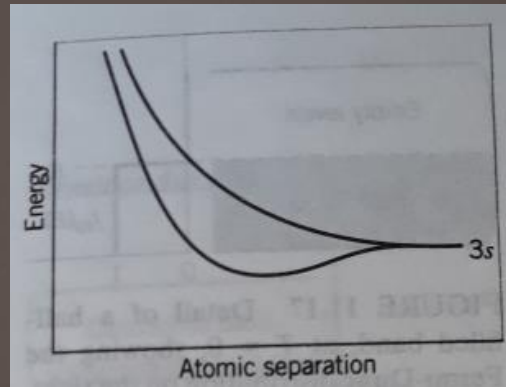


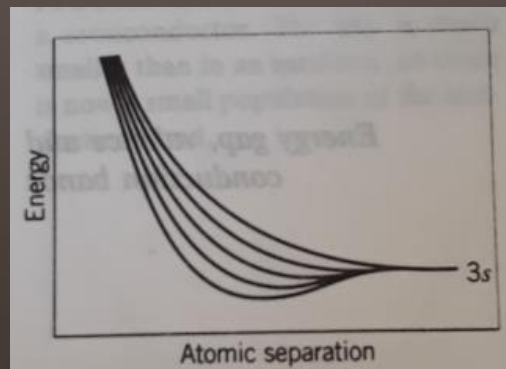
# Condensed matter physics

## Band structure in Solids-

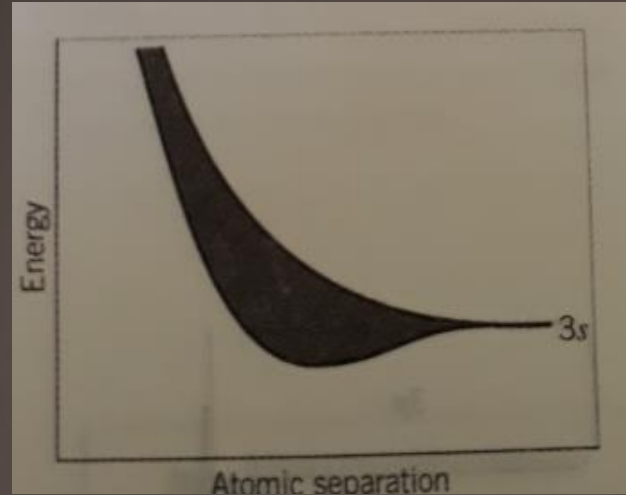
Consider a thought experiment in which 2 sodium atoms are brought close together. As two atoms are brought together, the wavefunction of electrons in 3S level of both atoms overlap. This leads two states of 3S level depending on addition and subtraction of wave functions of electrons.



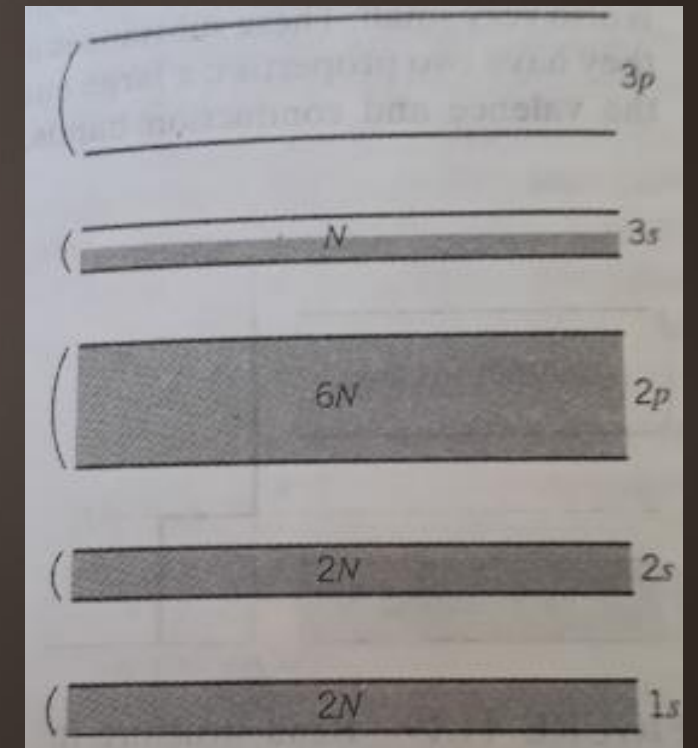
If 5 atoms of sodium are brought together, then 3S level will split into five energy levels.



If 'N', Avogadro's number of atoms are brought together, then the 3s level will split into N energy levels. These levels are closely spaced and thus, a band of N levels is observed for 3s level.

