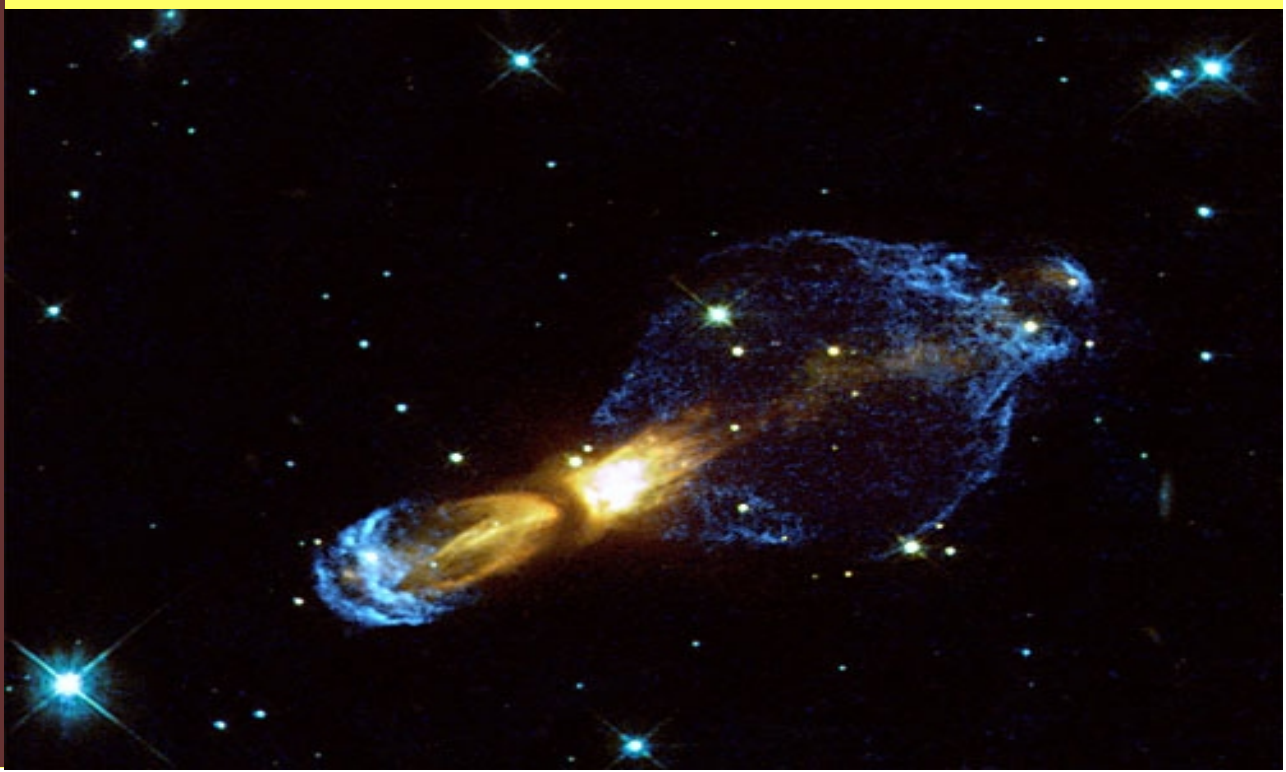Balasubramanyam
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Lecture Notes on
Engineering Physics
(As per latest VTU syllabus -2014)

LECTURE NOTES ON

ENGINEERING PHYSICS

Revised Edition 2014
As per latest VTU syllubus

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Part I

Module 1: Modern Physics and Quantum Mechanics

Chapter 1

Modern Physics

1.1   Introduction to Blackbody radiation spectrum

Emissive Power: The amount of radiation emitted per second per unit area of a surface is called emissive power of the surface. It is denoted by **E**. The unit of emissive power is Wm^{-2}

Absorptive power: It is defined as the ratio of the amount of radiation absorbed by the surface in the given time to that incident on it during the same interval of time.It is denoted by **a**. It has no unit.

Blackbody: A blackbody is one which absorbs all the radiations which is incident on it(Note that absorptive power of the blackbody is 1). Since a perfect blackbody cannot be realized in practice, a body which absorbs radiations in the visible region together with the radiation in the ultraviolet and infrared regions is considered as perfect blackbody. A good absorber of radiation is also a good emitter of radiation. Thus, when a blackbody is kept at high temperature compared to its surroundings it emits radiation in all regions.

1.2   Spectrum of blackbody radiation

Lummer and **Pringshein** studied energy distribution in the spectrum of blackbody radiation at various temperatures. The results are depicted in the graph shown. The observation of the curves reveals the following facts:

1. Energy distribution is not uniform. Energy emitted by the blackbody increases with wavelength, becomes maximum (E_m) for a particular wavelength λ_m and then de-
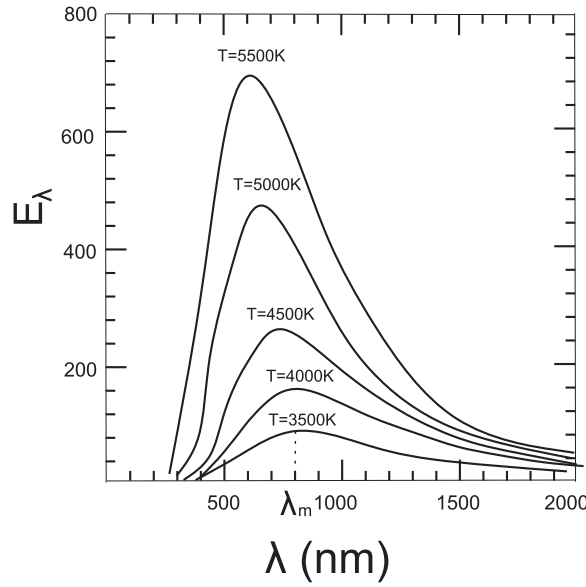
Figure 1.1: Energy distribution in blackbody radiation

creases with the wavelength.

2. Total energy emitted by the blackbody increases with the increase in temperature simultaneously the wavelength λ_m corresponding to the maximum energy E_m emitted by the blackbody decreases with the increase in temperature. This is called **Wein's displacement law**. Mathematically, $\lambda_m T$=Constant.

3. Total energy emitted by the blackbody is directly proportional to the fourth power of the absolute temperature. This is known as **Stefan's Law**. Mathematically, $E = \sigma T^4$

Where σ is called Stefan's constant which is equal to $5.67 \times 10^{-8} Wm^{-2}K^{-4}$.

1.3   Wein's distribution law

According to Wien's distribution law the energy emitted by the blackbody per unit volume in the range of wavelength from λ to $\lambda + d\lambda$ is given by

$$E_\lambda d\lambda = \frac{C_1}{\lambda^5} e^{-\frac{C_2}{\lambda T}} d\lambda \tag{1.1}$$

where C_1 and C_2 are constants and T is absolute temperature.

It was found that Wein's distribution law explains energy distribution only in the shorter wavelength region. Also, the energy emitted by the blackbody tends to zero at very high

temperature which is in contradiction with the experimental results.

1.4 Rayleigh-Jeans law

According to Rayleigh-Jeans law the energy emitted by the blackbody per unit volume in the range of wavelength from λ to $\lambda + d\lambda$ is given by

$$E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda \tag{1.2}$$

Where k is Boltzmann constant whose value is $1.38 \times 10^{-23} JK^{-1}$.

Rayleigh-Jeans formula was found to explain energy distribution only in the longer wavelength region of the blackbody radiation spectrum. Further, Rayleigh-Jeans law predicts that blackbody radiates enormous amount of energy in the shorter wavelength region so that no energy is available for emission in the longer wavelength region. But blackbody radiates mostly in the visible and infrared region. This discrepancy between experimental results and theoretical predictions was known as **'ultraviolet catastrophe'**.

1.5 Planck's Radiation Law

According to Planck's radiation law whenever radiation is emitted or absorbed by a blackbody energy exchange between matter and radiation takes place only in discrete steps, i.e emission or absorption of radiation of frequency ν takes place in steps of integrals multiples of packets of energy called quanta. Thus Energy emitted or absorbed by the body is given by

$$E = nh\nu$$

Where h = Planck's constant =6.254×10^{-34}Js and n = $1, 2, 3 \ldots$

According to Planck's radiation law the energy emitted by the blackbody per unit volume in the range of wavelength from λ to $\lambda + d\lambda$ is given by

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \, d\lambda \tag{1.3}$$

Where c = velocity of light = $3 \times 10^8 ms^{-1}$.

Planck's radiation law explains energy distribution in the spectrum of blackbody radiation over the wide range of wavelength, i.e., it is in agreement with the experimental results. Further, it can be shown that Planck's law reduces to Wein's distribution law in the shorter wavelength region while reduces to Rayleigh-Jeans law in the longer wavelength

region.

1.6   Reduction of Planck's law to Wein's law and Rayleigh-Jeans law

Planck's radiation law is given by equation (1.3).
(i) For shorter wavelengths, $e^{hc/\lambda kT}$ is very large $\Rightarrow e^{hc/\lambda kT} >> 1$ therefore

$$(e^{hc/\lambda kT} - 1) \approx e^{hc/\lambda kT} \tag{1.4}$$

Using equation (1.4) in equation (1.3) we get,

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}}} \ d\lambda \tag{1.5}$$

Substituting $8\pi hc = C_1$ and $\frac{hc}{k} = C_2$ where C_1 and C_2 are constants into equation (1.5) we get

$$E_\lambda d\lambda = C_1 \lambda^{-5} e^{-C_2/\lambda T} \ d\lambda \tag{1.6}$$

The equation (1.6) is the expression for Wein's law of radiation.

(ii) For longer wavelengths,
Expanding e^x as power series we have $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$
If x is small then its higher power terms could be neglected thus $e^x = 1 + x$ here $x = \frac{hc}{\lambda kT}$
and for longer wavelengths λ is large making x small

$$e^{hc/\lambda kT} = 1 + \frac{hc}{\lambda kT} \Rightarrow (e^{hc/\lambda kT} - 1) = \frac{hc}{\lambda kT} \tag{1.7}$$

Using equation (1.7) in (1.3) we get

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5 \left(\frac{hc}{\lambda kT}\right)} \ d\lambda = \frac{8\pi kT}{\lambda^4} \ d\lambda \tag{1.8}$$

This equation is Rayleigh-Jeans law of radiation. Thus, Wein's and Rayleigh-Jeans law turn out to be special case of Planck's raiation law.

1.7   Photoelectric effect

The phenomenon of emission of electrons by the surface of certain materials under the influence of radiation of suitable frequency is called photoelectric effect.

The materials which exhibit this property are called photosensitive while the electrons emitted are called photoelectrons.

The following observations are made during the experimental study of photoelectric effect.

1. Photoelectric effect is an instantaneous process. i.e., electrons are emitted by the surface as soon as radiation falls on the surface.

2. If frequency of the incident radiation is less than certain value, photoelectrons are not emitted however strong the intensity of the incident radiation may be. The frequency above which photoelectric effect takes place is called threshold frequency denoted by ν_o.

3. Kinetic energy of the photoelectrons emitted is directly proportional to the frequency of the incident radiation and independent of the intensity of radiation.

4. Number of photoelectrons emitted is directly proportional to the intensity of the incident radiation but independent of the frequency of the radiation.

5. For a particular value of negative potential given to anode, photoelectric current becomes zero. It is called stopping potential denoted by V_s. Kinetic energy of photoelectrons expressed in eV gives stopping potential

1.8   Einstein's explanation of photoelectric effect

Free electrons in metal are bound within the surface of the metal. Minimum energy required for the electron to escape from the surface of the metal is called Work Function and is denoted by W_0. According to Einsteins quantum theory of light, beam of light consists of stream of particles called quanta. The energy of each quanta which is also called as photon is given by $E = h\nu$, where h is Plancks constant and ν is frequency of the radiation. When light radiation of frequency $(> \nu_0)$ is made to fall on the surface of the metal, photon of the light interacts with electrons and transfers its energy completely to the electron. Since energy $h\nu$ absorbed by the electron is greater than the work function $W_0 = h\nu_0$, electron escape from the surface of the metal leading to photoelectric emission. According to the

law of conservation of energy, kinetic energy of the electrons emitted is given by

$$K.E = \frac{1}{2}m\nu^2 = h\nu - W_0 = h(\nu - \nu_0)$$

The above equation is called Einsteins photo-electric equation.

All the experimental results of photoelectric effect can be explained using above equation.

1.9   Compton Effect

The phenomenon in which the wavelength of X-rays increases when scattered from a suitable target is called Compton Effect.This kind of scattering is also called incoherent scattering.

Explanation: When x-ray photon interacts with matter, it collides with electrons and transfers its energy partially to the electron and get scattered with lesser energy. As a result frequency of the scattered x-ray is less than the incident beam and hence wavelength of the scattered beam is more than that of the incident beam. It can be shown that change in wavelength when X-ray undergo Compton scattering is given by

$$\Delta\lambda = \frac{h}{m_0c}(1 - cos\theta)$$

Where h is Plancks constant, m_0 is rest mass of the electron; c is velocity of light in free space and θ is the angle of scattering.
$\Delta\lambda$ is called Compton Shift (h/m_0c) and is called Compton wavelength.
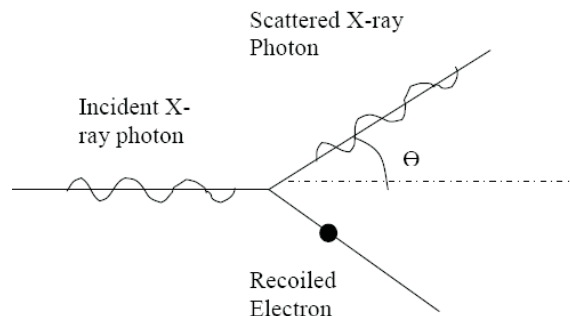
Figure 1.2: Electron scattering by X-ray

1.10   De-Broglie's hypothesis

We know that the phenomenon such as interference, diffraction, polarisation etc. can be explained only with the help of wave theory of light. While phenomenon such as photoelectric effect, Compton effect, spectrum of blackbody radiation can be explained only with the help of Quantum theory of radiation. Thus radiation is assumed to exhibit dual nature. i.e., both particle and wave nature.

In 1924, **Louis de-Broglie** made a bold hypothesis, which can be stated as follows

"If radiation which is basically a wave can exhibit particle nature under certain circumstances, and since nature likes symmetry, then entities which exhibit particle nature ordinarily, should also exhibit wave nature under suitable circumstances."

Thus according to De-Broglie's hypothesis, there is wave associated with the moving particle. Such waves are called **Matter waves** and wavelength of the wave associated with the particle is called **De-Broglie wavelength.**

1.11   Expression for De-Broglie wavelength (from analogy)

We know that radiation consists of stream of particles called Photon. Energy of each photon is given by

$$E = h\nu = hc/\lambda$$

Where h = Plancks constant, ν is the frequency and λ is wavelength of the radiation.

But according to Einsteins relativistic formula for energy of a particle

$$E^2 = p^2c^2 + m_0^2c^4$$

Where p is momentum of the particle and m_0 is rest mass of the particle.

For photon rest mass m_0 is zero. Therefore

$$E = pc = hc/\lambda$$

or

$$p = \frac{h}{\lambda}$$

is the momentum of the photon.

De-Broglie proposed that the same equation is applicable for matter waves also. Therefore, wavelength of the waves associated with the moving particle of mass **m** , moving with the velocity **v** is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

where the momentum is given by $p = mv$ and λ is the de Broglie wavelength.

De-Broglie wavelength of an electron accelerated by a potential difference of V volts:
Consider an electron accelerated by a potential difference of V volts Kinetic energy gained by the electron is given by

$$E = eV = \frac{1}{2}mv^2$$

$$v^2 = \frac{2E}{m} \quad \text{or} \quad v = \sqrt{\frac{2E}{m}}$$

$$\text{we have} \quad \lambda = \frac{h}{mv}$$

$$\therefore \quad \lambda = \frac{h}{m\sqrt{\frac{2E}{m}}} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

substituting for h, m, and e we get

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.602 \times 10^{-19} \times V}} = \frac{1.226}{\sqrt{V}} \, nm,$$

thus for $V = 100 \, volts$

$$\lambda = \frac{1.226}{\sqrt{100}} = 0.1226 \, nm$$

Note: For a particle of mass *M* and charge *q* accelerated through a potential difference of **V** volts, expression for De-Broglie wavelength is given by

$$\therefore \quad \lambda = \frac{h}{\sqrt{2MqV}}$$

1.12   Experimental verification of De-Broglie's hypothesis (Davisson and Germer experiment

The apparatus used by Davisson and Germer is shown in the Fig. 1.12. Apparatus consists of an electron gun E, which produces a narrow and collimated beam of electrons accelerated to known potential V. This narrow beam of electron is made to incident normally on a Nickel crystal C mounted on turntable T. Electrons get scattered from the crystal in all directions. The turntable T can be rotated about an axis perpendicular to both incident beam and scat-

tered beam. The scattered electron beam is received by an ionization chamber I mounted on an arm R capable of rotating about an axis same as that of turntable T. The angle of rotation can be measured with a help of circular scale S. Electrons entering the ionization chamber produces a small current, which can be measured with the help of galvanometer G connected to the ionization chamber. Keeping the accelerating potential constant at E,
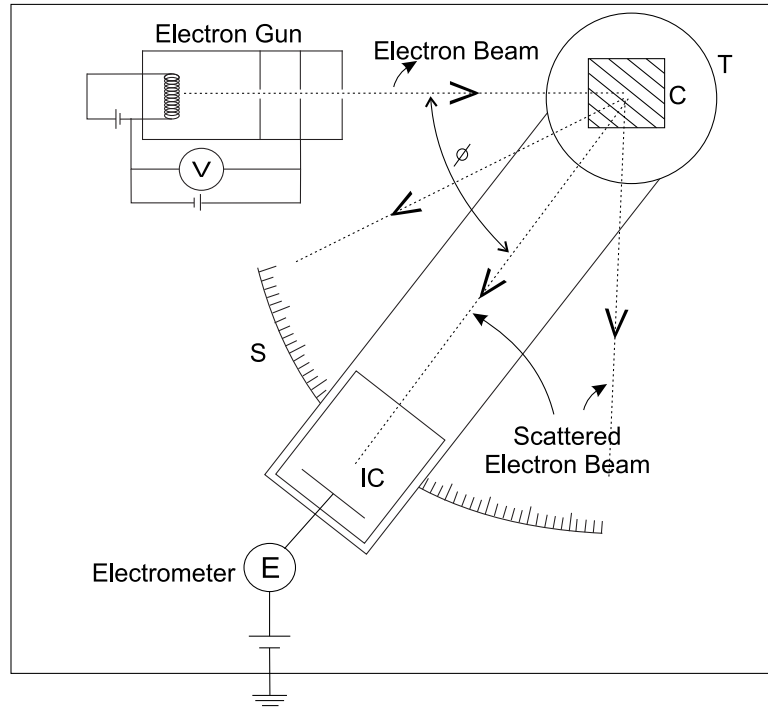
Figure 1.3: Davisson and Germer's experimental arrangement

electron beam is made to strike the nickel crystal C. Electrons scattered from the crystal is collected by the ionization chamber at various scattering angles ϕ and the corresponding value of ionization current I is noted. Experiment is repeated for various accelerating potentials V. A polar graph of representing ϕ and I for different values of V is plotted. The graphs obtained are shown in the fig 1.4.

When accelerating potential was $40\,V$, a smooth curve was obtained. When accelerating potential was increased to $44\,V$ a small bump appeared in the curve. Bump became more and more pronounced as the voltage was increased, to reach maximum for the accelerating potential of $54\,V$. Beyond $54\,V$, bump started diminishing and vanished after $68\,V$. The scattering angle corresponding the accelerating voltage $V = 54\,V$ when bump became maximum was found to be $\phi = 50^0$.

Davisson and Germer interpreted the result as follows: Electrons in the incident beam behave like waves. Thus when electrons strike the crystal they undergo Bragg's diffraction
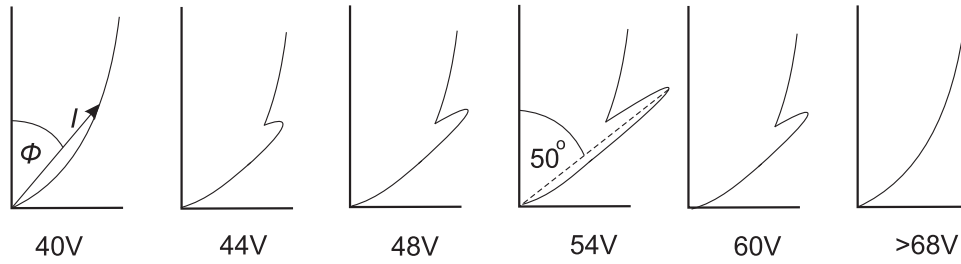
Figure 1.4: Plot for various acceleration voltages V

from the different planes of the crystal. The bump in the curve corresponds to constructive interference caused by the scattered electrons. According to Bragg's law the condition for constructive interference is given by

$$2dSin\theta = n\lambda$$

where d = Interplanar spacing for the crystal, θ = glancing angle made by incident beam with the crystal plane, n=order and λ is wavelength of the wave. Thus, when bump in the curve is maximum, $\theta = 65^0$ (see fig 1.5), n=1 and for nickel crystal **d = 0.091 nm**. Thus, wavelength of the waves associated with the electrons is given by
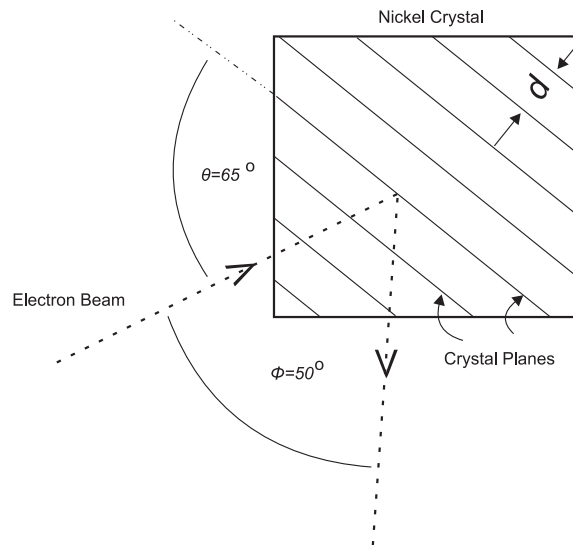
Figure 1.5: Angle between crystal plane and incident electrons

$$\lambda = \frac{2dsin\theta}{n} = \frac{2 \times 0.091 \times 10^{-9} \times sin65^0}{1}$$

$$\lambda = 1.65 \times 10^{-10}\,m$$

According to De-Broglie's hypothesis wavelength of the wave associated with the electrons accelerated by a potential difference of V volts is given by

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Since bump becomes maximum for the accelerating voltage of $54\,V$,

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 54}} = 1.66 \times 10^{-10}\,m$$

Thus there is an excellent agreement between experimental results and theoretical prediction. Thus Davisson-Germer experiment provides direct experimental evidence for the existence of matter waves.

1.13   Phase velocity and Group velocity

Phase velocity: The velocity with which a wave travels is called Phase velocity or wave velocity. It is denoted by v_p. It is given by

$$v_p = \frac{\omega}{\kappa}$$

$$\text{where } \omega = 2\pi\nu \text{ and } k = \frac{2\pi}{\lambda}$$

$$\text{thus } v_p = \nu\lambda$$

$$\text{or } v_p = \frac{h\nu}{h/\lambda} = \frac{E}{p} = \frac{mc^2}{mv}$$

$$\therefore \; v_p = \frac{c^2}{v}$$

Where c = velocity of light and v = is velocity of the particle.

The above equation gives the relationship between the phase velocity and particle velocity.

It is clear from the above equation that, Phase velocity is not only greater than the velocity of the particle but also greater than the velocity of light, which can never happen. Therefore phase velocity has no physical meaning in case of matter waves. Thus concept of group velocity was introduced.

Group velocity: Matter wave is regarded as the resultant of the superposition of large number of component waves all traveling with different velocities. The resultant is in the

form of a packet called wave packet or wave group. The velocity with which this wave group travels is called **group velocity.** The group velocity is represented by v_g.
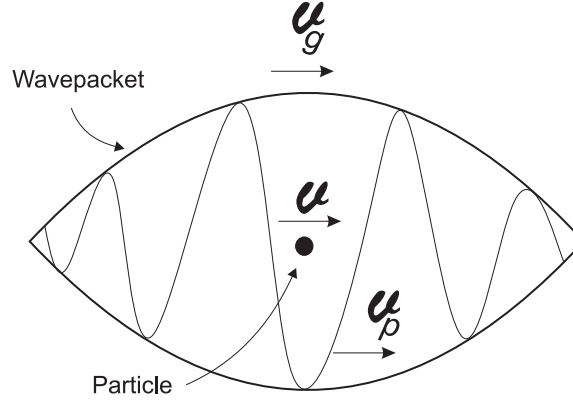
Figure 1.6: Wavepacket representation

1.14   Expression for group velocity

Consider two waves having same amplitude **A**, but of slightly different wavelengths and frequencies traveling in the same direction. The two waves are represented by

$$y_1 = Asin(\omega_1 t - \kappa_1 x) \tag{1.9}$$

$$y_2 = Asin(\omega_2 t - \kappa_2 x) \tag{1.10}$$

When these two waves superimpose upon one another, the resultant of the two waves is given by

$$y = y_1 + y_2 = Asin(\omega_1 t - \kappa_1 x) + Asin(\omega_2 t - \kappa_2 x)$$

$$\text{we have } sinA + sinB = 2cos\left(\frac{A-B}{2}\right) sin\left(\frac{A+B}{2}\right)$$

$$\therefore y = 2Acos\left[\frac{(\omega_1 - \omega_2)}{2}t - \frac{(\kappa_1 - \kappa_2)}{2}x\right] \times sin\left[\frac{(\omega_1 + \omega_2)}{2}t - \frac{(\kappa_1 + \kappa_2)}{2}x\right]$$

$$\therefore y = 2Acos\left(\frac{\Delta\omega}{2}t - \frac{\Delta\kappa}{2}x\right) sin(\omega t - \kappa x)$$

$$\text{since } \omega_1 \cong \omega_2, \omega_1 - \omega_2 = \Delta\omega \text{ and } \frac{\omega_1 + \omega_2}{2} = \omega$$

$$\text{similarly } \kappa_1 - \kappa_2 = \Delta\kappa \text{ and } \frac{\kappa_1 + \kappa_2}{2} = \kappa$$

Therefore

$$\therefore y = A' \, sin(\omega t - \kappa x)$$

The above equation represents a wave traveling with the velocity ω/κ. But amplitude of the wave is not constant which is given by

$$A' = 2A cos \left(\frac{\Delta\omega}{2}t - \frac{\Delta\kappa}{2}x \right)$$

The velocity with which variation in the amplitude is transmitted represents the group velocity, and is given by

Individual waves   Resultant wave
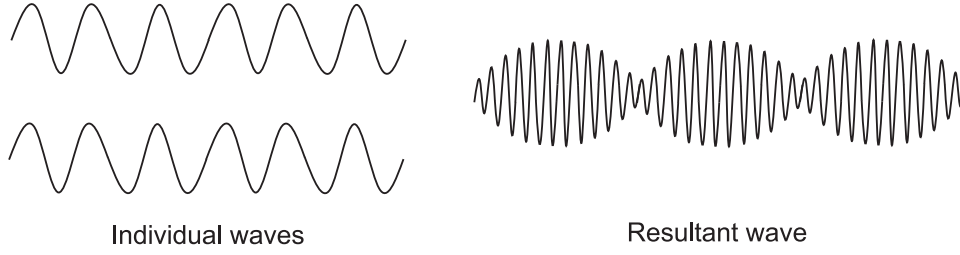
Figure 1.7: Super position of Waves

$$v_g = \frac{\Delta\omega/2}{\Delta\kappa/2} = \frac{\Delta\omega}{\Delta\kappa}$$

$$\text{in the limit} \Delta\kappa \longrightarrow 0$$

$$\left(\frac{\Delta\omega}{\Delta\kappa} \right) \rightarrow \left(\frac{d\omega}{d\kappa} \right)$$

$$\therefore \; v_g = \frac{d\omega}{d\kappa}$$

1.15   Relation between Group velocity and phase velocity

We know that Phase velocity is given by

$$v_p = \frac{\omega}{k} \quad \therefore \omega = v_p k$$

Group velocity is given by

$$v_g = \frac{d\omega}{dk} = v_p + k\frac{dv_p}{dk}$$

$$v_g = v_p + k\frac{dv_p}{d\lambda} \times \frac{d\lambda}{dk} \tag{1.11}$$

$$\text{but } k = \frac{2\pi}{\lambda} \quad \therefore \frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2}$$

$$\therefore \frac{d\lambda}{dk} = -\frac{\lambda^2}{2\pi}$$

Therefore equation (1.11) becomes

$$v_g = v_p + \frac{2\pi}{\lambda} \times \frac{dv_p}{d\lambda} \times \left(-\frac{\lambda^2}{2\pi}\right)$$

$$\therefore v_g = v_p - \lambda \frac{dv_p}{d\lambda} \tag{1.12}$$

Equation (1.12) gives the relation between phase velocity and group velocity.

1.16   Relation between group velocity and particle velocity

We know that group velocity is given by

$$v_g = \frac{d\omega}{d\kappa}$$

$$\text{but } \omega = 2\pi\nu = \frac{2\pi E}{h}$$

$$\therefore d\omega = \frac{2\pi dE}{h}$$

$$\text{and } k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$$

$$\therefore dk = \frac{2\pi dp}{h}$$

$$\therefore v_g = \frac{\frac{2\pi dE}{h}}{\frac{2\pi dp}{h}} = \frac{dE}{dp}$$

$$\text{but } E = \frac{p^2}{2m} \quad \therefore \frac{dE}{dp} = \frac{2p}{2m} = \frac{p}{m}$$

$$\therefore v_g = \frac{dE}{dp} = \frac{p}{m} = \frac{mv}{m} = v = \text{particle velocity}$$

Therefore group velocity = particle velocity

1.17 Expression for de-Broglie wavelength(From Group velocity)

We know that group velocity is given by

$$v_g = \frac{d\omega}{d\kappa}$$

$$\text{but } \omega = 2\pi\nu \quad \therefore \ d\omega = 2\pi \, d\nu$$

$$\text{and } \ k = \frac{2\pi}{\lambda} \therefore \ dk = 2\pi \, d\left(\frac{1}{\lambda}\right)$$

$$\therefore \quad v_g = \frac{d\nu}{d\left(\frac{1}{\lambda}\right)}$$

$$d\left(\frac{1}{\lambda}\right) = \frac{d\nu}{v_g} = \frac{d\nu}{v} \quad (\because v = v_g)$$

If m is the mass, v is the velocity and V is the potential energy of the particle, then total energy of the particle is given by

$$E = \frac{1}{2}mv^2 + V$$

$$\text{according to quantum theory } E = h\nu$$

$$\therefore \ h\nu = \frac{1}{2}mv^2 + V$$

$$\therefore \ \text{h} \, d\nu = \text{m} \, \text{v} \, d\text{v} + 0 \quad \because V = \text{Constant}$$

$$\therefore \ \frac{d\nu}{v} = \frac{m}{h}dv$$

$$\therefore \ d\left(\frac{1}{\lambda}\right) = \frac{m}{h}dv$$

$$\text{Integrating we get}$$

$$\frac{1}{\lambda} = \frac{\text{m}}{\text{h}}\text{v} + \text{CONSTANT}$$

$$\text{Taking constant as zero we get}$$

$$\frac{1}{\lambda} = \frac{m}{h}\nu$$

$$\frac{1}{\lambda} = \frac{m\nu}{h} \ or$$

$$\lambda = \frac{h}{m\nu} = \frac{h}{p}$$

This equation is the de-Broglie wave equation derived from group velocity.

1.18   Properties of Matter waves

Following are some of the important properties of matter waves:

1. Matter waves are waves associated with the moving particle

2. They are not electro magnetic in nature

3. Wavelength of the matter wave is given by $\lambda = \dfrac{h}{mv} = \dfrac{h}{p}$

4. The amplitude of the matter wave at the given point determines the probability of finding the particle at that point at the given instant of time.

5. There is no meaning for Phase velocity in case of matter waves. Only group velocity has meaning.

* *

Multiple Choice Questions

1. In photo-electric effect, the electrons are not emitted by photosensitive material unless

   (a) velocity of incident light exceeds a certain minimum velocity.

   (b) the frequency of incident light exceeds a certain minimum frequency.

   (c) the wave length of incident light exceeds a certain minimum wavelength.

2. What happens when the light intensity incident on a photoelectric surface is doubled?

   (a) The frequency of emitted photons is doubled.

   (b) The number of photons is doubled.

   (c) The number of photon becomes four times.

   (d) There is no effect at all.

3. Choose the incorrect statement:

   (a) The number of photoelectrons emitted is proportional to the intensity of light.

   (b) The velocity of photoelectrons is directly proportional to the wavelength of light.

   (c) The velocity of photoelectrons is directly proportional to the frequency of light.

   (d) None of these.

4. The phenomenon of ejection of electrons from the surface of metals, when light of a suitable wavelength fall on it, is known as

(a) electrolysis.   (b) photoelectric effect.   (c) ionization.   (d) radioactivity.

5. The idea of quantum nature of light has emerged in an attempt to explain

   (a) the thermal radiation of black body.

   (b) radioactivity.

   (c) fusion.

   (d) interference of light.

6. Photo electric cell is a device which converts

   (a) light energy in to electric energy.

   (b) chemical energy in to electric energy.

   (c) electric energy in to light energy.

   (d) magnetic energy in to electric energy.

7. The threshold wavelength for photoelectric emission from a material is 5200 Å photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a

   (a) 50 watt infrared lamp.

   (b) 1 watt infrared lamp.

   (c) 50 watt ultraviolet lamp.

   (d) 1 watt ultraviolet lamp.

8. The energy of a photon corresponding to the visible light of maximum wavelength is approximately

(a) $1\,\text{eV}$ (b) $1.6\,\text{eV}$ (c) $3.2\,\text{eV}$ (d) $7\,\text{eV}$

9. The work function of the metal is ϕ and the frequency of incident light is ν. There is no emission of photoelectrons when

   (a) $\nu < \phi/h$ (b) $\nu = \phi/h$ (c) $\nu > \phi/h$ (d) $\nu \geq \phi/h$

10. Light of two different frequencies, whose photons have energies 1 and 2.5 eV respectively, successively illuminated a metal whose work function is 0.5 eV. The ratio of the maximum speeds of the emitted electrons will be

    (a) $1 : 5$ (b) $1 : 4$
    (c) $1 : 2$ (d) $1 : 1$

11. The work function of photoelectric material is 3.3 eV. The threshold frequencies will be equal to

    (a) 8×10^{14} Hz   (b) 8×10^{10} Hz
    (c) 5×10^{20} Hz   (d) 4×10^{14} Hz

12. If the energy of a photon corresponding to a wavelength 6000Å is 3.32×10^{-19} Joule, the photon energy for a wavelength of 4000Å will be

    (a) 1.11×10^{-19} Joule

    (b) 2.22×10^{-19} Joule

    (c) 4.44×10^{-19} Joule

    (d) 4.98×10^{-19} Joule

13. Monochromatic light incident on a metal surface emits electron with K.E from zero to 2.6 eV. What is least incident energy if the tightly bound electron needs 4.2 eV to remove

    (a) 1.6 eV

    (b) from 1.6 eV to 6.8 eV

    (c) 6.8 eV

    (d) > 6.8 eV

14. The work function of a photo-electric material is 3.33 eV. The threshold frequency will be equal to

(a) 8×10^{14}Hz

(b) 5×10^{36}Hz

(c) 8×10^{10}Hz

(d) 4×10^{11}Hz

15. The photo-electric threshold of the photo-effect for a certain metal is 2750Å . The minimum energy of a photon producing the photo-effect is

   (a) 0.45 eV

   (b) 4.5 eV

   (c) 0.04 eV

   (d) 0.0045 eV

16. A foodball player kicked the ball into the goal post. The foodball did not behave as a wave to undergo diffraction because the de-Broglie wavelength corresponding to the football is

   (a) infinite.

   (b) much greater than the dimension of the post.

   (c) of the same order as the dimension of the goal post.

   (d) much less than the dimension of the goal post.

17. When photoelectrons produce by light in films are fed the loudspeaker, the loudness of sound depends upon

   (a) the mass of the electrons.

   (b) the number of electrons released.

   (c) the wavelength of the original light.

   (d) the charge of the electrons.

18. Light of two different frequencies whose photons have energies 0.5ev and 4.25eV respectively, successively illuminated a metal whose work function is 0.25eV. The ratio of the maximum speed of the emitted electrons will be

$$(a)\, 1:2 \quad (b)\, 1:4 \quad (c)\, 1:16 \quad (d)\, 1:1$$

19. In Compton scattering

    (a) whole of the incident photon is absorbed.

    (b) scattered photon has more energy than incident photon.

    (c) whole of the atom recoils.

    (d) change in the photon wavelength is independent of original wavelength of the photon.

20. If the surface temperature of the star is 10000 K and Wein's constant is $2.898 \times 10^{-3} mK$ then the wavelength corresponding to the maximum energy is

$$\text{(a) } 2898\text{Å} \quad \text{(b) } 2.898\text{Å} \quad \text{(c) } 2898 \times 10^{-7}\text{m} \quad \text{(d) } 1.449\text{Å}$$

21. When an incandescent bulb is switched on, the filament gets heated to $2000K$ then the energy emitted per unit area per second by the bulb is(Stefan's constant is $5.67 \times 10^{-8} Jm^{-2}s^{-1}$)

$$\text{(a) } 9.072 \times 10^5 \text{Jm}^{-2}\text{s}^{-1}\text{K}^4 \quad \text{(b) } 9.072 \times 10^5 \text{Jm}^{-2}\text{s}^{-1}$$
$$\text{(c) } 1.134 \times 10^{-4} \text{Jm}^{-2}\text{s}^{-1} \quad \text{(d) } 907.2 \text{Jm}^{-2}\text{s}^{-1}\text{K}^4$$

Descriptive Questions

1. State De-Broglie hypothesis and the conceptual reasoning on which it is based.

2. Define phase velocity and group velocity.Derive expression for phase velocity and group velocity on the basis of superposition of waves.

3. What are matter waves? Show that the electron accelerated by a potential difference V volts is $\lambda = 1.226/\sqrt{V}$ nm for non-relativistic case.

4. Explain the duality of matter waves from the inferences drawn from photoelectric effect and Davission -Germer effect.

5. Explain the energy distribution in the spectrum of black body. Give an account of the attempts made through various laws to explain the spectrum.

6. What is Plank's radiation law? Show how Wien's law and Rayleigh-Jeans law can be derived from it.

Exercise

1. Calculate the kinetic energy of an electron whose de-Broglie wavelength is equal to that of a $10keV$ photon.

2. Calculate the de-Broglie wavelength of an electron accelerated through a potential of 120 Volts.

3. Calculate the de-Broglie wavelength of a 0.3 kg cricket ball with a speed of $120km/hr$.

4. A particle of mass $0.5MeV/C^2$ has kinetic energy $100eV$. Find its de-Broglie wavelength, where C is the velocity of light.

5. Calculate the de-Broglie wavelength associated with 400gm cricket ball with a speed of 90 Km/hr.

6. A particle of mass $0.65MeV/C^2$ has a kinetic energy $80eV$. Calculate the de-Broglie wavelength, group velocity and phase velocity of the de-Broglie wave.

