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Complete Solution

CS = CF + PI

Lets consider the equation,

$$\frac{d^n y}{dx^n} + R_1 \frac{d^{n-1} y}{dx^{n-1}} + R_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + R_n y = X$$

$$(D^n + R_1 D^{n-1} + R_2 D^{n-2} + \dots + R_n) y = X$$

Particular Integral ie PI

$$PI = \frac{1}{(D^n + R_1 D^{n-1} + R_2 D^{n-2} + \dots + R_n)} \times X$$

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Rules to find Particular Integral

$\rightarrow \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$

If $f(a) = 0$ then $\frac{1}{f(D)} e^{ax} = x \cdot \frac{1}{f'(a)} e^{ax}$

If $f'(a) = 0$ then $\frac{1}{f(D)} e^{ax} = x^2 \cdot \frac{1}{f''(a)} e^{ax}$

$\rightarrow \frac{1}{f(D)} x^n = [f(D)]^{-1} x^n$ Expand $[f(D)]^{-1}$ & then operate

$\rightarrow \frac{1}{f(D}$

Problems:

1) Find, PI & CF of, $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = e^{2x}$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = e^{2x}$$

$$\frac{d}{dx} = D$$

$$D^2 - 2D + 10y = e^{2x}$$

$$AE = D^2 - 2D + 10 = 0$$

$$D = \frac{2 \pm \sqrt{4-40}}{2}$$

$$D = \frac{2 \pm \sqrt{-36}}{2}$$

$$D = \frac{2}{2} \pm \frac{6i}{2} \qquad D = 1 \pm 3i$$

Now, $\alpha \pm i\beta$, $\alpha = 1$ $i = \pm i$ $\beta = 3$

Its one pair of complex roots

$$CF = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$$

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$CF = e^{x}(C_1\cos 3x + C_2\sin 3x)$

$PI = \frac{1}{f(D)} . F(X)$ $\quad f(X) = X = e^{ax}$

$f(x = a = 2)$

$2^2 - 2(2) + 10 \neq 0$

$\Rightarrow PI = \frac{1}{f(2)} \times e^{2x}$

$PI = \frac{1}{2^2 - 2(2) + 10} \times e^{2x}$

$PI = \frac{e^{2x}}{10}$

$CS = PI + CF$ / $CS = CF + PI$

$CS = e^{x}(C_1\cos 3x + C_2\sin 3x) + \frac{e^{2x}}{10}.$

2] Find CS of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = e^{x}$

$\frac{d}{dx} = D$

$AE = D^2 + D - 2 = 0$

$D = \frac{-1 \pm \sqrt{1+8}}{2}$

$D = \frac{-1 \pm 3}{2}$

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$D = \frac{-1}{2} \pm \frac{3}{2}$

$D = \frac{-1 \pm 3}{2}$

$D = -2, 1$

$CF = C_1 e^{m_1 x} + C_2 e^{m_2 x}$

$CF = C_1 e^{-2x} + C_2 e^{x}$

$PI = \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$ $\quad (x = a = 1)$

