23.2 GEIGER-MULLER COUNTER

Nuclear radiations emitted by disintegrating nuclei cannot be sensed directly. Indirect methods are to be employed to detect them. Alpha, beta and gamma rays have the ability to ionize neutral atoms. This property is used in radiation detecting instruments.

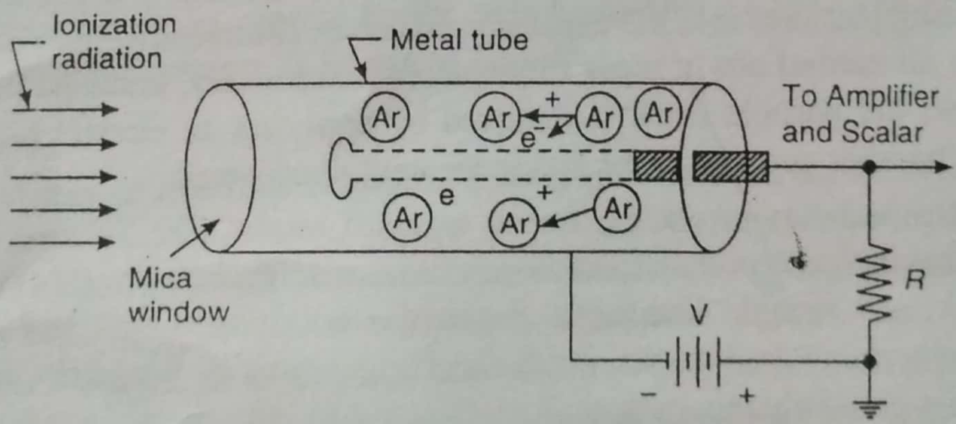


Fig. 23.1. A Schematic of Geiger -Muller counter. Radiation ionizes the argon gas molecules in the tube giving rise to electrical pulse, which is counted.

The Geiger-Muller counter is a radiation detector. It is a modified cathode ray tube with electrical circuits needed to amplify the current and detect it. The Geiger-Muller (G.M.) tube (Fig. 23.1) consists of a rugged metal case enclosed in a thin glass tube. The hollow metal case acts as cathode. A fine wire, usually of tungsten, runs through the center of the tube and is insulated from the metal. It acts as anode. The tube is evacuated and then partially filed with a mixture of 90% argon at 10 cm pressure and 10% ethyl alcohol vapour at 1 cm pressure. At one end of the tube a thin window of mica is arranged to allow the entry of radiation into the tube.

A dc potential of about 1200 volts is applied between the cathode and the wire. The value of the voltage is adjusted to be somewhat below the breakdown voltage of the gaseous mixture. A high resistance R is connected in series with battery.

A high energy particle entering through the mica window will cause one or more of the argon atoms to ionize. The electrons and ions of argon thus produced cause other argon atoms to ionize in a cascade effect. The result of this one event is a sudden, massive electrical discharge that causes a current pulse. The current through R produces a voltage pulse of the order of 10 μ V. An electronic pulse amplifier accepts the small pulse voltages and amplifies them to about 5 to 50 volts. The amplified output is then applied to a counter. As each incoming particle produces a pulse, the number of incoming particles can be counted.

The number of secondary electrons is independent of the number of the primary ions produced by incoming particle. The incoming particle acts as a trigger to release an avalanche of secondary electrons. The electrons reach the anode and cause ionization current in the circuit, whereas the positive ions move slowly and form a sheath around the anode for a short time. They reduce the potential difference to such a low value that the current in the circuit is stopped. Therefore, a brief pulse of current is produced by each incoming particle.

Fig. 23.2 shows a plot of counts per minute as a function of voltage. For voltages less than 1000 volts there is no discharge and hence no counts. Between 1000 to 1200 volts the number of pulses increases with the applied voltage almost linearly. Above 1200 volts, the number of counts remains constant over a certain region known as *plateau*. In this region, the magnitude of pulses becomes independent of the amount of original ionization. This plateau region is used for G.M. counter operation. If the voltage is increased above this region, a continuous discharge will take place, which is undesirable and is hence avoided.

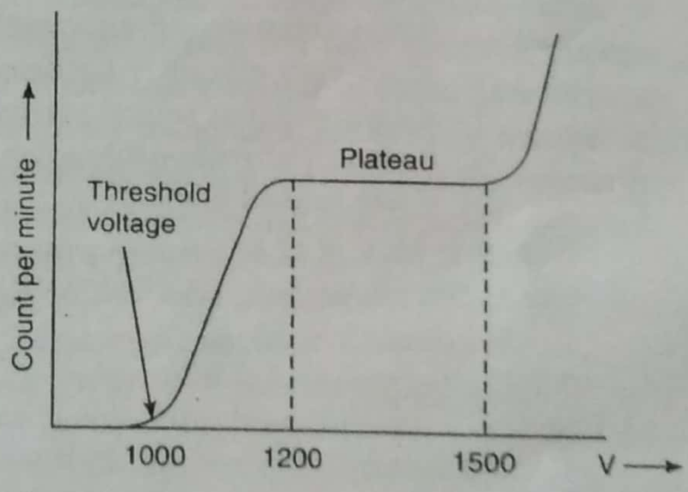


Fig. 23.2. A plot of potential difference applied across the electrodes in a G.M. counter versus count rate. G.M. counter is operated in the plateau region.

Quenching

When the positive ions reach the cathode, they dislodge secondary electrons from the cathode because of their high energies. These electrons move toward anode and produce unwanted avalanches. As a result the counter goes into a state of continuous avalanching. During the measurements, the counter fails to distinguish between the two types of pulses, one that is due to an incoming particle and the other due to unwanted avalanching. The process of preventing the undesirable continuous avalanching is known as *quenching*. In other words, quenching is the elimination of sheath of positive ions around the cathode.

Self-quenching

To cause internal automatic quenching, a small percentage of ethyl alcohol vapour is added to the argon gas in the tube which prevents undesirable continuous avalanching.

Counting rate

The G.M. counter can count about 5000 particles per second. The counting rate depends upon the dead time and recovery time of the GM counter.

Dead time

Dead time refers to the time taken by the tube to recover between counts. In the counter, the slowly moving positive ions take about 100 μs to reach the cathode. If a second particle enters the tube during this time, it will not be registered, as the potential difference across the electrodes is very low. Hence, the time interval is known as the dead time.