7. Compare the energy of a photon with that of an electron when both are associated with length of 0.2 nm.

Lois de-Broglie

Figure 1.8: Nobel laureate in 1929
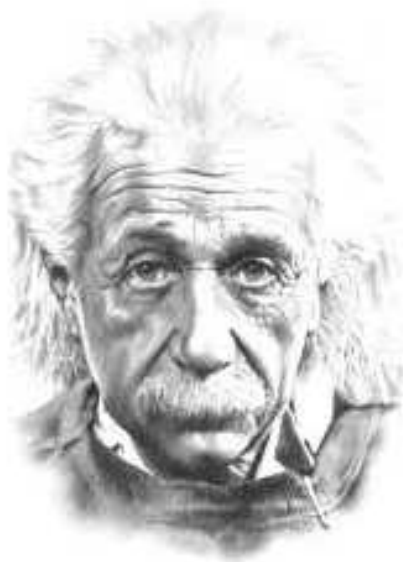
Albert Einstein

Figure 1.9: Nobel laureate in 1921

Chapter 2

Quantum Mechanics

2.1 Heisenberg's uncertainty Principle and its Physical significance

Statement: "It is impossible to determine both position and momentum of a particle simultaneously and accurately. The product of uncertainities involved in the determination of position and momentum simultaneously is greater or equal to $\frac{h}{4\pi}$".

Explanation: If we try to measure both position and momentum of a particle accurately and simultaneously, there will be always some error involved in the measurement. If we try to measure position accurately, error involved in the measurement of momentum increases and vice versa. If Δx is error (uncertainty) involved in the measurement of position and Δp_x is the error involved in the measurement of momentum then according to Heisenbergs Uncertainty Principle.

$$\Delta x \times \Delta p_x = \frac{h}{4\pi}$$

Significance of Heisenberg's uncertainty Principle:
We know that according to classical mechanics, if we know the position and momentum of a particle at some instant of time, its position and momentum at any later instant of time can be determined accurately. But according to Heisenberg's uncertainty Principle, if we try to measure position accurately, error in the measurement of momentum increases and vise-versa. Thus in quantum mechanics there is no place for the word exactness, and is replaced by the word probability. Heisenberg's uncertainty Principle implies that an event, which is impossible to occur according to classical physics, has finite probability of occurrence according to quantum mechanics.

2.2   Application of Uncertainty Principle

Non existing of electron in the nucleus

We know that the diameter of the nucleus is of the order of $10^{-14}m$. If an electron is to exist inside nucleus, then uncertainty in its position Δx must not exceed the size of the nucleus,

$$ie \ \ \Delta x \leq 10^{-14} m.$$

Then its uncertainty in the momentum is given by

$$\Delta p_x \geq \frac{h}{4\pi \Delta x},$$

$$ie., \ \ \Delta p_x \geq \frac{6.625 \times 10^{-34}}{4 \times 3.14 \times 10^{-14}},$$

$$ie., \ \ \Delta p_x \geq 5.275 \times 10^{-21} \, Ns,$$

Thus the momentum of the electron must be at least equal to the above value.

Then minimum energy that electron should have in order to exist inside the nucleus can be calculated using the formula,

$$E^2 = p^2 c^2 + m_0^2 c^4$$

Where p=momentum of the electron, c=velocity of light and m_0=rest mass of the electron. Substituting the values, we get

$$E \geq 1.585 \times 10^{-12} \, J$$

$$\text{or} \ \ E \geq 10 \, \text{MeV}$$

Explanation of β-decay So, in order to exist inside the nucleus, electron should posses energy greater or equal to $10\,MeV$. But experimentally determined values of energy of the electrons emitted by nucleus during the beta-decay were not greater than $4\,MeV$. This clearly indicates that electrons cannot exist inside the nucleus.

2.3   Time independent Schrodinger wave equation

Consider a particle of mass m, moving with the velocity v along the +ve x-axis. Then according to de-Broglie's hypothesis, wavelength of the wave associated with the particle is

given by

$$\lambda = \frac{h}{mv} \tag{2.1}$$

A wave traveling along x-axis can be represented by the equation

$$\Psi(x,t) = A\,e^{-i(\omega t - \kappa x)} \tag{2.2}$$

Where $\Psi(x,t)$ is called wave function. The equation (2.2) can be written as

$$\Psi(x,t) = \Psi(x)A\,e^{-i\omega t} \tag{2.3}$$

$$where\ \Psi(x) = e^{i\kappa x}$$

Differentiating equation (2.3) w.r.t x twice we get.

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = \frac{d^2 \Psi(x)}{dx^2}A\,e^{-i\omega t} \tag{2.4}$$

Differentiating equation (2.3) w.r.t t twice we get.

$$\frac{\partial^2 \Psi(x,t)}{\partial t^2} = -\omega^2 \Psi A\,e^{-i\omega t} \tag{2.5}$$

General wave equation is given by

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$$

From equation (2.4) and (2.5) we get

$$\begin{aligned}
\frac{d^2 \Psi}{dx^2}A\,e^{-i\omega t} &= -\frac{\omega^2}{v^2}\Psi A\,e^{-i\omega t} \\
\frac{d^2 \Psi}{dx^2} &= \frac{-\omega^2}{v^2}\Psi \ (\because \omega = 2\pi\nu \ \text{and}\ v = \nu\lambda) \\
\frac{d^2 \Psi}{dx^2} &= -\frac{4\pi^2\nu^2}{\nu^2\lambda^2}\Psi \\
\Rightarrow \frac{d^2 \Psi}{dx^2} + \frac{4\pi^2}{\lambda^2}\Psi &= 0 \tag{2.6}
\end{aligned}$$

The total energy of the particle is given by

$$E = \frac{1}{2}mv^2 + V$$

where V is the potential energy

$$
\begin{aligned}
mv^2 &= 2(E - V) \\
(mv)^2 &= 2m(E - V)
\end{aligned}
$$

substituting for $(m\nu)$ from equation (2.1) we get

$$
\begin{aligned}
\frac{h^2}{\lambda^2} &= 2m(E - V) \\
\frac{1}{\lambda^2} &= \frac{2m(E - V)}{h^2}
\end{aligned}
$$

substituting in equation (2.6) we get

$$
\begin{aligned}
\frac{d^2\Psi}{dx^2} + 4\pi^2 \frac{2m(E - V)}{h^2}\Psi &= 0 \\
\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\Psi &= 0
\end{aligned}
$$

The above equation is called one-dimensional Schrodinger's wave equation. In three dimension the Schrodinger wave equation becomes

$$
\frac{d^2\Psi}{dx^2} + \frac{d^2\Psi}{dy^2} + \frac{d^2\Psi}{dz^2} + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0
$$

$$
\nabla^2\Psi + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0
$$

2.4   Interpretation of wave function and its properties

The state of a quantum mechanical system can be completely understood with the help of the wavefunction Ψ. But wave function Ψ can be real or imaginary. Therefore no meaning can be assigned to wavefunction Ψ as it is. According to Max Born's interpretation of the wavefunction, the only quantity that has some meaning is $\mid \Psi \mid^2$, which is called **probability density**. Thus if Ψ is the wavefunction of a particle within a small region of volume $\mathbf{dv}$, then $\mid \Psi \mid^2 \mathbf{dv}$ gives the probability of finding the particle within the region $\mathbf{dv}$ at the given instant of time.

We know that electron is definitely found somewhere in the space

$$
\oint \mid \Psi \mid^2 dv = 1
$$

The wave function Ψ, which satisfies the above condition, is called normalized wave func-

tion.

Nature of Eigenvalues and Eigenfunctions (Properties of wave function)

A physical system can be completely described with the help of the wave function Ψ. In order to get wavefunction, first we have to set up a Schrodinger wave equation representing the system. Then Schrodinger wave equation has to be solved to get wavefunction Ψ as a solution. But Schrodinger wave equation, which is a second order differential equation, has multiple solutions. All solutions may not represent the physical system under consideration. *Those wavefunction which represent the physical system under consideration are acceptable and are called Eigenfunction.*

A wavefunction Ψ can be acceptable as wavefunction if it satisfies the following conditions.

1. Ψ should be single valued and finite everywhere.

2. Ψ and its first derivative with respect to its variables are continuous everywhere.

The solution of the Schrodinger wave equation gives the wavefunction Ψ. With the knowledge of Ψ we can determine the Energy of the given system. Since all wavefunctions are not acceptable, all the values of energies are not acceptable. Only those values of energy corresponding to the Eigenfunctions which are acceptable are called **Eigenvalues**.

2.5 Motion of an electron in an one dimensional potential well of infinite height (Particle in a box)

Consider an electron of mass m, moving along positive x-axis between two walls of infinite height, one located at $x = 0$ and another at $x = a$. Let potential energy of the electron is assumed to be zero in the region in-between the two walls and infinity in the region beyond the walls.

$$V = 0 \ \text{for} \ 0 \leq x \leq a$$

$$V = \infty \ \ \text{for} \ x < 0 \ \text{and} \ x > a$$

Region beyond the walls:

The Schrodinger wave equation representing the motion of the particle in the region beyond the two walls is given by

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - \infty)\Psi = 0 \tag{2.7}$$

The only possible solution for the above equation is $\Psi = 0$.

If $\Psi = 0$, the probability of finding the particle in the region $\mathbf{x = 0}$ and $\mathbf{x > a}$ is zero. i.e., particle cannot be found in the region beyond the walls.

Region between two walls:
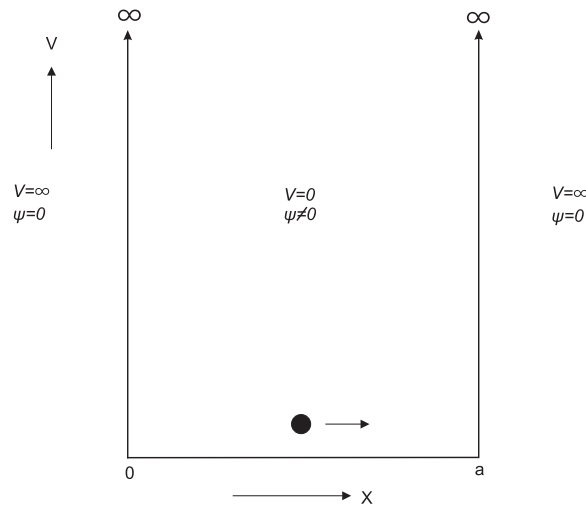
Figure 2.1: Particle in a infinite potential well

The Schrodinger's wave equation representing the motion of the particle in the region between the two walls is given by

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - 0)\Psi = 0 \quad (\because V = 0)$$

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 mE}{h^2}\Psi = 0$$

$$\frac{d^2\Psi}{dx^2} + \alpha^2\Psi = 0 \tag{2.8}$$

$$\text{where } \alpha^2 = \frac{8\pi^2 mE}{h^2} \tag{2.9}$$

Solution of the equation (2.8) is of the form

$$\Psi = A\sin\alpha x + B\cos\alpha x \tag{2.10}$$

Where A and B are unknown constants to be determined. Since particle cannot be found inside the walls

$$\text{at } x = 0, \ \Psi = 0 \qquad\qquad I$$

$$\text{and } x = a, \ \Psi = 0 \qquad\qquad II$$

The equations are called boundary conditions. Using the I boundary condition in equation (2.10), we get

$$0 = A \sin 0 + B \cos 0$$

$$\therefore B = 0$$

Therefore equation (2.10) becomes

$$\Psi = A \sin \alpha x \tag{2.11}$$

Using the boundary condition II in equation (2.11) we get

$$
\begin{aligned}
0 &= A \sin \alpha a \\
\sin \alpha a &= 0 \, (\because A \neq 0) \\
\therefore \alpha a &= n\pi = \sin^{-1} 0 \\
\text{where } n &= 0, 1, 2, 3 \ldots \\
\therefore \alpha &= \frac{n\pi}{a}
\end{aligned}
$$

Therefore correct solution of the equation (2.8) can be written as

$$\Psi_n = A \sin \frac{n\pi}{a} x , \tag{2.12}$$

The above equation represents Eigenfunction, where n=1, 2, 3 ...
(n=0 is not acceptable because, for n=0 the wavefunction Ψ becomes zero for all values of x. Then partial cannot be found anywhere)
Substituting for α in equation (2.9) we get

$$\left(\frac{n\pi}{a}\right)^2 = \frac{8\pi^2 mE}{h^2}$$

Therefore energy Eigenvalues are represented by the equation

$$E_n = \frac{n^2 h^2}{8ma^2} \tag{2.13}$$

Where $n = 1, 2, 3 \ldots$
It is clear from the above equation that particle can have only discrete values of energies. The lowest energy that particle can have corresponds to $n = 1$ and is called zero-point energy denoted by E_0. It is given by

$$E_0 = \frac{h^2}{8ma^2} \tag{2.14}$$

It is also called groud state energy. Therefore expression for energy can be written as

$$E_n = n^2 E_0 \qquad (2.15)$$

When $n = 2$ electron is said to be in first excited state, when $n = 3$ electron is said to be in second excited state, and so on.

Normalization of wavefunction:

We know that particle is definitely found somewhere in space between the walls at $x = 0$ to a

$$\int_0^a |\Psi|^2 \, dx = 1$$

$$\int_0^a A^2 \sin^2 \frac{n\pi}{a} x dx = 1$$

$$A^2 \int_0^a \frac{1}{2} \left[1 - \cos \frac{2n\pi}{a} \right] dx = 1$$

$$\frac{A^2}{2} \left[x - \frac{a}{2n\pi} \sin \frac{2n\pi}{a} x \right]_0^a = 1$$

$$\frac{A^2}{2} [(a - 0) - (0 - 0)] = 1$$

$$A^2 = \frac{2}{a} \quad \text{or} \quad A = \sqrt{\frac{2}{a}}$$

Therefore Normalised wavefunction is given by

$$\Psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x$$

Wavefunction, Probability density and energy of the particle for different energy levels:

For n=1 energy is given by $E_n = \frac{h^2}{8ma^2} = K$

Wavefunction and probability density for different values of x is given by

x	$\Psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x$	$p = |\Psi_n|^2 = \frac{2}{a} \sin^2 \frac{n\pi}{a} x$
0	0	0
$\frac{a}{2}$	$\sqrt{\frac{2}{a}}$	$\frac{2}{a}$
a	0	0

similarly for n=2

$$E_n = \frac{4h^2}{8ma^2} = 4K$$

x	$\Psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x$	$p = \mid \Psi_n \mid^2 = \frac{2}{a} \sin^2 \frac{n\pi}{a} x$
0	0	0
$\frac{a}{4}$	$\sqrt{\frac{2}{a}}$	$\frac{2}{a}$
$\frac{a}{2}$	0	0
$\frac{3a}{4}$	$\sqrt{\frac{2}{a}}$	$\frac{2}{a}$
a	0	0

Wavefunctions, probability density and energies are as shown in the figure.
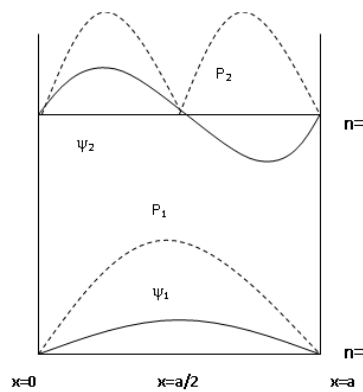
Figure 2.2: Graphical Representation of Wavefunctions and probability density at different energies

2.6 Energy eigenvalues of a Free particle

Consider a particle of mass m moving along positive x-axis. Particle is said to be free if it is not under the influence of any field or force. Therefore for a free particle potential energy can be considered to be constant or zero. The Schrodinger wave equation for a free particle

is given by.

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - 0)\Psi = 0$$

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 mE}{h^2}\Psi = 0$$

$$\frac{d^2\Psi}{dx^2} + \alpha^2\Psi = 0 \tag{2.14}$$

$$where \ \alpha^2 = \frac{8\pi^2 mE}{h^2} \tag{2.15}$$

The solution of the equation (2.14) is of the form

$$\Psi = A\sin\alpha x + B\cos\alpha x \tag{2.16}$$

where A and B are unknown constants to be determined. Since there are no boundary conditions A, B and α can have any values.

Energy of a particle is given by

$$E = \frac{\alpha^2 h^2}{8\pi^2 m} \tag{2.17}$$

Since there is no restriction on α there is no restriction on E.

Therefore energy of the free particle is not quantized. i.e., free particle can have any value of energy.

* *

Multiple Choice Questions

1. Which of the following is the correct statement:

    (a) only charged particles in motion are accompanied by matter waves

    (b) any particles in motion whether charged or uncharged is accompanied by matter waves

    (c) no particles whether at rest or in motion is ever accompanied by matter waves

    (d) only subatomic particles in motion are accompanied by matter waves

2. Davision and Germer experiments relates to

   (a) interference   (b) polarization   (c) electron diffraction   (d) phosphoresce

3. Which of the following statement about the energy in a quantum is true

   (a) varies directly with frequency.

   (b) varies inversely with frequency.

   (c) some for all frequency.

   (d) none of the above.

4. The wavelength λ associated with a particle of mass m moving with velocity v is given by

   (a) $\lambda = h/mv$ (b) $\lambda = mv/h$ (c) $\lambda = hv/m$ (d) $\lambda = m/hv$

5. The product of uncertainties between position and momentum is given by

   (a) $\Delta x \Delta p = \lambda$ (b) $\Delta x \Delta p = h$ (c) $\Delta x \Delta p = nh$ (d) $\Delta x \Delta p = mv$

6. The equation of motion of matter wave was derived by

   (a) Heisenberg   (b) Bohr   (c) deBroglie   (d) Schroedinger

7. The de Broglie wave associated with a moving particle is generally

   (a) a finite monochromatic wave train traveling with a velocity less than that of light

   (b) an infinite monochromatic wave train having a phase velocity greater than that of light

   (c) a wave packet having a group velocity equal to that of the moving particle

8. We do not observe matter waves in heavy particle because

   (a) the wavelength associated is very large

   (b) the wavelength associated is very small

   (c) they travel with lesser velocity

   (d) none of these

9. If E is the kinetic energy of the material partial of mass m, then the deBroglie wavelength is given by

$$\text{(a)}\, h/\sqrt{2mE} \quad \text{(b)}\, \sqrt{2mE/h} \quad \text{(c)}\, h\sqrt{2mE} \quad \text{(d)}\, h/2mE$$

10. When an electron is accelerated by a potential V volts, the de Broglie wavelength is given by

$$\text{(a)}\, 12.26/\sqrt{V}\,\text{nm} \quad \text{(b)}\, 26.12/\sqrt{V}\,\text{Å} \quad \text{(c)}\, 26.26/\sqrt{V}\,\mu\text{m} \quad \text{(d)}\, 12.26/\sqrt{V}\,\text{Å}$$

11. Velocity of matter wave is always

    (a) lesser than velocity of light   (b) equal to velocity of light
    (c) greater than velocity of light   (d) none of these

12. Existence of matter wave was experimentally first demonstrated by

    (a) Newton   (b) Planck
    (c) Davisson and Germer   (d) de Broglie

13. When an electron is accelerated,if de Broglie wavelength is 1Å, then the applied voltage is nearly equal to

$$\text{(a)}\, 15\,\text{volts} \quad \text{(b)}\, 12\,\text{volts} \quad \text{(c)}\, 500\,\text{volts} \quad \text{(d)}\, 150\,\text{volts}$$

14. If 100 volts is the potential with which an electron is accelerated, then the applied voltage is nearly equal to

$$\text{(a)}\, 1.226\text{Å} \quad \text{(b)}\, 1.226\text{nm} \quad \text{(c)}\, 12.26\text{nm} \quad \text{(d)}\, 0.1226\text{Å}$$

15. In Davission and Germer's experiment when 54 volts was applied to electrons, the pronounced scattering direction was found to be

$$\text{(a)}\, 50^0 \quad \text{(b)}\, 90^0 \quad \text{(c)}\, 120^0 \quad \text{(d)}\, \text{none of these}$$

16. For a particle in the ground state in an one-dimensional potential well of width a and

of infinite height, the probability of finding it will be maximum at a distance of

(a) a/2 from the wall   (b) a/4 from the wall
(c) 3a/4 from the wall   (d) a = 0 from the wall

17. Kinetic energy of electron accelerated by 50 volts is

(a) 50eV   (b) 10eV   (c) 5eV   (d) 15eV

Descriptive Questions

1. What are eigen values and eigen function?.

2. What is a wave function? Give its significance.

3. Obtain Schrodinger's description of particle in a deep potential well along with normalization condition. Specify wave functions, energies and wave pattern of de-Broglie waves.

4. Describe two application of uncertainty principle.

5. What are the properties of wave functions?

6. Find the eigen function and eigen value s for a particle in one dimensional potential well of infinite height and discuss the solutions.

7. A quantum particle confined to one dimensional potential well of finite width is known to be in its first excited state. What is the probability that the particle will be anywhere in the central half of the box?

8. Explain Heisenberg's uncertainty principle. Based on this, show the non-existence of electrons inside the nucleus.

9. Set up Schrodinger's wave equation and discuss the case of free particle.

10. What is the normalization of a wave function and give its significance. Obtain an expression for the energy of a particle inside a potential well of infinite height and calculate the gap between the ground state and the first two excited states.

Exercise

1. Calculate the kinetic energy of an electron whose de-Broglie wavelength is equal to that of a $10keV$ proton.

2. The speed of electron is measured to within an uncertainty of $2.2 \times 10^4 m/s$ in one dimension. What is the minimum width required by the electron to be confined in an atom.

3. Assuming that the radius of a nucleus of the order $10^{-14}m$ calculate the energy associated with a free particle of mass $9.1 \times 10^{-31}kg$ within this nucleus. Comment on the result obtained.

4. The ground state energy of an electron in an infinite well is 5.6×10^{-3} eV. What will be the ground state energy if the width of the well is doubled?

5. A quantum particle confined to one-dimensional box of width 'a' is known to be in its first exited state. What is the probability of the particle in central half?

6. A wave function is given by $\psi = A sin(n\pi x/L)$ for a motion of the particle in a zero potential well of breadth L. Calculate the value of A where x is the position of the particle along L.

7. An electron is confined to move between two rigid walls separated by $20\mathring{A}$. Find the de-Broglie wavelength representing the first three allowed energy state of the electron and the corresponding energies.

8. A spectral line of wavelength $4000\mathring{A}$ has a width of $8 \times 10^{-5}\mathring{A}$. Evaluate the minimum time spent by the electrons in the upper energy state between the excitation and deexcitation processes.

9. An electron has a speed of $6 \times 10^5 m/s$ with an inaccuracy of 0.01%. With what fundamental accuracy can we locate the position of the electron?

10. An electron is bound in one dimensional potential well of width $0.12nm$. Find the energy values in the ground state and also the first two excited state in eV.

Part II

Module 2:Electrical properties of materials

Chapter 3

Electrical Conductivity in Metals and Semiconductors

3.1 Classical free electron theory of metals

In order to explain electrical conductivity in metals, Lorentz and Drude put forward a theory called free electron theory of metals. It is based on the following assumptions.

1. Free electrons in a metal resemble molecules of a gas. Therefore, Laws of kinetic theory of gasses are applicable to free electrons also. Thus free electrons can be assigned with "average velocity $\overline{C}$", "Mean free path λ" and "mean collision time τ".

2. The motion of an electron is completely random. In the absence of electric field, number of electrons crossing any cross section of a conductor in one direction is equal to number of electrons crossing the same cross section in opposite direction. Therefore net electric current is Zero.

3. The random motion of the electron is due to thermal energy. Average kinetic energy of the electron is given by

$$\frac{3}{2}kT = \frac{1}{2}m\overline{C}^2$$

$$\text{or } \overline{C} = \sqrt{\frac{3kT}{m}}$$

$\overline{C}$=Average Velocity, k=Boltzman Constant, T=Absolute Temperature, m=mass of the electron.

4. Electric current in the conductor is due to the drift velocity acquired by the electrons in the presence of the applied electric field.

5. Electric field produced by lattice ions is assumed to be uniform throughout the solid.

6. The force of repulsion between the electrons and force of attraction between electrons and lattice ions is neglected.

3.2   The Drift Velocity

In the absence of the applied electric field, motion of free electron is completely random. During their motion electrons undergo collisions with the residual ions and during each collision direction and magnitude of their velocity changes in general. When electric field is applied, electrons experiences force in the direction opposite to the applied field. Therefore in addition to their random velocity, electron acquires velocity in the direction of the force. Since electrons continue to move in their random direction, with only a drift motion due to applied field, velocity acquired by the electrons in the direction opposite to the applied field is called **Drift velocity** and is denoted by v_d. Note the v_d is very small compared to $\overline{C}$, the average thermal velocity. Electric current in a conductor is primarily due to the drift velocity of the electrons.

3.3   Relaxation Time, Mean collision time and Mean free path

Mean free path (λ)**:** The average distance traveled by electrons between two successive collisions during their random motion is called mean free path, it denoted by λ.

Mean collision Time τ**:** The average time taken by an electrons between two successive collisions during their random motion is called mean collision time, it is denoted by τ. The relationship between λ and τ is given by $\lambda = \overline{C}\tau$.

Relaxation Time τ_r**:** In the presence of an applied electric field, electrons acquire drift velocity v_d in addition to the thermal velocity $\overline{C}$. if electric field is switched off, v_d reduces and becomes zero after some time. Let electric field is switched off at the instant t = 0, when drift velocity $v_d=v_0$. The drift velocity of the electron after the lapse of t seconds is given by

$$v = v_0 e^{\frac{-t}{\tau_r}}$$

where τ_r is called relaxation time. Suppose t=τ_r, then v_d=$v_0\ e^{-1} = \frac{1}{e}v_0$

Thus the relaxation time is defined as the time during which drift velocity reduces to 1/e times its maximum value after the electric field is switched off.

Relation between Relaxation time and Mean collision time:

The relationship between relaxation time and mean collision time is given by

$$\tau_r = \frac{\tau}{1 - \langle \cos \theta \rangle}$$

where θ is scattering angle, $\langle \cos \theta \rangle$ is average value of $\cos \theta$ taken over very large number of collision made by electrons.

It can be shown that, $\tau = \tau_r$ for metals.

3.4   Ohms Law :

Consider a conductor of length l and area of cross section A. Let ρ be the electrical conductivity of the conductor, then resistance of the conductor is given by

$$R = \rho \frac{l}{A} = \frac{1}{\sigma} \frac{l}{A}$$

Let a potential difference of V volts be applied between two ends of the conductor, which causes current I to flow through the conductor. Then, according to Ohms Law.

$$V = IR$$

$$V = \frac{Il}{\sigma A} = \frac{Jl}{\sigma}$$

$$\text{where } J = \frac{I}{A} \text{ is called current density}$$

$$\frac{V}{l} = \frac{J}{\sigma}$$

If "E" is the electric field established inside the conductor ,

then

$$E = \frac{V}{l}$$

$$\therefore \ E = \frac{J}{\sigma}$$

$$\text{or } J = \sigma E$$

The above equation also represents Ohm's law.

3.5   Expression for electrical conductivity

Consider a conductor of length l, area of cross section A, having n number of free electron per unit volume. Let a potential difference of V volt be applied between two ends of the

conductor and E be the electric field established inside the conductor. Then according to Ohms law

$$J = \sigma E$$

where σ is the electrical conductivity of the conductor and J is a current density given by

$$J = \frac{I}{A}$$

$$\sigma = \frac{J}{E} \qquad (3.1)$$

Consider a cross section 'X'' of the conductor. Let I be the current flowing through the conductor. Then according to the definition, current I is given by the quantity of charge flowing across the cross section X per second.

$$I = Ne$$

where N is the number of electrons crossing the cross section X in one second.
We know that, if v_d is drift velocity of the electrons. Electrons travel a distance equal to v_d in one second. Therefore number of electrons crossing the cross section X in one second is equal to number of electrons occupying the space between two cross sections X and Y separated by the distance v_d.
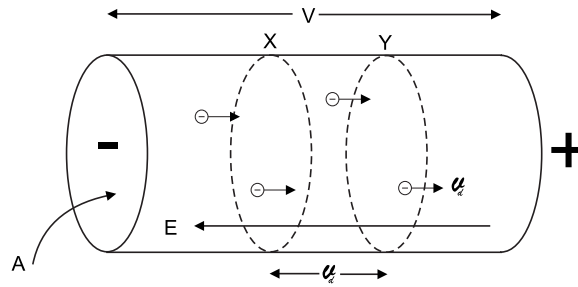
Figure 3.1: Current carrying conductor

$$N = n(Av_d)$$

where n=number of electrons per unit volume between the cross sections X and Y

$$I = nAv_d e$$

$$\text{But } J = I/A = nev_d$$

$$\therefore \sigma = J/E = \frac{nev_d}{E}$$

$$\sigma = \frac{nev_d}{E} \tag{3.2}$$

We know that free electrons undergo collisions with positive ions during their random motion. Let the drift velocity acquired by the electron becomes zero during each collision and let v_d be the drift velocity acquired by the electron just before next collision then

$$v_d = 0 + a\tau$$

Where a is the acceleration acquired by the electron in the presence of electric field E and τ is the average time taken by the electron between two successive collisions.

Force acting on the electron in the presence of electric field is given by

$$F = eE = ma$$

$$\therefore \quad a = \frac{eE}{m}$$

$$\therefore \quad v_d = \frac{eE}{m}\tau$$

$$\therefore \quad \text{equation } 6.2 \text{ becomes}$$

$$\sigma = \frac{ne}{E} \times \frac{eE\tau}{m}$$

$$\sigma = \frac{ne^2\tau}{m} \tag{3.3}$$

$$\text{but} \quad \tau = \frac{\lambda}{\overline{C}} \qquad (\lambda = \text{mean free path})$$

$$\therefore \quad \sigma = \frac{ne^2\lambda}{m\overline{C}}$$

from free electron theory

$$\overline{C} = \sqrt{\frac{3kT}{m}}$$

$$\therefore \quad \sigma = \frac{ne^2\lambda}{m\sqrt{\frac{3kT}{m}}}$$

$$\sigma = \frac{ne^2\lambda}{\sqrt{3mkT}} \tag{3.4}$$

Equation (3.4) gives expression for electric conductivity.

Note that, electrical conductivity of a conductor σ is proportional to

1. Number of electrons per unit volume and
2. Mean free path for free electrons
3. Inversely proportional to square root of the absolute temperature.

Mobility of Electrons:

The easiness with which electrons get drifted in the presence of applied electric field is called "Mobility". **It is defined as the drift velocity acquired by the electron per unit electric field. It is denoted by** μ

$$\text{Mobility } \mu = \frac{v_d}{E}$$

$$\text{We have } \sigma = \frac{nev_d}{E} = ne\mu$$

$$\sigma = ne\mu$$

$$\mu = \frac{\sigma}{ne}$$

$$\text{Also } \sigma = \frac{ne^2\tau}{m} = \mu ne$$

$$\therefore \quad \mu = \left(\frac{e}{m}\right)\tau$$

3.6 Effect of temperature and impurity on electrical resistivity of metals.

Resistance of a conductor can be mainly attributed to two reasons, namely

1. Scattering of electrons with the vibrating lattice ions:

During their random motion electrons undergo scattering from ions, which are in the continuous state of vibrations. This scattering is called phonon scattering and gives rise to resistance of metals. The resistance depends on the number of scattering that electrons undergo. As the temperature of the metal increases amplitude of vibrations of the ions increases, thereby increasing the number of collisions. This leads to increase in resistance. This resistivity is called ideal resistivity denoted by ρ_{ph} and is given by

$$\rho_{ph} = \frac{m}{ne^2\tau_{ph}} \tag{3.5}$$

τ_{ph}=mean collision time assuming that there are no impurities.

2. Scattering of electrons by the presence of impurities present in the metal:

Scattering of electrons from the sites of impurities present in the metals such as dislocation joints, grain Boundaries, impurity atom, etc gives rise to resistivity called residual resistivity. It exists even at absolute zero temperature. It is temperature independent and denoted by ρ_i . It is given by

$$\rho_i = \frac{m}{ne^2\tau_i}$$

where τ_i is the mean collision time assuming that scattering of electrons with the vibrating lattice ions is absent.

Thus net restivity of the conductor is given by

$$\rho = \rho_i + \rho_{ph}$$

$$\rho = \frac{m}{ne^2\tau_i} + \frac{m}{ne^2\tau_{ph}}$$

The above equation is called **Matthiessens Rule**, which states **that the net resistivity of conductor is equal to the sum of the resistivity due to the phonon scattering which is temperature dependent and resistivity due to the presence of impurity which is temperature independent.**

A graph of ρ against T is as shown in the fig.
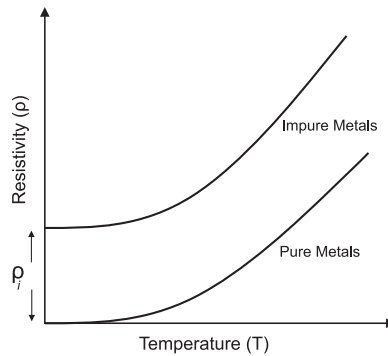
Figure 3.2: Variation of resistivity with temperature

At absolute zero temperature $\rho = \rho_i$ .As the temperature increases $\rho(T)$ increases gradually for lower temperatures. At higher temperatures, $\rho(T)$ increases rapidly overtaking the effect of ρ_i and becomes linearly dependent on temperature. Thus at higher temperatures the effect of impurity on resistivity becomes negligible and resistivity mainly depends on temperature.

3.7 Failure of classical free electron theory

Though classical free electron theory successfully explained electrical conductivity in metals, thermal conductivity and other phenomenon, it could not explain number of other experimentally observed facts. This led to the failure of classical free electron theory of metals. Some of the important reasons, which led to the failure of the theory, are,

1. **The specific heat of the Solids:** When heat is supplied to solids free electrons also absorb energy. Thus free electrons also contribute to the Specific heat of solids. This is called electronic specific heat. According to the free electron theory of metals energy of all the electrons in one-kilo mole of solid is given by

$$E = \frac{3}{2}N_A kT$$

Where N_A=Avagadro number, k=Boltzman constant and T=Absolute temperature. Therefore the electronic specific heat is given by

$$C_v = \frac{dE}{dT} = \frac{3}{2}N_A k$$

$$= \frac{3}{2} \times 6.023 \times 10^{26} \times 1.38 \times 10^{-23}$$

$$= 12.5 kJ/kmole/K$$

But experimentally determined value of C_v is 100 times less that the above value. Thus contribution to specific heat from electrons is negligible small.

2. **Dependence of conductivity on temperature:** According to free electron theory of metals, electrical conductivity of a conductor is given by

$$\sigma = \frac{ne^2\lambda}{\sqrt{3mkT}}$$

It is clear from the above equation that $\sigma \propto \frac{1}{\sqrt{T}}$
But experimentally it was found that $\sigma \propto \frac{1}{T}$

3. **Dependence of conductivity on Electron concentration:** According to free electron theory of metals, electrical conductivity of conductor is given by

$$\sigma = \frac{ne^2\lambda}{\sqrt{3mkT}}$$

It is clear from the above equation that $\sigma \propto n$

The value of electron concentration and conductivity for some of the metals are given below.

Metals	Electron concentration In $10^{28}/m^3$	Conductivity In $10^7/\Omega m$
Ag	5.85	6.30
Cu	8.45	5.88
Cd	9.28	0.15
Zn	13.10	1.09
Ga	15.30	0.67
Al	18.06	3.65

It is clear from the above table that there is no direct relationship between electron concentration and electrical conductivity.

From the above discussion it is clear that there exists discrepancy between theoretical predictions and experimental observation. This led to the failure of classical free electron theory of metals.

3.8   Quantum free electron theory of metal

Since classical free electron theory failed to account for the number of experimentally observed facts, Sommerfeld put forward a new theory, called quantum free electron theory, in the year 1928. The quantum free electron theory is based on following assumptions:

1. Though free electrons are free to move anywhere in the solid, they are bound within the boundary of the solid. Therefore, their energies are quantized according to quantum mechanics. Thus free electrons can have only discrete values of energy.

2. There exists large number of closely spaced energies for the electrons. Electrons are distributed among these energy levels according to Paulis exclusion principle, which states that there cannot be more than two electrons in any energy level.

3. The potential energy of the electrons remains uniform throughout the solid.

4. The force of repulsion between the electrons and force of attraction between electrons and lattice ions is neglected.

3.9 Fermi-Dirac Statistics

Fermi energy:

According to quantum free electron theory, free electron energies are quantised. Thus if there are N free electrons there exists N closely spaced energy levels. Since electrons are distributed among these energy levels according to Paulis exclusion principle. At absolute zero temperature, two electrons with least energy having opposite spins occupy lowest available energy level. The next two electrons with opposite spins occupy next energy level. And so on. In this way N/2 energy levels are occupied, and remaining N/2 energy levels are vacant. The top most energy level occupied at absolute zero temperature is called Fermi Energy level. The energy corresponding to that energy level is called **Fermi Energy. Fermi Energy can be defined as energy of that energy level below which all energy levels are completely occupied and above which all energy levels completely empty. It is denoted by** E_F Thus Fermi energy represents maximum energy that electrons can have at absolute zero temperature.

Fermi factor:

We know that at T=0 K, all energy levels below E_F are completely filled and all energy
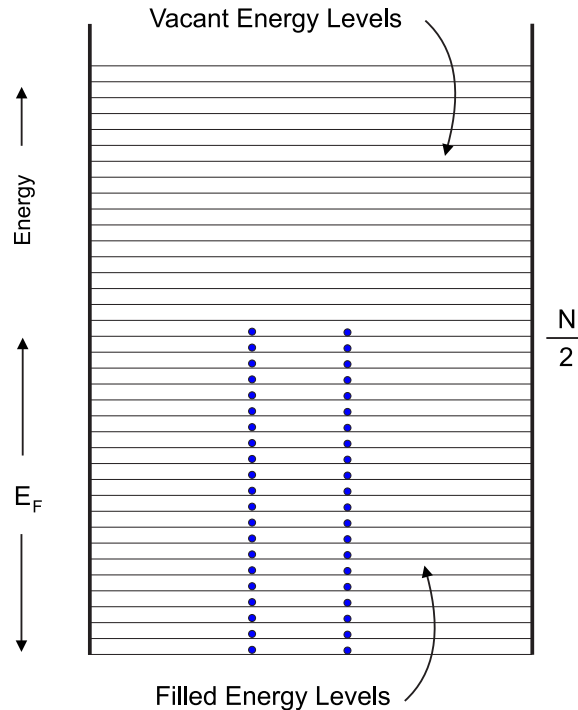

Figure 3.3: Distribution of Electrons at $T = 0K$

levels above E_F are completely empty. As the temperature is increased, some of the electrons absorb energy and start migrating to energy level above E_F. As a result there exists

some vacant energy levels below E_F and filled energy levels above E_F. Now it becomes difficult to say whether a particular energy level is occupied or vacant. The probability of occupation of any energy level is given by a mathematical function

$$f(E) = \frac{1}{e^{\frac{(E-E_f)}{kT}} + 1}$$

f(E)is called **Fermi factor.** It is defined as follows.

Fermi factor is a function, which gives the probability of occupation of an energy level E at the given temperature T for a material in thermal equilibrium.

