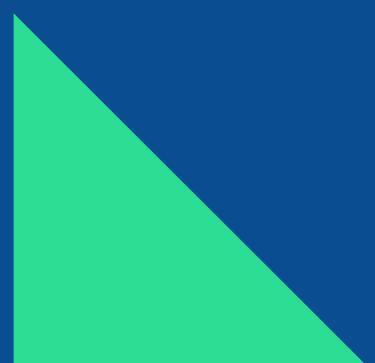




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Lecture Notes
Calculus & Linear Algebra (18MAT11)

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CALCULUS AND LINEAR ALGEBRA

MODULE - 05

LINEAR ALGEBRA

Introduction :-

Matrix :- The set of $m \times n$ elements arranged in a array of ' m ' rows and ' n ' columns is called the matrix of order $m \times n$ and the matrix can be denoted by Capital letters of Alphabets A, B, C.....

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_n \\ a_{21} & a_{22} & a_{23} & \dots & a_n \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

where a_{ij} can be called as element of a matrix of i^{th} row, j^{th} columns.

Rank of a matrix:- The rank of a matrix is a positive integer which can be evaluated of a matrix by taking number of non zero rows in a matrix after the elementary transformation.

The rank of a matrix 'A' can be denoted as, $R(A) = r$

Note:- If all the elements are zeros it is called zero row.

A. Find the rank of the matrix.

$$\textcircled{1}. \quad \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix} \quad R_2 : R_2 - 2R_1 \\ R_3 : R_3 + R_1 \\ R_4 : R_4 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix} \quad R_3 : R_3 + R_2$$

$$\sim \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_3 \leftrightarrow R_4$$

$$\Rightarrow A \sim \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_3 : \frac{1}{3} R_3$$

$\sim E$ (Echelon form)

$P(A) = \text{Number of non-zero rows} = 4$

(2) $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ By applying elementary row operation.

$$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & -11 & 5 & -3 \\ 0 & -11 & 5 & -3 \end{bmatrix} \quad R_3 : R_3 - 2R_1, \\ R_4 : R_4 - 4R_1$$

$$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 : R_3 + R_2, \\ R_4 : R_4 + R_2$$

$$\therefore P(A) = 2$$

(3) $A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$ Find the rank of matrix

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & -2 & -2 & -10 \\ 0 & 4 & 1 & -5 \end{bmatrix} \quad R_2 \rightarrow R_2 - R_1, \\ R_3 \rightarrow R_3 - 3R_1, \\ R_4 \rightarrow R_4 - 2R_1,$$

$$\sim \left[\begin{array}{cccc} 1 & 1 & 1 & 8 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -3 & -11 \\ 0 & 0 & -1 & -7 \end{array} \right] \quad R_3 \rightarrow R_3 - R_2$$

$$R_4 \rightarrow R_4 - 2R_2$$

$$\sim \left[\begin{array}{cccc} 1 & 1 & 1 & 3 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -3 & -11 \\ 0 & 0 & 0 & -10 \end{array} \right] \quad R_4 \rightarrow 3R_4 - R_3$$

$$P(A) = 4$$

④ Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -1 \end{bmatrix}$

Given,

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -1 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2$$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -1 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 3R_1$$

$$R_4 \rightarrow R_4 - 6R_1$$

$$\sim A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$$R_3 \rightarrow 5R_3 - 4R_2$$

$$R_4 \rightarrow 5R_4 - 9R_2$$

$$\sim A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 33 & 22 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_3$$

$$\sim A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \rho(A) = 3$$

⑤ Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$

Given,

$$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$$

$$R_3 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - 3R_1, R_4 \rightarrow R_4 - 2R_1$$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & -2 & -2 & -10 \\ 0 & -4 & 1 & -5 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2; R_4 \rightarrow R_4 - 2R_2$$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

$$R_4 \rightarrow 3R_4 - R_3$$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \rho(A) = 3$$

→ System of linear equation by Gauss elimination and Gauss jordan method.

Consider the system of equation having three unknowns

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\Rightarrow AX = B$$

Here the A is called Co-efficient matrix, X is variable matrix and B is called constant matrix.

The Augmented matrix

$$[A : B] = [AB] = \begin{bmatrix} a_{11} & a_{12} & a_{13} : b_1 \\ a_{21} & a_{22} & a_{23} : b_2 \\ a_{31} & a_{32} & a_{33} : b_3 \end{bmatrix}$$

Note :-

- ① If $\rho(A) = \rho(AB) = r = n$, number of unknowns, then the system of equation have unique solution.
- ② $\rho(A) = \rho(AB) = r < n$, then the system of equation may have infinite no. of solution.

