Applied physics

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Syllabus

Unit – I Waves

Wave equation, Principle of superposition, Stationary waves- difference between stationary and travelling waves.

Optics – interference and diffraction Concepts of interference, techniques for obtaining interference, interference due to thin film, wedge shaped film, Newton's rings, Michelson's interferometer Diffraction: Types of diffraction, diffraction due to N slits, resolving power of a grating,

Sound –Introduction to Ultrasonic waves, generation of ultrasonic waves by Piezoelectric technique, Non destructive testing (NDT) of materials (pulse echo technique).

Self learning topic: Applications of interference-Antireflection coating(No derivation), Determination of thickness of transparent sheet and wavelength of light by Michelson's interferometer

Unit II - Electromagnetism and Photonics

Maxwell's equations: Fundamentals of vector calculus, Maxwell's equations in vacuum, Physical significance of Maxwell's equations, Velocity of electromagnetic waves using Maxwell's equations.

Laser Introduction, interaction between radiation and matter, Einstein's coefficients for two level system, conditions for laser action, components of laser, CO2 laser, semiconductor laser, LIDAR.

Self learning topic: Industrial applications of laser (welding and drilling).

Optical fiber Total internal reflection in fiber, angle of acceptance, fractional index change, Numerical aperture, types of optical fibers, losses in optical fibers. Applications- fiber optic communication system.

Unit III - Quantum mechanics

Introduction, de Broglie hypothesis, G.P. Thomson's experiment, concept and relation between Phase velocity, Group velocity and particle velocity, Heisenberg's uncertainty principle and its elementary proof. Physical interpretation of wave function, Development of Schrödinger's time dependent wave equation (1D), normalisation condition, eigen values and eigen functions of a particle in an infinite potential well.

Self learning topic: Elementary operators in quantum mechanics.

Unit IV - **Condensed matter Physics**

Band theory of solids- energy bands in solids, Energy band formation in lithium, silicon and diamond, Fermi-Dirac distribution,

Semiconductors- electrical conductivity in intrinsic semiconductor, Fermi level in intrinsic semiconductor, Hall effect and applications. **Self learning topic**: Fermi level in extrinsic semiconductor at 0K and 300K

Superconductivity Introduction, resistivity as a function of temperature, Meissner effect, Critical magnetic field, Persistent current, Critical current density, London penetration depth, classification of superconductors, BCS theory (Qualitative), Josephson junction, SQUID. **Self learning topic**: High Tc superconductors and Maglev trains

Unit V -Advanced Physics

Nanomaterials: Introduction, Quantum confinement effect- Density of states, Top down and Bottom up approaches of synthesis of nanomaterials, High energy ball milling, Colloidal technique, Characterization techniques- X-ray diffraction (XRD), Scanning Electron Microscopy (SEM),

Nuclear Physics Introduction to Nuclear detectors and accelerators, G.M. Counter, Synchrotron, Nuclear reactors, Nuclear accelerators, working principle of accelerators, medical application of nuclear physics, radiation in our environment and its effects.

Text book: M. N. Avadhanulu and P. G. Kshirasagar. A text book of Engineering Physics, S. Chand and company limited, 9th Revised Edition (2014) onwards.

Sulabha K Kulkarni, Nanotechnology principles and practices, Capital Publishing Company, second edition (2011) onwards.

Waves

Wave is profile of disturbance that propagates in a medium. It is characterized by wavelength, frequency, velocity, amplitude, phase and intensity.

Wavefunction describes the profile. Example- $y = Asin\,(kx - \omega t)$

Wave equation describes the motion of a wave in a medium. (in 1-D)

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

The principle of superposition of waves $y = y_1 + y_2$

Interference – The superposition of 2 or more waves results in interference.

Beats are observed when the two waves that superpose differ little in wavelength and frequency.

Standing/stationary wave are observed when there is a superposition of two waves of same amplitude, wavelength and frequency moving in the opposite direction.

Condition for interference - Consider two waves that have phase difference of δ. The wavefunction of these waves $y_1 = Asin\ (kx - \omega t)$ and $y_2 = Asin\ (kx - \omega t + \delta)$.

The resultant wave has a wavefunction

$$y = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx - \omega t + \delta) = 2A\sin\left(kx - \omega t + \frac{\delta}{2}\right)\cos\delta/2$$

The maximum amplitude of the wave is obtained when $\cos\delta/2 = \pm 1$,i.e.,

when the phase difference is $\delta = 2n\pi$

The minimum amplitude of the wave is obtained when $\cos\delta/2 = 0$, i.e., when the phase difference is $\delta = (2n + 1)\pi$

For a stable interference monochromatic and coherent sources are necessary.

Optics – Interference and diffraction

The phenomenon of interference and diffraction can be understood by considering wave nature of light.

Techniques for obtaining interference-

1. Division of wavefront – In this technique, two coherent sources are generated by splitting the wavefront of the wave. Example- Young's double slit experiment.

2. Division of amplitude – In this technique, two coherent sources are produced by splitting the energy of wave. Example- Newton's rings

Important concepts-

Geometric path- It is physical distance 'd' covered by light.

Optical path- It is used to compare distance covered by light in different mediums. Optical path of light is $\Delta = \mu d$

If light covers a distance d in a medium of refractive index μ in time t, then $\mathrm{d} = \mathrm{vt} = \left(\frac{\mathrm{c}}{\mu}\right)\mathrm{t}$. Hence, $\Delta = ct = \mu d$

The optical path difference between two waves of wavelength λ is associated with the phase difference as

$$\delta = \frac{2\pi}{\lambda}\Delta$$

Thin film interference- A thin film of uniform thickness t and refractive index μ floats on material of refractive index μ’ such that μ>μ’.

Optical path difference between light reflected from the top and bottom surface of the film is

$\Delta = \mu(NO + OQ) - NT$ ---(1)

From $\Delta NOQ'$ and $\Delta QOQ'$,

$\cos r = \frac{t}{NO}$ and $\cos r = \frac{t}{OQ}$

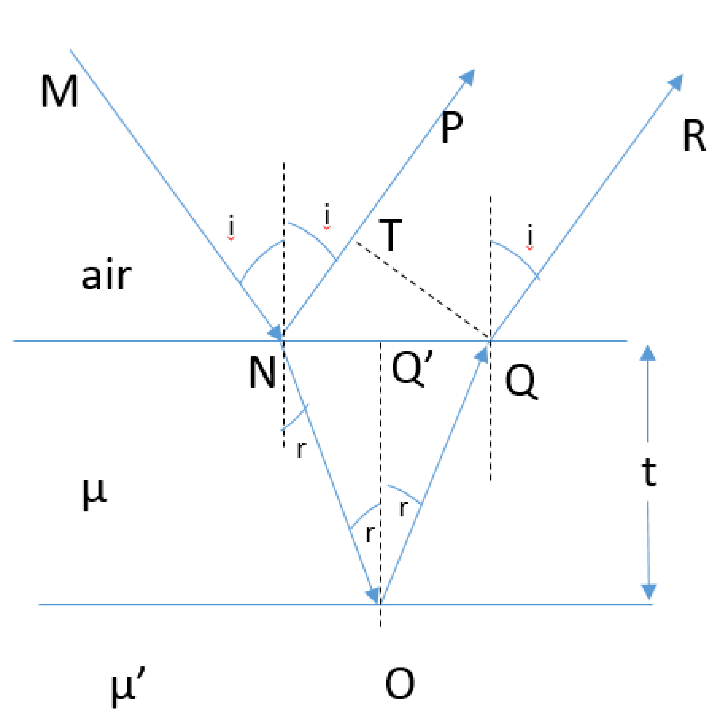
Thus, $NO + OQ = \frac{2t}{\cos r}$ ----(2)

In ΔNTQ, $\sin i = \frac{NT}{NQ}$

Thus, $NT = (NQ' + Q'Q) \sin i$

But from $\Delta NOQ'$ and $\Delta QOQ'$, $\tan r = \frac{NQ'}{t}$ and $\tan r = \frac{Q'Q}{t}$

Hence $NT = 2t \tan r \sin i$ ----(3)



Thus, from eq. (1), (2) and (3) ,

$\Delta = \frac{2\mu t}{\cos r} - 2t \tan r \sin i$ -----(4)

From Snell's law, $\sin i = \mu \sin r$

Thus, eq. (4) reduces to

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2t\ \mu \sin^2\ r}{\cos r} = \frac{2\mu t(1 - \sin^2 r)}{\cos r} = 2\mu t \cos r$$

Since the light gets reflected from denser medium at the top surface, a path difference of λ/2 gets introduced. Thus, total path difference is

$$\Delta = 2\mu t \cos r + \lambda/2$$

Thus, for light of wavelength λ, phase difference is

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}(2\mu t \cos r + \lambda/2)$$

Thus, for constructive interference as $\delta = 2n\pi$,

$2\mu t \cos r = \frac{\lambda}{2}(2n - 1)$, where $n = 1,2,3$

For destructive interference as $\delta = (2n+1)\pi$,

$2\mu t \cos r = n\lambda,$ where $n = 1,2,3$

Different colours on oil or soap film are due to constructive interference. The shape of pattern is due to variation in thickness of the film.

