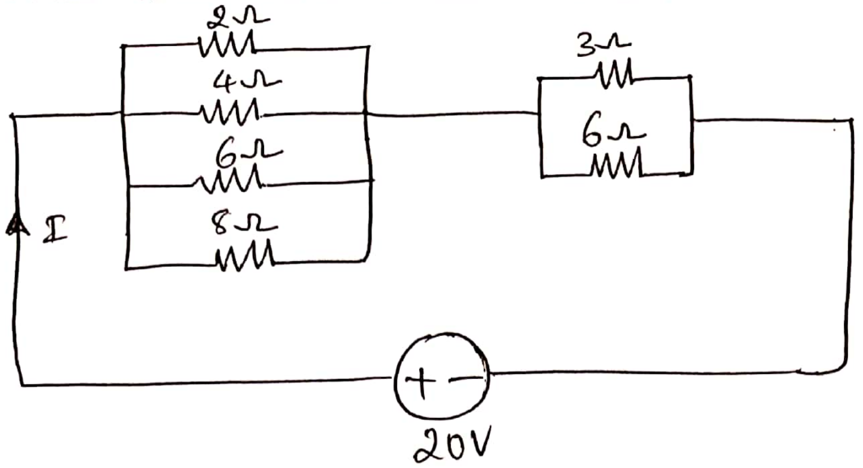
Total current $I = \frac{V}{R_{eq}} = \frac{10}{5} = 2\,A$

Using current division rule

$I_1 = \frac{I R_2}{(R_1+R_2)} = \frac{4\times2}{(4+4)} = 1A$ & $I_2 = \frac{I R_1}{(R_1+R_2)} = \frac{4\times2}{8} = 1A$

Power dissipated in 4Ω $= (1)^2 4 = 4$ Watts.

Ex2 Find the source current I.



Sol^n [The series parallel combination of resistances can be written in compact form symbolically as follows]

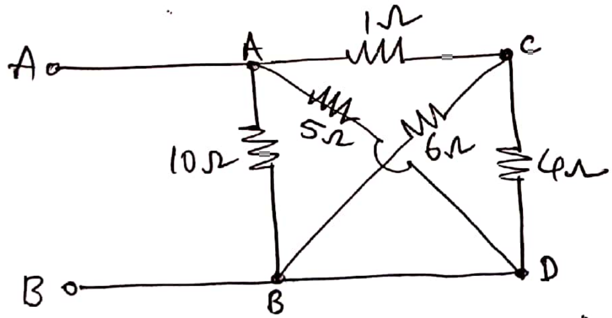
$$R_{eq} = \left[\left(\left(2\|4\right)\|6\right)\|8\right] + \left[3\|6\right] = \left[\left(\left(\frac{2\times4}{2+4}\right)\|6\right)\|8\right] + \left[3\|6\right]$$

$$= \left[\left(\frac{4/3\times6}{4/3+6}\right)\|8\right] + 2 = 2.92\,\Omega$$

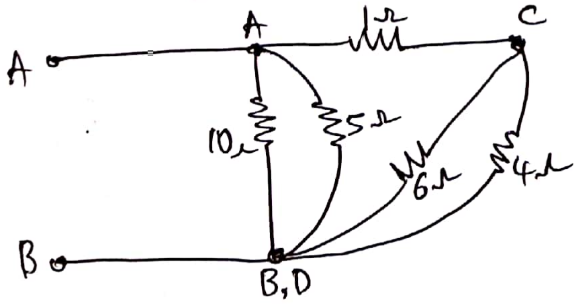
$\therefore\ R_{eq} = 2.92\,\Omega$

$\therefore$ source current $I = \frac{20}{2.92} = 6.75\,A$

Ex3 Find RAB ie equivalent resistance between A & B.



Sol^n Rewriting the circuit (merging B & D)

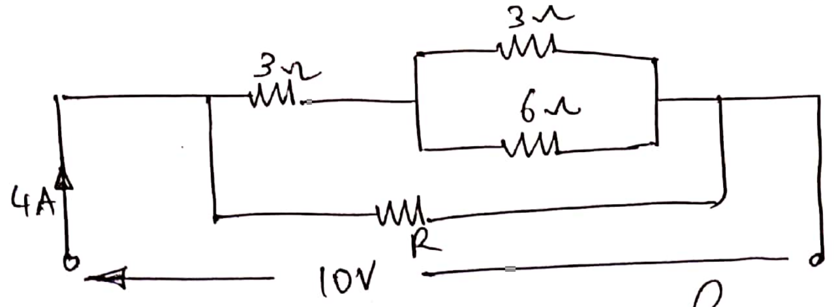


$$R_{eq} = \left(\left(\left(6\|4\right)+1\right)\|5\right)\|10$$

$$= \left(3.4\|5\right)\|10$$

on simplification $R_{eq} = 1.54\,\Omega$.

Ex 4 Determine R in the circuit shown below



Soln First determine expression for total resistance R_{eq} as

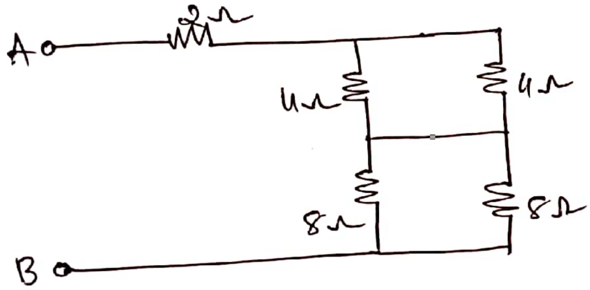
$$R_{eq} = ((3\|6)+3)\|R = (2+3)\|R = \frac{5R}{(5+R)}\ \Omega$$

Also $R_{eq} = \frac{10}{4} = 2.5 = \frac{5R}{(5+R)}$

$\therefore\ 2.5\times5 + 2.5R = 5R$ ie $2.5R = 12.5$

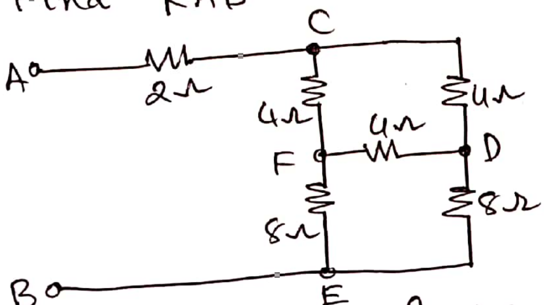
$\therefore\ \underline{R = 5\Omega}$

Ex 5 Find R_{AB}



Soln :- $R_{AB} = [(8\|8)+(4\|4)]+2$
$= 4+2+2 = 8\Omega$

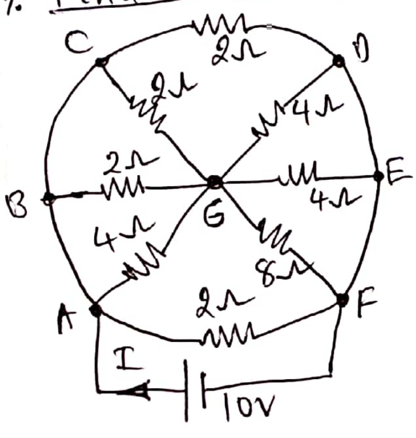
Ex 6 Find RAB



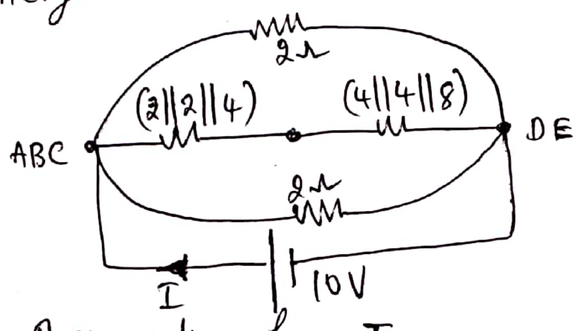
Soln :- In this problem 8Ω & 8Ω resistances can not be ~~comb~~ combined in parallel because of presence of 4Ω in FD branch

Hence to simplify this circuit star to delta OR delta to star conversion should be used without any other alternative.

Imp Ex 7. Find current delivered by the source



Soln :- Name the nodes as A, B, C, D, E, F.
Merge ABC & DEF.



Then solve for I.

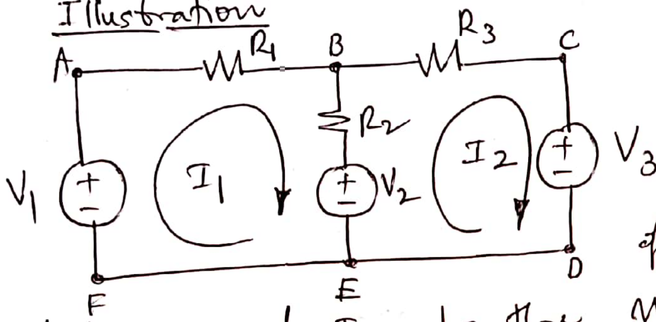
Kirchoff's laws

Kirchoff's laws are used for analysing electric networks to find currents in all the branches. Kirchoff's laws are used particularly in the networks having more than one electric source.

Kirchoff's voltage law (KVL)

In any electric network, the algebraic sum of voltages (Emfs and voltage drops) around a closed loop is zero.

Illustration



The closed paths ABEFA & BCDEB are called loops or Meshes.

Trace the mesh in the direction of mesh currents.

Let I_1 and I_2 be the mesh currents

By KVL for the Mesh ABEFA or Mesh 1

$$V_1 - I_1R_1 - (I_1 - I_2)R_2 - V_2 = 0 \quad \text{—— (1)}$$

By KVL for the Mesh BCDEB or Mesh 2

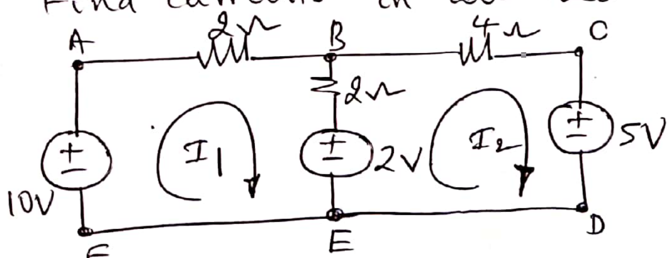
$$V_2 - (I_2 - I_1)R_2 - I_2R_2 - V_3 = 0 \quad \text{—— (2)}$$

Solving the equations (1) & (2) for I_1 & I_2, the currents in all the branches are determined.

This method is called Mesh analysis using KVL.

Example

Find currents in all the branches using KVL.



Soln Let I_1 and I_2 be the mesh currents.

By KVL for mesh 1

$$10 - 2I_1 - 2(I_1 - I_2) - 2 = 0.$$

On rearrangement

$$4I_1 - 2I_2 = 8 \quad \text{—— (1)}$$

By KVL for Mesh 2

$2 - 2(I_2 - I_1) - 4I_2 - 5 = 0$

on rearrangement

$2I_1 - 6I_2 = 3$ — (2)

From the Eqns 1 & 2 in matrix form

$$\begin{bmatrix} 4 & -2 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

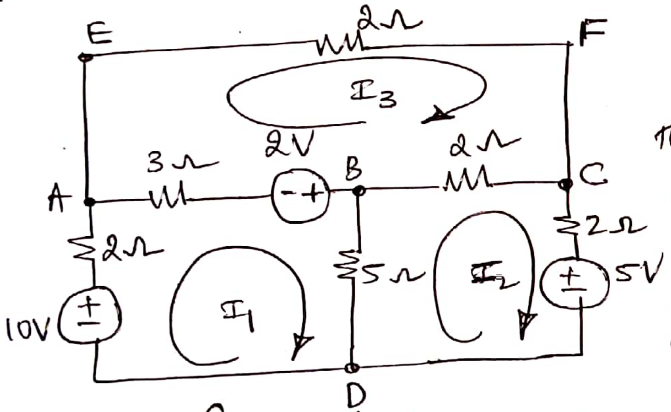
Solving for I_1 and I_2, $I_1 = 2.1A$ and $I_2 = 0.2A$

Hence the branch currents are (with direction)

$I_{AB} = I_1 = 2.1A$, (A to B) $\quad I_{BE} = I_1 - I_2 = 2.1 - 0.2 = 1.9A$ (B to E)

$I_{BC} = I_2 = 0.2A$ (B to C).

Ex 2 Find all the branch currents by using KVL



Let I_1, I_2 and I_3 be the Mesh currents, as shown.

(Applying KVL for each mesh in the direction of Mesh currents)

By KVL for Mesh 1 ie ABDA

$10 - 2I_1 - 3(I_1 - I_3) + 2 - 5(I_1 - I_2) = 0$

ie $\quad 10I_1 - 5I_2 - 3I_3 = 12$ — (1)

By KVL for Mesh 2 ie BCDB

$-5(I_2 - I_1) - 2(I_2 - I_3) - 2I_2 - 5 = 0$

ie $5I_1 - 9I_2 + 2I_3 = 5$ — (2)

By KVL for Mesh 3 ie AEFCBA

$-2I_3 - 2(I_3 - I_2) - 2 - 3(I_3 - I_1) = 0$

ie $3I_1 + 2I_2 - 7I_3 = 2$ — (3)

From Equations 1, 2 & 3 in matrix form

$$\begin{bmatrix} 10 & -5 & 3 \\ 5 & -9 & 2 \\ 3 & 2 & -7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 5 \\ 2 \end{bmatrix}$$

Solving for I_1, I_2 & I_3

$I_1 = 1.565A$ $\quad I_2 = 0.427A$, $\quad I_3 = 0.507A$

(PTO)

The Branch currents are

$I_{AD} = I_1 = 1.565$ A (A to D)

$I_{AB} = I_1 - I_3 = 1.565 - 0.507 = 1.058$ A (A to B)

$I_{BC} = I_3 - I_2 = 0.507 - 0.427 = 0.08$ A (C to B)

or $I_{BC} = I_2 - I_3 = 0.427 - 0.507 = -0.08$ A (B to C) or 0.08 A (C to B)

$I_{EF} = I_3 = 0.507$ A (E to F),

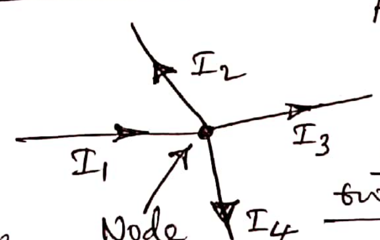
$I_{CD} = I_2 = 0.427$ A (C to D)

Kirchoff's current law (KCL)

In a electric network, the algebraic sum of currents at a node is zero.

consider the currents outgoing wrt a node as +ve and the currents incoming as −ve.

