→ Maxwell's equations of EM wave

1) gauss law:

for electric field $\nabla \cdot D = \rho$ —①

where $D = \epsilon E$ is the dielectric displacement

ρ is charge density ϵ - Permittivity of medium

E is Electric field

① can be written as $\nabla \cdot E = \frac{\rho}{\epsilon}$ Significance: Charge distribution generates a steady electric current

2) gauss law for magnetic field

$\nabla \cdot B = 0$ —②

where $B = \mu H$ is the magnetic flux density

H is applied magnetic field μ - permeability

② can also be written as $\nabla \cdot H = 0$

Significance: Magnetic monopole does not exist.

3) faraday's law:

$$\nabla \times E = -\frac{\delta B}{\delta t} \quad -③$$

$\because$ $B = \mu H$ ③ can also be written as

$$\nabla \times E = -\mu \frac{\delta H}{\delta t}$$

Significance: An electric field can also be generated by a time varying magnetic field.

4) Ampere's law:

$$\nabla \times H = J + \frac{\delta D}{\delta t} \quad -④$$

$\because$ $D = \epsilon E$ ④ can be written as

$$\nabla \times H = J + \epsilon \frac{\delta E}{\delta t}$$

where J is current density

Significance: Magnetic field is generated by time varying electric field

8) Show that EM wave propagate in vacuum with a velocity $c = 3\times10^8 m/s$

Proof: From maxwell's equation ④

$$\nabla \times H = J + \epsilon \frac{\delta E}{\delta t}$$

$J = \frac{I}{A} = \frac{1}{\text{Resistivity}} \frac{V}{d}$

$\text{Resistivity} = \frac{RA}{l}$

$\frac{I}{A} = \frac{l}{RA} \cdot \frac{V}{d}$

$I = \frac{V}{R} \Rightarrow V = IR$

$\because J = \sigma E$

$$\nabla \times H = \sigma E + \epsilon \frac{\delta E}{\delta t} \quad — ①$$

from Maxwell's equation ③

$$\nabla \times E = -\mu \frac{\delta H}{\delta t}$$

Taking curl on both sides we get

$$\nabla \times (\nabla \times E) = -\mu \frac{\delta}{\delta t} (\nabla \times H) \quad — ②$$

Substituting ① in ② we get

$$\nabla \times (\nabla \times E) = -\mu \frac{\delta}{\delta t} \left(\sigma E + \epsilon \frac{\delta E}{\delta t}\right)$$

$$\nabla \times (\nabla \times E) = -\mu\sigma \frac{\delta E}{\delta t} - \mu\epsilon \frac{\delta^2 E}{\delta t^2} \quad — ③$$

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 L \quad — ④ \text{ - vector identity}$$

from ③ and ④

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\mu\sigma \frac{\delta E}{\delta t} - \mu\epsilon \frac{\delta^2 E}{\delta t^2} \quad — ⑤$$

from Maxwell's equation ①

$$\nabla \cdot E = \frac{\rho}{\epsilon}$$

For free space which is a perfect dielectric, $\rho = 0$ and also not absorb wave so $\sigma = 0$

$$\therefore \quad \mu = \mu_0 \text{ and } \epsilon = \epsilon_0$$

Then the maxwell's equation ① becomes $\nabla \cdot E = 0$

Substituting these values in the above eq. ③ we get

$$-\nabla^2 E = -\mu_0 \epsilon_0 \frac{\delta^2 E}{\delta t^2} \quad \text{— (6a)}$$

IIIly for H we can write

$$+\nabla^2 H = \mu_0 \epsilon_0 \frac{\delta^2 H}{\delta t^2} \quad \text{— (6b)}$$

(6a) and (6b) are similar to general wave equation

$$\nabla^2 \xi = \frac{1}{V^2} \cdot \frac{\delta^2 \xi}{\delta t^2} \quad \text{— (7)}$$

where V is velocity of the wave

Comparing ⑥ and ⑦ we get

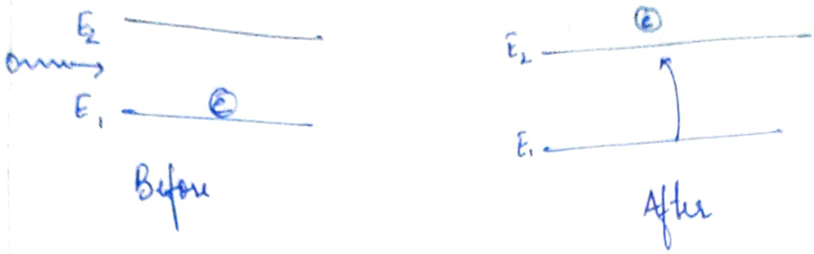
$$\frac{1}{V^2} = \mu_0 \epsilon_0$$

$$V = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \text{— (8)}$$

$\mu_0 = 4\pi \times 10^{-7}$ wb. A·m^2 and $\epsilon_0 = 8.85 \times 10^{-12}$ $C^2/N \cdot m^2$

$$\therefore \quad V = 3 \times 10^8 \, m/s = c$$

Thus EM waves propagate in vacuum with a velocity $C = 3 \times 10^8 \, m/s$



When an atom in its ground state E_1 interacts with an external photon of energy $h\nu = E_2 - E_1$ the atom absorbs the photon and gets excited to a higher state E_2. This process is known as stimulated absorption or induced absorption.