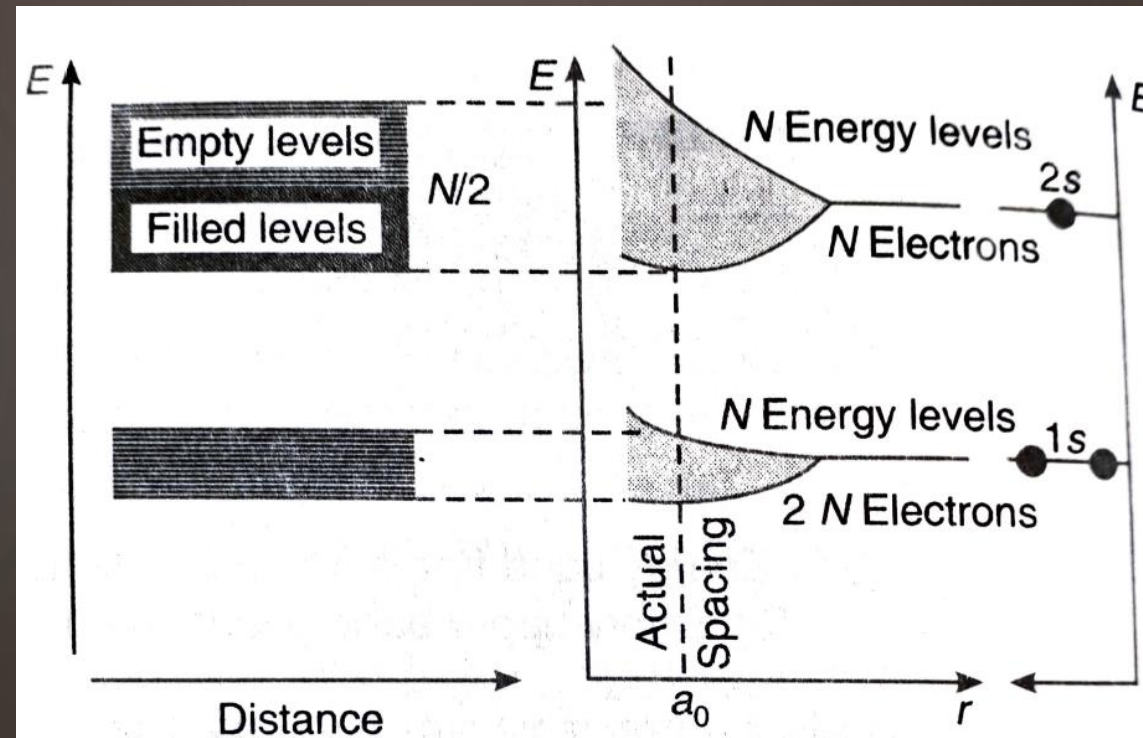


The overlap of wavefunction of electrons in innermost orbitals also takes place. The inner bands are narrower because of low overlap.



Band formation in lithium- The atomic number of lithium is 3 and its electron configuration of lithium is  $1S^2$  and  $2S^1$ . If  $N$  atoms of Li are brought together, then  $1S$  and  $2S$  levels split into a band that has  $N$  levels and  $2N$  energy states.

$1S$  band is completely occupied by  $2N$  electrons and  $2S$  band is occupied by  $N$  electrons. Thus,  $1S$  band is completely filled and  $2S$  band is half-filled. At room temperature, when Li is subjected to electric potential, energy of electrons in  $2S$  band increases. Since the band is half filled, these electrons move to empty energy levels and contribute to current. So, lithium is good conductor of electricity.

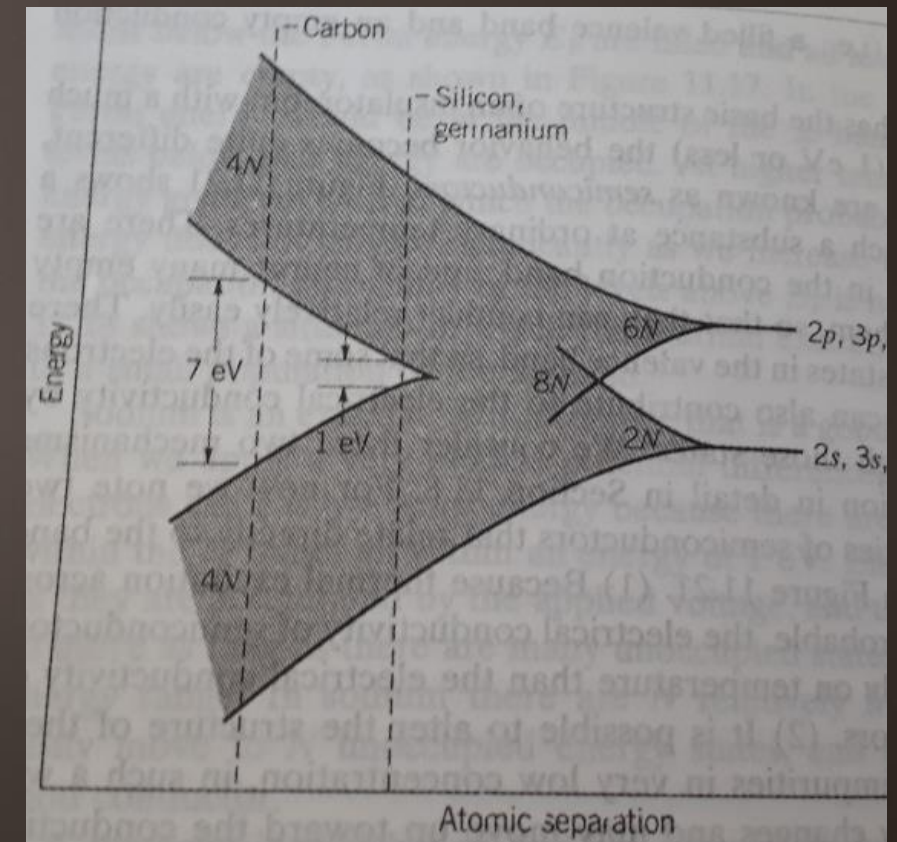


## Band formation in silicon and carbon/diamond –


The atomic number of Silicon and carbon is 14 and 6 respectively. Thus, electronic configuration is Si-  $1S^2, 2s^2, 2P^6, 3S^2, 3P^2$  and C-  $1S^2, 2s^2, 2P^2$

If N atoms of each are brought together, then 2S, 2P levels of C and 3S, 3P levels of Si split into bands as shown in diagram. The P levels splits into a band that has 6N states and S level splits into a band that has 2N states.