Recovery time

After dead time, the tube takes approximately 100 μs before it regains the original working conditions. This time interval is known as recovery time. Thus, *recovery time* is the time after which the original pulse levels are restored.

Paralysis time

The sum of dead time and recovery time is known as paralysis time, which is 200 μs. The tube can respond to the second incoming particle only after 200 μs.

Applications

The G.M. counter is very useful for detecting nuclear radiations and charged particles. It is largely used for recording cosmic ray events and measuring cosmic ray intensities.

Limitations

1. G.M. counter has a large 'dead time' and recovery time of the order of 200 μs. If a large number of particles enter the G.M. tube at a rapid rate, the tube will not have time to recover and some particles may not be counted.
2. It has very efficiency for detection of γ-radiation.
3. It cannot detect neutral particles.
4. It cannot provide information regarding the nature of the ionizing particle.
5. It has limited lifetime.

Example 23.1. *A G.M. counter wire collects 10^8 electrons per discharge. When the counting rate is 500 counts/min, what will be the average current in the circuit?*

Solution: Number of electrons collected in one minute, $n = 10^8 \times 500$

Charge per minute, $Q = ne = (10^8 \times 500)\,(1.602 \times 10^{-19}\,C)$ /min.

Average current, $$I = \frac{Q}{60\,s} = \frac{(10^8 \times 500)\,(1.602 \times 10^{-19}\,C)}{60\,s} = 1.3 \times 10^{-10}\,A.$$

21.26 NUCLEAR REACTORS

We can sum up the three important features of nuclear fission reaction as follows:

(*a*) **Energy Emission:** Each fission reaction produces about 200 MeV of energy and the total energy produced by a small sample of uranium, say 1 gm, is as enormous as 8.2×10^{10} J = 22.8 MWh. Most of the energy is carried by fission fragments in the form of kinetic energy.

(*b*) **Neutron Multiplicity:** Each fission event is accompanied by the emission of a number of neutrons which in turn cause fission of other nuclei.

(*c*) **Delayed Neutrons:** Many of the neutrons are emitted at the instant of fission. They are known as *prompt neutrons.* About 1% of the neutrons are emitted due to decays of fission fragments and they are called *delayed neutrons.*

The last feature enables mechanical control of the reaction rate and keeps the reaction from proceeding too rapidly. The second feature can be used to build a self-sustaining chain of nuclear fissions. The energy produced in the reactions can be extracted as heat and used to boil water, the resulting steam can then be used to drive a turbine to generate electrical power.

A system in which nuclear fission is produced in a controlled, self–sustaining chain reaction is known as ***nuclear reactor***. Enrico Fermi built the first nuclear reactor in 1942. A very large number of nuclear reactors are in operation in different countries as on today. They differ widely in design and construction depending on their and use. A nuclear reactor essentially consists of seven components arranged in different zones, each of which serves a definite purpose.

Fuel

Nuclear reactors can use pure fissile materials but it is easier and cheaper to use mixtures of isotopes. Naturally occurring uranium consists of 99.3% *U*-238 and only 0.7% *U*-235. *U*-238 is for all practical purposes not fissionable. Often natural uranium in which there is one *U*-235 atom per 140 *U*-238 atoms is used as nuclear fuel. In order to be useful as a fuel, the concentration of *U*-235 must be substantially increased (upto 3%) in naturally available uranium. This process is called *enrichment*. In many reactors, uranium enriched in *U*-235 is used as fuel. Another fissionable material is *Pu*-239. This does not occur in nature. It is produced through neutron capture by the non-fissionable *U*-238. The process of plutonium fuel production from uranium is known as *breeding.*

Moderators

The kinetic energy of neutrons emitted in fission process is of the order of a few MeV. Such high energy neutrons are known as *fast neutrons*. The fast neutrons have relatively low probability of inducing fission. Nuclear fission is much more probable with *slow neutrons*. The fast neutrons must be therefore, slowed down in order to increase their chances of initiating fission events. If a neutron is scattered from a heavy nucleus like uranium, the energy of the neutron is not changed. On the other hand in a collision with a very light nucleus, the neutron can lose substantial energy. Materials consisting of atoms of lower atomic mass are used for slowing down neutrons. Such materials are called ***moderators***.

The commonly used moderators are water, heavy water, graphite and beryllium. Heavy water is considered to be the best moderator, since it does not absorb neutrons. When a solid moderator is required, graphite is commonly used. Ordinary water is a good moderator, but has high neutron absorption cross-section. Beryllium and its oxide are also good moderators, but they are expensive, toxic and have poor mechanical property.

Reflectors

Some of the neutrons generated in the fission process may leak away without being absorbed. It is necessary to conserve neutrons so that we minimize the consumption of fissile material and keep the size of the reactors small. To reduce the neutron loss due to leakage the inner surface of the reactor is surrounded by a material which reflects the neutrons back into the core. Such materials are called ***reflectors***. Reflectors are made of nickel, thorium or other suitable materials.

Coolants

Intense heat is generated within the reactor core due to nuclear fission reactions. This heat must be removed using coolants for the safe operation of reactor. Ordinary water, heavy water and liquid metals are used as coolants. The ***coolant*** keeps the fuel assembly at a safe temperature; at the same time the heat carried away by the coolant is used in the heat exchange for utilization in power generation.

Control Rods

The rate of reactions in a nuclear reactor is controlled by **control rods**. Since the neutrons are responsible for the progress of the chain reactions, suitable neutron absorbers are employed to achieve control of reaction rate. Cadmium and boron are the most frequently used materials. The *control* procedure involves the insertion or withdrawal of these materials, taken in the form of rods, into or from the reactor core. With the control rod fully inserted, enough neutrons are absorbed so that the average number of neutrons available to cause new fissions is less than one per fission reaction. As the rod is slowly withdrawn, the average number of available neutrons increases until it is just equal to one per reaction. At that time the reactor is said to be *critical*. During the operation, the position of the control rod is continually adjusted so that energy is released at a steady rate.

Structural and Cladding materials

In a reactor, structural materials have to be used in the form of mechanical support for the various components. They are also used for holding the fuel, coolants, control rods and measuring instruments. Uranium readily reacts with air, water and other fluids. Hence ***cladding*** is required to pack fuel elements to isolate them. The cladding also prevents escape of the fission fragments. Zirconium has been found to be an excellent structural and cladding material. Titanium is also a good material but it is very expensive.

