

29.8 ENERGY BAND DIAGRAMS FOR SOME TYPICAL SOLIDS

29.8.1 Lithium

Let us consider the element lithium belonging to Group I in the periodic table. The electron configuration of lithium atom is $1s^2 2s^1$. The $1s$ shell is closed and there is only one electron at the $2s$ level. In solid lithium, $1s$ and $2s$ bands form corresponding to the $1s$ and $2s$ levels, as illustrated in Fig. 29.11.

Both $1s$ and $2s$ bands have N levels each. The $1s$ band is completely filled as $2N$ electrons occupy N energy levels whereas the $2s$ band is half-filled because the N available electrons fill $N/2$ lower levels in the band leaving the upper $N/2$ levels vacant. In general, the solids of Group-I elements form such half-filled energy bands at the top and therefore belong to the group of conductors.

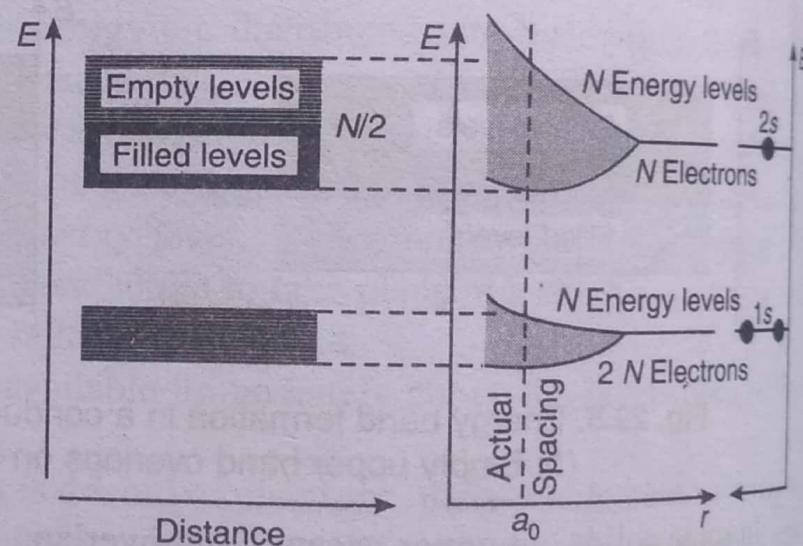


Fig. 29.11. Energy level splitting and energy band configuration in lithium solid showing half-filled $2s$ band.

therefore belong to the conductors group.

29.8.3 Energy Band Diagrams for Silicon and Diamond

Silicon belongs to Group IV elements in the periodic table. The electron configuration of silicon atom is $1s^2 2s^2 2p^6 3s^2 3p^2$. It is seen that the inner K and L shells are closed and the corresponding bands would be completely filled. In the outer subshells $3s$ and $3p$, $3s$ -subshell is closed. The $3p$ sub-shell is partially filled. Hence it is expected to behave as a good conductor. But because of formation of a hybrid band, which later branches out, the Si solid behaves as a semiconductor.

In the crystal formation process, when the atoms are very far apart, as at position 'd' in Fig. 29.13, the electrostatic interaction among them is negligible. Consequently, the electronic

energy levels of the crystal will be the same as those of isolated atoms. As the separation between atoms decreases, the $3s$ and $3p$ levels split and two bands are formed, as shown at position 'c' in Fig. 29.13. The band corresponding to $3s$ level has N energy levels and the band corresponding to $3p$ level has $3N$ levels. $2N$ electrons occupy N levels in $3s$ -band and $2N$ electrons occupy N levels in $3p$ -band. It may now be noted that there is an energy gap between the two bands. The energy gap is seen to decrease with the decrease in atomic spacing. At

position 'b' in Fig. 29.13 the two bands merge and form a composite band. The $3N$ upper levels merge with N lower levels, giving rise to a total of $4N$ levels. These levels have to be occupied by the $4N$ electrons available in total, and so the lowermost $2N$ levels are filled. When the atomic distance in our imaginary crystal is further reduced, the interaction among the atoms becomes very strong. Beyond the lattice spacing 'b' in Fig. 29.13, we find that the composite band branches out and once again two bands are formed, separated by a forbidden gap, E_g . The significant point is that the $4N$ energy levels are equally divided between the two branches. There is an equal distribution of levels, $2N$ in each, in the two bands. The $4N$ electrons available in total at $3s$ and $3p$ levels, now occupy the lower energy band and leave the upper band vacant. The lower band constitutes the valence band and the upper band the conduction band. This is the situation at the actual spacing ' a_0 ' in the silicon crystal. At position a_0 the two bands are not widely separated from each other. The value of E_g at 0 K is 1.12 eV . At normal temperatures, a significant number of electrons will be thermally excited from valence band to conduction band. The electrons excited to conduction band respond to the external voltage and produce a modest flow of current. Thus, Si behaves as a semiconductor.

Diamond

It is evident from Fig. 29.13 that the energy gap between the two branches goes on increasing with decreasing atomic distance. At the interatomic distance corresponding to line at 'a' in Fig. 29.13, the distance between the two bands becomes considerably large. In case of diamond the two bands are separated by 5.47 eV . Even at high temperatures, the thermal energy would be insufficient to excite enough number of electrons to the conduction band. Because of the non-availability of electrons in the conduction band electrical conduction cannot take place in the material and hence diamond behaves as insulator.

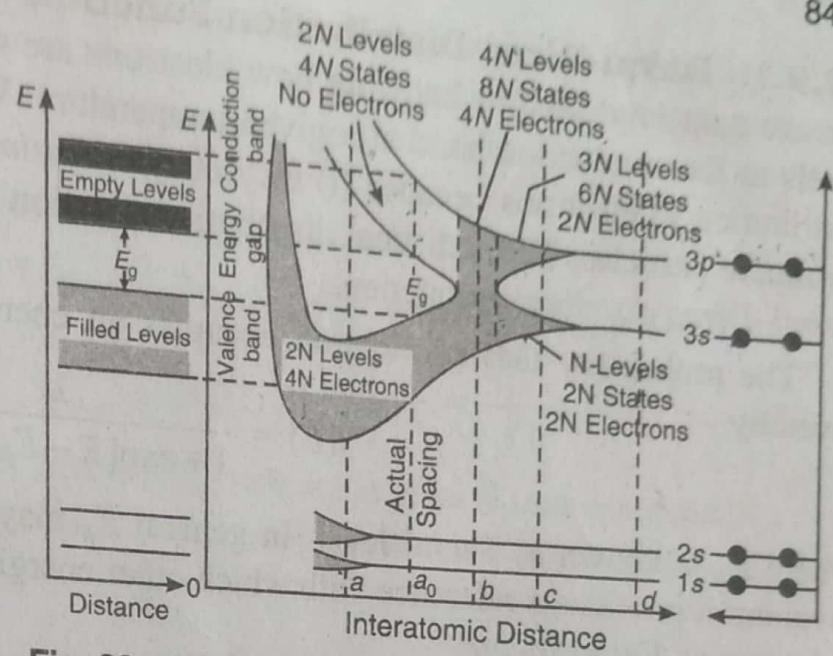


Fig. 29.13. Energy level splitting and band formation in crystals of Group IV A elements

29.9.1 Fermi-Dirac Distribution Function

We are next interested in knowing how electrons are distributed among the various energy levels in the conduction band at a given temperature. We cannot apply Maxwell-Boltzmann distribution to electrons because (i) they obey *exclusion principle* and (ii) they are *indistinguishable* particles. The statistical distribution function applicable to quantum particles is the *Fermi-Dirac distribution* function.

The probability that an electron occupies an energy level E at thermal equilibrium is given by

$$f(E) = \frac{1}{1 + \exp[(E - E_F)/kT]}$$

where E_F is known as **Fermi level**. In general E_F may or may not correspond to an energy level but it provides a reference with which other energies can be compared. The function $f(E)$ is known as **Fermi factor**.

The above equation is known as *Fermi-Dirac equation* or *Fermi-Dirac distribution function*. Note that the probability of the electron to occupy the energy level E increases with temperature. We first discuss about the distribution function and the related topics with reference to conductors. We shall find later that these concepts are equally applicable to other cases.

Example 29.1. Evaluate the Fermi function for energy kT above the Fermi energy.

Solution. The Fermi function is given by $f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$

$$\text{If } (E - E_F) = kT, \text{ then } f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} = \frac{1}{1 + e^1} = \frac{1}{1 + 2.78} = \frac{1}{3.78} = 0.269.$$

Example 29.2. In a solid, consider the energy level lying 0.01 eV below Fermi level. What is the probability of this level not being occupied by an electron?

Solution. $(E_F - E) = [E_F - (E_F - 0.01)] = 0.01 \text{ eV}$ and $kT = 0.026 \text{ eV}$ at $T = 300 \text{ K}$

The probability of an energy level E not being occupied by an electron is given by $[1 - f(E)]$.

$$\begin{aligned} [1 - f(E)] &= 1 - \frac{1}{1 + e^{(E-E_F)/kT}} = \frac{1}{1 + e^{(E_F-E)/kT}} = \frac{1}{1 + e^{0.01\text{eV}/0.026\text{eV}}} = \frac{1}{1 + e^{0.385}} \\ &= \frac{1}{1 + 1.47} = 0.405 \end{aligned}$$

29.9.2 Fermi Level

The occupancy of the energy levels by electrons in conductors is described by the Fermi-Dirac distribution function.

$$f(E) = \frac{1}{1 + \exp[(E - E_F)/kT]} \quad (29.9)$$

We distinguish two situations - one at absolute zero and the other at higher temperatures.

Case 1: $T = 0 \text{ K}$

Fig. 29.14 (a) depicts the conduction band of a conductor at 0K. At absolute zero, electrons occupy energy levels in pairs starting from the bottom of the band up to an upper level designated as E_F , leaving the upper levels vacant. **Fermi level** can be, therefore, defined as *the uppermost filled energy level in a conductor at 0K*. Correspondingly, **Fermi energy**

is defined as maximum energy that a free electron can have in a conductor at 0K. To use an analogy, the electron distribution in the conduction band can be likened to water at rest in a container. The Fermi level corresponds to the top surface of water. The Fermi function at 0K is shown in Fig. 29.14 (b).

Let us now apply equ. (29.9) to the solid taking the value of T as 0 K.

(i) For energy levels E lying below E_F , $E < E_F$, $(E - E_F)$ is a negative quantity.

$$\therefore f(E) = \frac{1}{1 + e^{-(E-E_F)/0}} = \frac{1}{1 + e^{-\infty}} = \frac{1}{1+0} = 1$$

$f(E) = 1$ indicates that all the energy levels lying below the level E_F are occupied.

(ii) For energy levels located above E_F , $E > E_F$, $(E - E_F)$ is a positive quantity.

$$\therefore f(E) = \frac{1}{1 + e^{(E-E_F)/0}} = \frac{1}{1 + e^{\infty}} = \frac{1}{1+\infty} = 0$$

The result $f(E) = 0$ implies that all the levels above E_F are vacant at $T = 0K$.

(iii) For $E = E_F$, the quantity $(E - E_F) = 0$.

$$\therefore f(E) = \frac{1}{1 + e^{0/0}} = \text{indeterminate}$$

The above result implies that the occupancy of Fermi level at 0K ranges from zero to one.