$1 + 1 - 2 = 0$

$0 = 0$

$\Rightarrow f'(D) = 2D + 1$

$PI = x \cdot \frac{1}{f'(a)} e^{ax}$ $\quad f'(a) = f'(1) = 2(1) + 1$

$= 3.$

$x \cdot \frac{1}{3} \cdot e^{x}$

$PI = \frac{x}{3} e^{x}$

$CS = CF + PI$

$CS = C_1 e^{-2x} + C_2 e^{x} + \frac{x}{3} e^{x}.$

5] Solve, $\frac{d^2y}{dx^2} - 4y = \sinh(2x+1) + 4^x$

$$\frac{d^2y}{dx^2} - 4y = \sinh(2x+1) + 4^x$$

$$\frac{d}{dx} = D$$

$$D^2 - 4y = \sinh(2x+1) + 4^x$$

$$AE = D^2 - 4 = 0$$

$$D^2 - 4 = 0$$

$$(D+2)(D-2) = 0$$

$$D = -2, 2.$$

$\cancel{CF = e^x(C_1 \cos}$ $\quad CF = C_1 e^{m_1 x} + C_2 e^{m_2 x}$

$$CF = C_1 e^{-2x} + C_2 e^{2x}$$

$$PI = \frac{1}{f(D)} \times f(x)$$

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\qquad 4^x = e^{\log 4^x}$

$= \frac{e^{(2x+1)} - e^{-(2x+1)}}{2}$ $\qquad e^{4\log x}$

$e^{\log 4x}$

e^{ax}

$a = \log 4$

$$PI = \frac{1}{D^2-4}\left[\sinh(2x+1) + 4^x\right]$$

$$PI = \frac{1}{D^2-4}\left[\frac{e^{2x+1} - e^{-(2x+1)}}{2} + e^{\log 4^x}\right]$$

$$PI = \frac{1}{2(D^2-4)}\left[e^{2x} \cdot e - e^{-2x} \cdot e^{-1} + 2e^{(\log 4)x}\right]$$

$$PI = \frac{e}{2} \cdot \frac{1}{D^2-4} e^{2x} - \frac{e^{-1}}{2} \cdot \frac{1}{D^2-4} e^{-2x} + \frac{1}{D^2-4} e^{(\log 4)x}$$

$$PI = x \frac{1}{2D} e^{2x} + x \cdot \frac{1}{2D} e^{-2x} + \frac{1}{D^2-4} e^{(\log 4)x}.$$

$D = 2$ $\quad$ $D = -2$ $\quad$ $D = \log 4$

$$\cancel{PI = x \frac{1}{4} e^{2x} + x \frac{1}{4}}$$

$$PI = \frac{e}{2} \frac{x e^{2x}}{4} + \frac{e^{-1}}{2} \frac{x e^{-2x}}{4} + \frac{e^{(\log 4)x}}{(\log 4)^2 - 4}$$

$$PI = \frac{e}{2}\frac{x}{4}$$

$$PI = \frac{x}{8}e^{2x+1} + \frac{x}{8}e^{-(2x+1)} + \frac{e^{(\log 4)x}}{(\log 4)^2 - 4}$$

$$PI = \frac{x}{8}\left[e^{2x+1} + e^{-(2x+1)}\right] + \frac{e^{x(\log 4)}}{(\log 4)^2 - 4}$$

$$PI = \frac{x}{4}\cos(2x+1) + \frac{4^x}{(\log 4)^2 - 4}$$

$$CS = c_1 e^{2x} + c_2 e^{-2x} + \frac{x}{4}\cos(2x+1) + \frac{4^x}{(\log 4)^2 - 4}$$

4) Solve, $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$

→ $\frac{d}{dx} = D$

$(D^2 - 6D + 9)y = 6e^{3x} + 7e^{-2x} - \log 2$

$AE = D^2 - 6D + 9$

$(D-3)^2 = 0$

$D = 3, 3.$ (Two real & equal roots)

$CF = (c_1 + c_2 x)e^{3x}$

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$$PI = \frac{1}{f(D)} \times F(x)$$

$$PI = \frac{1}{D^2-6D+9}\, 6e^{3x} + 7e^{-2x} - \log 2$$

$$PI = \frac{1}{D^2-6D+9}\, 6e^{3x} + \frac{1}{D^2-6D+9}\, 7e^{-2x} - \frac{1}{D^2-6D+9} \log 2$$

$PI =$ $f'(D) = 2D-6$ $\quad f(a) \neq 0 \quad f(0) \neq 0$

$f''(D) = 2$

$$PI = 6x^2 \frac{1}{2} e^{3x} + 7 \frac{1}{25} e^{-2x} - \log 2 \frac{1}{9} e^{0x}$$

$$PI = 3x^2 e^{3x} + \frac{7}{25} e^{-2x} - \log 2 \frac{1}{9} e^{0x}$$

$$CS = CF + PI$$

$$CS = (C_1 + C_2 x)e^{3x} + 3x^2 e^{3x} + \frac{7}{25} e^{-2x} - \frac{\log 2}{9}$$

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5) Solve, $(D^2+9)y = \sin 2x \cdot \cos x$

