Einstein Coefficients (theory)

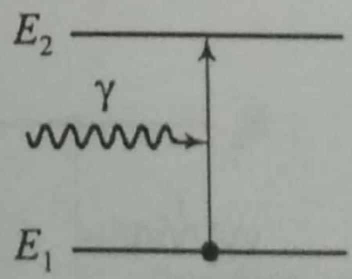
Expression for energy density :

Consider two energy states E_1 and E_2 of a system of atoms ($E_2 > E_1$). Let there be N_1 atoms with energy E_1 and N_2 atoms with energy E_2, per unit volume of the system. N_1 and N_2 are called the number density of atoms in the states 1 and 2 respectively. Let radiations with a continuous spectrum of frequencies be incident upon the system. Let there be radiation of frequency ν such that $\nu = (E_2 - E_1)/h$, and let U_ν be the energy density of radiations of frequency ν. Then $U_\nu d\nu$ will be the energy density of radiations whose frequencies lie in the range ν and $\nu + d\nu$.

Let us now consider the absorption, and the two emission processes case by case.

(i) Case of Induced Absorption:

In the case of induced absorption, an atom in the level E_1 can go to the level E_2 when it absorbs a radiation of frequency ν such that, $\nu = (E_2 - E_1)/h$ (Fig. 5). The number of such absorptions per unit time per unit volume, is called **rate of absorption.**



INDUCED ABSORPTION

Fig. 5

The rate of absorption depends upon,

a) the number density of lower energy state, *i.e.*, N_1, and,

b) the energy density *i.e.*, U_ν.

$\therefore$ Rate of absorption $\propto N_1 U_\nu$.

Or, we can write the same as,

$$\text{Rate of absorption} = B_{12} N_1 U_\nu, \qquad \ldots\ldots(2)$$

where, B_{12} is the constant of proportionality called **Einstein coefficient of induced absorption.**

(ii) Case of Spontaneous Emission:

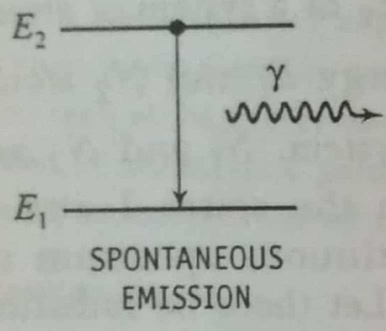


Fig. 6

In the case of spontaneous emission, an atom in the higher energy level E_2 undergoes transition to the lower energy level E_1 voluntarily by emitting a photon (Fig. 6). Since it is a voluntary transition, it is independent of the energy density of any frequency in the incident radiation. The number of such spontaneous emissions per unit time per unit volume, is called **rate of spontaneous emission** which is proportional to only the number density in the higher energy state, *i.e.*, N_2.

$$\therefore \text{ Rate of spontaneous emission} = A_{21} N_2, \qquad \ldots\ldots(3)$$

where, A_{21} is the constant of proportionality called the **Einstein coefficient of spontaneous emission.**

(iii) Case of Stimulated Emission:

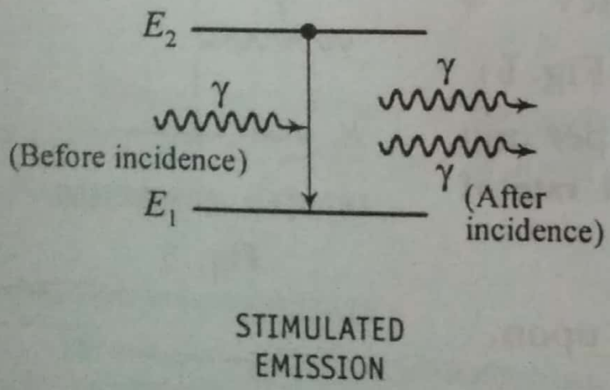


Fig. 7

Since the system requires an external photon of appropriate frequency $\nu \left[= (E_2 - E_1)/h\right]$, to stimulate the atom for the corresponding downward transition, and thereby cause emission of stimulated photons (Fig. 7), the energy density U_ν has a role to play in this case.

The number of stimulated emissions per unit time per unit volume, called **rate of stimulated emission**, is proportional to,

a) the number density of the higher energy state, *i.e.*, N_2, and,

b) the energy density, *i.e.*, U_ν.

$\therefore$ Rate of stimulated emission $\propto N_2 U_v$.

Or, rate of stimulated emission $= B_{21} N_2 U_v$, ...(4)

where, B_{21} is the constant of proportionality called the **Einstein coefficient of stimulated emission.**

Let the system be in thermal equilibrium which means that, the total energy of the system remains unchanged† in spite of the interaction that is taking place between itself and the incident radiation. Under such a condition, the number of photons absorbed by the system per second must be equal to the number of photons it emits per second by both the stimulated and the spontaneous emission processes.

$\therefore$ At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission.