Illustration



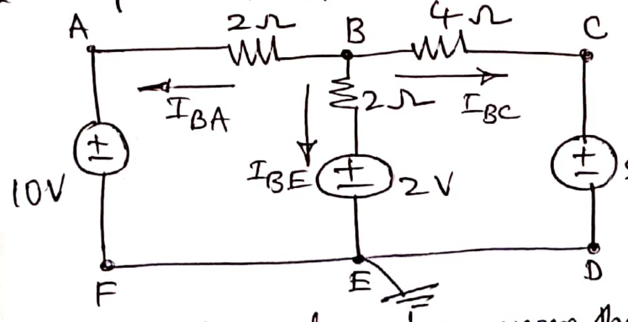
As per KCL

$$-I_1 + I_2 + I_3 + I_4 = 0.$$

Consider the nodes where more than two branches meet while using KCL.

One of the node is treated as GND with zero voltage.

Example:- Find currents in all the branches by KCL.



E is taken as ground node.
GND node voltage is zero.

Note
* Consider the nodes where more than two branches meet while using KCL for writing the current equations.

Soln By KCL at node B. E is the ground node (GND)

Let V_B be the Node voltage.

By applying KCL at node B, $I_{BA} + I_{BE} + I_{BC} = 0$

ie $$\frac{(V_B - 10 - 0)}{2} + \frac{(V_B - 2 - 0)}{2} + \frac{(V_B - 5 - 0)}{4} = 0$$

ie $$\frac{V_B}{2} - 5 + \frac{V_B}{2} - 1 + V_B/4 - 1.25 = 0$$

PTO

$\therefore V_B = \frac{7.25}{1.25} = 5.8$ Volt

$\therefore$ Branch currents are

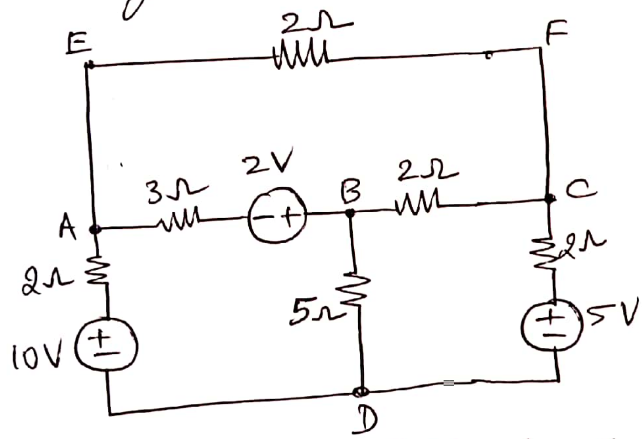
$I_{AB} = I_{AF} = \frac{(V_B - 10)}{2} = \frac{5.8 - 10}{2} = -2.1A$ (B to A) or 2.1 A (A to B)

$I_{BE} = \frac{(V_B - 2)}{2} = \frac{(5.8 - 2)}{2} = 1.9A$ (B to E)

$I_{BC} = I_{CD} = \frac{(V_B - 5)}{4} = \frac{(5.8 - 5)}{4} = 0.2A$ (B to C)

Note:- The node voltage V_B is assumed to be the highest & hence all the branch currents I_{BA}, I_{BE} & I_{BC} are outgoing. The same concept is applied all the other nodes while applying KCL at the nodes.

Ex Find currents in all the branches by Nodal analysis ie using KCL.



Sol^n Let V_A, V_B and V_C be the node voltages.

By KCL at node A,

$$\frac{(V_A - 10)}{2} + \frac{(V_A + 2 - V_B)}{3} + \frac{(V_A - V_C)}{2} = 0$$

ie $1.33V_A - 0.33V_B - 0.5V_C = 4.34$ —— (1)

By KCL at node B,

$$\frac{(V_B - 2 - V_A)}{3} + \frac{V_B}{5} + \frac{(V_B - V_C)}{2} = 0$$

ie $-0.33V_A + 1.03V_B - 0.5V_C = 0.66$ —— (2)

By KCL at node C

$$\frac{(V_C - V_A)}{2} + \frac{(V_C - V_B)}{2} + \frac{(V_C - 5)}{2} = 0$$

ie $-0.5V_A - 0.5V_B + 1.5V_C = 2.5$ —— (3)

From Equations 1, 2 & 3 in matrix form

PTO

$$\begin{bmatrix} 1.33 & -0.33 & -0.5 \\ -0.33 & 1.03 & -0.5 \\ -0.5 & -0.5 & 1.5 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_c \end{bmatrix} = \begin{bmatrix} 4.34 \\ 0.66 \\ 2.5 \end{bmatrix}$$

$V_A = 6.87,\ V_B = 5.68,\ V_c = 5.85$

solving for V_A, V_B and V_c find the branch currents as

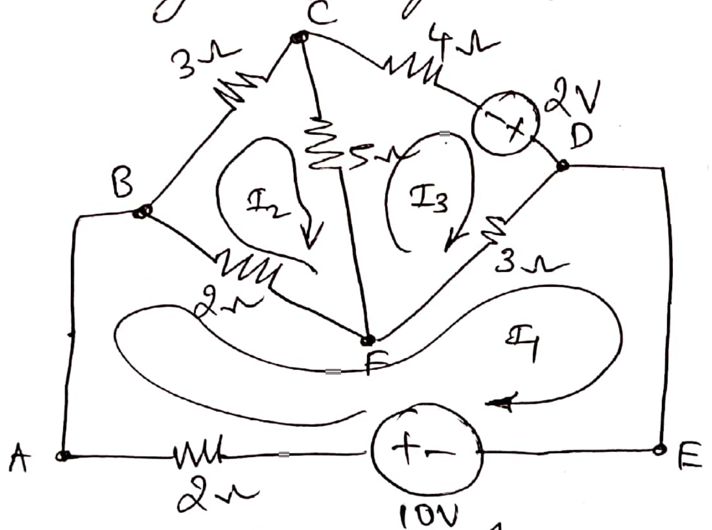
$I_{AB} = \frac{(V_A + 2 - V_B)}{3} = \frac{1.063A}{(A\,to\,B)}$ $\quad I_{BC} = \frac{(V_B - V_c)}{2} =$ ____

$I_{AD} = \frac{(V_A - 10)}{2} = \frac{-1.565A}{(A\,to\,D)}$ $\quad I_{CD} = \frac{(V_c - 5)}{2} =$ ____

$I_{BD} = V_B/5 =$ ____ $\quad I_{EF} = \frac{V_A - V_c}{2} =$ ____

complete the remaining substitutions.

Exe Find the current in all the branches by Mesh analysis (ie by KVL).



Sol^n Name the nodes as shown
Let I_1, I_2 & I_3 be the mesh currents.

By KVL for mesh 1

$$10 - 2I_1 - 2(I_1 - I_2) - 3(I_1 - I_3) = 0$$

By KVL for mesh 2

$$-2(I_2 - I_1) - 3I_2 - 5(I_2 - I_3) = 0$$

By KVL for mesh 3

$$-5(I_3 - I_2) - 4I_3 + 2 - 3(I_3 - I_1) = 0$$

simplify the eqns & Solve for I_1, I_2 & I_3 & then find the branch currents.

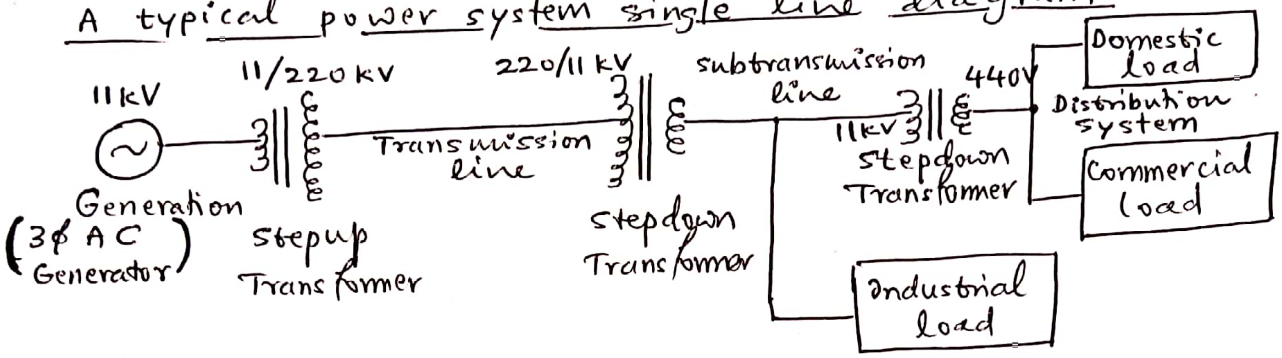
Basic Electrical & Electronics Engineering

Unit I

Typical Electrical System

An electric power system is a network of electrical components deployed to supply, transfer and use electric power.

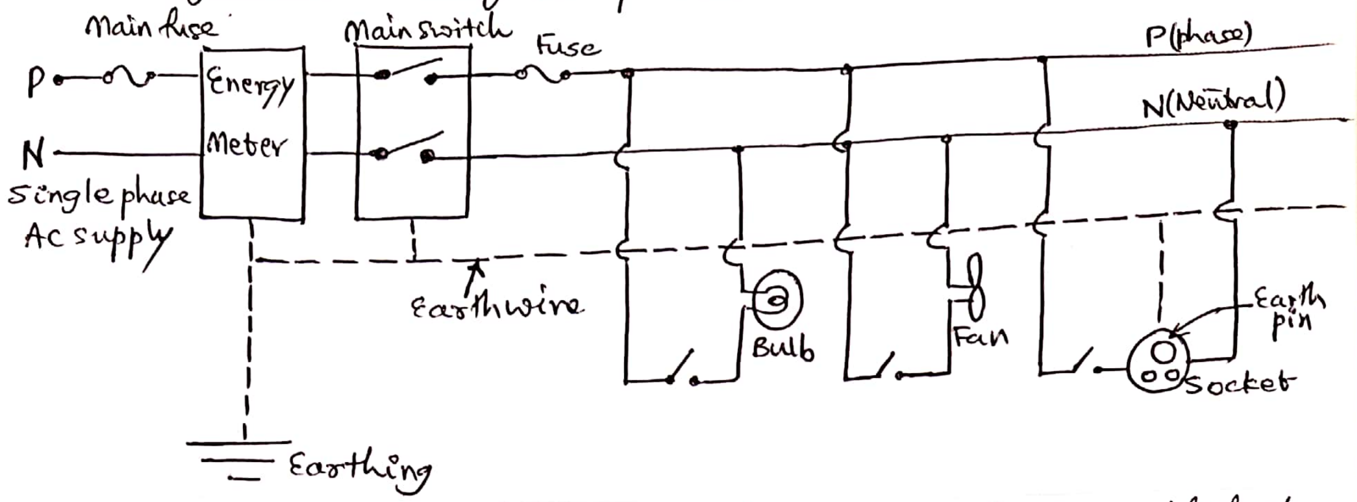
A typical power system single line diagram



In a typical power system 3 phase electric power is generated using 3ϕ AC generator (3ϕ Synchronous generator) at 11 kV voltage, in Generating stations of different types namely 1) Hydro electric power station 2) Thermal power station 3) Nuclear power station 4) Diesel power station. In addition electric power is generated using solar energy & wind energy resources called as Non conventional (or Renewable) energy resources.

Bulk amount of AC power generated at 11 kV is transmitted through transmission lines towards the load centres (consumers) located at long distances from the power stations. This power transmission is done at higher voltages of 220 kV or 400 kV. Hence step up transformers are used as shown. At the receiving end of the transmission line the voltage is reduced to 11 kV using step down transformers. Electric power is supplied to Industrial load at 11 kV. This part of the power system is called distribution system. The system voltage is further reduced to 440 V (3 phase) using step down transformers & then power is supplied to commercial & domestic loads.

Typical Domestic Wiring lay out



For domestic customers single phase AC supply is provided at 230V, 50 Hz. For commercial customers 3 phase AC supply is provided at 440V, 50Hz (line voltage). A typical domestic wiring layout consists of single phase AC supply, main fuse and MCB for protection, Energymeter, Main switch, sub fuses & MCBs, lamp load & fan load appliances with switches, extension load sockets. Single phase AC system has 2 wires namely phase and neutral. 3 phase AC system has 4 wires namely three phases ie R, Y and B and neutral.

* **Fuse :-** Fuse is an electrical safety device which is made up of lead-tin alloy having low melting point. It melts and interrupts the electric circuit when the current exceeds the normal value and thus protects the system from the overcurrent. If overcurrent is allowed to flow for along time then overheating occurs & the electrical system is damaged. The molten fuse element should be replaced by new element.
* **MCB (Miniature Circuit Breaker) :-** MCB is an electric protection device which is a switch which trips (opens) when the current exceeds the specified normal value. It is used as an alternative to the fuse. To restore the system supply only close the MCB switch & there is necessity to replace the link as in case of fuse. Hence MCB is more advantageous as compared to fuse.
* **Energy Meter :-** It is a measuring instrument used for measuring energy consumed by an electric load. Analog type (Induction type), Digital type and Electronic Energy meters are being used. Energy meters are integrating type meters which give cumulative reading. It indicates energy consumed in terms of kWh (Killowatt hour). 1 kwh is called one unit.
* **Main switch :-** This is a Double Pole Single Throw (DPST) switch with Iron clad or Plastic Clad with the highest current rating in the system. Used for isolating the domestic circuit from the main power supply during maintenance

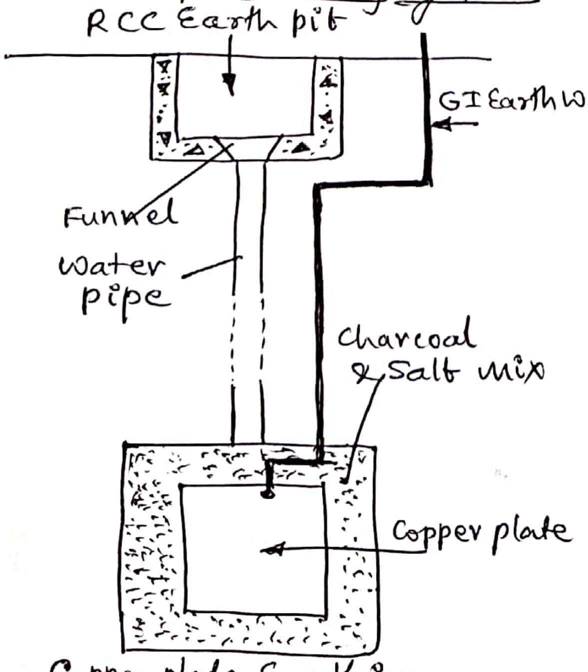
* **Switches :-** switches are used to connect or disconnect individual loads (appliances such as lamp, fan, motor etc). A switch is connected in series with each load appliance. All the load appliances are connected in parallel with the supply. (as shown in domestic wiring diagram).