Case 2: $T > 0K$

On heating the conductor, electrons are excited to higher energy levels. In general, $E_F \gg kT$. Therefore, for most of the electrons lying

deep in the conduction band, the thermal energy is not sufficient to cause a transition to an upper unoccupied level. At normal temperatures, only those electrons occupying the energy levels near the Fermi level can be thermally excited. These levels make up a narrow band of width kT directly adjacent to the Fermi level. Therefore, upon heating the solid, electrons having energy a little below E_F , jump into levels with energy somewhat above E_F and a new energy distribution of electrons is obtained.

Thus, as a result of thermal excitation, the probability of finding electrons in the levels immediately below E_F will decrease. On the same hand, the probability of finding electrons in

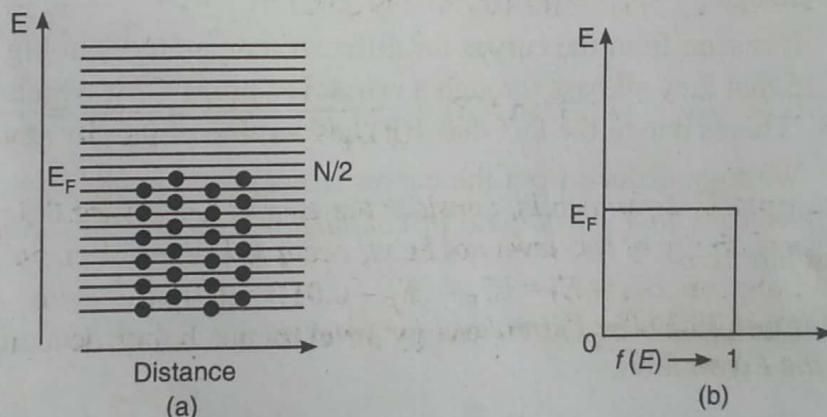


Fig. 29.14

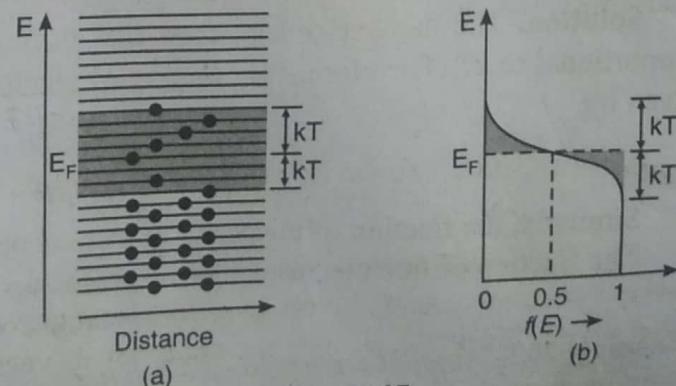


Fig. 29.15

the levels immediately above E_F increases. This fact is reflected in the graph (Fig. 29.15 b) as a blurring of the step plot.

At $T > 0\text{K}$, if we consider an electron at Fermi level, then $E = E_F$.

$$\therefore f(E) = \frac{1}{1 + e^{0/kT}} = \frac{1}{1+1} = \frac{1}{2}$$

This implies that the probability of occupancy of Fermi level at any temperature above 0K is 0.5 or 50%. Now we can define Fermi level as the energy level, which has a probability of occupancy of 0.5. An operational definition of Fermi energy can be given now. *Fermi energy is the average energy possessed by electrons participating in conduction in metals at temperatures above 0K.*

29.9.3 Effect of Temperature on Fermi Function

The Fermi-Dirac distribution curves for different temperatures are shown in Fig. 29.16. At $T = 0\text{ K}$, there is an abrupt jump in the value of $f(E)$ from 1 to zero at E_F . At $T > 0\text{K}$ the change is gradual. The higher the temperature, more gradual is the change.

It is seen from the curves for different temperatures in Fig. 29.16 that they all pass through a **crossover point C**, at which the probability of occupancy is 0.5. This is due to the fact that $f(E)$ has a value of 0.5 for any temperature greater than 0K.

We may deduce from the curves that *Fermi energy E_F is the average energy possessed by electrons that participate in conduction process in a conductor at temperatures above absolute zero.*

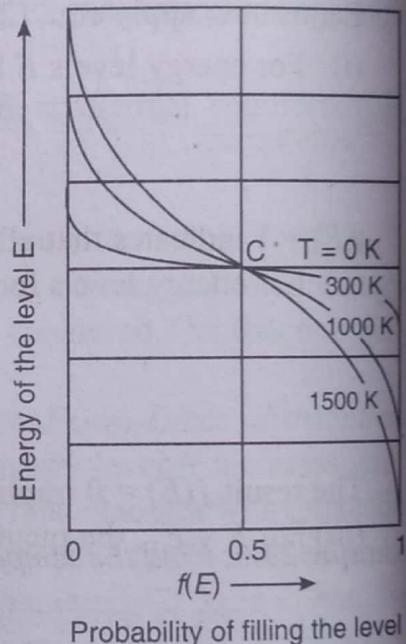


Fig. 29.16

Example 29.6. Find the temperature at which there is 1% probability that a state with energy 2 eV is occupied. Given that Fermi energy is 1.5 eV.

Solution. The probability of an energy state E being occupied by an electron is given by

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

$$E - E_F = 2 \text{ eV} - 1.5 \text{ eV} = 0.5 \text{ eV} \quad \text{and} \quad f(E) = 1\%$$

$$\frac{1}{100} = \frac{1}{1 + e^{0.5/kT}} \quad \text{or} \quad e^{0.5/kT} = \frac{0.99}{0.01} = 99$$

Taking logarithm on both the sides, we get

$$\frac{0.5 \text{ eV}}{kT} = 2.303 \log 99$$

$$\text{or} \quad \frac{0.5 \text{ eV}}{kT} = T = \frac{0.5 \text{ eV}}{2.303 \log 99 \times 8.61 \times 10^{-5} \text{ eV}} = 1262 \text{ K.}$$

Example 29.7. Show that the probability of finding an electron of energy ΔE above the Fermi level is same as probability of not finding an electron at energy ΔE below the Fermi level.

OR

Show that the probability that a state ΔE above the Fermi level E_F is filled equals the probability that a state ΔE below E_F is empty.

Solution. Let us consider an energy level E_2 that is above the Fermi level by an amount of energy ΔE . The probability that the energy level E_2 is occupied is given by

$$\begin{aligned} f(E_2) &= f(E_F + \Delta E) = \frac{1}{1 + \exp[(E_2 - E_F)/kT]} \\ &= \frac{1}{1 + \exp[(E_F + \Delta E - E_F)/kT]} \\ f(E_2) &= \frac{1}{1 + \exp(\Delta E / kT)} \end{aligned} \tag{1}$$

Next, let us consider the energy level E_1 that is below the Fermi level by energy ΔE . $[1 - f(E_1)]$ gives the probability that the level E_1 is not occupied.

$$\begin{aligned}
 [1 - f(E_1)] &= [1 - f(E_F - \Delta E)] \\
 &= 1 - \frac{1}{1 + \exp[(E_F - \Delta E - E_F) / kT]} \\
 &= 1 - \frac{1}{1 + \exp[-\Delta E / kT]} \\
 &= \frac{\exp[-\Delta E / kT]}{1 + \exp[-\Delta E / kT]}
 \end{aligned}$$

or

$$[1 - f(E_1)] = \frac{1}{1 + \exp[\Delta E / kT]} \quad (2)$$

The R.H.S of equation (1) and (2) are the same.

$$\therefore f(E_2) = [1 - f(E_1)]$$

It means that the probability of an energy level $[E_F + \Delta E]$ (ΔE above the Fermi level) being occupied is the same as the probability of an energy level $[E_F - \Delta E]$ (ΔE below E_F), being vacant.

Example 29.8. Show that the occupancy probabilities of two states whose energies are equally spaced above and below the Fermi energy add up to one.

Solution. Let us consider two energy levels E_2 and E_1 , which are equally spaced above and below the Fermi energy E_F .

Let

$$E_2 = E_F + \Delta E \quad \text{and}$$

$$E_1 = E_F - \Delta E$$

The probability of occupancy of the level E_2 is given by

$$\begin{aligned}
 F(E_2) &= F(E_F + \Delta E) = \frac{1}{1 + \exp[(E_2 - E_F) / kT]} \\
 &= \frac{1}{1 + \exp[(E_F + \Delta E - E_F) / kT]} \\
 \therefore F(E_2) &= \frac{1}{1 + \exp(\Delta E / kT)}
 \end{aligned}$$

The probability of occupancy of the level E_1 is given by

$$\begin{aligned}
 f(E_1) &= f(E_F - \Delta E) = \frac{1}{1 + \exp[(E_F - \Delta E - E_F) / kT]} \\
 \text{or} \quad f(E_1) &= \frac{1}{1 + \exp[-\Delta E / kT]}
 \end{aligned}$$

$$\begin{aligned}
 \therefore f(E_1) + f(E_2) &= \frac{1}{1 + \exp(\Delta E / kT)} + \frac{1}{1 + \exp(-\Delta E / kT)} \\
 &= \frac{1}{1 + \exp(\Delta E / kT)} + \frac{\exp(\Delta E / kT)}{1 + \exp(\Delta E / kT)} \\
 &= \frac{1 + \exp(\Delta E / kT)}{1 + \exp(\Delta E / kT)} \\
 \therefore f(E_1) + f(E_2) &= 1
 \end{aligned}$$

30.7 INTRINSIC CONDUCTIVITY

A single event of bond breaking in a pure semiconductor leads to generation of an **electron-hole pair**. At any temperature T , the number of electrons generated will be equal to the number of holes generated per unit volume. As the two charge carrier concentrations are equal, they are denoted by a common symbol n_i , which is called *intrinsic density* or *intrinsic concentration*. Thus,

$$n = p = n_i \quad (30.9)$$

In a semiconductor under thermal equilibrium condition, free electrons move in the conduction band and holes in the valence band, which are in a state of random motion. At a temperature T , they possess an average kinetic energy given by

$$\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT$$

where v_{th} is the mean **thermal velocity**. When a potential difference is applied across the solid, the equilibrium condition is disturbed. The electric field accelerates the electrons and holes but their motion is hindered due to interactions with the lattice vibrations. In the steady state condition there arises a net movement of electrons in a direction opposite to that of the electric field and movement of holes in the direction of the electric field. This net movement of electrons and holes is called **drift**, and the corresponding mean velocity is known as **drift velocity**, v_d . The drift motion is superposed on the random thermal motion of the charge carriers. The drift motion is directional and causes **drift current** flow, which is more often called **conduction current**.