③ If $\rho(A) < \rho(AB)$, then there will be no solution.

① Solve the system of equation by using Gauss elimination method $x+y+z=9$, $2x+y+z=0$, $2x+5y+7z=52$

Given,

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 52 \end{bmatrix}$$

$$\Rightarrow AX = B$$

$$\therefore (AB) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 9 \\ 2 & 1 & -1 & : 0 \\ 2 & 5 & 7 & : 52 \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 9 \\ 0 & -1 & -3 & : -18 \\ 0 & 3 & 5 & : 34 \end{array} \right] \quad R_2 : R_2 - 2R_1, \\ R_3 : R_3 - 2R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 9 \\ 0 & -1 & -3 & : -18 \\ 0 & 0 & -4 & : -20 \end{array} \right] \quad R_3 : R_3 + 3R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 9 \\ 0 & 1 & 3 & : 18 \\ 0 & 0 & 1 & : 5 \end{array} \right]$$

$\therefore \rho(A) = \rho(AB) = 3$ Number of unknown

$$\therefore AX = B$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 18 \\ 5 \end{bmatrix}$$

$$\Rightarrow x + y + z = 9 \rightarrow ①$$

$$y + 3z = 18 \rightarrow ②$$

$$z = 5$$

$$\therefore \textcircled{2} \Rightarrow y + 3z = 18$$

$$y + 3(5) = 3$$

$$y = 3$$

$$\textcircled{1} \Rightarrow x + 3 + 5 = 9$$

$$\Rightarrow x = 9 - 8$$

$$\Rightarrow x = 1$$

$$\therefore x = 1, y = 3, z = 5$$

② For what values of λ and μ , the system of equation $x+y+z=6$, $x+2y-3z=10$, $x+2y+\lambda z=\mu$

- a) unique solution b) Infinate Solution c) No solution

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$\Rightarrow AX = B$$

$$[AB] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 6 \\ 1 & 2 & -3 & : 10 \\ 1 & 2 & \lambda & : \mu \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 6 \\ 0 & 1 & -4 & : 4 \\ 0 & 1 & (\lambda-0) & : (\mu-6) \end{array} \right] \quad R_2 : R_2 - R_1$$

$$R_3 : R_3 - R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 6 \\ 0 & 1 & -4 & : 4 \\ 0 & 0 & (\lambda+3) & : (\mu-10) \end{array} \right] \quad R_3 : R_3 - R_2$$

→ For $\lambda \neq -3$ and $\mu \neq 10$, It may have unique Solution

→ For $\lambda = -3$ and $\mu = 10$, then the System have infinte Solution

→ For $\lambda = -3$ and $\mu \neq 10$ the system of equation may have no Solution.

- ③ For what values of λ and μ the system of equation $x+2y+3z=6$, $x+3y+5z=9$, $2x+5y+\lambda z=\mu$
- ① Unique solution
 - ② Infinitate solution
 - ③ No solution.

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ \mu \end{bmatrix}$$

$$\Rightarrow AX = B$$

$$[AB] = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & \lambda \end{bmatrix} \begin{bmatrix} 6 \\ 9 \\ \mu \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 : 6 \\ 1 & 3 & 5 : 9 \\ 2 & 5 & \lambda : \mu \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & : 6 \\ 0 & 1 & 2 & : 3 \\ 0 & 1 & (\lambda-6) & : (\mu-12) \end{bmatrix} \quad R_2 : R_2 - R_1 \\ R_3 : R_3 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & : 6 \\ 0 & 1 & 2 & : 3 \\ 0 & 0 & (\lambda-8) & : (\mu-15) \end{bmatrix} \quad R_3 : R_3 - R_2$$

- For $\lambda \neq 8$ and $\mu \neq 15$, It may have unique solution
- For $\lambda = 8$ and $\mu = 15$, It may have infinitate solution
- For $\lambda = 8$ and $\mu \neq 15$ the system of equation may have no solution.