* **Earthing in Electric power systems & Necessity of Earthing**

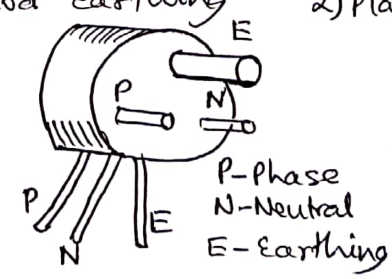
All the electrical appliances such as Electrical machines, Electric Iron, Refrigerators, washing Machine etc consist of Electric circuit assembly covered by Metallic or Non Metallic outer cover. Due to the failure of insulation the live electric conductor might have come in contact with outer metal cover surface of the appliance and the outer surface might be electrically charged. Hence the person touching such surfaces receives electric shock. Hence to avoid such incidences and protect the person the outer cover body of all the electric appliances are connected to earth through thick GI wire and earthing plate or Earthing rod. This earthing system has significantly less resistance than the human body. Hence the electric charge on the appliance surface tends to flow through earthing only bypassing the human body path. Thus the person is protected from the electric shock.

Also to protect Buildings from lightning, lightning arre--stors are used. Lightning arrestors are connected to earthing system where the electric charge readily flows to earth. Earth wire is run in all parts of the electrical systems as shown in domestic wiring diagram. Also to ensure proper earthing all the appliances are provided with 3 pin plug as shown. Earth pin is connected to earthing.

* **Types of Earthing systems** 1) Rod Earthing 2) Plate Earthing.



Copper plate Earthing




Symbol for Earthing

As per IEEE standards Earth resistance value should be 5 Ω or less. To maintain earth resistance at the specified value water is put in the Earth pit filled with salt and char--coal mixture, periodically.

* A problem on Energy Calculation

Ex 1 Calculate the energy consumption for a month of 30 days in a domestic setup with the following details.

Appliances	No.s	Rating	Number of hours of usage per day
Lamps	04	40W (each)	05
Fans	04	75W (each)	06
Fridge	01	300W	04
Water Heater	01	2kW	02
1 HP Motor Pump	01	1 HP	01

Calculate the energy bill if the tariff per unit energy is Rs.10/-

Soln Energy consumed = Power Rating of each appliance × No of appliances × Duration of Usage

1 HP = 746W

Total Energy consumed per day

$= 40 \times 4 \times 05 + 75 \times 04 \times 06 + 300 \times 1 \times 04 + (2 \times 10^3) \times 1 \times 02 + 746 \times 1 \times 1$

$= 8546$ Wh or 8.546 kWh (per day)

Energy consumed for one month of 30 days

$= 8.546 \times 30 = 256.38$ kWh or units.

Energy Bill per month = Energy Consumed in Units × Tariff Rs/Unit

$= 256.38 \times 10 = 2563.80$ Rs

Ex 2. Calculate the energy consumed during a month of August 2018 in a domestic set up consisting of 100 no.s of lamps of 40W, 40 fans of 75W, 5 no.s of A/c systems each of 2 kW and 2 electric heaters each of 2kW. Every day lamps for 6 hrs, fans for 6 hrs, A/c systems for 2 hrs and electric heaters for 2 hrs are used. If the energy tariff is Rs 5 per unit calculate the energy bill for the month.

Single phase AC circuits

Sinusoidal Voltage : Generation of sinusoidal Voltage

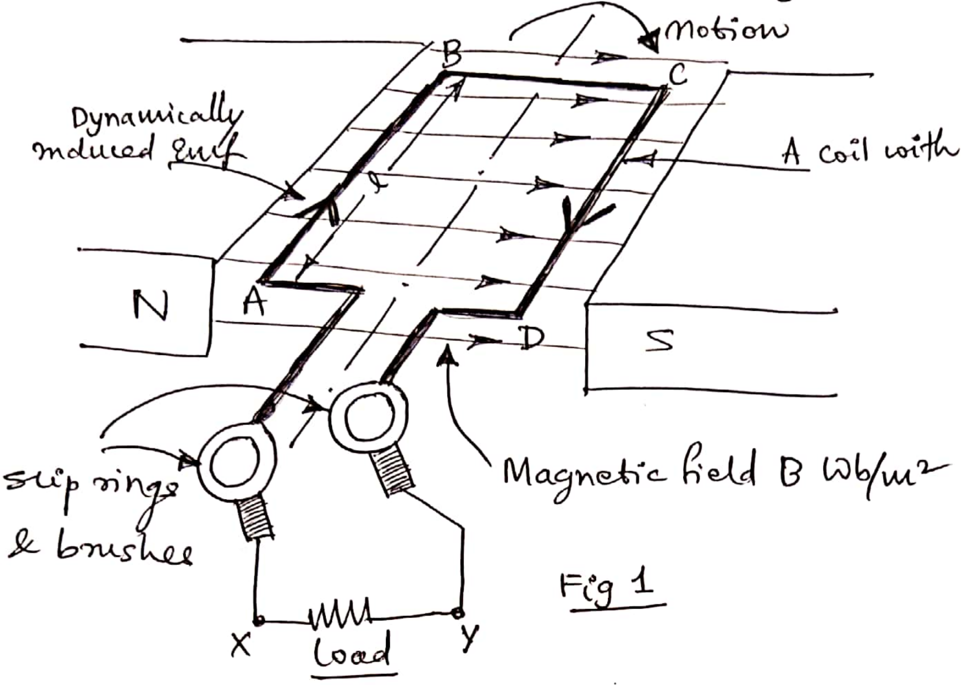


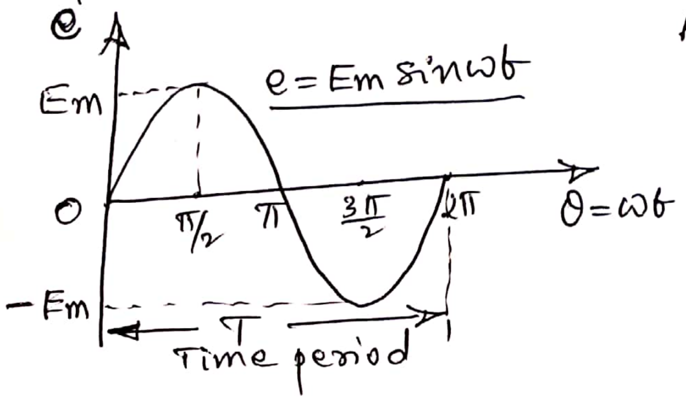
Fig 1

Consider a coil having N turns rotating in a magnetic field of flux density B Wb/m² at a velocity V m/sec. The coilsides are connected to two slip rings with carbon brushes mounted on the axis of the coil. Emf is generated in the active coilsides AB & CD.

The generation emf is due to dynamically induced emf and the emf generated is alternating in nature given by $e = 2NBlv\sin\theta$

where θ = Angle between plane of magnetic field & the plane perpendicular to the plane of the coil.

In fig 1 the induced current flows in load from Y to X. After 180° rotation the load current flows from X to Y. Thus the load current alternates as per the sinusoidal pattern. Thus sinusoid alternating emf is generated.



At $\theta = 0°$, $e = 0$ volt
$\theta = 90°$, $e = E_m$
$\theta = 180°$, $e = 0$.
$\theta = 270°$, $e = -E_m$
$\theta = 360°$, $e = 0$

[In a 2 pole system shown one cycle of emf is generated in one revolution.
In a 4 pole system 2 cycles of emf are generated in one revolution. For 6 pole → 3 cycles & so on.]

f **Frequency** :– The number of cycles generated in one second is called Frequency of alternating voltage. Expressed in Hertz (Hz) or cycles/second.

T **Time period** :– Time required for one cycle in seconds is called Time period.

[Since f cycles occur in one second, one cycle occurs in $\frac{1}{f} = T$ seconds.]

Instantaneous Value :– It is the magnitude of an alternating quantity (voltage or current) at any time instant t secs. Denoted by small letters v, i etc. (lower case)

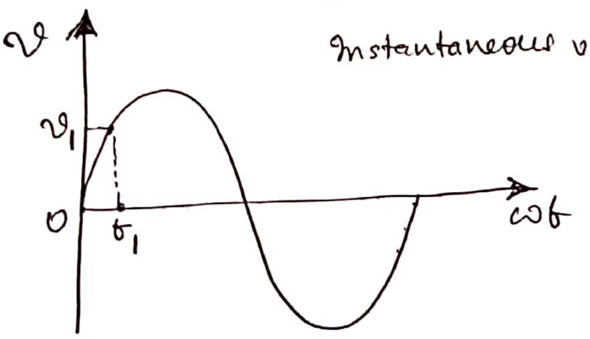
Represented by $v = V_m \sin \omega t = V_m \sin \theta$

where θ Angle traversed by the coil during its rotation at Angular velocity $\omega = \theta/t$ rad/sec. $\theta = \omega t$.

For one cycle $\theta = 2\pi$ rad & $t = T$ sec $\therefore \omega = \frac{2\pi}{T} = 2\pi f$

$v = V_m \sin \omega t = V_m \sin (2\pi f t)$.

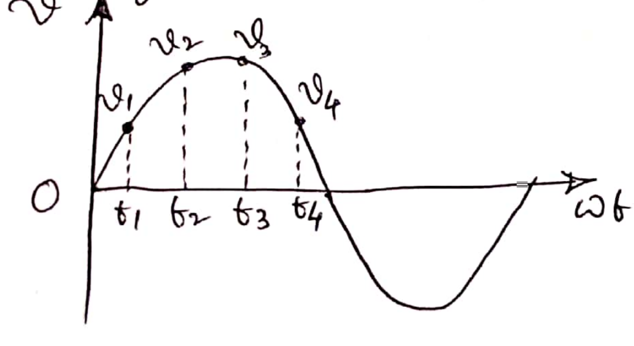
Instantaneous value at $t = t_1$ is $v_1 = V_m \sin(\omega t_1)$



Average Value of Alternating quantity

The average value of alternating quantity is given by the average of all the instantaneous values during one half cycle, for n instants of time.

$$V_{av} = \frac{v_1 + v_2 + v_3 + \cdots + v_n}{n}$$



Average value Mathematically given by

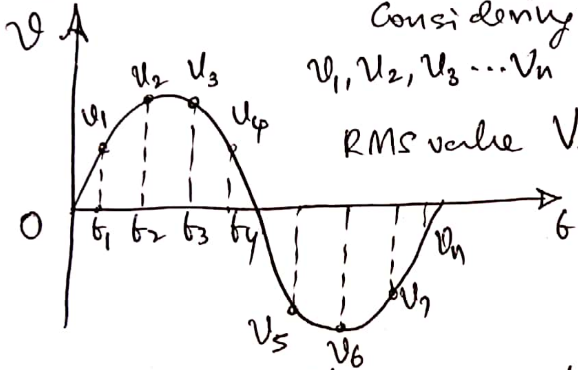
$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v \cdot d\omega t = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \cdot d\omega t$$

$$= \frac{1}{\pi} V_m \left[-\cos\omega t\right]_0^{\pi} = \frac{2 V_m}{\pi}$$

$$\therefore \boxed{V_{av} = \frac{2}{\pi} V_m} \qquad \boxed{I_{av} = \frac{2}{\pi} I_m}$$

RMS value of Alternating quantity

(Root Mean squared value) For $v = V_m \sin\omega t$



Considering the instantaneous values $v_1, v_2, v_3 \cdots v_n$ at n instants of time

$$\text{RMS value } V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \cdots + v_n^2}{n}}$$

Mathematically given by

For $v = V_m \sin\omega t$

$$\text{RMS value } V = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2 \cdot d\omega t} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\omega t \cdot d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\frac{(1-\cos 2\omega t)}{2} d\omega t} = \sqrt{\frac{V_m^2}{2\pi \times 2}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[2\pi\right]} = \frac{V_m}{\sqrt{2}}$$

$$\therefore \text{RMS voltage } V = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

$$\text{RMS current } I = \frac{I_m}{\sqrt{2}} = 0.707 V_m$$

In practice in AC circuits & systems the voltages and currents specified are RMS values.

Instantaneous voltage & currents are denoted by lower case letter v & i.

RMS voltage & current are denoted by upper case letters V and I.

Form factor: Ratio of RMS value & Average value of an alternating quantity. (For sinusoidal wave)

$$\text{Form factor } K_f = \frac{\text{RMS value}}{\text{Average value}} = \frac{V_m/\sqrt{2}}{2/\pi \, V_m} = 1.11$$

Peak factor (Amplitude factor):- Ratio of maximum value & RMS value of an alternating quantity. (For sinusoidal wave)

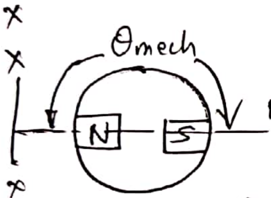
$$\text{Peak factor } K_p = \frac{\text{Maximum value}}{\text{RMS value}} = \frac{V_m}{V_m/\sqrt{2}} = 1.414$$

Phase Angle

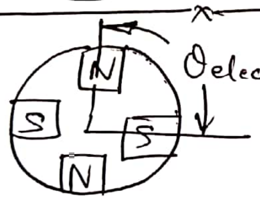
The state of an alternating quantity with reference phasor expressed in terms of electrical angle or time is called phase angle.

$$\text{Mechanical angle } \theta_{mech} = \frac{\text{Electrical Angle } \theta_{elect}}{\text{No of pair of poles}}$$

$$\text{ie } \boxed{\theta_{mech} = \frac{\theta_{elect}}{P/2}}$$



For $P=2$, $\theta_{mech} = \theta_{elect} = 180°$

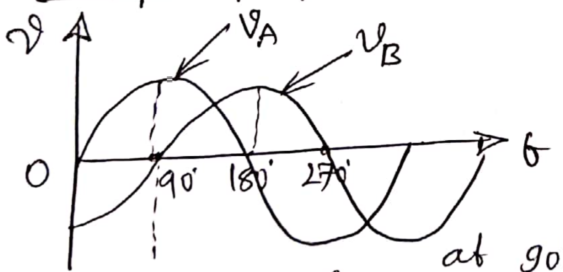


Angle betn 2 adjacent poles $\theta_{mech} = 90°$
$\theta_{elect} = 180°$

For $P = 4$

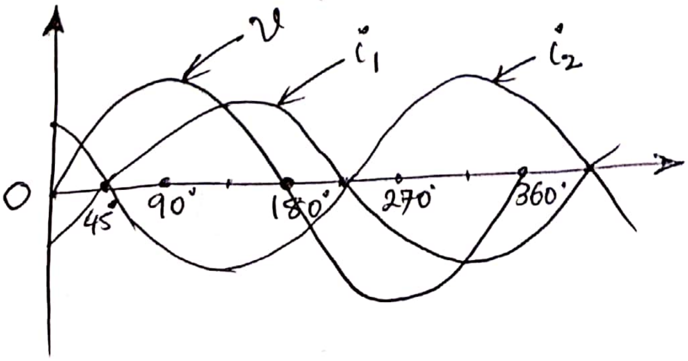
$$\therefore \theta_{mech} = \frac{\theta_{elect}}{P/2} = \frac{180°}{2} = 90°$$

Example of phase angle



Taking v_A as reference v_B has a phase difference of 90° elect. v_B lags v_A by 90°. First +ve peak of v_B occurs at 90° after the occurance of the first +ve peak of v_A.