Reactor Shielding

A nuclear reactor is a powerful source of highly penetrating neutrons and –radiation, which are very harmful to life. Therefore, a thermal shield and a biological shield are used to minimize the effects of the harmful radiation. The thermal shield, usually a wall of steel, is placed between the reactor and the biological shield to protect the later from excessive heating and damage. The biological shield is usually made of concrete. Concrete wall of about 2 m thick surrounding the reactor serves as an adequate shield.

21.26.1 Nuclear Reactor

A schematic of a nuclear reactor is shown in Fig. 21.16.

It consists of several zones each of which serves a definite purpose. The central part is called the **active zone** or **core**. The core is made up of moderator blocks with slots in which fuel channels are housed. A **fuel channel** is a metal tube containing the fuel elements. The **fuel element** consists of uranium containing slug clad in a stainless steel or zirconium metal casing. The fuel elements are grouped lengthwise in the fuel channel and form a common construction called the **fuel assembly**. Inlet and outlet pipes are incorporated in the fuel channel for circulation of coolant. The active zone is filled with a moderator. The fuel assembly and moderator are cooled by the coolant passing through the fuel channels. The heated coolant then passes through an outlet pipe to the collecting tank. The heat can be used to generated steam and drive the turbine.

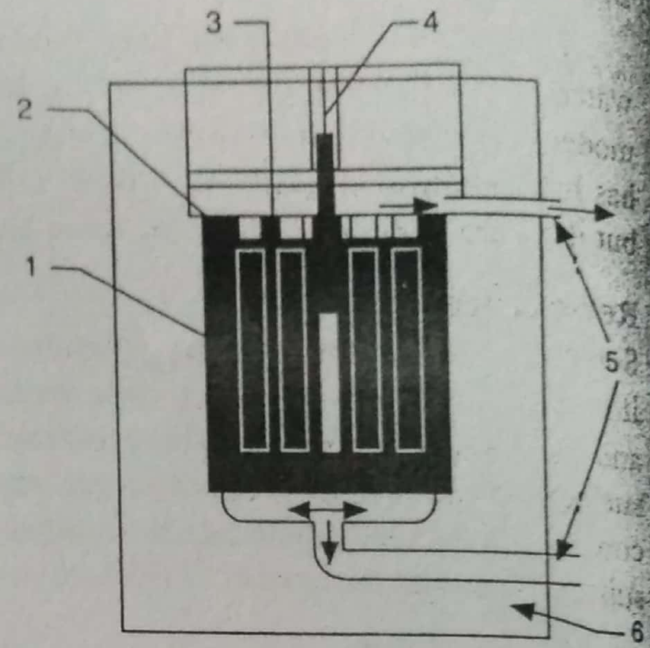


Fig. 21.16: Nuclear Reactor—1. Nuclear Fuel 2. Nuclear Reflector 3. Moderator 4. Control Rod 5. Coolant 6. Shield.

21.26.2 Types of Reactors

Reactors are designed and fabricated for different purposes, mainly for generation of power and for use in research.

Power Reactors

In a power reactor, heat produced in the nuclear reactor fuel is extracted to drive a turbine connected to an electrical generator. In practice two different types of reactors are used for producing power.

(*i*) **Boiling – Water Reactor:** In this type of reactors, a stream of water circulates through the core. The heat turns the water into steam, which is then directly fed to the turbine. Therefore, an external steam generators is not required. There is a considerable financial saving due to this. The disadvantage of this type of reactor is that the water can become radioactive; and a rupture of the pipes near the turbines could result in a serious accident with the spread of radioactive materials.

(*ii*) **Pressurized – Water Reactor:** In this type of reactor, heat is extracted in two steps. Water is circulated through the core under high pressure, which prevents the water turning into steam. This hot water in turn heats a secondary water system, which delivers steam to the turbine. Since steam is not taken from the reactor core, it is not radioactive. The main disadvantages of these reactors are the necessity of using highly enriched fuel and expensive structural materials.

Research Reactors

Research reactors produce high neutron fluxes for research. These neutrons are used as projectiles in nuclear reactions to produce new nuclides. These nuclides are used as tracers and for medical purposes.

23.13 PARTICLE ACCELERATORS

Particles are accelerated to very high energies using electric and magnetic fields. A charged particle acquires energy 'qV' when it is accelerated by a potential difference V. Choosing V to be very large, it should be possible to achieve any desired energy. In practice, the maximum voltage that one can produce limits the maximum energy of the particles. The production of maximum voltage in fact depends on the problem of providing adequate insulation. The Cockcroft-Walton and the Van de Graaff accelerators are examples of single-step high voltage accelerators. We can bypass the problem of insulation if we use small voltages and provide acceleration through small steps. By making the number of steps large, we can produce very high-energy charged particle beam. Two different types of small voltage accelerators are built which differ in their principle of operation. They are known as linear accelerators and cyclic accelerators.

(*i*) **Linear accelerator:** A linear accelerator (LINAC) accelerates charged particles to high energies without the need for very high voltages. In a linear accelerator, there is a succession of electrodes to which an alternating voltage is applied. Successive tubes have opposite voltages, but the voltage alternates with the frequency of the applied voltage. If the lengths of the tubes are correctly chosen, the motion of the charged particles is synchronised with the alternation of the voltage so that they cross the gap between the successive tubes at the right time to receive a push that increases their energy.

The largest proton linear accelerator is at the University of Minnesota, U.S.A., which can accelerate protons to 68 MeV. The largest electron linear accelerator is at Stanford University, U.S.A. It is 3.2 km long and can accelerate electrons to 25 GeV.

(*ii*) **Cyclic accelerator:** In cyclic accelerators, the particles are forced by a magnetic field to describe a curved path along which they receive increase in energy from electric fields at certain points on its path. Cyclotron is the first cyclic accelerator. The maximum energy that can be reached with a cyclotron is limited to several MeV. As demand arose for more energetic particles, cyclotrons were replaced by synchrotrons. The most powerful of these machines is the super proton synchrotron (SPS) at the European Centre for Nuclear Research (CERN), Switzerland, which can accelerate protons to an energy of 26 GeV.

Examples: Van de Graff generator and **LINAC** are examples of linear accelerators, while cyclotron and betatron are examples of cyclic accelerators.