- ③ Solve the system of equation Gauss elimination method $x+2y+z=3$, $3x+2y+2z=3$, $x-2y-5z=1$

Given,

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$$

$$\Rightarrow AX = B$$

$$[AB] = \begin{bmatrix} 1 & 2 & 1 : 3 \\ 3 & 2 & 1 : 3 \\ 1 & -2 & -5 : 1 \end{bmatrix} \quad R_2 \rightarrow R_2 - 3R_1, \\ R_3 \rightarrow R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 1 : 3 \\ 0 & -4 & -2 : -6 \\ 0 & -4 & -6 : -2 \end{bmatrix} \quad R_2 \rightarrow (-1)R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 1 : 3 \\ 0 & 4 & 2 : 6 \\ 0 & -4 & -6 : -2 \end{bmatrix} \quad R_3 \rightarrow R_3 + R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 1 : 3 \\ 0 & 4 & 2 : 6 \\ 0 & 0 & -4 : 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 2 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 4 \end{bmatrix}$$

$$-4z = 4$$

$$\boxed{z = -1}$$

$$x + 2(1) - 1 = 3$$

$$x + 2 - 1 = 3$$

$$x + 2y - z = 3$$

$$x + 2 - (-1) = 3$$

$$x + 1 = 3$$

$$2 + 2y + 1 = 3$$

$$x = 3 - 1$$

$$\boxed{x = 2}$$

$$2y = 3 + 1$$

$$\boxed{y = 2}$$

④ Solve the following System of equations by Gauss Jordan method $x+y+z=9$, $2x+y-z=0$

$$2x + 5y + 7z = 52$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 52 \end{bmatrix}$$

$$\Rightarrow AX = B$$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 2 & 1 & -1 & : & 0 \\ 2 & 5 & 7 & : & 52 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -1 & -3 & : & -18 \\ 0 & 3 & 5 & : & 34 \end{bmatrix} \quad R_2 : R_2 - 2R_1 \\ R_3 : R_3 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & : & -9 \\ 0 & -1 & -3 & : & -18 \\ 0 & 0 & -4 & : & -20 \end{bmatrix} \quad R_1 : R_1 + R_2 \\ R_3 : R_3 + 3R_2$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & : & -9 \\ 0 & -1 & -3 & : & -18 \\ 0 & 0 & 1 & : & 5 \end{bmatrix} \quad R_3 : \left(\frac{1}{4}\right)R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & -1 & 0 & : & -3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix} \quad R_1 : R_1 + 2R_3 \\ R_2 : R_2 + 3R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix} \quad R_2 : (-1)R_2$$

$$\therefore P(A) = P(AB) = 3 = n$$

$\therefore A \times B$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$x = 1, y = 3, z = 5$$

- ⑤ Solve the System of equation by Gauss Jordan method. $x+y+z=8$, $-x-y+2z=-4$, $3x+5y-7z=14$

Given,

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 3 & 5 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 14 \end{bmatrix}$$

$$AX = B$$

$$AB = \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 8 \\ -1 & 1 & 2 & : -4 \\ 3 & 5 & -7 & : 14 \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 8 \\ -1 & 1 & 2 & : -4 \\ 3 & 5 & -7 & : 14 \end{array} \right] \quad R_2 \rightarrow R_2 + R_1, \\ R_3 \rightarrow R_3 - 3R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 8 \\ 0 & 2 & 3 & : 4 \\ 0 & 2 & -10 & : -10 \end{array} \right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 8 \\ 0 & 2 & -10 & : -10 \\ 0 & 0 & 3 & : 4 \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & : 8 \\ 0 & 1 & -5 & : -5 \\ 0 & 0 & 3 & : 4 \end{array} \right] \quad R_2 = \frac{1}{2}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 6 & : 13 \\ 0 & 1 & -5 & : -5 \\ 0 & 0 & 3 & : 4 \end{array} \right] \quad R_1 \rightarrow R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 6 & : 13 \\ 0 & 1 & -5 & : -5 \\ 0 & 0 & 1 & : 4/3 \end{array} \right] \quad R_3 \rightarrow \frac{1}{3}R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & : 5 \\ 0 & 1 & 0 & : 5/3 \\ 0 & 0 & 1 & : 4/3 \end{array} \right] \quad R_1 \rightarrow R_1 - 6R_3 \\ R_2 \rightarrow R_2 + 5R_3$$

$$AX = B$$

$$\Rightarrow \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \left[\begin{array}{c} x \\ y \\ z \end{array} \right] = \left[\begin{array}{c} 5 \\ 5/3 \\ 4/3 \end{array} \right]$$