$\rightarrow \quad (D^2+9)y = \sin 2x \cdot \cos x$

$AE = D^2+9 = 0$
D^2+9

$f(D) = D^2+9 \qquad F(x) = \sin 2x \cos x$

$PI = \frac{1}{f(D)} \cdot F(x) = \frac{1}{D^2+9} \times \sin 2x \cos x$

$PI = \frac{1}{D^2+9} \cdot \frac{1}{2} [\sin 3x + \sin x]$

$PI = \frac{1}{2}\left[\frac{1}{D^2+9} \sin 3x + \frac{1}{D^2+9} \sin x\right]$

$\frac{1}{f(D^2)} \sin ax = \frac{1}{f(-a^2)} \sin ax$

$\frac{1}{2}\left[\frac{1}{-(3)^2+9} \sin 3x + \frac{1}{-(1)^2+9} \sin x\right]$

if $(-a^2) = 0$ then $\frac{1}{f(D^2)} \sin ax = x \cdot \frac{1}{f'(-a^2)} \sin ax.$

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$f'(D^2) = 2D$

$f'(-a^2) \neq 0.$

$= \frac{1}{2}\left[x \cdot \frac{1}{2D} \sin 2x + \frac{1}{-(1)^2+9} \sin x\right]$

$PI = \frac{1}{2}\left[\frac{x}{2D}\left(\frac{1}{D}\sin 3x\right) + \frac{1}{8}\sin x\right]$

$PI = \frac{1}{2}\left[\frac{x}{2}\left(-\frac{\cos 3x}{3}\right) + \frac{1}{8}\sin x\right]$

$PI = -\frac{x}{12}\cos 3x + \frac{1}{16}\sin x.$

$D^2+9=0$

$D^2=-9$

$D=\pm 3i$

$CF = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x)$

$\alpha=0,\ \beta=3\ \ i=\pm i$

$CF = e^0(C_1\cos 3x + C_2\sin 3x)$

$CF = C_1\cos 3x + C_2\sin 3x.$

$CS = CF + PI$

$CS = C_1\cos 3x + C_2\sin 3x - \frac{x}{12}\cos 3x + \frac{1}{16}\sin x.$

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6] Solve, $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$.

$\rightarrow$ $\frac{d}{dx} = D$

$(D^3 - 3D^2 + 4D - 2)y = e^x + \cos x$

$AE = D^3 - 3D^2 + 4D - 2 = 0$

$D^3 - 3D^2 + 4D - 2 = 0$

$D^2(D-3) + \cancel{2}$; 2 $D = x - 1, D^2 - 2D + 2$

$CF = C_1 e^x + e^x(C_2 \cos x + C_3 \sin 3x)$

$PI = \frac{1}{f(D)} \cdot F(X)$

$PI = \frac{1}{D^3 - 3D^2 + 4D - 2} \quad e^x + \cos x.$

$f(D) = (D-1)(D^2 - 2D + 2) \quad f(X) = e^x + \cos x$

$PI = \frac{1}{(D-1)(D^2 - 2D + 2)} \cdot e^x + \cos x.$

$PI = \frac{1}{(D-1)(D^2 - 2D + 2)} e^x + \frac{1}{(D-1)(D^2 - 2D + 2)} \cos x$

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$$PI = \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$$

$$\frac{1}{f(D^2)} \cos ax = \frac{1}{f(-a^2)} \cos ax$$

$f(a) = 0$

$$PI = \frac{1}{f(D^2)} e^{ax} = x \frac{1}{f'(a)} e^{ax}$$

$$\frac{1}{(D-1)}\left[\frac{1}{D^2-2D+2} \cdot e^{x}\right] + \frac{1}{(D^3-3D^2+4D-2)} \cos x$$

$$\frac{1}{D-1}\left(\frac{1}{1-2+2} e^{x}\right) + \frac{1}{D(-1)-3(-1)+4D-2} \cos x$$

$$\frac{1}{(D-1)} e^{x} + \frac{1}{(3D+1)} \cos x$$

$f(D) = 0$

$$= x \cdot \frac{1}{1} e^{x} + \frac{(3D-1)\cos x}{(3D+1)(3D-1)}$$

$$= x \cdot e^{x} + \frac{(3D-1)}{(9D^2-1)} \cos x \qquad D^2 = -(1)$$

$$= xe^{x} + \frac{(3D-1)}{10} \cos x$$

$$= xe^{x} - \frac{1}{10}[-3\sin x - \cos x]$$

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$PI = xe^x + \frac{1}{10}(3\sin x + \cos x)$