$\therefore$ From Eqs(2), (3) & (4), we have,

$$B_{12} N_1 U_v = A_{21} N_2 + B_{21} N_2 U_v .$$

Or,
$$U_v \left(B_{12} N_1 - B_{21} N_2\right) = A_{21} N_2 .$$

Or,
$$U_v = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}.$$

By rearranging the above equation, we get ,

$$U_v = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \right]. \qquad .....(5)$$

But, by Boltzmann's law, we have,

$$\frac{N_2}{N_1} = e^{-\frac{(E_2 - E_1)}{kT}} = e^{-\frac{h\nu}{kT}}.$$

$$\therefore \qquad \frac{N_1}{N_2} = e^{\frac{h\nu}{kT}}.$$

$\therefore$ Eq(5) becomes,

$$U_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1} \right]. \qquad \text{......(6)}$$

According to Planck's law, the equation for U_ν is,

$$U_\nu = \frac{8\pi h \nu^3}{c^3} \left[\frac{1}{e^{\frac{h\nu}{kT}} - 1} \right]. \qquad \text{......(7)}$$

Now, comparing the equations (6) and (7), term by term on the basis of positional identity, we have,

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3},$$

and,

$$\frac{B_{12}}{B_{21}} = 1,$$

or,

$$B_{12} = B_{21},$$

which implies that the probability of induced absorption is equal to the probability of stimulated emission. Because of the above identity, the subscripts could be dropped, and A_{21} & B_{21} can be represented simply as A and B, and Eq(6) can be rewritten.

∴ At thermal equilibrium the equation for energy density is,

$$U_\nu = \frac{A}{B\left[e^{\frac{h\nu}{kT}} - 1\right]}.$$

Production of Lasers

Pumping, Lasing, and Active System - the terminologies :

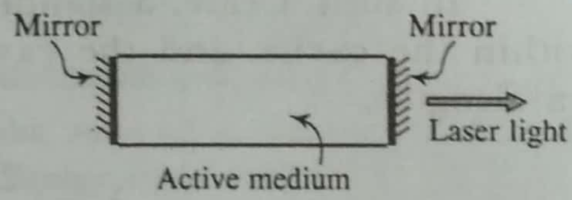
The act of exciting atoms from lower energy state to a higher energy state by supplying energy from an external source is called **pumping.**

The process which leads to emission of stimulated photons after establishing the population inversion is often referred to as **lasing.**

The quantum system between whose energy levels, the pumping and the lasing actions occur, is called an **active system.**

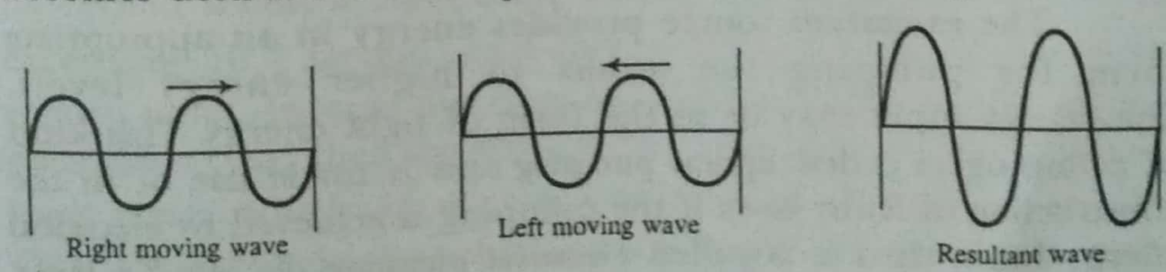
Laser cavity :

A laser device consists of an active medium bound between two mirrors (Fig. 8). The mirrors reflect the photons to and fro through the active medium. A photon moving in a particular direction represents a light wave moving in the same direction. Thus, the two mirrors along with the active medium form a **laser cavity.** Inside the cavity two types of waves exist; one type comprises of waves moving to the right, and the other one, to the left (Fig. 9a).

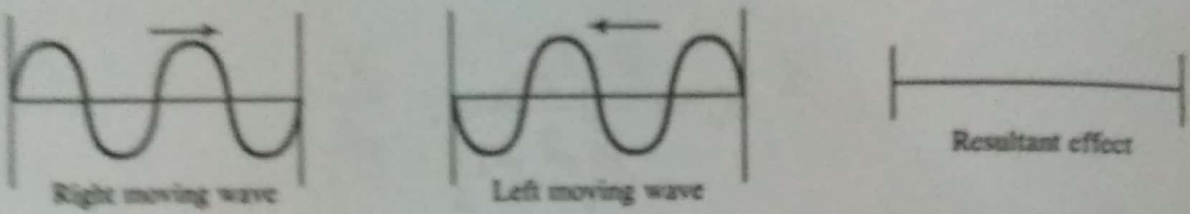


LASER CAVITY
Fig. 8

The two waves interfere constructively if there is no phase difference between the two (Fig. 9a). But, their interference becomes destructive if the phase difference is π (Fig. 9b).



CONSTRUCTIVE INTERFERENCE
Fig. 9a



DESTRUCTIVE INTERFERENCE
Fig. 9b

In order to arrange for constructive interference, the distance 'L' between the two mirrors should be such that the cavity should support an integral number of half wavelengths.

i.e.,

$$L = m\frac{\lambda}{2}, \qquad \text{......(1)}$$

where, m is an integer > 0, and λ is the wavelength of the laser light inside the material of the active medium.