3.10   The dependence of Fermi factor on Energy and Temperature

The variation of Fermi factor with Energy for various temperatures can be discussed as follows
(i) At absolute zero temperature (T=0):
(a) When T=0 and $E < E_F$

$$f(E) = \frac{1}{e^{-\infty} + 1} = \frac{1}{0 + 1} = 1$$
$$\therefore \quad f(E) = 1 \ \ for \ \ E < E_F$$

$\therefore$ since f(E)=1, at T=0, all energy levels below Fermi level are occupied.
(b) When T=0 and $E > E_F$

$$f(E) = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty + 1} = 0$$
$$\therefore f(E) = 0, \ \ for \ \ E > E_F$$

$\therefore$ At T=0 all the energy levels above Fermi energy level are vacant.
The graph of f(E) against E at T=0 is as shown in the fig
(ii) For temperature $T > 0$;
(a) When energy $E < E_F$

$$e^{\frac{E-E_f}{kT}} \ \ is \ \ > 0 \ But \ \ < 1$$

$\therefore$ f(E) is less than 1 but greater than 1/2
(b) When energy $E > E_F$
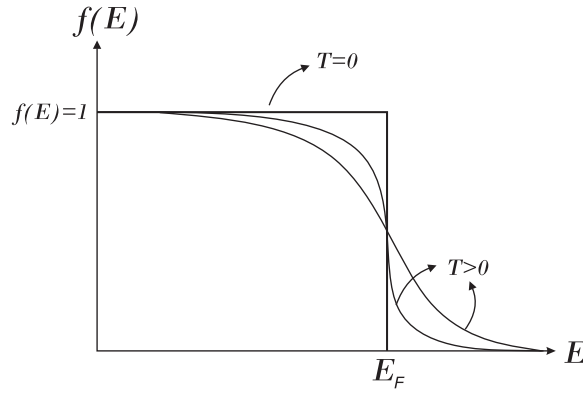
Figure 3.4: Variation of $f(E)$ with E

$$e^{\frac{E-E_f}{kT}} \quad is \quad > \quad 1$$

$\therefore$ f(E) is greater than 0 but less than 1/2

(c) When energy $E = E_F$

$$e^{\frac{E-E_f}{kT}} = 1$$

$$\therefore \quad f(E) = 1/2$$

The curve obtained is as shown in the fig.

It is clear from the above discussion that irrespective of the temperature the probability of occupation of the Fermi energy is always 1/2.

Therefore Fermi energy is considered as the most probable energy that electron possess in the solid.

3.11 Density of States

The density of states is defined as the number of energy levels available per unit volume per unit energy centered at E. It is denoted by g(E). The product g(E)dE gives the number of states per unit volume between the energy levels E and E+dE.

Then the number of electrons per unit volume having energies between E and E+dE is given by

$$N(E)dE = g(E)dE \times f(E)$$

Where f(E) is Fermi factor.

Expression for density of states:

The expression for the density of state is given by

$$\therefore \quad g(E)dE = \frac{\pi}{2}\left[\frac{8m}{h^2}\right]^{3/2}\sqrt{E}dE = (constant)\sqrt{E}dE$$

The above equation is called **density of energy states.** The graph of g(E) against E is as shown in the fig.
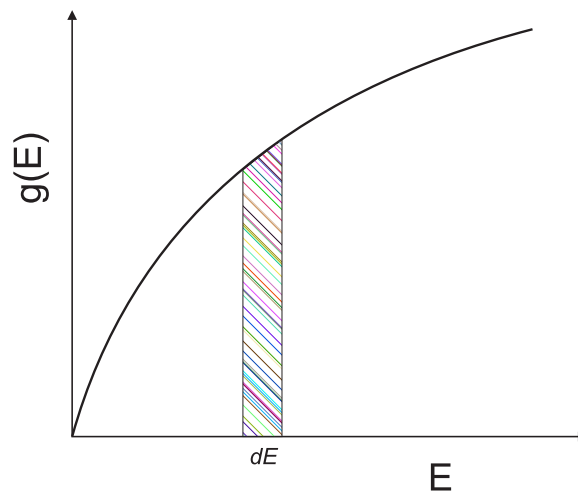
Figure 3.5: Variation of $g(E)$ with E

Expression for Fermi Energy at T=0K:

Number of electrons per unit volume having energies between E and E+dE is given by

$$N(E)dE = g(E)dE \times f(E)$$

At T=0 only energy levels up to E_F are filled. Therefore f(E)=1 for $E < E_F$

$$N(E)dE = g(E)dE = \frac{\pi}{2}\left[\frac{8m}{h^2}\right]^{3/2}\sqrt{E}dE$$

Therefore the number of electrons per unit volume is given by

$$n = \frac{\pi}{3}\left[\frac{8m}{h^2}\right]^{3/2}E_F^{3/2}$$

$$E_F^{3/2} = \left(\frac{3n}{\pi}\right)\left[\frac{h^2}{8m}\right]^{3/2}$$

$$E_F = \left[\frac{h^2}{8m}\right]\left(\frac{3n}{\pi}\right)^{2/3} = \left[\frac{h^2}{2m}\right]\left(\frac{3n}{8\pi}\right)^{2/3}$$

$$E_F = 3.65 \times 10^{-19} n^{2/3} \quad eV = 5.84 \times 10^{-38} n^{2/3} J$$

Fermi energy at $T > 0K$:

The Fermi energy at any temperature in general is given by

$$E_F = E_{F_0}\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{F_0}}\right)^2\right]$$

where E_{F_0} represents Fermi energy at T=0 K

Except at very high temperature it can be shown that

$$E_F = E_{F_0}$$

Fermi Temperature,(T_F):

The temperature at which the average thermal energy of the free electron in a solid becomes equal to the Fermi energy at T=0K is called Fermi temperature. Since thermal energy of an electron is given by kT.

When $T = T_F$

$$E_F = kT_F$$

$$\therefore \quad T_F = \frac{E_F}{k}$$

Fermi Velocity,(V_F):

The velocity of the electrons whose energy is equal to Fermi energy is called Fermi velocity.

$$\therefore \quad E_F = \frac{1}{2}mV_F^2$$

$$\text{or } V_F = \left(\frac{2E_F}{m}\right)^{1/2}$$

3.12   The Fermi-Dirac Distribution

The Fermi-Dirac Distribution gives the distribution of electrons among the permitted energies E and E+dE. It is given by

$$N(E)dE = g(E)dE \times f(E)$$

where g(E)dE is called density of states given by

$$g(E)dE = \frac{\pi}{2}\left[\frac{8m}{h^2}\right]^{3/2}\sqrt{E}dE$$

and f(E) is called Fermi factor given by

$$f(E) = \frac{1}{e^{\frac{(E-E_f)}{kT}} + 1}$$

The graph of N(E)dE against E gives the distribution of electrons among the available energy levels in the solid at the given temperature. The distribution is known as Fermi-Dirac Distribution. The graph representing the distribution is shown below.

It is clear from the graph that N(E) increase with the temperature becomes maximum for
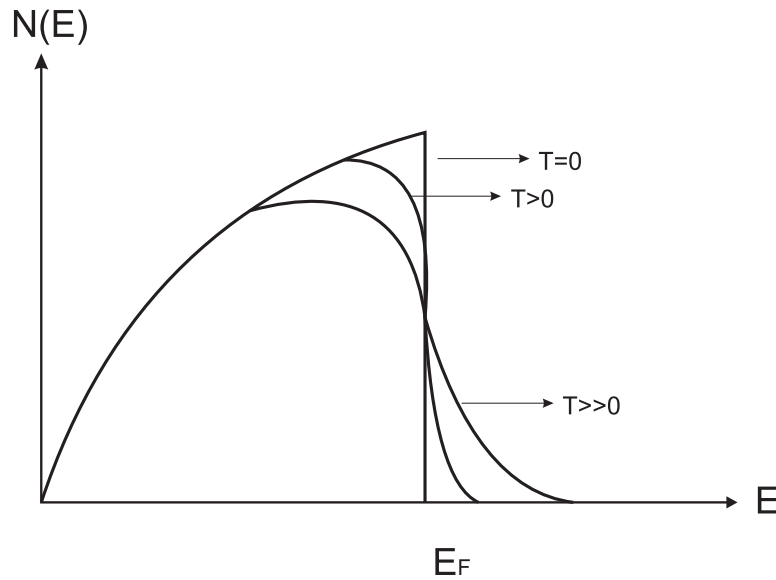
Figure 3.6: Fermi-Dirac distribution at different temperatures

$E = E_F$ and then decreases to zero for the energies $E > E_F$, at the temperature T=0 K. At higher temperatures some of the electrons occupying energy levels below Fermi energy absorb energy and move to higher energy levels above E_F. It is clear from the graph that even at very high temperatures only few electrons occupying energy levels closer to Fermi energy absorb energy and move to higher energy levels above Fermi energy. Electrons at lower energy levels are not at all disturbed.

3.13 Expression for electrical conductivity based on quantum theory

By applying Fermi-Dirac statistics and using Boltzmann transport equation, Sommerfield obtained the expression for electrical conductivity of metals. It is given by

$$\sigma = \frac{ne^2\lambda}{m * v_f}$$

where m^* is the effective mass of the electron in metals.

Expression for resistivity can be written as

$$\rho = \frac{m * v_f}{ne^2\lambda}$$

3.14 Merits of Quantum free electron theory

The drawbacks of classical free electron theory were removed by Quantum theory. Some examples are given below.

1. **Specific Heat:**
   According to classical theory electrons give significant contribution to specific heat of solids. But experimentally it was found that contribution to specific heat from electrons is negligibly small.
   According to quantum theory, When heat energy is supplied to solid, only those electron occupying energy levels closer to Fermi level absorb energy and get exited to higher energy levels. Thus only small percentage of electrons is capable of contributing to specific heat of the solid.
   It can be shown that only the fraction (kT/ E_F) of electrons contribute to specific heat of solid.
   Thus according to the quantum free electron theory of metals energy of all the electrons in one-kilo mole of solid is given by

$$E = \frac{3}{2}N_A kT \left(\frac{kT}{E_F}\right)$$

$$E = \frac{3}{2}N_A \frac{(kT)^2}{E_F}$$

Where N_A=Avogadro number,k=Boltzmann constant and T=Absolute temperature.
Therefore the electronic specific heat is given by

$$C_V = \frac{dE}{dT} = \frac{3}{2}N_A 2k^2 \frac{T}{E_F}$$

at room temperature and for $E_F = 5eV$

$$C_V = \frac{3}{2} \times 6.023 \times 10^{26} \times 2 \times (1.38 \times 10^{-23})^2 \times \frac{300}{5 \times 1.6 \times 10^{-19}}$$

$$= 0.129\, kJ/kmol/K$$

Which is in agreement with the experimental result.

2. **Temperature dependence of electrical conductivity:**
   According to classical theory $\sigma \propto 1/\sqrt{T}$
   But experimentally it is found that $\sigma \propto 1/T$
   Expression for electrical conductivity can be written as

   $$\sigma = \frac{ne^2 \tau_F}{m} \quad with \quad \tau_F = \frac{\lambda_F}{v_F}$$

   Since E_F is independent of temperature, v_F is also independent of temperature. Then only λ_F is dependent on temperature. We know that even at very high temperature only electrons closer to Fermi energy get exited. Thus only small percentage of electrons contributes to the electrical conductivity. It can shown that at very temperatures λ_F is inversely proportional to the temperature. Therefore it follows that

   $$\sigma \propto 1/T$$

   Which is agreement with the experimental results.

3. **Dependence of electrical conductivity on electron concentration:** According to quantum free electron theory expression for electrical conductivity is given by

   $$\sigma = \frac{ne^2 \lambda_F}{mv_F}$$

   Note that σ depends on n , but n depends on E_F , also v_F depends on E_F.
   Thus there is no direct relationship between σ and n.

Expression for electron concentration from molecular data:

Let M_A=Atomic weight, N=number of free electrons contributed by each atom to solid, D=is the density of the solid and N_A=Avogadro number.

$$M_A \text{ kg of solid contains } N_A \text{ atoms}$$

$$1 \text{ kg of solid contains } N_A/M_A \text{ atoms}$$

$$\text{Unit volume of a solid weighs } D \text{ kgs}$$

$$\text{Therefore unit volume of solid contains } D(N_A/M_A) \text{ atoms}$$

$$\text{Therefore number of electrons per unit volume is given by}$$

$$n = \frac{N N_A D}{M_A}$$

3.15 Conductivity of Semi conducting materials

For semiconductors, with an energy gap of 1.1 eV, at room temperature some electrons will be promoted into the conduction band which accounts for its intermediate value of conductivity. Furthermore, for every electron promoted into the conduction band, a "hole" is left in the valance band! Picture shows a hole in the valance band for every electron promoted into the conduction band. These holes are also considered to be charge carriers. So there are two types of carriers for electrical conduction: electrons and holes. Electrons are called n-type carriers (for negative) and holes are called p-type carriers (for positive).

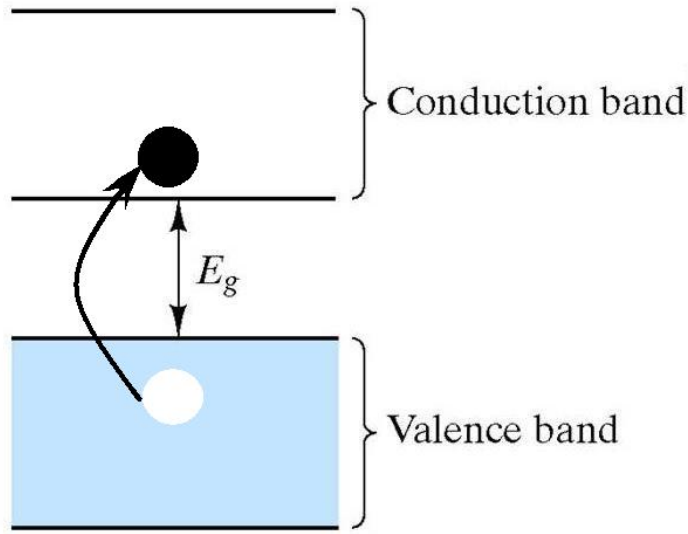For intrinsic semiconductors (no impurities), the number of electrons will be equal to the number of holes. $n_e = n_h$ Thus, the conductivity for an intrinsic semiconductor can be calculated:

$$\sigma = n_e q m_e + n_h q m_h \tag{3.5}$$

And since $n_e = n_h$ the conductivity for an intrinsic semiconductor is given by this equation:

$$\sigma = n_i q(m_e + m_h) \tag{3.5}$$

where i stands for intrinsic carrier concentration.

Semiconductors are semi-good electrical conductors because although their valence band is completely filled, the energy gap between the valance band and the conduction band is not too large. Hence some electrons can bridge it to become charge carriers. The difference between a semiconductors and an insulator is the magnitude of the energy gap. For semiconductors $Eg < 2eV$ and for Insulators $Eg > 2eV$.

3.16 Concentration of electrons and holes inintrinsic semiconductors

The concentration of electrons in conduction band is

$$n_o = \int_{E_c}^{\infty} f(E)N(E)dE \tag{3.5}$$

where $N(E)dE$ is the density of states (cm^{-3}) in the energy range dE. The number of electrons per unit volume in the energy range dE is the product of the density of states and the probability of occupancy $f(E)$. Thus the total electron concentration is the integration over the entire conduction band. The result of integration is the same as that obtained if we represent all of the distributed electron states in the conduction band edge E_c. Therefore, the conduction band electron concentration is the effective density of states at E_c represented by N_c times the probability of occupancy at E_c.

$$n_o = N_c f(E_c)dE \tag{3.5}$$

the Fermi function $f(E_c)$ can be simplified as

$$f(E_c) = \frac{1}{1 + e^{\frac{(E_c - E_F)}{kT}}} \approx e^{-\frac{(E_c - E_F)}{kT}} \tag{3.5}$$

This implies

$$n_o = N_c e^{-\frac{(E_c - E_F)}{kT}} \tag{3.5}$$

Similarly, the probability of finding an empty state (hole) in the valance band is

$$[1 - f(E)] = 1 - \frac{1}{1 + e^{\frac{(E_v - E_F)}{kT}}} \approx e^{\frac{-(E_F - E_v)}{kT}} \tag{3.5}$$

The concentration of holes in the valance band

$$p_o = N_v[1 - f(E_v)] = N_v e^{\frac{-(E_F - E_v)}{kT}} \tag{3.5}$$

Here, n_o and p_o represent electron and hole concentration at equilibrium and the equation is valid whether the material is intrinsic or doped. For intrinsic material, the fermi level E_F lies at some intrinsic level E_i near the middle of the band gap and the intrinsic electron and hole concentrations are

$$n_i = N_c e^{-\frac{(E_c - E_i)}{kT}}, \quad p_i = N_v e^{-\frac{(E_i - E_v)}{kT}} \tag{3.5}$$

The intrinsic electron and hole concentrations are equal (since the carriers are created in pairs), $n_i = p_i$. Thus the intrinsic concentration of electrons and holes is given by

$$n_i = \sqrt{N_c N_v} e^{\frac{-E_g}{kT}} \tag{3.5}$$

where

$$N_c = 2 \left(\frac{2\pi m_n kT}{h^2} \right)^{3/2} \tag{3.5}$$

and

$$N_v = 2 \left(\frac{2\pi m_p kT}{h^2} \right)^{3/2} \tag{3.5}$$

m_n and m_p are the effective mass of electron and hole respectively.

3.17 Fermi level in an intrinsic Semiconductor

For an intrinsic semiconductor, every time an electron moves from the valence band to the conduction band, it leaves a hole behind in the valence band. The density of electrons in the

conduction band equals the density of holes in the valence band.

$$n = p$$
$$N_c e^{-\frac{(E_c - E_i)}{kT}} = N_v e^{-\frac{(E_i - E_v)}{kT}} \tag{3.5}$$

Here N_c is the effective density of states in the conduction band, N_v is the effective density of states in the valence band, E_i is the Fermi energy of intrinsic semiconductor, E_c is the conduction band edge, E_v is the valence band edge, k is Boltzmann's constant, and T is the temperature in K.

Rearranging this equation yields,

$$e^{\left(\frac{2E_i - E_c - E_v}{kT}\right)} = \frac{N_v}{N_c} \tag{3.5}$$

taking the logarithm,

$$\frac{2E_i - E_c - E_v}{kT} = ln\left(\frac{N_v}{N_c}\right) \tag{3.5}$$

Solving for E_i,

$$E_i = \frac{E_c + E_v}{2} + \frac{kT}{2}ln\left(\frac{N_v}{N_c}\right) \tag{3.5}$$

3.18   Hall effect, Hall coefficient, Measurement of hall voltage and Hall angle

If a current carrying semiconductor placed in a perpendicular magnetic field, a potential difference will generate in the semiconductor which is perpendicular to both magnetic field and current. This phenomenon is called Hall Effect. In solid state physics, Hall effect is an important tool to characterize the materials especially semiconductors. It directly determines both the sign and density of charge carriers in a given sample. Consider a rectangular semiconductor of thickness t kept in XY plane. An electric field is applied in X-direction using Constant Current Generator (CCG), so that current I flow through the sample. If w is the width of the sample and t is the thickness. There fore current density is given by

$$J_x = \frac{I}{wt} \tag{3.5}$$

If the magnetic field is applied along negative z-axis, the Lorentz force moves the charge carriers (say electrons) toward the y-direction. This results in accumulation of charge carriers at the top edge of the sample. This set up a transverse electric field E_y in the sample. This develop a potential difference along y-axis is known as Hall voltage V_H and this effect is called Hall Effect.
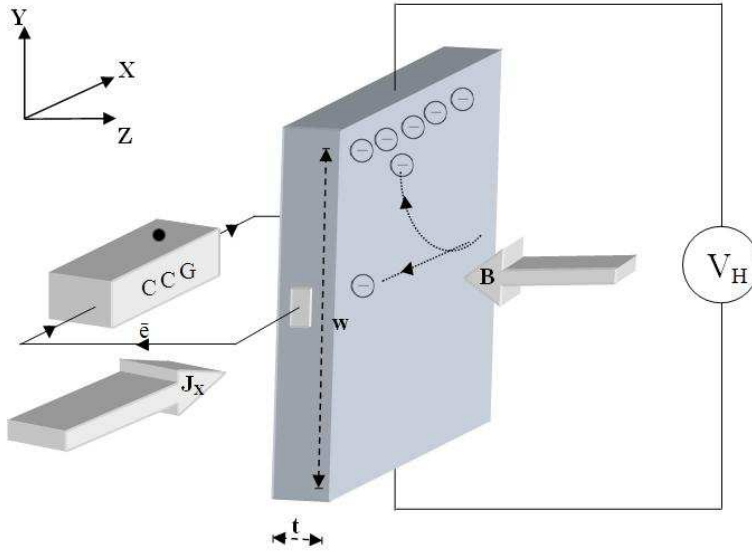
A current is made to flow through the sample material and the voltage difference between its top and bottom is measured using a voltmeter. When the applied magnetic field B=0,the voltage difference will be zero.

We know that a current flows in response to an applied electric field with its direction as conventional and it is either due to the flow of holes in the direction of current or the movement of electrons backward. In both cases, under the application of magnetic field the magnetic Lorentz force,

$$F_m = q(v \times B) \tag{3.5}$$

causes the carriers to curve upwards. Since the charges cannot escape from the material, a vertical charge imbalance builds up. This charge imbalance produces an electric field which counteracts with the magnetic force and a steady state is established. The vertical electric field can be measured as a transverse voltage difference using a voltmeter.

In steady state condition, the magnetic force is balanced by the electric force. Mathematically, we can express it as

$$eE = evB \tag{3.5}$$

Where 'e' the electric charge, 'E' the hall electric field developed, 'B' the applied magnetic field and 'v' is the drift velocity of charge carriers.

And the current 'I' can be expressed as,

$$I = neAv \tag{3.5}$$

Where 'n' is the number density of electrons in the semiconductor of length l ,breadth 'w'

and thickness 't'.

Using (3.18) and (3.18) the Hall voltage V_H can be written as,

$$V_H = Ew = vBw = \frac{IB}{net} \tag{3.5}$$

$$V_H = RH\frac{IB}{t} \tag{3.5}$$

by rearranging eq(3.18) we get

$$R_H = \frac{V_H t}{IB} \tag{3.5}$$

where R_H is called the Hall coefficient

$$R_H = 1/ne \tag{3.5}$$

Hall angle is the angle between induced electrical field and the current density

* *

Multiple Choice Questions

1. The classical expression for the electrical conductivity of a metal in terms of mass of the electron, charge concentration of electrons and collision time is given by

   (a) $mne\tau$ (b) $me\tau/n$ (c) $ne^2\tau/m$ (d) $ne^2\tau^2/m$

2. The collision time and the root mean square velocity of the electron at room temperature are 2.5×10^{-14}s and 1×10^5m/s respectively. The classical value of mean free path of the electron is

   (a) 2.5nm (b) 2.5×10^{-19}nm (c) 2.5Å (d) 100nm

3. If the mobility of electrons in a metal increases, the resistivity

   (a) decreases (b) increases (c) remain constant

4. Most widely used conducting materials are

   (a) germanium and silicon   (b) copper and aluminium

   (c) gold and silver   (d) tungsten and platinum

5. The temperature dependence of the classical expression for electrical resitivity of a metal is

   (a) $\rho \propto T^2$ (b) $\rho \propto 1/T^2$ (c) $\rho \propto T^{1/2}$ (d) $\rho \propto 1/T$

6. Mobility of electron is

   (a) flow of electron per unit electric field

   (b) reciprocal of conductivity

   (c) average electron drift velocity per unit electric field

   (d) none of these

7. The classical value of molar electronic specific heat is

   (a) $1.5R_u$ (b) $0.5R_u$ (c) $3R_u$ (d) $4.5R_u$

   where R_u is universal gas constant.

8. Ohm's law relates to the electric field E, conductivity σ and current density j as

   (a) $j = E/\sigma$ (b) $j = \sigma E^2$

   (c) $j = \sigma/E$ (d) $j = \sigma E$

9. Order of restivity of silver is

   (a) nano $-$ ohm m   (b) milli ohm m

   (c) ohm m   (d) micro ohm m

10. The average drift velocity v_x of electrons in a metal is related to the electric field E and collision time, τ as

    (a) $\sqrt{eE\tau/m}$ (b) $\sqrt{m/eE\tau}$

    (c) $eE\tau/m$ (d) $m/eE\tau$

11. If e, μ and n respectively represent the charge, mobility and concentration of electrons

respectively, then the electrical conductivity of the metal is given by

    (a) $n/\mu e$ (b) $\mu e/n$

    (c) ne   (d) $ne\mu$

12. The classical expression for molar electronic specific heat is $1.5R_u$. Express its value, giving the units

    (a) $12.5 \mathrm{Jkmol^{-1}K^{-1}}$ (b) $1250 \mathrm{kJkmol^{-1}K^{-1}}$

    (c) $125 \mathrm{kJkmol^{-1}K^{-1}}$ (d) $12.5 \mathrm{kJKmol^{-1}K^{-1}}$

13. The wave function of the motion of the particle in a one dimensional potential box of lengths is given by $\psi_n = A sin(n\pi x/a)$ where A is the normalization constant. The value of A is

    (a) $1/A$ (b) $\sqrt{2/a}$

    (c) a   (d) $\sqrt{a/2}$

where a is the length of the box

14. The energy of the lowest state in a one dimensional potential box of length a is

    (a) zero   (b) $2h^2/8ma^2$

    (c) $h^2/8ma^2$ (d) $h/8ma^2$

15. If E_1 is the energy of the lowest state of a one dimensional potential box of length a and E_2 is the energy of the lowest state when the length of the box is halved, then

    (a) $E_2 = E_1$ (b) $E_2 = 2E_1$

    (c) $E_2 = 3E_1$ (d) $E_2 = 4E_1$,

16. The general expression for the Fermi energy of a metal at 0 K is

    (a) $3.65 \times 10^{-19} \times n^{2/3}$ eV

    (b) $3.65 \times 10^{19} \times n^2$ eV

    (c) $3.65 \times 10^{19} \times n^{1/3}$ eV

    (d) $3.65 \times 10^{19} \times n^{3/2}$ eV

17. If the Fermi energy of a metal is 1.4 electron volt, the Fermi temperature of the metal

is approximately

(a) 1.6×10^3K (b) 1.6×10^4K

(c) 1.6×10^5K (d) 1.6×10^6K

18. The quantum mechanical expression for electrical conductivity is

(a) $\sigma = ne^2\lambda_F/m^*v_F$ (b) $\sigma = m^*v_F/\lambda_F ne^2$

(c) $\sigma = ne^2\lambda_F/m^*$ (d) $m^*/ne^2\lambda_F$

19. If E_F is the Fermi energy of a metal at 0 K and E_f is the Fermi energy at T K, then

(a) $E_F = E_f[1 + (\pi^2/12)(k_BT/E_F)^2]$

(b) $E_F = E_f[1 - (\pi^2/12)(k_BT/E_F)^2]$

(c) $E_f = E_F[1 + (\pi^2/12)(k_BT/E_F)^2]$

(d) $E_f = E_F[1 - (\pi^2/12)(k_BT/E_F)^2]$

20. E_F and E_f are the Fermi levels of sodium at 0 K and 10000K. E_F for sodium is 3 eV. Then E_f/E_F is

(a) 0.93 (b) 1.93

(c) 0.02 (d) 21

21. If ρ_o and $\rho(T)$ are the temperature independent and dependent parts of the resistivity of a metal, then the total resistivity is

(a) $\rho = \rho_o + \rho(T)$ (b) $\rho = \rho_o - \rho(T)$

(c) $\rho = \rho_o/\rho(T)$ (d) ρ_T/ρ_o

22. In thermionic emission a plot of $log[j/T^2]$ against 1/T is

(a) a straight line with positive slope

(b) a straight line with negative slope

(c) none of these

23. The expression for electric conductivity in terms of mobility of electrons μ density of electrons n and charge of electron e is given by

(a) $\sigma = 1/ne\mu$ (b) $\sigma = n/e\mu$

(c) $\sigma = n\mu/e$ (d) $\sigma = ne\mu$

24. The restivity of the material of the conductor is given by

    (a) $\rho = m/ne^2\tau_c$ (b) $\rho = ne^2\tau_c/m$

    (c) $\rho = n/me^2\tau_c$ (d) $\rho = me^2\tau_c/n$

25. At any temperature other than 0 K, the probability of finding an electron at Fermi level is

    (a) 1   (b) 0   (c) 0.5   (d) any positive value

Descriptive Questions

1. What is Fermi energy? Write expression for Fermi-factor. Obtain the expression for Fermi-energy (E_f) at $T = 0K$ in terms of electron concentration.

2. Outline the major successes of quantum free electron gas model of metals.

3. How does the electrical resistance of the metal change with impurity and temperature?

4. Derive the expression for electrical conductivity in metals in terms of relaxation time and explain any three drawbacks of classical theory of free electrons.

5. Bring out the salient features of Drude-Lorentz theory.

6. Obtain an expression for density of states of energy levels.

7. Describe Fermi energy and Fermi factor.

8. Explain Fermi energy and Fermi factor. Discuss the variation of Fermi factor with temperature and energy.

9. State Mathiessien's rule and give an account of the total resistivity both at high and low temperatures.

10. Using the free electron model derive an expression for electrical conductivity in metals.

11. Explain density of states.

12. Define Fermi factor and hence discuss the probability of occupation of various energy states by electrons at $T = 0K$ and $T > 0K$.

13. Describe Fermi-Dirac distribution and discuss the same for different temperature conditions.

14. Explain the terms: drift velocity, relaxation time and mean free path for free electron.

15. Explain the density of states. Derive an expression for number of allowed states per unit volume of a solid.

16. Define drift velocity, mobility and relaxation time for free electron, derive the expression for conductivity in terms of mean collision time.

17. Show that occupation probability at $E = E_f + \Delta E$ is same as non-occupation probability at $E = E_f - \Delta E$, E_f is the Fermi energy.

18. Discuss quantum free electron theory.

19. Show that sum of the probability of occupation ΔE below the Fermi level and the probability of occupation ΔE above the Fermi level is one.

20. Elucidate the difference between classical free electron theory and quantum free electron theory. Describe how quantum free electron theory has been successful in overcoming the failures of classical free electron theory.

21. Obtain the expression for intrinsic concentration of holes and electrons in semiconductors

22. Write a note on Fermi level in semiconductors.

23. Define Hall co-efficient. Obtain the expression for the same

Exercise

1. Find the temperature at which there is 1 percent probability that a state with energy 0.5eV above the Fermi-energy will be occupied by an electron.

2. Find the probability with which an energy level $0.2eV$ below Fermi level being occupied at room temperature of 300k and $1000k$.

3. Show that if the probability of occupancy is x at an energy level ΔE below the Fermi level, then x is also probability of non occupancy at an energy level V above the Fermi level.

4. Calculate the Fermi energy of electrons of mass $9.1 \times 10^{-31} kg$, given that density=$1.9 \times 10^3\ kg/m^3$, Avogadro number=6.02×10^{23} and atomic weight=$132.9 \times 10^{-3} kg/mol$ for monovalent metals.

5. Find the relaxation time of conduction electrons in a metal of resistivity $1.54 \times 10^{-8}\Omega - m$, if the metal has 5.8×10^{28} conduction electrons per m^3.

6. Calculate the drift velocity and thermal energy of electrons in a metal of thickness 1 mm across which a potential difference of 1 volt is applied, at the temperature of $300K$. The mobility of free electron is $40cm2/v - s$.

7. Calculate the probability of an electron occupying an energy level $0.02eV$ above the Fermi level at 200K and $400K$ in a material.

8. The Fermi level in potassium is $2.1eV$. What are the energies for which the probability of occupancy at $300K$ is 0.99, 0.01 and 0.5?

9. Calculate the Fermi energy in eV for a metal at $0K$, given that its $10500kg/m^3$. Atomic wt. is 107.9 and it has only one conduction electron per atom.

10. At what temperature we can expect 1% probability that an energy level $0.5eV$ above Fermi level will be occupied.

Chapter 4

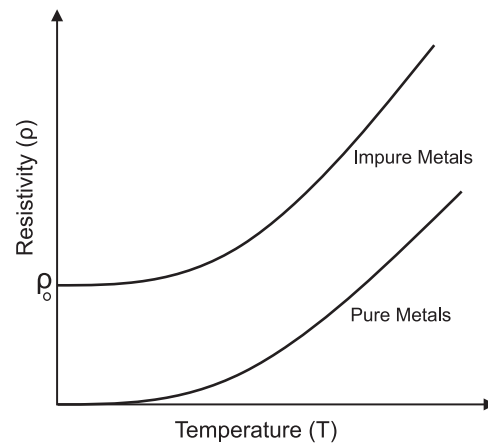# Superconductivity

Superconductivity

4.1 Temperature dependence of resistivity of metal

The variation of resistivity with temperature for a metal is as shown in the fig. Resistivity in the case of pure metal decreases with the decrease in temperature and becomes zero at absolute zero temperature. While in the case of impure metals the resistivity of metal will have some residual value even at absolute zero temperature. This residual resistance depends only on the amount of impurity present in the metal and is independent of the temperature. Thus net Resistivity of a metal can be written as

$$\rho = \rho_0 + \rho(T)$$

Thus net resistivity of conductor is equal to sum of temperature independent part and temperature dependent part as shown in the graph.

Superconductivity: Kamerlingh Onnes discovered the phenomenon of superconductivity in the year 1911. When he was studying the temperature dependence of resistance of Mercury at very low temperature he found that resistance of Mercury decreases with temperature with the decrease in temperature up to a particular temperature $T_c = 4.15K$. Below this temperature the resistance of mercury abruptly drops to zero. Between $4.15K$ and Zero degree Kelvin Mercury offered no resistance for the flow of electric current. The phenomenon is reversible and material becomes normal once again when temperature was increased above $4.15K$. He called this phenomenon as superconductivity and material which exhibited this property as superconductors.

Figure 4.1: Dependance of ρ on T

Thus the phenomenon of super conductivity is defined as:

"The phenomenon in which resistance of certain metals, alloys and compounds drops to zero abruptly, below certain temperature is called superconductivity"
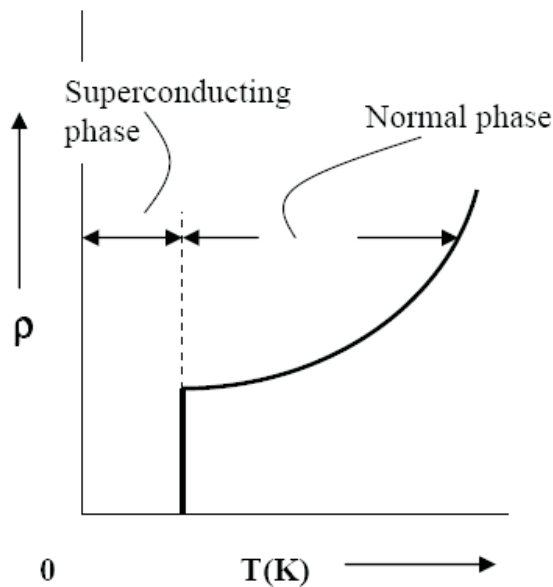
Figure 4.2: Dependance of ρ on T

The temperature, below which materials exhibit superconducting property is called critical temperature, denoted by T_c. Critical temperature T_c is different for different substances. The materials, which exhibit superconducting property, are called **superconductors**.

Above critical temperature material is said to be in **normal state** and offers resistance

for the flow of electric current. Below critical temperature material is said to be in **super-conducting state**. Thus T_c is also called as transition temperature.
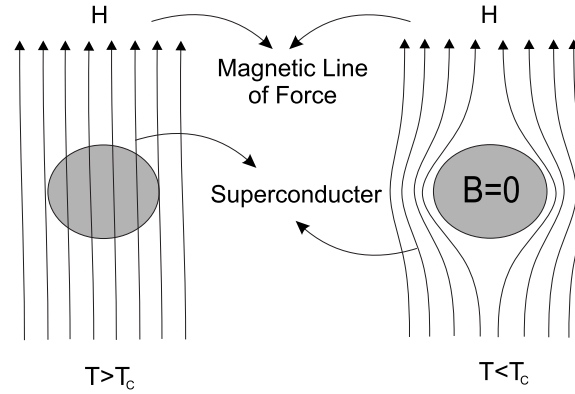

Figure 4.3: Meissner effect-expulsion of magnetic flux

4.2   Meissner Effect

In 1933, Meissner and Ochsenfeld showed that when a superconducting material is placed in a magnetic field, it allows magnetic lines of force to pass through, if its temperature is above T_c. If temperature is reduced below the critical temperature T_c ,it expels all the lines of force completely out of the specimen to become a perfect diamagnetic material. This is known as **Meissner effect**.

Since superconductor exhibits perfect diamagnetism below the critical temperature T_c, magnetic flux density inside the material is zero.

Therefore $B = 0$, for $T < T_c$

Relationship between flux density and the strength of the magnetizing field is given by

$$B = \mu_0(M + H)$$

μ_0 = Absolute permeability of free space

M = Intensity of magnetization of the material and

H = Strength of the magnetizing field

$$\because \quad B = o$$

$$0 = \mu_0(M + H)$$

$$or \ \ M = -H$$

Thus superconductor possesses negative magnetic moment when it is in superconducting state.

4.3 Critical field

We know that when superconductor is placed in a magnetic field it expels magnetic lines of force completely out of the body and becomes a perfect diamagnet. But if the strength of the magnetic field is further increased, it was found that for a particular value of the magnetic field, material looses its superconducting property and becomes a normal conductor. The value of the magnetic field at which superconductivity is destroyed is called the **Critical magnetic field**, denoted by H_c . It was found that by reducing the temperature of the material further superconducting property of the material could be restored. Thus, critical field doesnt destroy the superconducting property of the material completely but only reduces the critical temperature of the material.
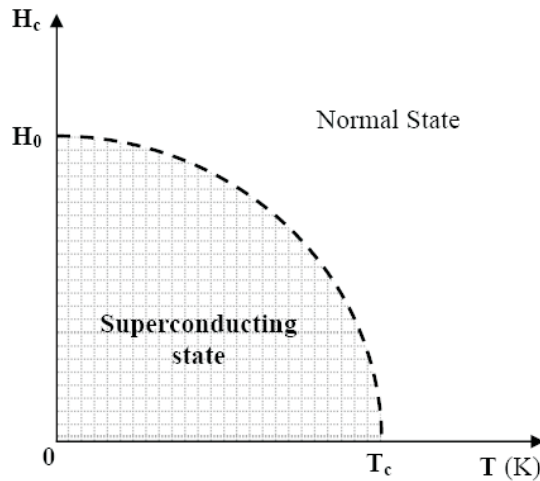

Figure 4.4: Dependance of H_c on T

Critical magnetic field H_c depends on the temperature of the material. The relationship between the two is given by

$$H_c = H_0 \left(1 - \frac{T^2}{T_c^2} \right)$$

4.4 BCS theory of Superconductivity

Bardeen,**Cooper** and **Schrieffer** explained the phenomenon of superconductivity in the year 1957. The essence of the BCS theory is as follows.

We know that resistance of the conductor is due to the scattering of electrons from the lattice ions. Consider an electron moving very close to a lattice ion. Due to coulomb interaction between electron and ion, the ion core gets distorted from its mean position. It is called **lattice-distortion**. Now another electron moving close to this lattice ion interacts with it. This results in the reduction in the energy of the electron. This interaction can be looked upon as equivalent to the interaction between two electrons via lattice. During the interaction exchange of phonon takes place between electron and the lattice. This interaction is called **electron-lattice-electron interaction via the phonon field**. Because of the reduction in energy between the two electrons, an attractive force comes into effect between two electrons. It was shown by Cooper that, this attractive force becomes maximum if two electrons have opposite spins and momentum. The attractive force may exceed coulombs repulsive force between the two electrons below the critical temperature, which results in the formation of bound pair of electrons called **cooper pairs**.