23.14 DRIFT TUBE ACCELERATOR

A **linear accelerator** (LINAC) accelerates charged particles to high energies without the need for very high voltages. In linear accelerators, there are a series of coaxial hollow cylindrical electrodes, known as *drift tubes* to which an alternating voltage is applied. Successive tubes have opposite voltages, but the voltage alternates with the frequency of the applied voltage. If the lengths of the tubes are correctly chosen, the motion of the charges particles is synchronized with the alteration of the voltage so that they cross the gap between the successive tubes at the right time to receive a push that increases their energy.

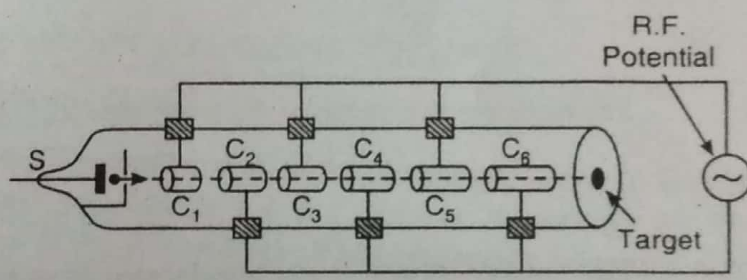


Fig. 23.14

Construction and working

The LINAC consists of a set of drift tubes of increasing lengths arranged linearly in an evacuated glass chamber (see Fig. 23.14). Alternate tubes are connected together. The odd numbered tubes are joined to one terminal and the even numbered tubes are joined to the other terminal of a high frequency power supply, which is in fact an r.f. oscillator. Thus, when tubes C_1, C_3, C_5 are positive, the other tubes C_2, C_4 and C_6 are negative and reversal of potential takes place in step with the frequency of the power supply. The positive ions produced by the source S travel along the axis of the tubes and are accelerated on crossing the gaps between the drift tubes. The ions do not receive acceleration while traveling inside the tubes because electric field does not exist there.

Initially, the positive ions move towards C_1 when it is negative. Then they travel through it with constant velocity. Length of C_1 is so selected in relation to the frequency of the oscillator that these ions arrive at the gap between C_1 and C_2 at the instant when C_2 has just become negative. It will happen only when the time taken by ions to travel through C_1 is exactly equal to half the time period of the oscillator. The ions are further accelerated across the gap and travel through C_2 at a higher uniform velocity. The length of C_2 is such that the ions arrive in the gap at the instant when C_3 has just become negative and are further accelerated. Thus, the ions are accelerated in the successive gaps and emerge from the final drift tube with extremely high velócities.

If N is the number of tubes, V is the maximum voltage of the oscillator, then the energy acquired by an ion of charge q is given by

$$E = \frac{1}{2} m\upsilon_N^2 = NqV \tag{23.20}$$

The time required to travel through any tube is equal to half time period of the applied r.f.voltage. Hence,

$$t = \frac{T}{2} = \frac{1}{2\nu}$$

If l_N is the length of the N^{th} tube and υ_N the velocity of the ion while traveling through it, then

$$t_N = \frac{l_N}{\upsilon_N}$$

$$\therefore \quad \frac{l_N}{\upsilon_N} = \frac{1}{2\nu}$$

or

$$l_N = \frac{\upsilon_N}{2\nu} \tag{23.21}$$

Substituting the value of υ_N from equ. (23.20) into the above equation, we get

$$l_N = \sqrt{\frac{NqV}{2m\nu^2}} \tag{23.22}$$

The largest proton linear accelerator is at the University of Minnesota, USA, which can accerlate protons to 68 MeV. The largest electron linear accelerator is at Stanford University, USA. It is 3.2 km long and can accelerate electrons to 25 GeV.

Example 23.5. *In a linear accelerator, proton accelerated thrice by a potential of 40 kV leaves a tube and enters an accelerating space of length 30 cm before entering the next tube. Calculate the frequency of the r.f.voltage and the length of the tube entered by the proton.*

Solution. Let υ_1 and υ_2 be the velocities of the proton entering and leaving the accelerating space. Let q and m be the mass and charge of the proton respectively. Then

$$\frac{1}{2} m\upsilon_1^2 = 3 \times 1.602 \times 10^{-19}\ \text{C} \times 4 \times 10^4\ \text{V}$$

$$\therefore \quad \upsilon_1 = \left[\frac{2\times3\times1.602\times10^{-19}\,\text{C}\times4\times10^4\,\text{V}}{1.67\times10^{-27}\,\text{kg}}\right]^{\frac{1}{2}} = 4.8 \times 10^6\ \text{m/s}$$

Similarly

$$\upsilon_2 = \left[\frac{2\times4\times1.602\times10^{-19}\,\text{C}\times4\times10^4\,\text{V}}{1.67\times10^{-27}\,\text{kg}}\right]^{\frac{1}{2}} = 5.5 \times 10^6\ \text{m/s}$$

Mean velocity $\quad \upsilon = 5.165 \times 10^6$ m/s.

The time taken to travel 30 cm = 0.3 m equals the half-period of the r.f.voltage.

$$\frac{T}{2} = \frac{0.3\text{ m}}{5.165\times10^{6}\text{ m/s}}$$

$\therefore$ Frequency of the r.f.voltage $\nu = \frac{1}{T} = \frac{5.165\times10^{6}\text{ m/s}}{2\times0.3\text{m}} = 8.6\text{ MHz.}$

Length of the next tube entered by the protons $l = \frac{\upsilon_2}{2\nu} = \frac{5.5\times10^{6}\text{ m/s}}{2(8.6\times10^{6}\text{ Hz})} = 0.32\text{ m.}$

23.15 CYCLOTRON

Cyclotron is the first **cyclic accelerator** built by E.O.Lawrence and M.S.Livingston in 1932.

Principle:

A moving charged particle describes a circular path in the presence of a transverse uniform magnetic field. The frequency of revolution of the charged particle is given by

$$\nu = \frac{qB}{2\pi m} \tag{23.23}$$

which is independent of the particle velocity. It means that a faster particle moving in a circle of bigger radius and a slower particle moving in a smaller circle take the same time for completing one revolution in a given uniform magnetic field. Hence, charged particles having different initial velocities can be uniformly accelerated to produce high-energy particle beam using a combination of crossed electric and magnetic fields.