In such a case, a standing wave pattern is established within the cavity, and the cavity is said to be resonant at wavelengths,

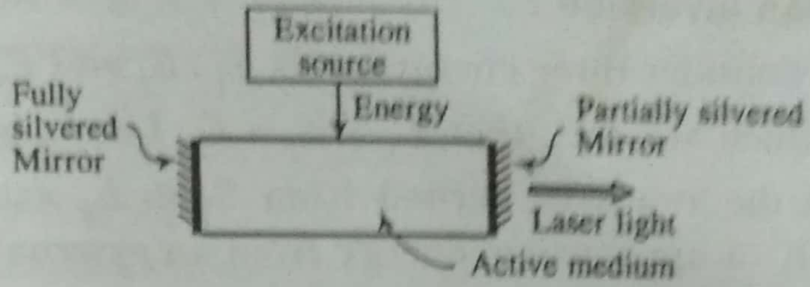
$$\lambda = \frac{2L}{m} \quad \text{[From Eq(1)].} \qquad \text{......(2)}$$

Requisites of a Laser System :

From the above discussion, we can say that, the 3 requisites of a laser system are,

1) an excitation source for pumping action,
2) an active medium which supports population inversion, and,
3) a laser cavity.

The excitation source provides energy in an appropriate form for pumping the atoms to higher energy levels. The energy input may be in the form of light energy. This kind of pumping is called **optical pumping** and is made use of in the construction of Ruby laser. If the pumping is achieved by electrical energy input then it is called **electrical pumping**. In He–Ne laser, the pumping is achieved by electrical discharge.



LASER SYSTEM
Fig. 10

A part of the input energy is absorbed by the active medium in which population inversion occurs at a certain stage(Fig. 10). Following this stage the medium attains capability to issue laser light.

The laser cavity provides the feedback necessary to tap certain permissible part of laser energy from the active medium. This is the basic structure of every laser device.

CONDITION FOR LASER ACTION

Population Inversion and the Metastable States :

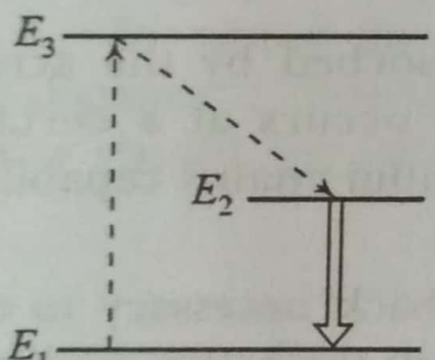
Population inversion is the state of a system at which the population of a particular higher energy state is more than that of a specified lower energy state.

Because of the condition stipulated in the Boltzmann factor, in a real physical system, the population inversion conditions don't exist under normal conditions. However, it is possible to achieve the population inversion condition in certain systems which possess a special kind of excited energy states called **metastable states.** A metastable state differs from any ordinary excited state in the following manner.

By providing energy, if an atom is made to go to one of its excited states, it stays there over a brief interval of time not exceeding 10^{-8} second, and then returns to one of the lower energy states. In case the state to which the atom is excited is a metastable state, then the atom stays in that state for an unusually long time, which is of the order of 10^{-3} to 10^{-2} second. This property helps in achieving the population inversion in the following way.

Population Inversion :

Consider three energy levels E_1 , E_2 and E_3 of a quantum system which are such that $E_3 > E_2 > E_1$. Let E_2 be a metastable state. Let the atoms be excited from E_1 to E_3 state (Fig. 11) by the supply of appropriate energy from an external source. From the E_3 state, the atoms undergo spontaneous downward transitions to E_1 and E_2 states rapidly. In spite of new arrival of atoms, the population of E_1 level cannot increase because of ongoing excitation of atoms to higher energy level.



POPULATION INVERSION
Fig. 11

But, since E_2 is a metastable state, those atoms which get into that state stay over a very long duration, because of which the population of E_2 state increases steadily. Under these conditions a stage will be reached wherein the population of E_2 state overtakes that of E_1, which is known as population inversion.

If the above conditions are arranged in a laser cavity, then once the population of E_2 exceeds that of E_1, the stimulated emissions outnumber the spontaneous emissions. Soon, stimulated photons all identical in respect of phase wavelength and direction, grow to a very large number which build up the laser light. Hence the condition for laser action is achieved by means of population inversion.

Construction: The schematic of typical CO_2 laser is shown in Fig. 24.21. It is basically a discharge tube having a bore of cross section of about 1.5 mm^2 and a length of about 260 mm. The discharge tube is filled with a mixture of carbon dioxide, nitrogen and helium gases in 1:4:5 proportions respectively. Other additives such as water vapour are also added. The active centres are CO_2 molecules lasing on the transitions between the vibrational levels of the electronic ground state.