$$\text{Atom} + \text{photon} \longrightarrow \text{Atoms}^*$$

where Atom represents the ground state atom
Atom* represents excited state atom.

Rate of stimulated absorption $\propto N_1 E(\nu)$
$= B_{12} N_1 E(\nu)$

where $N_1 \longrightarrow$ Population of atoms in the power state E_1
$B_{12} \longrightarrow$ Constant called Einstein coefficient of stimulated adsorption.
$E(\nu) \rightarrow$ Energy density of radiation.

→ Energy density of a radiation:

Rate of stimulated absorption $= B_{12} N_1 E(\nu)$ — ①
Rate of spontaneous emission $= A_{21} N_2$ — ②
Rate of stimulated emission $= B_{21} N_2 E(\nu)$ — ③

At thermal equilibrium

Rate of absorption = Rate of emission

$$B_{12} N_1 E(\nu) = A_{21} N_2 + B_{21} N_2 E(\nu)$$

$$B_{12} N_1 E(\nu) - B_{21} N_2 E(\nu) = A_{21} N_2$$

$$E(\nu)\{N_1 B_{12} - N_2 B_{21}) = A_{21} N_2$$

$$E(\nu) = \frac{A_{21} N_2}{N_1 B_{12} - N_2 B_{21}} = \frac{A_{21} N_2}{B_{21} N_2 \left(\frac{B_{12} N_1}{B_{21} N_2} - 1\right)} \quad -④$$

By Planks law the energy density of radiation is also given by $E(\nu) = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{(e^{\frac{h\nu}{KT}} - 1)}$ — ⑤

By Boltzman's equation: $\frac{N_2}{N_1} = e^{\frac{-h\nu}{KT}}$ — ⑥

where N_1 – Population density of atoms in higher state

K = Boltzman constant

T = Absolute temperature

Eq. ⑥ can also be written as $\frac{N_1}{N_2} = e^{\frac{h\nu}{KT}}$ — ⑥b

Substituting 6b in ④ we get

$$E(\nu) = \frac{A_{21} N_2}{B_{21} N_2 \left(\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{KT}} - 1\right)} \quad - ⑦.$$

comparing ⑤ and ⑦ coeff of $e^{\frac{h\nu}{KT}} = \frac{B_{12}}{B_{21}} = 1$

$\therefore B_{12} = B_{21} = B.$ — 8a

and $\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3}$ — 8b

Equation 8a indicates relation between Einsteins coefficient 8b can also be written by dropping subscripts as $\frac{A}{B} = \frac{8\pi h\nu^3}{c^3}$ — 8c

∴ Equation (8) can be written as

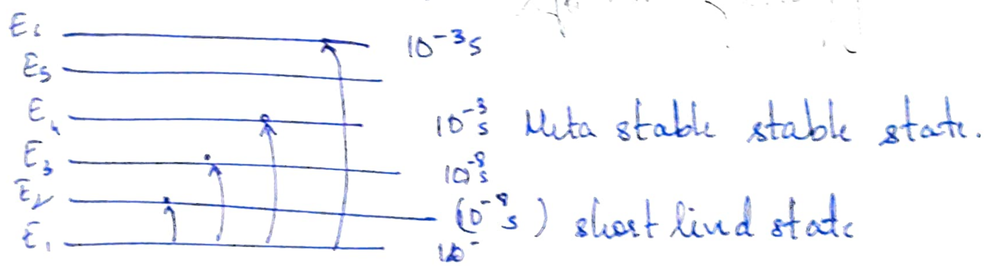
$$E(\nu) = \frac{A/B}{(e^{\frac{h\nu}{kT}} - 1)} \quad \text{—} (9).$$

which gives the expression for energy density of radiation in terms of Einsteins coefficients at thermal equilibrium

→ **Conditions for laser action:**

1) Higher state must be metastable state

E_2, E_1 10^{-8} s short lived state



Usually an excited state is characterised by the life time of atoms of the order of 10^{-8} s. Such energy states are called short lived states. However certain energy states are characterised by comparatively longer life times of atoms of order of 10^{-3} s. Such energy states are called meta stable states.

In order to start the laser action in any system some of the excited states must be meta stable states.