Construction:

The schematic of a cyclotron is shown in Fig. 23.15. A cyclotron consists of two hollow metal dees formed by cutting a short, cylindrical box along its diameter. The dees are separated by a few centimetres from each other. They are insulated from each other and are placed in a vacuum chamber located between the pole pieces of an electromagnet. The magnet produces a uniform magnetic field in a direction perpendicular to the semicircular faces of dees A high frequency oscillator is connected to the dees, which produces a r.f. electric field in the gap between the dees. A source of charged particles is located at the centre of the gap between the dees.

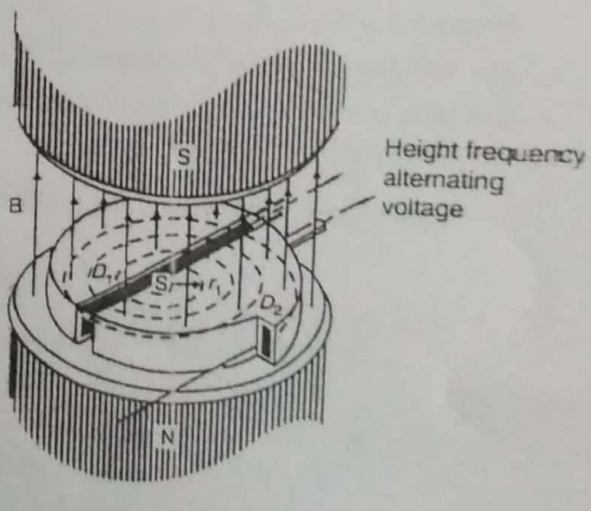


Fig. 23.15

Working:

Charged particles, say protons, are injected by the ion source S into the gap between the dees. The protons are accelerated by the r.f. electric field existing in the gap toward the dee, which is at a negative potential at that instant. However, the magnetic field that is acting perpendicular to the protons deflects them along a circular path. The protons travel in the hollow region of the dee and come back into the gap after completion of a half-revolution. The time taken for a half-revolution is given by

$$\frac{T}{2} = \frac{\pi m}{qB} \tag{23.24}$$

where T is the time period for the circular path in the magnetic field. The protons will be further accelerated if the dees reverse their polarity at the instant when the protons emerge into the gap. In such a case, the protons travel further in a semicircular path in the other dee and reach the gap in a time $T/2$. If, again at the same instant, the dees reverse their polarity, the protons receive another dose of acceleration. The process is repeated over and again many times. As the velocity increases with each dose of acceleration, the protons describe a spiral path in the dees, as shown in Fig. 23.15. During each revolution, each proton receives energy of $2qV$ electron volts and after about a hundred or more revolutions, the protons acquire energies of the order of several million electron volts. At the end of the journey, the proton beam is pulled out of its circular path by a negatively charged deflector plate and emerges out of the chamber through a narrow aperture.

Condition of Resonance:

In a cyclotron the protons are progressively accelerated provided that the time period T_0 of the r.f. electric field equals the time period T of revolution of protons in the magnetic field, B. Thus, it is required that the condition $T_0 = T$ is fulfilled. It means that

$$T_0 = \frac{2\pi m}{qB} \tag{23.25}$$

or

$$\nu_0 = \frac{qB}{2\pi m} \tag{23.26}$$

The above relation is known as the ***condition of resonance***.

Energy Acquired by the Charged Particles:

A proton makes N revolutions, receiving energy of $2qV$ during each revolution. The total kinetic energy acquired is

$$E = 2NqV \tag{23.27}$$

The radius R of the final orbit is given by $R = \dfrac{m\upsilon_{max}}{qB}$

The velocity in the final orbit is therefore $\upsilon_{max} = \dfrac{qBR}{m}$

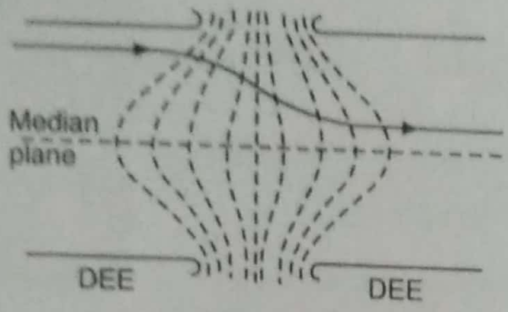
$$E = \frac{1}{2}m\upsilon_{max}^2 = \frac{B^2q^2R^2}{2m} \tag{23.28}$$

It is seen from the above expression that ***the final energy acquired by the particles in a cyclotron does not depend on the magnitude of the voltage applied across the dees***. A comparison of the two equations for final energy of the particles suggests that *the particles will have to execute a larger number of revolutions if the applied voltage is low or to make smaller number revolutions if the applied voltage is higher, to gain the same amount of kinetic energy.*

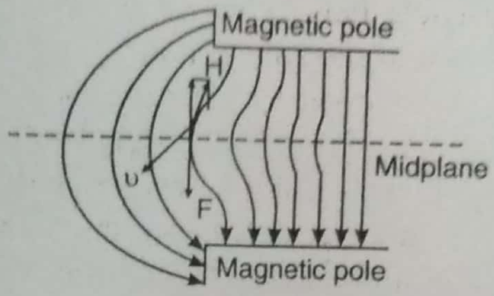
Role of Electric and Magnetic Fields:

The primary function of the r.f. electric field consists in imparting high kinetic energy to the charged particles. The primary function of magnetic field is to deflect the charged particles along circular path so that they repeatedly pass through the r.f. electric field region, and acquire more and more energy.

The second function of r.f. electric field is to focus the charged particles into a sharp beam. The non-uniform electric field in the gap between the dees plays the role of electron lens (Fig. 23.16 a.) and causes focussing of charged particles. The second function of magnetic field is to correct the paths of charged particles revolving nearer to periphery. Particles, which tend to stray from the median plane, are brought back to the median plane due to the Lorentz force component produced by non-uniform field at the outer edges of the magnet poles (Fig. 23.16b).



(a) Electric focussing in a cyclotron.



(b) Magnetic focussing in a cyclotron.

Fig. 23.16

Limitation of Cyclotron:

The kinetic energy acquired by charged particles in a cyclotron is given by

$$E = \frac{B^2R^2q^2}{2m}$$

According to this relation, it appears that the maximum energy of the particle beam is limited by the magnetic field B. Increasing the size of the magnet can increase B. However, there exists an ultimate limit for the size of magnet and the electric current to drive the electromagnet.

