The displacement of particle in the medium is therefore a function of space co-ordinates as well as a function of time

22-5-21

WAVES - wave equation

A simple harmonic plane travelling wave can be represented by

wave function $y = A\sin(Kx - \omega t) \rightarrow (1)$ + x-axis

where $\omega = 2\pi\nu$, the angular frequency

$K = \frac{2\pi}{\lambda}$, The propagation constant / wave no.

A - Amplitude

y - displacement.

Differentiating eq (1) twice partially w.r.t position x

$$\frac{\partial y}{\partial x} = A\cos(Kx - \omega t) K$$

$$\frac{\partial y}{\partial x} = KA\cos(Kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -KA\sin(Kx - \omega t).K$$

$$= -K^2 A\sin(Kx - \omega t) \rightarrow (2)$$

Using eq (1) again eq (2) can be written as

$$\frac{\partial^2 y}{\partial x^2} = -K^2 y. \rightarrow (3)$$

$||^{ly}$ differentiating eq ① partially w.r.t t (time) twice

$$\frac{\partial y}{\partial t} = A\cos(Kx-\omega t)(-\omega)$$

$$\frac{\partial y}{\partial t} = -\omega A\cos(Kx-\omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega A[-\sin(Kx-\omega t)](-\omega)$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(Kx-\omega t) \rightarrow ④$$

Now using eq^n ① again eq ④ can be written as

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 y \rightarrow ⑤$$

Dividing eq ③ by ⑤

$$\frac{\frac{\partial^2 y}{\partial x^2}}{\frac{\partial^2 y}{\partial t^2}} = \frac{-K^2 y}{-\omega^2 y} = \left(\frac{K}{\omega}\right)^2 = \left(\frac{\cancel{2\pi}}{\lambda}\frac{1}{\cancel{2\pi}\nu}\right)^2$$

$$= \left(\frac{1}{v_{wave}}\right)^2$$

$$\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} + \frac{\partial^2 y}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\boxed{\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}} \rightarrow ⑥$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z}\right) y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

wave eq^n is 3D

Eq (6) has no negative sign which indicates that 2nd order derivative is independent of direction

Eq (6) is 1 dim wave equation

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v_{wave}^2}\frac{\partial^2 y}{\partial t^2}} \rightarrow (6)$$

which is differential form of wave equation of travelling wave

SUPERPOSITION OF WAVES

Imp

Principle: when two or more no. of waves are super imposing on one another the displacement of resultant wave will be algebraic sum of displacements of individual waves at that instant

i.e $y = y_1 + y_2 + y_3 + \cdots$

where y_1, y_2, y_3 are displacement of individual waves.

Emerging case due to superposition of waves.

Case 1: If there are 2 waves of same frequency & same amplitude ~~superimpose~~ propagating in the same direction superimpose on each other, the resultant wave pattern give rise to the phenomenon of Interference.

$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$ which is laplacian operator

Case 2 : When two waves with different frequency moving in same direction super impose on each other the resultant wave pattern give rise to the phenomenon of $\boxed{\text{beats}}$.

Case 3 : when there are two waves with same frequency & Amplitude propagating in opposite direction. super impose on each other the resultant wave pattern give rise to the phenomenon of Stationary waves or $\boxed{\text{Standing waves}}$

Standing Waves

Two plane propagating waves which are moving in opposite direction and the can be represented by

$$y_1 = A \sin(Kx - \omega t) \rightarrow (1)$$
$$y_2 = A \sin(Kx + \omega t) \rightarrow (2)$$

By the principle of superposition, resultant displacement $y = y_1 + y_2$

$$y = A[\sin(Kx - \omega t) + \sin(Kx + \omega t)] \rightarrow (3)$$

By trigonometric identity

$$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \rightarrow (4)$$

Eq (3) can be written as.

$$y = 2A\sin\left(\frac{Kx - \omega t + Kx + \omega t}{2}\right)\cos\left(\frac{\cancel{Kx} - \omega t - (\cancel{Kx} + \omega t)}{2}\right)$$

$$y = 2A\sin(Kx)\cos(-\omega t)$$

$$y = 2A\sin(Kx)\cos(\omega t) \rightarrow (5)$$

$$y = A'\cos\omega t \rightarrow (6)$$

where $A' = 2A\sin(Kx) \rightarrow (7)$

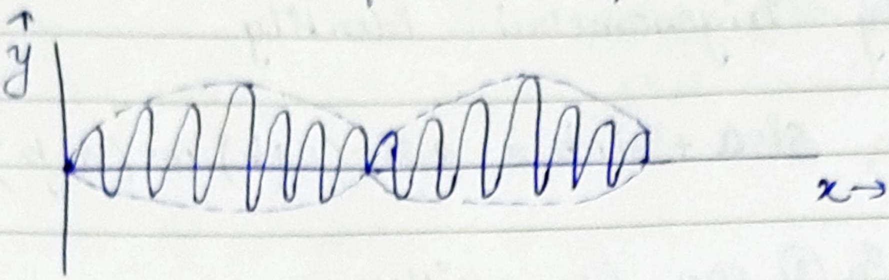
Note:

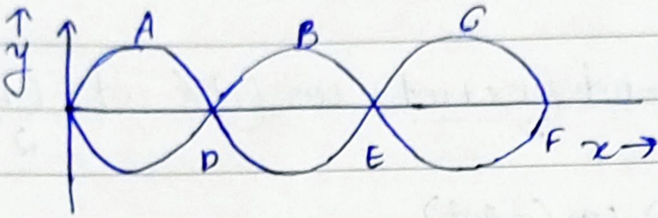
$\boxed{K=0 \text{ wave is not propagating}}$

Eq (7) indicates that amplitude of resultant wave is not constant but varying w.r.t position x according to sine function.

Eq (6) indicates that, the resultant of superposition of waves (1) & (2) is not travelling wave but stationary wave.

Characteristic of Standing Waves





ABC → Antinode

DEF → node

$A' = 2A \sin Kx$

<u>At node:</u> $A' = 0$ $\quad 2A \sin Kx = 0$

$\sin Kx = 0$

$Kx = 0, \pi, 2\pi, \dots \quad = n\pi$

$n = 0, 1, 2, 3$

<u>At Antinodes:</u> $A' \rightarrow max$

$2A \sin Kx = max$

$\sin Kx = max$

$\sin Kx = \pm 1$

$Kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$$kx = (2n+1)\frac{\pi}{2}$$

$$n = 0, 1, 2, 3 \ldots$$

Difference between

	Travelling wave	Standing wave
1.	A travelling wave propagates in a medium continuously with finite velocity	A standing wave is stationary and does not move in the medium
2.	A travelling wave transports energy from one location to the other. Hence there is energy flow across every plane in the across every plane in direction of wave propagation	There is no energy trans-fer in a standing wave There is no flow of energy across any plane The energy of oscillation periodically transforms from kinetic energy to potential energy of elastically deformed medium and vice-versa
3.	No particle on the wave is permanently at rest	Nodes are permanently at rest in standing wave

4	In a travelling wave all points oscillate with the same amplitude regardless of their location.	All the points of standing waves between 2 adjacent nodes oscillate with different amplitude
5	In a travelling wave different points oscillate with different phases	In standing wave all the points between two adjacent nodes oscillate oscillate in same phase
6	In travelling wave, all the particles do not pass through their mean position or reach the extreme positions simultaneously	In standing wave, all the particles pass through their mean position & reach their extreme position simultaneously twice in each cycle.