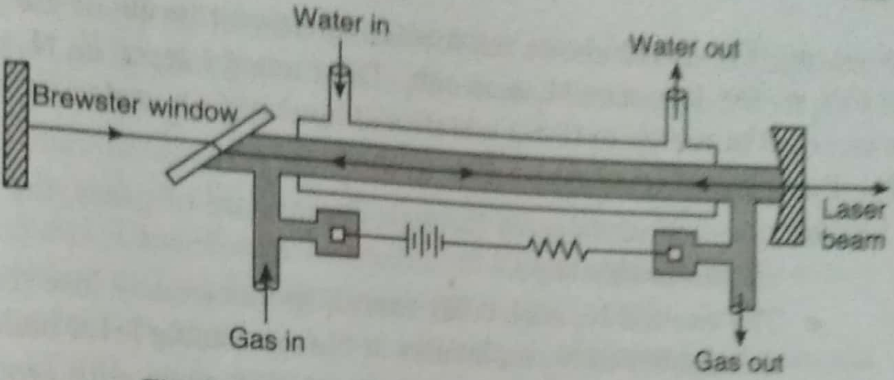


Fig. 24.21. Schematic of a carbon dioxide laser

Energy levels of CO_2 molecule

Fig. 24.22 shows the vibrational modes and rotations of CO_2 molecule.

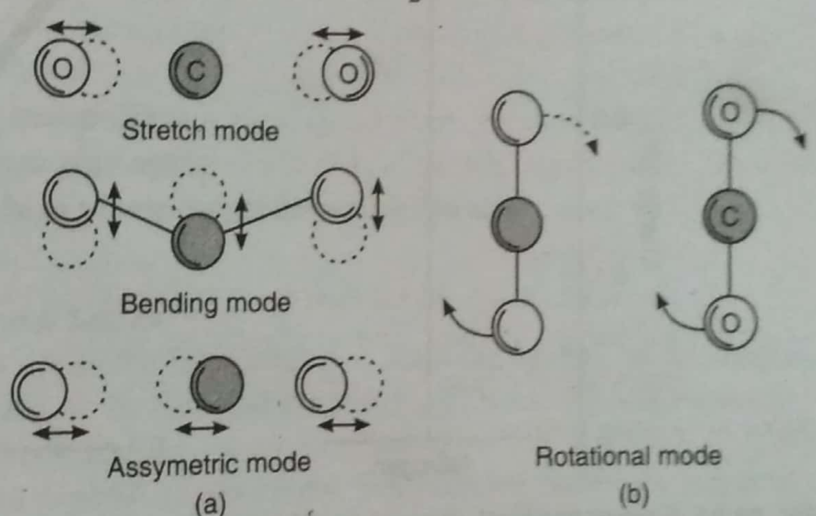


Fig. 24.22. Vibrational modes of a CO_2 molecule

- The electron energy levels of an isolated atom are discrete and narrow. However, in case of molecules the energy spectrum is complicated due to many additional features.
- Each electron energy level is associated with nearly equally spaced vibrational levels and each vibrational level in turn has a number of rotational levels.
- CO_2 molecule is a linear molecule consisting of a central carbon atom with two oxygen atoms attached one on either side.
- It undergoes three independent vibrational oscillations known as the **vibrational modes** (Fig.24.22). These vibrational degrees of freedom are quantized. At any one time, a CO_2 molecule can vibrate in a linear combination of three fundamental modes.
- The energy states of the molecule are represented by three quantum numbers (m n q).
- These numbers represent the amount of energy associated with each mode. For example, the number (020) indicates that the molecule in this energy state is in the pure bending mode with two units of energy.
- Each vibrational state is associated with rotational states corresponding to the rotation of CO_2 molecule about its centre of mass. The separations between vibrational – rotational states are much smaller on the energy scale compared to the separations between electron energy levels.

The nitrogen molecule N_2 is also characterized by similar vibrational levels.

Working: Fig. 24.23 shows the lowest vibrational levels of the ground electron energy state of CO_2 molecule and an N_2 molecule. The excited state of an N_2 molecule is metastable and it is identical in energy to (001) vibrational level of CO_2 molecule, indicated as E_5 in Fig. 24.23.

The Pumping Mechanism

- When current passes through the mixture of gases, the N_2 molecules get excited to the metastable state.
- The excited N_2 molecules cannot spontaneously lose their energy and consequently, the number of N_2 molecules at the metastable level builds up.
- The N_2 molecules undergo inelastic collisions with ground state CO_2 molecules and excite them to E_5 level. Some of the CO_2 molecules are also excited to the upper level E_5 through collisions with electrons.