What is a typical thickness of the film?

A film of few microns is required to produce interference pattern. A typical wavetrain emitted by incoherent light source is of few microns. So for a film of few microns, light reflected from top and bottom surface of the film belongs to same wavetrain and hence coherent. If thickness of film is more, then the reflected light belongs to different wavetrain and is incoherent.

Applications of thin film – Coating on lenses, mirrors of cameras and telescope. Coatings on lenses of eye glasses etc.

Problem – A parallel beam of light strikes an oil film (μ=1.4) floating of water. When viewed at an angle of 30^0 from the normal, destructive interference for n=6 is seen. Find the thickness of the film if wavelength of light is 5890 A^0. detectable radiation.

$$2\mu t\cos r = n\lambda$$

$$2\mu t\sqrt{1-\sin^2 r} = n\lambda$$

From Snell's law, $\mu_a \sin i = \mu \sin r$. As $\mu_a = 1$,

$$2\mu t\sqrt{1-\frac{\sin^2 i}{\mu}} = n\lambda$$

Thus, $t = \frac{n\lambda}{\sqrt{\mu^2-\sin^2 i}} = \frac{6\times5890\times10^{-10}}{\sqrt{1.4^2-\sin^2 30}} = 1.35\mu m$

Problem – Monochromatic light of wavelength 5893A^0 is reflecting at a normal incidence from a soap film of refractive index 1.42. What will be the least thickness for which the film will appear bright and dark?

For normal incidence r=0. For minimum thickness of the film n=1. Hence

For constructive interference $t = \frac{\lambda}{4\mu} = \frac{0.5893\mu m}{4\times1.42} = 0.108\mu m$

For destructive interference $t = \frac{\lambda}{2\mu} = \frac{0.5893\mu m}{2\times1.42} = 0.208\mu m$

Problem- A soap film of refractive index 1.33 is illuminated at 45^0 by white light. For the wavelength 5890A^0, find the minimum thickness at which destructive interference is observed.

(A: 0.26 μm)

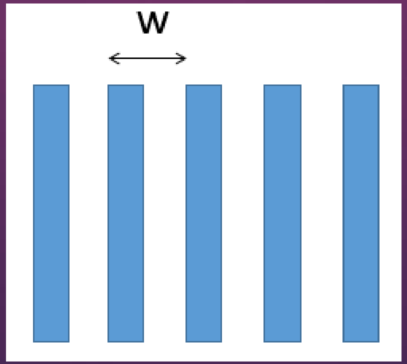
Wedge shaped film –

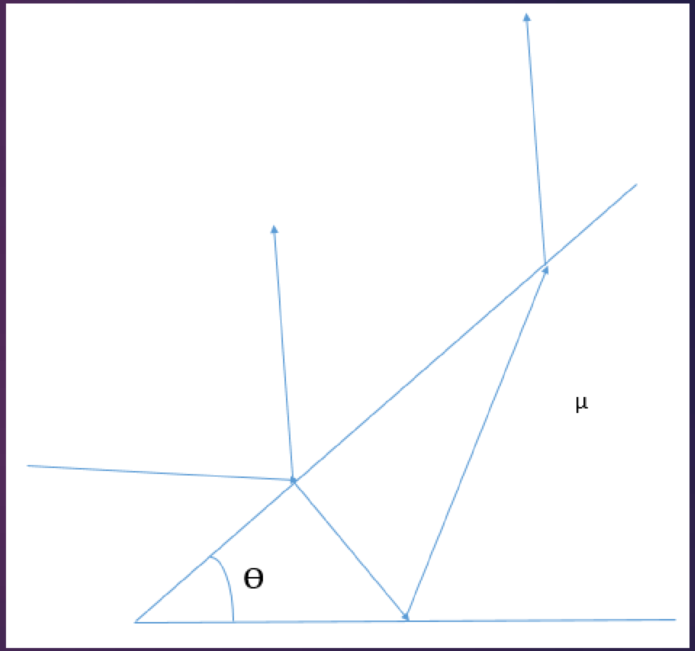
A wedge shaped film has top surface at an angle of θ with the bottom surface. The condition for interference is

Constructive interference $2\mu t\cos(r+\theta) = \frac{\lambda}{2}(2n-1)$

Destructive interference $2\mu t\cos(r+\theta) = n\lambda$

Fringewidth is the distance between consecutive bright or dark fringes.





Consider two observable consecutive dark fringes n^{th} and $n+1^{th}$ due to a film of thickness t_1 and t_2 respectively.

Thus, from diagram $\tan\theta = \frac{t_2 - t_1}{w}$

But since θ is small, above expression can be

$\theta = \frac{t_2 - t_1}{w}$ ---(1)

For n^{th} and $n+1^{th}$ dark fringe, the condition for destructive interference gives $2\mu t_1 \cos(r+\theta) = n\lambda$

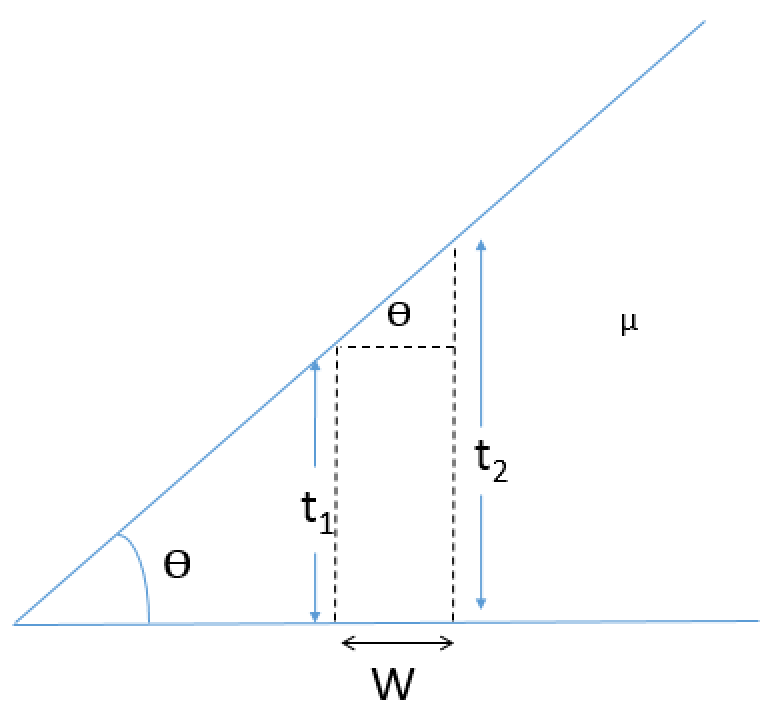
and $2\mu t_2 \cos(r+\theta) = (n+1)\lambda$

For small angles, $2\mu t_1 = n\lambda$ and $2\mu t_2 = (n+1)\lambda$

Thus, $(t_2 - t_1) = \lambda/2\mu$ ---(2)

Thus, eq. (1) and (2) gives,

$$w = \frac{\lambda}{2\mu\theta}$$



Problem – Monochromatic light of wavelength 550 nm is incident normally on a wedge shaped film of refractive index 1.5. If the distance between two consecutive fringes is 0.02 mm, find the angle of the wedge in degrees.

$$W = \frac{\lambda}{2\mu\theta}$$

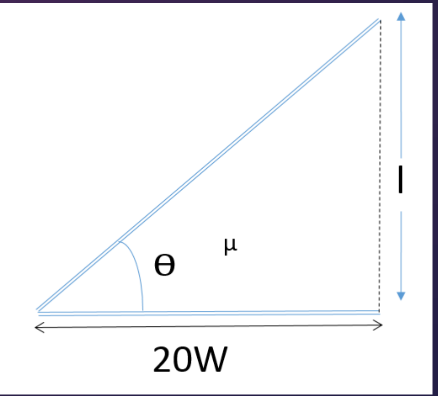
$$\theta = \frac{(550 \times 10^{-9})}{2 \times 1.5 \times 0.02 \times 10^{-3}} = 9.17 \times 10^{-3}\ rad$$

$$= 9.17 \times 10^{-3} \times \frac{180^0}{\pi} = 0.53^0$$

Problem: Two plane glass surfaces in contact with along one edge are separated at the opposite edge by a thin wire. If 20 dark fringes are observed for light of wavelength 5893 A^0, at normal incidence, what is the length of wire?