$$x = 5, y = 5/3, z = 4/3$$

$$AX = B$$

$$AB = \begin{bmatrix} 1 & 1 & 1 & : & 8 \\ -1 & 1 & 2 & : & -4 \\ 3 & 5 & -7 & : & 14 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 8 \\ -1 & 1 & 2 & : & -4 \\ 3 & 5 & -7 & : & 14 \end{bmatrix} \quad R_2 \rightarrow R_2 + R_1, \quad R_3 \rightarrow R_3 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 8 \\ 0 & 2 & 3 & : & 4 \\ 0 & 2 & -10 & : & -10 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 8 \\ 0 & 2 & -10 & : & -10 \\ 0 & 0 & 3 & : & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 8 \\ 0 & 1 & -5 & : & -5 \\ 0 & 0 & 3 & : & 4 \end{bmatrix} \quad R_2 = \frac{1}{2}R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 6 & : & 13 \\ 0 & 1 & -5 & : & -5 \\ 0 & 0 & 3 & : & 4 \end{bmatrix} \quad R_1 \rightarrow R_1 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 6 & : & 13 \\ 0 & 1 & -5 & : & -5 \\ 0 & 0 & 1 & : & 4/3 \end{bmatrix} \quad R_3 \rightarrow \frac{1}{3}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 5 \\ 0 & 1 & 0 & : & 5/3 \\ 0 & 0 & 1 & : & 4/3 \end{bmatrix} \quad R_1 \rightarrow R_1 - 6R_3, \quad R_2 \rightarrow R_2 + 5R_3$$

$$AX = B$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5/3 \\ 4/3 \end{bmatrix}$$

$$x = 5, \quad y = 5/3, \quad z = 4/3$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & -5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

$$AX = B$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & : 3 \\ 2 & 3 & 2 & : 5 \\ 3 & -5 & 5 & : 2 \end{array} \right]$$

$$R_2 : R_2 - 2R_1$$

$$R_3 : R_3 - 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & : 3 \\ 0 & -1 & 0 & : -1 \\ 0 & -11 & 2 & : -7 \end{array} \right]$$

$$R_3 : R_3 - 11R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & : 3 \\ 0 & -1 & 0 & : -1 \\ 0 & 0 & 0 & : 4 \end{array} \right]$$

$$\rho(A) < \rho(AB)$$

The system of equation have no solution.

→ Gauss Seidal iterative method :-

① Consider System of equation

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \quad \rightarrow ①$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \quad \rightarrow ②$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \quad \rightarrow ③$$

② Check the system of equation are diagonally dominate or not by the following way.

$$|a_{11}| > |a_{12}| + |a_{13}|$$

$$|a_{22}| > |a_{21}| + |a_{23}|$$

$$|a_{33}| > |a_{31}| + |a_{32}|$$

③ write $x_1 = \frac{1}{a_{11}} (b_1 - a_{12}x_2 - a_{13}x_3) \Rightarrow ①$

$$x_2 = \frac{1}{a_{22}} (b_2 - a_{21}x_1 - a_{23}x_3) \Rightarrow ②$$

$$x_3 = \frac{1}{a_{33}} (b_3 - a_{31}x_1 - a_{32}x_2) \Rightarrow ③$$

④ take the initial values of x_1, x_2, x_3 and continue the iteration until to get accuracy value.

⑤ If the given equation are not diagonal dominant
-at the rearrange the given equation according to diagonal dominate properly.

① Solve the system of equation by using gauss Seidal iterative method $10x + y + z = 12, x + 10y + z = 12$
 $x + y + 10z = 12$

Given,

$$10x + y + z = 12 \rightarrow ①$$

$$x + 10y + z = 12 \rightarrow ②$$

$$10x + y + z = 12 \rightarrow ③$$

∴ The given equation is diagonally dominate.