The drift velocity is given by

$$v_d = \mu E \quad (30.10)$$

Since the electrons move in relatively less populated conduction band the properties such as mobility, conductivity etc of electrons are larger compared to those of holes as the latter move in nearly full valence band. We designate the drift velocity of electron with v_{de} and that of hole with v_{dh} . Similarly, we denote the mobility of electron and hole with μ_e and μ_h respectively. Then, the current density due to electrons is given by

$$J_e = nev_{de} = ne\mu_e E \quad (30.11)$$

and the current density due to holes is

$$J_h = pev_{dh} = pe\mu_h E \quad (30.12)$$

Comparing the above expressions with Ohm's law $J = \sigma E$, we obtain the expressions for electronic and hole conductivities as follows.

$$\sigma_e = ne\mu_e \quad (30.13)$$

and

$$\sigma_h = pe\mu_h \quad (30.14)$$

Let us now consider a sample of semiconductor across which a potential difference V is applied. The potential difference V establishes an electric field E in the semiconductor. It causes a current I_e due to electrons drifting in the conduction band and a current I_h due to holes drifting in the valence band (see Fig. 30.7). The total current through the semiconductor is

$$I = I_e + I_h \quad (30.15)$$

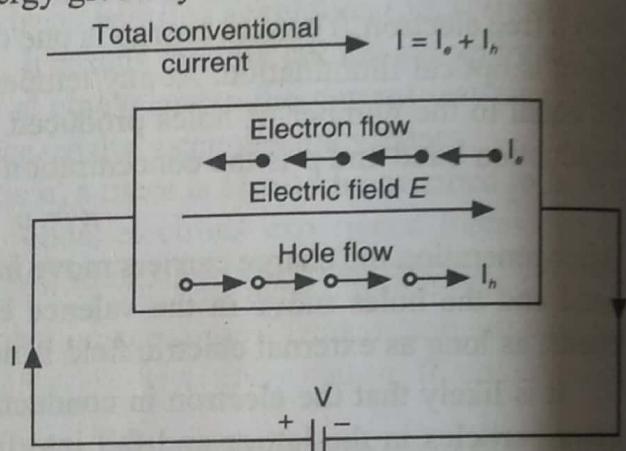


Fig. 30.7: Components of drift current due to electron and hole motion in an intrinsic semiconductor

\therefore Current density $J = \frac{I_e}{A} + \frac{I_h}{A}$ where A is cross-sectional area of the crystal.

or

$$\begin{aligned} J &= J_e + J_h \\ &= (ne\mu_e + pe\mu_h)E \end{aligned} \quad (30.16)$$

Therefore, the intrinsic conductivity is given by

$$\sigma = (ne\mu_e + pe\mu_h) \quad (30.17)$$

As $n = p = n_i$ in an intrinsic semiconductor, equ. (30.17) may be rewritten as

$$\sigma = en_i(\mu_e + \mu_h) \quad (30.18)$$

The equation (30.18) does not explain the temperature dependence of electrical conductivity in semiconductors. In general the variation of mobility with temperature is too small and the large variation in electrical conductivity in semiconductors is linked to the variation of electron concentration with temperature. Thus, as a first approximation, taking e , μ_e and μ_h as constants, we find that

$$\sigma(T) \propto n_i(T) \quad (30.19)$$

The expression for the intrinsic carrier concentration is derived in the band theory. Here, we take the final results from the band theory and use them to obtain an expression for $n(T)$, the variation of intrinsic carrier concentration with temperature.

30.8 CARRIER CONCENTRATIONS

30.11 FERMI LEVEL IN INTRINSIC SEMICONDUCTOR

In a pure semiconductor, the electrons in the conduction band cluster very close to the bottom edge of the band, and we assume that electrons are located right at the bottom edge of the conduction band, as shown in Fig. 30.9. Similarly, we assume that the holes are at the top edge of the valence band. The electron concentration in the conduction band is given by

$$n = N_C e^{-(E_C - E_F)/kT}$$

The hole concentration in the valence band is given by

$$p = N_V e^{-(E_F - E_V)/kT}$$

In an intrinsic semiconductor, the electron and hole concentrations are equal. Thus, $n = p$

$$N_C e^{-(E_C - E_F)/kT} = N_V e^{-(E_F - E_V)/kT} \quad (30.50)$$

Taking logarithm on both sides, we get

$$-\frac{(E_C - E_F)}{kT} = \ln \frac{N_V}{N_C} - \frac{(E_F - E_V)}{kT}$$

$$-E_C + E_F = kT \ln \frac{N_V}{N_C} - E_F + E_V$$

$$2E_F = (E_C + E_V) + kT \ln \frac{N_V}{N_C}$$

$$\therefore E_F = \frac{E_C + E_V}{2} + \frac{1}{2} kT \ln \frac{N_V}{N_C} \quad (30.51)$$

But $N_C = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2}$ and $N_V = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$

$$\therefore \frac{N_V}{N_C} = \left(\frac{m_h^*}{m_e^*} \right)^{3/2}$$

$$\therefore \ln \left(\frac{N_V}{N_C} \right) = \frac{3}{2} \ln \left(\frac{m_h^*}{m_e^*} \right)$$

$$\therefore E_F = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ln \left(\frac{m_h^*}{m_e^*} \right) \quad (30.52)$$

We can also write the above equation as

$$E_F = \frac{E_C + E_V}{2} - \frac{3}{4} kT \ln \left(\frac{m_e^*}{m_h^*} \right) \quad (30.53)$$

If the effective mass of a free electron is assumed to be equal to the effective mass of a hole, i.e.,

$$\begin{aligned} m_h^* &= m_e^* \\ \ln \left(\frac{m_h^*}{m_e^*} \right) &= 0 \end{aligned}$$

$$\therefore E_F = \frac{E_C + E_V}{2} \quad (30.54)$$

To make the meaning of the above equation more explicit, we write

$$E_F = \frac{E_C - E_V}{2} + E_V$$

$$E_F = \frac{E_g}{2} + E_V$$

If we denote the top of the valence band E_V as zero level, $E_V = 0$.

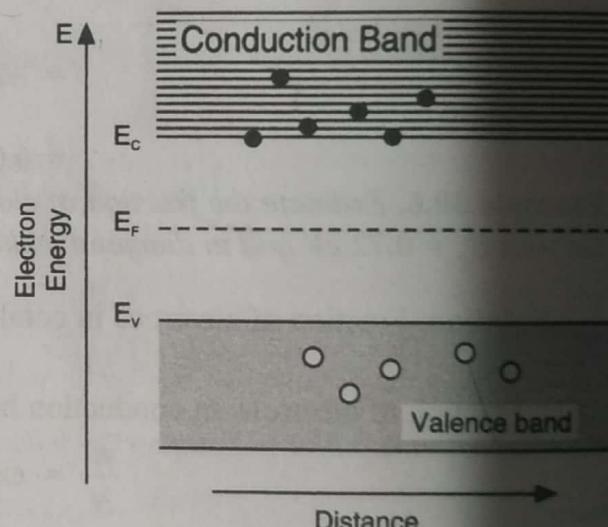


Fig. 30.9: Fermi level in intrinsic semiconductor

$$E_F = \frac{E_g}{2} \quad (30.55)$$

The above result shows that in an intrinsic semiconductor the Fermi level lies in the middle of the forbidden gap. We made the following assumptions in obtaining the relation (30.55).

- It was assumed that the electrons undergo transitions from the top edge level, E_V , of the valence band to the bottom edge level, E_C , of the conduction band. In reality, transitions are possible between the other levels also. However, the above result does not appreciably differ if other transitions are also taken into account.
- It was assumed that the effective mass of electrons in the conduction band, m_e^* , is exactly equal to the effective mass of the holes in the valence band, m_h^* . In practice, the effective masses differ from each other. However, the difference does not alter the above result significantly.

An important point to be noted here is that the **Fermi level is not an allowed energy level** in semiconductors. It only serves as a reference energy with reference to which we specify the energies of electrons and holes in a semiconductor.

30.28 HALL EFFECT

If a metal or a semiconductor carrying a current I is placed in a transverse magnetic field B , a potential difference V_H is produced in a direction normal to both the magnetic field and current directions. This is known as *Hall effect*. This effect was discovered by E.H. Hall in 1879 and showed that it is negatively charged particles that carry current in metals.

30.28.1 Importance of Hall Effect

The importance of Hall effect in the field of semiconductors is that it helps to determine

- (i) the type of semiconductor,
- (ii) the sign of majority charge carriers,
- (iii) the majority charge carrier concentration,
- (iv) the mobility of majority charge carriers, and
- (v) the mean drift velocity of majority charge carriers.

30.28.2 Experimental Arrangement

The experimental set up for the measurement of Hall voltage and determination of Hall coefficient is shown in Fig. 30.23 (a). A thin rectangular semiconductor wafer is mounted on an insulating strip and two pairs of electrical contacts are provided on opposite sides of the wafer. One pair of contacts is connected to a constant current source. And the other pair is connected to a sensitive voltmeter. This arrangement is mounted in between two pole pieces of an electromagnet such that the magnetic field acts perpendicular to the lateral faces of the semiconductor wafer.

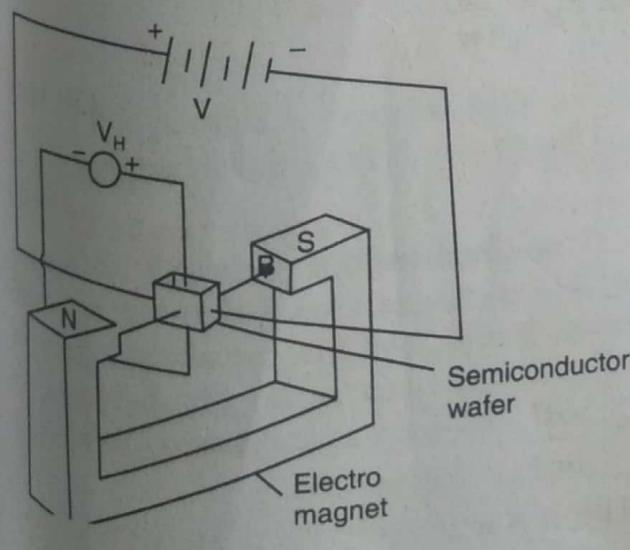
30.28.3 Hall Voltage

Let us assume that the semiconductor is a *p*-type semiconductor. Let a potential difference V be applied across its ends. A current of strength I flows through it along the x -direction (Fig. 30.23 b). Holes are the majority charge carriers in the *p*-type semiconductor. The current through the wafer is given by

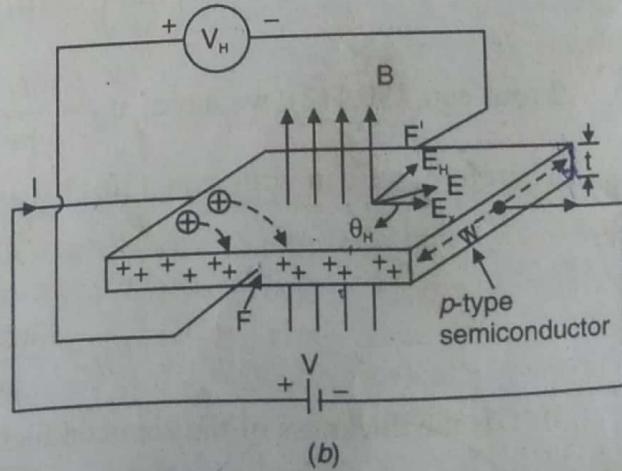
$$I = peAv_d \quad (30.111)$$

where p is the hole concentration

A is the area of cross-section of the end face of semiconductor wafer,



(a)



(b)

Fig. 30.23: (a) Basic experimental arrangement to study Hall effect (b) Generation of Hall voltage.

e is the electrical charge associated with a hole, and v_d is the average drift velocity of holes.

$$\text{The current density } J_x = \frac{I}{A} = pev_d \quad (30.112)$$

Any plane perpendicular to the current flow direction is an equipotential surface. Therefore, the potential difference between the front and rear faces F and F' is zero (see Fig. 30.24).