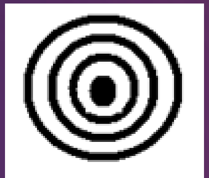
For wedge shaped film, $W = \frac{\lambda}{2\mu\theta}$. Let l be the length of wire.

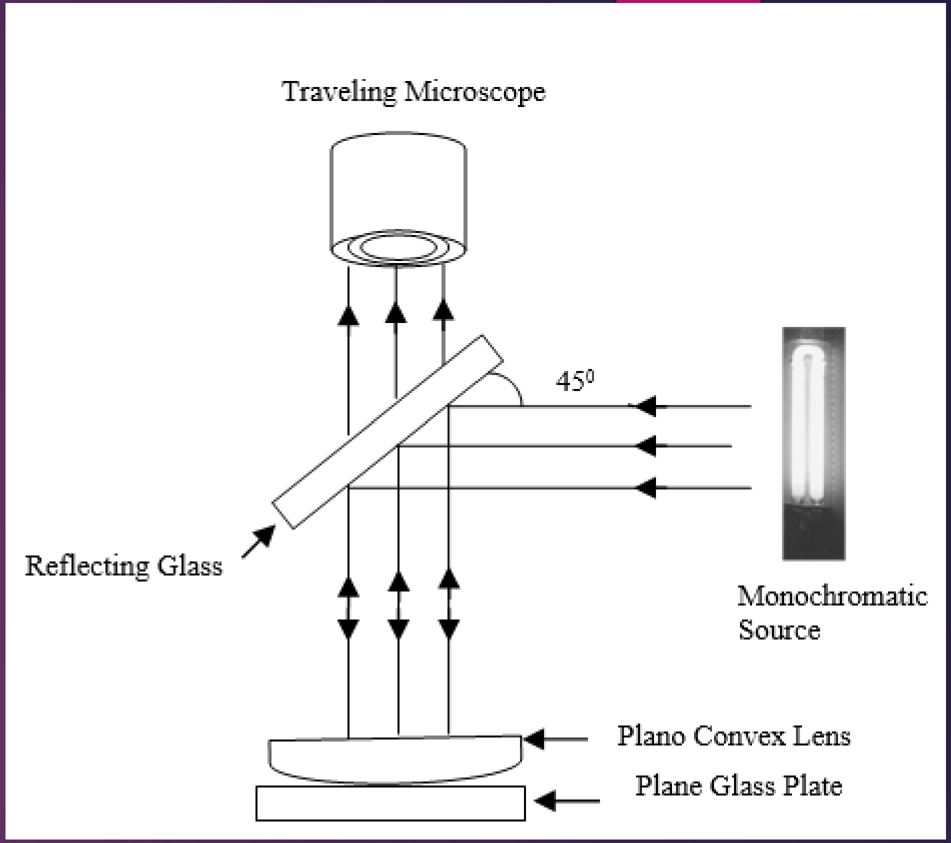
From diagram - $\theta = l/20W$ Hence, $W = \frac{\lambda}{2\mu(l/20W)} \Rightarrow l = \frac{10\lambda}{\mu} = 5.893\ \mu m$



Newton's rings-

A thin film of air is formed between plano-convex lens and glass plate. Light from monochromatic source is directed on the glass plate using a reflecting glass. Interference pattern of concentric rings, which is observed through microscope, which is known as Newton's rings.





How does thin film interference take place?

Light is reflected from the top and bottom surface of the thin air film sandwiched between plano-convex lens and glass plate. Superposition of this light produces interference pattern.

Why do we observe circular rings?

The thickness of the film is uniform at a certain distance from the point of contact between plano-convex lens and glass plate. The condition of constructive or destructive interference is satisfied by this film.

The thickness of the film decreases gradually. This is because the angle of the wedge goes on increasing as we move away from the point of contact between plano-convex lens and glass plate.

As W $\propto 1/\theta$, the thickness of the films decreases gradually.

In ΔOPQ, $(R-t)^2 + r_n^2 = R^2$

$$R^2 - 2Rt + t^2 + r_n^2 = R^2$$

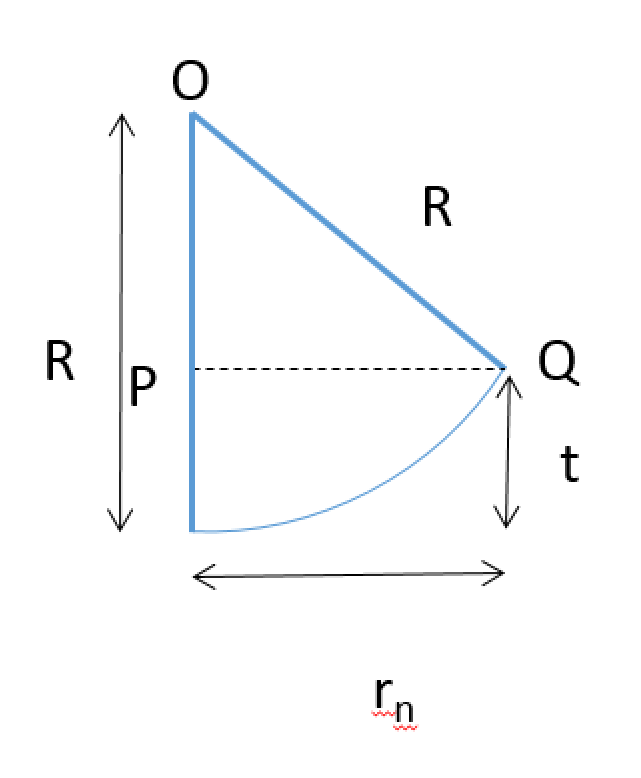
As t<<R, above equation can be simplified as

$r_n^2 = 2Rt$

Hence , $D_n^2 = 8Rt$ -----(1)

For a wedge shaped film, condition for constructive interference

gives, $2\mu t\cos(r+\theta) = (2n-1)\lambda/2$



For small angle approximation, $r \to 0$ and $\theta \to 0$.

Hence, $t = \frac{(2n-1)\lambda}{4\mu}$ -----(2)

Thus, from eq. (1) and (2), condition for a bright rings is

$$D_n^2 = \frac{2(2n-1)\lambda R}{\mu}$$

For a wedge shaped film, condition for destructive interference

gives, $2\mu t \cos(r+\theta) = n\lambda$.

For small angle approximation, $r \to 0$ and $\theta \to 0$.

Hence, $t = \frac{n\lambda}{2\mu}$ -----(3)

Thus, from eq. (1) and (3), condition for a dark rings is

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

Problem – In Newton's rings experiment, light of wavelength λ_1=6000 A^0 and λ_2=4800 A^0 is used. The radius of curvature of plano-convex lens is 0.96m. If n^{th} dark ring of λ_1 coincides with $n+1^{th}$ dark ring of λ_2 , find n and diameter of n^{th} ring.

As $D_n = D_{n+1}$ and $D_n^2 = 4n\lambda_1 R$, $D_{n+1}^2 = 4(n+1)\lambda_2 R$, these equations yield

$$n\lambda_1 = (n+1)\lambda_2 \Rightarrow \frac{\lambda_1}{\lambda_2} = 1 + \frac{1}{n}$$

Thus

$$\frac{1}{n} = \frac{\lambda_1}{\lambda_2} - 1 \Rightarrow \frac{1}{n} = \frac{\lambda_1 - \lambda_2}{\lambda_2}$$

Thus,

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{4800}{1200} = 4$$

The diameter of the nth ring is

$$D_n = \sqrt{4n\lambda_1 R} = \sqrt{4 \times 4 \times 6000 \times 10^{-10} \times 0.96} = 3.03\ \times 10^{-3} m = 3.03\ mm$$

Problem- In Newton's ring experiment, light of wavelength 5896A^0 is used. For a liquid medium, radius of 7th bright ring is 0.15 cm. If radius of plano-convex lens is 1m, calculate speed of light in the liquid.