As the atomic separation decreases, both bands merge together to form a single band that has 8N states. On further decrease in interatomic distance, the single band splits into two bands each having 4N states. The lower branch is called valence band and the upper branch is called conduction band. At equilibrium distance, the separation between these bands is 1 eV for Si and 7 eV for C.







At absolute zero temperature, the total  $4N$  electrons occupy  $4N$  states energy states of valence band for Si. As the temperature is raised to room temperature of 300K, electrons in the valence band move to the conduction band by crossing a band gap of 1eV. The number of electrons move to conduction band is not very large. Hence at room temperature, under influence of applied electric field, the electrons in the conduction and valence band move to empty energy levels that are available. Because of less number of charge carriers, the conductivity is moderate for Si.

For C, at absolute zero,  $4N$  electrons occupy  $4N$  state in the valence band and thus, conduction band is completely empty. At equilibrium, the separation between conduction band and valence band is 7eV. At room temperature, electrons in the valence band cannot move to conduction band due to large band gap energy. Hence, due to very low number of electrons in the conduction band, conductivity of C is found to be very low, which is characteristic of an insulator.

Fermi-Dirac distribution function/Fermi factor-

Fermi distribution function is applicable to indistinguishable particles that have half integral spin.

The function gives the probability of occupancy by a fermion of a level of energy  $E$  at temperature  $T$ . Mathematically, the function is defined as

$$F(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

where,  $E_F$  is the Fermi function.

At absolute zero,  $T=0K$ , for energy levels above Fermi energy i.e.,  $E > E_F$ , then  $E-E_F > 0$ , the distribution function becomes,

$$F(E) = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty} = 0$$

Thus, at absolute zero, the energy levels above Fermi level are empty.

At absolute zero, for energy levels below Fermi energy i.e.,  $E < E_F$ , then  $E - E_F < 0$ , the distribution function becomes,

$$F(E) = \frac{1}{e^{-\infty} + 1} = \frac{1}{1} = 1$$

Thus, at absolute zero, the energy levels below Fermi level are completely occupied.

At absolute zero, for Fermi energy i.e.,  $E = E_F$ , the distribution function becomes,

$$F(E) = \frac{1}{e^{0/0} + 1}, \text{ which gives } 0 < F(E) < 1.$$

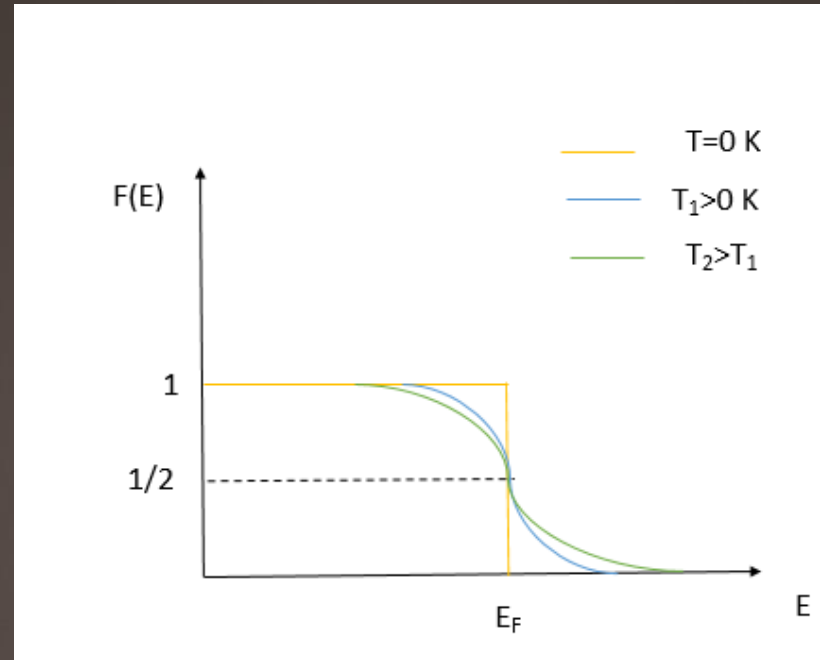
Thus, at absolute zero, the energy level below which all the energy levels are completely filled and all the energy levels above it are completely empty is called as Fermi energy.

For  $T \neq 0$  and  $E = E_F$ , the distribution function is

$$F(E) = \frac{1}{e^0 + 1} = \frac{1}{2}$$

Thus, for Fermi energy, the probability of occupancy is  $\frac{1}{2}$  for all temperatures except absolute zero.

## Temperature dependence of Fermi factor/ Fermi distribution function –



At absolute zero temperature, the probability of occupancy of energy levels below Fermi level is 1 and above is zero. In the graphs, the step function shows the Fermi distribution function for absolute zero.

For  $T_1$ , electrons just below the Fermi level move to energy levels that are above Fermi level. The graph for  $T_1$  indicates this. For further increase in temperature, more electrons move to energy levels above Fermi level which is shown by graph for  $T_2$ .



Conductivity of a conductor and intrinsic semiconductor- The free electrons in a conductor at temperature T move at a velocity known as thermal velocity in random directions.

The kinetic energy of these electrons is  $\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT$ .