Now, if a magnetic field B is applied normal to the wafer surface and hence to the direction of current flow in it, then a transverse potential difference is produced between faces F and F' . It is known as **Hall Voltage** V_H .

Before the application of magnetic field, holes move parallel to faces F and F' . On application of magnetic field B , the holes experience a sideways deflection due to the magnetic force F_L , which is given by

$$F_L = eBv_d \quad (30.113)$$

Holes are deflected toward the front face F and pile up there. Due to this, a corresponding equivalent negative charge is left on the rear face F' . These opposite charges produce a transverse electric field, E_H , whose direction is from the front to the rear face. Due to the action of E_H , holes experience an electric force in addition to the Lorentz force. When the force F_E due to this transverse electric field balances the magnetic force F_L , equilibrium condition is attained and the holes once again flow along x -direction parallel to the faces F and F' .

In the equilibrium condition

$$\begin{aligned} F_E &= F_L \\ eE_H &= ev_d B \end{aligned}$$

If ' w ' is the width of the semiconductor wafer, $E_H = \frac{V_H}{w}$

$$\therefore \left(\frac{V_H}{w} \right) = Bv_d \quad (30.114)$$

From equ. (30.112), we have $v_d = \frac{J_x}{pe}$

Therefore, we can write equ. (30.114) as

$$\therefore \frac{V_H}{w} = \frac{BJ_x}{pe} \quad (30.115)$$

$$V_H = \frac{wBJ_x}{pe} = \frac{wBI}{peA}$$

If 't' is the thickness of the semiconductor plate, $A = w t$.

$$\therefore V_H = \frac{BI}{pet} \quad (30.116)$$

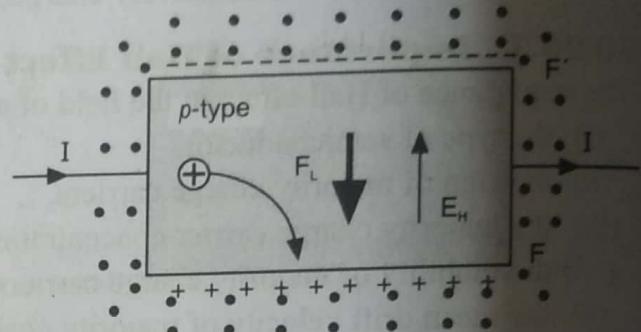


Fig. 30.24: Top view of the wafer—The directions of magnetic force and Hall field in *p*-type semiconductor

30.28.4 Hall Coefficient

Hall coefficient, R_H is defined as *Hall field per unit current density per unit magnetic induction*. Thus,

$$R_H = \frac{E_H}{J_x B} = \frac{V_H / w}{J_x B}$$

Using equ. (30.114), we get

$$R_H = \frac{BJ_x}{peJ_x B}$$

or

$$R_H = \frac{1}{pe} \quad (30.117)$$

Hall voltage, V_H can now be written as

$$V_H = R_H \frac{BI}{t} \quad (30.118)$$

$$\therefore R_H = \frac{V_H t}{BI} \quad (30.119)$$

The above equation is derived assuming that the *p*-semiconductor contains only holes. When the two types of charge carriers are taken into account, it is shown that the Hall coefficient is given by

$$R_H = \frac{(p\mu_h^2 - n\mu_e^2)}{e(p\mu_h + n\mu_e)^2} \quad (30.120)$$

From the above equation, we see that the Hall coefficient and Hall voltage are smaller for intrinsic materials than for extrinsic materials.

30.28.5 Drift Velocity

According to the equilibrium condition, namely $F_E = F_L$, we have

$$e\left(\frac{V_H}{w}\right) = eBv_d$$

$$v_d = \frac{V_H}{Bw} \quad (30.121)$$

As it is possible to experimentally measure the Hall voltage, magnetic field B and width of the wafer w , the mean drift velocity of the charge carriers can be obtained from the above equation.

30.28.6 Carrier Concentration

From the measurement of Hall voltage and current through the wafer and knowing the strength of magnetic induction and thickness of the wafer, R_H can be computed using equ. (30.119). Once R_H is known, the carrier concentration can be determined with the help of equ. (30.108), which may be rewritten as follows:

$$p = \frac{1}{R_H e}$$

In case of conductors and *n*-type semiconductors, the expression for Hall coefficient is

$$R_H = -\frac{1}{ne} \quad (30.122)$$

where n represents the concentration of electrons in the conductor or n -type semiconductor.

$$\therefore n = -\frac{1}{R_H e} \quad (30.123)$$

30.28.7 Doping Level

It is possible to estimate the doping concentration, N_A , from the value of p using the relation $p_p \approx N_A$.

30.28.8 Carrier Sign and Type of Semiconductor

With the direction of the magnetic field and the current as depicted in Fig. 30.23 (b), the sign of the Hall voltage is positive. For an n -type semiconductor, the Hall voltage will be negative when the directions of B and I are kept same. Therefore, knowing the sign of Hall voltage, the type of the semiconductor and hence the sign of the majority carrier can be determined.

30.28.9 Hall Mobility

Mobility is defined as the drift velocity acquired in unit electric field. We know that the current density J is given by the following expressions.

$$\begin{aligned} J &= pev_d \text{ and } J = \sigma E \\ \therefore pev_d &= \sigma E \\ \text{or} \quad \frac{v_d}{E} &= \frac{\sigma}{pe} \\ \therefore \mu_h &= \sigma R_H \end{aligned} \quad (30.124)$$

35.4 PROPERTIES OF SUPERCONDUCTORS

Superconductors exhibit many unusual and interesting properties.

35.4.1 Zero Electrical Resistance

A super conductor is characterized by zero electrical resistivity. It is not fundamentally possible to test experimentally whether the resistance is zero. A method devised by Onnes consists of measuring the decrease of the current in a closed ring of superconducting wire. The superconducting ring is kept in a magnetic field and it is cooled to below the critical temperature so that it goes into the superconducting state. When the external magnetic field is switched off, a current is induced in the ring. If the ring had a finite resistance, R , the current circulating in the ring would decrease according to the equation

$$I(t) = I(0)e^{-Rt/L} \quad (35.1)$$

where L is the inductance of the ring. The decay current is monitored by a change in the magnetic flux through a test coil held close to the superconducting ring. Any change in the magnetic flux of the superconducting ring will induce an emf in the test coil. Careful measurements established that the resistivity of superconductors could be taken as zero.

35.4.2 Persistent Current

Once a current is started in a closed loop of superconducting material, it will continue to keep flowing, of its own accord, around the loop as long as the loop is held below the critical temperature (see Fig. 35.4 b). Such a steady current, which flows without diminishing in strength, is called a **persistent current**. The persistent current does not need external power to maintain it because there do not exist I^2R losses. Calculations show that once the current flow is initiated, it persists for more than 10^5 years. Persistent current is one of the most important properties of a superconductor. Superconductor coils with persistent current flowing through them produce magnetic fields and can therefore act as magnets. Such a superconducting magnet does not require power supply to maintain its magnetic field.

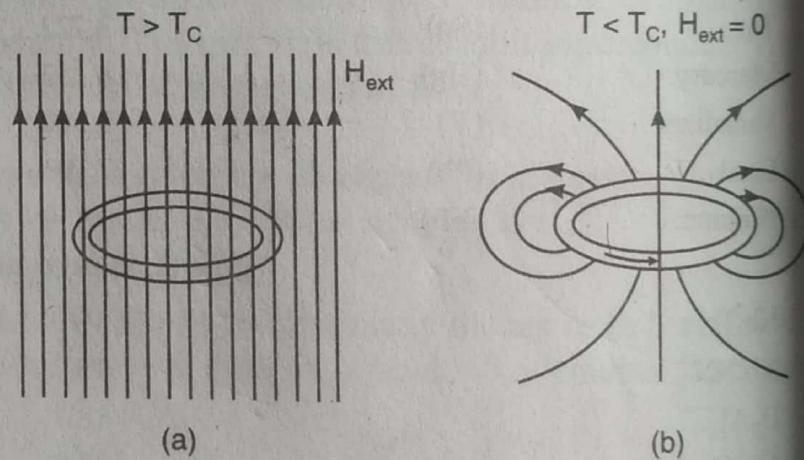


Fig. 35.4

35.4.3 Critical Temperature

When a superconducting material is cooled below a certain temperature, it goes into the superconducting state from normal state. The temperature at which a material in normal state goes into superconducting state is known as the **critical temperature**, T_c (see Fig. 35.1). Different materials have different critical temperatures. The transition is reversible. When the temperature of the material is increased above the critical temperature, it passes into the normal state. The transition is a thermodynamic phase transition. Just as the order in the arrangement of atoms increases at the transition of a material from liquid to solid state, a rearrangement of conduction electrons takes place leading to an increase in the order at the transition from normal to superconducting state. The superconducting transition is sharp for

a chemically pure and structurally perfect specimen while the transition range is broad for specimens which are structurally imperfect or which contain impurities.

35.4.4 Critical Magnetic Field

Superconducting state depends on the strength of the magnetic field in which the material is placed. Superconductivity vanishes if a sufficiently strong magnetic field is applied. The minimum magnetic field, which is necessary to regain the normal resistivity, is called the **critical magnetic field**, H_C . When the applied magnetic field exceeds the critical value H_C , the superconducting state is destroyed and the material goes into normal state.

The value of H_C varies with temperature. Fig.

35.5 shows the dependence of H_C on temperature in a typical superconductor. At temperatures below T_C in the absence of magnetic field, the material is in superconducting state. When a magnetic field is applied and as its strength reaches the critical value H_C the superconductivity in the material disappears. At any temperature $T < T_C$ the material remains superconducting until a corresponding critical magnetic field is applied. When the magnetic field exceeds the critical value, the material goes into normal state. The critical field required to destroy the superconducting state decreases progressively with increasing temperature. The dependence of critical field on temperature is governed by the following relation.

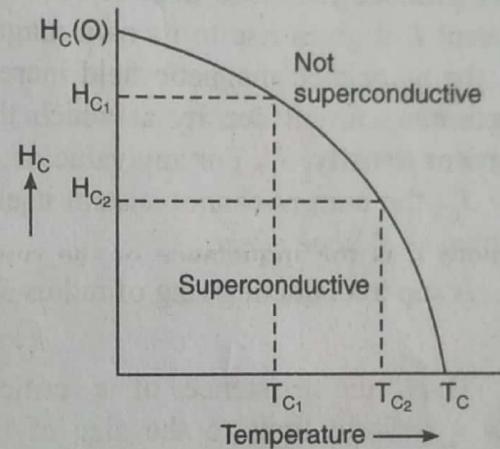


Fig. 35.5 : Schematic representation of the critical magnetic field as a function of temperature.

$$H_C(T) = H_C(0) \left[1 - \left(\frac{T}{T_C} \right)^2 \right] \quad (35.2)$$

where $H_C(0)$ is the critical field at 0 K.