For a bright ring,

$$D_n^2 = 2(2n-1)\lambda R/\mu$$

Thus,

$$\mu = \frac{2(2n-1)\lambda R}{D_n^2}$$

But $\mu = \frac{c}{v}$

Hence,

$$v = \frac{c}{\mu} = \frac{cD_n^2}{2(2n-1)\lambda R} = \frac{3\times 10^8 \times (2\times 0.15\times 10^{-2})^2}{2(14-1)\times 5896\times 10^{-10}\times 1} = 1.76\times 10^8 m/s$$

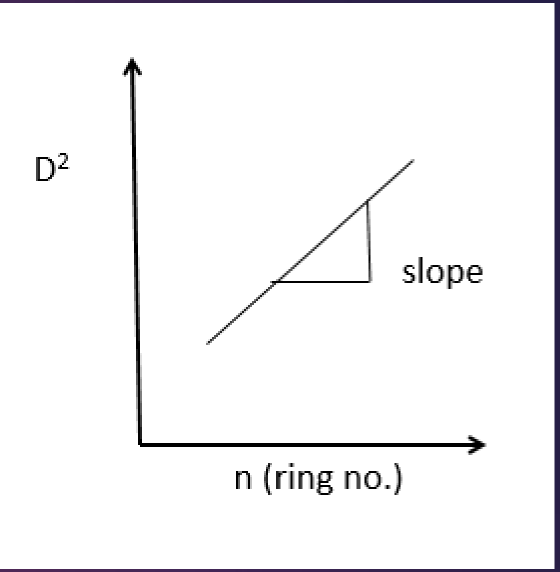
Application of Newton's rings –

1. Determination of wavelength

Diameter of pth dark ring $D_p^2 = 4p\lambda R$ and mth dark ring $D_m^2 = 4m\lambda R$

Thus, $D_p^2 - D_m^2 = 4(p-m)\lambda R$ gives

$$\lambda = \frac{D_p^2 - D_m^2}{4(p-m)R} = \frac{slope}{4R}$$



Problem- In Newton's rings experiment diameter of 15^{th} dark ring is 0.59 cm and of 5^{th} dark ring is 0.336 cm. Find wavelength of light if the radius of plano-convex lens is 1m.

P= 15, m=5. Thus,

$$\lambda = \frac{D_p^2 - D_m^2}{4(p-m)R} = \frac{(0.59 \times 10^{-2})^2 - (0.336 \times 10^{-2})^2}{4 \times 10 \times 1} = 5.88 \times 10^{-7} m = 0.588\ \mu m$$

Determination of refractive index of liquid.

If a liquid of refractive index μ is filled within the space between plano-convex lens and glass plate, then for dark rings,

$D_{\mu p}^2 = \frac{4p\lambda R}{\mu}$ and $D_{\mu m}^2 = \frac{4m\lambda R}{\mu}$

Thus, $D_{\mu p}^2 - D_{\mu m}^2 = \frac{4(p-m)\lambda R}{\mu}$ ---(1)

But for measurements of diameters of p^th^ and m^th^ ring with air as a medium, then $D_p^2 - D_m^2 = 4(p-m)\lambda R$ ----(2)

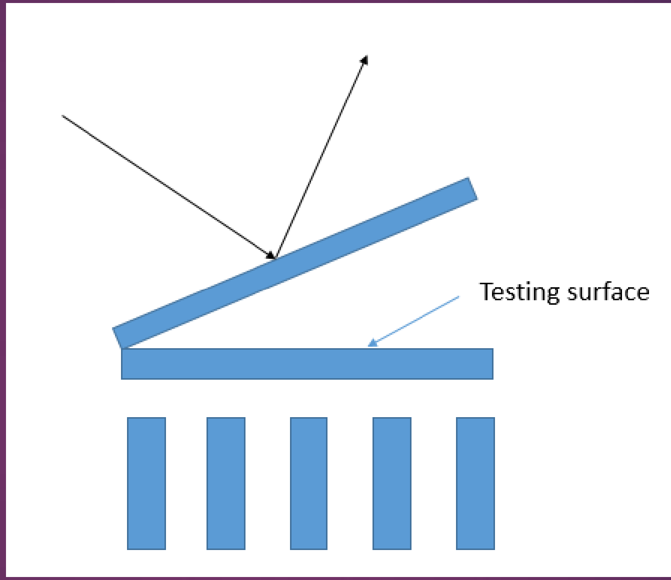
Thus, from eq. 1 and 2,

$\mu = \frac{D_p^2 - D_m^2}{D_{\mu p}^2 - D_{\mu m}^2}$

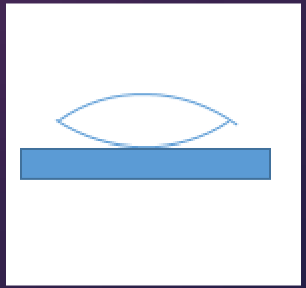
Applications- 1. Anti reflection coating – It is used to reduce the amount of reflected light from a lens of a camera, telescope. The reflected light from the top and bottom of the coated film should undergo destructive interference. The minimum thickness of coated film should be $t_m = \lambda/4\mu$.

The amplitude of reflected light should also be same for complete destructive interference. This is achieved when $\mu = \sqrt{\mu'}$.

2. Testing of optical flatness of surface- The optical flatness of surface can be done using wedge shaped film. Due to irregularities, the fringe pattern in not uniform. The surface is polished until it becomes optically flat. The uniform fringe pattern indicates optically flat surface.



3. Testing of lens surface – In similar way Newton's rings can be used to test surface of a lens. The concentric uniform pattern indicates perfect surface.



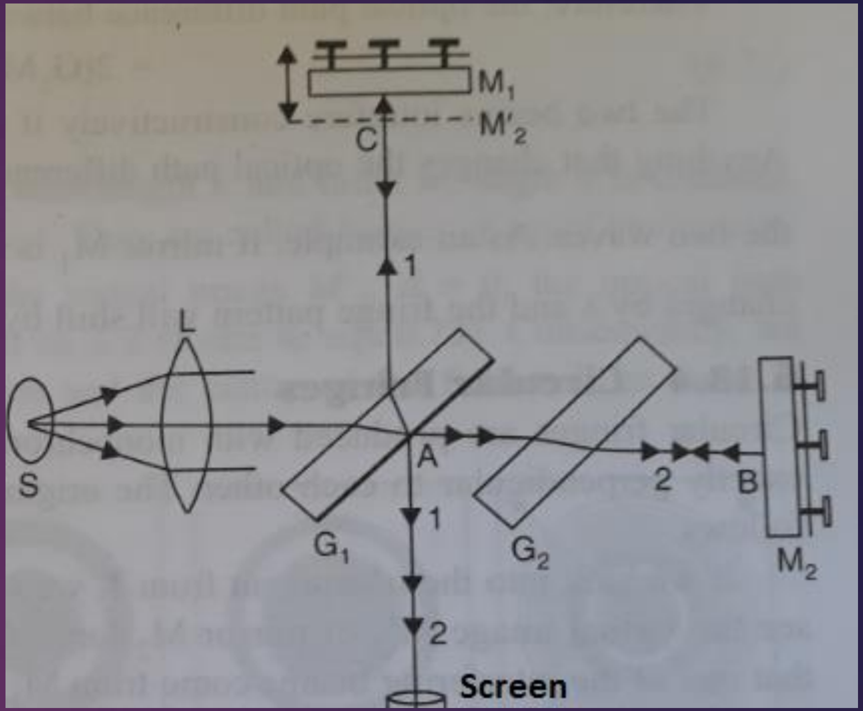
Michelson interferometer –

The interferometer has 1. monochromatic source (S)

2. Silver coated beam splitter (G_1) 3. Mirrors (M_1, M_2)

4. Compensator (G_2) 5. Screen

The beam splitter is at angle of 45^0 with the incoming ray from the source. Ray 1gets reflected from mirror M_1 (which can be moved) and retraces its path to reach the screen. It passes through G_1 three times. Ray 2 gets reflected from mirror M_2 (which is fixed) and retraces its path to reach the screen.