Ex Draw $v = 10 \sin 2t$, $i_1 = 5 \sin(2t - 45°)$ & $i_2 = 5 \sin(2t + 45)$



Phasor representation

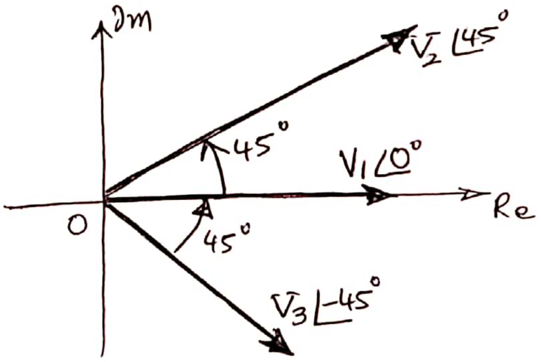
An alternating quantity voltage or current has magnitude and direction (phase angle) and hence they are represented by phasors (vectors). Each phasor is assumed to be rotating in anticlockwise direction at speed ω rad/sec.

Example

① $v_1 = V_{m_1} \sin \omega t$ is represented as $V_{m_1}\angle 0^\circ$ or $V_1\angle 0^\circ$
where magnitude may be Max. value or RMS value.
It is represented by a straight line of length proportional to the magnitude & its phase angle with respect to the +ve real axis (0° axis).

②
$v_2 = V_{m_2} \sin(\omega t + 45^\circ)$ is represented as $V_{m_2}\angle 45^\circ$ or $V_2\angle 45^\circ$

③ $v_3 = V_{m_3} \sin(\omega t - 45^\circ)$ is represented as $V_{m_3}\angle -45^\circ$ or $V_3\angle -45^\circ$



V_2 leads V_1 by 45°
V_3 lags V_1 by 45°

④ In general an alternating quantity $v = V_m \sin(\omega t \pm \phi)$ has phasor representation in 3 forms as

ⓐ $V\angle\phi$ Polar form.

ⓑ $V\cos\phi \pm jV\sin\phi$. Rectangular form. or Trigonometric form

ⓒ $Ve^{\pm j\phi}$. Exponential form.

Problem:- Represent $v = 10\sin(2t + 45^\circ)$ in i) Rectangular form ii) Polar form iii) Exponential form phasors.

(i) Rectangular form $10\cos(45^\circ) + j10\sin(45^\circ) \rightarrow 7.07 + j7.07$

(ii) Polar form $10\angle 45^\circ$

(iii) Exponential form $10e^{j45}$

Problem. Represent $i = 5\sin(3t - 60^\circ)$ in different phasor forms.

(i) $5\angle 60^\circ$ (ii) $0.25 - j0.43$ (iii) $5e^{-j60}$

(iv) Trigonometric form $5\cos 60^\circ - j5\sin 60^\circ$

Phasor Algebra

① **Phasor Addition:-** Phasors represented in Rectangular form are convinient for Addition. Hence it is necessary to convert phasors in to Rectangular form from Polar form.

Ex:- Add the phasors $10\angle 45^\circ$ and $1+j1$. First convert $10\angle 45^\circ$.

$10\angle 45^\circ + 1+j1 = 7.07+j7.07+1+j1 = \underline{8.07+j8.07}$

② **Phasor Multiplication:-** For phasor multiplication phasors in Polar form are convinient. Hence phasors should be converted from rectangular form to Polar form.

Ex:- Multiply $10\angle 45^\circ$ and $1+j1$. First convert $1+j1$ to Polar.

$10\angle 45^\circ \times 1+j1 \rightarrow 10\angle 45^\circ \times \sqrt{2}\angle 45^\circ \rightarrow 10\sqrt{2}\angle 45+45^\circ = 14.14\angle 90^\circ$

③ **Phasor division:-** For phasor division phasor in Polar form are convinient. Hence phasors should be converted from the other form to Polar form.

Ex:- Divide $10\angle 45^\circ$ by $1+j1$. First convert $1+j1$ to Polar.

$10\angle 45^\circ \div 1+j1 \rightarrow 10\angle 45^\circ \div \sqrt{2}\angle 45^\circ \rightarrow \frac{10}{\sqrt{2}}\angle 45-45 = \frac{10}{\sqrt{2}}\angle 0^\circ$

④ Perform the following calculations.

(i) $10\angle 45^\circ \times 5e^{j45} \div 5\angle 45^\circ \rightarrow \frac{10\angle 45 \times 5\angle 45^\circ}{5\angle 45^\circ} = 10\angle 45+45-45 = 10\angle 45^\circ$

(ii) $\frac{10e^{j10^\circ} + 5\angle 45^\circ}{3+j4} = \frac{10\angle 10^\circ + 5\angle 45^\circ}{(3+j4)} = \frac{9.84+j1.73+3.53+j3.53}{5\angle 53.13}$

(iii) Find i_1+i_2 if $i_1 = 10\sin(2t+30)$ & $i_2 = 5\sin(2t-30^\circ)$.

Soln $i_1 = 10\angle 30^\circ$ & $i_2 = 5\angle -30^\circ$ $\therefore i_1+i_2 = 10\angle 30^\circ + 5\angle -30^\circ = 13+j2.5$

$= 5\sqrt{7}\angle 10.89^\circ$

$= 13.22\angle 10.89^\circ$

(iv) Find (i_1-i_2). Soln: $10\angle 30 - 5\angle -30 = 4.33+j7.5 = 8.66\angle 60^\circ A$

Note:- Using the latest calculators the above calculation can be performed directly in complex mode. Refer Calculator Mannual. Verify the following using Calculator directly

(i) $\frac{(1+j1)(1-j1)}{(3+j4)} = 0.4\angle -53.13^\circ$ (ii) $\frac{(1+j1)+(1-j1)}{(1+j1)-(1-j1)} = 1\angle -90^\circ$

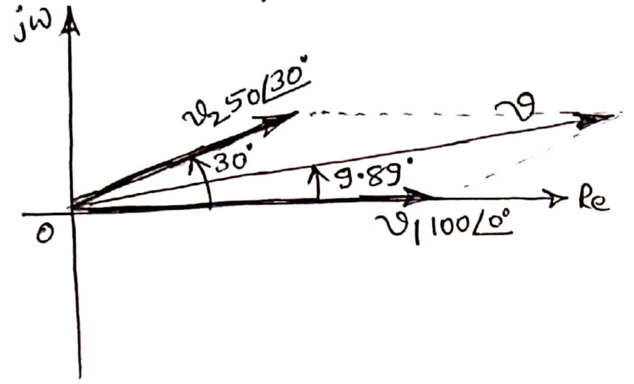
Ex: If $v_1 = 100 \sin 5t$ & $v_2 = 50 \sin(5t + 30°)$ find $v_1 + v_2$. Draw phasor diagram.

Solⁿ:- Writing in phasor form as $v_1 = 100\angle 0°$ & $v_2 = 50\angle 30°$

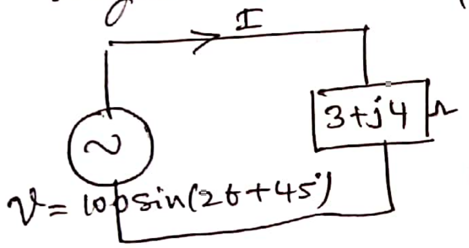
$v = v_1 + v_2 = 100\angle 0° + 50\angle 30° = 145.46\angle 9.89°$ V

The answer in instantaneous form $v = 145.46 \sin(5t + 9.89°)$ Volt

Phasor diagram   All phasors are referred wrt 0° axis.



Ex:- Find current I if the applied voltage is $v = 100 \sin(2t + 45°)$ Volts and impedence of the circuit is $3 + j4\ \Omega$. Draw phasor diagram. Find the power dissipated in the circuit

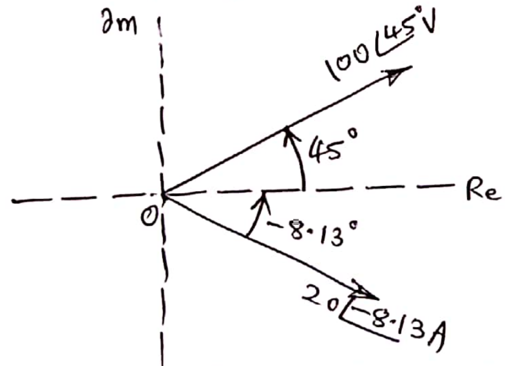


Solⁿ:- Applied voltage $V = 100\angle 45°$ V

Impedence $Z = 3 + j4\ \Omega$

current $I = \frac{V}{Z} = \frac{100\angle 45}{(3+j4)} = 20\angle -8.13$ A

Note:- All phasors have angles referred to reference axis ie 0° axis, Re axis.



Phasor diagram

This portion is not in the syllabus

To find power dissipated.

Complex power $= VI^*$ where I^* Complex conjugate

$= \frac{100}{\sqrt{2}}\angle 45 \times \left(\frac{20}{\sqrt{2}}\angle -8.13\right)^* = \frac{100}{\sqrt{2}}\angle 45 \times \frac{20}{\sqrt{2}}\angle 8.13$

$= \frac{2000}{2}\angle 45 + 8.13 = 1000\angle 53.13$ Volt-Amp

$= VI\angle\phi$

Power dissipated ie True power ie Active power $P = VI\cos\phi = 1000 \cos(53.13)$

$= 600$ W

Reactive power $Q = VI\sin\phi = 1000 \sin(53.13)$

$= 800$ W

Ex: Find instantaneous current at $t = 2$ sec if $i = 20\sin 5t$ & time period and frequency. Find RMS & Average value

Soln At $t=2$, $i = 20\sin 5t = 20\sin(5\times2) = 3.47$ A.

Comparing $i = 20\sin 5t$ with $i = I_m \sin\omega t$.

Amplitude $I_m = 20$, RMS value $= \frac{I_m}{\sqrt{2}} = \frac{20}{\sqrt{2}}$ A

Average value $= \frac{2}{\pi} I_m = \frac{2}{\pi}\times 20$

$\omega = 2\pi f = 5$. Then frequency $f = \frac{5}{2\pi} = 0.8$ Hz

Time period $T = 1/f = \frac{1}{0.8} = 1.25$ sec.

Ex: If $v = 100\sin(50t)$ find the time at which voltage is 10V.

Soln: $v = 100\sin(50t)$. $\therefore$ $10 = 100\sin(50t)$ then $50t = \sin^{-1}\left[\frac{10}{100}\right]$

ie $50t = \sin^{-1}\left[\frac{10}{100}\right] =$

$\therefore$ $t = \frac{0.1}{50} = \underline{2\times10^{-3}\text{ sec}}$ at which $v = 10$ volts.

[Refer Fig 1] [Fig. not required for the solution]

Ex:- If $v = 100\sin(50t + 45^\circ)$, find the time at which voltage is 10 Volt.

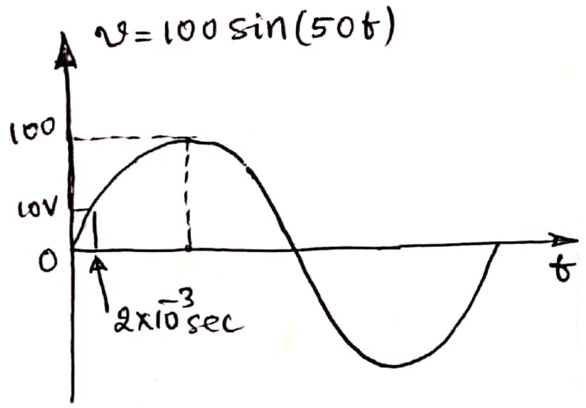
Soln $v = 100\sin(50t + 45^\circ) = 10$

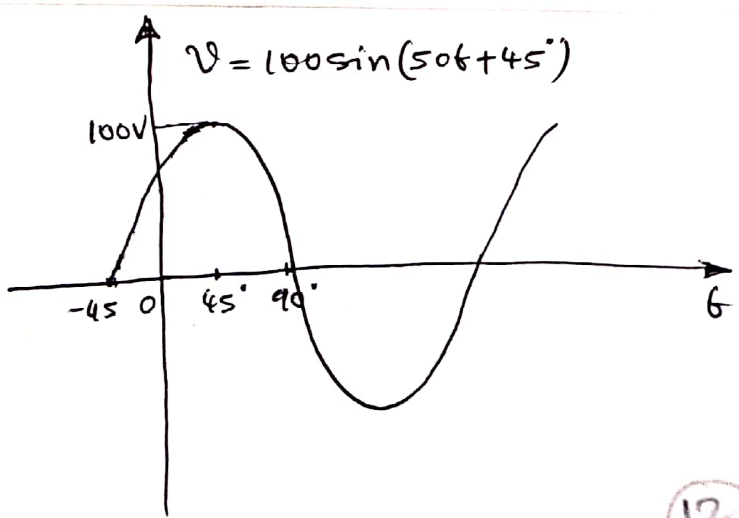
$\therefore$ $\sin(50t + 45) = 10/100$

$\therefore$ $(50t + 45^\circ) = \sin^{-1}\left[\frac{10}{100}\right] = 5.73^\circ$ or 0.1 rad

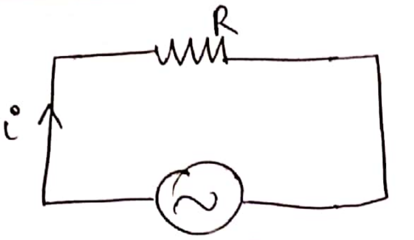
ie $50t + \pi/4 = 0.1$

$\therefore$ $50t = 0.1 - \pi/4$ hence $t = -0.013$ sec





(i) AC circuit with Pure resistance



$v = V_m \sin\omega t$

Applied voltage $v = V_m \sin\omega t$

current $i = \frac{v}{R} = \frac{V_m \sin\omega t}{R} = I_m \sin\omega t$

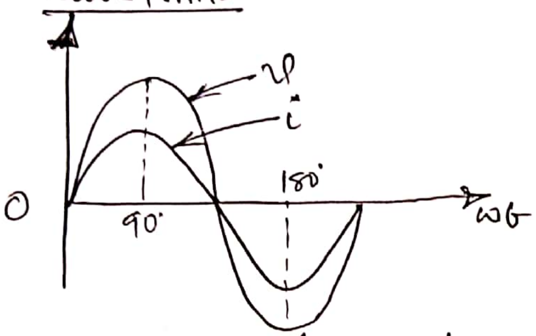
where $I_m = \frac{V_m}{R}$.

Hence $v = V_m \sin\omega t$ ie $V\angle 0^\circ$

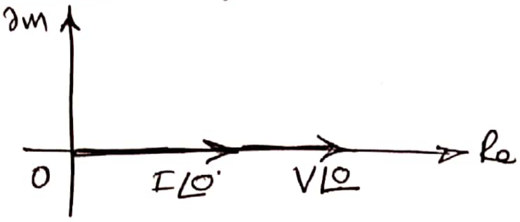
$i = I_m \sin\omega t$ ie $I\angle 0^\circ$

$\therefore$ v and i are in phase.