When electric field is applied to a conductor, these electrons move under the influence of applied electric field with a velocity called as drift velocity, which is given as  $v_d = \mu E$ , where  $\mu$  is the mobility of electrons and E is the applied electric field.

The current density in a conductor is defined as  $J = nev_d$

$J = ne\mu E$  — — — (1), from the definition of drift velocity.

According to Ohm's law,  $J = \sigma E$  — — — — (2).

Thus, from eq. (1) and (2), conductivity of metal is given as

$$\sigma = ne\mu$$

For intrinsic semiconductor, both electrons and hole contribute to the current. Thus,

$$J = J_e + J_h = ne\mu_e E + pe\mu_h E$$

For intrinsic semiconductor,  $n = p = n_i$ , Thus, the current density is

$$J = n_i e (\mu_e + \mu_h) E$$

Thus, conductivity of intrinsic semiconductor is  $\sigma = n_i e (\mu_e + \mu_h)$ .

Fermi energy in intrinsic semiconductor-

For intrinsic semiconductor, number density of electrons in the conduction band is given by

$$n = N_C e^{(E_C - E_F)/kT}$$

And number density of holes in the valence band is given by

$$p = N_V e^{(E_F - E_V)/kT}$$

where  $N_C = 2 \left( \frac{2\pi m_e^* kT}{h^2} \right)^{\frac{3}{2}}$  and  $N_V = 2 \left( \frac{2\pi m_h^* kT}{h^2} \right)^{\frac{3}{2}}$

Here  $E_C$  is the lowest level of conduction band,  $E_F$  is the Fermi energy  $k$  is Boltzmann constant,  $T$  is temperature  $E_V$  is the highest level of valence band,  $m_e^*$  is the effective mass of the electron and  $m_h^*$  is the effective mass of the hole.

Since the electron density and hole density is same for intrinsic semiconductor,  $n=p$  implies

$$2 \left( \frac{2\pi m_e^* kT}{h^2} \right)^{\frac{3}{2}} e^{(E_C - E_F)/kT} = 2 \left( \frac{2\pi m_h^* kT}{h^2} \right)^{\frac{3}{2}} e^{(E_F - E_V)/kT}$$

$$e^{(E_C + E_v - 2E_F)/kT} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

$$\frac{E_C + E_v - 2E_F}{kT} = \frac{3}{2} \ln \frac{m_h^*}{m_e^*}$$

If the effective mass of electron and hole is same,  $m_h^* \cong m_e^*$ . Thus,  $\frac{(E_C + E_v)}{2} = E_F$  which gives  $E_F = E_g/2$ .

Thus, Fermi energy lies in the forbidden band for intrinsic semiconductor.

Problem- Show that Fermi energy distribution function is symmetric about Fermi energy for all temperatures, i. e.,  $F(E_F + \Delta E) + F(E_F - \Delta E) = 1$

$$\text{Since, } F(E) = \frac{1}{e^{(E - E_F)/kT} + 1}, \quad F(E_F + \Delta E) = \frac{1}{e^{(E_F + \Delta E - E_F)/kT} + 1} = \frac{1}{e^{\Delta E/kT} + 1} \text{ ---- (1)}$$

$$F(E_F - \Delta E) = \frac{1}{e^{(E_F - \Delta E - E_F)/kT} + 1} = \frac{1}{e^{-\Delta E/kT} + 1} \text{ ---- (2)}$$

$$\begin{aligned} \text{Thus, } F(E_F + \Delta E) + F(E_F - \Delta E) &= \frac{1}{e^{\Delta E/kT} + 1} + \frac{1}{e^{-\Delta E/kT} + 1} = \frac{1}{e^{\Delta E/kT} + 1} + \frac{e^{\Delta E/kT}}{e^{\Delta E/kT} + 1} \\ &= \frac{e^{\Delta E/kT} + 1}{e^{\Delta E/kT} + 1} = 1 \end{aligned}$$

Problem – The Fermi energy of Li is 4.72eV. (a) Calculate the Fermi velocity. (b) Calculate de-Broglie wavelength of electron moving at the Fermi velocity.

$$E_F = \frac{1}{2}mv_F^2. \text{ Thus, } v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2 \times 4.72 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.29 \times 10^6 \text{ m/s}$$

de-Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.29 \times 10^6} = 0.56 \text{ nm}$$

Problem – At what temperature can we expect a 10% probability that electrons in silver have an energy which is 1% above Fermi energy of silver.  $E_F = 5.5 \text{ eV}$ .

$$F(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

But  $E = E_F + 0.01E_F$ . Thus,  $E - E_F = 0.01E_F = 0.01 \times 5.5 = 0.055 \text{ eV}$ .

$$0.1 = \frac{1}{e^{(0.055 \text{ eV})/kT} + 1}$$

Hence,  $e^{(0.055 \text{ eV})/kT} + 1 = 10$

$$\text{Thus, } T = \frac{0.055 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times \ln 9} = 290 \text{ K}$$

