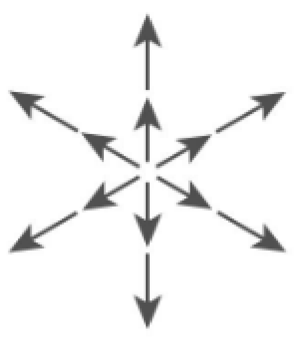
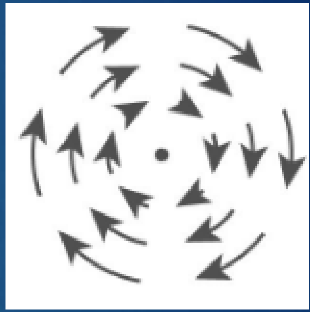
Electromagnetism and photonics

Divergence of a vector explains how the vector spreads out from a point under consideration.



$$\nabla \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Curl of a vector explains how the vector swirls around a point under consideration.



$$\nabla \times \vec{V} = \begin{pmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \boldsymbol{\partial/\partial x} & \boldsymbol{\partial/\partial y} & \boldsymbol{\partial/\partial z} \\ \boldsymbol{V_x} & \boldsymbol{V_y} & \boldsymbol{V_z} \end{pmatrix}$$

Maxwell's equations-

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0$$

It (Gauss law) implies that the divergence of electric field through a closed Gaussian surface is proportional to the charge density enclosed within that surface

$$\nabla \cdot \vec{B} = 0$$

The divergence of magnetic field through a closed surface is always zero. It also implies that magnetic monopoles cannot exist.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

(Faraday's law) It implies that the changing magnetic field (with respect to time) through a closed loop generated electric field along the loop.

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

(Ampere's law) It implies that a constant current density or time varying electric field through a closed loop generate magnetic field around the loop.

For a charge free and current free region, Maxwell's equations are

$$\nabla \cdot \vec{E} = 0 \;--(1) \qquad \nabla \cdot \vec{B} = 0 \;---(2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} ---(3) \qquad \nabla \times \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} --(4)$$

Curl of eq. 3 gives, $\nabla \times \left(\nabla \times \vec{E}\right) = -\nabla \times \frac{\partial \vec{B}}{\partial t} \quad \Longrightarrow \quad \nabla\left(\nabla \cdot \vec{E}\right) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$

Using eq. (1) and (3), above equation reduces to

$$\nabla^2\vec{E} = \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2} - -(5)$$

Curl of eq. 4 gives, $\nabla \times \left(\nabla \times \vec{B}\right) = \varepsilon_0\mu_0\nabla \times \frac{\partial\vec{E}}{\partial t} \quad \Longrightarrow \quad \nabla\left(\nabla \cdot \vec{B}\right) - \nabla^2\vec{B} = \varepsilon_0\mu_0\frac{\partial}{\partial t}(\nabla \times \vec{E})$

Using eq. (2) and (4), above equation reduces to

$$\nabla^2\vec{B} = \varepsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2} - - -(6)$$

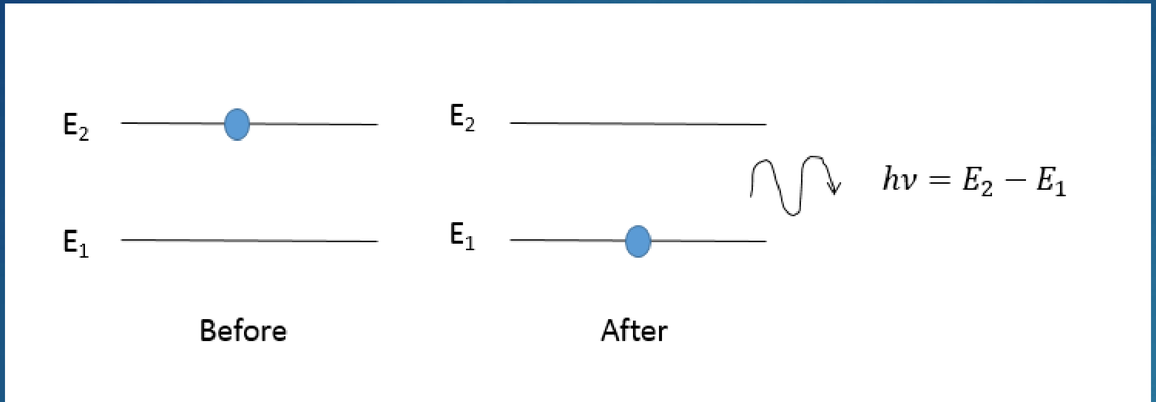
Comparing eq. (5) and (6) with wave equation $\nabla^2\varphi = \frac{1}{v^2}\frac{\partial^2\varphi}{\partial t^2}$

The speed of light (electromagnetic wave) is

$$v = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = \frac{1}{\sqrt{8.85\times10^{-12}\times1.26\times10^{-6}}} = 2.99 \times 10^8 m/s$$

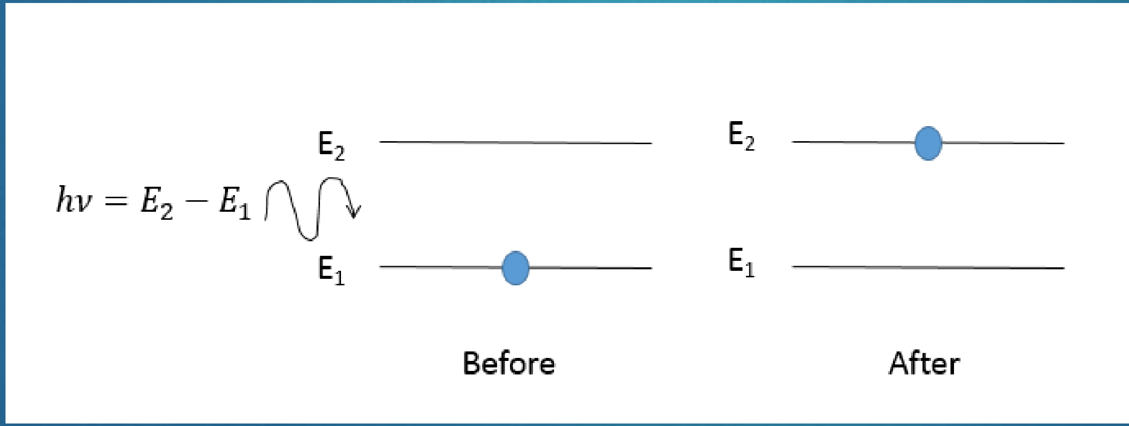
Interaction between matter and radiation –