Wave forms

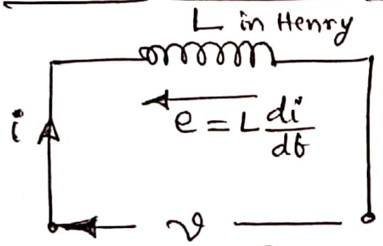


Phasor diagram



Impedence of the ckt $Z = \frac{V\angle 0^\circ}{I\angle 0^\circ} = R = Z\angle 0^\circ$ Pure resistance in Ω

(ii) Pure Inductive AC circuit



$v = V_m \sin\omega t$ is the applied voltage

e = self induced emf

By KVL, $v - e = 0$ ie $v = e = L\frac{di}{dt}$

$\therefore v = L\frac{di}{dt}$ Hence $i = \frac{1}{L}\int v \cdot dt$

$\therefore i = \frac{1}{L}\int V_m \sin\omega t \cdot dt = -\frac{V_m}{\omega L}\cos\omega t = \frac{V_m}{\omega L}\sin(\omega t - 90^\circ)$

$= I_m \sin(\omega t - 90^\circ)$

$\therefore i = I_m \sin(\omega t - 90^\circ)$ ie $I_m\angle -90^\circ$

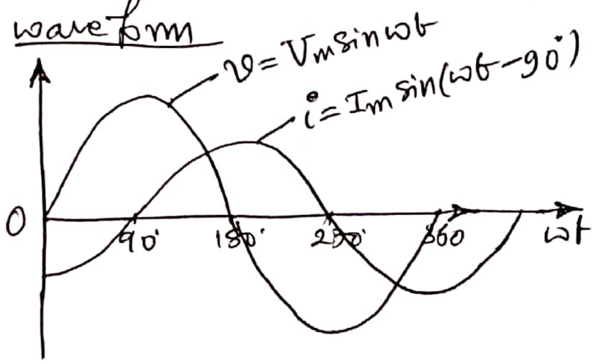
$v = V_m \sin(\omega t)$ ie $V_m\angle 0^\circ$

$\therefore$ Impedence $Z = \frac{V_m\angle 0^\circ}{I_m\angle -90^\circ} = \frac{V_m\angle 0^\circ}{\frac{V_m}{\omega L}\angle -90^\circ} = \omega L\angle 90^\circ = j\omega L = jX_L$

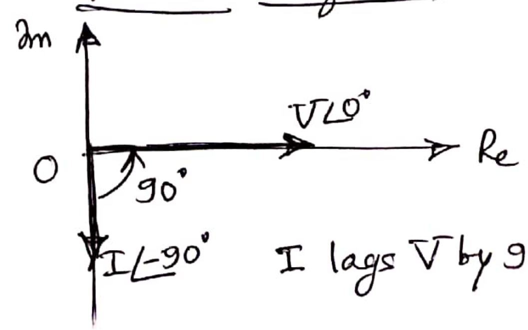
where $X_L = \omega L = 2\pi f L$ is called Inductive reactance in Ohm (Ω).

i lags v by 90°

waveform

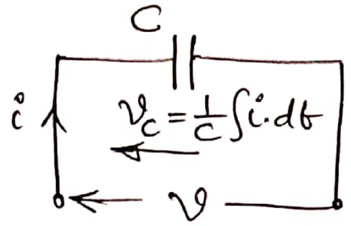


Phasor diagram



I lags V by 90°

(iii) Pure Capacitive AC circuit



$v = V_m \sin\omega t$ is the applied voltage
v_c = Voltage developed across C
By KVL, $v - v_c = 0$ ie $v = v_c$
ie $v = \frac{1}{C}\int i \cdot dt$ Hence $i = C\frac{dv}{dt}$

$\therefore i = C\frac{d}{dt}\left[V_m \sin\omega t\right] = C V_m \omega \cos\omega t = V_m \omega C \sin(\omega t + 90^\circ)$

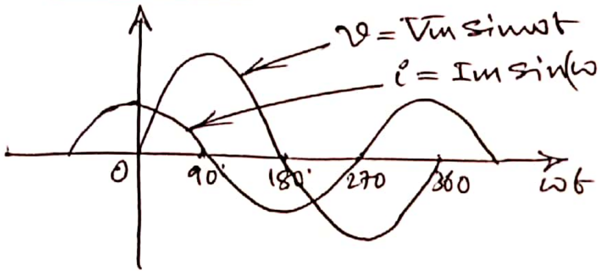
$\therefore i = I_m \sin(\omega t + 90^\circ)$ $\therefore$ i leads v by 90°

Hence $v = V_m \sin\omega t$ ie $V_m\angle 0^\circ$
$i = I_m \sin(\omega t + 90)$ ie $I_m \angle 90^\circ$

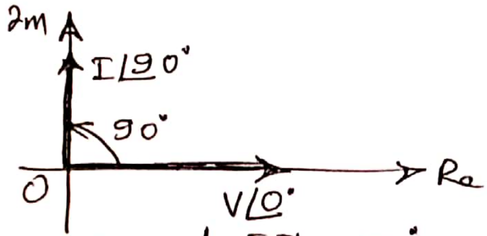
$\therefore$ Impedence $Z = \frac{V_m\angle 0^\circ}{I_m\angle 90^\circ} = \frac{V_m\angle 0^\circ}{\omega C V_m \angle 90^\circ} = \frac{V_m\angle 0^\circ}{\frac{V_m \angle 90^\circ}{1/\omega C}} = \frac{1}{\omega C}\angle -90^\circ = -j\frac{1}{\omega C}$
$= -jX_C$

$\therefore X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ is called Capacitive reactance in Ω.

waveforms

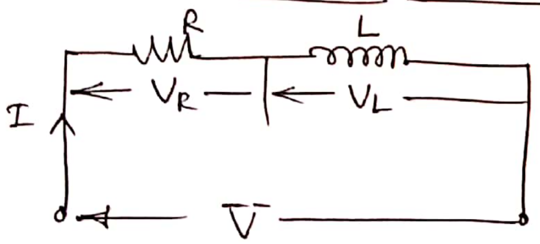


Phasor diagram



I leads V by 90°

R-L series AC circuit



V = Applied voltage (rms) in Volts
I = Current (rms) in Amps
$V_R = IR$ & $V_L = IjX_L$
By KVL,
$V - V_R - V_L = 0$ $\therefore V = V_R + V_L$

$\therefore V = IR + jIX_L = I(R + jX_L) = IZ$

ie $Z = R + jX_L$ is Impedence of the circuit

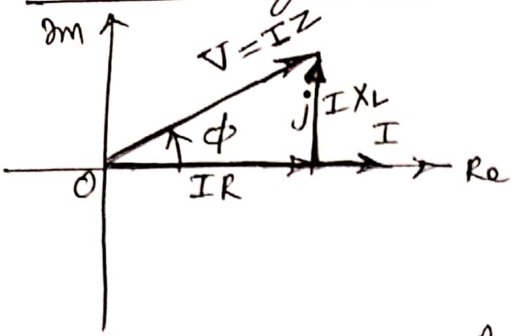
* Combination of Resistance & Reactance is called Impedence

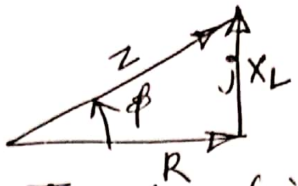
Impedence $Z = \underset{\uparrow}{R + jX_L} = \sqrt{R^2 + X_L^2}\angle\left(\tan^{-1}\frac{X_L}{R}\right) = \underset{\uparrow}{Z\angle\phi}\ \Omega$
Rectangular form — Polar form

Taking Applied voltage as $V\angle 0$ volt
current $I = \frac{\vec{V}}{\vec{Z}} = \frac{V\angle 0^\circ}{Z\angle\phi} = \frac{V}{Z}\angle -\phi$ Amps.

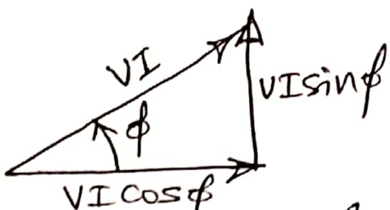
$\therefore$ I lags V by angle ϕ less than 90°
$\therefore$ In instantaneous form $v = V_m \sin\omega t$ & $i = I_m \sin(\omega t - \phi)$

Phasor diagram





Impedance triangle



Power triangle

$$\phi = \tan^{-1}\left[\frac{X_L}{R}\right]$$

$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

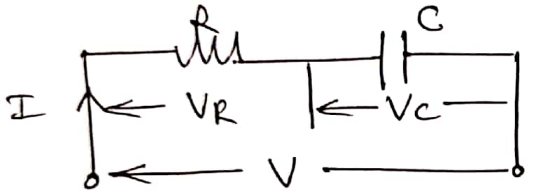
VI = Apparent power or Total power in Volt Amp ie VA

$VI\cos\phi$ = True or Active or Useful power in watts ie W.

$VI\sin\phi$ = Reactive power in volt-Amp Reactive ie VAR.

$$\text{Apparent power} = \sqrt{(\text{True Power})^2 + (\text{Reactive power})^2}$$

R-C series AC circuit



V = Applied Voltage (rms) in Volts

I = Current (rms) in amps.

$V_R = IR$ and $V_C = -jX_C$

By KVL, $V - V_R - V_C = 0$ $\therefore V = V_R + V_C$

$\therefore V = IR - jX_C = I(R - jX_C) = IZ$

ie $Z = R - jX_C$ is impedence of the circuit

* Combination of resistance & Reactance is called Impedence

Impedence $Z = R - jX_C = \sqrt{R^2 + X_C^2}\,\angle \tan^{-1}\left(\frac{-X_C}{R}\right) = Z\angle{-\phi}\ \Omega$

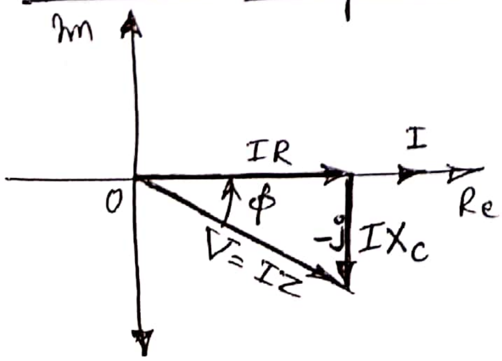
Taking Applied Voltage as $V\angle 0^\circ$ volt

current $I = \frac{\vec{V}}{\vec{Z}} = \frac{V\angle 0^\circ}{Z\angle{-\phi}} = \frac{V}{Z}\angle\phi$ Amps

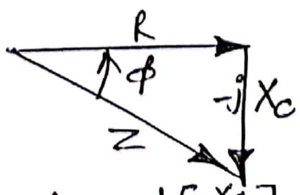
$\therefore$ I leads V by angle less than 90°

$\therefore$ In instantaneous form $v = V_m \sin\omega t$ and $i = I_m \sin(\omega t + \phi)$

Phasor diagram

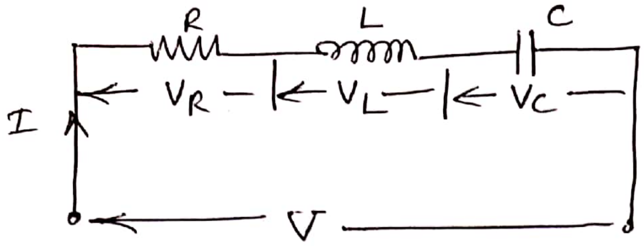


Impedence triangle



$$\phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$$

RLC Series AC Circuit



V = Applied voltage (rms) in Volts
I = current (rms) in Amps
$V_R = IR$, $V_L = jIX_L$, $V_C = -jIX_C$

By KVL, $V - V_R - V_L - V_C = 0$ ie $V = V_R + V_L + V_C$

$\therefore V = IR + jIX_L - jIX_C = I(R + jX_L - jX_C) = IZ$

ie $Z = R + j(X_L - X_C)$ is impedence of the circuit.

* Combination of resistance & Reactance is called Impedence

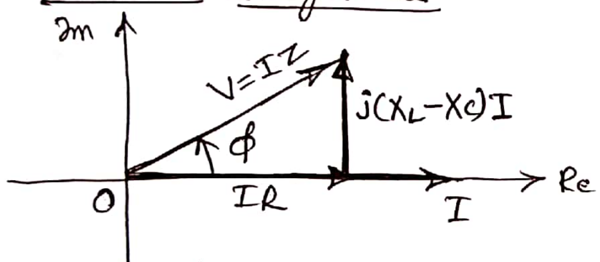
Impedence $Z = R + j(X_L - X_C) = \sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = Z\angle\phi$

If $X_L > X_C$ then the ckt is effectively inductive
If $X_C > X_L$ then the ckt is effective capacitive

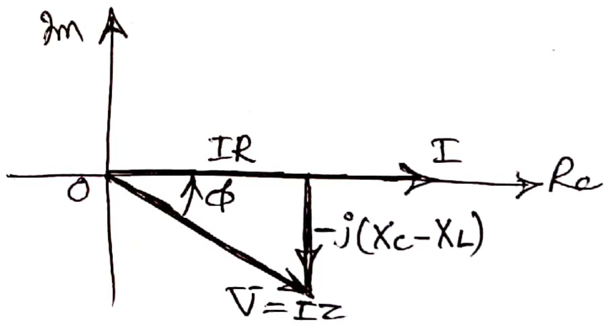
~~Phasor~~ Taking applied voltage as $V\angle 0°$ volt

current $I = \frac{\vec{V}}{\vec{Z}} = \frac{V\angle 0°}{Z\angle\phi} = \frac{V}{Z}\angle\phi = \frac{V}{Z}\angle\pm\phi$ + if $X_C > X_L$, − if $X_L > X_C$

Phasor diagrams



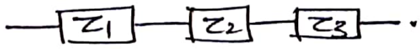
For $X_L > X_C$, I lags V



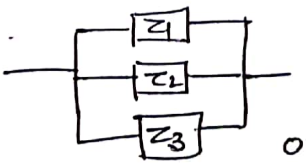
For $X_C > X_L$, I leads V

* If $X_L = X_C$, $Z = R$ ∴ The RLC series circuit is purely resistive.

Reciprocal of impedence Z is Admittance Y ie $Y = \frac{1}{Z}$

Impedences in series :  $Z_{eq} = Z_1 + Z_2 + Z_3 + \cdots$

Impedences in parallel:-

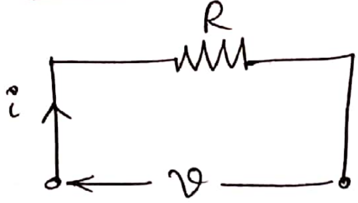


$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots$

or $Y_{eq} = Y_1 + Y_2 + Y_3 + \cdots$

Thus impedences in series can be added.
Admittances in parallel can be added.