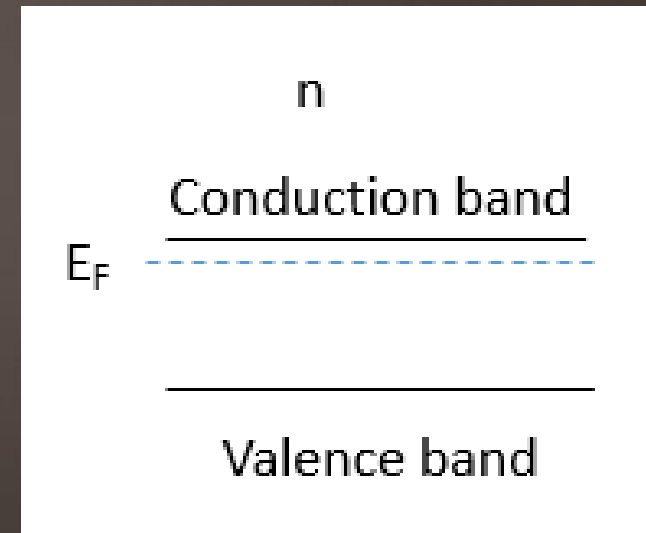
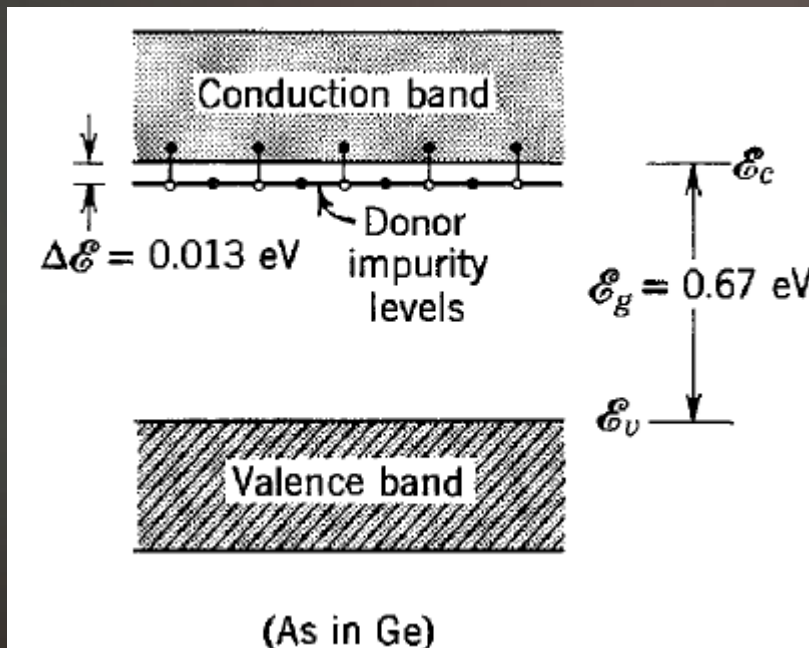
Problem – Find the temperature at which there is 1% probability that a state of energy 2 eV is occupied by electron. ( $E_F=1.5$  eV)

$$F(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$0.01 = \frac{1}{e^{(2-1.5) \times 1.6 \times 10^{-19} / (1.38 \times 10^{-23} T)} + 1}. \text{ Thus, } T = (0.5 \times 1.6 \times 10^{-19}) / (1.38 \times 10^{-23} \times \ln 99) = 1262 \text{ K}$$

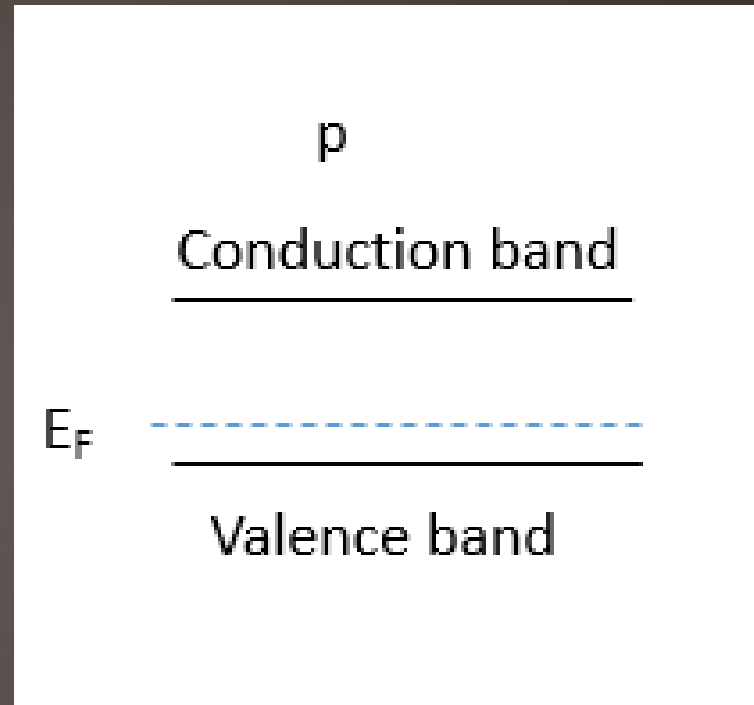
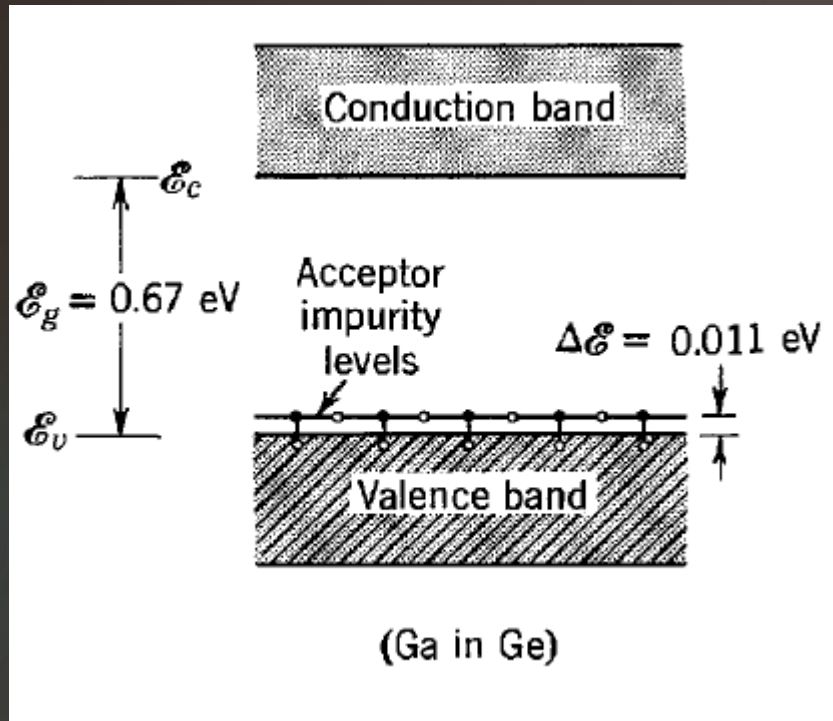
Fermi level in extrinsic semiconductor-