At temperatures below the critical temperature large number of electron lattice electron interaction takes place and all electrons form a cloud of cooper pairs. Cooper pairs in turn move in a cohesive manner through the crystal, which results in an ordered state of the conduction electrons without any scattering on the lattice ions. This results in a state of zero resistance in the material.
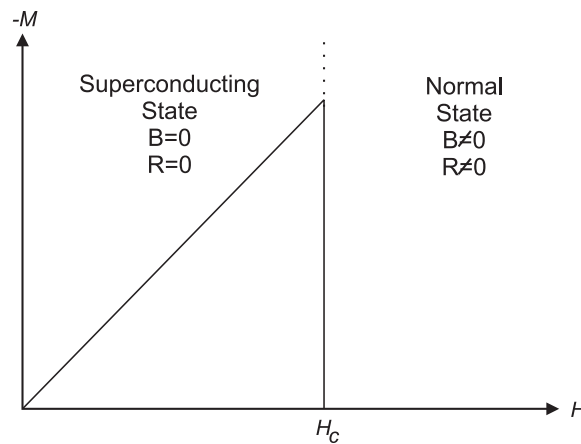

Figure 4.5: Dependence of magnetic moment on H for type I super conductors

4.5   Types of Superconductors

Type I or Soft Superconductors: Superconducting materials, which exhibit, complete Meissner effect are called Soft superconductors. We know that below critical temperature, superconductors exhibit perfect diamagnetism. Therefore they possess negative magnetic

moment.

Ex: Sn, Hg, Nb.

The graph of magnetic moment Vs magnetic field is as shown in the Fig 4.5. As field strength increases material becomes more and more diamagnetic until H becomes equal to H_c. At H_c , material losses both diamagnetic and superconducting properties to become normal conductor. It allows magnetic flux to penetrate through its body. The value of H_c is very small for soft superconductors. Therefore soft superconductors cannot withstand high magnetic fields. Therefore they cannot be used for making superconducting magnets. They are used fro making superconductiong switches.

Type II or Hard Superconductors

Superconducting materials, which can withstand high value of critical magnetic fields, are called Hard Superconductors.

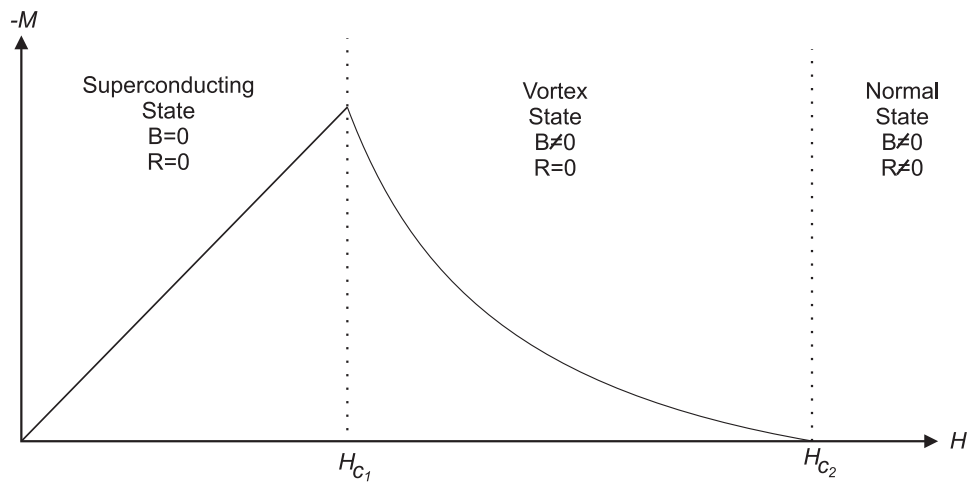Ex: $Nb_3,\ Sn,\ Nb_3Ge,\ YBa_2Cu_3O_7$

Figure 4.6: Dependence of magnetic moment on H for type II super conductors

The graph of magnetic moment Vs magnetic field is as shown in the Fig. Hard superconductors are characterized by two critical fields H_{c_1} and H_{c_2}.When applied magnetic field is less than H_{c_1} material exhibits perfect diamagnetism. Beyond H_{c_1} flux penetrates and fills the body partially. As the strength of the field increases further, more and more flux fills the body and thereby decreasing the diamagnetic property of the material. At H_{c_2} flux fills the body completely and material losses its diamagnetic property as well as superconducting property completely.

BetweenH_{c_1} and H_{c_2} material is said to be in **vortex state**. In this state though there is flux penetration, material exhibits superconducting property. Thus flux penetration occurs
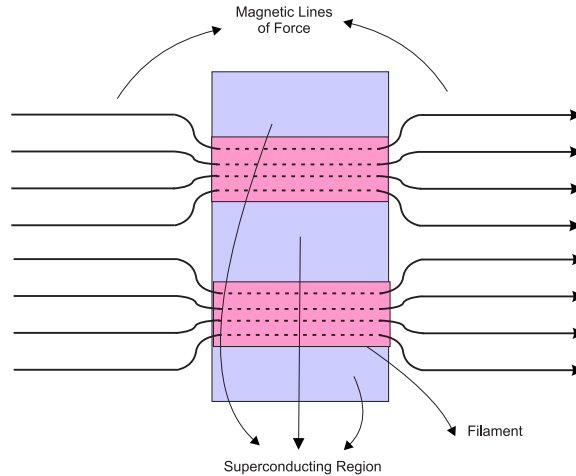
Figure 4.7: Vortex state

through small-channelised regions called filaments. In filament region material is in normal state. As H_{c_2} the field strength increases width of the filament region increases at they spread in to the entire body, and material becomes normal conductor as a whole. The value of H_{c_2} is hundreds of times greater than H_c of soft superconductors. Therefore they are used for making powerful superconducting magnets.

4.6 High Temperature Superconductivity

Superconducting materials, which exhibit superconducting property at higher temperatures, are called high temperature superconductors. Thus high temperature superconductors possess higher value of critical temperature compared to conventional superconductors. Most of the high temperature superconductors are found to be non-metals and intermetallics compounds, but are oxides, that fall into the category of ceramics. In 1986 a compound containing barium, lanthanum, copper, and oxygen having $T_c = 30K$ was developed. In 1987, scientists developed a compound which is an oxide of the form $YBa_2Cu_3O_7$ often referred to as $1-2-3$ compound having $T_c = 77K$.

All high temperature superconductors are oxides of copper, and bear a particular type of crystal structure called Perovskite crystal structure. Such crystal structures are characterized by large number of copper-oxygen layers. It was found that addition of extra copper-oxygen layer pushes the critical temperature T_c to higher values.

It was also found that formation of super currents in high superconductors is direction dependent. The super currents are strong in the copper-oxygen layer and weak in the direction perpendicular to the planes.

4.7 Applications of Superconductivity

1. Superconducting Magnets: We know that in ordinary electromagnet strength of the magnetic field produced depends on the number of turns (**N**) in the winding and the strength of the current (**I**) flowing through the winding. To produce strong magnetic field either **N** or **I** should be increased. If **N** is increased size of the magnet increases and if **I** is increased power loss ($I^2 R$) increases, which results in production of heat. Therefore there are limitations to increase **N** and **I**. If superconducting wires are used for winding in electromagnets, even with small number of turns strong magnetic fields can be produced by passing large current through the winding, because there is no loss of power in superconductors.

The type II superconductors, which have high H_c and T_c values, are commonly used in superconducting magnets. Ex: Niobium-tin, Niobiumaluminium, niobium-germanium and vanadium-gallium alloys.

The superconducting magnets are used in Magnetic Resonance Imaging (MRI) systems, for plasma confinement in fusion reactors, in magnetohydrodynamic power generation, in Maglev vehicles, etc.

2.Maglev Vehicles:

Vehicles, which float under the magnetic effect, are called **MAGnetically LEVitated vehicles** or simply **MAGLEV** vehicles. Such vehicle offer benefits like no friction, less power and noiseless transportation.

Cross section of the Maglev vehicle is as shown in the Fig 4.8. The system uses direct current superconducting magnets fixed under the carriage, which in turn induce eddy currents in the aluminium guide way. Eddy currents in the guide way generate repulsive force, which pushes the carriage up. Then forward motion of the vehicle is achieved by the principle of synchronous linear induction. The superconducting magnets and the cryogenic system for cooling the magnet are mounted below the carriage isolating them from the passenger compartment.

3.SQUIDS:

SQUID is an acronym for Superconducting Quantum Interface Device. SQUIDs are highly sensitive device, which can detect even weakest magnetic fields of the order of $10^{-14}T$. Squid is formed using two Josephsons junctions in the form of a loop. When a magnetic field is applied to this loop it induces circulating current in the loop which produces magnetic field which is just enough to exclude the magnetic flux inside the loop. It was found that the super current I_s induced in the loop is periodic in nature with the variation of applied magnetic field.

When squid is brought under the influence of an external magnetic field the flux through the loop changes and this causes the change in loop current. This variation of current in
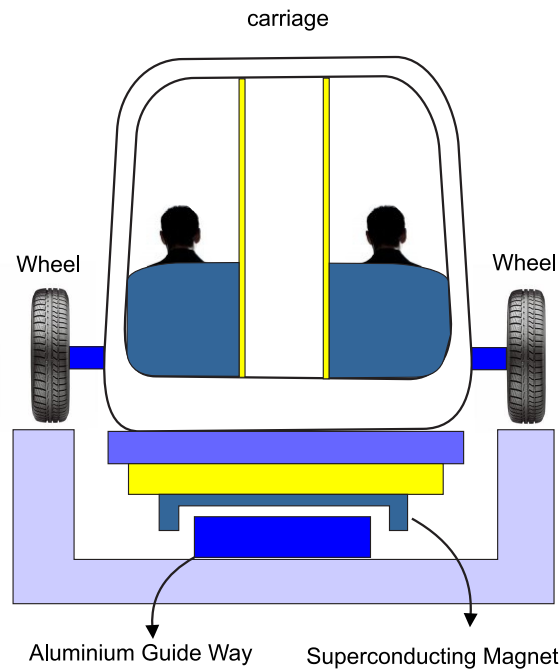
carriage

Wheel   Wheel

Aluminium Guide Way   Superconducting Magnet

Figure 4.8: Schematic diagram of MAGLEV vehicle (cross-sectional view)
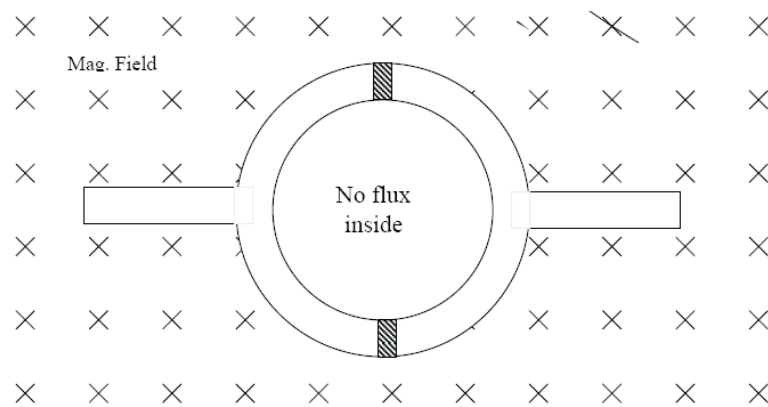
Mag. Field

No flux
inside

Figure 4.9: Current flow in a SQUID

the loop can be detected with the help of an induction coil kept adjacent to the loop. Thus SQUID can detect even a very sensitive magnetic field.

Squids are used to measure very small magnetic fields produced by heart and brain. They are used as voltmeter to measure the Hall effect and Thermoelectricity. They are used to detect ore and oil deposits.

Multiple Choice Questions

1. Superconductors are generally

    (a) ferromagnetic and antiferromagnetic metals

    (b) monovalent metals

    (c) amorphous thin films of Be and Bi

    (d) thin films of barium titanate

2. Hard superconductors observe

    (a) breakdown of Silsbee's rule

    (b) incomplete Meissner effect

    (c) high critical field and transition temperature

    (d) all of these

3. Soft superconductors observe

    (a) Meissner effect

    (b) Silsbee's rule

    (c) both (a) and (b)

    (d) none of these

4. The temperature at which a conductor becomes a superconductor is called

    (a) superconducting temperature

    (b) Curie temperature

    (c) Onne's temperature

    (d) transition temperature

5. The transition temperature of most superconducting elements lie in the range.

    (a) zero to 10K   (b) 10K to 20K

    (c) 20K to 50K   (d) above 50K

6. The transition temperature of mercury is

    (a) 1K   (b) 1.14K

    (c) 4.12K   (d) 9.22K

7. A superconducting material when placed in a magnetic field will

   (a) attract the magnetic field toward its center

   (b) repel all the magnetic lines of forces passing through it

   (c) attract the magnetic field but transfer it into a concentrated zone

   (d) not influence the magnetic field

8. A superconducting material on being subjected to the critical field changes to

   (a) critical conductivity

   (b) superconducting which is independent of temperature

   (c) normal state

   (d) remains uninfluenced

9. Transition temperature T_c and critical field H_c for a superconductor are related as

   (a) $H_c = H_o(T_c - 1)$ (b) $H_c = H_o(T_c + 1)$
   (c) $T_c = T_o[1 - (H_o/H_c)^2]$ (d) $H_c = H_o[1 - (T/T_c)^2]$

10. As the isotopic mass of mercury decreases

    (a) the critical temperature increases slightly

    (b) the critical temperature decreases slightly

    (c) remains constant

11. Cooper pairs are formed

    (a) at very low temperatures as the thermal energy is not sufficient to disrupt the binding

    (b) at high temperatures as the thermal energy is sufficient to form the cooper pair

    (c) none of these

12. The favorable condition both for superconductivity and high restistance is

    (a) a weak electron-phonon interaction

    (b) a strong electron-phonon interaction

    (c) a weak phonon-phonon interaction

    (d) none of these

13. Below transition temperature, a superconducting material exhibits

    (a) only zero resistance

    (b) only diamagnetic property

    (c) zero resistance and diamagnetism

    (d) zero resistance and ferromagnetism

14. The magnetization of a superconductor,is

$$(a)\, 0 \quad (b)\, H \quad (c)\, 1 \quad (d)\, -H$$

15. When a material becomes superconductor

    (a) the properties of a lattice structure do not change

    (b) the properties of a lattice structure do change

    (c) it becomes ferromagnetic in nature

    (d) magnetic property does not change

16. In type-II superconductor

    (a) the magnetic flux pass through the entire material

    (b) the magnetic flux does not pass through the material

    (c) the magnetic flux does not suddenly drop to zero but decreases exponentially

    (d) none of these

Descrptive Questions

1. Explain the effect of magnetic field on superconductors and meissner effect.

2. Explain the BCS theory of superconductivity.

3. Give a brief account of high temperature superconductors.

4. Describe Type I and Type II superconductors.How does a superconductor differ from a normal conductor?

5. Describe how Cooper pairs are formed and explain the salient features of superconductivity.

6. Explain the working of SQUID.

7. Write short note on Maglev vehicles.

Part III

Module 3: Lasers and Optical Fibers

Chapter 5

Lasers and Holography

Lasers

Laser is an acronym for Light Amplification by Stimulated Emission of Radiation. Laser is a highly "monochromatic coherent beam of light of very high intensity". In 1960 Mainmann built the first "LASER" using Ruby as active medium.

5.1   Interaction of Radiation with matter

1. Stimulated Absorption:
When an atom in the ground state say E_1 absorbs a photon of energy $(E_2 - E_1)$ it makes transition into exited state E_2. This is called Stimulated or Induced absorption. It is represented as follows,

$$\text{Photon} + \text{Atom} = \text{Atom*}.$$

2. Spontaneous Emission: Spontaneous emission is one in which atom in the excited state emits a photon when it returns to its lower energy state without the influence of any external energy.
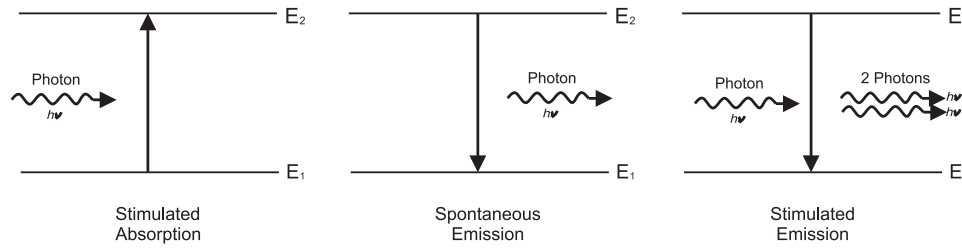
Figure 5.1: Emission and Absorption of Radiation

Consider an atom in the excited state E_2. Excited state of an atom is highly unstable. With in a short interval of time, of the order of 10^{-8}sec,atom returns to its lower energy state say E_1 and emits difference in energy in the form of photon of energy $h\nu = E_2$ - E_1 spontaneously.

If the two atoms are in the same excited state and returns to some lower energy states two photons of having same energy are emitted. These two photons may not travel in the same direction. They produce in-coherent beam of light. Spontaneous emission is represented as follows,

$$\text{Atom* = Atom + Photon.}$$

3. Stimulated Emission:

Consider an atom in the excited state E_2. If a photon of energy E_2 - E_1 is made to incident on the atom in the excited state E_2.

The incident photon forces (stimulates) the atom in the excited state to make transition in to ground state E_1 by emitting difference in energy in the form of a photon. This type of emission in which atom in the excited state is forced to emit a photon by the influence of another photon of right energy is called stimulated emission. Stimulated emission can be represented as follows.

$$\text{Photon + atom* = Atom + (photon + photon).}$$

When stimulated emission takes place, incident photon and the emitted photon are in phase with each other and travel along the same direction. Therefore they are coherent.

5.2 Einsteins A and B Coefficients and Expression for energy density

Let E_1 and E_2 be the two energy states of the system of atoms. Let N_1 and N_2 be the number of atoms with energy states E_1 and E_2 respectively. Let $U\nu$ represent the energy density of incident photon having frequency (ν). Hence the system absorbs and emits energy through the following processes

(a)**Stimulated Absorption:** In case of Stimulated absorption, the rate of absorption is given by number of absorptions per second per unit volume. Rate of absorption depends upon,
(1) Number of atoms in the lower energy state. (N_1)
(2) The energy density $(U\nu)$.
Hence, the Rate of absorption $\propto N_1 U\nu$ and Rate of absorption $= B_{12} N_1 U\nu$, where B_{12} is proportionality constant called Einsteins coefficient of induced absorption.

(b)**Spontaneous emission:** In case of Spontaneous emission, the rate of spontaneous emission is given by number of spontaneous emissions per second per unit volume.
Rate of Spontaneous emission depends upon,
(1) Number of atoms in the higher energy state (E_2). i.e. (N_2)
Hence, the Rate of Spontaneous emission $\propto N_2$
Rate of Spontaneous emission $= A_{21} N_2$.
Where A_{21} is proportionality constant Called Einsteins coefficient of Spontaneous emission.

(c)**Stimulated Emission:** In case of Stimulated Emission The rate of Stimulated emission is given by number of Stimulated emissions per second per unit volume. Rate of Stimulated Emission depends upon,
(1) Number of atoms in the higher energy state (E_2). i.e. (N_2)
(2) The energy density $(U\nu)$

Hence, the Rate of Stimulated emission $\propto N_2 U\nu$
Rate of Stimulated emission $= B_{21} N_2 U\nu$, where B_{21} is proportionality constant Called Einsteins coefficient of Stimulated Emission.

Under Thermal Equilibrium the total Energy of the System remains unchanged. Hence number of photons absorbed is equal to number of photons emitted.
Rate of Induced Absorption = Rate of Spontaneous emission + Rate of Stimulated Emission.
Therefore

$$B_{12} N_1 U\nu = A_{21} N_2 + B_{21} N_2 U\nu$$

$$\Rightarrow U\nu(B_{12} N_1 - B_{21} N_2) = A_{21} N_2.$$

After simplification

$$\Rightarrow U\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1} \right] \tag{5.1}$$

But according to the Boltzman's law of distribution the number of atoms in the higher energy state E_2 at any temperature "T" is given by

$$N_2 = N_1 e^{-\left(\frac{E_2 - E_1}{kT}\right)} = N_1 e^{-\frac{h\nu}{kT}}$$

$$\Rightarrow \quad \frac{N_1}{N_2} = e^{\frac{h\nu}{kT}}$$

Substituting for N_1/N_2 in equation (5.1)

$$\Rightarrow \ U\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1} \right] \tag{5.2}$$

According to Planck's Law of Radiation. The equation for Energy density is given by

$$U\nu = \frac{8\pi h\nu^3}{C^3} \left[\frac{1}{e^{\frac{h\nu}{kT}} - 1} \right] \tag{5.3}$$

Comparing (5.2) and (5.3) we have

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{C^3} \text{ and } \frac{B_{12}}{B_{21}} = 1$$

$$\text{or } B_{12} = B_{21}$$

(i.e. Probability of Stimulated absorption is equal to Probability of Stimulated emission.) Hence A_{21} and B_{21} can be replaced by A and B and the expression for the Energy density is given by,

$$U\nu = \frac{A}{B \left[e^{\frac{h\gamma}{kT}} - 1 \right]}$$

Where h is Plancks constant, k is Boltzmann Constant, T is Absolute Temperature, and c is the velocity of light.

5.3   Conditions for Laser Action

Let an atom in the excited state be stimulated by a photon of right energy so that atom makes stimulated emission. Two coherent photons are obtained. These two coherent photons
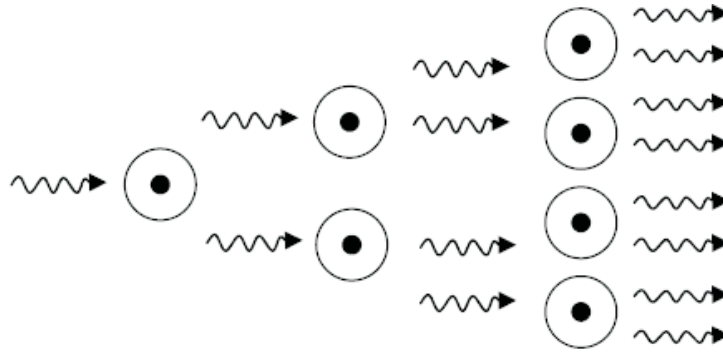
Figure 5.2: Laser action

if stimulate two atoms in the exited state to make emission then four coherent photons are produced. These four coherent photons then stimulates 4 atoms in the excited state to produce 8 coherent photons and so on. As the process continues number of coherent photons increases. These coherent photons constitute an intense beam of laser. This phenomenon of building up of number of coherent photons so as to get an intense laser beam is called lasing action.

Population inversion and optical pumping:
In an order to produce laser beam there should be more number of stimulated emissions when compared to spontaneous emission. It is possible only if number of atoms in the exited states is greater than that is in the ground state. When system is in thermal equilibrium, then number of atoms in the higher energy level is always less than the number of atoms in the lower energy level. If by some means number of atoms in the exited slate is made to exceed number of atoms in the ground state then **population inversion** is said to have established between excited state and ground state. The method of achieving the population inversion is called **pumping**. If light is used to pump electrons to the higher energy level then the method is called **Optical Pumping**.

The essential Conditions for population inversion are
(1) Higher energy state should posses a longer life time
(2) The number of atoms in the lower energy state must be greater than the number of atoms in the higher energy state.

Metastable State: Population inversion can be created with the help of three energy levels as follows.

Let E_1 is the ground state of an atom. Let E_2 and E_3 are the two excited states. If an atom is excited into the energy state, within a short inter of time of 10^{-8} sec, atom makes a transition into the energy state E_2. Let lifetime of the atom in the energy level E_2 is of

the order of 10^{-2} to 10^{-3} sec. Then atoms stay in the excited state E_2 for sufficiently long time without making any spontaneous emission.
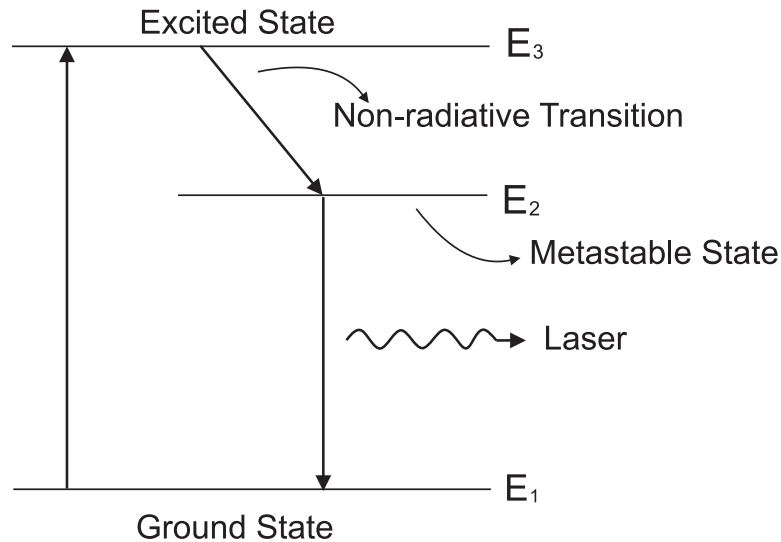

Figure 5.3: Population Inversion, E_2 is the metastable state

As more and more atoms are excited from the ground state to E_3 more and more atoms are transferred from E_3 to E_2. As a result, within a short interval of time population inversion is established between energy level E_2 and E_1. The energy level E_2 in which atoms remain for unusually longer time is called **Metastable state**. When transition from E_3 to E_2 takes place excited atom looses energy in the form of heat without emitting any radiation. Such transitions are called radiation less transition (Non-radiative transitions).

5.4   Requisites of a Laser System

The Three requisites of a Laser system are

(1) Energy Source or Excitation Source for Pumping action

(2) Medium Supporting population inversion called Active Medium

(3) The Laser Cavity

Appropriate amount of energy is to be supplied for the atoms in order excite them to higher energy levels. If the input energy is in the form of light energy then pumping is called optical pumping. If it is in the form of electrical energy then pumping is called electrical pumping.

Population inversion occurs at certain stage in the Active medium due to the absorption of energy. After this stage the Active medium is capable of Emitting laser light.

The Laser Cavity consists of an active medium bound between two mirrors. The Mirrors reflect the light to and fro through the active medium. This also helps to tap certain permissible part of laser energy from the active medium.

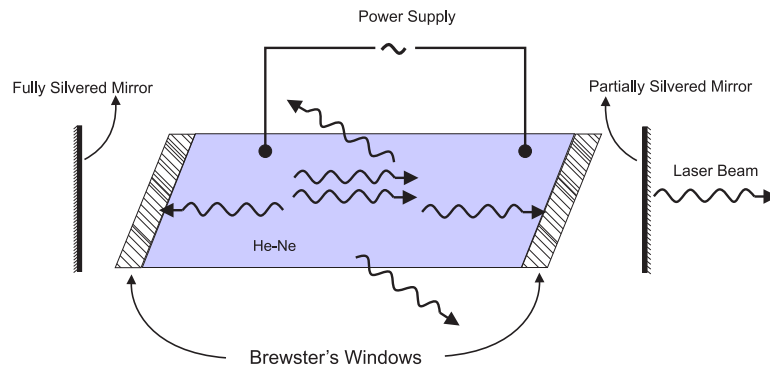## 5.5   Helium-Neon Laser (Gaseous state laser)

Figure 5.4: Helium-Neon laser

Construction: It consists of quartz discharge tube of length 50cm and diameter 0.5cm fitted with Brewsters windows on either side and filled with the mixture of He and Ne gas in the ratio of 10:1. It is placed between two highly parallel plane mirrors one of which is completely silvered while the other is partially silvered.

Working: The energy level diagram for He and Ne atoms are as shown in the fig. When discharge is produced in the tube large numbers of electrons are produced. These highly energetic electrons collide with His atoms, which are abundant and excite them to energy level 2s. This type of collision is called collision of first kind and represented as follows,

$$e_1 + He \rightarrow He^* + e_2$$
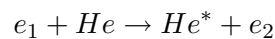
Where e_1 and e_2 are energies of electron before and after collision.

2s energy level of He is relatively metastable with the energy of about 20.61eV. 5s energy level of Neon with the energy 20.66eV is very close to 2s energy level of the He

atom. Therefore when the He atoms in the excited state 2s collide with less abundant Neon Atoms, they transfer their energies completely to neon atom in the ground state so that neon atom get excited to 5s energy level.
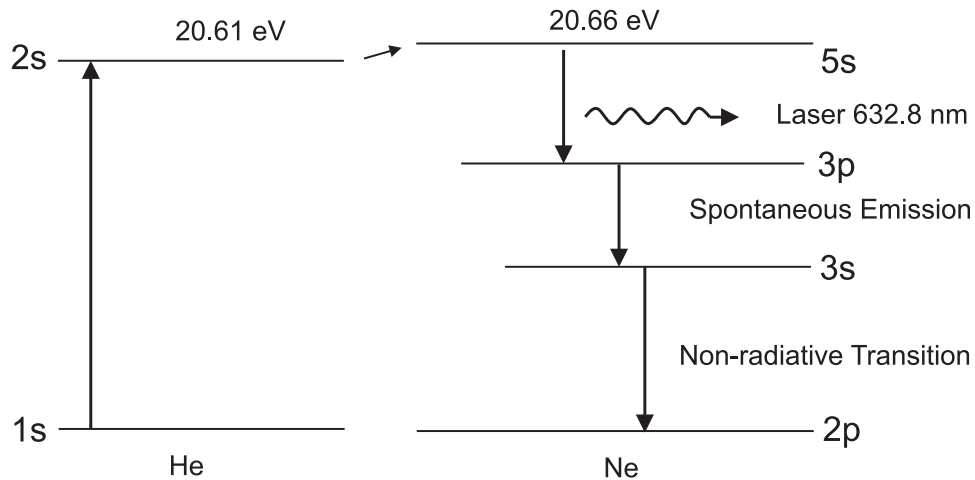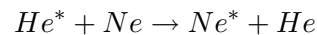

Figure 5.5: Helium-Neon energy level diagram

After transfering energy to Ne atom the He atom returns to its ground state. This type of energy transfer is called resonance transfer and collisions are called collision of second kind represented as below.

$$He^* + Ne \rightarrow Ne^* + He$$

Since number of Ne atoms is less when compared to number of He atoms, within a short interval of time population inversion is created between 5s and 3p energy level. The transition from 5s to 3p results in laser beam of wavelength $6328\mathring{A}$, transition from 3p to 3s takes place in the form of spontaneous emission and transition from 3s to 2p in the form of non- radiative transitions.

5.6   Semiconductor Laser (Injection Laser)

Light emitting diodes are basically semiconductor lasers. A widely used semiconductor laser is GaAs Laser (Gallium Arsenide).

Construction: The figure shows a typical Semiconductor laser. It consists of a heavily doped PN junction with a depletion layer of thickness 0.1 micrometre. Diode used is a cube with each edge 0.4mm long with the junction lying horizontal as shown in the figure. The current is passed through the ohmic contacts provided to the top and bottom faces. The front

and back faces are polished and made highly parallel to each other to have a laser cavity. The other two faces are roughened.
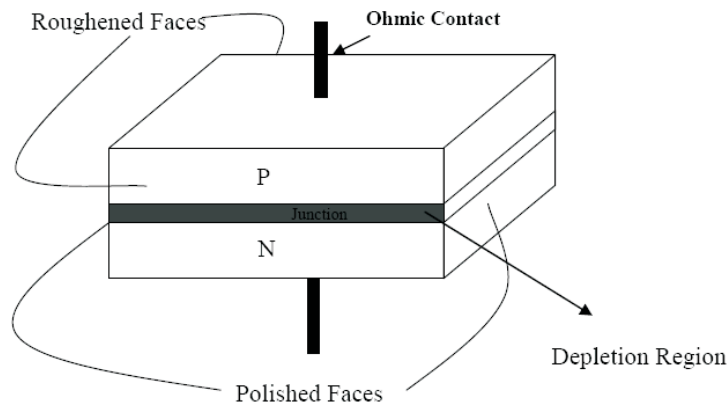

Figure 5.6: Semiconductor laser

Working: The Diode is forward biased using an external source. Therefore electrons and holes flow across junction. Hence the current flows through the diode. The semiconductor used in LED and Semiconductor LASER is a direct band gap semiconductor. In case of a direct band gap semiconductor when a hole meets an electron it recombines with electron emitting a photon. This could be considered as the transition of electron from conduction band to valance band. When the current is low spontaneous emission is predominant. If the current is sufficiently high population inversion is achieved. The photons liberated initially due to spontaneous emissions induce further stimulated emissions. The laser cavity helps in the amplification of light. Finally this results in an avalanche of photons and hence the laser action is achieved. If the GaAs semiconductor is used then the wavelength of the laser emitted is 840nm.

5.7   Applications of Laser

Engineering applications: The engineering applications of laser are
(1) Cutting, Drilling and Welding
(2) Measurement of pollutants in the atmosphere
(3) Holography etc.,

Laser Welding, Cutting and Drilling:
(1) **Welding:** focusing a beam of laser on the welding spot does laser welding. The heat generated melts the material over a tiny area on which the beam is focused. The impurities such as oxides float on the surface of the melt. Hence when cooled the welded region

becomes homogeneous solid structure. This makes the joint stronger. Since the Laser beam can be controlled with a high precision the welding is also done with high precision. Laser welding is a contact less process. Hence no foreign material can get into the welded joint. Laser welding is used in microelectronics in which components are sensitive to heat. Carbon dioxide lasers with a power output of 10 KW are used for this purpose.

(2) **Cutting:** In metals Laser cutting is done with the assistance of gas blowing. A nozzle through which oxygen gas is blown surrounds the focusing part of the Laser. Hence a fine jet of gas is also focused on the cutting spot to where the Laser beam is focused. The combustion of the gas burns the metal. The oxygen jet will blow the tiny splinters along with the molten part of the metal away. The Laser beam controls the accuracy of cutting not the burning gas. Laser cutting is used in Textile industry etc., The advantages of the laser cutting are (a) High quality Cutting (b) No thermal damage and chemical change etc., Low power carbon dioxide laser is used for cutting purposes.

(3) **Drilling:** Subjecting the material to pulses does Laser Drilling. The duration of the pulses will be of 0.1 ms to 1 ms. The intense heat generated over a short duration by the pulses evaporates the material locally. Hence the hole is left. Nd-YAG Laser is used in case metals but Carbon Dioxide Laser is used in case of both Metals and Non metals. The advantages of Laser drilling are (a) No tools wear out (b) Drilling can be achieved at any oblique angle (c) Very fine holes of dimension 0.2 to 0.5 mm can be drilled.

5.8   Measurement of pollutants in the atmosphere:

There are different types of pollutants in the atmosphere. They are
(1) Gasses like (a) Oxides of Nitrogen (b) Carbon monoxide (c) Sulphur dioxide
(2) Particulate matter such as (a) dust (b) smoke etc.
The measurement of pollutants is done using Laser and is referred to as LIDAR (Light detection and ranging). The **Lidar** system consists of a transmitter and receiver. The laser beam is sent through the atmosphere. The receiver receives the back-scattered light. Distance of the congestion from the measuring station is calculated on the time-delay between the pulse emission and the reception of the back-scattered light. The concentration of pollutants can be mapped for different vertical sections of the atmosphere by scanning space around the station. In this method the composition of the pollutant cannot be determined. The following methods can be employed to know the composition of the pollutants. In both the methods laser beam is passed through the sample of polluted air collected from the desired region.

1. **Absorption technique:** When the laser beam passes through the collected sample it

undergoes absorption of various degrees depending on the pollutant. Depending on the characteristic absorption pattern of the transmitted light the composition of the atmospheric pollutant can be determined.

2. **Raman scattering:** In this method the Raman-Spectrum of the transmitted light is obtained. This spectrum consists of two side bands in addition to the incident wavelength. The side bands are symmetrically spaced on both sides of the incident wavelength. The change in wavelength of the side bands is called Raman Shift. Based on Raman Shift the composition of the pollutant can be determined.
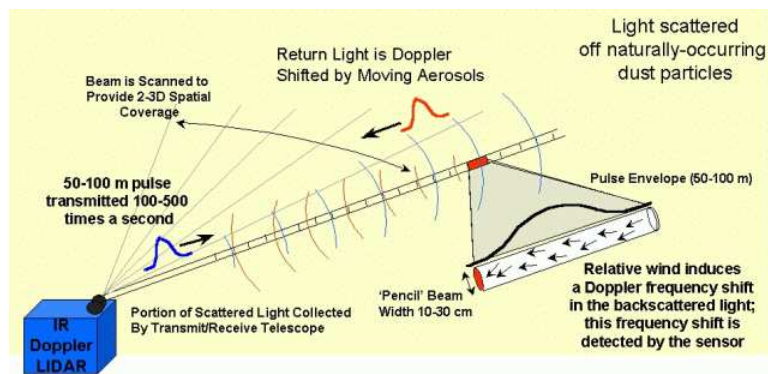

Figure 5.7: LIDAR effectively detects and characterizes air contaminants

Holography

HOLOGRAPHY is the technique of recording an image in which a light wave is a carrier of information and recording is done in terms of wave parameters like amplitude and phase.

5.9   Principle of Hologram Construction

When two highly coherent parallel light beams are incident on a photographic plate, one reflected from the object and the other incident directly, then an interference fringe pattern is recorded on a photographic plate due to interference which holds the 3D pictorial details of the object and the image of the of the object can be reconstructed by diffracting the light beam passing through the photographic plate.
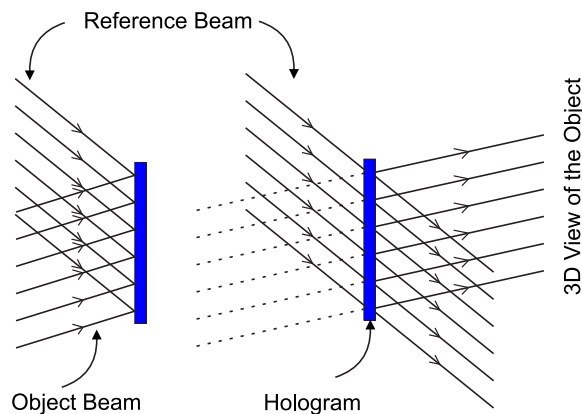

Figure 5.8: Interference of beams

Construction of a HOLOGRAM: The Object Beam which is the Coherent beam of light from a 3-D object is incident on the photographic plate. The reference beam, which is a coherent laser beam, is incident directly on the photographic plate. Due to interference, fringes are recorded on the plate. The photographic plate is developed. If the Reference beam is incident on the photographic in the same direction as it was earlier, the 3-D view of the object appears even though there is no object beam. This is because of the diffraction of light by the interference fringes on the photographic plate.

Phase Recording: Since the light waves which are reflected from different parts of the object travel different distances, the path difference will be different for such waves and

during interference the fringes formed depend on the depth of the point on the object. Hence the phase recording is automatically achieved.