$CS = CF + PI$

$CS = C_1e^x + e^x(C_2\cos x + C_3\sin 3x) + xe^x$
$+ \frac{1}{10}(3\sin x + \cos x).$

7] Solve, $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$

$\rightarrow$

$\frac{d}{dx} = D$, $D^2 + D = x^2 + 2x + 4$

$AE = D^2 + D = 0$
$D(D+1) = 0$
$D = 0, -1.$ (Two real & different roots)
$CF = C_1e^{0x} + C_2e^{-1x}$ / $C_1e^{-x} + C_2$

$PI = \frac{1}{f(D)} \cdot F(X)$

$PI = \frac{1}{D^2+D} \cdot x^2 + 2x + 4$

$\frac{1}{f(D)} x^n = [f(D)]^{-1} x^n$

$\cancel{PI} = (1+D)^{-1} = 1 - D + D^2 - D^3 + \ldots$

$PI = \frac{1}{D}(1+D)^{-1}(x^2+2x+4)$

$PI = \frac{1}{D}(1 - D + D^2 - D^3 \ldots)(x^2+2x+4)$

$PI = \frac{1}{D}[(x^2+2x+4) - (2x+2) + 2]$

$PI = \frac{1}{D}[x^2+4]$

$PI = \int x^2+4 \qquad PI = \cancel{\int}\frac{x^3}{3} + 4x$

$CS = CF + PI$

$CS = c_1 e^{-x} + c_2 + \frac{x^3}{3} + 4x.$

8] Solve, $y'' - 2y' + 2y = x + e^x \cos x.$

$\rightarrow \quad \frac{d}{dx} = D$

$D^2 - 2D + 2y = x + e^x \cos x$

$AE = D^2 - 2D + 2 = 0$

$D = \frac{2 \pm \sqrt{4-8}}{2}$

$D = 1 \pm i\cancel{8}$

$$CF = e^{x}(c_1 \cos x + c_2 \sin x)$$

$$PI = \frac{1}{f(D)} . F(X)$$

$$PI = \frac{1}{D^2 - 2D + 2} . (x + e^{x}\cos x)$$

$$PI = \frac{1}{D^2 - 2D + 2} x + \frac{1}{D^2 - 2D + 2} e^{x}\cos x$$

$$PI = (D^2 - 2D + 2)^{-1} x + \frac{1}{(D^2 - 2D + 2)} e^{x}\cos x$$

$$PI = \frac{1}{2} \frac{1}{[1 - (D - D^2/2)]} x + e^{x} \frac{1}{(D+1)^2 - 2(D+1) + 2} \cos x$$

$$PI = \frac{1}{2} \left[1 - \left(D - \frac{D^2}{2}\right)\right]^{-1} x + e^{x} \frac{1}{D^2 + 1} \cos x$$

$$\cancel{PI = \frac{1}{2}\left[1 - D + \frac{D^2}{2}\right]^{-1} x +}$$

$$PI = \frac{1}{2}\left[1 + \left(D - \frac{D^2}{2}\right) + \ldots\right] x + e^{x} x \frac{1}{2D} \cos x$$

$$PI = \frac{1}{2}\left[x + xD - \frac{D^2 x}{2}\right] + \frac{x e^{x}}{2} \frac{1}{D} \cos x$$

$$PI = \frac{1}{2}[x + 1 + 0 + \ldots] + \frac{x e^{x} \sin x}{2}$$

$$PI = \frac{1}{2}[x + 1] + \frac{x e^{x}}{2} \sin x.$$

$$CS = e^{x}(c_1 \cos x + c_2 \sin 2x) + \frac{1}{2}[x + 1] + \frac{x e^{x}}{2} \sin x.$$

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9] Solve, $\frac{d^2y}{dx^2} - 4y = x\sinh x$.