Example 35.1. The transition temperature for Pb is 7.2 K. However, at 5 K it loses the superconducting property if subjected to magnetic field of 3.3×10^4 A/m. Find the maximum value of H which will allow the metal to retain its superconductivity at 0 K.

$$\text{Solution. } H_C(0) = \frac{H_C(T)}{1 - (T^2 / T_C^2)} = \frac{3.3 \times 10^4 \text{ A/m}}{1 - (25 / 51.28)} = 6.37 \times 10^4 \text{ A/m.}$$

Example 35.2. The critical field of niobium is 1×10^5 A/m at 8 K and 2×10^5 A/m at 0 K. Calculate the transition temperature of the element.

Solution.

$$T_C = \frac{T}{\left[1 - \frac{H_C(T)}{H_C(0)} \right]^{\frac{1}{2}}} = \frac{8K}{\left[1 - \frac{1 \times 10^5 \text{ A/m}}{2 \times 10^5 \text{ A/m}} \right]^{\frac{1}{2}}} = 11.3 \text{ K}$$

Example 35.3. The transition temperature for lead is 7.26 K. The maximum critical field for the material is 8×10^5 A/m. Lead has to be used as a superconductor subjected to a magnetic field of 4×10^4 A/m. What precaution will have to be taken?

$$\text{Solution. } T = T_C \left[1 - \frac{H_C(T)}{H_C(0)} \right]^{\frac{1}{2}} = 7.26K \left[1 - \frac{4 \times 10^4 \text{ A/m}}{8 \times 10^5 \text{ A/m}} \right]^{\frac{1}{2}} = 7.08 \text{ K.}$$

Therefore, the temperature of the metal should be held below 7.08 K.

35.4.5 Critical Current Density

The critical magnetic field required to destroy superconductivity need not be necessarily applied externally. An electric current flowing through the superconducting material itself may produce magnetic field of requisite strength. Thus, if a superconducting ring carries a current I , it gives rise to its own magnetic field. As the current increases to a critical value, I_C , the associated magnetic field increases to H_C and the superconductivity disappears. The maximum current density at which the superconductivity disappears is called the **critical current density**, J_C . For any value of $J < J_C$, the current can sustain itself whereas for values $J > J_C$, the current cannot sustain itself. This effect was observed in 1916 by Silsbee and is known as *Silsbee effect*.

A superconducting ring of radius R ceases to be a superconductor when the current is

$$I_C = 2\pi R H_C \quad (35.3)$$

Thus, the existence of a critical current sets a definite limit to the size of the current that can flow through a superconducting coil without disturbing its superconducting state. The maximum current that a superconductor can carry decreases as the temperature is raised and falls to zero at the transition temperature of the material. Since the critical current falls with temperature, the critical magnetic field will also decrease as the transition temperature is approached. The variation of critical current density J_C and critical magnetic field H_C with temperature is shown in Fig. 35.6.

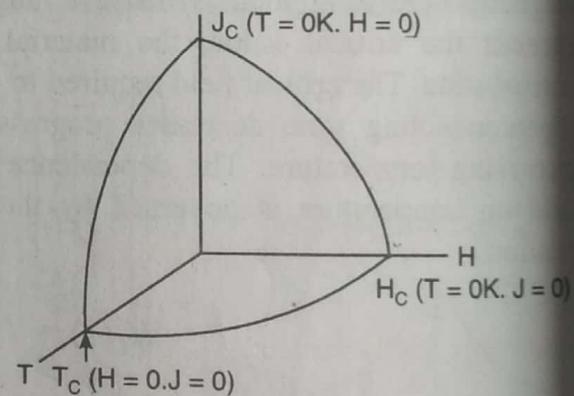


Fig. 35.6

Example 35.4. The critical magnetic field at 5 K is $2 \times 10^3 \text{ A/m}$ in a superconductor ring of radius 0.02 m. Find the value of critical current.

$$\text{Solution. } I_C = 2\pi R H_C = 2 \times 3.143 \times 0.02 \text{ m} \times 2 \times 10^3 \text{ A/m} = 251.4 \text{ A}$$

Example 35.5. Calculate the critical current for a wire of lead having a diameter of 1 mm at 4.2 K. The critical temperature for lead is 7.18 K and $H_C(0) = 6.5 \times 10^4 \text{ A/m}$.

$$\text{Solution. } H_C(T) = H_C(0) \left[1 - \left(\frac{T}{T_C} \right)^2 \right] = 6.5 \times 10^4 \text{ A/m} \left[1 - \left(\frac{4.2 \text{ K}}{7.18 \text{ K}} \right)^2 \right] = 4.28 \times 10^4 \text{ A/m}$$

$$\text{The critical current } I_C = 2\pi r H_C = \pi d H_C = 1 \times 3.14 \times 10^{-3} \text{ m} \times 4.28 \times 10^4 \text{ A/m} = 134.5 \text{ A.}$$

35.4.6 Perfect Diamagnetism – Meissner Effect

In 1933 W.Hans Meissner and Robert Ochsenfeld found that when superconductors are cooled below their critical temperature in the presence of a magnetic field, the magnetic flux is expelled from the interior of the specimen and the superconductor becomes a perfect diamagnetic. This phenomenon is known as **Meissner effect** (see Fig. 35.7). Meissner and

Ochsenfeld found that as the temperature of the specimen is lowered to T_C , the magnetic flux is suddenly and completely expelled from it. The flux expulsion continues for $T < T_C$. The effect is reversible. When the temperature is raised from below T_C , the flux suddenly penetrates the specimen at $T = T_C$ and the material returns to the normal state.

The magnetic induction inside the specimen is given by

$$B = \mu_0(H + M) = \mu_0(1 + \chi)H$$

where H is the magnetic field applied externally and M is the magnetization produced within the specimen.

At $T < T_C$

$$B = 0 \text{ and therefore } \mu_0(H + M) = 0$$

It follows that

$$M = -H$$

The susceptibility of the material is

$$\chi = \frac{M}{H} = -1 \quad (35.4)$$

The specimen is therefore diamagnetic and the state in which magnetization cancels the external magnetic field completely is referred to as *perfect diamagnetism*.

The Meissner effect contradicts the fundamental principles of electromagnetism. The condition of perfect diamagnetism cannot be explained from the simple definition that superconductivity is a state of zero resistivity. Meissner effect shows that

in the superconductor not only $\frac{dB}{dt} = 0$

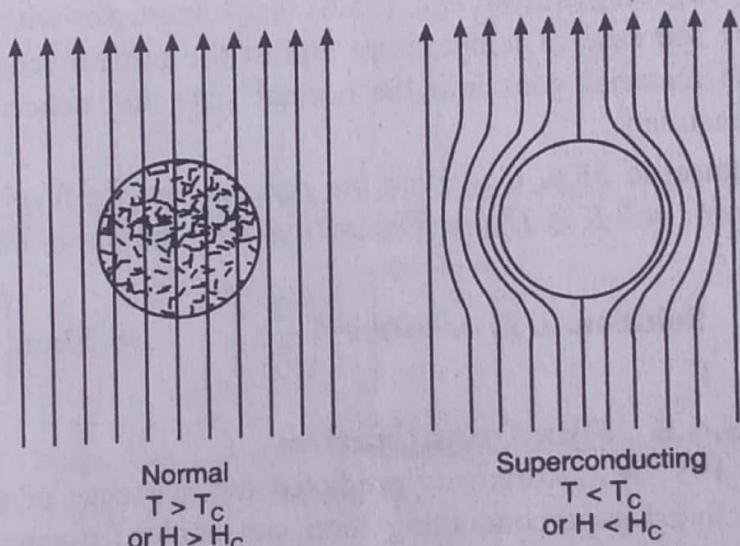


Fig. 35.7

but also $B = 0$. Thus, two mutually independent properties, namely zero resistivity and perfect diamagnetism are the essential properties that characterize the superconducting state.

Applications of Meissner effect

- The Meissner effect is the standard test used to conclusively prove whether a particular material is a superconductor or not.
- A material in superconducting state is a perfect dia-magnet and hence strongly repels external magnets. A smaller magnet repelled by a bigger superconductor hovers in air. This is known as **levitation effect**. In a similar way, a small chip of superconducting material hangs on to a bigger magnet and this effect is known as **suspension effect**. The levitation effect is utilized in the operation of Maglev trains.

35.4.7 London Penetration Depth

When a magnetic field is applied to a superconductor, the applied field does not suddenly drop to zero at the surface. Instead the field decays exponentially according to the formula

$$H(x) = H(0)e^{-x/\lambda} \quad (35.5)$$

where $H(0)$ is the field applied at the surface at $x = 0$ and x is the distance from the surface. The length λ is called the **London penetration depth**. It may be defined as the effective depth to

which a magnetic field penetrates a superconductor. The penetration depth λ ranges from 300 to about 5000 Å depending on the material. It is independent of frequency of the magnetic field but it strongly depends on temperature.

The temperature dependence of λ is given by the relation

$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1 - \left(\frac{T}{T_C}\right)^4}} \quad (35.6)$$

where $\lambda(T)$ and $\lambda(0)$ are the penetration depths at T and 0 K.

It follows from equ. (35.6) that λ increases with the increase in temperature and at the critical temperature, it becomes infinite. At $T = T_C$ the material goes into the normal state and hence the magnetic field penetrates the whole specimen.

Example 35.6. Calculate the penetration depth of lead at 5.2 K if the London penetration depth at 0 K is 37 nm. The critical temperature of lead is 7.193 K.

$$\text{Solution. } \lambda(T) = \lambda(0) \left[1 - \left(\frac{T}{T_C} \right)^4 \right]^{-1/2} = 37 \text{ nm} \left[1 - \left(\frac{5.2 \text{ K}}{7.193 \text{ K}} \right)^4 \right]^{-1/2} = 43.4 \text{ nm.}$$

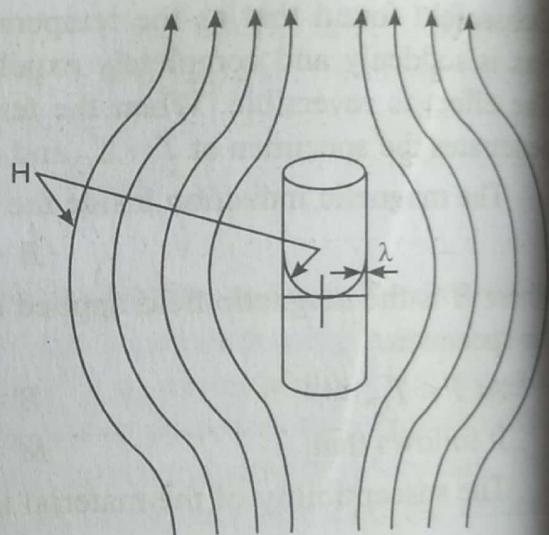


Fig. 35.8

35.6 TYPE-I AND TYPE-II SUPERCONDUCTORS

Superconductors are divided into two categories depending on the way in which the transition from superconducting to normal state proceeds when the externally applied magnetic field exceeds H_C .

Type-I Superconductors:

In type I superconductors, the transition from superconducting state to normal state in the presence of magnetic field occurs sharply at the critical value of H_C , as shown in Fig. 35.12 (a).

Type I superconductors are perfectly diamagnetic below H_C and completely expel the magnetic field

from the interior of the superconducting phase. Up to the critical field strength, the magnetization of the material grows in proportion to the external field and then abruptly drops to zero at the transition to the normal state, as shown in Fig. 35.12 (b). The magnetic field can penetrate only the surface layer and current can flow only in this layer. Consequently, type I superconductors are poor carriers of electrical current.