5.10 Recording and Reconstruction of the image:

Recording of the image of the object.: Recording is done either by wave front division technique or amplitude division technique.
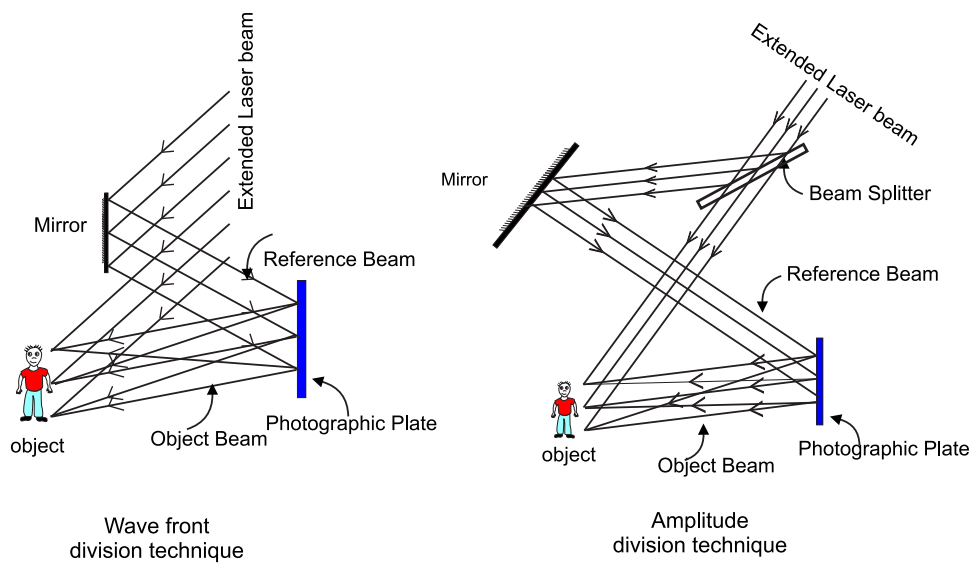

Figure 5.9: Hologram recording

Wave front division technique: In this technique an expanded laser beam is obtained from the source. This is incident on the mirror and object simultaneously (as shown in fig.). The mirror and object are so placed that, the mirror and the object reflect a part of the beam (wave front division). The reflected beams from mirror (reference beam) and from the object (object beam) are made to incident on the photographic plate as shown in the figure. Due to which interference fringes are formed on the photographic plate. The photographic plate is developed.

Amplitude division technique: In this Technique a Beam splitter is used to split the incident expanded laser beam into two parts (Amplitude division). One part is incident on the mirror and reflected onto the photographic plate (reference beam). The other part is incident on the object and reflected on to the photographic plate (object beam). Due to which interference fringes are formed on the photographic plate. The photographic plate is developed. Interference fringes are seen on the developed photographic plate.
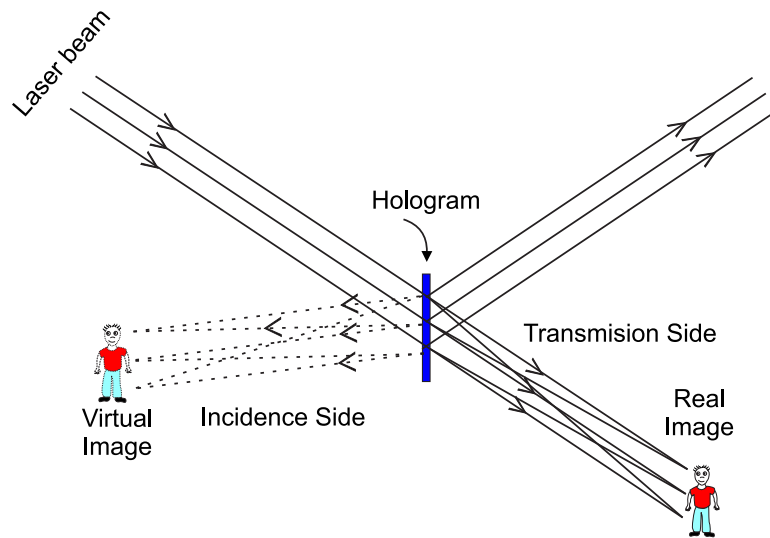
Figure 5.10: Reconstruction of image

Reconstruction of the image: The Original laser beam is directed at the hologram in the same direction as the reference beam in order to reconstruct the image. The reference beam undergoes diffraction in the hologram. Secondary wavelets originate due to diffraction. These secondary wavelets interfere constructively in certain direction and generate both a real and a virtual image of the corresponding point of the object on the transmission side of the hologram. It is as shown in the figure. The image can be photographed by keeping a photographic plate in the plane of the image formation of the convergent beam.

5.11   Applications of Holography

(1)**Holographic Diffraction Gratings;**
In a conventional grating rulings are made by the diamond tip fixed to the ruling engine. More uniform rulings can be produced by interference of two laser beams with plane wavefronts on a hologram.

(2)**Information coding**
Holograms are also used in the cases where the information has to be guarded. In this case an encoding mask is placed on the path of reference beam. This encoding mask generates special wave fronts which gets recorded on the photographic plate. In order to get back the information holograms has to be illuminated with the same kind of wave front only. If we

keep the masking details as secret then the hologram is guarded.

$* **$

Multiple Choice Questions

1. According to Boltzman's law of distribution the number of atoms in the higher energy state E_2 at any temperature "T" is given by

$$(a)\, N_2 = N_1 e^{-[\frac{E_2 - E_1}{kT}]} \quad (b)\, N_2 = N_1 e^{[\frac{E_2 - E_1}{kT}]}$$
$$(c)\, N_2 = N_1 e^{-[\frac{kT}{E_2 - E_1}]} \quad (d)\, N_2 = N_1 e^{[\frac{kT}{E_2 - E_1}]}$$

2. Metastable state is a state in which electrons can stay for

   (a) same time as the other excited state without making spontaneous emission

   (b) lesser time without making spontaneous emission

   (c) longer time without making spontaneous emission

   (d) none of these

3. Brewsters window is used to

   (a) reflect the incident beam of light

   (b) polarize the incident beam of light

   (c) diffract the incident beam of light

   (d) scatter the incident beam of light

4. Electrodes in He-Ne laser are used as

   (a) energy source for pumping action

   (b) medium supporting population inversion called active medium

   (c) the lasing cavity

   (d) none of these

5. Metastable state increases

   (a) spontaneous emission

   (b) stimulated emission

    (c) induced absorption

    (d) none of these

6. Einstein's A and B coefficients

    (a) are probability of spontaneous emission and probability of (induced absorption or stimulated emission) respectively

    (b) are the energies of photons A and B

    (c) both options (a) and (b) are correct

    (d) both options (a) and (b) are wrong

7. Rate of induced absorption depends on

    (a) no. of atoms in the lower energy state

    (b) the energy density

    (c) no. of atoms in the higher energy state

    (d) both options (a) and (b) are correct

8. Rate of spontaneous emission depends on

    (a) no. of atoms in the lower energy state

    (b) the energy density

    (c) no. of atoms in the higher energy state

    (d) both options (b) and (c) are correct

9. Rate of stimulated emission depends on

    (a) no. of atoms in the lower energy state

    (b) the energy density

    (c) no. of atoms in the higher energy state

    (d) both options (b) and (c) are correct

10. In Semiconductor laser the material used is

    (a) any semiconductor

    (b) direct band gap semiconductor

    (c) indirect band gap semiconductor

    (d) not a semiconductor

11. While reconstructing the image from hologram

(a) we need the object

(b) image can be reconstructed without the object and only virtual image is produced

(c) image can be reconstructed without the object and only real image is produced

(d) image can be reconstructed without the object, both real and virtual images are produced

12. In recording the image on the photographic plate the reference beam and the object beam undergo

(a) diffraction at the photographic plate

(b) reflection at the photographic plate

(c) interference at the photographic plate

(d) polarization at the photographic plate

Descriptive Questions

1. Explain the process of spontaneous and stimulated emission.

2. Obtain an expression for energy density of radiation under equilibrium condition in terms of Einstein's co-efficients

3. Write a note on measurement of pollutants in an atmosphere using laser.

4. Explain the construction and working of He-Ne gas LASER with energy level diagram.

5. Write a note on Holography.

6. Mention the characteristic of a laser beam.

7. What are semiconductor diode lasers? Describe with energy band diagram the construction and working of semiconductor diode laser. Mention the uses of diode lasers.

8. Describe briefly the application of lasers in welding, cutting and drilling. Mention the nature and property of the lasers used.

9. Explain with sketches the basic principle of operation of lasers.

10. Describe the recording and reconstruction processes in Holography with the help of suitable diagrams.

11. With a neat sketch label the requisites of a typical laser system.Explain why two-level energy system is not feasible to obtain laser action at thermal equilibrium?

12. Write the features of He-Ne laser system with respect to the following:

    (i) Active medium   (ii) Energy level diagram   (iii) Laser cavity
    (iv) Operating wavelengths and   (v) Applications

Exercise

1. The ratio of population of two energy levels out of which upper one corresponds to a metastable state is 1.059×10^{-30}. Find the wavelength of light emitted at $330K$.

2. Find the number of modes of the standing waves and their frequency separation in the resonant cavity of length1 mm of He-Ne laser operating at wavelength $632.8nm$.

3. A laser medium at thermal equilibrium temperature $300K$ has two energy levels with a wavelength separation of 1μ. Find the ratio of population densities of the upper and lower levels.

4. A He-Ne laser is emitting a beam with an average power of $4.5mW$. Find the number of photons emitted per decond by the laser. The wavelength of the emitted radiation is $6328\mathring{A}$.

5. Find the ratio of population of two energy levels in a Laser if the transition between them produces light of wavelength $694.3nm$. Assume the ambient temperature to be 27^0C.

Figure 5.11: Reconstruction of 3D image

Chapter 6

Optical fibers

Optical fibers are transparent Dielectrics, which guide light over long distances with the help of successive **Total Internal Reflections**.
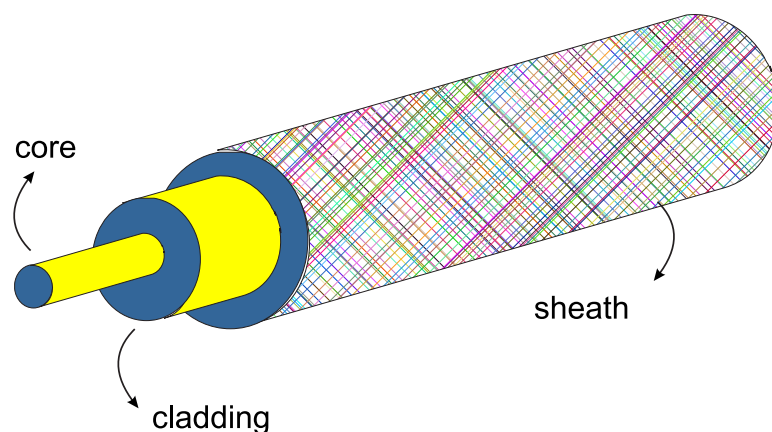

Figure 6.1: Optical fiber

It consists of three regions
(1) The innermost Light guiding region called **Core**.
(2) The middle region-covering core made of material similar to Core is called **Cladding**. The RI of Cladding is less than that of Core.
(3) The outermost protecting layer for Core and Cladding from moisture, crushing and chemical reaction etc., is called **Sheath**. The Optical Fibers are either made as a single fiber or a **flexible bundle or Cables**. A **Bundle fiber** is a number of fibers in single jacket.

Principle of Optical fibres: It is based on the principle of Total Internal reflection. Con-

sider a ray of light passing from denser medium to rarer medium. As the angle of incidence increases the angle of refraction also increases. For a particular angle of incidence called Critical Angle the refracted ray just grazes the interface (Angle of refraction is 90^0). If the angle of incidence is greater than Critical Angle then the ray reflected back to the denser medium. This phenomenon is called **Total Internal Reflection**.
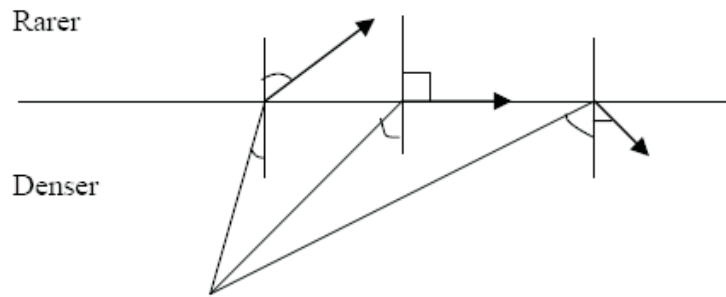

Figure 6.2: Total internal reflection

6.1   Propagation of light through fiber(Optical fiber as a light guide):

The main function of the Optical fiber is to accept maximum light and transmit the same with minimum attenuation.

The incident light enters the core and strikes the interface of the Core and Cladding at large angles as shown in fig. Since the Cladding has lower RI than Core the light suffers multiple Total Internal Reflections. This is possible since by geometry the angle of incidence at the interface is greater than the Critical angle. Since the Total internal reflection is the reflection at the rarer medium there is no energy loss. Entire energy is transmitted through the fiber. The propagation continues even if the fiber is bent but not too sharply. Since the fiber guides light it is called as **fiber light guide** or **fiber waveguide**.

6.2   Angle of Acceptance and Numerical Aperature

Light gathering ability of a fiber depends on two facts.
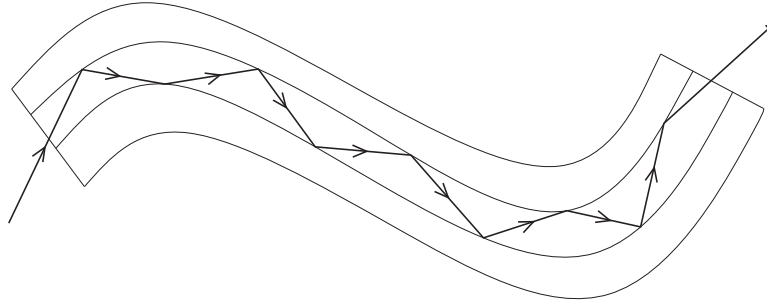(1) Core size.
(2) Numerical Aperture.

Figure 6.3: Fiber as a waveguide

Numerical aperture of fiber is determined by **Acceptance angle**. Let a light beam enter at an angle "i" to the axis of the fiber (On the launching face). The ray gets refracted at an angle "r" and strikes the Core Cladding interface at an angle θ. If $\theta > \theta_c$ (Critical angle) for the system T I R takes place. It means as long as θ is greater than Critical angle light stays within the fiber.

In order to maintain θ to be greater than Critical angle, the angle of incidence relative to Axis of the fiber (At the launching face) should not be greater than a value θ_0. The ray corresponding to θ_0 can be used to describe a Conical surface. The cone formed is called **Acceptance cone** and the angle θ_0 is called **Acceptance angle**. The sine of acceptance angle (ie. $\sin \theta_0$) is called **Numerical aperture**.
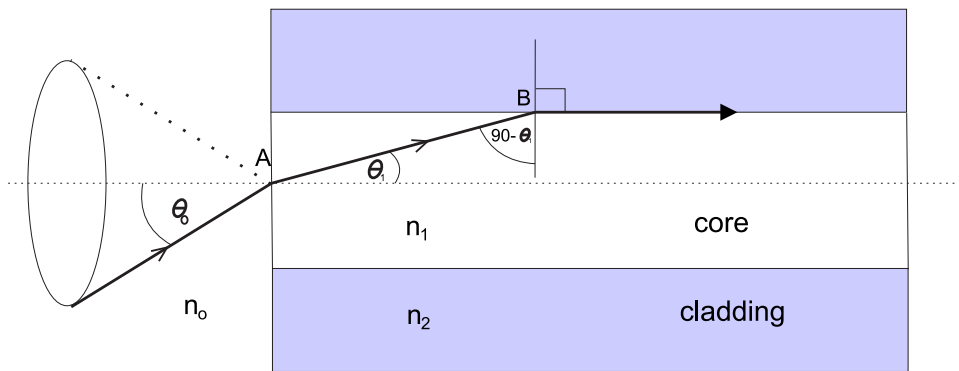

Figure 6.4: Acceptance cone

6.3 Expression for Numerical Aperture

Let n_0, n_1 and n_2 be the Refractive indices of Surrounding, Core and Cladding respectively. Also RI of Core is Greater than RI of Cladding (i.e $n_1 > n_2$).
By applying Snell's law for the Launching face of Fiber

$$n_0 \sin \theta_0 = n_1 \sin \theta_1$$

$$\therefore \;\; \sin \theta_0 = \frac{n_1}{n_0} \sin \theta_1 \tag{6.1}$$

By applying Snell's law at 'B'

$$n_1 \sin(90 - \theta_1) = n_2 \sin(90)$$

Where $(90 - \theta_1)$ is Critical angle

$$\cos \theta_1 = \frac{n_2}{n_1}$$

$$\sin \theta_1 = \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin \theta_1 = \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

Substituting for $\sin \theta_1$ in (6.1)

$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$(\text{NumericalAperture}) NA = \sin \theta_0 = \sqrt{\frac{n_1^2 - n_2^2}{n_0^2}}$$

for air $n_0 = 1$

$$\therefore NA = \sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

If 'i' is the angle of incidence at the launching face then the condition for propagation of light through the optical fiber is

$$i \leq \theta_0$$

$$\sin i \leq \sin \theta_0$$

$$\sin i \leq NA$$

sine of the angle of incidence must be less than or equal to numerical aperture
Fractional RI Change($\triangle$):
It is the ratio of RI difference between Core and Cladding to the RI of core

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Relation between NA and Δ

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\Delta n_1 = n_1 - n_2 \tag{6.2}$$

we know that

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$NA = \sqrt{(n_1 - n_2)(n_1 + n_2)}$$

from equation (6.2)

$$NA = \sqrt{(n_1 + n_2)(\Delta)n_1}$$

if $n_1 \cong n_2$

$$NA = \sqrt{\Delta n_1 (2n_1)}$$

$$\therefore NA = n_1\sqrt{2.\Delta}$$

6.4   Modes of Propagation

Though optical fiber should support any numbers of rays for propagation practically it is found that it allow only certain restricted number of rays for propagation. The maximum number of rays supported by the fiber is called **Modes of propagastion**. Modes of propagation can be determined using parameter called V-number.

V-number (Normalised Frequency of the fibre):
An Optical fibre may be characterized by one more parameter called V-number. This determines the Number of modes supported for propagation in the fibre.

$$V = \frac{\pi d}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{\pi d}{\lambda}(NA)$$

where "d is the diameter of the core, λ is wavelength, n_1 is the refractive index of the core and n_2 is the refractive index of the cladding. NA is numerical Aperture. If the fiber is

surrounded by a medium of refractive index n_0, then

$$V = \frac{\pi d}{\lambda} \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

If $V \gg 1$, the number of modes supported by fiber can be determined using the formula

$$Number of Modes \approx V^2/2$$

6.5   Types of Optical Fibers

Based on R I profile and Core size and Modes of Propagation Optical fibers are classified into 3 types,

(1) Single mode step index fiber.

(2) Multi mode step index fiber.

(3) Graded index multi mode or Grain index fiber.

Single mode step index fiber: A single mode step index fiber consists of a very fine thin
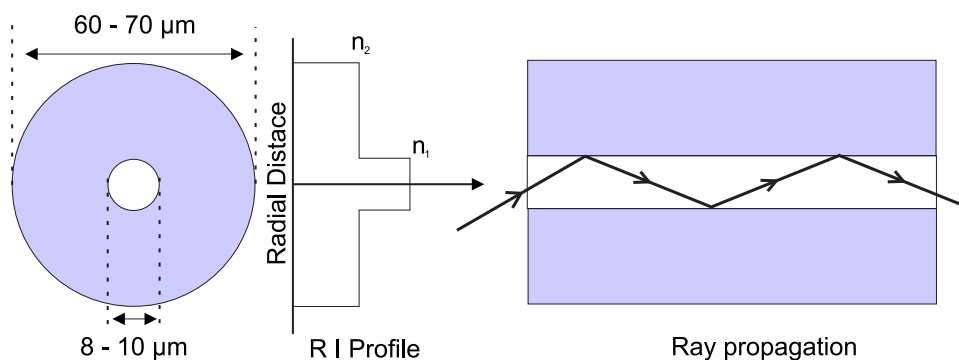

Figure 6.5: Single mode step index fiber

core of uniform RI surrounded by Cladding of RI lower than that of Core. Since there is abrupt change of RI of Core and Cladding at the interface it is called Step index fiber. Since the Core size is small the Numerical aperture is also small. They accept light from laser source. Splicing is difficult. They are used in submarine cables.

Multimode step index fiber:

This is similar to single mode step index fiber with the exception that it has a larger core diameter. The core diameter is very large as compared to wavelength of light transmitted. A typical multimode step index fiber is as shown above. The numerical aperture is
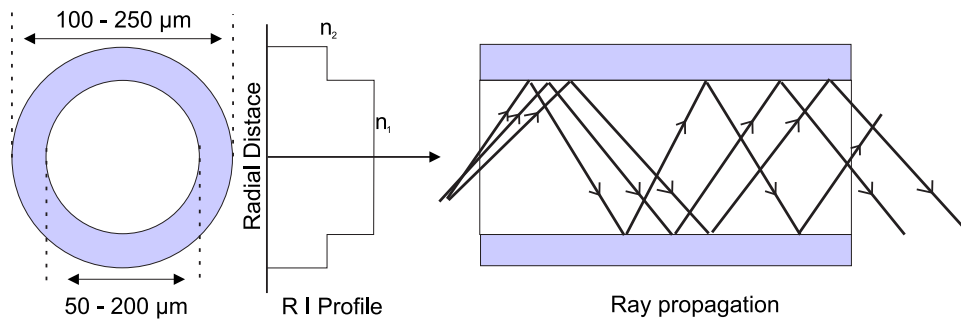
Figure 6.6: Multi mode step index fiber

large because of large core size. They accept light from both laser as well as from LED. They are used in data links.

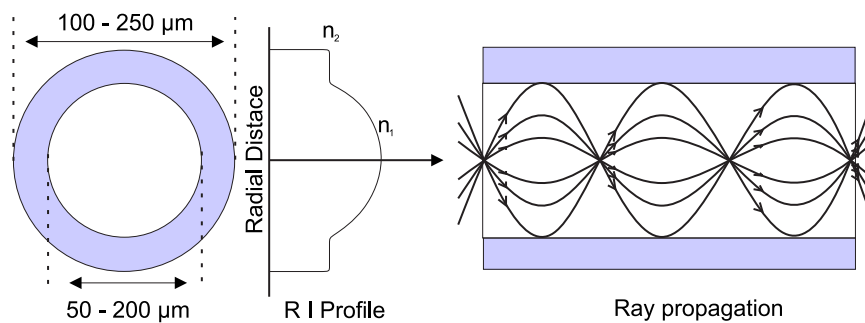**Graded index multimode fiber(GRIN fiber):**

Figure 6.7: Graded index multimode fiber

The multimode fiber, which has concentric layers of RI, is called GRIN fiber. It means the R I of the Core varies with distance from the fiber axis. It has high R I at Center and falls rapidly as radial distance increases from the axis. The R I profile is as shown in fig. In GRIN fibers the acceptance angle and numerical aperture diminishes with radial distance. The light transmission is as shown above. They accept light from both laser as well as from LED. They are used for medium distance communication for example telephone link between central offices.

6.6   Attenuation and Fiber losses

An Optical signal passing through a fiber will get progressively reduced. This reduction or attenuation of signal may be defined as the ratio of the optical output power from a fibre of length L to the input optical power. It is expressed in terms of decibel / km.

$$\alpha = -\frac{10}{L} \log_{10} \left(\frac{P_2}{P_1} \right)$$

Where $P_1 \rightarrow$ Power of optical signal at launching end (input power)

$P_2 \rightarrow$ Power of optical signal at receiving end (output power)

The attenuation is wavelength dependent and so the wavelength must also be specified.

The attenuation in fibers gives rise to the following three losses

(1) Absorption losses

(2) Geometric Effects

(3) Rayleigh Scattering.

Absorption: Even very pure glass absorbs light of a specific wavelength. Strong electronic absorption occurs in **UV** region and vibrational absorption occurs in **IR** region of wavelength 7 micrometer to 12 micrometer. These losses are attributed due to inherent property of the glass and are called intrinsic absorption. However, this loss is insignificant.

Impurities are major extrinsic source of losses in fiber. Hydroxyl radical ions (OH) and transition metals like Nickel, Chromium, Copper, Manganese etc. have electronic losses near visible range of spectrum. These impurities should be kept as minimum as possible in the fiber. Intrinsic as well as extrinsic losses are found to be minimum at about 1.3 micrometer.

Geometric effects: These may occur due to manufacturing defects like irregularities in fibre dimensions during drawing process or during coating, cabling or insulation processes.

The microscopic bends are the bends with radii greater than fiber diameter. The micro bends couple light between the various guided modes of the fiber and some of them then leak through the fiber.

Rayleigh Scattering: As glass has disordered structure having local microscopic variation in density which may also cause variation in RI. So light traveling through these structures may suffer scattering losses due to Rayleigh ie., scattering $\alpha = 1/\lambda^4$. It means Rayleigh scattering sets a lower limit on wavelength that can be transmitted by a glass fibre at 0.8 micrometer below which scattering loss is appreciably high.

Dispersion: A pulse launched with a fibre gets attenuated due to losses in fibre. Moreover the incoming pulse also spreads during the transit through the fibre. So a pulse at the output is wider than the pulse at the input i.e., the pulse gets distorted as it moves through the fibre. This distortion of pulse is due to dispersion effects, which is measured in terms nanoseconds per km. There are three phenomena that may contribute towards the distortion effect.

(1) Material dispersion

(2) Wave guide dispersion

(3) Intermodal dispersion.

In optical fibers the Cladding material has uniform RI. But the RI of Core may remain constant or subjected to variation in a particular way. The curve representing the variation of RI with the Radial distance from the axis of the Core is called **Refractive index profile**. Modes of Propagation is the number paths of light rays along which the waves are in phase inside the fibre. The number of modes, a fibre can support depends on the ratio d/λ where d is the diameter of the core and λ is the wavelength of the wave transmitted.

6.7   Applications:

(1) **Communication:** Optical fibers are used in communication to transmit signals for long distances. An optical signal derived from electrical analog signal is transmitted through the optical fiber. At the other end again the optical signal is converted into electrical signal.
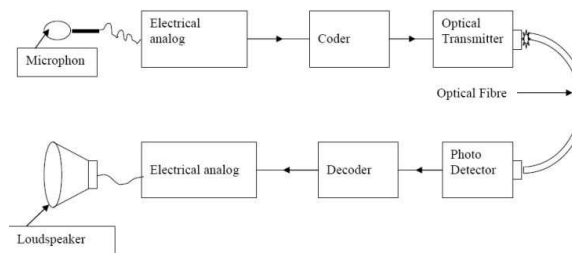

Figure 6.8: Typical point to point fiber optic communication system

For example: First voice is converted into electrical signal using a transducer. It is digitized using a Coder. The digitized signal, which carries the voice information, is fed to an optical transmitter. The light source in optical transmitter (L E D or Laser Diode) emits modulated light, which is transmitted through optical fiber.

At the other end the modulated light signal is detected by a photo detector and is decoded using a decoder. Finally the information is converted into analog electrical signal and is fed to a loud speaker, which converts the signal to voice (sound).

(2) **Fiber Endoscope:** Endoscope is an optical instrument used to visualize or photograph Human internal body parts. The light needed for photographing the part is carried by the optical fiber.

(3) Optical fibers are used in local area networks.

(4) They are used in Data Links.

Advantages:

(1) Optical fibers can carry very large amounts of information in either digital or analog form.

(2) The raw material for optical fiber is of low cost and abundant.

(3) It has low cost /meter/ channel.

(4) Cables are very compact.

(5) Signals are protected from radiation from lightning or sparking.

(6) There is no energy radiation from fiber.

(7) No sparks are generated.

Disadvantages:

(1) The optical connectors are very costly.

(2) Maintenance cost is high.

(3) They cannot be bent too sharply.

(4) They under go structural changes with temperature.

$* **$

Multiple Choice Questions

1. Propagation of light through fiber core is due to

    (a) diffraction   (b) interference

    (c) total internal reflection   (d) refraction

2. In an optical fiber if n_1 is the refractive index of core and n_2 the refractive index of cladding then

    (a) $\frac{n_1 - n_2}{n_1} < 1$ (b) $\frac{n_1 - n_2}{n_1} > 1$

    (c) $\frac{n_1 - n_2}{n_1} = 0$ (d) $\frac{n_1 - n_2}{n_1} = \infty$

3. Numerical aperture (NA) of the fiber is related to acceptance angle by

(a) $\text{NA} = \sin\alpha_i = \frac{\sqrt{n_2^2 - n_1^2}}{n_0}$ (b) $\text{NA} = \sin\alpha_i = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

(c) $\text{NA} = \sin^{-1}\alpha_i = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$ (d) $\text{NA} = \sin^{-1}\alpha_i = \frac{\sqrt{n_2^2 - n_1^2}}{n_2}$

4. Choose the correct statement

   (a) cladding has higher refractive index than core

   (b) core has higher refractive index than cladding

   (c) cladding is for providing mechanical strength to the fiber

   (d) the loss in the waveguides without cladding is very low

5. Step index fiber can be a

   (a) monomode fiber only

   (b) multimode fiber only

   (c) monomode as well as multimode fiber

   (d) either monomode or multimode(cannot be both)

6. Choose the correct statement

   (a) Intermodal dispersion occurs in graded index fibers only

   (b) Intermodal dispersion is minimum in graded index fibers

   (c) Intermodal dispersion can not occurs in step index fibers

   (d) none of the above is correct

7. The process of mixing the signal with the carrier is called

   (a) dispersion   (b) attenuation
   (c) modulation   (d) demodulation

Descrptive Questions

1. With neat diagram explain different types of optical fiber, along with the refractive index profile and mode propagation sketches.

2. What is attenuation? Explain the factors contributing to the fiber loss.

3. Explain the principle of light propagation in an optical fiber. Derive the expression for numerical aperture in term of refractive indices of core and cladding.

4. Discuss the advantages and disadvantages of optical fiber communication system over conventional communication system.

5. With the neat diagram explain the terms:

$$(i)\,Acceptance\,angle\,and\quad(ii)\,Numerical\,aperture.$$

6. What is total internal reflection?

7. What is acceptance cone half angle?

Exercise

1. The N.A of an optical fiber is 0.2 when surrounded by air. Determine R.I of core given R.I of cladding is 1.59 also find acceptance angle when the fiber is surrounded by water.(given RI of water = 1.33)

2. Calculate the number of modes that can be propagated inside an optical fiber, given $n_{core} = 1.53$, $n_{clad} = 1.50$, core radians $50\mu m$, $\lambda = 1\mu m$.

3. An optical signal has lost 85% its power after traversing 400 m of fiber. What is the fiber loss?

4. Calculate the number of modes an optical fiber can transmit, given the following data: wavelength of light=$1\mu m$, radius of the core=$50\mu m$, Refractive index of the core=1.50, Refractive index of the cladding 1.48.

5. The attenuation of a optical fiber is $-3.6dB/km$. What is the fraction of light intensity that remains after (i)$1km$ (ii) after $3km$?

6. Calculate the numerical aperture,fractional index change and V-number for a fiber of core diameter $40\mu m$ and with refractive indices of 1.55 and 1.50 respectively for core and cladding. The wavelength of the propagation wave is $1400nm$. Assume that the fiber is in air.

Part IV

Module 4: Crystal Structure

Chapter 7

Crystal Structure

7.1 Introduction

A crystal is a solid composed of atoms or other microscopic particles arranged in an orderly repetitive array. The study of crystal physics aims to interpret the macroscopic properties in terms of properties of the microscopic particles of which the solid is composed. The arrangement of atoms in a Crystal is called **Crystal Structure**.

Lattice points and Space Lattice:
Points can be imagined in space about which atoms or molecules are located. Such points are called **Lattice Points.** The totality of such points is called **Space Lattice** or **Crystal Lattice.**The Three-Dimensional space lattice (3-D Space lattice) may be defined as a finite array of lattice points in three-dimension in which each and every lattice point has identical surrounding in the array.
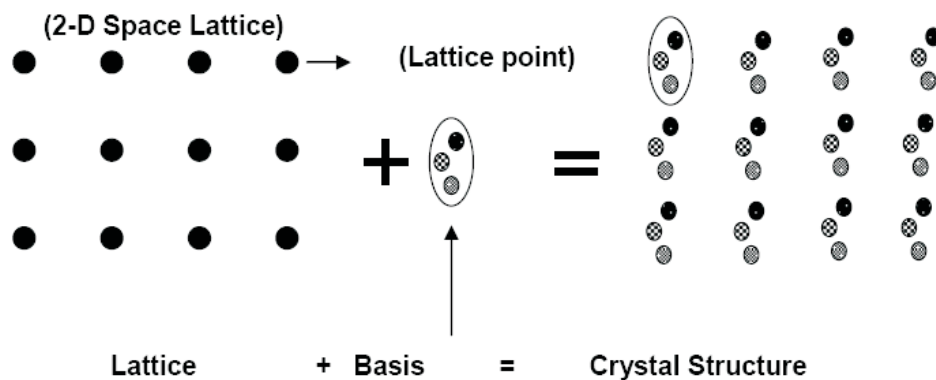

Figure 7.1: Crystal structure

Basis and Crystal structure: Every lattice point can be associated with one or unit assembly of atoms or molecules identical in composition called **Basis or Pattern.** The regular periodic three-dimensional arrangement of Basic is called **Crystal Structure.** Space lattice is imaginary. Crystal structure is real.

Bravais and Non-Bravais lattice: A Bravais lattice is one in which all lattice points are identical in composition. If the lattice points are not identical then lattice is called Non-Bravais lattice.

The set of lattice points $\otimes's$ together constitutes a Bravais lattice. Similarly the set of lattice
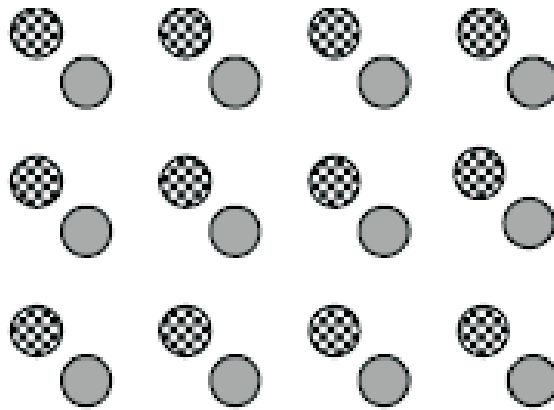

Figure 7.2: Non-Bravais lattice

points $\bigcirc's$ together constitutes a Bravais lattice. But set of all lattice points $\otimes's$ and $\bigcirc's$ together constitutes a Non-Bravais lattice. Hence a Non-Bravais lattice could be considered as the superposed pattern of two or more interpenetrating Bravais lattices.

Unit cell and Lattice parameters:

In every crystal some fundamental grouping of particles is repeated. Such fundamental grouping of particles is called **unit cell**. A unit cell is chosen to represent the symmetry of the crystal. Hence the unit cell with maximum symmetry is chosen. They are the basic building blocks of the crystal. When these unit cells are transferred in three dimension that will generate the crystal.

Each crystal lattice is described by type of unit cell.But each unit cell is described three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ when the length of the vectors and the angles (α, β, γ) between them are specified. They are nothing but the intercepts of the faces and the interfacial angles. All together they constitute **lattice parameters**.

Primitive Cell: Some times reference is made to a primitive cells. Primitive cell may be
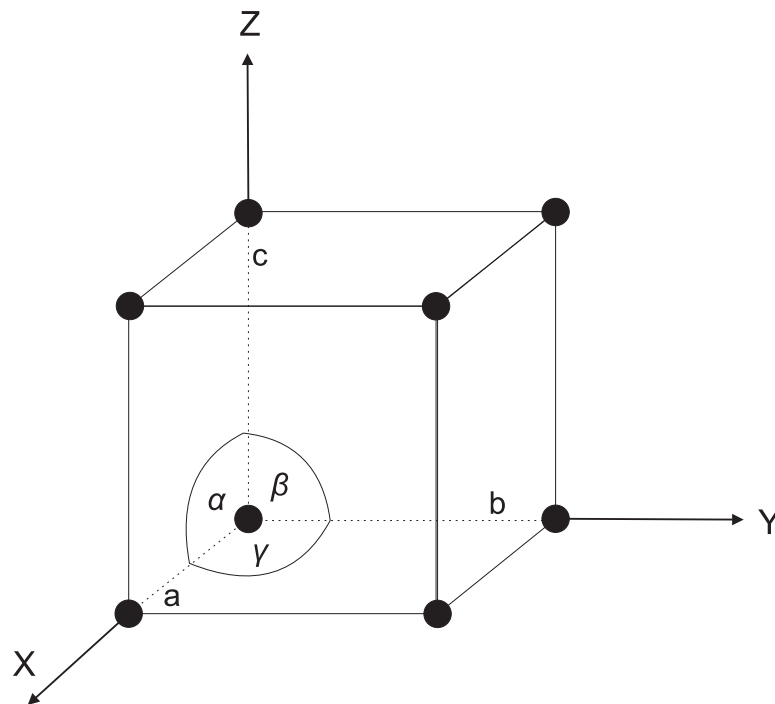
Figure 7.3: Unit cell

defined as a geometric shape which, when repeated indefinitely in three dimensions, will fill all space and it consists of lattice points only at corners. It consists of only one atom per cell. Therefore unit cells may be primitive (simple) or Non-primitive.

7.2 Crystal system and Bravais space lattice

Based on lattice parameters crystals are classified into seven basic systems. If atoms are placed only at corners seven crystal system yield seven lattices. But few more lattice could be constructed by placing atoms at face center, body center and base centred., Bravais showed that there are 14 such lattices that exits in nature. Hence the name of the Bravais space lattice. Each crystal system differs from the other in lattice parameters.

(1) **Cubic Crystal System(Isometric)** $\quad a = b = c \ and \ \alpha = \beta = \gamma = 90^0$.

(Simple or primitive, Face centered (FCC)and Body centered (BCC)).

(2) **Tetragonal Crystal System** $\quad a = b \neq c \ and \ \alpha = \beta = \gamma = 90^0$

(Simple and body centered).

(3) **Orthorhombic Crystal System** $\quad a \neq b \neq c \ and \ \alpha = \beta = \gamma = 90^0$

(Simple, Face centered (FCC), Body centered(BCC) and Base centered.

(4)**Monoclinic Crystal System** $\quad a \neq b \neq c \ and \ \alpha = \beta = 90^0 \neq \gamma$
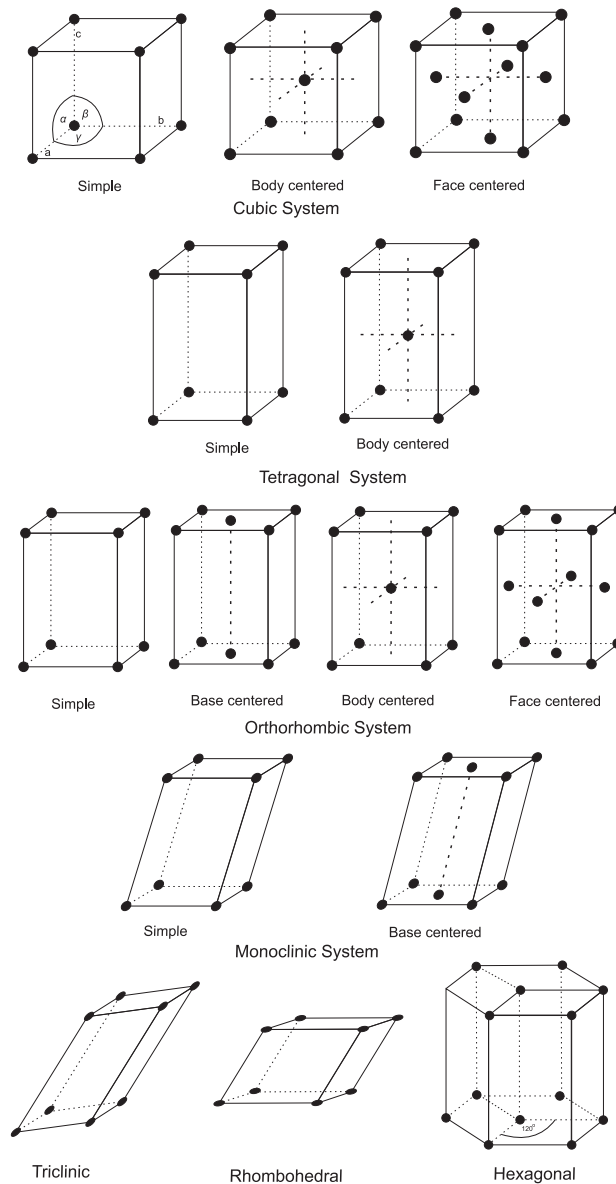
(Simple and Base centered).