There arises a more basic limitation due to the relativistic variation of particle mass at velocity $\upsilon \approx c$. It is known that the mass 'm' of a particle increases with velocity according to the relation

$$m = \frac{m_0}{\sqrt{1-\upsilon^2/c^2}} \tag{23.29}$$

where m_0 is known as the rest mass of the particle.

Therefore, the time taken by the particle to complete a revolution increases from

$$T_0 = \frac{2\pi m_0}{qB}$$

to

$$T = \frac{2\pi m}{qB}, \text{ at velocities comparable to } c.$$

$$= \frac{2\pi m_0}{qB}\left[\frac{1}{\sqrt{1-\upsilon^2/c^2}}\right] \tag{23.30}$$

We can express T as

$$T = T_0\left(\frac{m}{m_0}\right) = T_0\left(\frac{mc^2}{m_0c^2}\right) = T_0\left(\frac{E}{E_0}\right)$$

where E_0 is the energy of the particle at $\upsilon << c$ and E is the energy of the particle at $\upsilon \approx c$.

Writing

$$E = E_0 + K,$$

$$T = \left(1+\frac{K}{E_0}\right) \tag{23.31}$$

As K increases with increasing particle velocity, $T > T_0$, the above inequality shows that the period of particle revolution significantly increases with the increasing velocity. The result is that the particle fails to reach the gap at the right moment when the electric field reversal occurs. Because of the failure, it gets decelerated instead of being accelerated.

In case of electrons, for example, the rest mass energy

$$E_0 = m_0c^2 = (9.1 \times 10^{-31}\text{kg})\ (3 \times 10^8 \text{ m/s})^2 = 0.51 \text{ MeV}$$

It implies that even at low energies of the order of less than 1MeV the period of revolution of electrons is doubled and hence they cannot show up in the gap at the required instant. Therefore, they drop out of synchronism and cease to be accelerated further. Hence, electrons cannot be accelerated to high energies in a cyclotron.

Example 23.6. *A cyclotron with its dees of radius 2 m has a magnetic field of 0.75 wb/m². Calculate the maximum energies to which (i) protons and (ii) deuterons can be accelerated.*

Solution.

The maximum energy to which particles are accelerated in a cyclotron is given by

$$E_{max} = \frac{B^2q^2R^2}{2m}$$

(*i*) In case of protons

$$E_{max} = \frac{(0.75 \text{ wb/m}^2)^2(1.602\times10^{-19}\text{C})^2(2\text{m})^2}{2(1.67\times10^{-27}\text{kg})}$$

$$= 1.73\times10^{-11}\frac{wb^2\cdot c^2\cdot m^2}{m^4\cdot kg} = 1.73\times10^{-11}\frac{V^2\cdot s^2\cdot C^2}{m^2\cdot kg}$$

$$= 1.73\times10^{-11}\text{J} = (1.73\times10^{-11})(6.24\times10^{18}\text{eV})$$

$$= 107.9\times10^6\text{eV}$$

$$\therefore \quad \mathbf{E_{max} = 108\ MeV}$$

(*ii*) In case of deuterons

$$E_{max} = \frac{(0.75 \text{wb/m}^2)^2(1.602\times10^{-19}\text{C})^2(2\text{m})^2}{2(3.34\times10^{-27}\text{kg})}$$

$$= 8.64\times10^{-12}\text{J} = (8.64\times10^{-12})(6.24\times10^{18}\text{eV})$$

$$= \mathbf{53.91\ MeV.}$$

Example 23.7. *Protons are accelerated in a cyclotron. The magnetic field strength is 1.3 wb/m² and the radius of the last semicircle is 0.5 m.*

(i) What must be the frequency of the oscillator supplying power to the dees?

(ii) What is the final energy acquired by the proton beam?

(iii) If the total transit time of a proton is 3.3 μs, how much energy is imparted to protons in each passage from one dee to the other?

Solution. (*i*) Frequency

$$\nu = \frac{Bq}{2\pi m} = \frac{1.3 \text{ wb/m}^2\times1.602\times10^{-19}\text{C}}{2\times3.143\times1.67\times10^{-27}\text{kg}} = \mathbf{19.85\ MHz.}$$

(*ii*) Final energy

$$E_{max} = \frac{B^2q^2R^2}{2m} = \frac{(1.3 \text{ wb/m}^2)^2(1.602\times10^{-19}\text{C})^2(0.5\text{m})^2}{2(1.67\times10^{-27}\text{kg})}$$

$$= \mathbf{20.28\ MeV.}$$

(*iii*) Number of revolutions $N = 2\nu T = 2\times19.85\times10^6\text{ Hz}\times3.3\times10^{-6}\text{ s} = 131$

Energy gained by proton during one transit $= \frac{2NeV}{N} = \frac{20.28\ MeV}{131} =$ **155 keV.**

Example 23.8. *The magnetic field strength in a certain cyclotron is 0.9 Wb/m2. If light hydrogen ions (protons) are accelerated in the cyclotron*

(i) What must be the frequency of the oscillator supplying power to the dees?

(ii) If each passage of ions across the accelerating gap increases the energy of the ion by 60,000 eV, how long does it take for the ion introduced at the centre of the dees to emerge at the rim of the dee with energy of 6 MeV?

(iii) Calculate the radius of the last semicircle before the ion emerges from the cyclotron.

Solution. (*i*) Frequency $\nu = \frac{Bq}{2\pi m} = \frac{0.9\,\text{wb/m}^2 \times 1.602\times10^{-19}\,\text{C}}{2\times3.143\times1.67\times10^{-27}\,\text{kg}} = \mathbf{13.72\ MHz.}$

(*ii*) Transit time $T = \frac{N}{2\nu} = \frac{E_{max}/E_1}{2\nu} = \frac{6\times10^6(1.602\times10^{-19})\text{J}}{6\times10^4(1.602\times10^{-19})\text{J}\times2\times13.72\times10^6\,\text{Hz}}$

$= \mathbf{3.64\ \mu s.}$

(*iii*) Maximum Energy $E_{max} = \frac{B^2q^2R^2}{2m} \quad \therefore \quad R = \frac{\sqrt{2mE_{max}}}{bq}$

$$\therefore \quad R = \frac{(2\times1.67\times10^{-27}\,\text{kg}\times6\times10^6\,\text{eV}\times1.602\times10^{-19}\,\text{J/eV})^{1/2}}{(0.9\,\text{wb/m}^2)(1.602\times10^{-19}\,\text{C})} = \mathbf{0.39\ m.}$$