Power in Pure resistive circuit (having R only)



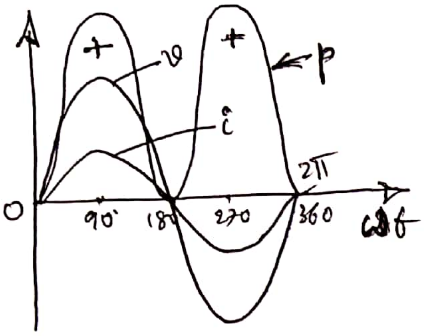
Applied voltage $v = V_m \sin\omega t$

current $i = I_m \sin\omega t$

v and i are in phase

Instantaneous power $p = vi$

$\therefore$ Average power (over 0 to 2π ie one cycle) $P = \frac{1}{2\pi}\int_0^{2\pi} p \cdot d\omega t = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \cdot I_m \sin\omega t \cdot d\omega t$



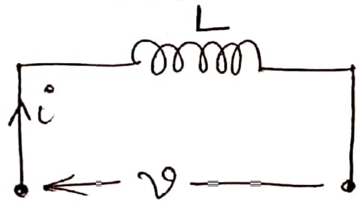
$$P = \frac{1}{2\pi} V_m I_m \int_0^{2\pi} \sin^2\omega t \cdot d\omega t = \frac{1}{2\pi}\int_0^{2\pi} \frac{(1-\cos 2\omega t)}{2} d\omega t$$

$$P = \frac{V_m I_m}{4\pi}\left[\omega t \Big|_0^{2\pi} - \frac{\sin(2\omega t)}{2}\Big|_0^{2\pi}\right] = \frac{V_m I_m}{4\pi} 2\pi = \frac{V_m I_m}{2}$$

$$\therefore P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI \text{ watts} \quad (V \,\&\, I \text{ are rms})$$

Since V & I are in phase, $\phi = 0°$, power factor $\cos\phi = 1$

Power in Pure Inductive circuit (having L only)



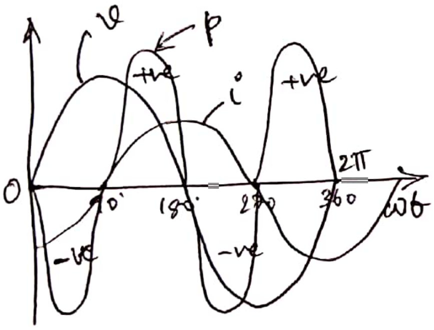
Applied voltage $v = V_m \sin\omega t$

current $i = I_m \sin(\omega t - 90°)$

i lags v by 90°

Instantaneous power $p = vi$

$\therefore$ Average power (over 0 to 2π) $P = \frac{1}{2\pi}\int_0^{2\pi} p \cdot d\omega t = \frac{1}{2\pi}\int_0^{2\pi} vi\, d\omega t$



$$\therefore P = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \cdot I_m \cdot \sin(\omega t - 90°) d\omega t$$

$$\therefore P = \frac{1}{2\pi}\int_0^{2\pi} V_m I_m \sin\omega t \cdot \sin(\omega t - 90°) d\omega t$$

$$\therefore P = \frac{V_m I_m}{2\pi}\int_0^{2\pi} \left[\frac{\cos(\omega t - (\omega t - 90°)) - \cos(\omega t + \omega t - 90°)}{2}\right] d\omega t$$

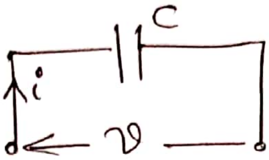
$$\therefore P = \frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\cos 90° - \cos(2\omega t - 90°)}{2} d\omega t = \frac{V_m I_m}{4\pi}\int_0^{2\pi} \frac{\sin 2\omega t \cdot d\omega t}{2}$$

$$= \frac{V_m I_m}{8\pi}\Big[\cos 2\omega t\Big]_0^{2\pi} = 0$$

$\therefore$ **Average Power consumption in L is zero.** ie L does not consume power

over 0 to 2π, p has two +ve half & two -ve half which sum to zero

Power in pure capacitive circuit (having only C)

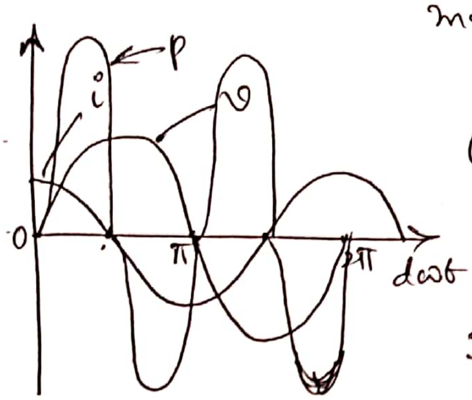


Applied voltage $v = V_m \sin\omega t$

current $i = I_m \sin(\omega t + 90^\circ)$

i leads v by 90°

Instantaneous power $p = vi$



Average power (over 0 to 2π) $P = \frac{1}{2\pi}\int_0^{2\pi} p \cdot d\omega t = \frac{1}{2\pi}\int_0^{2\pi} vi \cdot d\omega t$

$$\therefore P = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \cdot I_m \sin(\omega t + 90^\circ)\, d\omega t$$

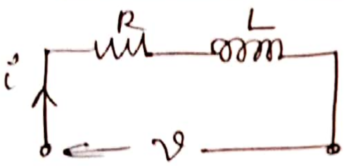
$$\therefore P = \frac{V_m I_m}{2\pi}\int_0^{2\pi} \sin\omega t \cdot \cos\omega t \cdot d\omega t$$

$$\therefore P = \frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\sin 2\omega t}{2}\, d\omega t = \frac{V_m I_m}{4\pi}\int_0^{2\pi} \sin 2\omega t \cdot d\omega t$$

$$\therefore P = \frac{V_m I_m}{8\pi}\left[\cos 2\omega t\right]_0^{2\pi} = 0$$

$\therefore$ **Average Power consumption in pure Capacitor is Zero.**

Power in R-L series AC circuit



Applied voltage $v = V_m \sin\omega t$

current $i = I_m \sin(\omega t - \phi)$

i lags v by $\phi < 90^\circ$

$\therefore$ Instantaneous power $p = vi = V_m \sin\omega t \cdot I_m \sin(\omega t - \phi)$

$\therefore$ Average power (over 0 to 2π) $P = \frac{1}{2\pi}\int_0^{2\pi} p \cdot d\omega t = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \cdot I_m \sin(\omega t - \phi)\, d\omega t$

$$\therefore P = \frac{1}{2\pi}\int_0^{2\pi} V_m I_m \cdot \sin\omega t \cdot \sin(\omega t - \phi)\, d\omega t$$

$$= \frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\cos[\omega t - (\omega t - \phi)] - \cos(\omega t + \omega t - \phi)}{2}\, d\omega t$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} \left[\cos\phi \cdot d\omega t - \left[\cos 2\omega t \cdot \cos\phi + \sin 2\omega t \cdot \sin\phi\right]\right] d\omega t$$

$$= \frac{V_m I_m}{4\pi} \cos\phi \left[\omega t\right]_0^{2\pi} - \cos\phi\left[\frac{\sin 2\omega t}{2}\right]_0^{2\pi} - \left|\frac{\sin\phi \cos 2\omega t}{2}\right|_0^{2\pi}$$

$$= \frac{V_m I_m}{4\pi}\cos\phi\,(2\pi) - 0 - 0 = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi$$

$$\therefore \boxed{P = VI\cos\phi} \text{ watts.}$$

Power in R-C series ckt

$v = V_m \sin\omega t$, $\quad i = I_m \sin(\omega t + \phi)$ where i leads v by $\phi < 90°$

Instantaneous power $p = vi = V_m \sin\omega t \cdot I_m \sin(\omega t + \phi)$

Average power (over 0 to 2π) $P = \frac{1}{2\pi}\int_0^{2\pi} p \cdot d\omega t = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \cdot I_m \cdot \sin(\omega t+\phi) d\omega t$

$$\therefore P = \frac{V_m I_m}{2\pi}\int_0^{2\pi} \sin\omega t \cdot \sin(\omega t + \phi) \cdot d\omega t = \frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\cos(\omega t - \omega t - \phi) - \cos(2\omega t + \phi)}{2}$$

$$\therefore P = \frac{V_m I_m}{4\pi}\int_0^{2\pi}\left[\cos\phi - \cos(2\omega t + \phi)\right] d\omega t = \frac{V_m I_m}{4\pi}\int_0^{2\pi}\cos\phi \, d\omega t - \frac{V_m I_m}{4\pi}\int_0^{2\pi}\cos(2\omega t+\phi) d\omega t$$

$$\therefore P = \frac{V_m I_m \cos\phi}{4\pi}\left[\omega t\right]_0^{2\pi} - \frac{V_m I_m}{4\pi}\int_0^{2\pi}(\cos 2\omega t \cdot \cos\phi - \sin 2\omega t \cdot \sin\phi) d\omega t$$

$$\therefore P = \frac{V_m I_m}{4\pi}\cos\phi\,[2\pi] - 0 + 0 = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi$$

$$\therefore \underline{P = VI\cos\phi \text{ watts}}$$

Power consumed by R-C series ckt is $P = VI\cos\phi$ watts

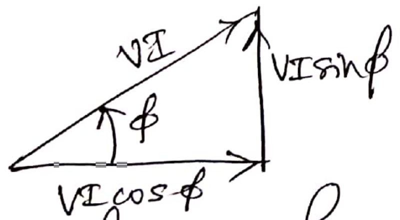
similarly Power consumed by RLC series ckt is $P = VI\cos\phi$ watts

$VI\cos\phi$ = Active power or True power in watts W

$VI\sin\phi$ = Reactive power in Volt Amp Reactive VAR

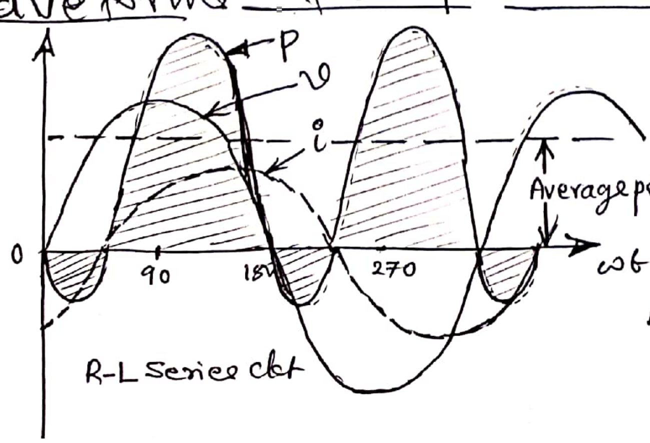
VI = Apparent power $= \sqrt{(VI\cos\phi)^2 + (VI\sin\phi)^2} = VI$ in Volt Amp VA

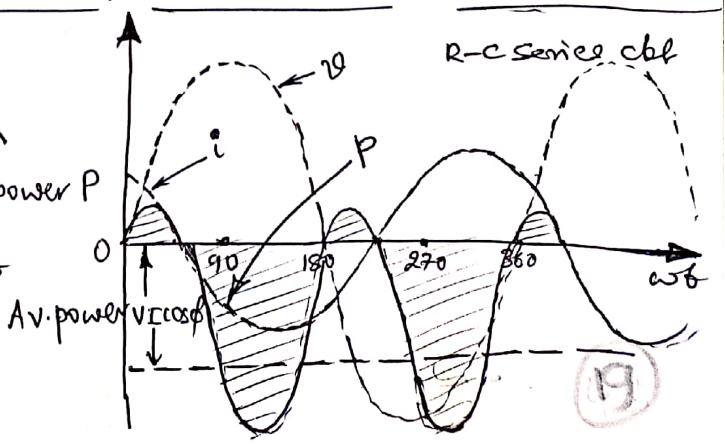
Product of RMS voltage V & RMS current I in AC circuit is called Apparent power.



Also $VI\cos\phi = I^2R$ watts.

Waveforms for power in R-L series ckt & RC series ckt





Problems on single AC circuits

Ex 1. Find current, power consumed & power factor in a single phase AC circuit with $R = 5\Omega$ in series with $L = 5mH$ with applied voltage $v = 100\sin 314t$ volt. Draw phasor diagram and wave forms.

Sol^n Applied voltage $v = 100\sin 314t$ volt & $V = \frac{100}{\sqrt{2}}$ volt

$\omega = 314$ rad/sec. $R = 5\Omega$, $X_L = \omega L = 314 \times 5 \times 10^{-3} = 1.57\Omega$

$\therefore$ Impedence $Z = 5 + j1.57 = 5.24\angle 17.43\,\Omega$

$\therefore$ current $I = \dfrac{100/\sqrt{2}\angle 0°}{5.24\angle 17.43} = \dfrac{V}{Z} = 13.49\angle -17.43\,A$

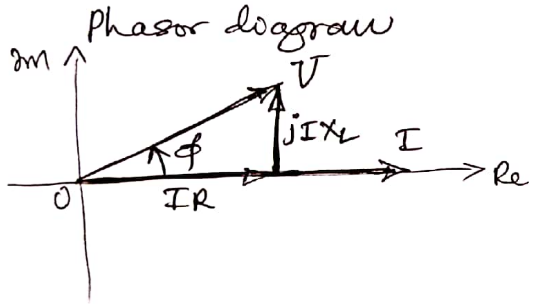
(-ve indicates lagging current)

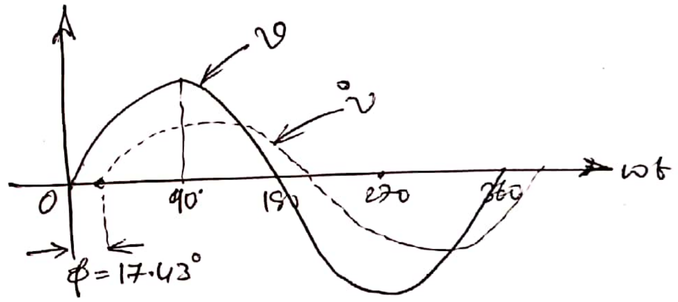
power factor angle $\phi = -17.43°$

Power factor $\cos\phi = \cos(17.43°) = 0.9540$ (lagging)

Power consumed $P = VI\cos\phi = \frac{100}{\sqrt{2}} \times 13.49 \times 0.9540 = \underline{910\,W}$

Phasor diagram





Ex 2 Find current, power factor and power consumed in a single phase AC circuit with 230V, 50Hz supply with resistance 100Ω connected with a capacitor 25µF in series.