Aluminium, lead and indium are examples of Type I superconductors. The critical field is relatively low for type I superconductors. They would generate magnetic fields of about 100 to 2000 G only. Hence, they are not of much use in production of high magnetic fields. Type I superconductors are also called **soft superconductors**. We summarize here the characteristics of Type I superconductors.

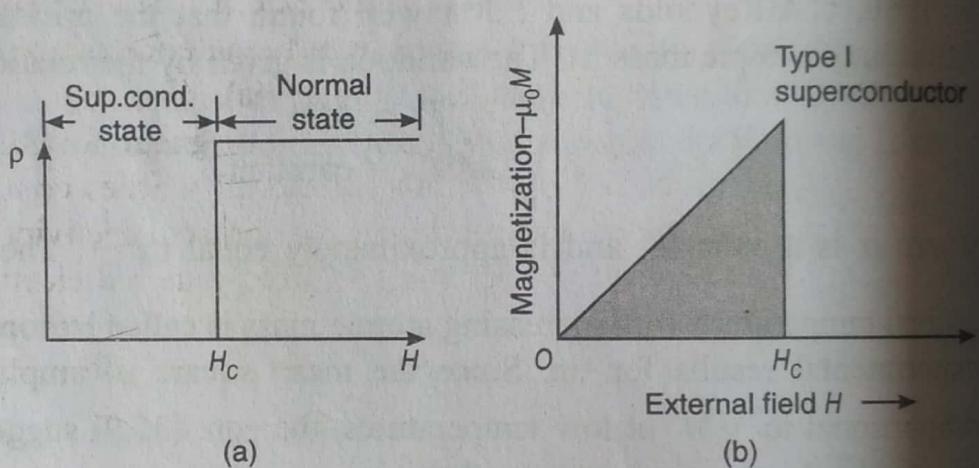


Fig. 35.12

Characteristics of Type-I superconductors

1. They are perfectly diamagnetic and exhibit complete Meissner effect.
2. They have only one critical field. At the critical field the magnetization drops to zero.
3. The maximum critical field for type I superconductor is of the order of 0.1 Wb/m^2 .
4. The transition at H_C is reversible. Below H_C the material behaves as a superconductor, and above H_C it behaves as a normal conductor.

Disadvantages

Type I superconductors cannot carry large currents and hence are not of much use in producing high magnetic fields.

Type-II Superconductors

Type II superconductivity was discovered by Schubnikov in 1930s and was explained by Abrikosov in 1957. Type II superconductors are characterized by two critical fields H_{C1} and

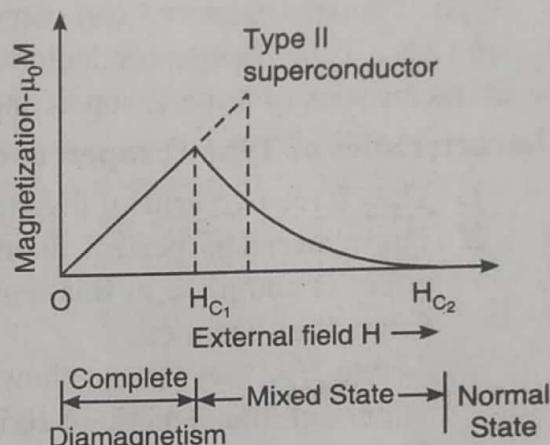
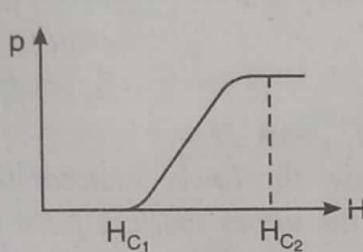


Fig. 35.13

H_{C2} . The transition from superconducting state to normal state occurs gradually as the magnetic field is increased from H_{C1} to H_{C2} , as shown in Fig. 35.13. The magnetization of the material grows in proportion to the external field up to the lower critical field H_{C1} . The external magnetic flux is expelled from the interior of the material till then. At H_{C1} the magnetic field lines begin penetrating the material.

As the magnetic field increases further, the magnetic flux through the material increases. At the upper critical field H_{C2} , the magnetization vanishes completely and the external field has completely penetrated and destroyed the superconductivity. In the region between H_{C1} and H_{C2} , the material is in a magnetically mixed state but electrically it is a superconductor. H_{C2} can be as high as 20 to 50 Wb/m^2 and the retention of superconductivity in such high magnetic fields make type II materials very useful in applications of creating very high magnetic fields.

Transition metals and alloys consisting of niobium, silicon and vanadium exhibit type II superconductivity. Ceramic superconductors also belong to this category.

A distinguishing feature of type II superconductors is that super currents arising in an external magnetic field can flow not only over the surface of a conductor but also in the bulk. Above the lower critical field H_{C1} , it becomes energetically more favourable to admit a

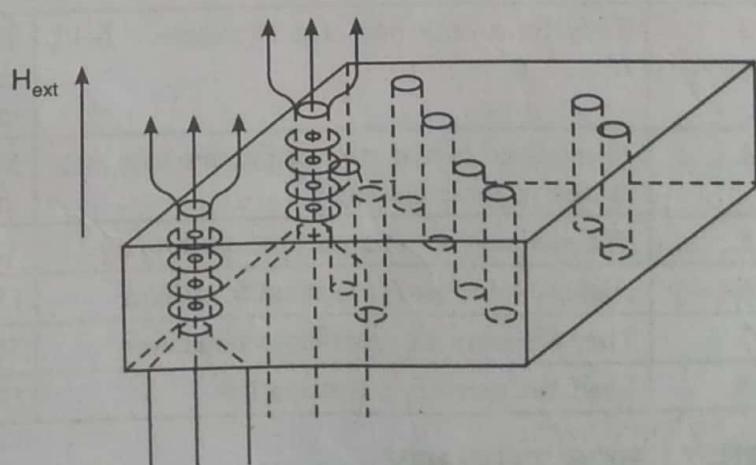


Fig. 35.14

single flux quantum rather than have the superconductor exclude H_{C1} . The superconductor passes into a mixed state where the bulk of the material is superconducting but is threaded by very thin filaments of normal material (see Fig. 35.14). The thin filaments of normal material serve as the paths along which the magnetic field penetrates. In the center of the filament superconductivity is absent. These normal regions are surrounded by vortices of super currents. A flux line together with its current vortex is called a **fluxoid**. At H_{C1} , fluxoids appear in the material and increase in number as the magnetic field is increased. An increase in the magnetic field will not cause an increase of the flux in each vortex line but will cause an increase in the number of fluxoids threading the superconductor. At H_{C2} the fluxoids fill the entire specimen and superconductivity disappears.

Type II superconductors can carry larger currents when the magnetic field lies between H_{C1} and H_{C2} . Type II superconductors are called **hard superconductors**. We summarize here the characteristics of Type II superconductors.

Characteristics of Type-II superconductors

1. They have two critical magnetic fields, H_{C1} and H_{C2} .
2. The material is perfect diamagnetic below the *lower critical field*, H_{C1} . Meissner effect is complete in this region. Above the *upper critical field*, H_{C2} , magnetic flux enters the specimen.
3. Above H_{C1} they do not show complete Meissner effect and therefore do not behave as perfect diamagnetic materials.
4. They exist in an *intermediate state* in between the critical fields, H_{C1} and H_{C2} . The intermediate state is a mixture of the normal and superconducting states, magnetically but electrically the material is a superconductor.
5. At H_{C2} the magnetization vanishes and the specimen returns to normal conducting state.
6. The upper critical field is very high and is of the order of 30 Wb/m^2 .

Applications

They are used in applications of generating very high magnetic fields.

COMPARISON BETWEEN TYPE I AND TYPE II SUPERCONDUCTORS

SL.No.	Type-I Superconductors	Type-II Superconductors
1.	They exhibit complete Meissner effect	They do not exhibit complete Meissner effect
2.	They show perfect diamagnetic behaviour	They do not show perfect diamagnetic behaviour
3.	They have only one critical magnetic field, H_C	They have two critical magnetic fields, lower critical magnetic field, H_{C1} and upper critical magnetic field, H_{C2}
4.	There is no mixed state or intermediate state in case of these materials	Mixed state or intermediate state is present in these materials
5.	The material loses magnetization abruptly	The material loses magnetization gradually
6.	Highest value for H_C is about 0.1 Wb/m^2	Upper critical field is of the order of 30 Wb/m^2
7.	They are known as soft superconductors	They are known as hard superconductors
8.	Lead, tin, mercury are examples	Nb-Sn, Nb-Ti, Nb-Zr, Va-Ga are examples

35.7 BCS THEORY

In 1957, the American physicists J. Bardeen, L.N. Cooper and J.R. Schrieffer developed the quantum theory of superconductivity, which came to be known as BCS theory. Starting from

the two experimental results, namely the isotope effect and the variation of electronic specific heat with temperature, the BCS theory assumed interaction of two electrons through quanta of lattice vibrations. It successfully explained the effects like zero resistivity, Meissner effect etc. The two principal features of BCS theory are

1. Electrons form pairs, called **Cooper pairs**, which propagate throughout the lattice and
2. Such propagation is without resistance because the electrons move in resonance with phonons.

To appreciate the formation of Cooper pair, let us consider the model in Fig. 35.15, in which two electrons propagate along a single lattice row. Each electron experiences an attraction towards its nearest positive ion. When the electrons get very close to each other in the region between ions, they repel each other due to Coulomb force. In an equilibrium condition, a balance between attraction and repulsion is established and the two electrons combine to form a Cooper pair. At normal temperatures, the attractive force is too small and pairing of electrons does not take place. However, at lower temperatures, such pairing is energetically advantageous. In a typical superconductor, the dense cloud of Cooper pairs form a collective state and the motion of all the Cooper pairs is correlated. As such the pairs drift cooperatively through the material. Thus, the superconducting state is an *ordered state of the conduction electrons*. Since the density of Cooper pairs is very high, even large currents require only a small velocity. The small velocity of ordered Cooper pairs minimize collision processes and leads to zero resistivity.

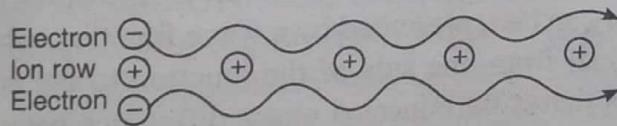


Fig. 35.15

The electrons of a Cooper pair have a lower energy than two unpaired electrons. The theory predicted the existence of an energy gap between the ground state (superconducting state) and first excited state (see Fig. 35.16). The energy gap represents the energy required to break up a Cooper pair. Hence, larger energy gaps correspond to more stable superconductors. According to BCS theory, the energy gap at 0K is given by

$$E_g(0) = 2\Delta \approx 3.52kT_C \quad (35.11)$$

The energy gap is generally of the order of 10^{-3} eV. The existence of energy gap can be proved experimentally. One such experiment involves studying the absorption of microwaves by a superconductor. At temperatures close to absolute zero, a superconductor does not absorb energy until the energy quanta of incident radiation is equal to greater than 2Δ . The absorption then grows fast to a value typical for the normal metal, because electrons can now absorb photons and go to higher energy states that lie above the energy gap.

The BCS theory also predicted flux quantization.