Population Inversion

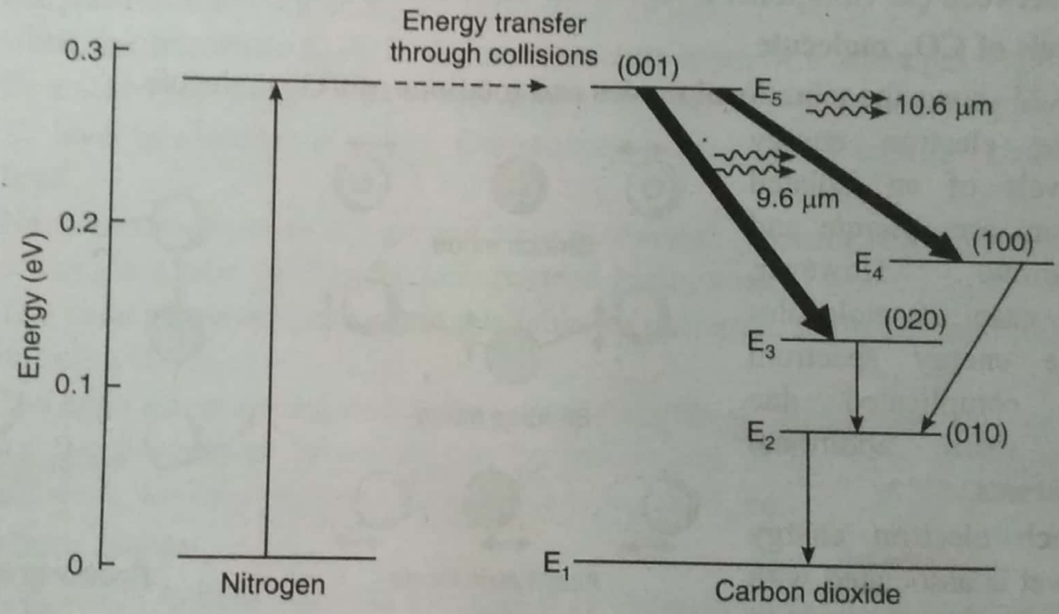


Fig. 24.23. Energy levels of nitrogen and carbon dioxide molecules and transitions between the levels

- The E_5 level is the upper lasing level while the (020) and (100) states marked as E_3 and E_4 levels act as the lower lasing levels.
- As the population of CO_2 molecules builds up at E_5 levels population inversion is achieved between E_5 level and the levels at E_4 and E_3.

Lasing

- Random photons are emitted spontaneously by a few of the atoms at the energy level E_5.
- The spontaneous photons traveling through the gas mixture prompt stimulated emission of photons.
- The photons bounce back and forth between the end mirrors, causing more and more stimulated emission during each passage. The strength of the stimulated photons traveling along the axis of the optical cavity (discharge tube) builds up rapidly while the photons traveling at angles to the axis are lost.
- The laser transition between $E_5 \rightarrow E_4$ levels produces far IR radiation at the wavelength 10.6 μm (1,06,000Å).
- The lasing transition between $E_5 \rightarrow E_3$ levels produces far IR radiation at 9.6 μm (96,000Å) wavelength.

- E_3 and E_4 levels are also metastable states and the CO_2 molecules at these levels fall to the lower level E_2 through inelastic collisions with normal (unexcited) CO_2 molecules.
- This process leads to accumulation of population at E_2 level. As the gaseous mixture heats up, the E_2 level, which is close to the ground state, tends to be populated through thermal excitations. Thus, the de-excitation of CO_2 molecules at the lower lasing level poses a problem and inhibits the laser action.
- The helium atoms de-excite CO_2 molecules through inelastic collisions and decrease the population density of CO_2 at E_2 level. It also aids cooling the gaseous mixture through heat conduction.
- The CO_2 molecules are once again available for excitation to higher state and participate in lasing action.
- CO_2 molecules are excited to the upper lasing level continuously through collisions. As the population inversion can be maintained in the face of continuous laser emission, the laser operates in continuous wave mode.

Salient Features

- Uses four-level pumping scheme
- The active centers are CO_2 molecules
- Electrical discharge is the pumping agent
- High efficiency (40%) and high power output (several kilowatts)
- Operates in CW mode

24.11.5.1 Homojunction semiconductor laser

Construction: Fig. 24.24 shows the schematic of a homojunction diode laser. Starting with a heavily doped n-type GaAs material, a p-region is formed on its top by diffusing zinc atoms into it. A heavily zinc doped layer constitutes the heavily doped p-region. The diode is extremely small in size. Typical diode chips are 500 μm long and about 100 μm wide and thick. The top and bottom faces are metallized and metal contacts are provided to pass current through the diode. The front and rear faces are polished parallel to each other and perpendicular to the plane of the junction. The polished faces constitute the Fabry-Perot resonator. In practice there is no necessity to polish the faces. A pair of parallel planes cleaved