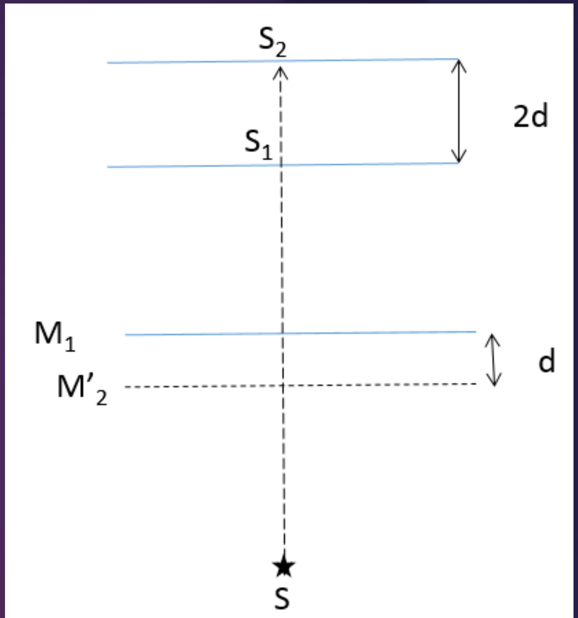
The compensator is placed in the path of ray 2 to make the optical paths of both rays (1 and 2) identical. The compensator has same thickness as beam splitter.

The superposition of ray 1 and 2 produces concentric bright and dark rings on the screen.

Working of Michelson interferometer -

The mirror M_1 and image of mirror M_2 are separated by distance d. The light from the source S gets reflected from these mirrors.

The light reflected from mirror M_1 can be imagined as coming from source S_1 while light reflected from image of mirror M_2 can be imagined as coming from source S_2. Since light moves twice the distance d between M_1' and M_2' to get reflected from M_1, the sources are separated by a distance of 2d. Thus, optical path difference is 2d. For constructive interference this must be equal to $n\lambda$ and for destructive interference it must be equal to $(2n-1)\lambda/2$.



A bright ring is observed when

$$2d\cos\theta = (2n-1)\lambda/2$$

A dark ring is observed when

$$2d\cos\theta = n\lambda$$

Applications- 1. Determination of wavelength – The movement of mirror M_1 causes either collapse or emergence of fringes. The wavelength of monochromatic source can be found using this property.

Let X_i be the initial position of the mirror. A vertical line can be marked on a circular fringe on the screen. The mirror is moved through a distance to final position X_f. During this movement if N fringes cross the line, then

$$N\lambda = 2|X_f - X_i|$$

Hence

$\lambda = \frac{2|X_f - X_i|}{N}$ gives the wavelength.

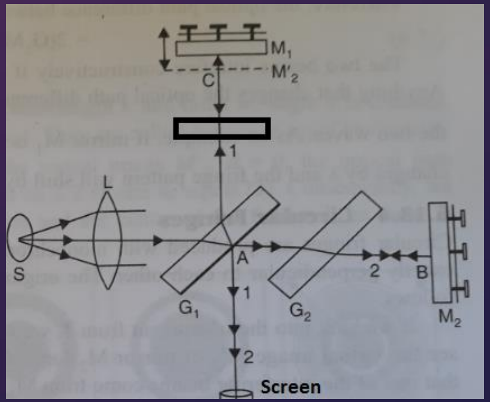
2. Find the thickness or refractive index of a plate –

If a plate of refractive index μ is introduced in the path of ray 1, then the fringe pattern on the screen changes by N rings on the screen. The change in optical path of ray 1 is $\Delta = 2\mu t - 2t$.

But this change is equal to $N\lambda$. Thus,

$$N\lambda = 2t(\mu - 1)$$

Thus, measurement of t can be done if μ is known or vice-versa.



3. Calculation of difference in wavelength – If the light source used in the interferometer has two closely spaced wavelengths such that $\lambda_1 > \lambda_2$, then for a specific 'x', n^{th} dark ring of λ_1 will coincide with n^{th} dark ring of λ_2. This is called as position of maximum distinctness where the sharpness of the pattern is maximum.

As the mirror M_1 is moved, the fringes move on the screen. Since $\lambda_1 > \lambda_2$, the fringes for λ_2 will move at a faster rate on the screen than fringes for λ_1.

For next position of maximum distinctness, n^{th} dark ring of λ_1 will coincide with $n+1^{th}$ dark ring of λ_2. If the mirror has been moved through a distance d, then

$2d = n\lambda_1$ ----(1) and $2d = (n+1)\lambda_2$ -----(2)

Hence $n\lambda_1 = (n+1)\lambda_2 \Rightarrow n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{\lambda_2}{\Delta\lambda}$---(3)

From equation, 1 and 3,

$$2d = \frac{\lambda_1 \lambda_2}{\Delta\lambda}$$

Since the wavelengths are close,

$$\Delta\lambda = \frac{\lambda_{avg}^2}{2d}$$

Problem : A mirror in Michelson interferometer is moved through 0.05 mm and 200 fringes cross the field of view. Calculate the wavelength of light used.

$|X_f - X_i| = 0.05 \times 10^{-3}$m and N=200. Thus,

$$\lambda = \frac{2|X_f - X_i|}{N} = \frac{(2 \times 0.05 \times 10^{-3})}{200} = 0.5\mu m$$

Problem – When a thin film of glass (μ=1.5) is inserted in a path of one of the beams of Michelson interferometer, 30 fringes cross the field of view. If thickness of film is 0.018 mm calculate wavelength of light.

$\mu = 1.5, N = 30, t = 0.018 \times 10^{-3} m$

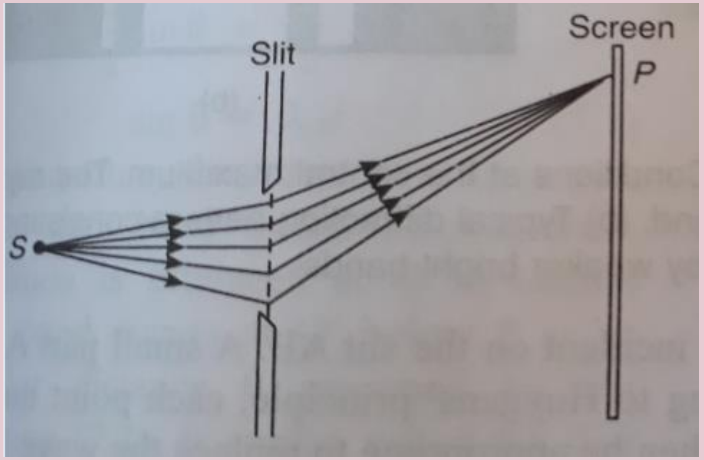
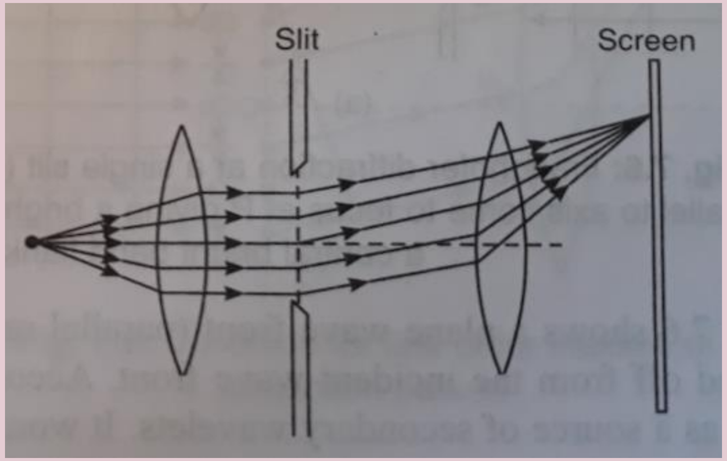
As

$$2t(\mu - 1) = N\lambda$$

then $\lambda = \frac{2t(\mu-1)}{N} = \frac{2\times0.018\times10^{-3}\times(1.5-1)}{30} = 0.6\mu m$

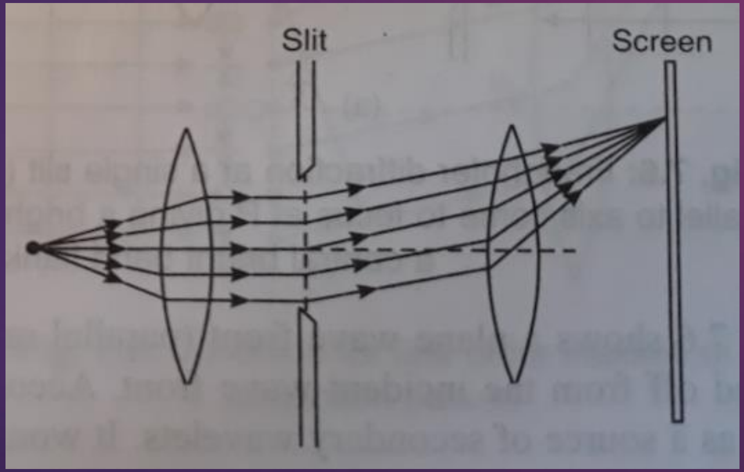
Diffraction - The deviation of a light beam from its linear path occurs when the beam comes across an obstacle, which could be opaque or transparent. Due to such obstacle, the energy distribution of the deviated light gets modified. This effect is called as diffraction.