n type semiconductor – The donor atoms causes increase in concentration of electrons. This results in shift of Fermi level towards conduction band.

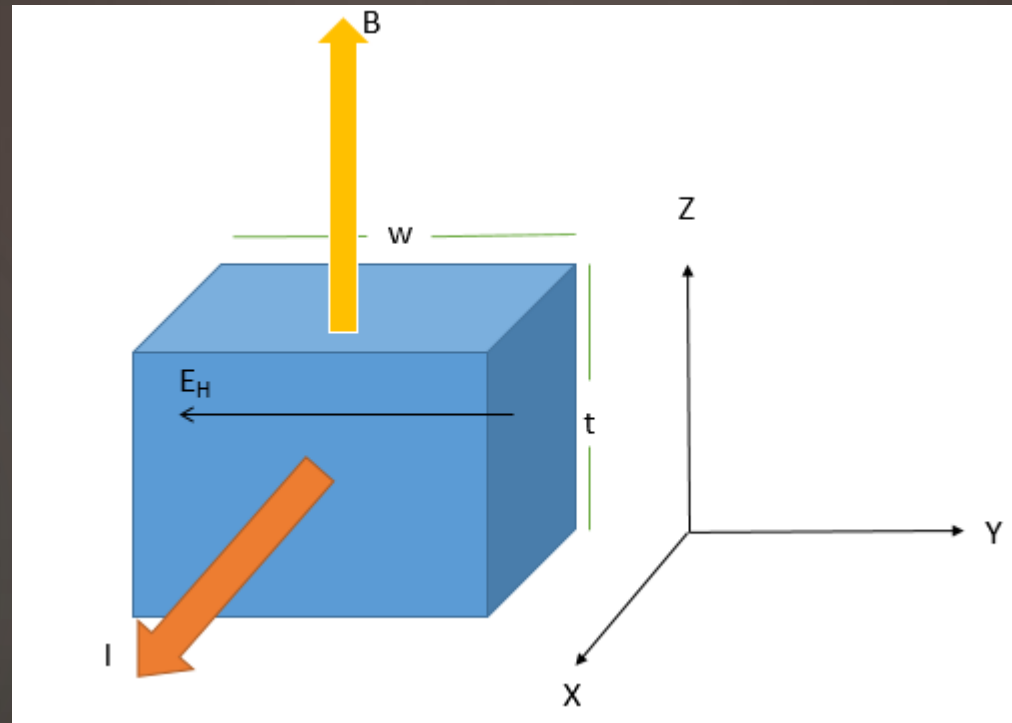




p type semiconductor – The acceptor atoms causes increase in concentration of holes in the valence band. This results in shift of Fermi level towards valence band.



Hall effect – Consider n type semiconductor of width  $w$  and thickness  $t$ . The current flows along X axis as shown in the diagram. A magnetic field is applied along Z axis. An electric field gets established in the semiconductor due to application of magnetic field. The electric field is perpendicular to both current and magnetic field. This is known as Hall effect. The voltage established is called as Hall voltage.



Electrons are the majority carriers in n type semiconductor.

The force acting on a charged particle moving in electric and magnetic field is given by Lorentz force,

$$\vec{F}_L = q(\vec{E} + \vec{v} \times \vec{B})$$

Since the electron moves in the applied magnetic field, the Lorentz force is given as  $\vec{F}_L = -e(-v_d \hat{i} \times B \hat{k}) = ev_d B(-\hat{j}) = -ev_d B \hat{j}$

The electrons move towards negative Y axis because of this force and start accumulating on the face of the semiconductor. To maintain charge neutrality in the semiconductor, equal and opposite charge build up on the other face of the semiconductor. Due to this, electric field gets established.

The field also exerts a force on electron. The process of accumulation of electron continues until this force balances Lorentz force. The force due to electric field is

$$\vec{F}_E = eE_H \hat{j}$$

Thus, at equilibrium,  $\vec{F}_L + \vec{F}_E = 0$ , i. e. ,  $-ev_d B \hat{j} + eE_H \hat{j} = 0$ . Hence,

$$v_d = \frac{E_H}{B} \text{ ---- (1)}$$

From the definition of current density,  $J = nev_d$ . So, from eq. (1),  $J = \frac{neE_H}{B} \text{ --- (2)}$

But for the above n type semiconductor,  $J = \frac{I}{wt}$  --- (3).