Sol^n Applied voltage $V = 230V$, Frequency $f = 50Hz$

$R = 100\Omega$, $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 50 \times 25 \times 10^{-6}} = 127.32\Omega$

$\therefore$ Impedence $Z = R - jX_C = 100 - j127.32 = 161.89\angle -51.85\,\Omega$

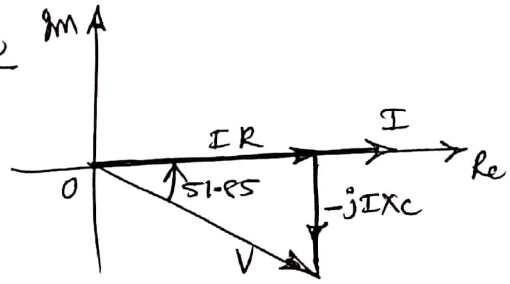
current $I = V/Z = \dfrac{230\angle 0°}{161.89\angle -51.85} = 1.42\angle 51.85\,A$

$\therefore$ Power factor angle $\phi = 51.85$ I leads V by $51.85°$

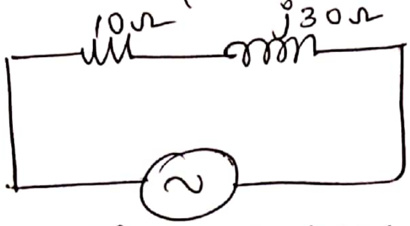
$\therefore$ Power consumed $P = VI\cos\phi = 230 \times 1.42 \times \cos(51.85) = 201.74\,W$

Power factor $\cos\phi = \cos(51.85) = 0.6177$

Phasor diagram



Ex 3. Find the current, active power, Reactive power and power factor in the circuit shown.



$v = 50 \sin(314t + 30^\circ)$

Solⁿ Applied voltage $V = 50\angle 30^\circ$ V, Impedance $Z = 10 + j30 = 31.62\angle 71.56\,\Omega$

(i) current $I = \frac{V}{Z} = \frac{50\angle 30^\circ}{31.62\angle 71.56} = 1.58\angle -41.56$ A ($V_m = 50V$ & $I_m = 1.58$)

ie $i = 1.58 \sin(314t - 41.56)$ A

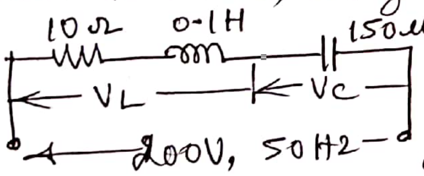
(ii) Active power $P = VI\cos\phi = \frac{50}{\sqrt{2}} \times \frac{1.58}{\sqrt{2}} \cos(71.56) = 12.34W$

(iii) Reactive power $Q = VI\sin\phi = \frac{50}{\sqrt{2}} \times \frac{1.58}{\sqrt{2}} \sin(71.56) = 37.47$ VAR

(iv) Power factor $\cos\phi = \cos(71.56) = 0.316$ (lag)

Ex 4 A coil of resistance 10Ω and inductance 0.1H connected in series with a 150µF across a 200V, 50Hz supply. calculate
i) Inductive reactance ii) capacitive reactance iii) impedance iv) current
v) power factor vi) voltage across coil and capacitor respectively.

Solⁿ



Impedance $Z = R + jX_L - jX_C$

where $X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.41\,\Omega$

Also $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}} = 21.23\,\Omega$

∴ Total impedance $Z = 10 + j31.41 - j21.23 = 10 + j10.17 = 14.26\angle 45.48^\circ\,\Omega$

current $I = V/Z = \frac{200\angle 0^\circ}{14.26\angle 45.48} = 14.02\angle -45.48$ A (I lags V)

p.f $\cos\phi = \cos(-45.48^\circ) = 0.701$ (lag

voltage across Coil $= I(R + jX_L) = 14.02\angle -45.48 \times (10 + j31.41)$

$= 461.96\angle 26.85$ Volt

voltage across C $= I(-jX_C) = 14.02\angle -45.48 \times 21.23\angle -90^\circ$

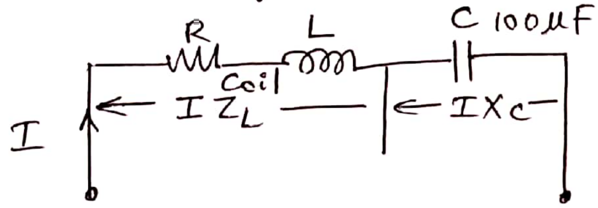
ie $= 297.64\angle -135.48$ V

∴ i) Ind. reactance $jX_L = j31.41\,\Omega$ ii) cap. reactance $-jX_C = -j21.23\,\Omega$

iii) Impedance $Z = 14.26\angle 45.48\,\Omega$ iv) current $I = 14.02\angle -45.48$ A

Ex 4 A coil of power factor 0.6 is series with a 100 μF capacitor. At 50 Hz, the pd across the coil is equal to the p.d across the capacitor. Find resistance and inductance of the coil. [Hint:- A coil has resistance R & inductance L]

Soln



$f = 50\,Hz, \quad C = 100\,\mu F$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.85\,\Omega$

Let R & L be resistance & inductance of the coil.

~~From data~~

From data, voltage across coil ie IZ_L = voltage across C ie IX_C

$\therefore Z_L = X_C$ ie $Z_L = 31.85\,\Omega = |R + jX_L|$

p.f of the coil $\cos\phi = 0.6, \quad \phi = \cos^{-1}(0.6) = 53.13$

$\therefore R = Z_L \cos\phi = 31.85 \times 0.6 = 19.11\,\Omega$

$X_L = Z_L \sin\phi = 31.85 \times 0.8 = 25.48\,\Omega$

$\therefore X_L = 2\pi f L = 2\pi \times 50 \times L = 25.48$

$\therefore$ Inductance of the coil $L = \frac{25.48}{2\pi \times 50} = 0.081\,H$

Ex 5 A coil having resistance R and inductance L is connected across a variable frequency alternating current supply of 110V. An ammeter showed 15.6A at 80Hz and 19.7A at 40 Hz. Find R and L.

Soln At 80 Hz, $Z = V/I = \frac{110}{15.6} = 7.05 = \sqrt{R^2 + X_L^2}$

where $X_L = 2\pi f L = 2\pi \times 80 \times L$

$\therefore (7.05)^2 = R^2 + (2\pi \times 80)^2 L^2$ ——— ①

At 40 Hz, $Z = V/I = \frac{110}{19.7} = 5.58 = \sqrt{R^2 + X_L^2}$

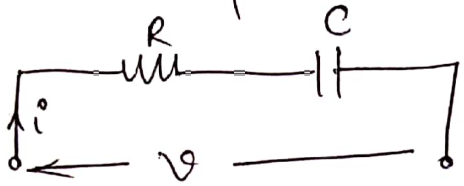
where $X_L = 2\pi f L = 2\pi \times 40 \times L$

$\therefore (5.58)^2 = R^2 + (2\pi \times 40)^2 L^2$ ——— ②

From ① and ② solving for R^2 and L^2 & then R & L

$R = \sqrt{R^2} = 5\,\Omega, \quad L = \sqrt{L^2} = 9.9\,mH$

Ex. 7 A current of $i = 4\sin(314t - 100^\circ)$ produces a voltage drop of $v = 220\sin(314t + 200^\circ)$ in series ~~AC~~ circuit. Find the circuit parameters.



Soln:-

Applied voltage $\bar{V} = 220\angle 200^\circ$ V

current $I = 4\angle -100^\circ$

$\therefore$ Impedence $Z = \frac{V}{Z} = \frac{220\angle 200^\circ}{4\angle -100} = 55\angle 300^\circ\ \Omega$

$= 27.5 - j47.63\ \Omega = R - jX_C$

$\therefore$ Resistance $R = 27.5\ \Omega$

Capacitive Reactance $-jX_C = -j47.63$

$\therefore X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} = \frac{1}{314 C} = 47.63$

$\therefore C = 1/(47.63 \times 314) = 66.86\ \mu F$.

$\therefore$ Ans:- The ckt parameters are $R = 27.5\ \Omega$, $C = 66.86\ \mu F$

Ex 8 A 100V, 50 Hz inductive circuit takes a current of 15 A lagging the voltage by 30°. Find the resistance & inductance of the ~~ckt~~ circuit.

Soln Applied voltage $V = 100\angle 0^\circ$ $f = 50$ Hz

current $I = 15\angle -30^\circ$ A. (I lags V by 30°)

Impedence $Z = V/I = \frac{100\angle 0^\circ}{15\angle -30^\circ} = 6.66\angle +30^\circ\ \Omega = R + jX_L$

$\therefore$ Resistance $R = Z\cos\phi = 6.66\cos 30^\circ = 5.77\ \Omega$ //

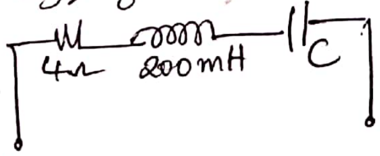
Inductive reactance $X_L = Z\sin\phi = 6.66\sin 30^\circ = 3.33\ \Omega$

Also $X_L = 2\pi f L = 2\pi \times 50 \times L = 3.33$.

$\therefore$ Inductance $L = \frac{3.33}{2\pi \times 50} = 10.6$ mH //

Ex 9:- A coil having an inductance of 200 mH and resistance of 4 Ω is connected to a capacitor in series across a 50 Hz supply. Calculate the capacitor required to make circuit p.f as 0.5 lagging.

Soln:-



$R = 4\ \Omega$, $X_L = 2\pi \times 50 \times 200 \times 10^{-3} = 1.25\ \Omega$

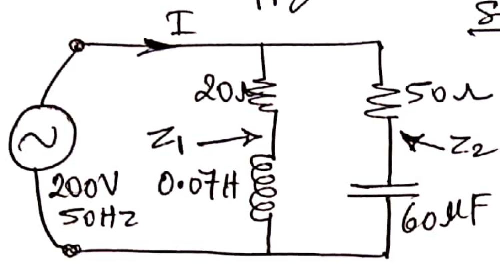
Impedence $Z = 4 + j1.25 - jX_C$

$\therefore$ p.f $\cos\phi = \frac{R}{|Z|} = \frac{4}{|(4 + j1.25 - jX_C)|} = 0.5$

$\therefore \frac{4}{\sqrt{4^2 + (1.25 - X_C)^2}} = 0.5$. Find X_C and C.

$C = 57.943\ \mu F$

Ex10. Two circuits connected in parallel have (i) a coil of resistance 20Ω and inductance 0.07H (ii) a capacitance of 60μF in series with 50Ω resistance. Calculate the total current & power factor when connected to 200V 50Hz supply.



Soln

$X_L = 2\pi f L = 2\pi \times 50 \times 0.07$
$= 22\,\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}} = 53.05\,\Omega$

Branch Impedences Z_1 & Z_2

$\therefore Z_1 = 20 + j22\,\Omega$ & $Z_2 = 50 - j53.05\,\Omega$

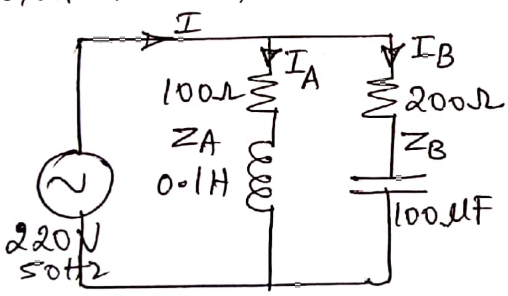
$\therefore$ Total Impedence $Z_{eq} = Z_1 \| Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(20+j22)(50-j53.05)}{(20+j22+50-j53.05)}$

$= 28.30\angle 24.95\,\Omega$

Total current $I = \frac{V}{Z_{eq}} = \frac{200\angle 0^\circ}{28.30\angle 24.95} = 7.067\angle -24.95\,A$

Power factor $\cos\phi = \cos(-24.95) = 0.9066$ (lagging)

Ex11. Find the total current and current in each branch



Soln

$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.41\,\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\,\Omega$

$\therefore Z_A = 100 + j31.41\,\Omega = 104.81\angle 17.43\,\Omega$

$Z_B = 200 - j31.83\,\Omega = 202.51\angle -9.04\,\Omega$

Total impedence $Z_{eq} = Z_A \| Z_B = \frac{Z_A Z_B}{(Z_A + Z_B)} = \frac{(104.81\angle 17.43)(202.51\angle -9.04)}{300\angle -0.08}$

$= 70.75\angle 8.47\,\Omega$

current $I = \frac{V}{Z} = \frac{220\angle 0^\circ}{70.75\angle 8.47} = 3.109\angle -8.47\,A$

$I_A = \frac{I \times Z_B}{(Z_A + Z_B)} = \frac{3.109\angle -8.47 \times 202.51\angle -9.04}{(104.81\angle 17.43 + 202.51\angle -9.04)} = 2.099\angle -17.43\,A$

$I_B = \frac{I \times Z_A}{(Z_A + Z_B)} = \frac{3.109\angle -8.47 \times 104.81\angle 17.43}{(104.81\angle 17.43 + 202.51\angle -9.04)} = 1.086\angle 9.042\,A$

Ex 11. A resistance 12 Ω, an inductance 0.15 H and a capacitance of 100 μF are connected in series across a 100 V, 50 Hz supply. Calculate i) Impedence ii) Current (iii) phase difference between current & supply voltage ie power factor angle iv) power consumed.

Sol^n $R = 12\,\Omega$, $L = 0.15\,H$ & $C = 100\,\mu F$

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.1\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.8\,\Omega$$

(i) Impedence $Z = R + jX_L - jX_C = 12 + j47.1 - j31.8$

$$= 12 + j15.3\,\Omega = 19.43\angle 52\,\Omega$$

In phasor form

(ii) Current $I = \frac{V}{Z} = \frac{100\angle 0^\circ}{19.43\angle 52} = 5.15\angle -52\ A$

(iii) Phase difference betn i & v ie $\phi = 52^\circ$
current lags supply voltage by 52°.

(iv) power consumed $P = VI\cos\phi = 100 \times 5.15 \times \cos 52$
$= 371.1\ W$

Another method of solution

$R = 12\,\Omega$, $L = 0.15\,H$, $C = 100\,\mu F$

$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.15 = 47.1\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.8\,\Omega$$

(i) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{12 + (47.1 - 31.8)^2} = 19.43\,\Omega$

(ii) current $i = \frac{V}{Z} = \frac{100}{19.43} = 5.15\ A$ (only magnitude)

(iii) phase difference betn current & supply voltage

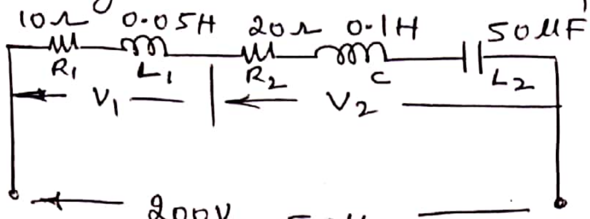
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \text{ or } \cos^{-1}\left[\frac{R}{Z}\right]$$

$$\phi = \tan^{-1}\left[\frac{15.3}{12}\right] = 52^\circ$$

$$\text{or } \phi = \cos^{-1}\left[\frac{12}{19.43}\right] = 52^\circ\,(\text{lag})$$

(iv) Power consumed $P = VI\cos\phi = 100 \times 5.15 \times \cos 52^\circ$
$= 371.1\ W$

Ex 12: For the circuit shown find the values of current I, Voltages V_1 & V_2 and power factor.