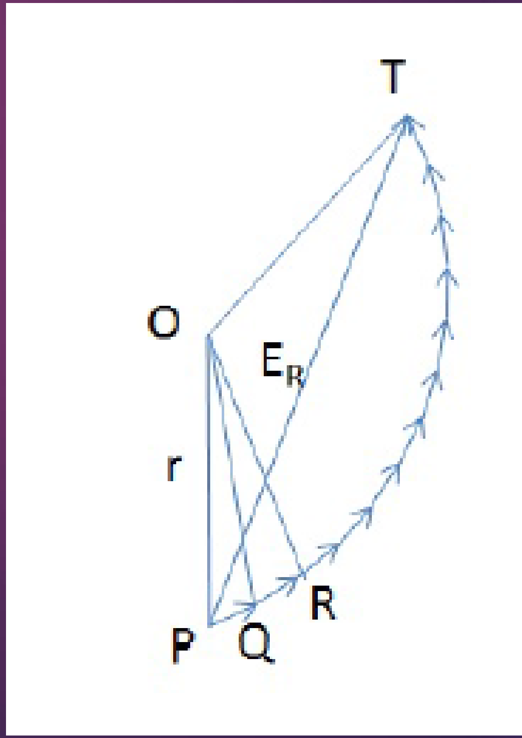
Types of diffraction-

Fresnel diffraction	Fraunhoffer diffraction
	
Source and screen are at a finite distance from slit	Source and screen are at infinite distance from slit
Cylindrical or spherical wavefront	Plane wavefront
Mathematically complex	Mathematically simple

Diffraction due to single slit- A single slit of width b can be split into n number of small slits of equal widths. These coherent sources has φ as the phase difference between the waves emitted by neighbouring slits. Due to same width of each slit the magnitude of electric field vector, E, is same. At a point on the screen, electric field vectors due to these slits add up to resultant electric field vector, E_R . The magnitude of resultant electric field vector governs the intensity of light observed at a point

For large number of slits, n, addition of vectors, which is an equi-angle polygon, forms a circle of radius r.



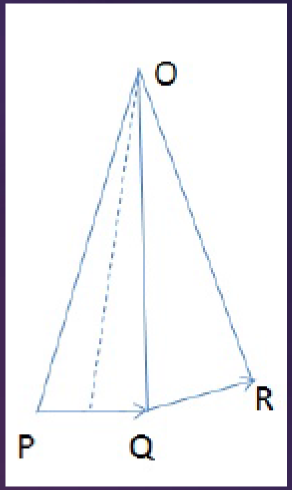


In ΔOPQ, $PQ = QR = E$, $OP = OQ = OR = r$ and $\angle POQ = \angle QOP = \phi$

A perpendicular is drawn from O to PQ. From geometry of diagram,

$$\sin \phi/2 = \frac{E/2}{r}$$

$$r = \frac{E/2}{\sin\frac{\phi}{2}} ----(1)$$

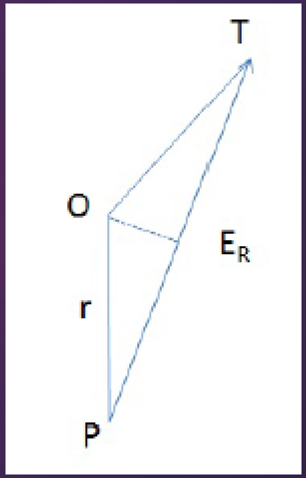


In ΔOPT, $PT = E_R$, $OP = OT = r$ and $\angle POT = n\phi$

From geometry of diagram,

$$\sin n\phi/2 = \frac{E_R/2}{r}$$

$$r = \frac{E_R/2}{\sin\frac{n\phi}{2}} ----(2)$$

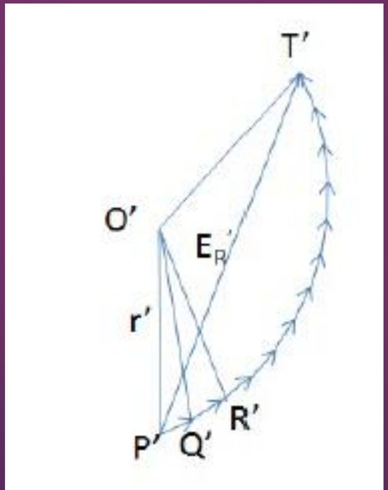


From eq. (1) and (2), $E_R = E\ \frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}$ Since φ is small, $\sin \phi/2 = \phi/2$.

$$E_R = nE\ \frac{\sin\frac{n\phi}{2}}{n\varphi/2} \Rightarrow E_R^2 = (nE)^2 \sin^2\alpha\,/\alpha^2 \Rightarrow I = \frac{I_0 \sin^2\alpha}{\alpha^2}$$

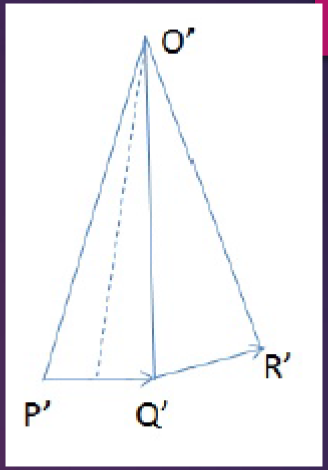
Diffraction due to N slits- Let N slits of width b be separated be a distance a. These coherent sources has δ as the phase difference between the waves emitted by neighbouring slits. Due to same width of each slit the magnitude of electric field vector, E', is same. At a point on the screen, electric field vectors due to these slits add up to resultant electric field vector, E'_R . The magnitude of resultant electric field vector governs the intensity of light observed at a point

For large number of slits, N, addition of vectors, which is an equi-angle polygon, forms a circle of radius r'.



In $\Delta O'P'Q'$, $P'Q' = Q'R' = E'$, $O'P' = O'Q' = O'R' = r'$ and $\angle P'O'Q' = \angle Q'O'P' = \delta$

A perpendicular is drawn from O' to P'Q'. From geometry of diagram,

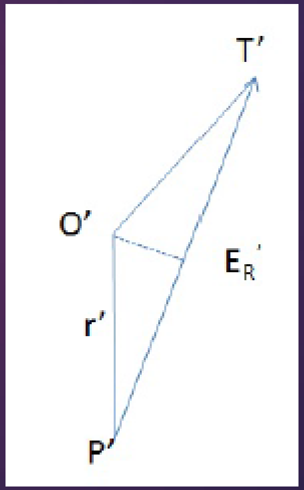


$$\sin \delta/2 = \frac{E'/2}{r'}$$

$$r' = \frac{E'/2}{\sin\frac{\delta}{2}} ----(1)$$

In $\Delta O'P'T'$, $P'T' = E_R{}'$, $O'P' = O'T' = r'$ and $\angle P'O'T' = N\delta$

From geometry of diagram,



$$\sin N\delta/2 = \frac{E_R{}'/2}{r'}$$

$$r' = \frac{E_R{}'/2}{\sin\frac{N\delta}{2}} ----(2)$$

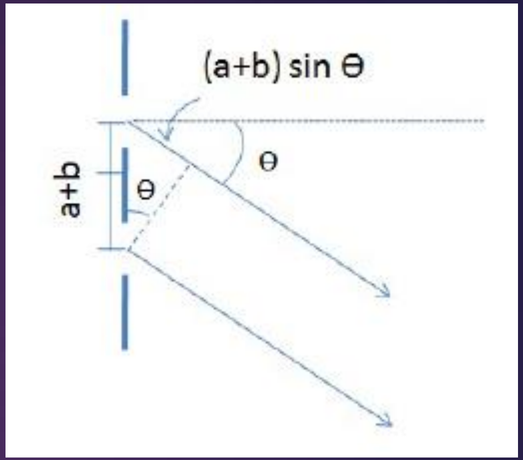
From eq. (1) and (2), $E_R{}' = E' \frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}$

$E_R'^2 = E'^2 \sin^2 N\beta \,/\, \sin^2 \beta \Longrightarrow I' = I\frac{\sin^2 N\beta}{\sin^2 \beta} \Longrightarrow$

$$I' = \frac{I_0 \sin^2 \alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2 \beta}$$

The optical path difference between rays emerging from neighbouring slits is

$$\Delta = (a+b)\sin\theta$$



As phase difference $\delta = \frac{2\pi}{\lambda}\Delta$

$$\delta = \frac{2\pi}{\lambda}(a+b)\sin\theta$$

As $\beta = \delta/2$, $\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$

For $\beta = n\pi,\ n = 0, \pm 1, \pm 2, \ldots$ which implies

$$\lim_{\beta \to n\pi} \frac{\sin N\beta}{\sin\beta} = \pm N$$

Hence, the maximum intensity is $I' = \frac{I_0 \sin^2\alpha}{\alpha^2} N^2$.