From eq. (2) and (3)  $\cdot \frac{I}{wt} = \frac{neE_H}{B}$

From definition of electric field,  $E_H = \frac{V_H}{w}$ . Thus,

$$\frac{I}{wt} = \frac{neV_H}{wB}, \text{ which gives } n = \frac{IB}{V_H t e} \text{ --- (4)}$$

All the quantities on RHS of eq. (4) are measurable. Hence, above equation can give the concentration of electrons in n type semiconductor.

Hall coefficient is defined as  $R_H = \frac{1}{ne}$ . From eq. (4), Hall coefficient is expressed as  $R_H = \frac{V_H t}{IB}$

Applications – 1. Determination of type of semiconductor can be done from the sign of electric potential.

2. Determination of carrier concentration in semiconductors and metals can be done.

3. Measurement of mobility- From definition  $\sigma = ne\mu$ . But  $\rho = 1/\sigma$ . Hence  $\frac{1}{\rho} = \frac{\mu}{R_H}$ . Thus  $\mu = R_H/\rho$ . Thus, determination of Hall coefficient and resistivity gives mobility of carriers.

4. Since  $V_H \propto B$ , measurement of magnetic field can be done using a Hall probe.

Problem- Calculate the drift velocity of electrons in aluminium wire of diameter 0.9 mm and carrying a current of 6 A. Assume that  $4.5 \times 10^{28} \text{ m}^{-3}$  electrons are available for conduction.

$$J = nev_d \Rightarrow v_d = \frac{I}{Ane} = \frac{I}{\pi r^2 ne} = \frac{6}{\pi \times (4.5 \times 10^{-3})^2 \times 4.5 \times 10^{28} \times 1.6 \times 10^{-19}} = 1.29 \times 10^{-3} \text{ m/s}$$

Problem- Resistivity of p type silicon is  $9 \times 10^{-3} \Omega \text{ m}$ . The Hall coefficient is  $3.6 \times 10^{-4} \text{ m}^3/\text{C}$ . Find mobility of charge carriers and density of charge carriers.

$$\text{Mobility of charge carriers is } \mu = \frac{R_H}{\rho} = \frac{3.6 \times 10^{-4}}{9 \times 10^{-3}} = 0.04 \text{ m}^2/\text{Vs}.$$

$$\text{Density of charges } n = \frac{1}{R_H e} = \frac{1}{3.6 \times 10^{-4} \times 1.6 \times 10^{-19}} = 1.72 \times 10^{22} \text{ m}^{-3}.$$

Problem – A copper strip 4 cm thick and 0.55 mm wide carries a current of 100 A along its length. If placed in magnetic field of 2T along the thickness of the strip, Hall voltage of 29.7  $\mu\text{V}$  appears across width. Find Hall electric field and carrier concentration.

$$\text{Hall field } E_H = \frac{V_H}{w} = \frac{29.7 \times 10^{-6}}{0.55 \times 10^{-3}} = 5.4 \times 10^{-2} \frac{\text{V}}{\text{m}}$$

$$\text{Carrier concentration } n = \frac{IB}{V_H t e} = \frac{100 \times 2}{29.7 \times 10^{-6} \times 4 \times 10^{-2} \times 1.6 \times 10^{-19}} = 1.05 \times 10^{27} \text{ m}^{-3}$$



Problem- At room temperature, carrier concentration in intrinsic semiconductor is  $1.5 \times 10^{16} \text{ m}^{-3}$ . Find resistivity if  $\mu_e = 0.135 \text{ m}^2/\text{Vs}$  and  $\mu_h = 0.048 \frac{\text{m}^2}{\text{Vs}}$

For intrinsic semiconductor,  $\sigma = n_i e (\mu_e + \mu_h)$  But since

$$\rho = \frac{1}{\sigma} = \frac{1}{1.5 \times 10^{16} \times 1.6 \times 10^{-19} (0.135 + 0.048)} = 2.3 \times 10^3 \Omega \text{m}$$

Problem- In Hall experiment, a current of 0.25 A flows through a sample of 0.2 mm thick and 5 mm wide. For magnetic field of 0.2 T, the Hall voltage is 0.15 mV. (a) What is carrier concentration? (b) What is drift velocity of carriers?

$$(a) \quad n = \frac{IB}{V_H t e} = \frac{0.25 \times 0.2}{0.15 \times 10^{-3} \times 1.6 \times 10^{-19} \times 0.2 \times 10^{-3}} = 1.04 \times 10^{25} \text{ m}^{-3}$$

$$(b) \quad v_d = \frac{J}{ne} = \frac{I}{Ane} = \frac{0.25}{0.2 \times 10^{-3} \times 5 \times 10^{-3} \times 1.04 \times 10^{25} \times 1.6 \times 10^{-19}} = 0.15 \text{ m/s}$$

Problem – A Hall probe records Hall voltage if 0.2 mV for magnetic field of 1T. What will be the magnetic field if Hall voltage is 0.25 mV?