1. Spontaneous emission- In spontaneous emission, the atom is initially in the excited level of energy E_2. It makes a transition to ground level of energy E_1 by emitting a photon of energy $h\nu = E_2 - E_1$



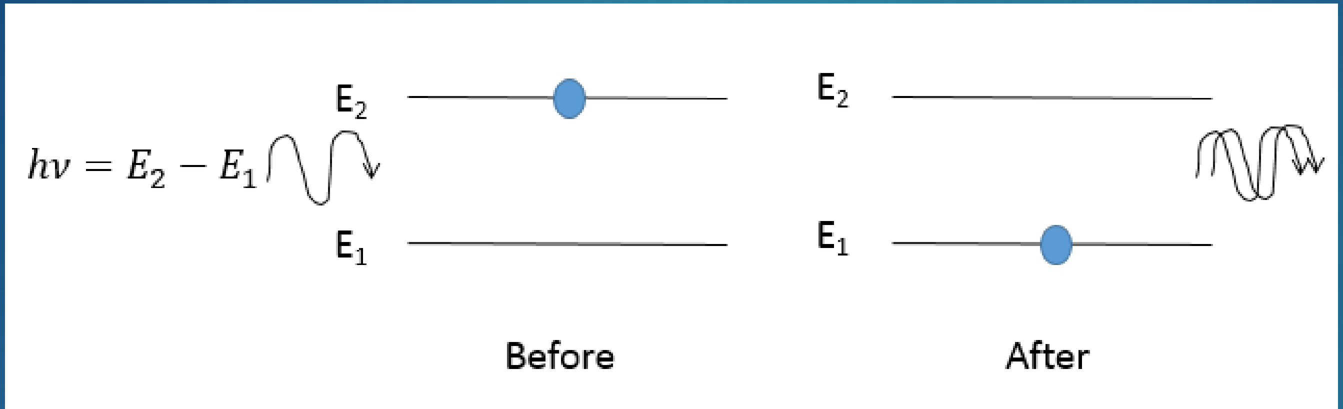
$$Atom^* \rightarrow Atom + photon$$

2. Stimulated absorption- In stimulated absorption, the atom is initially in the ground level of energy E_1. It makes a transition to excited level of energy E_2 by absorbing a photon of energy $h\nu = E_2 - E_1$



$$Atom + photon \rightarrow Atom^*$$

3. Stimulated emission- In stimulated absorption, the atom is initially in the excited level of energy E_2. It interacts with an incoming photon of energy $h\nu = E_2 - E_1$ and makes a transition to ground level of energy E_1 by emitting a photon of energy $h\nu = E_2 - E_1$.



$$Atom^* + photon \rightarrow Atom + 2\ photons$$

The emerging 2 photons travel in the same direction and the associated electromagnetic waves are in phase.

Imagine a collection of atoms where all the atoms are in the excited state. The interaction of an atom with An incoming photon of appropriate energy will result in emission of 2 photons. These two photons will lead to emission of 4 photons. Thus, the number of photons will go on increasing as 2, 4 , 8, 16, 32… This will result in amplification.

Einstein coefficients – An atom can undergo stimulated absorption, spontaneous emission or stimulated emission process during its interaction with a photon.

Einstein coefficients A_{21}, B_{21}, B_{12} have the following physical significance.

A_{21}-probability per atom per unit time that an atom undergoes spontaneous emission from level 2 to level 1.

$B_{12}\rho(\nu)$ - probability per atom per unit time that an atom undergoes stimulated absorption from level 1 to level 2.

$B_{21}\rho(\nu)$- probability per atom per unit time that an atom undergoes stimulated emission from level 2 to level 1.

$\rho(\nu)$ is the spectral density of radiation applied to the atoms.

Let n_1 and n_2 be the number of atoms in level 1 and 2.

If the atoms are in thermal equilibrium with radiation, then rate of emission and rate of absorption of photons must be equal.

Rate of emission $= n_2(A_{21} + B_{21}\rho(\nu))$

Rate of absorption = $n_1 B_{12}\rho(\nu)$

Thus, at equilibrium, $n_2\left(A_{21}+B_{21}\rho(\nu)\right)=n_1B_{12}\rho(\nu)$

Dividing above equation by n_2 and B_{21},

$$\frac{A_{21}}{B_{21}}+\rho(\nu)=\frac{n_1}{n_2}\frac{B_{12}}{B_{21}}\rho(\nu)$$

Thus, $\rho(\nu)\left(\frac{n_1}{n_2}\frac{B_{12}}{B_{21}}-1\right)=\frac{A_{21}}{B_{21}}$.

$$\rho(\nu)=\frac{\left(\frac{A_{21}}{B_{21}}\right)}{\left(\frac{n_1}{n_2}\frac{B_{12}}{B_{21}}-1\right)}---(1)$$

From Boltzmann law, the ratio of number of atoms in level 1 and 2 is

$$\frac{n_1}{n_2}=e^{(E_2-E_1)/kT}=e^{h\nu/kT}--(2)$$

Thus, eq. (1) and (2) gives

$$\rho(\nu)=\frac{\left(\frac{A_{21}}{B_{21}}\right)}{\left(\frac{B_{12}}{B_{21}}e^{h\nu/kT}-1\right)}--(3)$$

Comparing eq. (3) with the Plank's law of radiation, $\rho(\nu)=\left(\frac{8\pi h\nu^3}{c^3}\right)\frac{1}{\left(e^{h\nu/kT}-1\right)}$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}, \qquad B_{12} = B_{21}$$

These are known as Einstein's relations. The coefficients for stimulated absorption and emission are equal. The ratio of coefficient for spontaneous emission and stimulated emission is proportional to ν^3. Thus, for higher energy difference between level 1 and 2, spontaneous emission is more likely to happen than stimulated emission process.