Thus, constructive interference is observed when

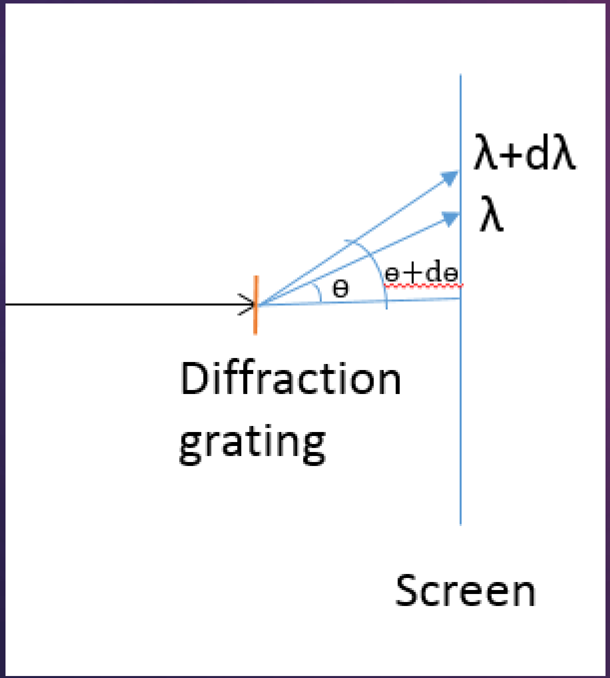
$$(a+b)\sin\theta = n\lambda$$

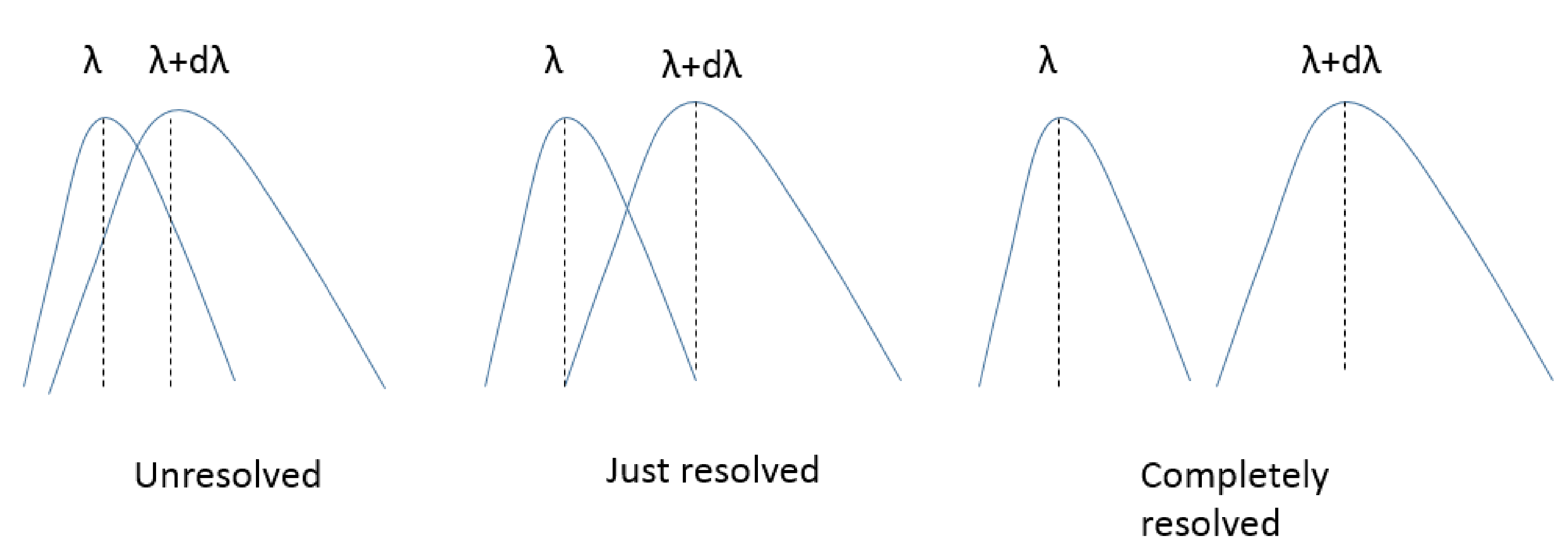
For minimum intensity $\sin N\beta = 0$. Hence destructive interference occurs at $\beta = \frac{m\pi}{N}$

Thus, destructive interference is observed when $N(a+b)\sin\theta = m\lambda\ ,\ m \neq nN$

Resolving power of grating –Consider a source emitting light of wavelength λ and λ+dλ. The n^{th} orders of λ and λ+dλ are observed at angle ɵ and ɵ +dɵ respectively.

In case of diffraction grating resolving power refers to ability of grating to separate two closely space wavelengths. According to Rayleigh criterion, the maximum and minimum of λ and λ+dλ coincide, then the wavelength are just resolved. The cases of unresolved and completely resolved are as below.





If the wavelengths are just resolved, then nN+1th minimum of λ coincides with nth maximum of λ+dλ. Hence from condition of destructive interference for λ,

$N(a+b)\sin(\theta+d\theta) = (nN+1)\lambda$---(1)

and constructive interference for λ+dλ,

$(a+b)\sin(\theta+d\theta) = n(\lambda+d\lambda)$---(2)

From eq. (1) and (2),

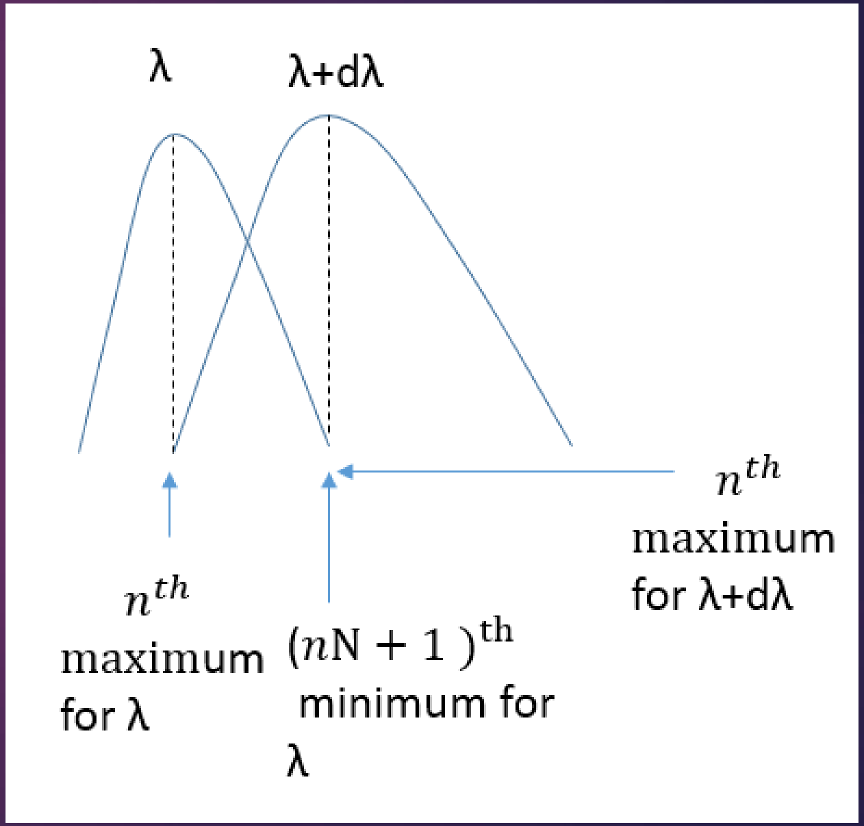
$$N[n(\lambda+d\lambda)] = (nN+1)\lambda$$

$$Nn\lambda + Nnd\lambda = nN\lambda + \lambda$$

Thus,

$$\frac{\lambda}{d\lambda} = Nn$$

which is resolving power of the grating.