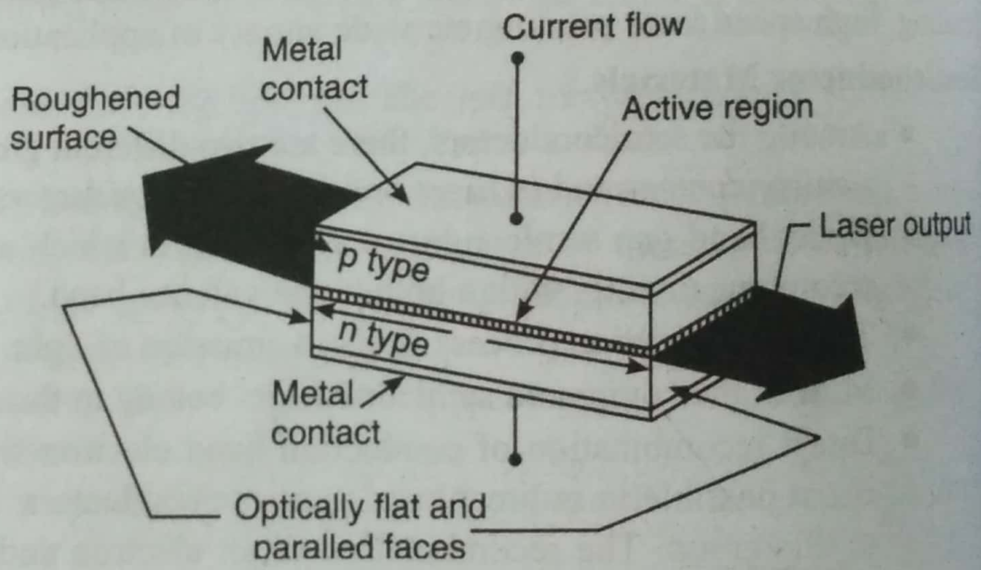


Fig. 24.24. Schematic of homojunction diode laser

at the two ends of the pn junction provides the required reflection to form the cavity. The two remaining sides of the diode are roughened to eliminate lasing action in that direction. The entire structure is packaged in small case which looks like the metal case used for discrete transistors.

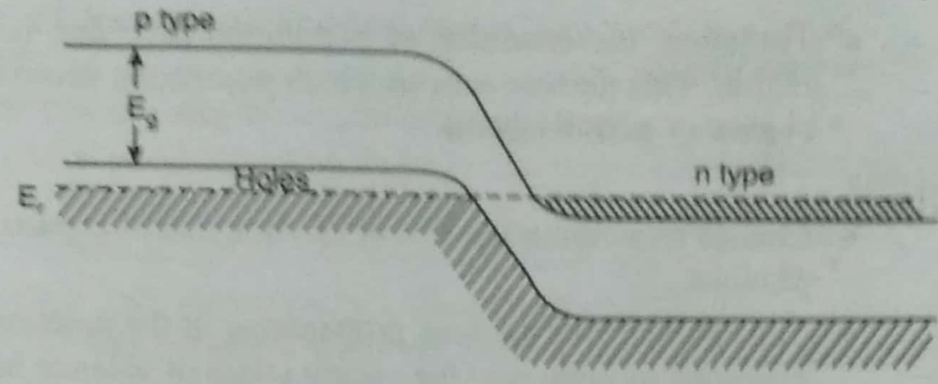


Fig. 24.25. Energy band diagram of a heavily doped p-n junction without bias

Working: The energy band diagram of a heavily doped p-n junction is shown in Fig. 24.25.

- Heavily doped p- and n- regions are used in making a laser diode.
- Because of very high doping on n-side, the donor levels are broadened and extend into the conduction band. The Fermi level also is pushed into the conduction band.
- Electrons occupy the portion of the conduction band lying below the Fermi level.
- Similarly, on the heavily doped p-side the Fermi level lies within the valence band and holes occupy the portion of the valence band that lies above the Fermi level.
- At thermal equilibrium, the Fermi level is uniform across the junction.

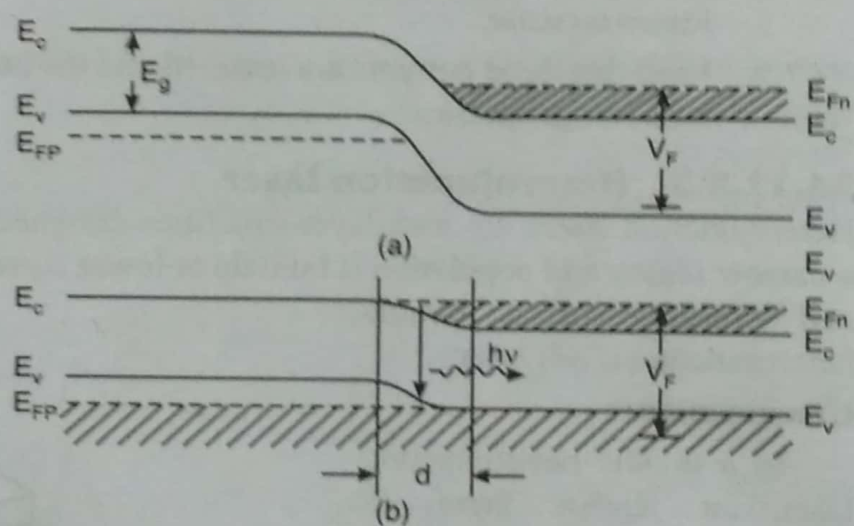


Fig. 24.26. Laser diode under forward bias

The Pumping Mechanism

- When the junction is forward-biased, electrons and holes are injected into the junction region in high concentrations (Fig. 24.26a).
- In other words, charge carriers are *pumped* by the dc voltage source.
- When the diode current reaches a threshold value (see Fig. 24.26b), the carrier concentrations in the junction region will rise to a very high value.