$$① \Rightarrow x = \frac{1}{10} [12 - y - z]$$

$$② \Rightarrow y = \frac{1}{10} [12 - x - z]$$

$$③ \Rightarrow z = \frac{1}{10} [12 - x - y]$$

Let $x=0, y=0, z=0$

$$\text{Iterative } ① \Rightarrow x^{(1)} = \frac{1}{10} [12 - 0 - 0] = 1.2$$

$$y^{(1)} = \frac{1}{10} [12 - 1.2 - 0] = 1.08$$

$$z^{(1)} = \frac{1}{10} [12 - 1.2 - 1.08] = 0.972$$

$$\text{Iterative } ② \Rightarrow x^{(2)} = \frac{1}{10} [12 - 1.08 - 0.972] = 0.9948$$

$$y^{(2)} = \frac{1}{10} [12 - 0.9948 - 0.972] = 1.0033$$

$$z^{(2)} = \frac{1}{10} [12 - 0.9948 - 1.0033] = 1.002$$

$$\text{Iterative } ③ \Rightarrow x^{(3)} = \frac{1}{10} [12 - 1.0033 - 1.0002] = 0.999$$

$$y^{(3)} = \frac{1}{10} [12 - 1 - 1.0002] = 1$$

$$z^{(3)} = \frac{1}{10} [12 - 1 - 1] = 1$$

\therefore The Solution $x=1, y=1, z=1$

② Solve the System of equation by Gauss Seidal method
 $2x + y - 2z = 17, 3x + 20y - z = 18, 2x - 3y + 2z = 25$
 Carry out three iterations

Given,

$$2x + y - 2z = 17 \rightarrow ①$$

$$3x + 20y - z = 18 \rightarrow ②$$

$$2x - 3y + 2z = 25 \rightarrow ③$$

\therefore The equation are diagonally dominate.

$$\textcircled{1} \Rightarrow x = \frac{1}{20} [17 - y + 2z]$$

$$\textcircled{2} \Rightarrow y = \frac{1}{20} [-18 - 3x + z]$$

$$\textcircled{3} \Rightarrow z = \frac{1}{20} [25 - 2x + 3y]$$

let $x=0, y=0, z=0$

$$\text{Iterative } \textcircled{1} \therefore x^{(1)} = \frac{1}{20} [17 - 0 + 0] = 0.85$$

$$y^{(1)} = \frac{1}{20} [-18 - 3(0.85) + 0] = -1.0275$$

$$z^{(1)} = \frac{1}{20} [25 - 2(0.85) - 3(-1.0275)] = 1.0109$$

$$\text{Iterative } \textcircled{2} \therefore x^{(2)} = \frac{1}{20} [17 + 1.0275 - 12(1.0109)] = 1.0025$$

$$y^{(2)} = \frac{1}{20} [-18 - 3(1.0025) + 1.0109] = -0.999$$

$$z^{(2)} = \frac{1}{20} [25 - 2(1.0025) + 3(-0.999)] = 0.9998$$

$$\text{Iterative } \textcircled{3} \therefore x^{(3)} = \frac{1}{20} [17 + 0.9998 + 2(0.9998)] = 1$$

$$y^{(3)} = \frac{1}{20} [-18 - 3(1) + 0.9998] = -1$$

$$z^{(3)} = \frac{1}{20} [25 - 2(1) + 3(-1)] = 1$$

$$\therefore x = 1, y = -1, z = 1$$

- (3) Solve the System of equation by using Gauss Seidal Iterative method $12x + y + z = 31, 2x + 8y + z = 24, 3x + 4y + 10z = 58$

\Rightarrow Given equations are diagonally dominate

$$x = \frac{1}{12} [31 - y - z]$$

$$y = \frac{1}{8} [24 - 2x + z]$$

$$z = \frac{1}{10} [58 - 3x - 4y]$$

Iterative ① :- $x^{(1)} = \frac{1}{12} [31 - 0 - 0] = \frac{31}{12} = 2.583$

$$y^{(1)} = \frac{1}{8} [24 - 2(2.583) + 0] = 2.354$$

$$z^{(1)} = \frac{1}{10} [58 - 3(2.583) - 4(2.354)] = 4.0835$$

Iterative ② :- $x^{(2)} = \frac{1}{12} [31 - (2.354) - (4.0835)] = 2.046$

$$y^{(2)} = \frac{1}{8} [24 - 2(2.046) + 4.0835] = 2.99 = 3$$

$$z^{(2)} = \frac{1}{10} [58 - 3(2.046) - 4(3)] = 3.98 = 4$$

Iterative ③ :- $x^{(3)} = \frac{1}{12} [31 - (3) - (4)] = 2$

$$y^{(3)} = \frac{1}{8} [24 - 2(2) + 4] = 3$$

$$z^{(3)} = \frac{1}{10} [58 - 3(2) - 4(3)] = 4$$

$$\therefore x = 2, y = 3, z = 4$$

\rightarrow Eigen value and Eigen vectors

Suppose 'A' be a square matrix of the order 2×2 or 3×3 for any ' λ ' such that $(A - \lambda I)$ is