The expression for the spectral density

$$\rho(\nu) = \frac{\left(\frac{A_{21}}{B_{21}}\right)}{(e^{h\nu/kT} - 1)}$$

$$e^{\frac{h\nu}{kT}} - 1 = \frac{A_{21}}{B_{21}\rho(\nu)}$$

If $h\nu \gg kT$, spontaneous emission process is a dominant process.

If $h\nu \cong kT$, stimulated emission process becomes significant.

If $h\nu \ll kT$, stimulated emission process is dominant.

The ratio of emission rate and absorption rate is

$$\frac{\text{Emission rate}}{\text{absorption rate}} = \frac{\mathrm{n_2}\left(\mathrm{A_{21}} + \mathrm{B_{21}}\rho(\nu)\right)}{n_1 B_{12}\rho(\nu)} = \frac{n_2}{n_1}\left(1 + \frac{A_{21}}{B_{12}\rho(\nu)}\right)$$

For stimulated emission, as $\frac{A_{21}}{B_{12}\rho(\nu)} \ll 1$. $\frac{\text{Emission rate}}{\text{absorption rate}} \cong \frac{n_2}{n_1}$

Thus, the rate of emission will be more for stimulated emission process when the ratio, $\frac{\text{Emission rate}}{\text{absorption rate}}$>1 , i.e.,

$$\frac{n_2}{n_1} > 1.$$

This is known as population inversion where more number of atoms are found is level 2 than in level 1. Population inversion is the necessary condition for working of a laser.

Metastable state- The usual lifetime of an electron in excited state is of order of few nano-seconds. But some excited states have a lifetime of milli-seconds. These are known as metastable states and used to achive population inversion in lasers.

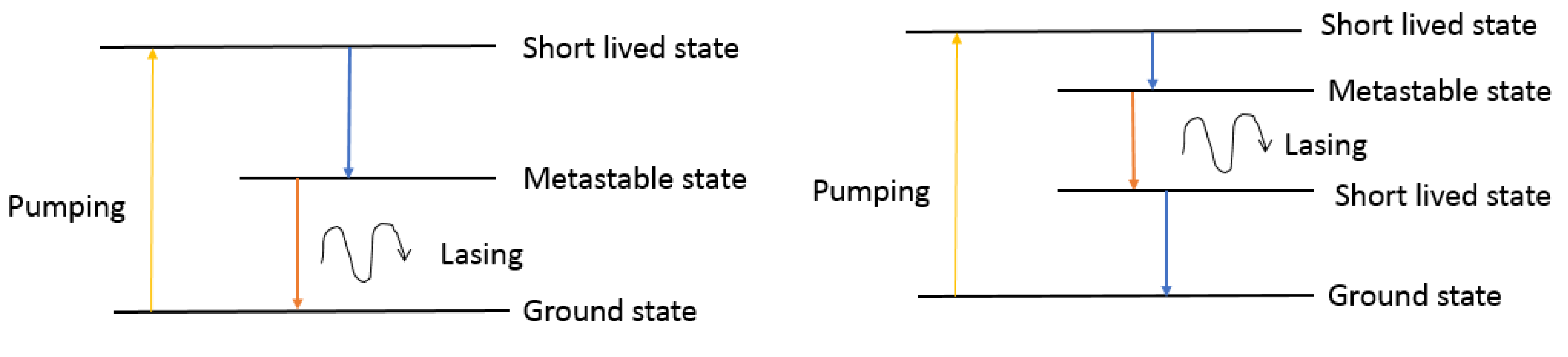
The stimulated emission from a metastable state gives rise to highly monochromatic emission of light. From the uncertainty principle,

$$\Delta E \Delta t \geq \frac{h}{2\pi}$$

Thus, as $\Delta t = 10^{-3}$s, $\Delta E \approx 10^{-31}$ J.But $\Delta E = h\Delta\nu$, which leads to $\Delta\nu = 10^{3}\ Hz$

This is a narrow width in frequency, which implies that the emitted light in highly monochromatic.

A laser system is a 3 or 4 level system.



3 level system - Ruby laser

4 level system – He-Ne laser

Laser structure – Any laser consists of three main components.

1. Gain medium – The gain medium consists of atoms or molecules that undergo stimulated emission. The population inversion is achieved in gain medium. The gain medium can be in solid, liquid or gaseous state.

2. Pumping mechanism- The pumping mechanism of a laser achieves and maintains population inversion in the gain medium. The pumping mechanism supplies energy in the form of optical or electrical energy to achieve population inversion.

3. Optical cavity- The purpose of optical cavity is to produce intense collimated beam. To produce intense output beam, a long gain medium will be required through which photons can move. Such long medium will result in cumbersome laser system. To avoid it, a gain medium is placed within optical cavity. This cavity consists of 2 mirrors, one of which has 100 % reflectivity and another mirror has partly reflecting property.

Problem – He-Ne laser has average out power of 4.5mW at 632.8 nm. Find the photons emitted per second by the laser.

Energy of a single photon is $E = \frac{hc}{\lambda}$. Let N be the number of photons emitted per second by the laser. Then the power of the laser is

$$P = NE = \frac{Nhc}{\lambda}$$

Thus, $N = \frac{P\lambda}{hc} = \frac{4.5\times10^{-3}\times632.8\times10^{-9}}{6.6\times10^{-34}\times3\times10^{8}} = 1.43 \times 10^{16}$

Problem- A pulse laser operating at 780 nm delivers 20mW of average power. If each pulse lasts for 10ns, calculate number of photons in each pulse.

Let N be the number of photons in each pulse, t be the duration of pulse and E is the energy of each photon.

Energy of each pulse is Pt=NE. Thus,

$$N = \frac{Pt}{E} = \frac{Pt}{\frac{hc}{\lambda}} = \frac{Pt\lambda}{hc} = \frac{20 \times 10^{-3} \times 10^{-8} \times 780 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = 7.8 \times 10^{8}$$

Problem- A laser medium at thermal equilibrium temperature of 300 K has 2 energy levels with wavelength separation of 1μm. Find the ration of population densities of upper and lower levels.