35.8 JOSEPHSON EFFECT

In 1962, Brian Josephson predicted that Cooper pairs could tunnel through an insulating layer, which separates two superconductors. The superconductor-insulator-superconductor layer

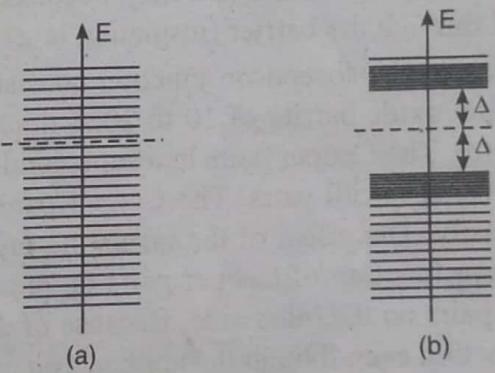


Fig. 35.16

constitutes the **Josephson junction**, as shown in Fig. 35.17 (b). The insulating layer is of the order of 1 nm thickness. Josephson predicted that the tunneling can occur without any resistance, giving rise to a direct current when the voltage applied across the junction is zero and an alternating current when the applied voltage is a dc voltage.

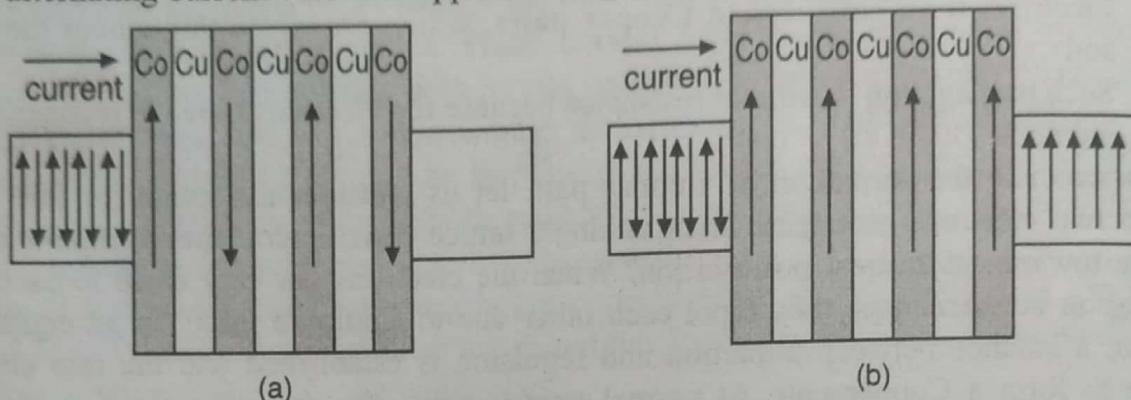


Fig. 35.17

35.8.1 The dc Josephson Effect

Two superconductors separated by a thick insulating layer, say of 10 nm thickness, are essentially two independent superconductors without any joint properties. When the insulator layer is thin, say 1 nm thick, they become a system of coupled conductors. The cooper pairs tunnel through the barrier (insulating layer) as a single unit.

Consider a Josephson junction consisting of two superconducting metal films separated by a thin oxide barrier of 10 to 20 Å thick. Let it be connected in a circuit as shown in Fig. 35.17 (c). The Cooper pairs in a superconductor can be represented by a wave function, which is the same for all pairs. The Cooper pairs tunnel from one side of the junction to the other side easily. The effect of the insulating layer is that it introduces a phase difference between the wave function of Cooper pairs on one side of the insulating layer and the wave function of the pairs on the other side. Because of this phase difference, a super current appears across the junction even though the applied voltage is zero. This is known as the dc Josephson effect. Josephson showed that the super current through the junction is given by

$$I_S = I_C \sin \phi_0 \quad (35.12)$$

where ϕ_0 is the phase difference between the wave functions describing Cooper pairs on both sides of the barrier, and I_C is the critical current at zero voltage condition. I_C depends on the thickness and width of the insulating layer and the temperature.

35.8.2 The ac Josephson Effect

If we apply a dc voltage across the Josephson junction, it introduces an additional phase on Cooper pairs during tunneling. As a result a strikingly new phenomenon will be observed. The dc voltage generates an alternating current I given by

$$I = I_C \sin(\phi_0 + \Delta\phi) \quad (35.13)$$

Because of the dc voltage V applied across the barrier, the energies of Cooper pairs on both sides of the barrier differ in energy by $2eV$. Using the quantum mechanical calculations, it can be shown that

$$\Delta\phi = 2\pi t \left(\frac{2eV}{h} \right) \quad (35.14)$$

$$I = I_C \sin \left[\varphi_0 + 2\pi t \left(\frac{2eV}{h} \right) \right] \quad (35.15)$$

The current given by eq. (35.15) represents an alternating current of frequency

$$\nu = \frac{2eV}{h} \quad (35.16)$$

Equ. (35.16) shows that a photon of frequency ν is emitted or absorbed when a Cooper pair crosses the junction. Thus, when a dc voltage is applied across a Josephson junction, an ac current is produced by the junction. This is known as the **ac Josephson effect**. At $V = 1 \mu V$, ac current of frequency 483.6 MHz is produced.

Example 35.8. A Josephson junction with a voltage difference of $650 \mu V$ radiates electromagnetic radiation. Calculate its frequency.

Solution. $\nu = \frac{2eV}{h} \text{ Hz} = \frac{2(1.602 \times 10^{-19} \text{ C})(650 \times 10^{-6} \text{ V})}{6.626 \times 10^{-34} \text{ J}} = 3 \times 10^{11} \text{ Hz.}$

35.9 HIGH SUPERCONDUCTORS

Superconductors are divided into low T_C and high T_C superconductors based on their transition temperature. Broadly, materials having T_C below 24 K are regarded as low T_C superconductors and those having T_C above 27 K are regarded as high T_C superconductors. However, in practice, materials for which liquid nitrogen cooling can cause transition to superconducting state may be regarded as high T_C superconductors while those that require liquid helium coolant are considered as low T_C superconductors. The maximum transition temperature that could be achieved before 1980's was 23.2 K in Nb_3Ge , which is a metallic alloy. Therefore, it was hoped that metallic alloy systems could be made to have higher transition temperatures but such systems were not discovered. Therefore, the focus has shifted to ceramic oxides, which are insulating materials at normal temperatures. In 1986 Bednortz and Muller discovered superconductivity in ceramic materials. They found that the mixed metallic oxide of lanthanum-barium-copper ($\text{La}_1\text{Ba}_2\text{Cu}_3\text{O}_7$) exhibited superconductivity at about 30 K. The superconductivity of the oxide was linked with the deficiency of oxygen ions in the oxide compound. When this deficiency of oxygen was carefully controlled, by keeping the samples in oxygen atmosphere, it was found that the material would exhibit superconductivity in the temperature range 30 to 40 K. In 1987 Chu and coworkers replaced lanthanum with yttrium and prepared $\text{YBa}_2\text{Cu}_3\text{O}_7$ with transition temperature of about 95 K. This was a major breakthrough, as this can be maintained in the superconducting state with far less expensive liquid nitrogen coolant (nitrogen exists in liquid state below 77 K) and marks the beginning of preparation of high T_C superconductors. This oxide also contains a deficiency of oxygen with the chemical formula $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ where δ indicates the deficiency of oxygen and is in the range 0 to 0.1. $\text{YBa}_2\text{Cu}_3\text{O}_7$ is the most extensively studied high T_C superconductor and exhibits a defective perovskite structure with three perovskite cubic unit cells stacked on top of each other.

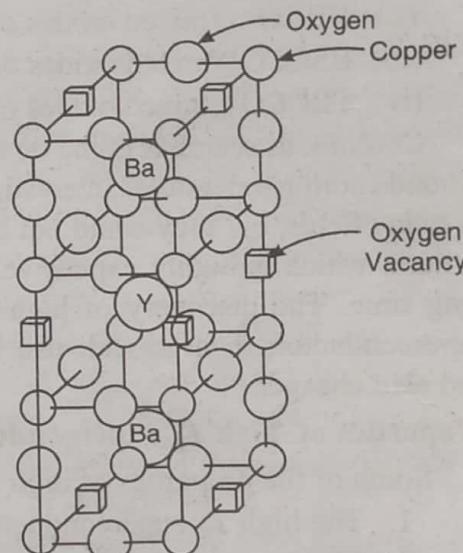


Fig. 35.18

(Fig. 35.18). For an ideal stack of three perovskite cubic unit cells, the $\text{YBa}_2\text{Cu}_3\text{O}_x$ compound should have the composition $\text{YBa}_2\text{Cu}_3\text{O}_9$, in which x would be equal to 9. However, analysis shows that x ranges from 6.65 to 6.90 for this material to exhibit superconductivity. At $x = 6.90$, its T_C is highest (≈ 90 K) and at $x = 6.65$, superconductivity disappears. Thus, oxygen vacancies are found to play a key role in the superconducting behaviour of ceramic oxides. If the cell contains one atom of rare earth metal, two barium atoms, three copper atoms and seven oxygen atoms, then such compounds are called 1-2-3 superconductors. These high T_C copper oxide superconductors belong to type II and their upper critical field is very high (of the order of 150 to 200 tesla). Apart from $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ compounds, the compounds based on bismuth like Bi-Sr-Cu-O or Ba-Ca-Cu-O systems are also superconducting. For example, the compound $\text{Bi}_2\text{CaSr}_2\text{Cu}_2\text{O}_{8+x}$ exhibits superconductivity and has the transition temperature 85 K and $\text{Bi}_2\text{Ca}_2\text{Sr}_2\text{Cu}_3\text{O}_{10+x}$ has the transition temperature 110 K. In 1993, a still higher of 133 K was achieved in mercury based copper oxide $\text{HgBa}_2\text{Ca}_2\text{Cu}_3\text{O}_{1+x}$.

The above discussed mixed oxide materials may be arranged into four major families.

- (i) LBCO (mixed oxide of lanthanum, barium and copper)
- (ii) YBCO (mixed oxides of yttrium, barium and copper)
- (iii) BSCCO (mixed oxides of bismuth, strontium, calcium and copper)
- (iv) TBCCO (mixed oxides of thallium, barium, calcium and copper)

Continuous search is going on to discover materials that may exhibit superconducting state around room temperature. Interesting practical applications of superconductors are visualized in many fields, but they could not be used on large scale because of the requirement of liquid helium, which is highly expensive. Therefore, they remained as a laboratory curiosity for a long time. The discovery of high temperature materials opened up possibilities of putting superconductors to large-scale use. They require only liquid nitrogen, which is easily available and also cheaper.

Properties of high T_C superconductors

Some of the properties of high T_C superconductors are as follows:

1. The high T_C superconductors are brittle in nature.
2. The properties of the normal state of these materials are highly anisotropic.
3. The Hall coefficient is positive indicating that the charge carriers are holes.
4. Their behaviour cannot be explained by BCS theory.
5. The isotope effect is almost absent in these materials.
6. The magnetic properties of these materials are highly anisotropic.
7. The effect of pressure is different on different materials. For example the application of pressure increases the critical temperature of LBCO compounds but decreases the critical temperature of YBCO compounds.