the Eigen value and x is called the Eigen Vector
 To evaluate the eigen values and respective
 Eigen Vectors to write the characteristics equation

$$|A - \lambda I| = 0$$

① Diagnolysi the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

$$\therefore A - \lambda I = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\Rightarrow A - \lambda I = \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}$$

$$\therefore |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (5-\lambda)(2-\lambda) - 4 = 0$$

$$\Rightarrow 10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$$

$$\Rightarrow \lambda^2 - 7\lambda + 6 = 0$$

$$\Rightarrow \lambda^2 - \lambda - 6\lambda + 6 = 0$$

$$\Rightarrow \lambda(\lambda-1) - 6(\lambda-1) = 0$$

$$\Rightarrow (\lambda-1)(\lambda-6) = 0$$

$$\Rightarrow \lambda = 1, \lambda = 6$$

If, $\lambda = 1$

$$(A - \lambda_1 I)x_1 = 0$$

$$\Rightarrow \begin{bmatrix} 5-1 & 4 \\ 1 & 2-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 4x_1 + 4x_2 = 0$$

$$\Rightarrow x_1 + x_2 = 0$$

$$\Rightarrow x_1 = -x_2$$

$$\Rightarrow \frac{x_1}{1} = \frac{x_2}{1}$$

$$\therefore \lambda_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\text{if } \lambda = 6$$

$$(A - \lambda_2 I) x_2 = 0$$

$$\Rightarrow \begin{bmatrix} 5-6 & 4 \\ 1 & 2-6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -x_1 + 4x_2 = 0$$

$$\Rightarrow x_1 = 4x_2$$

$$\Rightarrow \frac{x_1}{4} = \frac{x_2}{1}$$

$$\therefore \lambda_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\therefore P = [x_1 \ x_2] = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{5} \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{5} \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$$

$$\therefore P^{-1}AP = \frac{1}{5} \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{5} \begin{bmatrix} 1 & -4 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{5} \begin{bmatrix} 5 & 0 \\ 0 & 30 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}$$

② diagonalise the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$

$$\therefore A - \lambda I = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -19 - \lambda & 7 \\ -42 & 16 - \lambda \end{bmatrix}$$

$$\therefore |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -(\lambda + 19) & 7 \\ -42 & (-\lambda - 16) \end{vmatrix} = 0$$

$$\Rightarrow (\lambda + 19)(\lambda - 16) + 294 = 0$$

$$\Rightarrow \lambda^2 + 19\lambda - 16\lambda - 304 + 294 = 0$$

$$\Rightarrow \lambda^2 - 2\lambda + 5\lambda - 10 = 0$$

$$\Rightarrow \lambda(\lambda - 2) + 5(\lambda - 2) = 0$$

$$\Rightarrow (\lambda - 2)(\lambda + 5) = 0$$

$$\Rightarrow \lambda = 2, -5$$

$$\text{where } \lambda = 2$$

$$(A - \lambda_1 I) x_1 = 0$$

$$\Rightarrow \begin{bmatrix} -21 & 7 \\ -42 & 14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -21x_1 + 7x_2 = 0$$

$$\Rightarrow -3x_1 + x_2 = 0$$

$$\Rightarrow 3x_1 = x_2$$

$$\Rightarrow \frac{x_1}{1} = \frac{x_2}{3}$$

$$\therefore \lambda_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

when $\lambda = -5$

$$(A - \lambda_2 I) x_2 = 0$$

$$\Rightarrow \begin{bmatrix} -14 & 7 \\ -42 & 21 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -14x_1 + 7x_2 = 0$$

$$\Rightarrow -2x_1 + x_2 = 0$$

$$\Rightarrow \frac{x_1}{1} = \frac{x_2}{2}$$

$$\therefore x_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\therefore P = [x_1 \ x_2]$$

$$P = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -2 & -1 \\ 3 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$$

$$\therefore P^{-1}AP = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 2 \\ -15 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix} = D$$