The ratio of levels is given as

$$\frac{n_1}{n_2} = e^{\frac{h\nu}{kT}}$$

Hence, $\frac{n_2}{n_1} = e^{-\frac{h\nu}{kT}} = e^{-\frac{hc}{\lambda kT}} = \exp\left(\frac{-6.6\times10^{-34}\times3\times10^{8}}{10^{-6}\times1.38\times10^{-23}\times300}\right) = e^{-48.0} = 1.36 \times 10^{-21}$

Problem- If the ratio of population of 2 energy levels is 1.05×10^{-30}, find the wavelength of light emitted at 300 K.

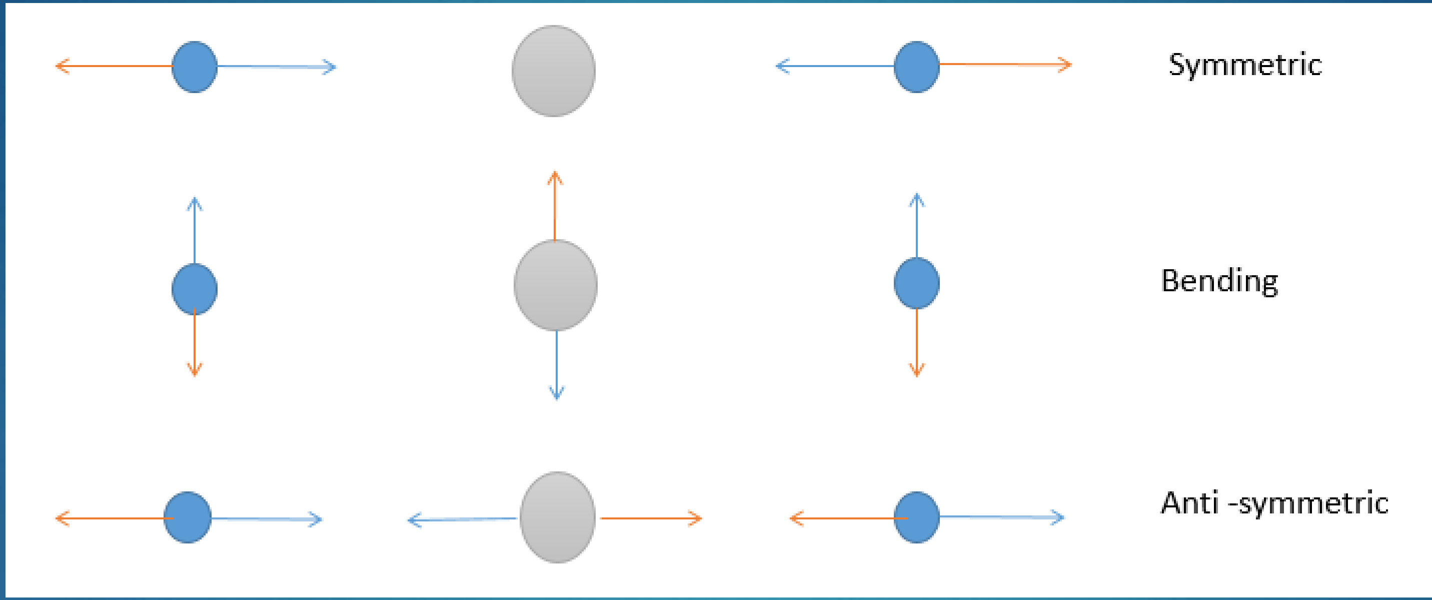
$$\frac{n_2}{n_1} = e^{-\frac{hc}{\lambda kT}}$$

Hence,

$$-\frac{hc}{\lambda kT} = \ln\left(\frac{n_2}{n_1}\right)$$

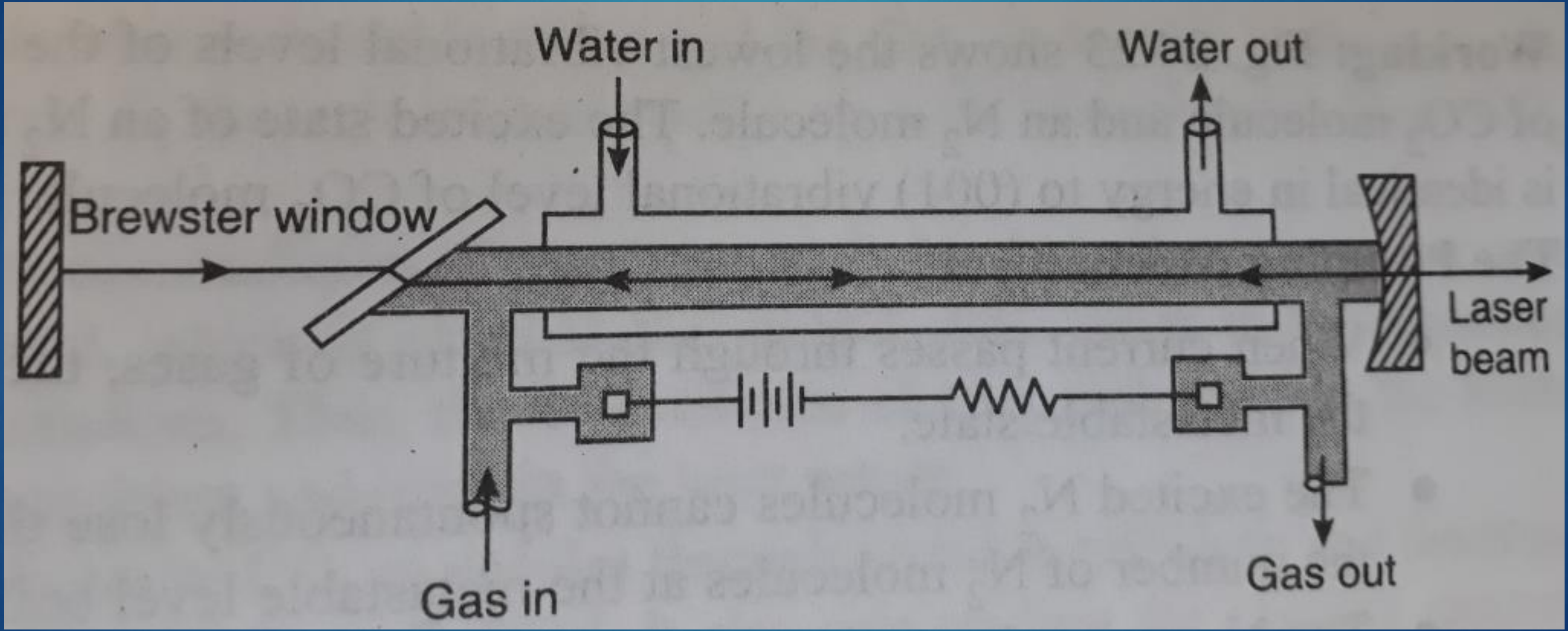
$$\lambda = -\frac{hc}{kT\ln\left(\frac{n_2}{n_1}\right)} = -\frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{1.38 \times 10^{-23} \times 300 \times \ln(1.059 \times 10^{-30})} = 696\ nm$$

CO_2 laser – Carbon dioxide molecule vibrates in three different ways that are known as vibrational modes. The vibrational energies of the molecules are quantized.