Example 23.9. *Deuterons are accelerated in a fixed frequency cyclotron to a maximum dee orbit radius of 0.88 m. The magnetic field is 1.4 T. Calculate the energy of the emerging deuteron beam and the frequency of the dee voltage. What change in magnetic flux density is necessary if doubly charged helium ions are accelerated?*

Given atomic masses: $H^2 = 2.014102$ amu *and* $He^4 = 4.002603$ amu

Solution. (*i*) Frequency $\nu = \frac{Bq}{2\pi m} = \frac{1.4\text{wb/m}^2\times1.602\times10^{-19}\,\text{C}}{2\times3.143\times2.014102\times1.67\times10^{-27}\,\text{kg}} = \mathbf{10.61\ MHz.}$

(*ii*) Energy $E_{max} = \frac{B^2q^2R^2}{2m}$

$$= \frac{(1.4\text{wb/m}^2)^2(1.602\times10^{-19}\,\text{C})^2(0.88\text{m})^2}{2\times2.014102\times1.67\times10^{-27}\,\text{kg}}$$

$= \mathbf{36.19\ MeV.}$

(*iii*) Magnetic flux density $B_{He} = \frac{2\pi m\nu}{q}$

$$= \frac{2\times3.143\times4.002603\times1.67\times10^{-27}\,\text{kg}\times10.61\times10^6\,\text{Hz}}{2\times1.602\times10^{-19}\,\text{C}}$$

$= \mathbf{1.39\ T}$

Example 23.10. *A particle cyclotron is designed with dees of radius 75 cm and with magnets that can provide a field of 1.5 T.*

(i) To what frequency should the oscillator be set if deuterons are to be accelerated?

(ii) What is the maximum energy of deuterons that can be obtained?

Solution. (*i*) Frequency $\nu = \dfrac{Bq}{2\pi m} = \dfrac{1.5\,\text{wb}/\text{m}^2 \times 1.602\times10^{-19}\,\text{C}}{2\times3.143\times2\times1.67\times10^{-27}\,\text{kg}} = \mathbf{11.45\ MHz.}$

(*ii*) Energy
$$E_{max} = \frac{B^2 q^2 R^2}{2m} = \frac{(1.5\text{wb}/\text{m}^2)^2 (1.602\times10^{-19}\,\text{C})^2 (0.75\text{m})^2}{2\times2\times(1.67\times10^{-27}\,\text{kg})}$$
$$= \mathbf{30.26\ MeV.}$$

23.18 ELECTRON SYNCHROTRON

The electron synchrotron is based on the principle of the combined working of betatron and cyclotron. In the electron synchrotron, the electrons are first accelerated by using the action of the betatron to energy of about 2 MeV. Then they have a velocity of 0.98 c. Subsequently, the electrons travel at practically constant speed, but increase in mass as energy is imparted to them. For an electron traveling with an angular velocity ω in a circular orbit of radius r

$$m\omega^2 r = Be\,\omega\, r$$

or

$$\omega = \frac{Be}{m} \tag{23.40}$$

where B is the magnetic flux density at the orbit. If ω is to remain constant, B must increase in the same ratio as 'm'. In order to keep the electrons in a stable orbit, a small magnet is used inside the dough-nut tube. The magent is less massive as the acceleration of the electrons beyond 2 MeV energy is achieved by radio-frequency (r.f.) electric field. This r.f. electric field is obtained between silver electrodes deposited over a short length along the arc of the tube (see Fig. 23.19). The silver coating has a short gap in it across which the output of an r.f. oscillator is connected. The frequency of the r.f. supply is adjusted to be equal to the time of one revolution of the electron in the circular orbit. Thus, the electrons are accelerated each time they cross the gap and gain additional energy. The r.f. supply is kept on while the magnetic flux is increasing and is automatically cut off when the electrons attain the required energy.

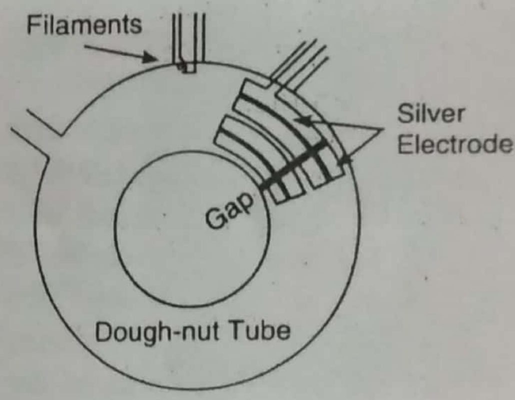


Fig. 23.19

Maximum energy

In an electron synchrotron the maximum energy of electrons depends upon the radius r of the orbit and on maximum magnetic field strength B and is given by

$$E = Berc \tag{23.41}$$

or

$$E = \frac{(3\times10^8\,\text{m/s})(1.602\times10^{-19}\,\text{C})}{1.602\times10^{-13}}\, rB\, MeV = 300\, rB\ \text{MeV} \tag{23.42}$$

Frequency of the r.f. electric field

As the synchrotron acceleration starts when the velocity of electrons are very close to c, the frequency is given by

$$\nu = \frac{c}{2\pi r} = \frac{4.7}{r}\ \text{MHz} \tag{23.43}$$

The r.f. accelerating electric field must have a frequency equal to the above frequency. Electrons can be accelerated upto 1 BeV using the electron-synchrotron.

23.19 PROTON SYNCHROTRON

The principle of proton synchrotron is essentially the same as that of the electron synchrotron. A proton synchrotron consists of a dough-nut shaped vacuum chamber which has the form of a race-track (Fig. 23.20).

It is made of stainless steel, porcelain or plastic supported in the gap of an annular magnet. The annular magnet is made of four quadrants separated by straight gaps. The straight sections are free from magnetic field and are used for injecting, accelerating and ejecting the protons. Thus, the synchrotron consists of four sections joined up by arc shaped segments.