$\rightarrow \frac{d}{dx} = D$, $(D^2-4)y = x\sinh x$

$AE = D^2 - 4 = 0$

$(D+2)(D-2) = 0$

$D = 2, -2$. (Two real & different roots)

$CF = C_1e^{2x} + C_2e^{-2x}$

$PI = \frac{1}{f(D)} \cdot f(x)$

$PI = \frac{1}{D^2-4} x\sinh x$

$PI = \frac{1}{D^2-4} x\left(\frac{e^x - e^{-x}}{2}\right)$

$PI = \frac{1}{D^2-4} \frac{xe^x}{2} - \frac{1}{D^2-4} \frac{xe^{-x}}{2}$

$PI = \frac{1}{2}\left[\frac{1}{D^2-4} e^x x - \frac{1}{D^2-4} e^{-x}x\right]$

$PI = \frac{1}{2}\left[e^x \frac{1}{(D+1)^2-4} x - e^{-x}\frac{1}{(D-1)^2-4} x\right]$

$PI = \frac{1}{2}\left[e^x \cdot \frac{1}{D^2+2D-3} x - e^{-x}\frac{1}{D^2-2D-3} x\right]$

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$$\frac{1}{f(D)}x^n = [f(D)]^{-1}x^n$$

$$PI = \frac{1}{2}\left[\frac{e^x}{-3}\frac{1}{(D^2/-3+2D/-3+1)}x - \frac{e^{-x}}{-3}\frac{1}{(D^2/-3-2D/-3+1)}x\right]$$

$$PI = \frac{1}{2}\left[\frac{e^x}{-3}\left[1-\left(\frac{D^2}{3}+\frac{2D}{3}\right)\right]^{-1}x + \frac{e^x}{3}\left[1-\left(\frac{D^2}{3}+\frac{2D}{3}\right)\right]^{-1}x\right.$$

$$PI = \frac{1}{2}\left[\frac{e^x}{-3}\left(1-D\cdots\right]x + \frac{e^x}{3}\right.$$

$$PI = \frac{1}{2}\left[\frac{e^x}{-3}\left[1+\left(\frac{D^2}{3}+\frac{2D}{3}\right)\cdots\right]x + \frac{e^{-x}}{3}\left(1+\left(\frac{D^2}{3}-\frac{2D}{3}\right)\cdots x\right]\right.$$

$$PI = -\frac{x}{3}\sinh x - \frac{2}{9}\cosh x$$

$$CS = C_1e^{2x} + C_2e^{-2x} - \frac{x}{3}\sinh x - \frac{2}{9}\cosh x.$$

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Applications of differential equations

→ Simple Harmonic Motion. (SHM).

When the acceleration of a particle is proportional to its displacement from a fixed point and is always directed towards it, then the motion is said to be simple harmonic.

If the displacement of the particle at any instant t, from fixed point O is x, then,

$$\frac{d^2x}{dt^2} = -\mu^2 x \quad \text{or} \quad (D^2 + \mu^2)x = 0$$

$$x = c_1 \cos \mu t + c_2 \sin \mu t$$

Velocity, $v = \frac{dx}{dt} = \mu(-c_1 \sin \mu t + c_2 \cos \mu t)$

If particle starts from rest at A, where $OA = a$

(at $t = 0$, $x = a$) $\quad a = c_1$

(at $t = 0$, $dx/dt = 0$) $\quad 0 = c_2$

$x = a \cos \mu t$

$$\frac{dx}{dt} = -a\mu \sin \mu t = -\sqrt{(a^2 - x^2)}$$

which gives the displacement & the velocity of the

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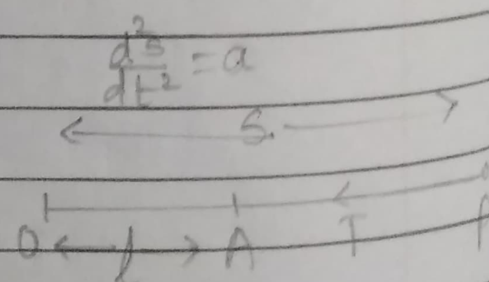
1) In the case of a stretched elastic horizontal string which has one end fixed & a point of mass 'M' attached to the other, find the equation of motion of the particle given that l, is the natural length of the string & 'e' is its elongation due to weight mg. Also find the displacement of particle when initially $s=0$, $v=0$.

→ Let OA be the elastic horizontal string with the end 'O' fixed & having a particle of mass m attached to the end.