Problem: A light of wavelength 550 nm falls normally on a grating having grating constant 2.2 μm. Determine angular position of 2nd and 3rd order.

a+b=$2.2 \times 10^{-6}m$, λ=550×10^{-9}m.

Since $(a+b)\sin\theta = n\lambda$, $\theta = \sin^{-1}(\frac{n\lambda}{a+b})$

For n=2,

$$\theta = \sin^{-1}\left(\frac{2 \times 550 \times 10^{-9}}{2.2 \times 10^{-6}}\right) = \sin^{-1} 0.5 = 30^0$$

For n=3,

$$\theta = \sin^{-1}\left(\frac{3 \times 550 \times 10^{-9}}{2.2 \times 10^{-6}}\right) = \sin^{-1} 0.75 = 48.6^0$$

Problem : For a diffraction grating, 2nd order is observed at 30^0 at 500 nm. Find number of lines per cm on the grating.

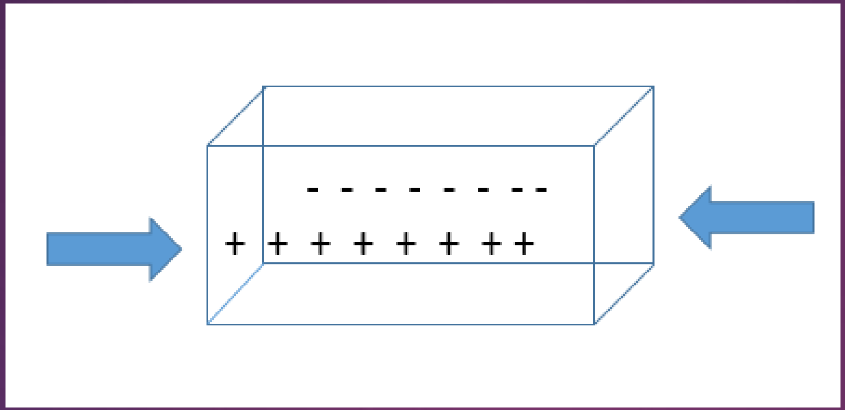
$$(a+b) = \frac{n\lambda}{\sin\theta} = \frac{2 \times 500 \times 10^{-9}}{\sin 30^0} = 2 \times 10^{-6}m = 2 \times 10^{-4}cm$$

Hence no. of lines per cm $= \frac{1}{a+b} = \frac{1}{2\times10^{-4}} =$ 5000 lines per cm

Ultrasonic- Sound that has frequency more than 20kHz.

Ultrasonic waves can be generated using inverse piezoelectric effect.

In piezoelectric effect, when force is applied on along the two faces of piezoelectric crystal, equal and opposite charges appear on the other two faces of the crystal.

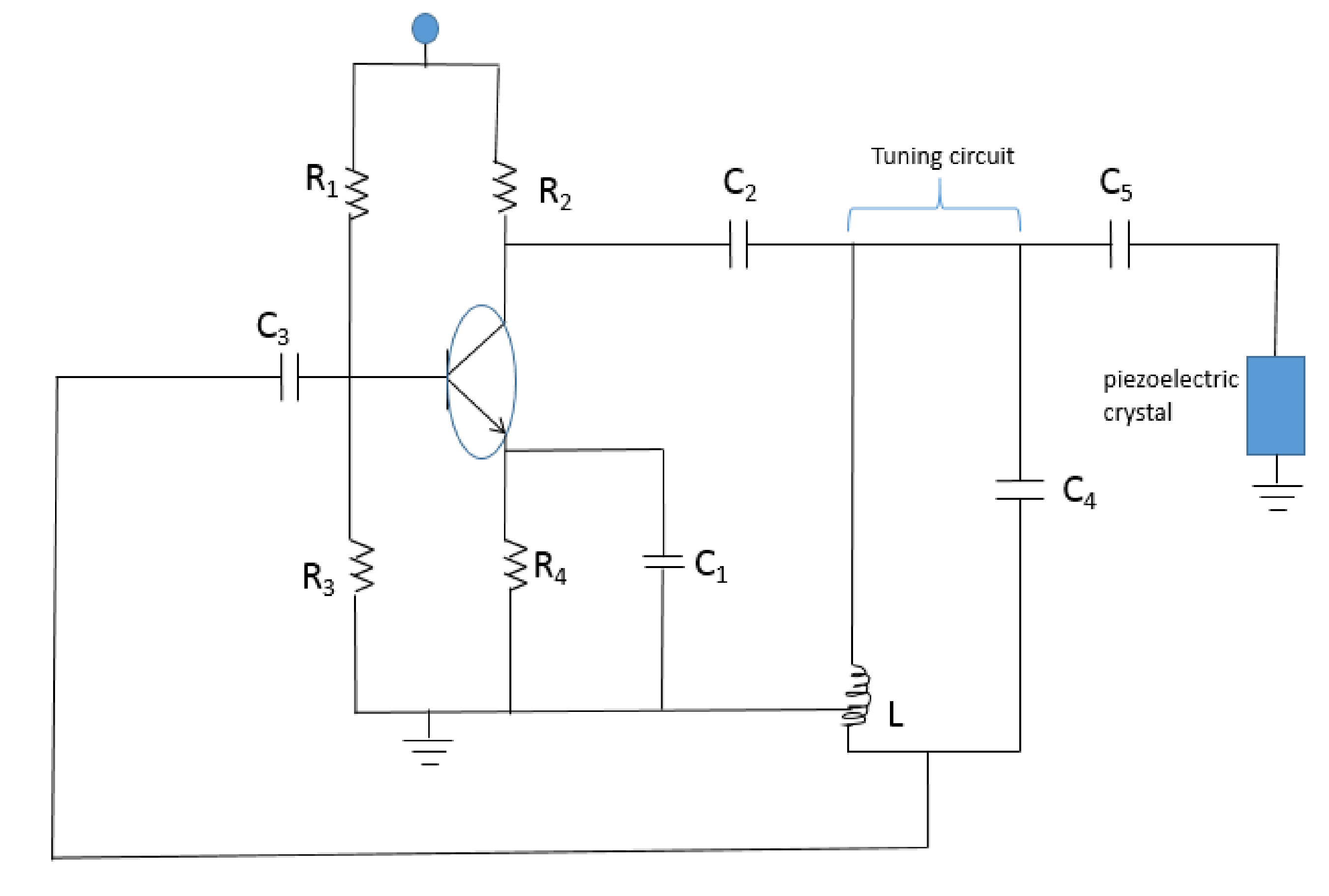


In inverse piezoelectric effect, if voltage is applied along two faces of the crystal then then dimension of the crystal changes along other two faces of the crystal.

If polarity of voltage is varied with a known frequency, then the other two faces vibrate with the same frequency generating longitudinal waves in the surrounding medium. For frequency greater than 20KHz, ultrasonic waves are generated in the surrounding medium.

In the ultrasonic wave generator circuit, capacitor C_3 provides positive feedback to amplifier circuit. The npn transistor operates only when the voltage reaches 0.7 V.

The tuning circuit controls the frequency of oscillation. The amplifier circuit controls the applied voltage. This in turn controls the intensity of ultrasonic waves. Ultrasonic waves upto frequency 800 MHz can be generated using this circuit.



Applications of ultrasonic waves- Non destructive testing (NDT) is a technique for detecting defects in a material without affecting its future utility.

Normal beam pulse echo testing – A probe, which acts as transmitter and receiver is used in this technique. The ultrasonic waves are incident normally on the surface of the specimen. The ultrasonic pules is reflected from the rear surface is recorded in CRO. If a defect comes across the path of the waves, then a pulse of lower amplitude is recorded at an earlier instant than the reflected pulse from the rear surface. This gives location of the defect as well.