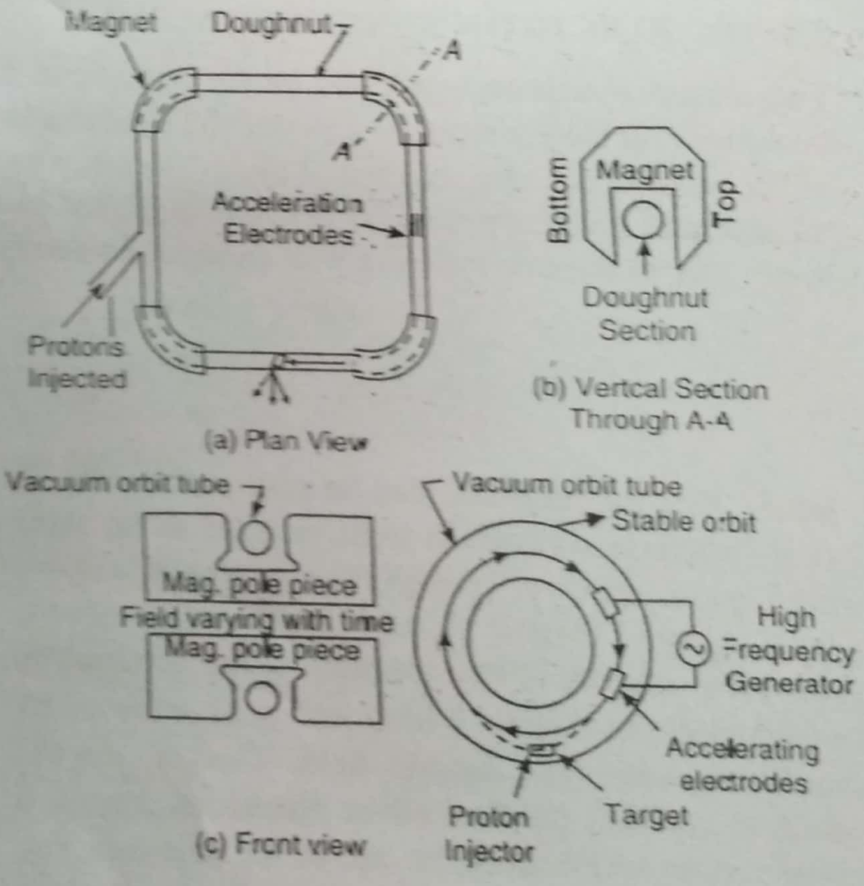


Fig. 23.20

The protons are first accelerated up to 10 MeV in a Van de Graff accelerator and then fed into the synchrotron. Protons at low energy are injected in periodic pulses into the orbit. The protons are made to go in circular orbit by the magnetic field. They are accelerated once in each revolution when they pass between the electrodes connected to an r.f. oscillator. The magnet is excited periodically from 300 gauss to 15,000 gauss and the protons are accelerated during the time the magnetic field is increasing. Simultaneous control over the variation of magnetic field strength and the frequency of the r.f. oscillator is maintained in such a way that the protons travel in an orbit of constant radius and arrive at the electrodes when the applied r.f. voltage is in phase of acceleration

When the protons have attained maximum energy, the frequency is distorted so that the orbit does not remain stable. By a suitable adjustment, the high energy protons are allowed to strike the target.

Medical Radiation Physics

One of the most important applications of nuclear physics has been in medicine, both for diagnostic and therapeutic purposes. The use of X rays for producing images for medical diagnosis is well known, but X rays are of limited value. They show distinct and detailed images of bones, but they are generally less useful in making images of soft tissue. Radioactive isotopes can be introduced into the body in chemical forms that have an affinity for certain organs, such as bone or the thyroid gland. A sensitive detector (called a "gamma-ray camera") can observe the radiations from the isotopes that are concentrated in the organ and can produce an image that shows how the activity is distributed in the patient. These detectors are capable of determining where each gamma-ray photon originates in the patient. Figure 13.20 shows an image of the brain, taken after the patient was injected with the radioactive isotope ^{99}Tc ($t_{1/2} = 6$ h). The images clearly show an area of the brain where the activity has concentrated. Ordinarily the brain does not absorb impurities from the blood, so such concentrations often indicate a tumor or other abnormality.

Another technique that reveals a wealth of information is *positron emission tomography* (PET), in which the patient is injected with a positron-emitting isotope that is readily absorbed by the body. Examples of isotopes used are ^{15}O ($t_{1/2} = 2$ min), ^{13}N ($t_{1/2} = 10$ min), ^{11}C ($t_{1/2} = 20$ min), and ^{18}F ($t_{1/2} = 110$ min). These isotopes are produced with a cyclotron, and because of the short half-lives the cyclotron must be present at the site of the diagnostic facility. When a positron emitter decays, the positron quickly annihilates with an electron and produces two 511-keV gamma rays that travel in opposite directions. By surrounding the patient with a ring of detectors, it is possible to determine exactly where the decay occurred, and from a large number of such events, the physician can produce an image that reconstructs the distribution of the radioisotope in the patient. One advantage of the PET scan over X-ray techniques such as the CAT (computerized axial tomography) scan, is that it can produce a dynamic image—changes in the patient during the measuring time can be observed. Color Plate 13 shows a brain scan of a patient who was injected with glucose labeled with ^{18}F. Active areas of the brain metabolize glucose more rapidly, and so they become more concentrated with ^{18}F. The figure shows which areas of the brain are more active for language or music.

Radiation therapy takes advantage of the effect of radiations in destroying unwanted tissue in the body, such as a cancerous growth or an overactive thyroid gland. The effect of the passage of radiation through matter is often to ionize the atoms. The ionized atoms can then participate in chemical reactions that lead to their incorporation into molecules and subsequent alteration of their biological function, possibly the destruction of a cell or the modification of its genetic material. For example, an overactive thyroid gland is often treated by giving the patient radioactive ^{131}I, which collects in the thyroid. The beta emissions from this isotope damage the thyroid cells and ultimately lead to their destruction. Certain cancers are treated by implanting needles or wires containing radium or other radioactive substances. The decays of these radioisotopes cause localized damage to the cancerous cells.

Other cancers can be treated using beams of particles that cause nuclear reactions within the body at the location of the tumor. Pions and neutrons are used for this purpose. The absorption of a pion or a neutron by a nucleus causes a nuclear reaction, and the subsequent emission of particles or decays by the reaction products again causes local damage that is concentrated at the site of the tumor, inflicting maximum damage to the tumor and minimum damage to the surrounding healthy tissue.