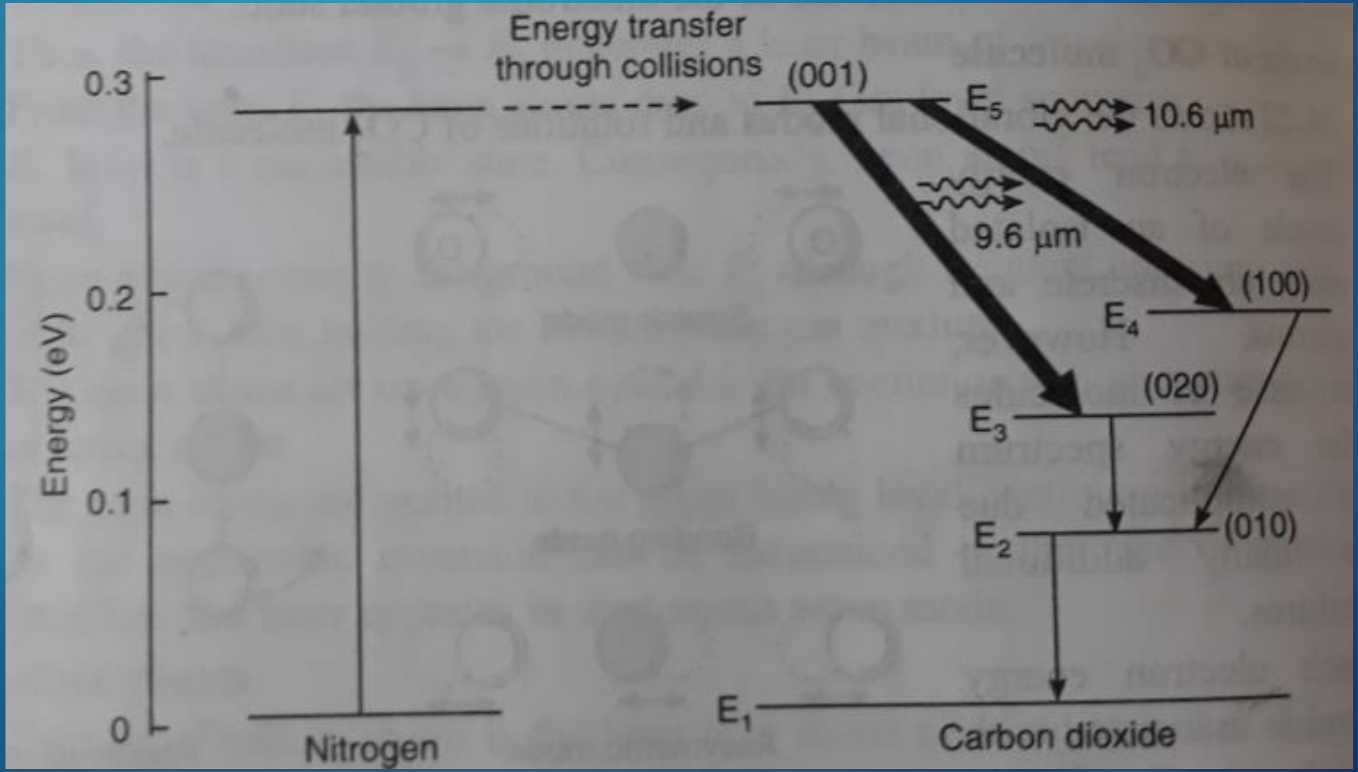


v_1, v_2, v_3 are the symmetric, bending and anti-symmetric vibrational mode quantum numbers that can be equal to 0,1,2,…

Structure - CO_2 laser consists of discharge tube of cross section 1.5 mm^2 and 26 cm long. The tube is fixed with partly reflecting mirror at one end to get output beam and 100% reflecting mirror at the other end. The tube is filled with a mixture of CO_2, N_2 , He in 1:4:5 proportion and the discharge tube is fitted with water cooling system.



Working- The discharge of electrons through the mixture of gases excites v=1 vibrational state of N_2 molecule. The CO_2 molecule are excited to (001) state by collision with N_2 molecules. CO_2 molecule makes a transition from (001) state to (100) state by emitting radiation of 10.6 μm. The decay rate of (100) is 20 times more than (001) state which helps to achieve population inversion. (001) state also decays to (020) state by emitting radiation of 9.6μm. (100) and (020) make a transition to (010) state. The (010) state decays to ground state. The thermal excitation populates (010) state. This affects population inversion. Hence, the laser is cooled with water. The laser operates in continuous mode or pulsed mode and has an output of 1 to 10^4 W.