③ diagonalysis the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(A - \lambda I) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix}$$

$$|A - \lambda I| = (1-\lambda)(1-\lambda) - 4 = 0$$

$$= 1 - \lambda - \lambda + \lambda^2 - 4 = 0$$

$$= \lambda^2 - 2\lambda - 3 = 0$$

$$= \lambda^2 - 3\lambda + \lambda - 3 = 0$$

$$= \lambda(\lambda - 3) + 1(\lambda - 3) = 0$$

$$\lambda = -1, \lambda = 3$$

When $\lambda = -1$

$$[A - \lambda_1 I] x_1 = 0$$

$$\begin{bmatrix} 1+1 & 2 \\ 2 & 1+1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1 + 2x_2 = 0$$

$$\frac{x_1}{1} = \frac{x_2}{1}$$

$$x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

when $n = 3$

$$\begin{bmatrix} 1-3 & 2 \\ 2 & 1-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 2x_2 = 0$$

$$\frac{x_1}{1} = \frac{x_2}{1}$$

$$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$P[x_1 \ x_2] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} AP = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -3 & 3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -2 & 0 \\ 0 & 6 \end{bmatrix}$$

$$P^{-1} AP = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} = D$$

④ Diagnolysi the matrix, $A = \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix}$

Given,

$$A = \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -1 - \lambda & 3 \\ -3 & 4 - \lambda \end{bmatrix}$$

$$(A - \lambda I) = (1 + \lambda)(4 - \lambda) + 9 = 0$$

$$= (4 - \lambda + 4\lambda - \lambda^2) + 9 = 0$$

$$= (-\lambda^2 + 3\lambda + 4) + 9 = 0$$

$$= -3\lambda - 4\lambda^2 + 6 = 0$$

$$= \lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$= \lambda(\lambda - 2) - 1(\lambda - 2) = 0$$

$$\lambda = 1, \lambda = 2$$

If $\lambda = 2$

$$\begin{bmatrix} 1 & -2 & -3 \\ 2 & 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + 3x_2 = 0$$

$$\frac{x_1}{1} = \frac{x_2}{1}$$

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

If $\lambda = 1$

$$\begin{bmatrix} -1 & -1 & 3 \\ 2 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 3x_2 = 0$$

$$2x_1 = 3x_2$$

$$\frac{x_1}{3} = \frac{x_2}{2}$$

$$x_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$P = [x_1 \ x_2] = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$$

$$P^{-1} = \frac{1}{2-3} \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$$

$$\begin{aligned} P^{-1}AP &= \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -4 & 6 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = D \end{aligned}$$

Rayleigh's power method :-

Step ① :- let A be a square matrix of order $m \times m$ consider the initial eigen vector of the order $m \times 1$

Step ② :- $Ax^{(0)} = \lambda^{(1)}x^{(1)}$ where λ indicates here numerically large in the product of two matrix and x is respectively eigen vector.

Step ③ :- $Ax^{(1)} = \lambda^{(2)}x^{(2)}$

$Ax^{(2)} = \lambda^{(3)}x^{(3)}$

Continue the process until to get the value is accurately equal and which is said to be largest eigen value , and which is said to be largest eigen

① Find the largest eigen value $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ with $x^{(0)} = [1 \ 0 \ 0]^T$ by using power value Given,

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}, x^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$Ax^{(0)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \lambda^{(1)} x^{(1)}$$

$$Ax^{(1)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2 \end{bmatrix} = 2.5 \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \lambda^{(2)} x^{(2)}$$

$$Ax^{(2)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8 \begin{bmatrix} 1 \\ 0 \\ 0.9286 \end{bmatrix} = \lambda^{(3)} x^{(3)}$$

$$Ax^{(3)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.9286 \end{bmatrix} = \begin{bmatrix} 2.9286 \\ 0 \\ 2.8572 \end{bmatrix} = 2.9286 \begin{bmatrix} 1 \\ 0 \\ 0.9756 \end{bmatrix} = \lambda^{(4)} x^{(4)}$$

$$Ax^{(4)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.9756 \end{bmatrix} = \begin{bmatrix} 2.9756 \\ 0 \\ 2.9512 \end{bmatrix} = 2.9756 \begin{bmatrix} 1 \\ 0 \\ 0.9918 \end{bmatrix} = \lambda^{(5)} x^{(5)}$$

$$Ax^{(5)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.9918 \end{bmatrix} = \begin{bmatrix} 2.9918 \\ 0 \\ 2.9836 \end{bmatrix} = 2.9918 \begin{bmatrix} 1 \\ 0 \\ 0.9973 \end{bmatrix}$$

\therefore The large Given value $\lambda = 2.9918 \sim 3$ and $\lambda^{(6)} x^{(6)}$
it's eigen value $\begin{bmatrix} 1 \\ 0 \\ 0.9973 \end{bmatrix} \sim \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

② Find the largest eigen value corresponding eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking initial eigen vector for $x = [1 \ 1 \ 1]^T$.