Figure 7.4: Crystal Systems

(5)**Triclinic Crystal System** $\qquad a \neq b \neq c \ \ and \ \ \alpha \neq \beta \neq \gamma \neq 90^0$
(Simple).

(6)**Trigonal Crystal System(Rhombohedral)** $a = b = c \ \ and \ \ \alpha = \beta = \gamma \neq 90^0$
(Simple)

(7)**Hexagonal Crystal System** $\qquad a = b \neq c \ \ and \ \ \alpha = \beta = 90^0, \gamma = 120^0$
(Simple)

The fourteen Bravais space lattice are as shown in Figure 7.4

7.3 Directions and Planes in a crystal

Directions: In crystals there exists directions and planes in which contain concentration of atoms. It is necessary to locate these directions and planes for crystal analysis. Arrows in two dimensions show directions.**The directions are described by giving the coordinates of the first whole numbered point ((x, y) in two dimension,(x,y,z) in three dimension) through which each of the direction passes**. Directions are enclosed within square brackets.
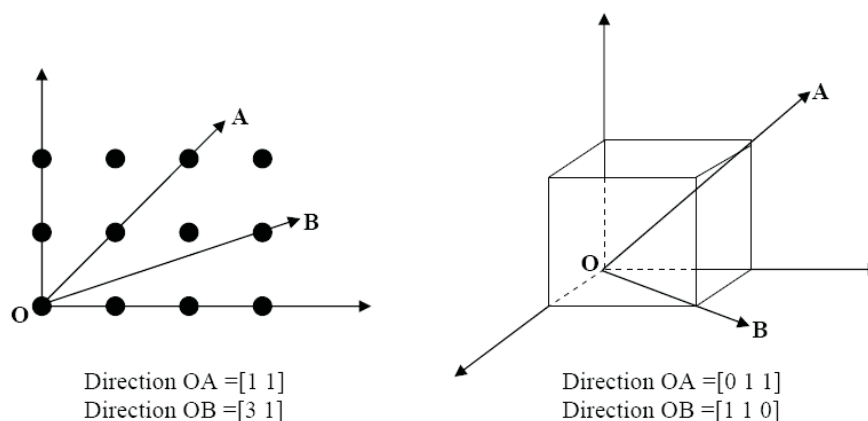
Eg.

Direction OA =[1 1]
Direction OB =[3 1]

Direction OA =[0 1 1]
Direction OB =[1 1 0]

Figure 7.5: Direction of line in a lattice

Planes: The crystal may be regarded as made up of an aggregate of a set of parallel equidistant planes, passing through the lattice points, which are known as **lattice planes**. These lattice planes can be chosen in different ways in a crystal. The problem in designating these planes was solved by **Miller** who evolved a method to designate a set of parallel planes in a crystal by three numbers (h k l) called **Miller Indices**.
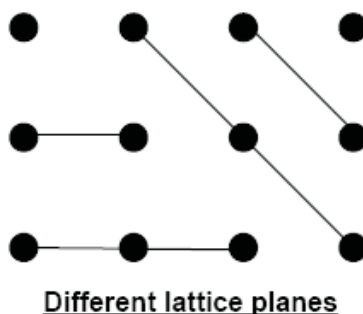

Different lattice planes

Figure 7.6: Plane orientations in a lattice

7.4   Miller indices

Miller indices may be defined as the reciprocals of the intercepts made by the plane on the crystallographic axes when reduced to smallest numbers.

Steps to determine Miller Indices of given set of parallel planes: Consider a plane ABC which is one of the planes belonging to the set of parallel planes with miller indices (h k l). Let x, y and z be the intercepts made by the plane along the Three crystallographic axes X, Y and Z respectively.

(1) Determine the coordinates of the intercepts made by the plane along the three crystallographic axes.

(2) Express the intercepts as multiples of the unit cell dimensions, or lattice parameters along the axes.

(3) Determine the reciprocals of these numbers.

(4) Reduce them into the smallest set of integral numbers and enclose them in simple brackets. (No commas to be placed between indices)

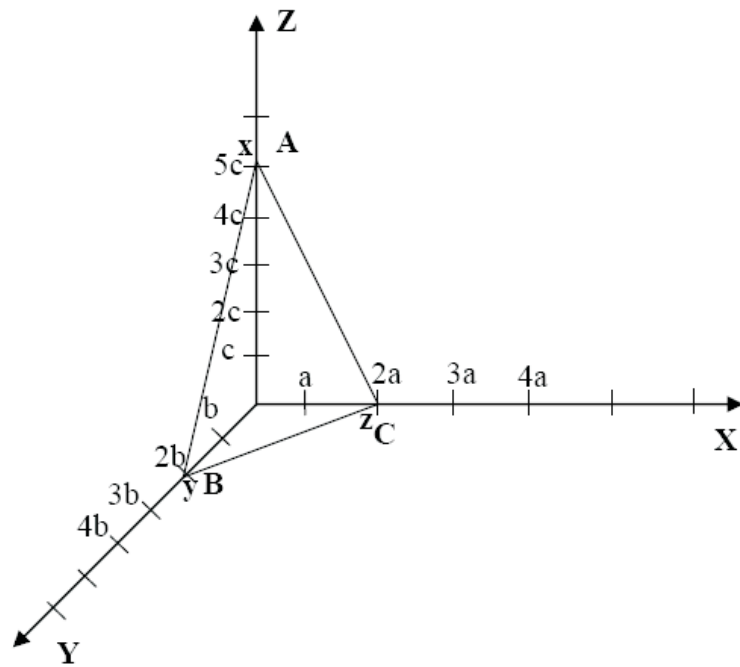

Figure 7.7: Miller plane orientation

Example:

Lets determine the Miller indices for the plane shown in the fig (7.7)

(1) The intercepts $x = 2a$, $y = 2b$ and $z = 5c$. In general, $x = pa$, $y = qb$, $z = rc$.

(2) The multiples of lattice parameters are

$$\frac{x}{a} = \frac{2a}{a} = 2, \frac{y}{b} = 2, and \frac{z}{c} = 5$$

(3) Taking the reciprocals

$$\frac{a}{x} = \frac{1}{2}, \frac{b}{y} = \frac{1}{2}, and \frac{c}{z} = \frac{1}{5}$$

(4) Reducing the reciprocals to smallest set of integral number by taking LCM.

$$10 \times \frac{1}{2}, 10 \times \frac{1}{2}, 10 \times \frac{1}{5}$$

$$\underline{\quad 5 \qquad 5 \qquad 2 \quad}$$

Miller indices of plane ABC=(h k l)=(5 5 2)

Note:

(a) All parallel equidistant planes have the same Miller indices.

(b) If the Miller indices have the same ratio, then the planes are parallel.

(c) If the plane is parallel to any of the axes, then the corresponding intercepts is taken to be ∞.

7.5 Expression for Interplanar spacing in terms of Miller Indices

Consider a Lattice plane ABC, which is one of the planes belonging to the set of planes with Miller indices (h k l). Let x, y and z be the intercepts made by the plane along the Three crystallographic axes X, Y and Z respectively.

Let OP be the perpendicular drawn form the origin to the plane. Let α', β' and γ' and be the angles made by OP with the crystallographic axes X, Y and Z respectively. Let another consecutive plane parallel to ABC pass through the origin. Let a, b and c be the lattice parameters. OP is called interplanar spacing and is denoted by d_{hkl}.

From right angled triangle OCP

$$\cos \alpha' = \frac{OP}{OC} = \frac{d_{hkl}}{x}$$

From right angled triangle OBP

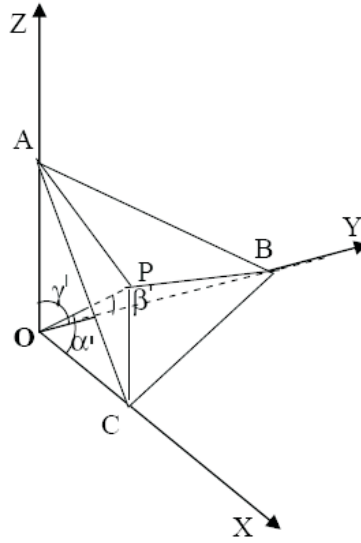$$\cos \beta' = \frac{OP}{OB} = \frac{d_{hkl}}{y}$$

Figure 7.8: Interplanar spacing

From right angled triangle OAP

$$\cos \gamma' = \frac{OP}{OA} = \frac{d_{hkl}}{z}$$

but we know that

$$h = \frac{a}{x}, k = \frac{b}{y} \, and \, l = \frac{c}{z}$$

Therefore

$$\cos \alpha' = \frac{h}{a} d_{hkl}$$

$$\cos \beta' = \frac{k}{b} d_{hkl}$$

$$\cos \gamma' = \frac{h}{c} d_{hkl}$$

for the rectangle Cartesian coordinate system we have
$(\cos \alpha')^2 + (\cos \beta')^2 + (\cos \gamma')^2 = 1$

$$d_{hkl}^2 \left(\frac{h^2}{a^2}\right) + d_{hkl}^2 \left(\frac{k^2}{b^2}\right) + d_{hkl}^2 \left(\frac{l^2}{c^2}\right) = 1$$

$$d_{hkl}^2 = \frac{1}{\left(\frac{h^2}{a^2}\right) + \left(\frac{k^2}{b^2}\right) + \left(\frac{l^2}{c^2}\right)}$$

$$d_{hkl} = \sqrt{\frac{1}{\left(\frac{h^2}{a^2}\right) + \left(\frac{k^2}{b^2}\right) + \left(\frac{l^2}{c^2}\right)}}$$

is the expression for interplanar spacing.

For a cubic lattice a=b=c,.Therefore

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

7.6   Coordination number and Atomic packing factor

Definition

(1)**Coordination number:** It is the number of equidistant nearest neighbors that an atom has in a crystal structure.

(2)**Nearest neighbour distance:** It is the distance between two nearest neighbors in a crystal structure.

(3)**Atomic packing factor (APF) or Packing fraction:** It is the fraction of space occupied by atoms in a unit cell. It is defined as the ratio of volume occupied by atoms in unit cell to the volume of the unit cell. If the number of atoms per unit cell are n and if V_a is the volume of atoms in the unit cell and V is the volume of the unit cell then,

$$APF = \frac{n \times V_a}{V}$$

(4)**Lattice Constant:** In a cubic lattice the distance between atoms remains constant along crystallographic axes and is called Lattice Constant.

7.7   Calculation of Atomic packing factor for different crystal systems

Simple Cubic Structure

In simple cubic structure each atom consists of 6 equidistant nearest neighbours. Hence its co-ordination number is 6.

 Eight unit cells share each atom at the corner. Hence only $1/8th$ of the volume of the atom lies in each cell. Since the atoms are present only at corners, the number of atoms per unit cell is given by
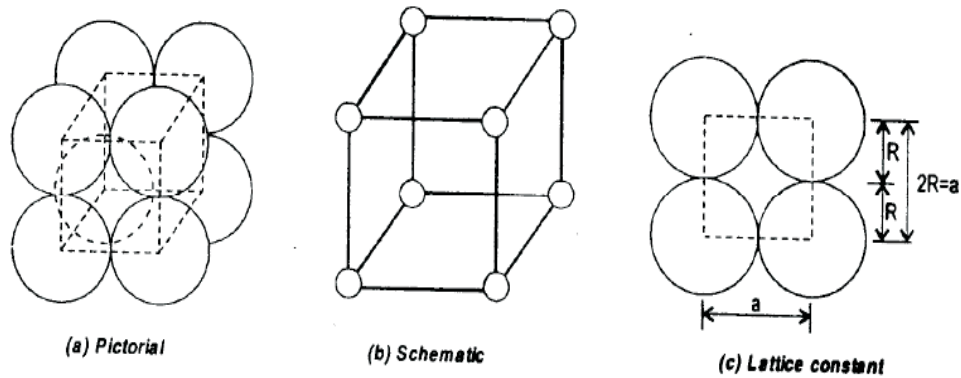
Figure 7.9: Simple cubic structure

$n = \frac{1}{8} \times 8 = 1 atom$

We know that the APF is given by

APF=$\frac{n \times V_a}{V}$

APF=$\frac{1 \times \frac{4\pi R^3}{3}}{a^3}$

In this structure the atoms touch each other along the sides of the cube. Therefore a=2R,where R is the radius of each atom.

$APF = \frac{4\pi R^3}{3(2R)^3}$

$APF = \frac{4\pi R^3}{3(8R^3)}$

$APF = 0.5235$

Hence atoms occupy 52.35% off the volume of the unit cell.

Body Centered Cubic (BCC) Structure:

Each atom has 8 equidistant nearest neighbors. Hence the co-ordination number is 8. Since
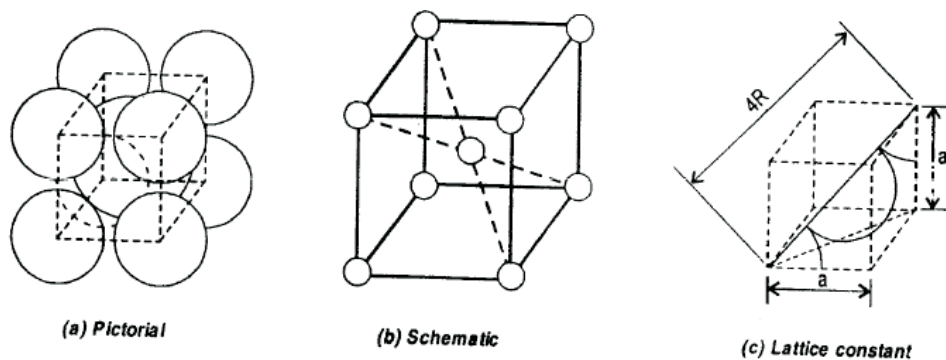

Figure 7.10: Body centered cubic structure

there are eight atoms at corners and 1 atom at the body center, the no of atoms per unit cell is given by

$n = 1 + \frac{1}{8} \times 8 = 2 \, atoms$

Also in this structure the atoms touch each other along the body diagonal. Therefore

$(4R^2) = (\sqrt{2}a)^2 + a^2$ Where R is the radius of the atom

$16R^2 = 2a^2 + a^2 = 3a^2$

$a^2 = \frac{16R^2}{3}$

$a = \frac{4R}{\sqrt{3}}$

Now the APF is given by

$APF = \frac{n \times V_a}{V}$

APF=$\frac{2 \times \frac{4\pi R^3}{3}}{a^3}$

APF=$\frac{2 \times \frac{4\pi R^3}{3}}{\left(\frac{4R}{\sqrt{3}}\right)^3}$

APF=$\frac{\sqrt{3}\pi}{8}$

APF=0.6802

Hence atoms occupy 68.02% of the volume of the unit

Face Centered Cubic (FCC) Structure:

In FCC structure in addition to atoms at corners, atoms are present at face centers. Each
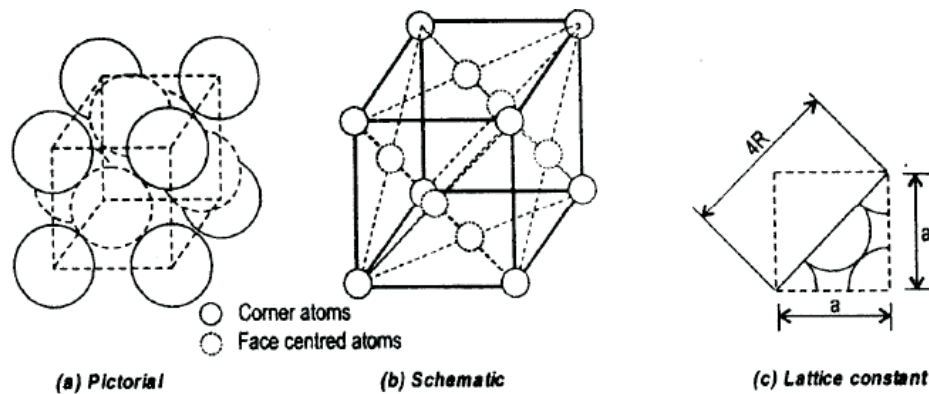
Figure 7.11: Face centered cubic structure

atom consists of 12 equidistant nearest neighbours. Hence the coordination number is 12.

The number of atoms per unit cell is

$n = \frac{1}{2} \times 6 + \frac{1}{8} \times 8 = 4$ atoms

In this structure atoms touch each other along the face diagonal. Therefore

$(4R)^2 = a^2 + a^2$

$16R^2 = a^2 + a^2 = 2a^2$ Where R is the atomic radius

$a^2 = \frac{16R^2}{2}$

$a = \frac{4R}{\sqrt{2}} = 2\sqrt{2}R$

The APF is given by

APF=$\frac{n \times V_a}{V}$

APF=$\frac{4 \times \frac{4\pi R^3}{3}}{a^3}$

APF=$\frac{16\pi R^3}{3(2\sqrt{2}R)^3}$

APF=$\frac{\sqrt{2}\pi}{6}$

APF=0.7405

Hence atoms occupy 74.05% of the volume of the unit cell.

7.8   X-Ray Diffraction and Bragg's Law

X-Ray Diffraction

The wavelength of x-ray is of the order of Angstrom (Å). Hence optical grating cannot be used to diffract X-rays. But the dimension of atoms is of the order of few angstrom and also atoms are arranged perfectly and regularly in the crystal. Hence crystals provide an excellent facility to diffract x-rays.

Bragg considered crystal in terms of equidistant parallel planes in which there is regularity in arrangement of atoms. These are called as Bragg planes. There are different families of such planes that exist in the crystal and are inclined to each other with certain angle.

In Braggs diffraction the crystal is mounted such that an X-ray beam is inclined on to the crystal at an angle θ. A detector scans through various angles for the diffracted X-rays. It shows peaks for (maximum current) for those angles at which constructive interference takes place. Braggs law gives the condition for constructive interference.
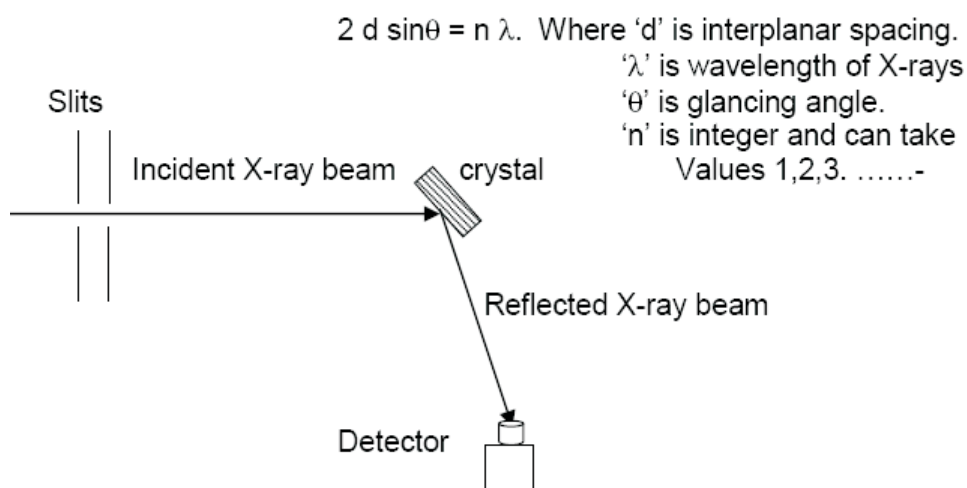

Figure 7.12: Braag's diffraction

Derivation of Bragg's Law:

Consider Monochromatic beam of X-Rays. It is incident on the crystal with glancing angle θ. Ray AB, which is a part of the incident beam, is scattered by an atom at B along BC. Similarly the ray DE is scattered by an atom in the next plane at E along EF. The two scattered rays undergo constructive interference if path difference between the rays is equal to integral multiple of wavelength.
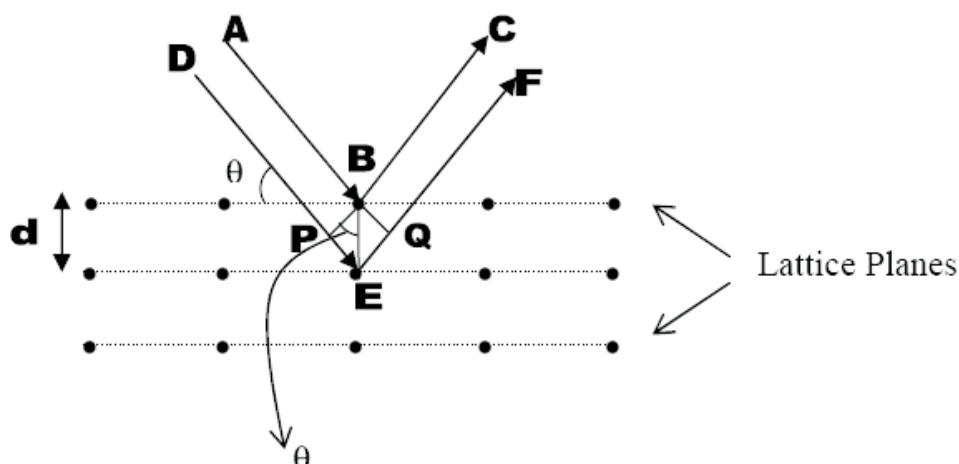
Figure 7.13: X-ray scattering by crystal

Construction:BP and BQ are the perpendicular drawn as shown in fig.

The path difference

$$\delta = PE + EQ = n\lambda \qquad (7.1)$$

From Right angled triangle PBE

$$\sin\theta = \frac{PE}{BE}$$

Where BE=d(Interplanar spacing)=d_{hkl}

Therefore $PE = BE\sin\theta = d\,\sin\theta$

Similarly from Right angled triangle QBE

$QE = BE\sin\theta = d\,\sin\theta$

Substituting in (7.1) $\delta = d\sin\theta + \sin\theta = n\lambda$

$$\delta = 2d\sin\theta = n\lambda$$

Therefore the condition for constructive interference is integral multiple of wavelength of X-rays which is given by

$$2d \sin \theta = n\lambda$$

Hence Bragg's Law.

Since Bragg diffraction satisfies the laws of reflection it is also called Bragg reflection.

7.9 Bragg's X-ray Spectrometer(Determination of wavelength and Interplanar spacing)

It is an instrument devised by Bragg to study the diffraction of X-rays using a crystal as Grating. It is based on the principle of **Bragg Reflection. Construction:** Monochromatic
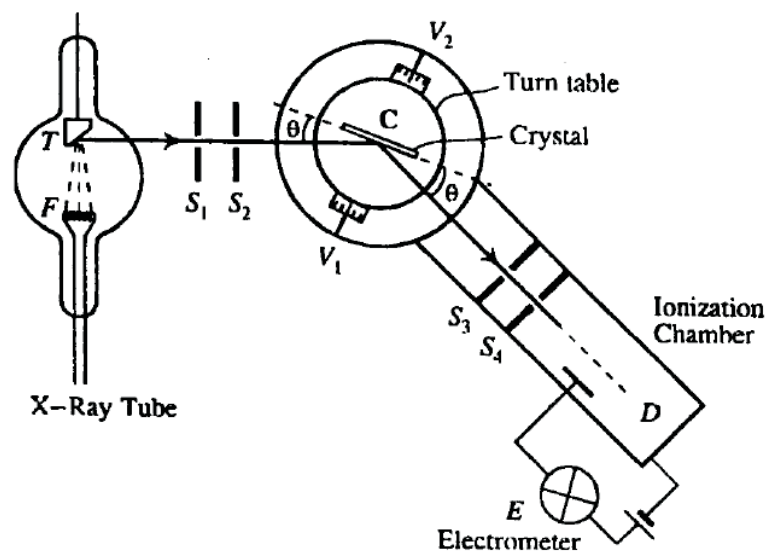
Figure 7.14: Bragg's x-ray spectrometer

X-Ray Beam from an X-Ray tube is collimated by slits s1 and s2 and is incident on the crystal mounted on the turntable at a glancing angle θ. The crystal can be rotated using the turntable. The reflected X-Ray beam is again collimated by slits s3 and s4 and allowed to pass through ionization chamber fixed on the **Mechanical Arm**. Due to ionization in the medium current flows through the external circuit, which is recorded by the **Quadrant Electro Meter (E)**. In order to satisfy the laws of reflection the coupling between the turntable and the mechanical arm is so made that, if the turntable is rotated through an angle q then mechanical arm rotates through an angle 2θ.

Experiment: Rotating the turntable increases glancing angle. Ionization current is measured as a function of glancing angle. The Ionization current is plotted versus glancing angle. It is as shown below. The angles corresponding to intensity maximum are noted
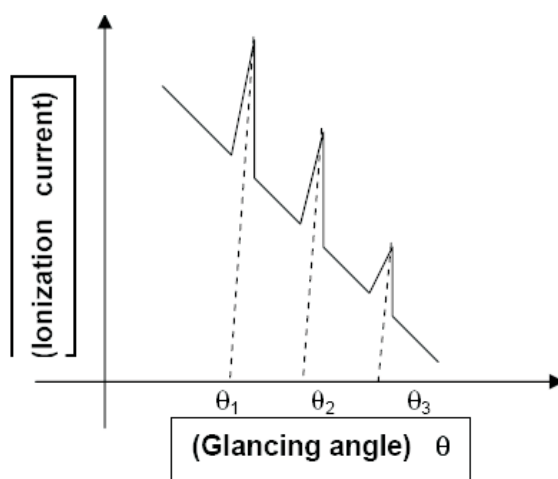
Figure 7.15: Variation of ionization current

down. The lowest angle θ corresponding to maximum intensity corresponds to the path difference λ.

$$2d\sin\theta_1 = n_1\lambda = \lambda$$

Similarly for next higher angles

$$2d\sin\theta_2 = n_2\lambda = 2\lambda$$

$$2d\sin\theta_3 = n_3\lambda = 3\lambda$$

and so on

$$\sin\theta_1 : \sin\theta_2 : \sin\theta_3 = 1 : 2 : 3 \tag{7.2}$$

If equation (7.2) is satisfied for $\theta_1, \theta_2, \theta_3$ etc, then the Bragg's law is verified.

By determining θ using Bragg's Spectrometer and by knowing the value of interplanar separation (d), Wavelength (λ) of X-ray beam can be calculated.

By determining θ using Bragg's Spectrometer and by knowing the value of Wavelength (λ) of X-ray beam Interplanar separation (d) can be calculated.

7.10   Crystal Structure of Sodium Chloride(NaCl)

NaCl is a ionic compound. Hence both Na and Cl are in ionic state. The molecule is under equilibrium because, the attractive force due to ions is balanced by repulsive force due to electron clouds. The Bravais lattice of NaCl is FCC with the basis containing one Na ion
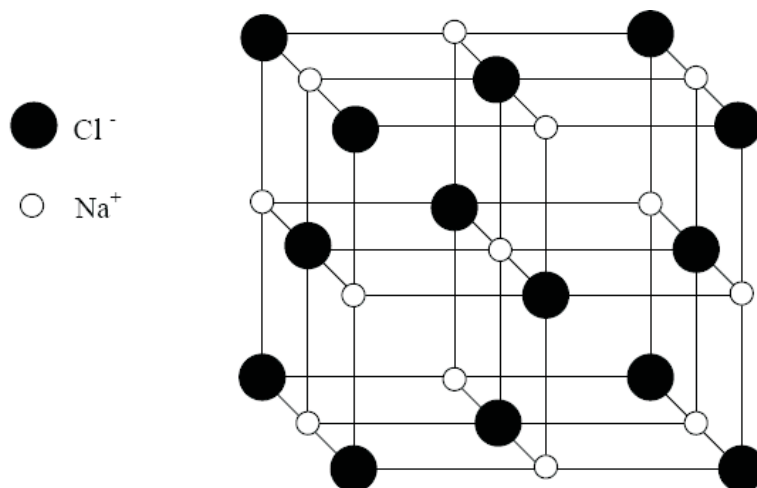
Figure 7.16: Crystal structure of NaCl

and one Cl ion. The bond length is $2.813\mathring{A}$. For each atom there are 6 equidistant nearest neighbours of opposite kind. Hence the coordination number is 6. There are 12 next nearest neighbors of the same kind. The conventional cell which consists of four molecules of NaCl is as shown in the figure. The coordinates of the ions in the conventional cell is as given below. (taking Na ion as origin)

This structure could be considered as the superposed pattern of two interpenetrating Bra-

Cl⁻   ½ 0 0, 0 0 ½, 0 ½ 0, ½ ½ ½.

Na⁺   0 0 0, ½ ½ 0, 0 ½ ½, ½ 0 ½.
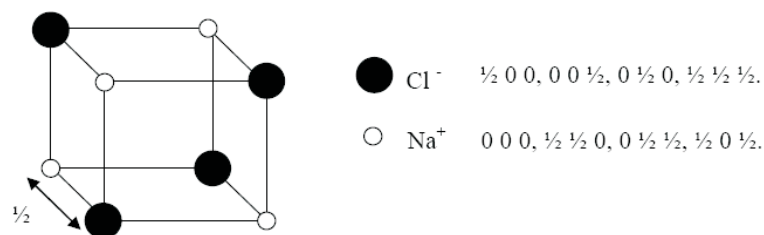
Figure 7.17: Co-ordinates of ions in NaCl

vais lattice each made of one type of ion.

7.11   Crystal structure of Diamond(Allotropic form of carbon)

Diamond is an allotropic form of carbon. The Bravais lattice is an FCC similar to ZnS. There are 18 carbon atoms in the unit cell. 8 at corners, 6 at face centers and 4 at intermediate tetrahedral positions. The unit cell is as shown in the fig. In the unit cell, each carbon
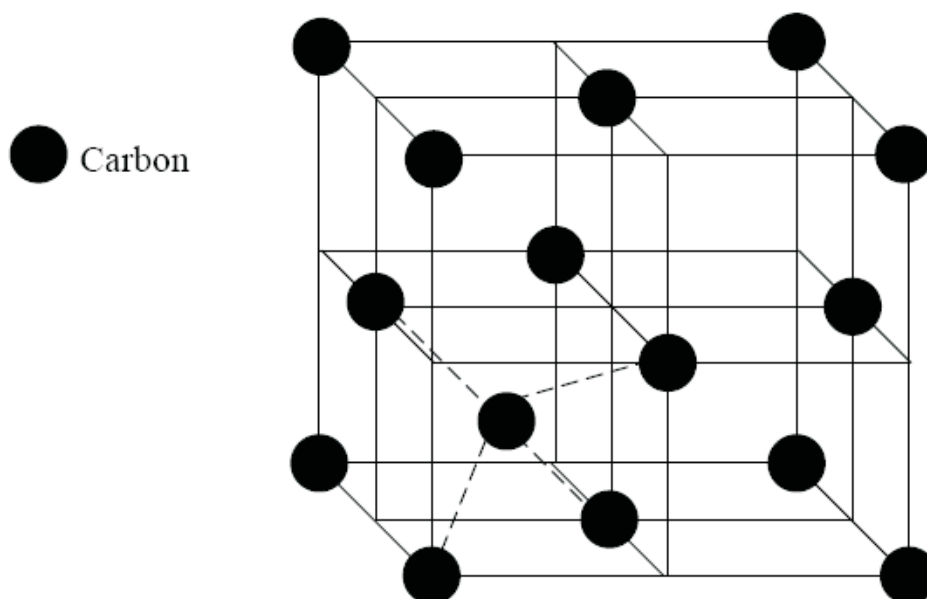
Figure 7.18: Diamond structure

atom bonds to four other carbon atoms in the form of a tetrahedron. Since each atom has four equidistant nearest neighbors the coordination number is 4.This structure could be considered as the superposed pattern of two interpenetrating Bravais FCC lattices each made of Carbon with one displaced from the other along $1/4th$ of the body diagonal. The interatomic distance is $1.54\mathring{A}$ and the lattice constant $3.56\mathring{A}$.

The Co-ordinates of the atoms in
conventional cell are
¼ ¼ ¼ , ½ 0 0, 0 ½ 0, 0 0 ½ , ½ ½ ½

½
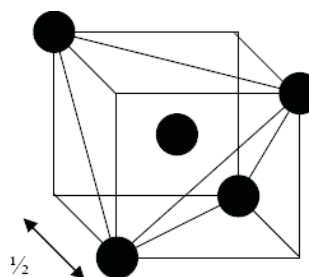
Figure 7.19: Tetrahedron formation by C atoms

* **

Multiple Choice Questions

1. The relation of length of axes of unit cell in monoclinic crystal system is

   (a) $a = b = c$ (b) $a = b \neq c$
   (c) $a \neq b \neq c$ (d) $a \neq b = c$

2. The relation of angles between axes of a triclinic crystal system is

   (a) $\alpha = \beta = \gamma = 90$ (b) $\alpha \neq \beta = \gamma = 90$
   (c) $\alpha \neq \beta \neq \gamma \neq 90$ (d) $\alpha = \beta = \gamma \neq 90$

3. The packing density of body centered cube is

   (a) $\pi/6$ (b) $\sqrt{3}\pi/8$
   (c) $\sqrt{2\pi/6}$ (d) $8/\sqrt{3}\pi$

4. The atomic radius for simple cubic lattice is

   (a) $a/2$ (b) $\sqrt{3}a/4$
   (c) $\sqrt{2}a/4$ (d) $a/4$

5. The Miller indices of a plane which cuts the intercepts of 2,3 and 4 units along the three axes are

   (a) $6, 4, 3$ (b) $2, 3, 4$
   (c) $3, 2, 1$ (d) $2, 3, 2$

6. If r is the radius of the atom in a crystal crystallizing in the simple cubic structure, then the nearest neighbour distance is

   (a) $r/2$ (b) $4r$
   (c) $2r$ (d) none of these

7. Classify the following unit cell into proper crystal system: a=1.08 nm, b=0.947 nm, c=0.52 nm and $\alpha = 41^0$, $\beta = 82^0$, $\gamma = 95^0$.