At any time 't'
$OP = S$, so elongation
$AP = OP - OA$
$= S - l$
since $T = mg$
For elongation $= (S-l)$ then tension,

$$T = \frac{mg}{e}(S-l) \quad -①$$

Tension being the only horizontal force,

$$\frac{d^2s}{dt^2} = \frac{-T}{M} = -\frac{mg}{e}(S-l) = \left(-S + \frac{lg}{e}\right.$$

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$$\frac{d^2s}{dt^2} = -\frac{gs}{e} + \frac{gl}{e}$$

$$\frac{d^2s}{dt^2} + \left(\frac{g}{e}\right)s = \frac{gl}{e} \quad \text{—(2)}$$ $\Rightarrow$ Required equation of motion.

$P = \frac{g}{e}$ $\quad Q = \frac{gl}{e}$ $\quad \frac{d}{dt} = D$

$$\left(D^2 + \frac{g}{e}\right)s = \frac{gl}{e}$$

$$AE = D^2 + \frac{g}{e} = 0$$

$D^2 = -\frac{g}{e}$ $\quad D = \pm i\sqrt{\frac{g}{e}}$

$\alpha = 0, \; i = \pm i \;\&\; \beta = \sqrt{\frac{g}{e}}$

$$CF = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$$

$$CF = e^0\left(C_1 \cos\sqrt{\frac{g}{e}}\,x + C_2 \sin\sqrt{\frac{g}{e}}\,x\right)$$

$$CF = C_1 \cos\sqrt{\frac{g}{e}}\,x + C_2 \sin\sqrt{\frac{g}{e}}\,x.$$

$$PI = \frac{1}{f(D)} \cdot f(X)$$

$$PI = \frac{1}{D^2 + g/e} \cdot \frac{gl}{e}$$

$$PI = \frac{gl}{e} \cdot \frac{1}{D^2 + g/e} e^{0t}$$

$$PI = \frac{gl}{e} \cdot \frac{1}{g/e}$$

$$PI = l.$$

$$CS = CF + PI$$

$$CS = C_1 \cos\left(\sqrt{\frac{g}{e}}\right)x + C_2 \sin\left(\sqrt{\frac{g}{e}}\right)x + l$$

$$S = C_1 \cos\left(\sqrt{\frac{g}{e}}\right)t + C_2 \sin\left(\sqrt{\frac{g}{e}}\right)t + l \quad \text{--- (3)}$$

at $t=0$, $S = S_0$ & $t=0$, $\frac{ds}{dt} = 0$

$$S_0 = C_1 + l$$
$$C_1 = S_0 - l$$

$$\cancel{S_0 = C_1 + l}$$
$$\frac{ds}{dt} =$$

$$\frac{ds}{dt} = C_1\left(-\sin\sqrt{\frac{g}{e}}\,t\right)\left(\sqrt{\frac{g}{e}}\right) + C_2\left(\cos\sqrt{\frac{g}{e}}.t\right)\left(\sqrt{\frac{g}{e}}\right) \neq 0$$

$$C_2 = 0$$

$$S = (S_0 - l)\cos\left(\sqrt{\frac{g}{e}}\right)t + l$$

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Solution of complimentary functions

If two roots are equal (ie $m_1 = m_2$)

$$y = (C_1 + C_2)e^{m_1 x} + C_3 e^{m_3 x} + \dots + C_n e^{m_n x}$$

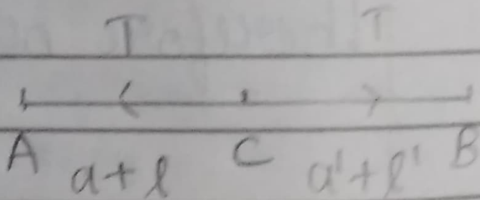
$$y = Ce^{m_1 x} + C_3 e^{m_3 x} + \dots + C_n e^{m_n x}$$

It has only n-1 arbitrary constants & is, therefore not the complete

2) A particle of mass m executes SHM in the line joining the point A & B on a smooth table & is connected with these points by elastic strings whose tension in equillibrium are each T. If l, l' be the extension of the strings beyond their natural lengths, find the time of an oscillations.

