Maxwell's eqns for free space

Considering free space for which, the charge density $\rho = 0$ and current density $J = 0$.

Maxwell's eqns for free space become

Gauss law $\nabla \cdot E = 0$ — (1)

Gauss law for magnetism $\nabla \cdot B = 0$ or $\nabla \cdot H = 0$ — (2)

Faraday's law $\nabla \times E = -\frac{\partial B}{\partial t}$ — (3)

& Ampere law $\nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t}$ — (4)

Since $H = \frac{B}{\mu_0}$. eqn (4) can be written as

$$\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \quad \text{— (5)}$$

Expression for velocity of light :

Taking curl of equation (3)

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times B)$$

using equation eqn (5)

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\right)$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{— (6)}$$

By laplace identity

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E \quad \text{— (7)}$$

But by eqn (1) $\nabla \cdot E = 0$

$$\therefore \nabla \times (\nabla \times E) = -\nabla^2 E \quad \text{— (8)}$$

from eqn (6) and (8)

$$\nabla^2 E = \mu_0 \varepsilon_0 \, \partial^2 E / \partial t^2 \quad \text{— (9a)}$$

Similarly for B,

$$\nabla^2 B = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \quad \text{— (9b)}$$

The Equations represents the typical wave equations whose phase velocity is given by the square root of the quantity that is the reciprocal of the co-efficients of the time derivative

i.e $$V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{(4 \times \pi \times 10^{-7}\, wb/Amp)(8.9 \times 10^{-12}\, C^2/N\text{-}m^2)}}$$

$$= 3.0 \times 10^{8}\, m/s$$

$\therefore$ $V = C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ the velocity of light