      (a) triclinic   (b) monoclinic

      (c) orthorhombic   (d) hexagonal

8. The nearest neighbour distance in the case of bcc structure is

      (a) $(a\sqrt{3}/2)$ (b) $(a\sqrt{2}/2)$

      (c) $2a/\sqrt{3}$ (d) $2a/\sqrt{2}$

9. The co-ordination number in the case of simple cubic crystal structure is

      (a) 12   (b) 6

      (c) 2   (d) 1

10. If n is the number of atoms in the unit cell of the cubic system, N_A and M_A are the Avogadro's number and atomic weight respectively and ρ is the density of the element, then the lattice constant a is given by

      (a) $\left[\frac{M_A\rho}{nN_A}\right]^{1/3}$ (b) $\left[\frac{nN_A}{M_A\rho}\right]^{1/3}$

      (c) $\left[\frac{nM_A}{N_A\rho}\right]^{1/3}$ (d) $\left[\frac{\rho N_A}{M_A n}\right]^{1/3}$

11. The maximum radius of the interstitial sphere that can just fit into the void between the body centered atom of bcc structure is

      (a) $r[(2/\sqrt{3}) - 1]$ (b) $r[(\sqrt{3}/2) - 1]$

      (c) $r[\sqrt{3} - 1]$ (d) $r(\sqrt{2} - 1)$

12. The packing factor of the fcc structure is

      (a) 52%   (b) 68%

      (c) 92%   (d) none of these

13. The Miller indices of the plane parallel to the x and y axes are

    (a) $(1\,0\,0)$ (b) $(0\,1\,0)$
    (c) $(0\,0\,1)$ (d) $(1\,1\,1)$

14. If (3 2 6) are the Miller indices of a plane, the intercepts made by the plane on the three crystallographic axes are

    (a) $(2a\,3b\,c)$ (b) $(a\,b\,c)$
    (c) $(a\,2b\,3c)$ (d) (none of these)

15. The Miller indices of the plane parallel to y and z axes are

    (a) $(1\,0\,0)$ (b) $(0\,1\,0)$
    (c) $(0\,0\,1)$ (d) $(1\,1\,1)$

16. A plane intercepts at a, b/2, 3c in a simple cubic unit cell. The Miller indices of the plane are

    (a) $(1\,3\,2)$ (b) $(2\,6\,1)$
    (c) $(3\,6\,1)$ (d) $(1\,2\,3)$

17. In a cubic lattic $d_{100} : d_{110} : d_{111}$ is

    (a) $6 : 3 : 2$ (b) $6 : 3 : \sqrt{2}$
    (c) $\sqrt{6} : \sqrt{3} : \sqrt{2}$ (d) $\sqrt{6} : \sqrt{3} : \sqrt{4}$

18. The number of atoms per unit area of the plane (0 1 0) of a simple cubic crystal is

    (a) $1/4r^2$ (b) $4r^2$
    (c) $1/4r$ (d) none of these

where r is the radius of the atom

19. The tiny fundamental block which,when repeated in space indefinitely generates a

crystal is called

    (a) primitive cell   (b) lattice cell
    (c) unit cell   (d) none of these

20. There are _____ basic crystal systems

    (a) four   (b) five
    (c) six   (d) seven

21. There are _____ distinguishable ways of arranging points in three dimensional space called Bravais lattices

    (a) 7   (b) 14
    (c) 8   (d) 5

22. Effective number of atoms belonging to the unit cell of simple cubic structure is

    (a) 8   (b) 1   (c) 4   (d) 6

23. Atomic packing factor of simple cubic structure is

    (a) 0.68   (b) 0.74   (c) 1.00   (d) 0.52

24. The relation between atomic radius r and lattice constant a in the case simple cubic structure is

$$\text{(a) } a = 2r \quad \text{(b) } a = \frac{r}{2} \quad \text{(c) } a = \frac{4}{\sqrt{3}}r \quad \text{(d) } a = 2\sqrt{2}r$$

25. The coordination number of BCC structure is

    (a) 6   (b) 8   (c) 12   (d) 4

26. Effective number of atoms belonging to the unit cell of BCC structure is

  (a) 8 (b) 1 (c) 2 (d) 9

27. Atomic packing factor of BCC structure is

  (a) 0.68 (b) 0.74 (c) 0.52 (d) 1.00

28. The relation between atomic radius r and lattice constant a in the case of BCC structure is

  (a) $a = 2r$ (b) $a = \frac{4}{\sqrt{3}}r$ (c) $a = 2\sqrt{2}r$ (d) $\frac{\sqrt{3}}{4}r$

29. The coordination number of FCC structure is

  (a) 6 (b) 8 (c) 12 (d) 4

30. Effective number of atoms belonging to the unit cell of FCC structure is

  (a) 14 (b) 8 (c) 4 (d) 2

31. Atomic packing factor of FCC structure is

  (a) 0.68 (b) 0.74 (c) 0.52 (d) 1.00

32. The relation between atomic radius r and lattice constant a in the case of FCC structure is

  (a) $a = 2r$ (b) $a = \frac{4}{\sqrt{3}}r$ (c) $a = 2\sqrt{2}r$ (d) $\frac{r}{2\sqrt{2}}$

33. The number of atoms present in the unit cell of diamond cubic crystal structure is

  (a) 2 (b) 4 (c) 8 (d) 16

34. The coordination number of NaCl structure is

(a) 6   (b) 8   (c) 12   (d) 4

35. The number of molecules present in each unit cell of NaCl is

(a) 8   (b) 4   (c) 27   (d) 12

36. The direction that connects the origin and $\left(\frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right)$ point is

(a) [112]   (b) [322]   (c) [211]   (d) [333]

37. [211] is the direction that connects the orgin (0,0,0) and

(a) $(1, 2, 2)$ (b) $(1/2, 1, 1)$ (c) $(1, 1, 2)$ (d) $(1, 1/2, 1/2)$

38. In a cubic crystal a plane makes intercepts 1,-3, 1 on the x, y, and z axes respectively. The Miller indices of the plane are

(a) $(\bar{3}\,1\,\bar{3})$ (b) $(3\,\bar{1}\,3)$ (c) $(1\,\bar{3}\,1)$ (d) $(\bar{1}\,3\,\bar{1})$

39. In a crystal cell a, b and c represent unit translational vectors along x, y and z axes. A plane makes intercepts $2a$, $3b$ along x and y axes and runs parallel to z-axis. Miller indices corresponding to this plane is

(a) (23∞) (b) (230) (c) (321) (d) (320)

40. The planes 112 and 224 are

(a) perpendicular to each other

(b) parallel to each other

(c) intersecting at an angle other than 90^0

(d) none of these

41. Because of which property of the crystal,x-axis can be diffracted from the crystals.

(a) random arrangements of atoms

(b) color of the crystals

(c) periodic array of atoms

(d) none of these

42. Which one of the following statements is correct ?

(a) X-rays are electromagnetic radiation of long wavelength

(b) X-rays produce in a gas tube are different from those in Coolidge tube

(c) the wavelength of the scattered x-ray by a material is longer than that of the incident beam

(d) X-ray spectral lines are produced when fast moving electrons are retarded by a material

43. If 50 kV is the applied potential in an x-ray tube, then the minimum wavelength of x-rays produced is

(a) 0.2nm (b) 2nm
(c) 0.2Å (d) 2Å

44. X-rays consist of

(a) negatively charged partials

(b) electromagnetic radiation

(c) positively charged particles

(d) a stream of neutrons

45. X-rays are produced when an element of high atomic weight is bombarded by high energy

(a) protons (b) photons
(c) neutrons (d) electrons

46. When the potential difference between the electrodes of an x-rays tube is increased, there take place an increase in the

(a) intensity (b) frequency
(c) wavelength (d) speed of $x-$ rays

47. x-rays can be deflected by

   (a) magnetic field   (b) electric field
   (c) none of these

48. As the wavelength of x-rays is smaller than that of visible light, the speed of x-rays in vacuum is

   (a) same as that of visible light

   (b) larger than that of visible light

   (c) smaller than that of visible light

49. If the angle between the direction of the incident x-ray and the diffracted one is 16^0, the angle of incidence will be

   (a) 32^0 (b) 24^0
   (c) 90^0 (d) 82^0

50. The short wavelength limit of x-rays depend upon

   (a) nature of the target

   (b) p.d across the filament used

   (c) nature of the filament used

   (d) none of these

51. If the plane intercepts at $x = 0.75$, $y = 0.25$ and parallel to $z-$axis then the Miller indices are

   (a) $\left(\frac{3}{4} \frac{1}{4} \infty\right)$ (b) $(4\,6\,0)$ (c) $(4\,12\,0)$ (d) $(4\,12\,\infty)$

52. X-ray beam of wavelength $1\mathring{A}$ undergoes minimum order Bragg reflection from the plane $(2\,0\,1)$ of a cubic crystal at the glancing angle of 45_0, the lattice constant is

   (a) 0.707×10^{-10}m   (b) 7.07×10^{-7}m   (c) 1.58×10^{-7}m   (d) 1.58×10^{-10}m

53. The maximum radius of the interstitial sphere that can first fit into the void between the face centered atoms of structure

$$(a)\, r\left(\frac{2}{\sqrt{3}} - 1\right) \quad (b)\, 2r(\sqrt{2} - 1) \quad (c)\, r(\sqrt{2} - 1) \quad (d)\, r(\sqrt{3} - 1)$$

54. The planes 100 and 020 are

   (a) intersecting at an angle of 45° (b) intersecting at an angle of 30°
   (c) intersecting at an angle of 90° (d) parallel to each other

55. The direction that connects the origin and $(\frac{4}{5} \frac{1}{5} \frac{3}{5})$ point is

$$(a)\, [4\,1\,3] \quad (b)\, [5\,2\,4] \quad (c)\, [3\,1\,4] \quad (d)\, [5\,5\,5]$$

56. The nearest neighbour distance in the case of FCC structure is

$$(a)\, \frac{2\,a}{\sqrt{3}} \quad (b)\, \sqrt{2}\,a \quad (c)\, \frac{\sqrt{3}\,a}{2} \quad (d)\, \frac{a}{\sqrt{2}}$$

57. In order to get X-rays of more intensity

   (a) large current is required.

   (b) large voltage is required.

   (c) both the current and voltage are to be increased.

   (d) current and voltage have no effect on the intensity of X-rays.

58. The lattice constant of a crystal is $10^{-7}cm$, the upper limit for the wavelength of X-rays with this crystal which can be studied is

$$(a)\, 2\text{Å} \quad (b)\, 10\text{Å} \quad (c)\, 20\text{Å} \quad (d)\, 1\text{Å}$$

Descriptive Questions

1. Derive an expression for inter planar spacing of a crystal in terms of Miller indices.

2. Discuss Bravais lattice and crystal system with the help of illustration.

3. Define: (i) Packing factor (ii) Coordination number. Calculate the coordination number and packing factor for SC,BCC and FCC structures.

4. What is meant by crystallographic planes? Describe the crystal structure of diamond.

5. Define lattice points, bravais lattice and primitive cell. Explain in brief the seven crystal system with neat diagrams.

6. Explain how Braggs X-ray spectrometer can be used to determine the interplanar spacing.

7. Define crystal lattice.

8. Explain Miller indices.

9. With neat figure describe the structure of NaCl.

10. Explain (i) Bravais lattices, (ii) Lattice parameters, and (iii) Difference between unit cell and primitive cell

Exercise

1. Copper has fcc structure of atomic radius $0.127nm$. Calculate the inter planar spacing for [322] plane.

2. Sketch the following plane in a cubic unit cell $(101), (121), (010)$

3. A current of 5 mA flows in an X-ray tube operating under a potential of $10kV$. Calculate the rate at which the electrons are bombarding the target and the maximum speed they could attain.

4. Calculate the glancing angle for incidence of X-ray of wavelength $0.58\mathring{A}$ on the plane (132) of NaCl which result in $2nd$ order diffraction maxima taking the lattice spacing as $3.81\mathring{A}$.

5. Draw the following planes in a cubic unit cell $(132), (1\bar{1}0), (010), (1\bar{2}1)$.

6. An X-ray beam of wavelength $0.7\mathring{A}$ undergoes minimum order Bragg reflection from the plane (302) of a cubic crystal at glancing angle 35^0. Calculate the lattice constant.

7. Explain the equation governing the diffraction of X-rays. Find the miller indices of a plane making the intercepts $2a$, $3b$,and $6c$ respectively on [100], [010] and [001] directions. a, b and c represent basis vectors.

8. X-rays are diffracted in the first order from a crystal with d spacing $2.82 \times 10^{-10} m$ at a glancing angle 6^0. Calculate the wavelength of X-rays.

9. A monochromatic X-ray beam of wavelength $0.7 \overset{\circ}{A}$ undergoes first order Bragg reflection from the plane $[302]$ of a cubic crystal at a glancing angle of 35^0. Calculate the lattice constant.

10. Sketch the following planes in a cubic crystal: (i)(213) (ii)(110) (iii)(TTO).

11. Calculate miller indices for a plane which intercepts at $[4, -3, 1/2]$ along the axes.

12. A plane intercepts the crystal axes at x = 0.75, y = 0.5, and parallel to z axis. What are the miller indices? Also draw the plane on a cubic unit cell.

13. In a crystal, lattice plane intercepts 1 unit on x-axis, and parallel to y,z axes. What are miller indices?

14. Draw planes for given set of intercepts in a cubic unit cell $[\infty, \infty, 1/2]$, $[2/3, 1/2, 3/4]$,$[1, 1/2, 1]$.

Part V

Module 5: Shock waves and Science of Nano Materials

Chapter 8

Shockwaves

8.1 Mach number

Consider an object moving in a fluid. Let v_o be the velocity of the object in the medium. The velocity of sound in the medium is v_sIt is defined as the ratio of speed of the object to the speed of the sound in the surrounding medium. It is a dimensionless quantity. Mathematically Mach number M is given by

$$M = \frac{v_o}{v_s} \tag{8.0}$$

Mach 0.5 represents half the speed of sound and Mach 2 represents twice the speed of sound. The Mach number is named after the scientist Ernst Mach. Mach number is used in both the cases

1. When an object is moving in a fluid

2. When the fluid is moving around an object.

8.2 Classification of waves and Mach regimes

The classification of waves is as follows

Types of waves

Acoustics : It is the study which deals with the study of waves through gases, liquids and solids.

Ultrasonics : It is the study of waves which have frequencies higher than the audible sound (>20kHz).

Subsonic : Waves which travel with speeds less than the speed of sound in a medium are called subsonic waves.

Supersonic : Waves which travel with speeds greater than the speed of sound in a medium are called supersonic waves.

Mach regimes

The following is a table representing the regimes of fluid flow of moving object in a fluid with regard to Mach number.

Mach No.	<0.8	0.8 - 1.2	1.0	1.0 - 5.0	5.0-10.0	>10
Regime	Subsonic	Transonic	Sonic	Supersonic	Hypersonic	High-Hypersonic

8.3   Shock Waves and its applications

Shock Waves

When the speed of a moving object in a medium becomes equal to the velocity of sound in the medium the wavefronts cannot escape from the source and pile up. This results in a large amplitude sound barrier.When the speed of the moving object or source exceeds the speed of sound in the medium then the wave fronts lag behind the source forming a cone-shaped region with the source at a vertex. The edge of the cone forms a supersonic wave front with an unusually large amplitude called a "shock wave". A sonic boom is heard when shock waves reach an observer. Shock waves generated when a fighter jet attains the speed of sound is as shown in the figure 8.3.

In comparison with the speed of sound waves the speed of a shock wave varies with its amplitude. The speed of a shock wave is always greater than the speed of sound in the fluid and decreases as the amplitude of the wave decreases. If the speed of the shock wave equals the normal speed then the shock wave dies and is reduced to an ordinary sound wave.

The shock wave from a supersonic object is a cone composed of overlapping spherical wavefronts. As any one of these wavefronts forms, it propagates radially outward at speed v_o and acquires a radius $v_o t$. At the same time the source, traveling at speed v_s moves forward $v_s t$. The two displacements form the base and hypotenuse of the triangle used to determine Mach angle at the vertex of the shock cone.

Figure 8.1: Shockwaves

$$\sin\theta = \frac{v_s}{v_o} \tag{8.0}$$

If the object is Traveling at the speed of sound then the Mach number is equal one and results in a Mach angle of ninety degrees. if the speed of the object is transonic speeds then the shock wave is a wall of high pressure moving along with the object in a direction perpendicular to its velocity. For supersonic speed Mach number is less than one and the Mach angle is less than ninety degrees. The faster the object moves, the narrower the cone of high pressure behind it becomes. Measuring the vertex angle is thus a simple way to determine the speed of a supersonic object.

8.4 Applications of Shock waves

The following are the applications of the shock waves.

1. Mach reflection of a shock wave is used to remove micron size dust particles from the surface of silicon wafers.

2. Shock waves are used in medical therapy in orthopedics and for breaking kidney stones.

3. Shock waves are used in pencil industries to impregnate preservatives into wood slats.

4. Shock waves are used in sandal oil extraction.

5. Shock waves are used in cell transformation.

6. Shock waves are used in geophysics.

8.5 Basics of conservation of Mass, Energy and Momentum

Conservation of Mass

The basic law of conservation is mass can neither be created nor destroyed. For a closed system the total mass of the system remains constant. Mathematically

$$m_{sys} = Constant \tag{8.0}$$

Hence

$$\frac{dm_{sys}}{dt} = 0 \tag{8.0}$$

Control volume is the mathematical abstraction used for creating a mathematical model for a physical process. For an inertial frame of reference it is a volume fixed in space or moving with constant velocity through which the fluid flows. The surface enclosing the control volume is referred to as the control surface. For a control volume or open system conservation of mass is expressed in the rate form as

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{cv}}{dt} \tag{8.0}$$

Here $\dot{m}_{in}$ and $\dot{m}_{out}$ are the mass flow into and out of the control volume respectively. $\frac{dm_{cv}}{dt}$ is the rate of change of mass within the control volume boundaries. In fluid mechanics, the conservation of mass relation written for a differential control volume is usually called the continuity equation.

Conservation of Momentum

It is given by the total momentum of the system is always conserved. By applying Newton's second law of motion to the control volume the conservation of momentum is "any change in momentum of the fluid within a control volume be due to the net flow of fluid into the volume and the action of external forces on the fluid within the volume."

Conservation of energy

It deals with energy can neither be created nor destroyed. In other words total energy of a system remains constant. In case of a fluid system under adiabatic conditions the total energy entering the system must be same as the total energy leaving the system. The total energy of the control volume remains constant.

8.6 Rankine-Hugonit equations for normal shock waves

Normal Shock Waves

Consider a flow with speeds greater than sonic speed and hence shock waves are generated. Normal shock waves are the basic type of shock waves which are in a direction perpendicular to the flow.

Rankine–Hugonit equations

Consider a rectangular region of fluid (say air) around a normal shock wave as shown in the figure 8.6. The left of the shock wave we have the initial conditions and final conditions to the right. M_1, V_1, p_1, T_1 and ρ_1 represent the initial conditions of the flow and M_2, V_2, p_2, T_2 and ρ_2 represent the final conditions of the flow.
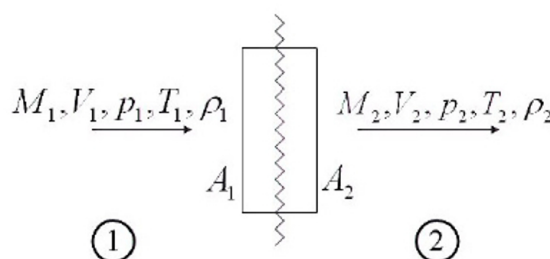

Figure 8.2: Normal Shock Wave

The following are the assumptions with regard to the flow

1. It is a steady flow.

2. It is an adiabatic system.

3. There are no viscous forces acting between the system and boundaries.

4. There are no body forces acting.

Let us determine the mass flow of the rectangular fluid system using the equation of continuity. The mass flow into the rectangular system is given by $\rho_1 u_1 A_1$. Here u_1 is the in flow velocity along the x-direction. Similarly the mass flow out of the rectangular system is given by $\rho_2 u_2 A_2$. Here u_2 is the out flow velocity along the x-direction. Applying the law of conservation of energy we have

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2 \tag{8.0}$$

For a rectangular system $A_1 = A_2$ hence equation 8.6 becomes

$$\rho_1 u_1 = \rho_2 u_2 \tag{8.0}$$

Let us derive the momentum equation using the law of conservation of momentum. The rate of momentum flow entering the system is given by $(\rho_1 u_1 A_1) u_1$. Similarly the rate of momentum flow our of the system is given by $(\rho_2 u_2 A_2) u_2$. Hence the net change in momentum is given by

$$(\rho_1 u_1 A_1) u_1 - (\rho_2 u_2 A_2) u_2 \tag{8.0}$$

The net force acting on the system in terms of pressure and area is given by

$$p_1 A_1 - p_2 A_2 \tag{8.0}$$

Since the rate of flow is considered per second and using the law of conservation of momentum we have

$$(\rho_1 u_1 A_1) u_1 - (\rho_2 u_2 A_2) u_2 = p_1 A_1 - p_2 A_2 \tag{8.0}$$

Simplifying the above equation results in

$$(p_1 + \rho_1 u_1{}^2) A_1 = (p_2 + \rho_2 u_2{}^2) A_2 \tag{8.0}$$

Since $A_1 = A_2$ for a rectangular system we get

$$p_1 + \rho_1 u_1{}^2 = p_2 + \rho_2 u_2{}^2 \tag{8.0}$$

Lastly, considering the law of conservation of energy to the flow across the shock wave. Since the flow is one-dimensional and there are no changes in the flow properties in any direction that is normal to that of the flow and also because the upstream and downstream faces of the control volume lie upstream and downstream of the shock wave, there are no temperature gradients normal to any face of the control volume. Hence the flow through the control volume is adiabatic. Thus the energy equation is given by

$$\frac{u_1^2}{2} + c_p T_1 = \frac{u_2^2}{2} + c_p T_2 \tag{8.0}$$

Here c_p is the specific heat at constant pressure for the fluid system. The above equation states that the total enthalpy is the same on both sides of the shock wave. The equations 8.6, 8.6 and 8.6 are called Rankine-Hugonit equations for normal shock waves.

8.7 Generation of shock waves using shock tubes

Methods of creating shock waves in laboratory

Shock waves find applications extending into the fields of engineering, manufacturing, medical,biological and scientific research. Shock waves are produced by sudden release of energy like in explosions, supersonic flows , impact of projectiles at high speeds etc.

Even though shock waves have such a wide range of applications and common occurrence it is not a topic in the school or college studies. This is due to the safety issues involved with regard to the high pressure associated with the production and propagation of shock waves. Thus by developing systems for generating shock waves in the laboratories of schools and colleges without the any associated risks of high pressure would be of great concern.

Shock Tubes

A device used to produce shock waves of required strength in the laboratory is called Shock tube. It consists of a long simple tube which is separated into two regions (sections) by a meta diaphragm. The regions are driver and driven sections. The metal diaphragm is commonly driven by compressed high pressure driver gas using either a compressor or by detonation of small quantity of explosive. Thus driver side ruptures the diaphragm creating a shock wave which travels into the driven section along the length of the shock tube. Such shock tubes can be used for classroom teaching. The following are some of the types of shock tubes.

1. Compression Driven

2. Blast Driven

3. Piston Driven

4. Reddy Tube

The compression driven shock tube is as shown in the figure 8.7.

Reddy Tube

Appratus and Operation: Dr. K.P.J. Reddy and associates have developed a miniaturized simple hand-operated pressure-driven shock tube named Reddy tube very simple to operate. The main aim of this capabilities of this convenient to operate simple, mini shock tube which is more versatile than the traditional compression-driven and blast-driven shock tubes. Reddy shock tube is a special hand-held piston driven shock tube capable of operating in traditional compression-driven mode using manual energy. Reddy tube is the first
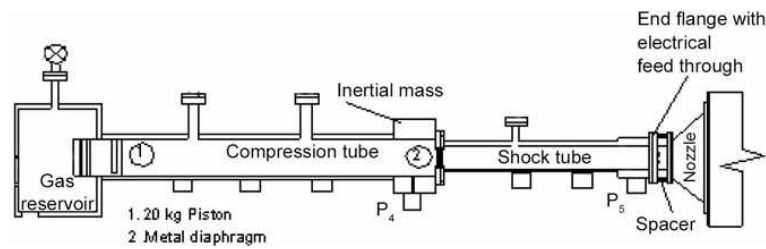
Figure 8.3: Compression driven shock tube

such devise which is capable of producing shock waves with Mach numbers exceeding 1.5 by simple hand operation. The reddy shock tube is as shown in the figure 8.7
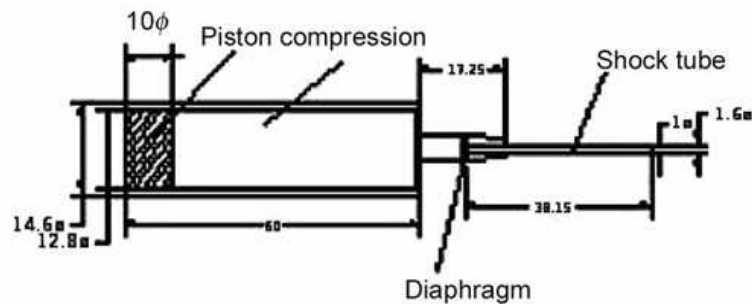

Figure 8.4: Reddy Shock Tube

Construction :   Reddy tube operates on the principle of conventional piston driven shock tube. Reddy shock tube is made of simple medical syringe with the following dimensions

- Compression chamber is of length 60mm and dia 12.8mm.

- A plastic piston with plunger with outer dia 12.8mm fitted closely inside the compression chamber.

- O-rings on the piston head prevent the leakage of leaking of gas.

- The driven tube is a 30 mm long SS tube of 1 mm internal diameter and wall thickness of 0.8 mm.

- The compression tube and the shock tube (hypodermic needle) are separated by a 50 μm thick plastic diaphragm.

- The elliptic free end of the hypodermic needle (shock tube) is made circular by grinding and is open to atmospheric air.

Working : Diaphragm and the needle are detached from the compression tube and the piston is moved outwards to fill the compression chamber with ambient air at atmospheric pressure. The diaphragm and the needle are attached to the compression tube. The piston is pushed into the compression chamber so that the air inside is compressed and pressure builds. As the piston moves the pressure increases and finally the diaphragm ruptures resulting in the very high speed flow of the compressed air. Schlieren images of the flow at the exit of the Reddy tube confirm the generation of shock waves. Thus shock waves of mach 1.5 to 2.0 are generated at the tip of the needle.

Experimental analysis of the performance characteristics

The analysis and performance of reddy tube are as follows.The pressure measured at the 1mm exit of the reddy tube is as shown in the figure 8.7. The plot shows pressure produced by a typical blast wave and the peak over pressure generated by this simple hand operated shock tube is about 3.5 bar.
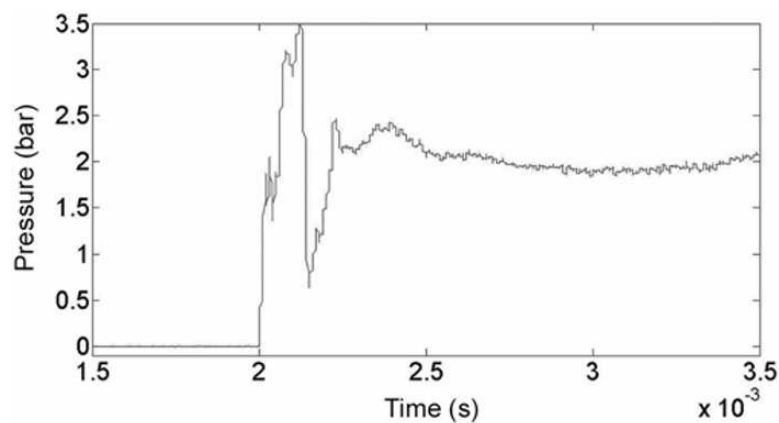

Figure 8.5: Pressure measured at the 1mm exit of reddy tube

Comparison of the theoretically predicted values of the shock Mach number with the measured values of the ratio of diaphragm rupture pressure to the shock tube initial pressure are in agreement.

Comparison of the measured pressure jump across the primary shock wave with the theoretically estimated values for different shock Mach numbers are in agreement

Comparison of the measured pressure jump across the reflected shock wave with the theoretically estimated values for different shock Mach numbers are in agreement.

Descriptive Questions

1. What is Mach's number? What is its importance?

2. Explain how sound waves are classified? What is Mach's regime?

3. What is shock wave? Mention important application of shock waves

4. Obtain Rankine- Hugonit equations

5. Explain construction and working of Reddy's shock tube

Chapter 9

Nanotechnology

Nanotechnology

9.1 What is Nanoscience?

Nanoscience is study of material behavior at nanoscale.

9.2 What is nanoscale?

Dimension which is of the order of 1nm to 100nm is called nanoscale. Any material whose dimension is less than 100nm can be considered as nano material. $1nm = 10^{-9}m$

9.3 What is nanotechnology?

The term nanotechnology was coined by Norio Taniguchi in 1947. Nanotechnology is the manipulation of matter at nanoscale to create novel structure, devices and systems.
Ex:
Structure:- Carbon nano tubes, graphene, bucky ball.
Devices:- nanodiodes, capacitors,..etc..
Systems:- NEMS-nano electro mechanical systems.

9.4   What do we look forward in nanotechnology?

In nano technology we look forward to

1. Explore more about the nature of the matter.

2. Develop new theories.

3. Find new questions and answers in many areas.

4. Develop new technologies to make human life more comfortable.

9.5   What is the basic principle of nanotechnology?

The properties of a material depends on the arrangement of atoms and molecules. Arrangement of atoms and molecules in nanoscale materials differ from that of bulk arrangement of same material. Hence when arrangement changes material behavior changes.

Number of physical phenomenon became noticeably different as the size of the system decreases. This is the basic principle of nanotechnology. Ex:- Nano gold and Bulk gold exhibit different properties.

Bulk Gold

1. It is yellow in colour.

2. It has the property of luster.

3. It is inert.

4. It is a good conductor.

Nano Gold

1. Depending on scale, they turn red, blue, yellow and other colours.

2. It looses lusture property.

3. It is a good catalyst.

4. It is a semiconductor.

9.6 Mention different types of nanomaterials. Give an example for each.

There are three types. Namely,

1. One dimensional nanomaterials. Ex:- Graphene.

2. Two dimensional nanomaterials. Ex:- nanofibres, wires, CNT.

3. Three dimensional nanomaterials. Ex:- Fullernes, quantum dots, quantum drivers.

9.7 Explain different types of preparation of nanomaterials?

Basically there are two methods of preparation of nanomaterials, namely

1. Bottom-up approach.

2. Top-down approach.

Synthesis of nanoparticles can also be classified into physical and chemical methods. Physical Methods

Ball milling method:-
This method is basically used to prepare nanoparticles of metals and alloys in the form of powder. Ball mills are equipped with grinding media composed of hardened steel balls. Ball mills rotate around a horizontal axis, partially filled with the material to be ground plus the grinding medium. The balls rotate with high speed inside a drum and then fall on the solid with the gravity force and crush the solid into nano crystallites.

The significant advantage of this method is that it can be readily implemented commercially. Ball milling can be used to make carbon nanotubes. It is commonly used to prepare metal oxides nano crystals like Cerium Oxide(CeO2) and Zinc oxide(ZnO).

Vapour deposition Method:-
There are two types namely physical vapour deposition method and chemical vapour deposition method.In physical vapour deposition method material is directly deposited on a surface. In chemical method, it involves a chemical reaction followed by the deposition.

In physical method, material kept in crucible is heated so that it gets evaporated. It is blown towards a substrate with the help of a inert gas to avoid formation of large particles. Atoms / molecules blown by inert gas are allowed to condense on a substrate. The particles grow while traveling from crucible to substrate. Hence the particle size can be controlled by

changing the distance between crucible and substrate and also the vapour pressure. Particles deposited on substrate is removed using scrapers.

In CVD method vapour condensed on substrate undergo chemical reaction to form a nano film.

Sol-Gel method:-

This method is also known as thermal decomposition. This process involves the formation of colloidal suspension or solution(sol) and gelation of the sol to form nano powders is a continuous liquid phase(gel). The solution is kept at suitable temperature. Gelling agents are added under controlled viscosity and PH value.

The particles in sol are polymerized through the removal of the stabilizing components and produces a gel. The final heat treatment (evaporation of liquid) pyrolyze the remaining organic and inorganic components and form a nano particles coating on a substrate.

Molecular manufacturing

Molecular manufacturing is a future technology based on Feynman's vision of factories using nanomachines to build complex products, including additional nanomachines. It promises to bring great improvements in the cost and performances of manufacture goods,while making possible a range of products impossible today.

Every manufacturing method is a method for arranging atoms. Most methods arrange atoms crudely; even the finest commercial microchips are irregular at the atomic scale. Many of today's nanotechnologies face the same limit.

The basic idea is simple: Where chemists mix molecules in solution, allowing them to wander and bump together at random, nanomachines will instead position molecules, placing them in specific locations in a carefully chosen sequence. Letting molecules bump at random leads to unwanted reactions - a problem that brows worse as products get larger. By holding and positioning molecules, nanomachine will control how molecules react, building up complex structures with atomically precise control.

9.8   What are Carbon nanotubes(CNT)? Mention different types of CNTs.

Carbon nanotubes is a graphite sheet consisting of covalently bonded carbon atoms in hexagonal type arrangement. The sheet is rolled up into a cylinder with the ends closed

by hemispherical graphite domes. Carbon nanotubes can be devided into two basic classes namely
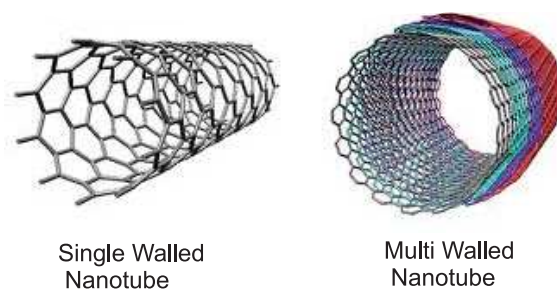

Figure 9.1: SWNT and MWNT

1. Single wall nanotubes (SWNT)

2. Multi wall nanotubes (MWNT)

SWNT's are formed from the single graphite layer with typical diameter in the range 0.4 to 2nm. MWNT's include structures formed in coaxial arrangement of several (2-100) graphite cylinders and their external diameter ranges from 10 to 100nm. Based on the chirality CNT are classified into 3 types. Namely,

1. Zig-zag structure

2. Armchair structure
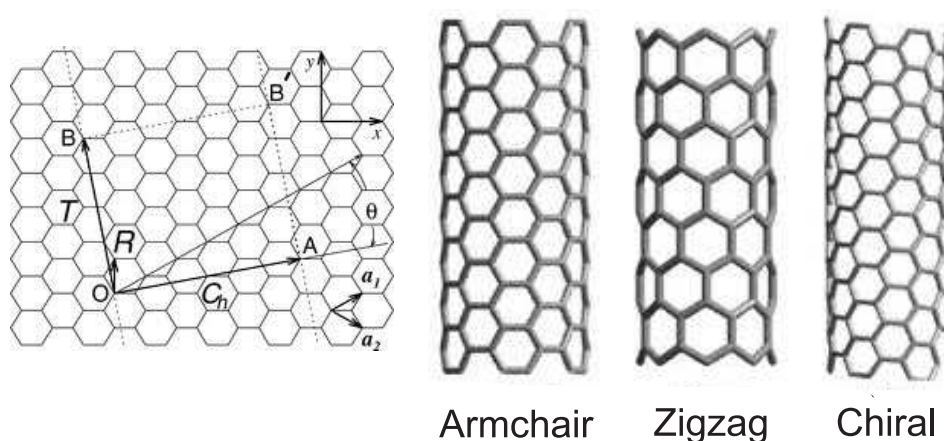
3. Chiral structure.

Armchair   Zigzag   Chiral

Figure 9.2: Types of nanotubes

9.9 Mention important properties of CNT?

1. Stronger and most flexible molecular material because of C-C covalent bonding and hexagonal network architecture.

2. Young's modulus of over 1 Tpa vs 70 Gpa for Aluminium. 100 GPA for carbon fiber.

3. Strength to weight ratio 500 times greater than that for Al. similar improvements on steel and Titanium. One order of magnitude improvement over graphite.

4. Maximum strain is 10% much higher than any material.

5. Thermal conductivity is 3000W/mk in the axial direction with small values in the radial direction.

6. Electrical conductivity in order of magnitude higher than copper.

7. Can be metallic or semiconducting depending on chirality. Band gap can be adjusted. Electrical properties can be controlled through application of external magnetic field and mechanical force.

8. Very high current carrying capacity.

9.10 What are potential applications of CNT?

1. Because of high mechanical strength CNT are used to make spherical stab-proof and bullet proof clothes.

2. They are used to make field effect transistors which are capable of digital switching using a single electron.

3. Because of high electrical conductivity they are used for thermal management of electronic circuits.

4. They are used for making high efficient solar cells.

5. In medicine, SWNT are inserted around cancerous cells, and then excited with radio waves, which cause them to heat up and kill the cells.

6. They are used for making NEM's.

9.11   Bucky Balls:Properties and Applications

Buckminster fullerene commonly known as buckyball c onsists of 60 carbon atoms arranged in the form of cage like fused- ring structure which resembles the soccer ball. **Properties:-**

Figure 9.3: Buckyball

1. It is extremely stable and can withstand high temperature and pressure.

2. It can entrap atoms and small molecules, which don't react with the fullerene molecule.

3. Fullerenes are sparingly soluble in many aromatic solvents such as Toluene and Carbon Disulfide, but not in water.

4. When electron donating elements such as alkali or other metals are doped into the C60 they convert from a semiconductor in to a conductor or even super conductor.

 Applications:-

1. C60 molecules can exchange and transfer atoms and molecules the human body therefore they are used for drug delivery system.

2. They are used for inhibiting HIV virus.

3. They are used for making highly efficient lead acid batteries.

4. They are used for making photovoltaic cells.

Ultrasonics

Sound waves whose frequency is higher than 20000 Hz is called Ultrasonic waves.

Non -Destructive testing: The method by which the internal flaw of the given material can be detected without damaging the material is called Non destructive testing. Out of various methods used, Ultrasonic methods are more commonly used in Non-destructive testing.

9.12   Ultrasonic method for Non-Destructive testing

If the material has an internal flaw then the material is non-homogeneous. When the ultrasonic waves are passed through this material, some part of the incident waves are reflected back at the region of change in density and the remaining part is transmitted.

The ratio of the reflected and transmitted intensities of ultrasonic waves are given by the expression

$$\frac{I_2}{I_1} = \left(\frac{\rho_2 C_2 - \rho_1 C_1}{\rho_2 C_2 + \rho_1 C_1} \right)^2$$

where, I_2 and I_2 are the intensities of reflected and transmitted ultrasonic waves respectively.

ρ_2 and ρ_1 are the density of medium 2 and 1 respectively.

C_2 and C_1 are the Ultrasonic wave velocities in medium 2 and 1 respectively.

If $C_1 \cong C_2$ then, the expression becomes

$$\frac{I_2}{I_1} \cong \left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \right)^2$$

$$I_2 \cong I_1 \left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \right)^2$$

Thus, the intensity of the reflected wave is dependent on the difference between the densities of the mediums for a given incident wave intensity.

Principle:

When the ultrasonic waves travel from one medium to other of different density then, some part of incident waves gets reflected. By noting the time elapsed between the instant of transmission and reflected wave we can find out the location of change in density.

If the change in density is unexpected at that location then it indicates the presence of an internal flaw. The reflected wave can can tell us about the location of an internal flaw. The reflected wave is echo. Hence this testing is called pulse-echo method.
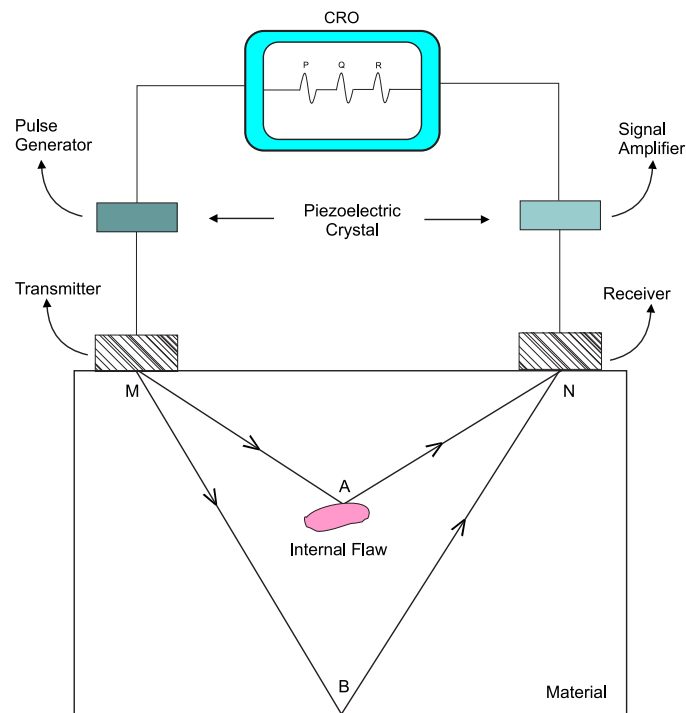
Working procedure:

Figure 9.4: Non-destructive method of testing by using ultrasonic waves

The testing set up consist of the pulse generator, piezoelectric crystal, signal amplifier and CRO (Cathode Ray Oscilloscope). The experimental setup is as shown.

The pulse generator will generate a pulse of signal whose frequency is same as the Ultrasonic frequency. The pulse generator is connected to CRO (time base) which detects a pip 'P' the incident pulse.

This signal is fed to a transducer (Piezoelectric crystal) which converts the electrical signal to mechanical vibration. The piezoelectric crystal is in contact with the material which is being tested. If there are any air gaps between the contact of piezoelectric crystal and the material then the energy is dissipated. To avoid this dissipation of energy a thin film of oil is placed between the piezoelectric crystal and the material.

The vibrations are passed through the material and they are reflected wherever there is change in density or discontinuity. There is a discontinuity at the boundary hence the wave gets reflected at B. Point A has internal damage which reflects some amount of light.

After the pip P the Q and R are due to reflection at the region of change in density at the location of internal flaw and at boundary. The distance MAN is less than MBN hence we get pip Q before R. By noting down the spacing of Q and R from P we can calculate the location of the internal flaw.

9.13 Experimental determination of elastic constants in solids

Determining the velocity of ultrasonic waves in solids:

In solids sound waves can exists as longitudinal and transverse waves hence there are two velocities
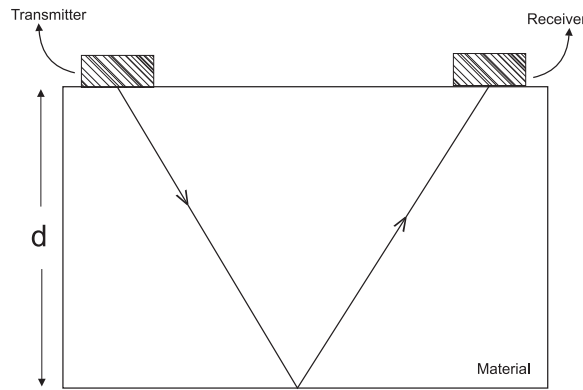(i) Longitudinal velocity(C_L)
(ii) Transverse velocity(C_S)

Figure 9.5: Setup to determine the velocity of ultrasonic waves in solids

By cutting the piezoelectric crystal in a suitable manner we can either generate longitudinal or a transverse waves. Therefore, we should find both the Longitudinal Velocity (C_L) and the Transverse velocity (C_S) for ultrasonic waves in case of solids.

If the piezoelectric crystal is cut in such a way that it generates longitudinal waves. This longitudinal wave which is incident gets reflected back at the boundary due to change in density. By knowing the dimension of the specimen we can know the distance d travelled from the point of incidence to the boundary where it gets reflected. The reflected wave travels a distance d before it is received by the piezoelectric crystal. If t is is the time taken for the transmitted wave to be received, then in time t the total distance travelled is 2d from the transmitting end to the receiving end. Therefore, the velocity $C_L = \frac{2d}{t}$. Similarly the shear velocity C_S can be obtained if the piezoelectric crystal is cut in such a way so as to give out transverse waves.