6. **MagLev Trains:** The most spectacular application would be the so-called ‘MagLev’ trains. The coaches of the train do not slide over steel rails but float on a four inch air cushion above the track using superconducting magnets; this eliminates friction and energy loss as heat, allowing the train to reach high speeds of the order of 500 km/hr. Such magnetic levitation trains would make train travel much faster, smoother, and more efficient due to the lack of friction between the tracks and train.

Operation

A typical plan of Maglev train is shown in Fig. 35.20. The train has superconducting magnets built into the base of its carriages. An aluminium guideway is laid on the ground and carries electric current. The repulsion between the two powerful magnetic fields, namely the field produced by the superconductor magnet and the field produced by the electric current in the aluminium guideway causes magnetic levitation of the train. A levitation of about 10 to 15 cm is achieved. The train is fitted with retractable wheels, which act in a way similar to the wheels of an airplane. Once the train is levitated in air, the wheels are retracted into the body and the train glides forward on the air cushion. When the train is to be halted, the wheels are drawn out and the train descends slowly onto the guideway and runs forward till it stops.

7. SQUIDS

A *superconducting quantum interference device* (SQUID) is a device used to measure extremely weak magnetic flux. Thus, it is basically a sensitive magnetometer. The heart of a SQUID is a superconducting ring, which contains one or more Josephson junctions. There

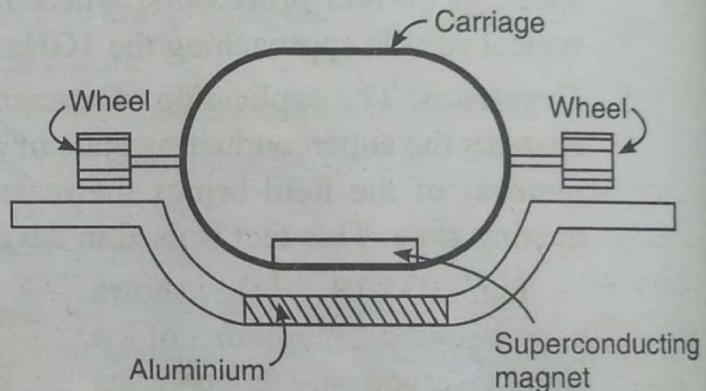


Fig. 35.20

are two main types of SQUID: DC SQUID and RF (or AC) SQUID. The DC SQUID was invented in 1964 by Robert Jaklevic, John Lambe, Arnold Silver, and James Mercereau. The RF SQUID was invented in 1965 by J.E. Zimmerman and Arnold Silver at Ford. An RF SQUID is made of one Josephson junction and a dc SQUID consists of two Josephson junctions in parallel and relies on the interference of the currents from each junction. We study here the dc SQUID. DC SQUIDS are more difficult and expensive to produce, but DC SQUID magnetometers are much more sensitive.

Fabrication

SQUIDs are usually fabricated from lead or pure niobium. The lead is taken in the form of an alloy with 10% gold or indium. A thin niobium layer deposited on to it acts as the base electrode of the SQUID and the tunnel barrier is oxidized onto this niobium surface. The top electrode is a layer of lead alloy deposited on top of the other two, forming a sandwich arrangement. The entire device is then cooled to within a few degrees of absolute zero with liquid helium.

The schematic of a two-junction SQUID [direct current (DC) SQUID] is shown in Fig. 35.21 (a). It consists of two Josephson junctions arranged in parallel so that electrons tunneling through the junctions demonstrate quantum interference.

Working

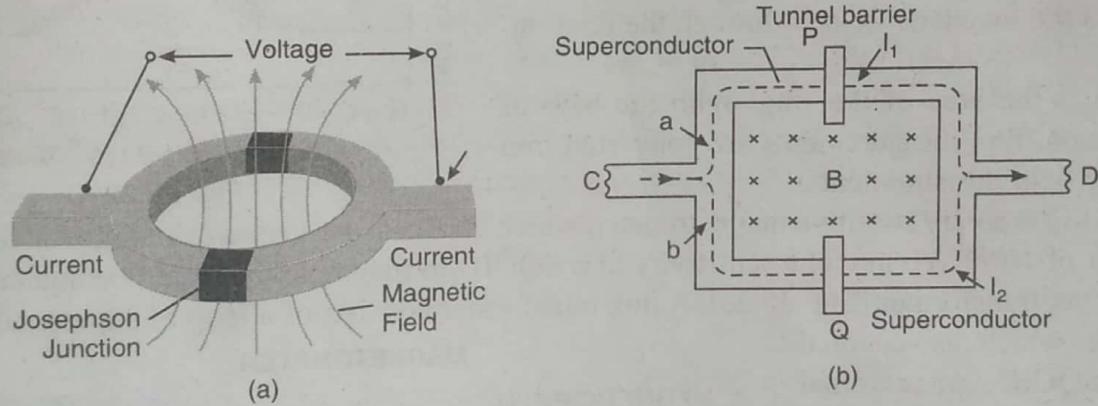


Fig. 35.21

A dc supercurrent is applied to the SQUID (Fig. 35.21 b). This current, known as *bias* current, enters the device through the arm C. It is divided along two paths *a* and *b* and again merge into one and leaves through the arm D. P and Q are the Josephson junctions and the insulating layers at P and Q are of different thickness. I_1 and I_2 are the currents tunneling through the junctions P and Q respectively. In a superconductor, a single wave function describes all the Cooper pairs. The wave function experiences a phase shift at the junctions P and Q. Let the phase difference between points C and D taken on a path through junction P be δ_a and the phase difference between points C and D taken on a path through junction Q be δ_b . In the absence of magnetic field these two phases are equal. That is, $\delta_b - \delta_a = 0$. When a magnetic field B is applied perpendicular to the loop, the flux passes through the loop, and changes the quantum mechanical phase difference across each of the two junctions. The wave functions at the two Josephson junctions interfere with each other. In other words, the supercurrents flowing along the paths *a* (PD) and *b* (QD) interfere. Hence, the device is named SQUID. The interference closely resembles the optical interference observed with Young's double slit. In the case of light, the phase difference between light waves is due to the difference in optical path lengths. In case of supercurrent interference, the waves are the de Broglie waves of Cooper pairs, and the phase difference is caused by the applied magnetic

field. According to Josephson's theory, the phase difference between the reunited currents is directly proportional to the magnetic flux, Φ , through the ring. It can be shown that the total current through two parallel Josephson junctions is given by

$$I_T = 2(I_0 \sin \delta_0) \cos \frac{e\Phi}{\hbar c} \quad (35.17)$$

The above relation indicates that a progressive increase or decrease of the magnetic flux, causes the current to oscillate between a maximum and a minimum value. Maxima in the current occur whenever the magnetic flux increases by one flux quantum. Thus, the period of these oscillations is one flux quantum Φ_0 .

$$\Phi_0 = \frac{\hbar}{2e} = 2.06 \times 10^{-15} \text{ webers}$$

Fig. 35.22 shows the variation of the current through a pair of Josephson junctions as a function of the magnetic flux applied.

In practice, instead of the current, we measure the voltage across the SQUID, which also oscillates with the changing magnetic field. Thus, the SQUID is a flux-to-voltage transducer, converting a tiny change in magnetic flux into voltage. The flux Φ is related to the magnetic field B through the relation

$$\Phi = BA$$

where A is the area of the ring. With the help of this relation, flux measurements are converted into magnetic field measurements.

SQUID is a very sensitive magnetometer, which can measure very weak magnetic fields of the order of 10^{-13} Wb/m^2 . The sensitivity of a SQUID to magnetic fields can be enhanced by using a flux transformer (Fig. 35.23). A flux transformer consists of a loop of superconducting material, which is coupled to the SQUID. An external magnetic field produces a persistent supercurrent in the loop and this current induces a flux in the SQUID. As the loop encloses a much larger area than can a SQUID, the sensitivity of the device gets enhanced.

Applications of Squids

SQUIDS are used to measure very small magnetic fields. Since the current is sensitive to very small changes in the magnetic field, the SQUID acts as a very sensitive magnetometer. As ordinary magnetometers, SQUIDS are capable of measuring magnetic fluctuations of the order of 10^{-13} T . Because of their extreme sensitivity, SQUIDS find applications in many fields, engineering, medicine and many other fields. For example, geologists use them for measuring rock magnetism and continental drift. Physical processes, such as muscular or neural activity, in humans (and other animals) create magnetic fields as small as a thousand billionth of a tesla (as a comparison, a fridge magnet generates about a tenth of a tesla). Human heart generates magnetic fields of about 10^{-14} wb/m^2 and the human brain generates

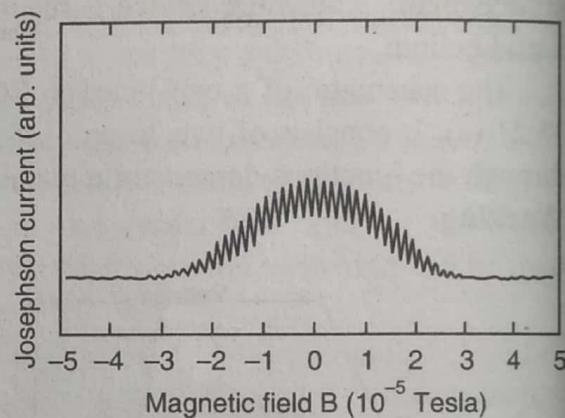


Fig. 35.22

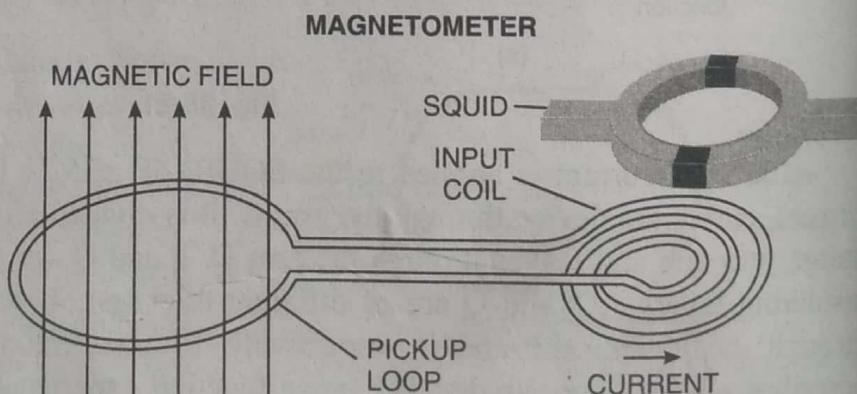


Fig. 35.23

magnetic fields of about 10^{-14} Wb/m². SQUIDs can detect these feeble fields and an array of SQUIDs is used in *magnetoencephalography* (MEG), the process of brain imaging. The SQUIDs are also used in nondestructive testing. In testing for corrosion of aluminum sheets riveted together in aircraft, the SQUID measures the influence of the aircraft skin on an applied oscillating magnetic field; a change in electrical conductivity reveals the defects.

Example 35.9. For a certain metal the critical magnetic field is 5×10^3 A/m at 6K and 2×10^4 A/m at 0K. Determine its transition temperature.

$$\text{Solution. } T_C = \frac{T}{\left[1 - \frac{H_C(T)}{H_C(0)}\right]^{\frac{1}{2}}} = \frac{6K}{\left[1 - \frac{5 \times 10^3 \text{ A/m}}{2 \times 10^4 \text{ A/m}}\right]^{\frac{1}{2}}} = 6.93 \text{ K.}$$