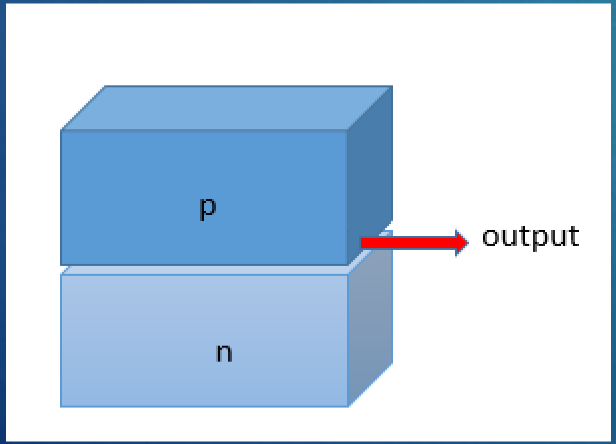
Semiconductor laser - A semiconductor laser is build using a direct bang gap semiconductor. Direct recombination of electron and hole is possible in direct band gap semiconductor without dissipation of energy. Example: GaAs (Gallium arsenide)

A semiconductor laser can be of two types –

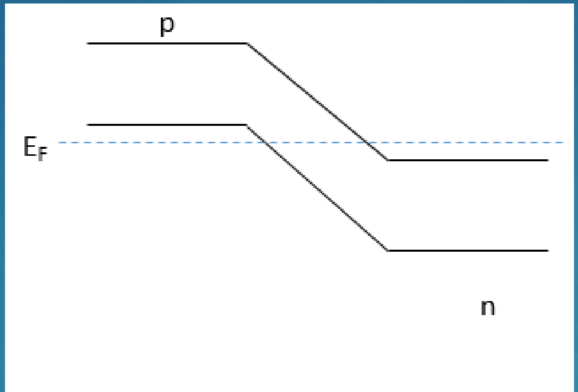
1. Homo-junction laser

2. Hetero-junction laser.

A homo-junction laser is 0.5 mm long and 0.1 mm thick and wide. Metal contact at the top p and bottom n provide a facility for conduction of current. Optically flat surfaces are used to get output from the laser.

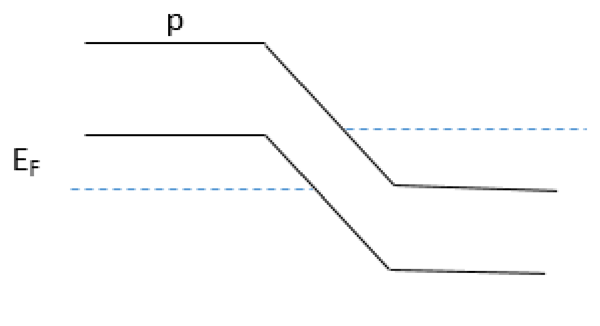


p and n regions are heavily doped. This leads to penetration of donor levels in conduction band and acceptor levels in the valence band.



Without bias

At threshold current, high concentration of electrons and holes results in population inversion in active region. Initially spontaneous emission occurs due to recombination of holes and electrons. The photons emitted lead to stimulated emission process in the active region.



With bias

GaAs semiconductor laser operates at 900 nm. The length of optical cavity determined the output wavelength.

Disadvantages –

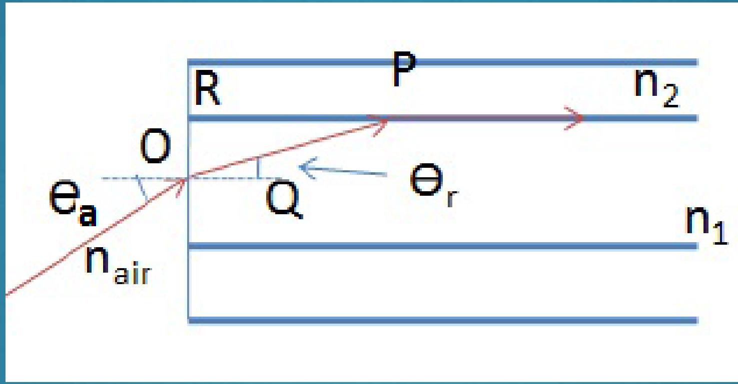
1. Active region is not well defined.
2. Cavity losses are more
3. It requires high current for operation and thus cannot be operated continuously at room temperature.

LIDAR - Light detection and ranging implies the working principle of RADAR. In LIDAR, concentration of pollutant particles as a function of distance is measured. However, information about nature of particles cannot be obtained. A photodetector detects light from the pollutant like CO. SO_2. The distance and concentration of pollutants can be estimated from time taken for detecting a signal and its intensity respectively.

The type of pollutants ca be detected by study of absorption spectrum or Raman spectrum.

Optical fibre- Optical fibre is made up of a core and cladding where refractive index of core is larger than refractive index of cladding. The cladding is covered by plastic jacket that protects core and cladding. In optical fibre, light propagates inside the core because of multiple total internal reflection.

Total internal reflection occurs when light passes from a denser medium to a rarer medium and has angle greater than the critical angle.



At air-core boundary , $n_{air} \sin\theta_a = n_1 \sin\theta_r$.

$$\sin\theta_a = n_1 \sin\theta_r ----(1)$$

For core-cladding boundary, $n_1 \sin(90-\theta_r) = n_2 \sin 90 = n_2$

$$\cos\theta_r = n_2/n_1 -----(2)$$

The light gathering capacity of optical fibre is called as numerical aperture and is given as

$$NA = \sin\theta_a$$

From eq. (1),

$$NA = n_1\sqrt{1-\cos^2\theta_r}$$

And eq. (2) ,

$$NA = n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

The light coupled within the angle of acceptance undergoes multiple total internal reflections and reaches the other end of fibre. The light coupled at an angle greater then angle of acceptance gets refracted at core-cladding boundary and is lost.

Fractional change in refractive index –

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Since $NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1 - n_2)(n_1 + n_2)}$