Sol^n Total resistance $R = 10+20 = 30\Omega$

Total inductance $L = 0.05+0.1 = 0.15H$

$\therefore$ Inductive reactance $X_L = 2\pi f L = 2\times\pi\times50\times0.15 = 47.1\Omega$

Capacitive reactance $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\times50\times50\times10^{-6}} = 63.7\Omega$

$\therefore$ Impedence $Z = R + jX_L - jX_C = 30 + j47.1 - j63.7 = 30 - j16.6\ \Omega$

$= 34.3\angle{-28.95}\ \Omega$

Current $I = \frac{V}{Z} = \frac{200\angle 0^\circ}{34.3\angle{-28.95}} = 5.83\angle 28.95$ A

[This current I leads V by $\phi = 28.95$]

To find V_1

Impedence of that part $Z_1 = R_1 + jX_{L1}$

$X_{L1} = 2\pi\times50\times0.05 = 15.7\Omega$ & $R_1 = 10\Omega$

$\therefore Z_1 = 10 + j15.7\Omega = 18.6\angle 57.5\Omega$

$V_1 = IZ_1 = 5.83\angle 28.95 \times 18.6\angle 57.5 = 108.4\angle 86.45$ Volt

To find V_2

Impedence of that part $Z_2 = R_2 + jX_{L2}$

$X_{L2} = 2\pi\times50\times0.1 = 31.4\Omega$ & $R_2 = 20\Omega$, $X_C = 63.7\Omega$

$Z_2 = 20 + j31.4 - j63.7 = 20 - j32.3\Omega = 38\angle{-58.22}\ \Omega$

$V_2 = IZ_2 = 5.83\angle 28.95 \times 38\angle{-58.22} = 221.54\angle{-29.27}$ Volt

To find powerfactor of the ckt $\cos\phi$

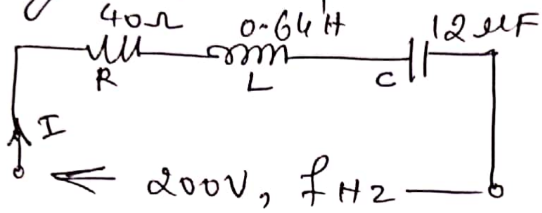
cos of phase angle of I ie $\cos(28.95) = 0.875$ (lead)

OR $\cos\phi = \frac{R}{Z} = \frac{30}{34.3} = 0.875$

* Note:- Solving such problems using phasors is more advantageous because all quantities are in phasor form. (I, V_1, V_2, Z etc) This is useful in drawing phasor diagrams.

Ex 13 A choke coil of 40Ω resistance & 0.64H Inductance is connected in series with a capacitance of 12μF. 200V, AC supply of variable frequency is applied. Find the frequency at which resonance occurs. & the current & Voltages across the Coil & capacitor at resonance.

Solⁿ * Note: Draw the circuit diagram first & then analyse the problem.



Solⁿ $R = 40\Omega$, $L = 0.64H$ & $C = 12\mu F$

Resonance frequency $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.64 \times 12 \times 10^{-6}}} = 57.4 Hz$

To find current at resonance

At resonance $X_L = X_C$ & hence total reactance $X = X_L - X_C = 0$

∴ Impedence at resonance $Z = R + j(X_L - X_C) = R\ \Omega = 40\Omega$

∴ current I at resonance $= \frac{V}{Z} = \frac{200}{40} = 5A$

Voltage across Coil at resonance, $V_L = I Z_{coil}$

where $Z_{coil} = 40 + j(2\pi \times 57.4 \times 0.64) = 40 + j230.81 = 234.25 \angle 80.16\ \Omega$

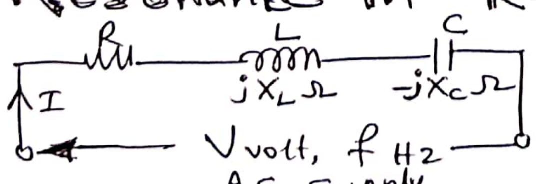
∴ $V_L = I Z_{coil} = 5 \times 234.25 = 1171.25 V$

Voltage across Capacitor at resonance $V_C = I X_C$

where $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 57.4 \times 12 \times 10^{-6}} = 231.06\ \Omega$

∴ $V_C = I X_C = 5 \times 231.06 = 1155.3 V$

Resonance in R-L-C Series circuit



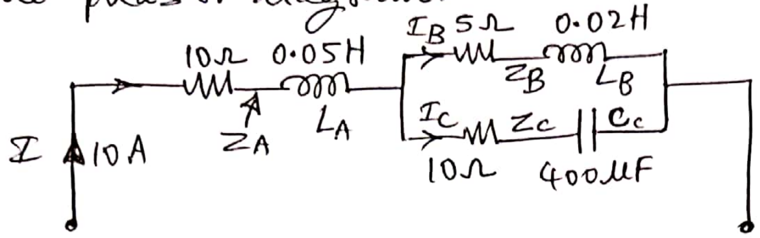
$Z = R + j(X_L - X_C)$

By giving variable frequency AC supply to a RLC series circuit, vary the frequency till X_L becomes equal X_C where $f = \frac{1}{2\pi\sqrt{LC}}$ called Resonant frequency.

At Resonance Impedence of the ckt $Z = R$ since $X_L = X_C$.

At resonance Z is minimum & hence current $I = V/R$ is maximum. P.f of the ckt at resonance is Unity.

Ex 14 Determine the voltage at frequency of 50 Hz to be applied across AB, so that current in the circuit is 10 A. Draw the phasor diagram.



Let Z_A, Z_B & Z_C be the impedences.

Soln $X_{LA} = 2\pi f L_A = 2\pi \times 50 \times 0.05 = 15.70\,\Omega$

$X_{LB} = 2\pi f L_B = 2\pi \times 50 \times 0.02 = 6.28\,\Omega$

$X_{C_C} = 2\pi f C_c = \dfrac{1}{2\pi f C_c} = \dfrac{1}{2\pi \times 50 \times 400 \times 10^{-6}} = 7.95\,\Omega$

$\therefore Z_A = 10 + j15.70\,\Omega, \quad Z_B = 5 + j6.28\,\Omega, \quad Z_C = 10 - j7.95\,\Omega$

$\therefore$ Total impedence $Z_{eq} = Z_A + Z_B \| Z_C$

$\therefore Z_{eq} = 10 + j15.70 + (5+j6.28) \| (10-j7.95)$

$= 10 + j15.70 + \dfrac{(5+j6.28)(10-j7.95)}{(5+j6.28+10-j7.95)} = 10 + j15.70 + 6.79\angle 19.34$

$Z_{eq} = 24.31\angle 47.56\,\Omega$

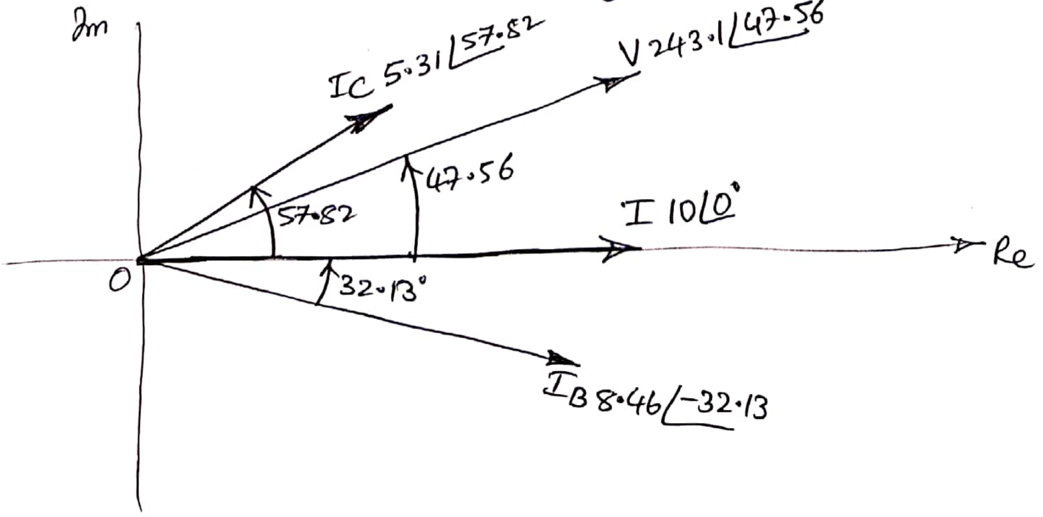
$\therefore$ Applied Voltage $V = IZ = 10\angle 0^\circ \times 24.31\angle 47.56 = 243.1\angle 47.56\,V$

From data current $I = 10\angle 0^\circ$

Branch currents $I_C = I \times \dfrac{Z_B}{(Z_B+Z_C)} = \dfrac{10\angle 0^\circ \times (5+j6.28)}{(5+j6.28+10-j7.95)} = 5.31\angle 57.82\,A$

$I_B = I \dfrac{Z_C}{(Z_B+Z_C)} = \dfrac{10\angle 0^\circ\,(10-j7.95)}{(5+j6.28+10-j7.95)} = 8.46\angle -32.13\,A$

Phasor diagram (Taking $I = 10\angle 0^\circ$ as reference)



PRACTICE PROBLEMS

Unit Single Phase AC circuits

Ex1 A rectangular coil of 30x20cm dimensions has 40 turns rotating about an axis parallel to the longer side at 1500rpm in a uniform magnetic field of flux density 0.075 T (means Tesla or Wb/m^2). Find the maximum emf induced in the coil.

[Hint:- $e = Blv \sin\theta$ volts. $l = 30$ cm ie 30×10^{-2} m

$v = \frac{\pi DN}{60} = \frac{\pi \times 20\times10^{-2}\times1500}{60}$ D = Diameter of rotation $= 20\times10^{-2}$ m

$= 15.71$ m/s

$e_{max} = Blv$ where $\theta = 90°$]

Ex2 An alternating current is represented by $i = 10\sin(942t)$ amperes. Determine the frequency, time period, the time taken to reach 6A for the first time. [Ans: 150 Hz, Time period $T = 6.67$ms
$t = 0.68$ms]

Ex3. An alternating current varying sinusoidally with a frequency of 50 Hz has a RMS value of 20A. Write down the equation for the instantaneous value & find the instantaneous current at (i) 0.0025 sec (ii) 0.0125 A. [Ans: $i = 20\sqrt{2}\sin(2\pi\times50t) = 20\sqrt{2}\sin(314t)$]
$t = 1.667$ msec
(iii) Find the time at which the current reaches 14.14 A first time in the cycle.

Ex4 Represent the following AC quantities in phasor form.
(i) $v = 100\sin(314t)$ (ii) $v = 50\sin(314t - 45°)$ (iii) $i = 10\sin(314t)$
(iv) $i = 10\sin(314t + 45°)$
[Ans: i) $100\angle 0°$ (ii) $50\angle -45°$ (iii) $10\angle 0°$ (iv) $10\angle 45°$

Ex5 In an AC circuit if the applied voltage is $v = 100\sin(314t)$V & current is $i = 10\sin(314t - 45°)$A find circuit parameters: Ans: $R = 7.07\Omega$, $L = 0.0225$ H

Ex6. A coil has an inductance of 0.05H with a resistance of 10Ω. It is connected to single phase 200V, 50Hz supply. Find the impedence, current, power consumed & power factor. Ans [18.6Ω, 10.75A, 1155W, 0.537 lag]

Ex7 A resistance of 12Ω, inductance 0.15H and capacitance of 130 μF are connected in series across a 100V, 50Hz supply. Calculate the impedence, current and power factor. Ans [25.6Ω, 3.9A, 0.4687]

Ex8 A coil takes a current of 10A at a phase angle of −30° ie $10\angle{-30^\circ}$ A when connected to a 250V, 50Hz Calculate the resistance and inductance of the coil.

[Ans: 21.65Ω, 0.0398 H]

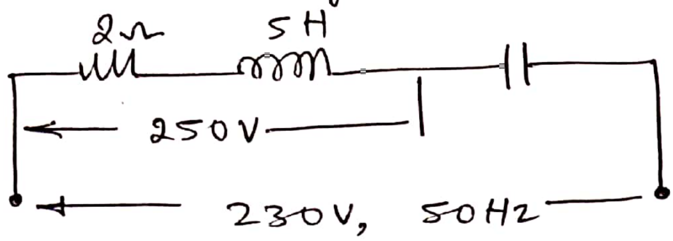
Ex9. A choke coil takes a current of 2.5A when connected to a single phase 250V, 50Hz ac supply & consumes 400W. Calculate the resistance, inductance & powerfactor of the coil. [Ans: 64Ω, 0.245H, 0.64 lag]

Ex10. A resistor and an inductor are connected in series across a 200V, 50Hz supply. The voltage across the inductor is 150V. If the circuit takes 2A at 0.8 pf lagging. Calculate i) resistance of the resistor ii) resistance & inductance of the inductor.

Ex11. An alternating voltage of 80+j60V is applied to a circuit and the current flowing is −4+j10 A. Find (a) Impedence of the circuit (b) the power consumed (c) Apparent power (d) Reactive power. [Ans: $9.285\angle{-74.9}$ Ω, 280.5W, 1078.84 VA, 1041.73 VAR]

Ex12. A choke coil of inductance 0.08H and resistance 12Ω is connected in parallel with a 120μF capacitor to a 240V, 50Hz supply. Determine the current drawn from the supply & its phase angle. Also find currents in each branch. [Ans:- $3.923\angle{18.9}$ A]

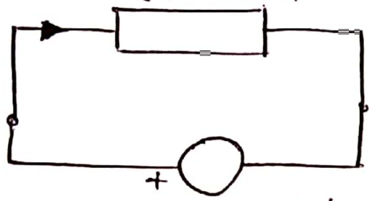
Ex 13. Find the value of C in the circuit shown below.



Ans: 25.3 μF

Ex 14 Find the parameters of the unknown passive networks in the figs below

$i = 1\sin(t-30)$ A

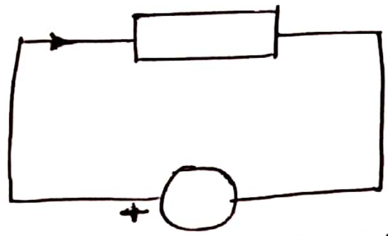


$v = 10\sin t$ volt

Ans: $R = 8.77\ \Omega$

$L = 5$ H

$i = 1\sin(t+30^\circ)$ A



$v = 10\sin t$ volts

Ans: $R = 8.77\ \Omega$

$C = 1/5$ F

Ex 15 Two impedences $(14+j5)\,\Omega$ and $(18+j10)\,\Omega$ are connected in parallel across a 200V, 50Hz supply. Determine (1) admittance of each branch (2) total current, power and power factor. (3) Total admittance

(4) Also find the capacitance which must be connected in parallel with the original circuit so that the resultant power factor ~~is unity~~ of the ckt is unity.

Ans: (1) $0.067\angle{-19.65}\ \Omega$, $0.0485\angle{-29.05}\ \Omega$

(2) Total admittance $0.1154\angle{-23.59}\ \Omega$

(3) Total current $23.09\angle{-23.59}$ A

(4) power factor 0.916 (5) 4232.08 W

Ex 16. Draw the phasor diagrams for the following circuits

(i) R-L series circuit (ii) R-C series circuit

(iii) R-L-C series circuit.