Population Inversion

- As a result, the region (region 'd' in Fig. 24.26b) contains a large concentration of electrons within the conduction band and *simultaneously* a large number of holes within the valence band.
- Holes represent absence of electrons.
- Thus, the upper energy levels in the narrow region are having a high electron population while the lower energy levels in the same region are vacant.

- Therefore, the condition of population inversion is attained in the narrow junction region. This narrow zone in which population inversion occurs is called an **inversion region** or **active region**.

Lasing

- Chance recombination acts of electron and hole pairs lead to emission of spontaneous photons.
- The spontaneous photons propagating in the junction plane stimulate the conduction electrons to jump into the vacant states of valence band.
- This stimulated electron-hole recombination produces coherent radiation.
- GaAs laser emits light at a wavelength of 9000 Å in IR region.

Drawbacks of homojunction lasers:

1. In homojunction lasers, the active region is not well defined due to the diffusion length of the carriers.
2. The semiconductor has nearly uniform refractive index throughout. Therefore, light can diffuse from active layer into the surrounding medium. As a result the cavity losses increase.
3. High threshold currents are required and the laser cannot be operated continuously at room temperature.

The large intensity that is possible in the focused output of a laser beam and its directionality makes laser an extremely useful tool for a variety of industrial applications.

Welding: Welding is the joining of two or more pieces into a single unit. If we consider welding of two metal plates, the metal plates are held in contact at their edges and a laser beam is made to move along the line of contact of the plates. The laser beam heats the edges of the two plates to their melting points and causes them to fuse together where they are in contact. The main advantage of the laser welding is that it is a contact-less process and hence there is no possibility of introduction of impurities into the joint. In the process, the work-pieces do not get distorted, as the total amount of input is very small compared to conventional welding processes. The heat-effected zone is relatively small because of rapid cooling. Laser welding can be done even at difficult to reach place. CO_2 lasers are used in welding thin sheets and foils.

Drilling: The principle underlying drilling is the vaporization of the material at the focus of the beam. With lasers, one can drill holes as small as 10 μm in diameter. For drilling, the energy must be supplied in such a way that rapid evaporation of material takes place without significant radial diffusion of heat into the work piece. The vaporized material is removed with the help of a gas jet. Pulsed ruby and neodymium lasers are commonly used for drilling holes of small l/D ratio, where l is the thickness of the work and D is the hole diameter.

Hardening: Heat treatment is the process, which is done for sometime to harden metals and certain other materials. Heat treatment is common in the tooling and automotive industry. Heat-treating converts the surface layer to a crystalline state that is harder and more resistant to wear. In general CO_2 lasers of about 1 kW output power operating in cw mode are used for heat treatment. As metals are more reflecting at IR frequencies, a heat absorbing coating such as graphite or zinc phosphate is applied on the surface of the work piece to help it absorb laser energy more efficiently. Laser heat treatment requires a low amount of energy input to the work piece. Laser processing is advantageous as it can provide selective treatment of the desirable areas. Heat treatment is used to strengthen cylinder blocks, gears, camshafts etc in the automobile industry. As the method is a non-contact method, stress is not induced in the work-pieces.

Electronics Industry: Electronics industry uses lasers in the manufacture of electronic components and integrated circuits. Lasers have been used to perforate and divide silicon slices having several hundred circuits. They are also used for the isolation of faulty components in a large integrated circuit by disconnecting the conducting paths by evaporation. Trimming of thick and thin film resistors using lasers is a very common application.

Measurement of atmospheric pollutants: Laser is a very useful tool for the measurement of the concentrations of various atmospheric pollutants such as N_2, CO, SO_2 etc gases and particulate matter such as dust, smoke and flyash. Conventional methods of pollution measurements require that samples of pollutants are to be collected for chemical analysis. Therefore, these methods cannot give real-time data. In contrast, laser methods permit measurements by remotely sensing the composition of atmosphere without the necessity of sample collection or chemical processing.

In one of the laser techniques, the light scattered by pollutants is studied. A pulsed laser is used as the source of light and the light scattered back is detected by a photodetector. The distance to particulate matter and the concentrations of particulate matter is obtained in this method. The distance is inferred from the time that light takes to travel up to the pollutant region and to return back. This technique is known as LIDAR which stands for light detection and ranging. The principle is very much similar to that of RADAR. The method helps in determining the concentration of particulate matter as a function of distance. However, this method cannot provide any information regarding the nature of the scattering particles. It is mainly useful in knowing the distribution of atmospheric pollutants in different vertical sections and in monitoring their variations. Environmental agencies measure concentrations of harmful gases such as SO_2 and NO_2 using this method.

Another technique uses study of absorption of light beam by pollutants. The existence of specific gases in the atmosphere is detected using absorption spectroscopy techniques. A laser beam is transmitted through polluted sample and the attenuation of intensity of light due to absorption in the sample is detected and recorded. Each chemical absorbs light of characteristic wavelengths and from the absorption spectrum, its existence can be inferred.