But from definition of fractional change in refractive index ,

$n_1 - n_2 = n_1\Delta$

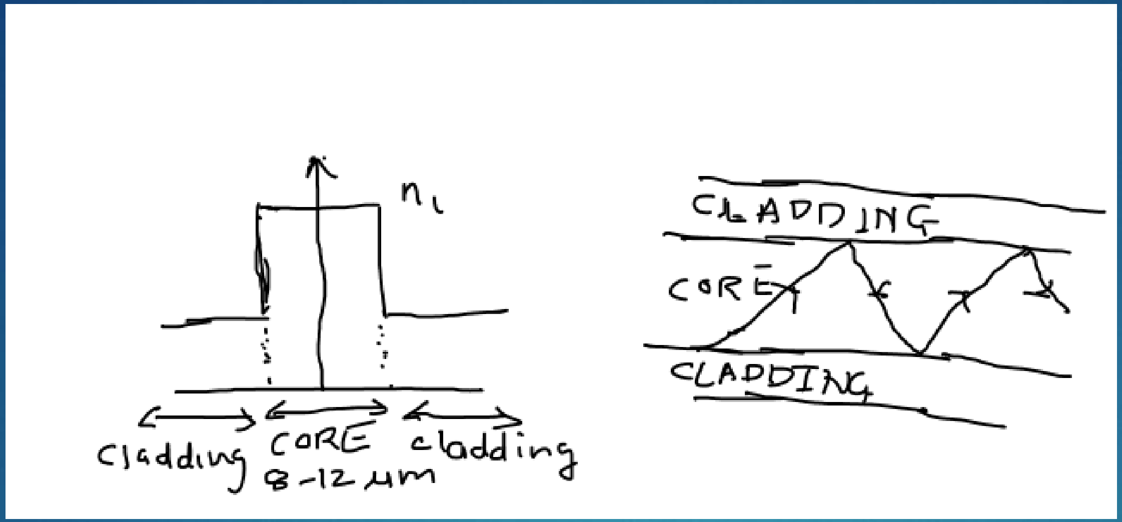
Thus, $NA = \sqrt{n_1\Delta(n_1 + n_2)}$. As $n_1 \approx n_2$,

$$NA = n_1\sqrt{2\Delta}$$

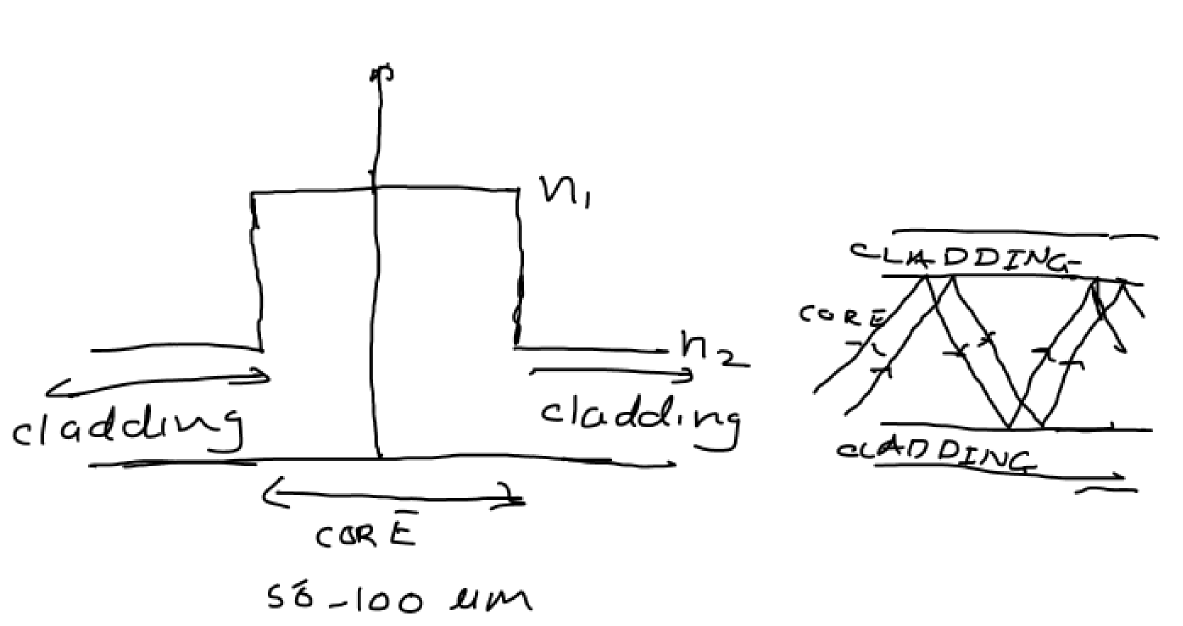
Mode of a fibre – In optical fibre a mode refers to a form of propagation of guided wave that is characterised by a particular field pattern in a plane transverse to the direction of propagation.

Type of fibres – Fibres are classified on the basis of modes that can fit in the core and the refractive index profile.

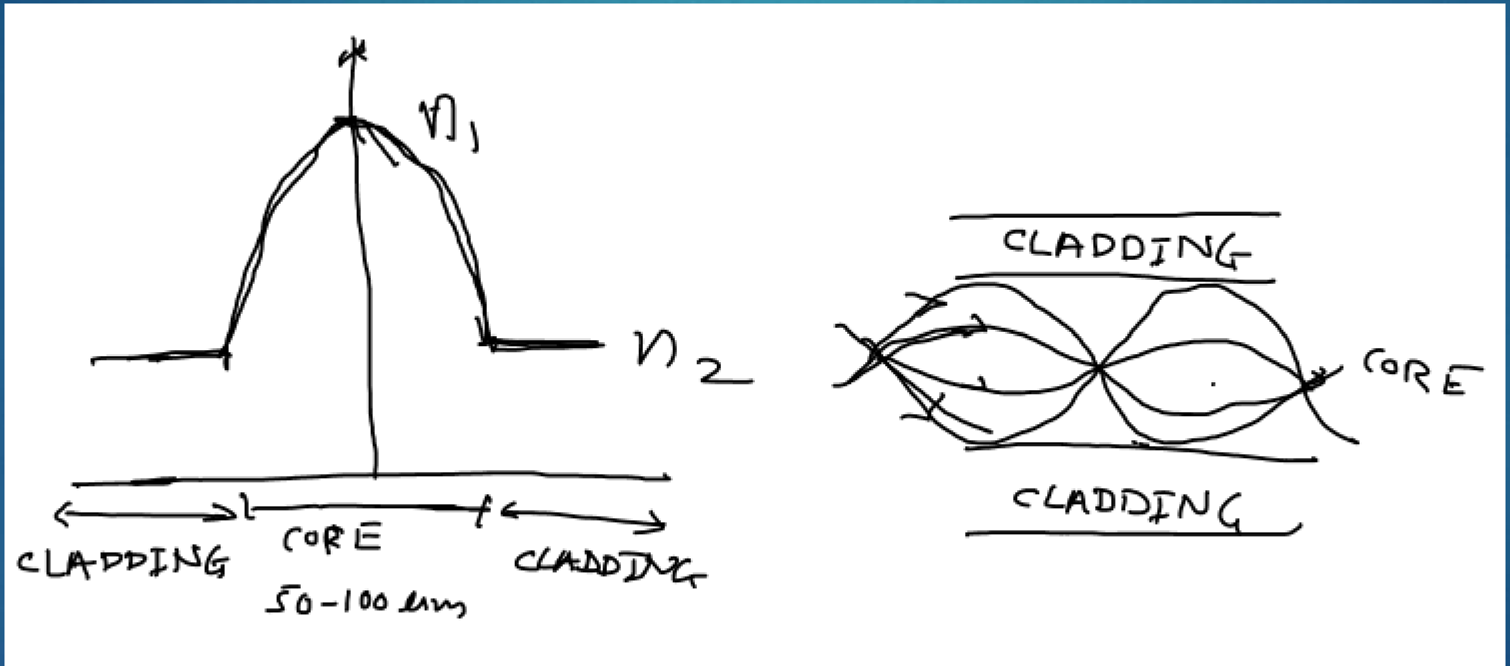
1. Step index single mode fibre- In step index fibre the fibre supports only one kind of mode. In this fibre, typical diameter of core is about 8-12 microns and refractive index is uniform in core and cladding region.