Given,

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}, x^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$AX^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0 \\ 0.6667 \end{bmatrix} \lambda^{(1)} x^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.3636 \\ 0.5455 \end{bmatrix} = \begin{bmatrix} 7.8182 \\ -3.6363 \\ 4.0003 \end{bmatrix} = 7.8182 \begin{bmatrix} 1 \\ 0.4651 \\ 0.5117 \end{bmatrix} \lambda^{(2)} x^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.4631 \\ 0.5117 \end{bmatrix} = \begin{bmatrix} 7.9536 \\ -3.9070 \\ 4.0002 \end{bmatrix} = 7.9536 \begin{bmatrix} 1 \\ -0.4912 \\ 0.5029 \end{bmatrix} \lambda^{(3)} x^{(3)}$$

$$AX^{(3)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.4631 \\ 0.5117 \end{bmatrix} = \begin{bmatrix} 7.9536 \\ -3.9070 \\ 4.0002 \end{bmatrix} = 7.9536 \begin{bmatrix} 1 \\ -0.4912 \\ 0.5029 \end{bmatrix} \lambda^{(4)} x^{(4)}$$

$$AX^{(4)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.4912 \\ 0.5029 \end{bmatrix} = \begin{bmatrix} 7.9882 \\ -3.9765 \\ 4 \end{bmatrix} = 7.9882 \begin{bmatrix} 1 \\ -0.4978 \\ 0.5007 \end{bmatrix} \lambda^{(5)} x^{(5)}$$

$$AX^{(5)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.4918 \\ 0.5007 \end{bmatrix} = \begin{bmatrix} 7.9970 \\ -3.9941 \\ 4 \end{bmatrix} = 7.9970 \begin{bmatrix} 1 \\ -0.4994 \\ 0.5002 \end{bmatrix} \sim \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix}$$

\therefore The large given value of $\lambda = 7.9970 \sim 8$ and it's eigen value $\begin{bmatrix} 1 \\ -0.4994 \\ 0.5002 \end{bmatrix}$

Q.3) Find the largest Eigen value of $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$
 by taking $x^0 = [1, 0, 0]^T$ as initial eigen value.

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \quad x^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$AX^{(0)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 25 \\ 1 \\ 2 \end{bmatrix} = 25 \begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix} \quad \lambda^{(1)} x^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix} = \begin{bmatrix} 25.2 \\ 1.12 \\ 1.66 \end{bmatrix} = 25.2 \begin{bmatrix} 1 \\ 0.0454 \\ 0.0688 \end{bmatrix} \quad \lambda^{(2)} x^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.0444 \\ 0.0666 \end{bmatrix} = \begin{bmatrix} 25.1778 \\ 1.1332 \\ 1.7332 \end{bmatrix} = 25.1778 \begin{bmatrix} 1 \\ 0.0450 \\ 0.0668 \end{bmatrix}$$

$$AX^{(3)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.0450 \\ 0.0668 \end{bmatrix} = \begin{bmatrix} 25.1526 \\ 1.1350 \\ 1.7246 \end{bmatrix} = 25.1526 \begin{bmatrix} 1 \\ 0.0451 \\ 0.0685 \end{bmatrix} \quad \lambda^{(3)} x^{(3)}$$

$$AX^{(4)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.0451 \\ 0.0685 \end{bmatrix} = \begin{bmatrix} 25.1826 \\ 1.1350 \\ 1.7248 \end{bmatrix} = 25.1826 \begin{bmatrix} 1 \\ 0.0450 \\ 0.0685 \end{bmatrix} \quad \lambda^{(4)} x^{(4)}$$

$$AX^{(5)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.0450 \\ 0.0685 \end{bmatrix} = \begin{bmatrix} 25.1820 \\ 1.1350 \\ 1.7260 \end{bmatrix} = 25.1820 \begin{bmatrix} 1 \\ 0.0450 \\ 0.0685 \end{bmatrix} \quad \lambda^{(5)} x^{(5)}$$

\therefore The largest Eigen value $\lambda = 25.1820$ and Eigen vector $x = \begin{bmatrix} 1 \\ 0.0450 \\ 0.0685 \end{bmatrix}$