A third method uses Raman effect to detect the pollutants. The Raman effect involves scattering of light by gas molecules accompanied by a shift in the wavelength of light. Raman shifts are characteristic of each molecular species. Hence, analysis of backscattered laser light reveals the constituents of the gas sample. The ozone concentration high in the atmosphere is determined using this technique.

Example 2 :

The average output power of laser source emitting a laser beam of wavelength 6328 Å is 5 mW. Find the number of photons emitted per second by the laser source.

V.T.U. Jan 11

Data :

Wavelength of the emitted light, $\lambda = 6328$ Å $= 6328 \times 10^{-10}$ m,

Power output $= 5$ mW $= 5 \times 10^{-3}$ W,

To find :

No. of photons emitted/second, $N = ?$

Solution :

We know that the energy difference,

$$\Delta E = h\nu = \frac{hc}{\lambda} \text{ Joule},$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{6328 \times 10^{-10}},$$

$$= 3.143 \times 10^{-19} \text{ J}.$$

This energy difference becomes the energy of each of the emitted photon. If N is the number of photons emitted per second to give a power output of 5 mW, then,

$$N \times \Delta E = 5 \text{ mW} = 5 \times 10^{-3} \text{ J/s}.$$

$$\therefore \quad N = \frac{5 \times 10^{-3}}{3.143 \times 10^{-19}},$$

$$= 1.59 \times 10^{16}.$$

$\therefore$ The number of photons emitted per second = 1.59×10^{16}.

Example 3 :

A pulsed laser emits photons of wavelength 780 nm with 20 mW average power/pulse. Calculate the number of photons contained in each pulse if the pulse duration is 10 ns.

Data :

Wavelength of the photon, $\lambda = 780 \text{ nm} = 780 \times 10^{-9}$ m.

Power of each pulse, $p = 20 \text{ mW} = 20 \times 10^{-3}$ J/s.

Duration of each pulse, $t = 10 \text{ ns} = 10 \times 10^{-9}$ s.

To find :

No. of photons in each pulse, $N = ?$

Solution :

We have, wavelength of the photons, $\lambda = 780 \times 10^{-9}$ m.

$\therefore$ Energy of each photon, $\Delta E = \frac{hc}{\lambda}$,

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{780 \times 10^{-9}},$$

$$= 2.55 \times 10^{-19} \text{ J}.$$

Now, we have, energy of each pulse,

$$E = \text{power} \times \text{duration of the pulse},$$
$$= P \times t = 20 \times 10^{-3} \times 10 \times 10^{-9},$$
$$= 2 \times 10^{-10} \text{ J}.$$

If N is the number of photons (each of energy ΔE) in the pulse, then,

$$N \times \Delta E = E.$$

$$\therefore \quad N = \frac{E}{\Delta E} = \frac{2 \times 10^{-10}}{2.546 \times 10^{-19}} = 7.86 \times 10^{8}.$$

$\therefore$ **Number of photons in each pulse is 7.86×10^8.**

Example 5 :

A medium in thermal equilibrium at temperature 300 K has two energy levels with a wavelength separation of 1 μm. Find the ratio of population densities of the upper and lower levels.

V.T.U. Jan 07

Data :

Temperature, $T = 300$ K,

Wavelength separation*, $\lambda = 10^{-6}$ m.

To find :

The ratio of population densities, $\frac{N_2}{N_1} = ?$

Solution :

We have the Boltzmann factor,

$$\frac{N_2}{N_1} = e^{-\frac{\Delta E}{kT}} = e^{-\frac{hc}{\lambda kT}},$$

$$= e^{-\left(\frac{hc}{k}\right)\left(\frac{1}{\lambda T}\right)} = e^{-\frac{0.014413}{\lambda T}},$$

$$= e^{-\frac{0.014413}{10^{-6} \times 300}},$$

$$= e^{-48.043},$$

$$\therefore \quad \frac{N_2}{N_1} = 1.365 \times 10^{-21}.$$

$\therefore$ **The ratio of population densities = 1.365×10^{-21}.**

Example 6 :

The ratio of population of two energy levels is 1.059×10^{-30}. Find the wavelength of light emitted by spontaneous emissions at 330 K. V.T.U. June 12

Data :

The ratio of population, $(N_2/N_1) = 1.059 \times 10^{-30}$.

Ambient temperature, $T = 330$ K.

To find :

The wavelength of light emitted, $\lambda = ?$

Solution :

We have the Boltzmann factor, $\frac{N_2}{N_1} = e^{-\Delta E/kT}$.

By taking natural log on both sides, we have,

$$\ln\left(\frac{N_2}{N_1}\right) = -\frac{\Delta E}{kT} = -\frac{hc}{\lambda kT} = -\left(\frac{hc}{k}\right)\left(\frac{1}{\lambda T}\right),$$

$$= -\frac{0.014413}{\lambda T}.$$

$$\therefore \quad \lambda = -\frac{0.014413}{\ln\left(\frac{N_2}{N_1}\right)T} = -\frac{0.014413}{\ln\left(1.059 \times 10^{-30}\right) \times 330},$$

$$= 632 \text{ nm}.$$

$\therefore$ **The wavelength of light emitted by spontaneous emissions is 632 nm.**