Step index multimode fibre- The step index multimode fibre supports more than one kind of mode. In this fibre, the typical diameter of fibre is 50-100 microns and refractive index is uniform in the core and cladding region.



Graded index multimode fibre – The multimode fibre supports more than one kind of mode. The typical diameter is of order of 50-100 microns. The refractive index varies gradually in the core of the fibre. The refractive index is highest at the centre of the core and decreases towards the core cladding boundary. The light rays undergo refraction in the core as the refractive index varies in the core.



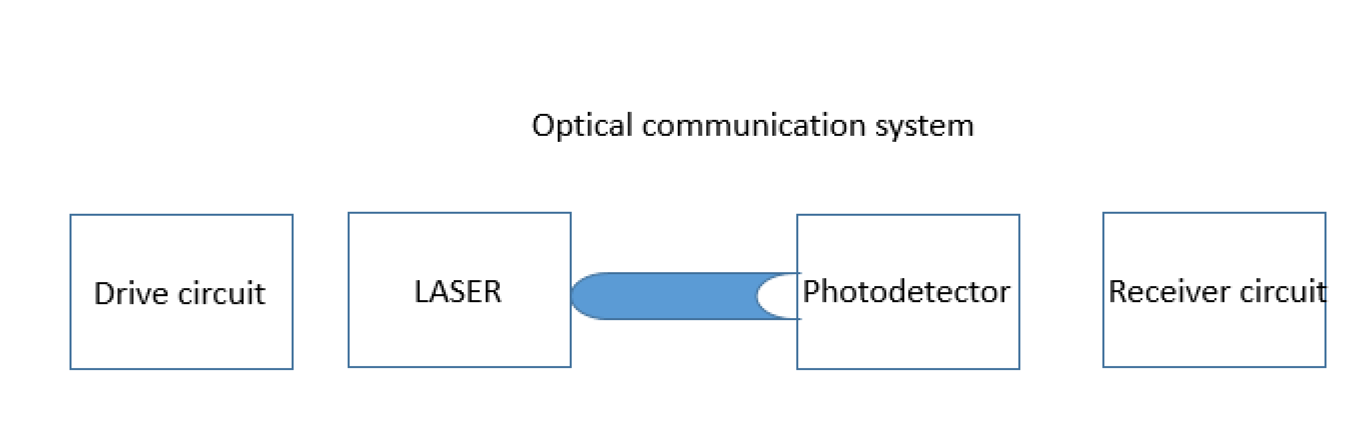
Losses or attenuation in optical fibre– As optical signal moves through the core of the fibre, the strength of the signal decreases due to the following mechanism.

1. Absorption – The absorption of light by impurities leads to the loss of optical signal. The impurity atoms absorb photons and they either emit photons in random direction or they undergo non-radiative decay transition.
2. Scattering- The Rayleigh scattering takes place for optical signal. The scattering is proportional to λ^{-4} and is caused by inhomogeneous structure.
3. Bending losses- The losses occur due to macroscopic bending and microscopic bending. The bend in the optical fiber leads to refraction of signal at core-cladding boundary in case of macroscopic bending losses. For microscopic bending losses, the irregularities at core-cladding boundary hampers total internal reflection and leads to loss of optical signal.

All these losses are quantified by coefficient of attenuation

$$\alpha = \frac{10}{L} \log\left(\frac{P_{in}}{P_{out}}\right)$$

Optical fibre communication system -



In optical fibre communication system, electrical signal is converted to optical signal and it is coupled to optical fibre cable for transmission over a long distance.

At regular interval optical signal is amplified by various techniques for the losses that occur during transmission. At the receive end, optical signal is converted to electrical signal.

Problem – An optical fibre has a core of material of refractive index 1.55 and cladding of refractive index 1.50. Calculate numerical aperture, angle of acceptance and fractional index change.

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.55^2 - 1.50^2} = 0.39$$

As, $NA = \sin\theta_a$,

we get
$$\theta_a = \sin^{-1} NA = \sin^{-1} 0.39 = 22.9^0$$

$$\Delta = \frac{n_1 - n_2}{n_2} = \frac{1.55 - 1.50}{1.50} = 0.032$$

Problem- Find the attenuation in a optical fibre of length 500 m when light signal of 100 mW emerges out of fibre with a power of 90 mW.

$$\alpha = \frac{10}{L}\log\left(\frac{P_{in}}{P_{out}}\right) = \frac{10}{500}\log\left(\frac{100}{90}\right) = 9.15 \times 10^{-4} dB/m = 0.915\ dB/km$$

Industrial application of laser- The large intensity and directionality makes laser very useful for industrial application.

Welding- In laser welding, the two metal pieces that are to be weld are held in contact. A laser beam is made to move along the line of contact.

Due to deposition of large heat, the metals are heated up to melting point temperature and it causes the pieces to fuse together. The laser welding is a contactless process. A relatively small area is affected because of this process. This technique also avoids deformation of the pieces that are weld together and avoids possibility of introduction of impurity at the joint.

Drilling – The vaporization of material takes place in laser drilling. The energy of laser should be supplied in such a way that it evaporates material without significant diffusion of heat in the drilled piece. A gas jet is used to remove evaporated material.