Determination of elastic constants in Solids:

In case of transverse waves, the velocity is $C_s = \sqrt{\frac{\eta}{\rho}} \Rightarrow \eta = \rho C_s^2 = \frac{q}{2(1+\sigma)}$

$$\text{Where} \quad \eta = \frac{q}{2(1+\sigma)} \Rightarrow C_s = \sqrt{\frac{q}{2(1+\sigma)\rho}} \tag{9.1}$$

$$C_L = \sqrt{\frac{q(1-\sigma)}{(1+\sigma)(1-2\sigma)\rho}} \tag{9.2}$$

From equation 9.1 and 9.2 we get,

$$\frac{C_s^2}{C_L^2} = \frac{\frac{q}{2(1+\sigma)\rho}}{\frac{q(1-\sigma)}{(1+\sigma)(1-2\sigma)\rho}} = \frac{(1-2\sigma)}{2(1-\sigma)}$$

$$\Rightarrow (1-2\sigma) = \frac{2C_s^2(1-\sigma)}{C_L^2} = \frac{2C_s^2}{C_L^2} - \frac{2C_s^2\sigma}{C_L^2}$$

$$\therefore \quad 1 - \frac{2C_s^2}{C_L^2} = \left(2 - \frac{2C_s^2}{C_L^2}\right)\sigma \Rightarrow \sigma = \frac{1 - \frac{2C_s^2}{C_L^2}}{\left(2 - \frac{2C_s^2}{C_L^2}\right)}$$

From equation 9.1 we get,

$$q = 2(1+\sigma)\rho C_s^2 \tag{9.3}$$

Thus by knowing C_s and C_L we can find q, η and σ where

q=Young's modulus

η=Rigidity modulus

$\sigma = \text{Poisson's ratio} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$

9.14   Experimental determination of elastic constants in liquids

Velocity of Ultrasonic waves in liquids:

To find the velocity of ultrasonic waves in liquids we set up an experiment as shown. Here, we have a water tight flask which is transparent and filled completely with water. Inside the flask at the bottom we keep a quartz crystal between two plates. . The plates are connected to a function generator (oscillator). When the frequency of the oscillator matches with the natural frequency of the quartz crystal it gives resonant vibration. These vibrations
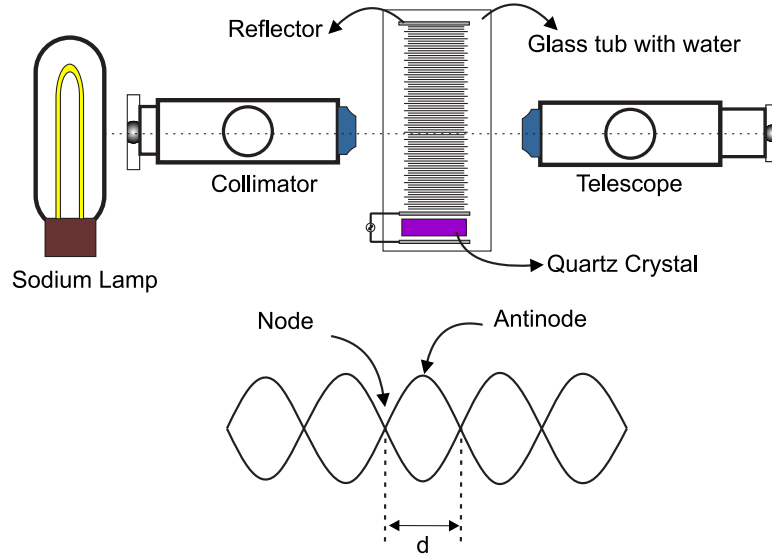
travel through the liquid.

Figure 9.6: Setup to determine the velocity of ultrasonic waves in liquids

The waves are reflected back by a reflector at the top of the flask. The reflected waves superimpose with the incident waves and they form stationary waves. The density of water increase at the nodes and decreases at the antinodes. Due to increase in density at the nodes they become opaque (doesn't allow light) where as antinodes allow light to pass through them. Thus, the liquid in the flask behaves like a diffraction grating with slits closely spaced. Hence, diffraction takes place such that

$$\frac{\sin\theta}{nN} = \lambda_L$$

where $\frac{1}{N}$=d=grating constant(distance between 2 lines on the grating)

In this the grating constant d is the distance between two nodes which is $\frac{\lambda_u}{2}$, where λ_u and λ_L are wavelengths of ultrasonic waves and sodium light respectively.

$$\frac{d\sin\theta}{n} = \lambda_L$$

and distance between two nodes d is given by

$$\frac{1}{N} = d = \frac{\lambda_u}{2}$$

Hence

$$\frac{\frac{\lambda_u}{2}\sin\theta}{n} = \lambda_L = \frac{\lambda_u\sin\theta}{2n}$$

Therefore

$$\lambda_u = \frac{2n\lambda_L}{\sin\theta}$$

and the velocity of ultrasonic waves is

$$v_u = \lambda_u f = \frac{2nf\lambda_u}{\sin\theta}$$

Thus we have determined the velocity of ultrasonic waves.

Determination of elastic constants in liquids:

In liquids the velocity of ultrasonic waves are given by $v_u = \sqrt{\frac{k}{\rho}}$

$\therefore$ the bulk modulus $v_u^2 = \frac{k}{\rho} \Rightarrow K = \rho v_u^2$, thus knowing the density ρ and the velocity of the ultrasonic waves in the liquid we can find the bulk modulus of that liquid.

* **

Multiple Choice Questions

1. Which of following is organic nanomaterial?

   (a) fullerene   (b) nanowires
   (c) carbon nanotubes   (d) protein based nanostructures

2. Molecular manufacturing is

   (a) allowing molecules to wander and bump together at random

   (b) method of arranging atoms precisely

   (c) method of arranging atoms crudely

   (d) none of these

3. In a nanomechanical bearing the bond between the atoms is

   (a) metallic bond   (b) ionic bond
   (c) chemical bond   (d) covalent bond

4. Nanofabrication might be done by

(a) removing one atom at a time until desired structure emerges

(b) assembling of a chip atom by atom resembling bricklaying

(c) a process resembling cell division in living things

(d) all of the above

5. Which of the following is a steady state system?

(a) isolated capacitor such that it cannot charge or discharge

(b) a wire carrying constant dc current

(c) both options (a) and (b) are examples for a steady state system

(d) LCR circuit

6. Electrical conductivity of carbon nanotubes depends on

(a) twist   (b) diameter

(c) both twist and diameter   (d) independent of twist and diameter

7. As the temperature is increased the thermal conductivity of carbon nanotubes

(a) remains same   (b) increases

(c) decreases   (d) can't say

8. In a carbon nanotube the bond between the carbon atoms is

(a) metallic bond   (b) ionic bond

(c) chemical bond   (d) covalent bond

9. Carbon nanotube is

(a) a sheet of carbon atoms rolled up into long tubes

(b) sixty carbon atoms arranged in the shape of a football

(c) one dimensional array of atoms

(d) three dimensional array of atoms

10. Fullerene is

(a) a sheet of carbon atoms rolled up into long tubes

(b) sixty carbon atoms arranged in the shape of a football

(c) one dimensional array of atoms

(d) three dimensional array of atoms

11. Nanowires are

    (a) a sheet of carbon atoms rolled up into long tubes

    (b) sixty carbon atoms arranged in the shape of a football

    (c) one dimensional array of atoms

    (d) three dimensional array of atoms

12. In classical scaling

    (a) the role of fields and currents in the system are expected to change

    (b) the material properties such as density, elastic moduli, coefficient of friction remain unchanged

    (c) both the options (a) and (b) are correct

    (d) both the options (a) and (b) are wrong

13. In classical electromagnetic scaling

    (a) The electrostatic field in the system and its effects remain unchanged with scaling

    (b) It is assumed that there is magnetic force

    (c) both the options (a) and (b) are correct

    (d) both the options (a) and (b) are wrong

14. Ultrasonic sound frequencies are typically considered to be

    (a) above 20,000 kilohertz

    (b) below 20,000 kilohertz

    (c) above 20,000 Hertz

    (d) below 20,000 Hertz

15. Waves used in ultrasonic testing of materials are _____ in nature.

    (a) mechanical   (b) magnetic

    (c) electromagnetic   (d) harmonious

16. The smallest distance between two points on an elastic wave where the particles are in the same state of motion is the:

 (a) period   (b) wavelength
 (c) frequency   (d) hypotenuse

17. Particle motion in a longitudinal wave is:

 (a) parallel to the direction of wave propagation

 (b) at right angles to the direction of wave propagation

 (c) retrograde

 (d) in counterclockwise ellipses

18. Compared to water, the acoustic impedance of steel is:

 (a) higher   (b) lower
 (c) about the same   (d) higher or lower depending on the wave mode

19. In ultrasonic testing a piezoelectric material is used to:

 (a) convert electric energy to mechanical energy

 (b) convert mechanical energy to electrical energy

 (c) both a and b

 (d) none of the above

20. While measuring the velocity of Ultrasonic waves in liquid the stationary waves in the liquid

 (a) decreases the density of liquid at antinodes and increases the density of liquid at nodes

 (b) increases the density of liquid at antinodes and decreases the density of liquid at nodes

 (c) density does not change

 (d) none of these

21. Thin layer of oil is placed between the piezoelectric crystal and the solid being tested in Non-destructive testing using ultrasonic waves to

 (a) reduce friction

(b) remove air gap so as to avoid dissipation of energy

(c) oil amplifies the amplitude of ultrasonic waves

(d) all of the above

22. Ultrasonic waves undergo reflection only if

(a) there is change in density of the medium

(b) there is no change in density of the medium

(c) there is mirror inside the medium

(d) the medium is homogeneous

23. Velocity of ultrasonic waves is

(a) same as ultraviolet light

(b) same as visible light

(c) same as sound waves

(d) higher than sound waves

24. Ultrasonic waves cannot undergo

(a) reflection   (b) diffraction
(c) polarization   (d) refraction

Descriptive Questions

1. Describe the different Nano-scale systems.

2. Discuss nano-technology system giving at least one application in detail.

3. Describe the concept of self-organization and explain its need in nano technology.

4. Write a brief note on (i) Nanotechnology (ii) Carbon nanotubes.

5. What are ultrasonic waves? Describe a method of measuring the velocity of ultrasonic waves in solids.

6. What is an acoustic grating? Explain how an acoustic grating is used to determine the velocity of ultrasonic waves in liquid. Also mention how the bulk modulus of a liquid can be evaluated.

7. What is non-destructive testing? Explain with principle how flaw in a solid can be detected by non-destructive method using ultrasonics?

8. Explain the density of states for various quantum structures.

9. Describe quantum tunneling.

Part VI

Solutions for MCQs and Problems

Answers to MCQ

Chapter 1
Modern Physics

1. (b)	2. (b)	3. (b)	4. (b)	5. (a)
6. (a)	7. (c), (d)	8. (b)	9. (a)	10. (c)
11. (a)	12. (d)	13. (c)	14. (a)	15. (b)
16. (d)	17. (b)	18. (b)	19. (d)	20. (a)
21. (b)				

Chapter 2
Quantum Mechanics

1. (b)	2. (c)	3. (a)	4. (a)	5. (b)	6. (d)
7. (c)	8. (b)	9. (a)	10. (d)	11. (c)	12. (c)
13. (d)	14. (a)	15. (a)	16. (a)	17. (a)	

Chapter 3
Electrical Conductivity in Metals and Semiconductors

1. (c)	2. (a)	3. (a)	4. (b)	5. (c)
6. (c)	7. (a)	8. (d)	9. (a)	10. (c)
11. (d)	12. (d)	13. (b)	14. (c)	15. (d)
16. (a)	17. (b)	18. (a)	19. (d)	20. (a)
21. (a)	22. (b)	23. (d)	24. (a)	25. (c)

Chapter 4
Superconductivty

1. (c)   2. (d)   3. (c)   4. (d)   5. (a)
6. (c)   7. (b)   8. (c)   9. (d)   10. (a)
11. (a) 12. (b) 13. (c) 14. (d) 15. (a)
16. (c)

Chapter 5
Lasers and Holography

1. (a)   2. (c)   3. (b)   4. (a)   5. (b)
6. (a)   7. (d)   8. (c)   9. (d)   10. (b)
11. (d) 12. (c)

Chapter 6
Optical Fiber

1. (c)   2. (a)   3. (b)   4. (b)   5. (d)
6. (b)   7. (c)

Chapter 7
Crystal Structure

1. (c)	2. (c)	3. (b)	4. (a)	5. (a)
6. (c)	7. (a)	8. (a)	9. (b)	10. (c)
11. (a)	12. (d)	13. (c)	14. (a)	15. (a)
16. (c)	17. (c)	18. (a)	19. (c)	20. (d)
21. (b)	22. (b)	23. (d)	24. (a)	25. (b)
26. (c)	27. (a)	28. (b)	29. (c)	30. (c)
31. (b)	32. (c)	33. (c)	34. (a)	35. (b)
36. (a)	37. (d)	38. (b)	39. (d)	40. (b)
41. (c)	42. (c)	43. (c)	44. (b)	45. (d)
46. (b)	47. (c)	48. (a)	49. (d)	50. (b)
51. (c)	52. (d)	53. (c)	54. (c)	55. (a)
56. (d)	57. (a)	58. (c)		

Chapter 9
Nanomaterials

1. (d)	2. (b)	3. (c)	4. (d)	5. (b)
6. (c)	7. (c)	8. (c)	9. (d)	10. (a)
11. (b)	12. (c)	13. (b)	14. (a)	15. (c)
16. (a)	17. (b)	18. (a)	19. (a)	20. (c)
21. (a)	22. (b)	23. (a)	24. (c)	25. (c)

Solutions

Chapter 1:Modern Physics

1. $\dfrac{P^2}{2m_P} = 10KeV = 1.602 \times 10^{-15}J$

   $m_P = 1.672 \times 10^{-27}Kg \Rightarrow P^2 = 5.36 \times 10^{-42}Kg^2m^2s^{-2}$

   $\Rightarrow P = 2.315 \times 10^{-21}Kgms^{-1}$

   $P = \dfrac{h}{\lambda} \Rightarrow \lambda = \dfrac{h}{P} = 2.86 \times 10^{-13}m$

   KE of the electron $= \dfrac{P^2}{2m_e}$ and $P = \dfrac{h}{\lambda}$

   since de Broglie wavelength is same P is same

   $KE = \dfrac{5.36 \times 10^{-42}}{2m_e} = 2.94 \times 10^{-12}J$

   KE=18.36 MeV (for electron)

2. $\lambda = \dfrac{h}{\sqrt{2meV}} = 1.12 \times 10^{-10}m$

3. $\lambda = \dfrac{h}{mv} = \dfrac{6.625 \times 10^{-34}}{0.3 \times 120 \times \left(\dfrac{1000}{3600}\right)} = 6.626 \times 10^{-35}m$

4. mass$=0.5 \times 10^6 eV/C^2$ equivalent energy is $0.5 \times 10^6 eV$

   $E = 8.01 \times 10^{-14}J$

   $\Rightarrow m = \dfrac{8.01 \times 10^{-14}}{9 \times 10^{16}} = 8.91 \times 10^{-31}Kg$

   $v = \sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2 \times 100 \times 1.6 \times 10^{-19}}{8.91 \times 10^{-31}}} = 5.996 \times 10^6 ms^{-1}$

   $\lambda = \dfrac{h}{mv} = 1.24 \times 10^{-10}m$

5. Practice problem

6. $m = 0.65 MeV/C^2 \qquad KE = 80eV$

   $m = 1.158 \times 10^{-30}Kg \qquad E = 80eV = 1.28 \times 10^{-17}J$

   $\Rightarrow v = \sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2 \times 1.28 \times 10^{-17}}{1.158 \times 10^{-30}}} = \sqrt{\dfrac{2.56 \times 10^{-17}}{1.158 \times 10^{-30}}} = 4.70 \times 10^6 ms^{-1}$

   $\lambda = \dfrac{h}{mv} = 1.21 \times 10^{-10}m$ (de-Broglie wavelength)

   $v = v_g$ i.e., particle velocity=group velocity

   $v_g = 4.70 \times 10^6 ms^{-1}$

   Phase velocity, $v_p = \dfrac{c^2}{v} = \dfrac{9 \times 10^{16}}{4.70 \times 10^6 ms^{-1}} = 1.91 \times 10^{10}ms^{-1}$

7. $E_P = h\nu = \dfrac{hc}{\lambda} = 9.94 \times 10^{-16} J$

$E_{electron} = \dfrac{P^2}{2m} = \dfrac{h^2}{2m\lambda^2} = E_e = 6.025 \times 10^{-18} J$

$\dfrac{E_P}{E_e} = 164.98 \quad E_P \approx 165 E_e$

Chapter 2:Quantum Mechanics

1. $\Delta V = 2.2 \times 10^4 ms^{-1}$

$m\Delta V = \Delta P$ and $\Delta P.\Delta x = \dfrac{\hbar}{2}$

$\Rightarrow \Delta x = \dfrac{h}{4\pi\Delta P} = \dfrac{h}{4\pi m\Delta V} = \dfrac{6.625 \times 10^{-34}}{4\pi \times 9.11 \times 10^{-31} \times 2.2 \times 10^4}$

$= 2.63 \times 10^{-9} m$

2. $L = 10^{-8} m \Rightarrow \Delta x = 10^{-8} m$, $\Delta V = \dfrac{\hbar}{2\Delta x}$

$\Delta V = \dfrac{\hbar}{2m\Delta x} = 5.78 \times 10^3 ms^{-1}$

This tells us that the minimum velocity that an electron should have in this box is 5.78×10^3

3.

$$ie \quad \Delta x \leq 10^{-14} m.$$

Then its uncertainty in the momentum is given by

$$\Delta p_x \geq \dfrac{h}{4\pi\Delta x},$$

$$ie., \quad \Delta p_x \geq \dfrac{6.625 \times 10^{-14}}{4 \times 3.14 \times 10^{-14}},$$

$$ie., \quad \Delta p_x \geq 5.275 \times 10^{-21} \, Ns,$$

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$E \geq 1.585 \times 10^{-12} \, J$$

$$\text{or} \ \ E \geq 10 \, \text{MeV}$$

So, in order to exist inside the nucleus, electron should posses energy greater or equal to $10\,MeV$. But experimentally determined values of energy of the electrons emitted by nucleus during the beta-decay were greater than $4\,MeV$. This clearly indicates that electrons cannot exist inside the nucleus.

4. $E_0 = 5.6 \times 10^{-3} eV = 3.495 \times 10^{16} J$

$E_0 = \dfrac{h^2}{8ma^2}$; if width is doubled $E' = \dfrac{h^2}{4(8ma^2)} = \dfrac{E_0}{4} = \dfrac{3.495 \times 10^{16}}{4} J = 8.738 \times 10^{15} J$

will be the ground state energy after the width is doubled.

5. $\varphi = \sqrt{\dfrac{2}{a}} \sin \dfrac{n\pi}{a} x \Rightarrow | \varphi |^2 \sin^2 \dfrac{n\pi}{a} x$

$n = 2$, (first extended state) $x = \dfrac{a}{2} \Rightarrow \dfrac{2}{a} \sin^2 \pi = 0 = | \varphi |^2$

$P = 0$

6. $\varphi = A \sin \left(\dfrac{n\pi x}{L} \right) \qquad V = 0,$

$\int_0^L | \varphi |^2 \; dx = 1 = \int_0^L A^2 \sin^2 \dfrac{n\pi}{L} x \, dx = 1$

$= \dfrac{A^2}{2} \int_0^L \left[1 - \cos \dfrac{2n\pi}{L} x \right] dx = 1 = \dfrac{A^2}{2} \left[2 - \left(\dfrac{L}{2n\pi} sin \dfrac{2n\pi}{L} x \right) \right] = 1$

$\Rightarrow A^2 = \dfrac{2}{L} \Rightarrow A = \sqrt{\dfrac{2}{L}}$

7. $a = 20 \overset{\circ}{A}, \qquad E = \dfrac{n^2 h^2}{8ma^2} = n^2 E_0 = n^2 \left(1.51 \times 10^{-20} J \right)$

$\lambda = \dfrac{h}{\sqrt{2mE}} = \dfrac{h}{n\sqrt{2mE_0}}$

$\lambda = \dfrac{6.625 \times 10^{-34}}{n \times 1.658 \times 10^{-25}} = \dfrac{4 \times 10^{-9}}{n} m$

$n = 1 \Rightarrow \lambda = 4 \times 10^{-9} m$

$n = 2 \Rightarrow \lambda = 2 \times 10^{-9} m$

$n = 3 \Rightarrow \lambda = 1.33 \times 10^{-9} m$

$E_1 = n^2 E_0 = E_0 = 1.51 \times 10^{-20} J$

$E_2 = 2^2 E_0 = 6.04 \times 10^{-20} J$

$E_3 = 9 E_0 = 13.59 \times 10^{-20} J$

8. $\Delta E \times \Delta t = \dfrac{\hbar}{2} = \dfrac{h}{4\pi} \quad \text{and} \quad \Delta E = h \Delta \gamma = h(\gamma_2 - \gamma_1)$

$\Delta E = h \left(\dfrac{c}{\lambda_2} - \dfrac{c}{\lambda_1} \right) = hc \left(\dfrac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right)$

$\lambda_2 = 4000 \times 10^{-10} + 4 \times 10^{-15} m \cong 4000 \overset{\circ}{A}$

$\lambda_1 = 4000 \times 10^{-10} - 4 \times 10^{-15} m \cong 4000 \overset{\circ}{A}$

$\lambda_1 \lambda_2 = (4000 \times 10^{-10})^2 = 1.6 \times 10^{-13} m^2$

$\lambda_1 - \lambda_2 = \Delta \lambda = 8 \times 10^{-5} \overset{\circ}{A} = 8 \times 10^{-15} m$

$\Delta E = hc \left(\dfrac{\Delta \lambda}{\lambda_1 \lambda_2} \right) = 6.625 \times 10^{-34} \times 3 \times 10^8 \times \left(\dfrac{8 \times 10^{-15}}{1.6 \times 10^{-13}} \right)$

$$\Delta E = 9.93 \times 10^{-27} J$$
$$\Delta t = \frac{h}{4\pi \Delta E} = 5.3 \times 10^{-9} s = 5.3 ns$$

9. $v = 6 \times 10^5 ms^{-1}$ Error=0.01%
$$\Delta V = 120 ms^{-1} = \frac{V \times 2 \times 0.01}{100} \Rightarrow \Delta P = m \Delta V = 1.093 \times 10^{-28} Kgms^{-1}$$
$$\Delta P \Delta x = \frac{\hbar}{2} = \frac{h}{4\pi} \Rightarrow \Delta x = \frac{h}{4\pi \Delta P}$$
$$\Delta x = 4.82 \times 10^{-7} m$$

10. $a = 0.12 nm$ and $E = \dfrac{n^2 h^2}{8ma^2} = n^2 E_0$
$$E = n^2 E_0 \text{ and } E_0 = \frac{h^2}{8ma^2} = \frac{(6.625 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (0.12 \times 10^{-9})^2}$$
$E_0 = 4.814 \times 10^{-18} J$ (ground state $n = 1$)=26.11eV
$E_1 = 1.925 \times 10^{-17} J$ $(n = 2) E_1 = 120.35 eV$
$E_2 = 4.33 \times 10^{-17} J$ $(n = 3) E_2 = 270.42 eV$

Chapter 3:Electrical conductivity in metals and Semiconductors

1. $f(E) = \dfrac{1}{e^{E-E_P/KT} + 1} = 0.01$
$E - E_P = 0.5 eV = 8.01 \times 10^{-20} J$
$\Rightarrow e^{E-E_F/KT} + 1 = 100$ $e^{E-E_P/KT} = 99$
$ln(e^{E-E_F/KT}) = ln 99 = 4.595$
$\Rightarrow \dfrac{E - E_F}{kT} = 4.595$ $T = \dfrac{E - E_F}{k(4.6)} = 1261.2K$

2. $E - E_F = -0.2 eV$ because$(E \ is \ 0.2 eV \ below \ E_F)$
$= -3.2 \times 10^{-20} J$
$f(E) = \dfrac{1}{e^{(E-E_F/KT)} + 1}$,
if $T = 300K$ then $f(E) = \dfrac{1}{e^{-7.736} + 1} = 0.9996$
$ie., 99.96\%$
at $T = 1000K, f(E) = \dfrac{1}{e^{-3.2 \times 10^{-20}/K(1000)} + 1} = 0.9103$
therefore probability is 91.03%

3. $f(E) = x$
$E - E_F = -\Delta E \ (if \ E \ is \ \Delta E \ below \ E_F)$
and $E - E_F = \Delta E \ (if \ E \ is \ \Delta E \ above \ E_F)$

The probability of occupancy at an energy level ΔE below E_F is

$$f(E) = \frac{1}{e^{-\Delta E/kT} + 1} = x$$

Probability of occupancy + Probability of non-occupancy is always 1

Therefore probability of non-occupancy = 1-probability of occupancy

therefore probability of non-occupancy $= 1 - \dfrac{1}{e^{\Delta E/kT} + 1}$

$$\Rightarrow 1 - \frac{1}{e^{\Delta E/kT} + 1} = \frac{e^{\Delta E/kT} + 1 - 1}{\Delta E/kT} + 1$$

$$= \frac{1}{1 + \dfrac{1}{e^{\Delta E/kT}}} = \frac{1}{e^{-\Delta E/kT} + 1} = x$$

Hence proved

4. Non covalent $\Rightarrow$ 1 electron per atom = N

$$n = \frac{NDN_A}{M_A} = \frac{(1)(10^3 \times 1.9)Kgm^3(6.022 \times 10^{23})mol^{-1}}{132.9 \times 10^{-3}Kgmol^{-1}}$$

$n = 8.61 \times 10^{27} electronsm^{-3}$

$$E_F = \left(\frac{3n}{\pi}\right)^{2/3} \left(\frac{h^2}{8m}\right) = (4.07 \times 10^{18})\left(\frac{h^2}{8m}\right) = 2.45 \times 10^{-19}J$$

$E_F = 1.53eV$

5. $\rho = 1.54 \times 10^{-8}nm$

$n = 5.8 \times 10^{28}m^{-3}$

$$\Rightarrow \sigma = \frac{1}{\rho} = 6.49 \times 10^7 \, \text{hom}\, m^{-1}$$

$$\frac{\sigma m}{ne^2} = \rho \Rightarrow \rho = \frac{6.49 \times 10^7 \times 9.11 \times 10^{-31}}{5.8 \times 10^{28} \times (1.602 \times 10^{-19})^2} = 3.97 \times 10^{-14}s$$

for metals $\rho = \rho_r = 3.97 \times 10^{-14}s$

6. $v = 1V, \quad d = 1mm,$

$\mu = 40cm^2v^{-1}s^{-1} = 40 \times 10^{-4}m^2V^{-1}s^{-1}$

$$\mu = \frac{V_d}{E} \Rightarrow \mu E = V_d \Rightarrow 40 \times 10^{-4} \times E = V_d$$

$$E = \frac{v}{d} = \frac{1}{10^{-3}} = 1000Vm^{-1}$$

Therefore $V_d = 4ms^{-1}$

Thermal energy$=\dfrac{3}{2}kT$

$$=\frac{3}{2} \times 1.38 \times 10^{-23} \times 300$$

$= 6.21 \times 10^{-21}J = 0.039eV$

7. $E - E_F = 0.02eV = 3.204 \times 10^{-21}J$

$$f(E) = \frac{1}{e^{3.204 \times 10^{-21}/kT} + 1}$$

at $T = 200K$

$$f(E) = \frac{1}{e^{1.16} + 1} = 0.2386 = 23.86\%$$

at $T = 400K$

$$f(E) = \frac{1}{e^{0.58} + 1} = 0.3589 = 35089\%$$

8. $E_F = 21eV = 3.37 \times 10^{-19} J$

$T = 300k, find E$

$$\frac{1}{e^{E-E_F/kT} + 1} = f(E)$$

$$\Rightarrow \frac{1}{f(E)} = e^{E-E_F/kT} + 1$$

$$e^{E-E_F} kT = \frac{1}{f(E)} - 1 = \frac{1 - f(E)}{f(E)}$$

$$\frac{E - E_F}{kT} = \ln\left(\frac{1 - f(E)}{f(E)}\right)$$

$$E = E_F + kT \ln\left(\frac{1 - f(E)}{f(E)}\right)$$

if $f(E) = 0.99$

$$E = 3.37 \times 10^{-19} + K(300) \ln\left(\frac{0.01}{0.99}\right)$$

$$E = 3.18 \times 10^{-19} J = 1.985 eV$$

if $f(E) = 0.01$

$$E = E_F + kT \ln\left(\frac{0.99}{0.01}\right) = 3.56 \times 10^{-19} J = 2.22 eV$$

if $f(E) = 0.5$

$$E = E_F + 0 = E_F = 3.37 \times 10^{-19} J = 2.1 eV = E_F$$

9. $N = 1$,

$M_A = 107.9 Kg/Kmol$

$N_A = 6.022 \times 10^{26} Kmol^{-1}$

$D = 10500 Kgm^{-3}$

Therefore $n = \dfrac{N N_A D}{M_A} = 5.86 \times 10^{28} m^{-3}$

$$E_F = \left(\frac{3n}{\pi}\right)^{2/3} \left(\frac{h^2}{8m}\right)$$

$$= (5.6 \times 10^{28})^{2/3} (6.025 \times 10^{-38})$$

$$= 1.464 \times 10^{19} \times 6.025 \times 10^{-38}$$

$$E_F = 8.8206 \times 10^{-19} J = 5.5 eV$$

10. $f(E) = \dfrac{1}{e^{E-E_F/kT} + 1}$

$E - E_F = 0.5 eV = 8.01 \times 10^{-20} J$

$$0.01 = \frac{1}{e^{5802.3/T} + 1} \quad \Rightarrow \quad e^{5.802 \times 10^3 /T} + 1 = 100$$

$$e^{5.802 \times 10^3 /T} = 99 \text{ and } \frac{5.802 \times 10^3}{T} = ln\,99 = 4.595$$

Therefore $T = \dfrac{5.802 \times 10^3}{4.595} = 1262.7K$

Chapter 4:Superconductivity

Chapter 6:Optical fibers

1. $NA = \sqrt{n_1^2 - n_2^2} = 0.2$ (because for air $n_0 = 1$)
   $\Rightarrow n_1^2 - n_2^2 = 0.04 \quad \Rightarrow \quad n_1^2 = n_2^2 + 0.04 = 2.568$
   $\Rightarrow n_1 = 1.603$ (RI of core)
   for water RI=1.33
   $NA = \dfrac{\sqrt{n_1^2 - n_2^2}}{1.33} = \dfrac{0.2}{1.33} = \sin\theta_0$
   $\Rightarrow \theta_0 = \sin^{-1}(0.15) = 8.65^0$ if surrounded by water

2. $V = \dfrac{\pi d}{\lambda}\sqrt{n_1^2 - n_2^2}$
   $n_1 = 1.53, \quad n_2 = 1.5 \quad d = 2 \times$ core radius $= 100\mu m$
   $\lambda = 1\mu m \quad$ therefore$V = \frac{\pi(100 \times 10^{-6})\sqrt{1.53^2 - 1.5^2}}{1 \times 10^{-6}} = 94.7$
   number of modes $= \dfrac{V^2}{2} = 4484\,\text{modes}$

3. $P_{out} = 0.15\,P_{in}, \quad L = 0.4\,Km$
   Attenuation $\alpha = -\dfrac{10}{L}\,log_{10}\left(\dfrac{0.15\,P_{in}}{P_{in}}\right) = 20.6\,dB/Km$

4. Practice problem

5. $\alpha = 3.6\,dB/Km$
   (i) After 1 Km distance , $\dfrac{P_{out}}{P_{in}} =?$
   (ii) After 3 Km distance , $\dfrac{P_{out}}{P_{in}} =?$
   $\alpha = -\dfrac{10}{L}\,log_{10}\left(\dfrac{P_{out}}{P_{in}}\right)$
   $\Rightarrow log_{10}\left(\dfrac{P_{out}}{P_{in}}\right) = -\left(\dfrac{L\alpha}{10}\right) \quad \Rightarrow \quad \dfrac{P_{out}}{P_{in}} = 10^{-\frac{\alpha L}{10}}$
   After 1 $Km \dfrac{P_{out}}{P_{in}} = 10^{-3.6/10} = 0.437$

After $3\,Km\,\dfrac{P_{out}}{P_{in}} = 10^{-10.8/10} = 0.0832$

6. Core diameter$=40 \times 10^{-6}m$

RI of core $=1.55$

RI of cladding$=1.5$

$\lambda = 1400 \times 10^{-9}m$

n_0 for air $= 1$

$NA = \sqrt{n_1^2 - n_2^2} = 0.391$

fractional index change $\Delta = \dfrac{n_1 - n_2}{n_1}$

$\Delta = \dfrac{1.55 - 1.5}{1.55} = 0.0323$

V-number$=\dfrac{\pi d}{\lambda}\sqrt{n_1^2 - n_2^2} = \dfrac{\pi \times 40 \times 10^{-6}}{1400 \times 10^{-9}} \times 0.391$

$= 35$

Chapter 7:Crystal Physics

1. For FCC

$\sqrt{2}a = 4R \;\Rightarrow\; a = 2\sqrt{2}R = 2\sqrt{2} \times 0.127nm$

$\Rightarrow\; a = 3.59 \times 10^{-10}m$

$d_{hkl} = \dfrac{3.59 \times 10^{-10}}{\sqrt{9+4+4}} = 8.71 \times 10^{-11}m$

2. $(101), (121), (010)$

$(101) \;\Rightarrow\; 1 \infty 1$ (see fig1)

$(121) \;\Rightarrow\; 1\ 0.5\ 1$ (see fig2)

$(010) \;\Rightarrow\; \infty\ 1\ \infty$ (see fig3)

3. $, \;\Rightarrow\; n = \dfrac{I}{e} =$ number of electrons bombarding the target per second

$n = \dfrac{5 \times 10^{-3}}{1.6 \times 10^{-19}} = 3.12 \times 10^{16}s^{-1}$

maximum speed v $= \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2 \times 1.6 \times 10^{-19} \times 10^4}{9.1 \times 10^{-31}}} = 59.34 \times 10^6 ms^{-1}$

4. $a = 3.18\text{Å}, \quad \lambda = 0.58 \times 10^{-10}m, \quad n = 2$

$(h\,k\,l) = (1\,3\,2)$

$d = \dfrac{a}{\sqrt{h^2 + k^2 + l^2}} = 1.018 \times 10^{-10}m$

$$2\,d\sin\theta = 2\,\lambda \quad \Rightarrow \quad \sin\theta = \frac{\lambda}{d} = displaystyle\frac{0.58 \times 10^{-10}m}{1.018 \times 10^{-10}m}$$

Therefore $\sin\theta = 0.57 \quad \Rightarrow \quad \theta = $ glancing angle $= \sin^{-1}(0.57)$

$\theta = 34.75^0$

5. Draw planes in the cubical cell $(1\,3\,2)\,(1\,\bar{1}\,0)\,(1\,\bar{2}\,1)$

   (see figures 4,5,6)

6. $n = 1 \quad \lambda = 0.7\mathring{A} \quad$ plane $(h\,k\,l) = (3\,0\,2)$

   $\theta = 35^0, \quad 2d\sin\theta = n\lambda$

   $$\Rightarrow \quad d = \frac{n\lambda}{2\sin\theta} = \frac{(1)(0.7 \times 10^{-10})}{2 \times \sin 35} = 6.1 \times 10^{-11}m$$

   $d = 0.61\mathring{A} \quad \Rightarrow \quad a = d\sqrt{h^2 + k^2 + l^2}$

   Therefore $a = 61.02 \times 10^{-12} \times \sqrt{3^2 + 2^2} = 2.2\mathring{A}$

7. $x = 2a, \quad y = 3b, \quad z = 6c$

   $[1\,0\,0] \to x - axis, \quad [0\,1\,0] \to y - axis \quad [0\,0\,1] \to z - axis$

   Therefore $\dfrac{x}{a} = 2, \quad \dfrac{y}{b} = 3, \quad \dfrac{z}{c} = 6$

   $\dfrac{a}{x} = \dfrac{1}{2} \quad \dfrac{b}{y} = \dfrac{1}{3} \quad \dfrac{c}{z} = \dfrac{1}{6}$

   $LCM = 6$

   $(h\,k\,l) = (3\,2\,1)$

8. $n = 1, \quad d = 2.82 \times 10^{-10}m, \quad \theta = 6^0, \quad \lambda = ?$

   $2d\sin\theta = n\lambda$

   $2 \times 2.82 \times 10^{-10} \times \sin 6 = 5.89 \times 10^{-11} = 0.59\mathring{A}$

9. Repeated

10. Practice problem

11. $[4\,,-3\,,1/2\,]$

    $h \to \dfrac{1}{4}, \quad k \to -\dfrac{1}{3}, \quad l \to 2$

    $LCM = 12$

    $\Rightarrow (h\,k\,l) = (3\ 4\ 24)$

12. $x = 0.75, \quad y = 0.5, \quad z = \infty$

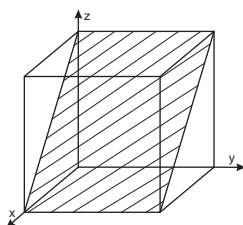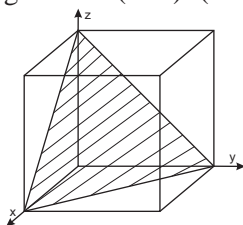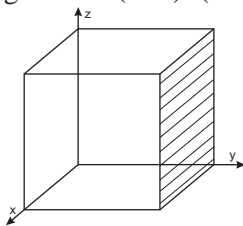$\quad x = 3/4, \quad y = 1/2, \quad z = \infty$
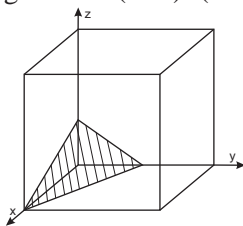
take reciprocal

$\dfrac{4}{3} \quad 2 \quad 0$

Therefore $(h\,k\,l) = (4\,6\,0)$ (see fig7)

13. $x = 1, \quad y = \infty, \quad z = \infty$

$\quad (h\,k\,l) = (1\,0\,0)$

14. Practice problem

Figure 9.7: (h k l)=(1 0 1)

Figure 9.8: (h k l)=(1 2 1)

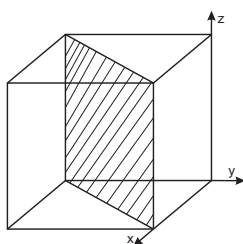Figure 9.9: (h k l)=(0 1 0)

Figure 9.10: (h k l)=(1 3 2)

Figure 9.11: (h k l)=$(1\,\bar{1}\,0)$

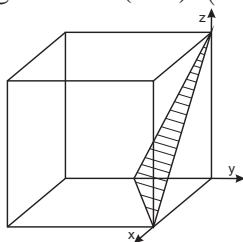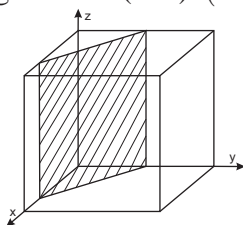Figure 9.12: (h k l)=$(1\,\bar{2}\,1)$

Figure 9.13: (h k l)=$(4\,6\,0)$