→ In the equillibrium position, let the particle be at C so that

$AC = a + l$
$BC = a' + l'$



where a & a' are natural length of the strings, l & l' are extensions of natural lengths of the strings.

Then, the tension are given by,

$T = \frac{\lambda l}{a}$ —(1) & $T' = \frac{\lambda' l'}{a'}$ —(1)
(Hook's law)

$\lambda \rightarrow$ weight
$l \rightarrow$ length
$a \rightarrow$ elongation

$T = \underline{\lambda x}$

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In equillibrium, $T = T'$

at any time t, let the particle be at P,

$AP = l + x \qquad PB = CB - CP = l' - x$

$$T_1 = \frac{\lambda(l+x)}{a} \qquad T_2 = \frac{\lambda'(l'-x)}{a'}$$

Equation of motion, $F = ma$

$$m \cdot \frac{d^2x}{dt^2} = T_2 - T_1 = \frac{\lambda'(l'-x)}{a'} - \frac{\lambda(l+x)}{a}$$

$$= \frac{a\lambda'(l'-x) - a'\lambda(l+x)}{a' \cdot a}$$

$$= \frac{a\lambda' l' - a\lambda' x - a'\lambda l - a'\lambda x}{a' a}$$

$$= \frac{\lambda' l'}{a'} - \frac{\lambda' x}{a'} - \frac{\lambda l}{a} - \frac{\lambda x}{a}$$

$$= \left(\frac{\lambda' l'}{a'} - \frac{\lambda l}{a}\right) - \left(\frac{\lambda'}{a'} + \frac{\lambda}{a}\right) x.$$

$$= (T' - T) - \left(\frac{T'}{l'} + \frac{T}{l}\right) x$$

$$= -\left(\frac{T}{l'} + \frac{T}{l}\right) x$$

$$= -T\left(\frac{1}{l'} + \frac{1}{l}\right) x.$$

$$= -\left(\frac{l+l'}{l l'}\right) T x.$$

$$m\frac{d^2x}{dt^2} = -\left(\frac{l+l'}{ll'}\right) Tx$$

$$\frac{d^2x}{dt^2} = \left[-\left(\frac{l+l'}{ll'}\right)\frac{T}{m}\right] x$$

Let $\mu = \left(\frac{l+l'}{ll'}\right)\frac{T}{m}$

$$\frac{d^2x}{dt^2} = -\mu x.$$

Periodic Time $= \frac{2\pi}{\mu} = 2\pi\left[\frac{M}{T}\cdot\left(\frac{ll'}{l+l'}\right)\right]$

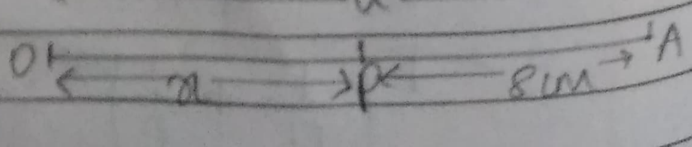
3] A particle moves with SHM of period 12s. travells 8cm from the position of rest 2s. find the amplitude, the Maximum velocity & velocity at the end of 2s.

→ Time = $\cancel{2\pi}\ \frac{2\pi}{\sqrt{\mu}} = 12$ $\quad \sqrt{\mu} = \frac{2\pi}{12} = \frac{\pi}{6}$

AP = $a - x$

OA = a

OP = x = OA − AP



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$AP = 8\,cm$ at $t = 2s$

By defination of SHM having amplitude,

$x = a\cos\sqrt{\mu}\,t$

$a - 8 = a\cos\sqrt{\mu}\,t$

$a - 8 = a\cos\left(\frac{\pi}{6}\right)2$

$a - 8 = a\left(\frac{1}{2}\right)$

$a = 16.$

Max velocity $= \sqrt{\mu}\,a$

$= \frac{\pi}{6} \times 16$ $\qquad x = a-$

$= \frac{8\pi}{3}$

velocity v at the end of 2s.

$v = \sqrt{\mu}\sqrt{a^2 - x^2}$

$v = \frac{\pi}{6}\sqrt{256-64}$

$v = \frac{\pi}{6}\sqrt{192}$. 8 / $\frac{4\pi\sqrt{3}}{3}$ / $\frac{4\pi}{\sqrt{3}}$ cms^{-1}.