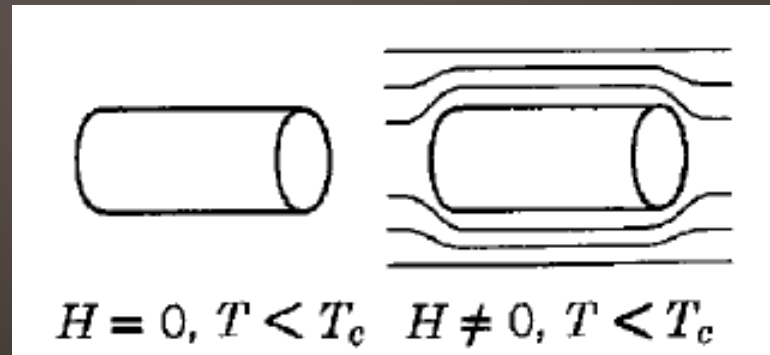
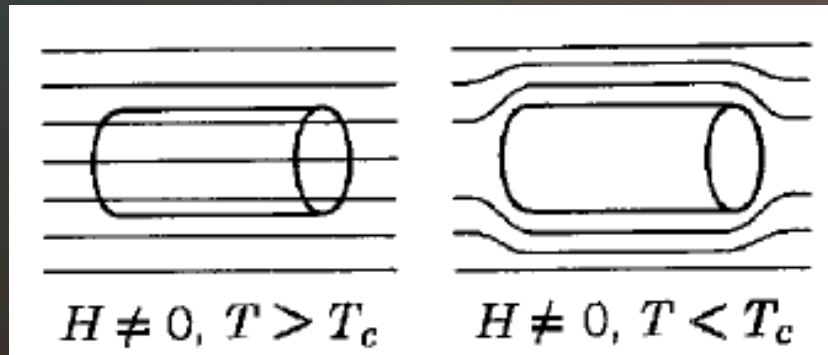
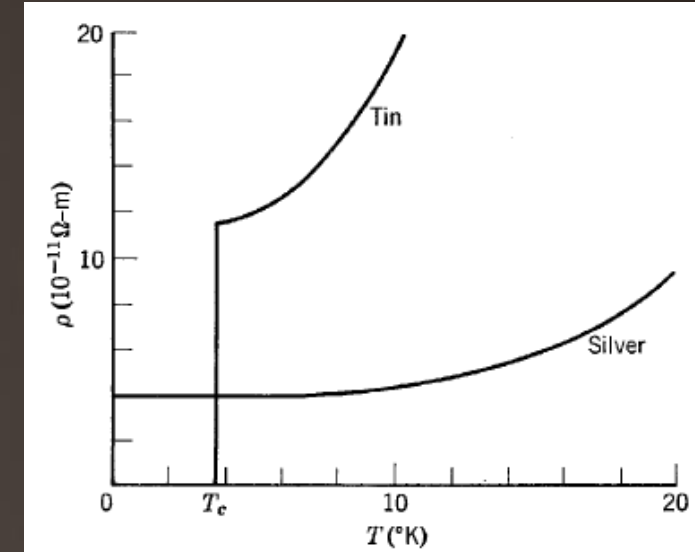
$$\frac{V_{H1}}{V_{H2}} = \frac{B_1}{B_2} \text{ Hence } B_1 = \left( \frac{V_{H1}}{V_{H2}} \right) B_2 = \left( \frac{0.25}{0.2} \right) 1 = 1.25 \text{ T}$$

# Superconductors

<https://www.youtube.com/watch?v=g9hWML3NwN8> (5:00 to 7:25)

The vanishing of resistance (or resistivity) in a material below certain temperature (known as critical temperature) is known as superconductivity.

Meissner effect – Consider a superconductor at temperature  $T > T_c$  in a magnetic field. If the superconductor is cooled below the critical temperature, then magnetic field gets expelled from the superconductor. This is known as Meissner effect.



As  $B = \mu_0(M + H)$ . But as  $M = \chi H$ ,

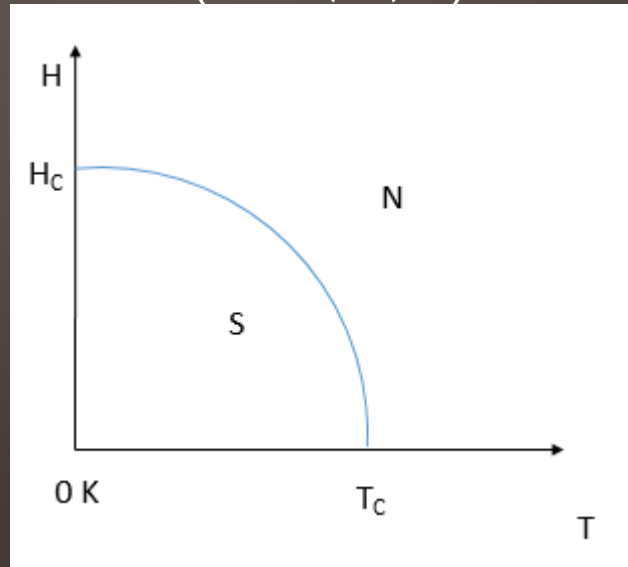
$$B = \mu_0(1 + \chi)H$$

For  $T < T_C$ ,  $B=0$  implies,  $\chi = -1$ .

Thus, below critical temperature, superconductors turn into a perfect dia-magnet. Meissner effect is a quantum mechanical effect that is observed on macroscopic level.

Critical magnetic field- When a superconductor is at a temperature  $T < T_C$ , and if strength of the external magnetic field goes on increasing, then superconducting state gets destroyed beyond a certain magnetic field, which is given by

$$H = H_C \left( 1 - \left( \frac{T}{T_C} \right)^2 \right)$$



Critical current – It is the maximum current that can flow through a superconducting wire of radius  $r$  and is given as

$$I_C = 2\pi r H_C$$

where  $H_C$  is the critical magnetic field of that material.

London penetration depth- It is the depth inside a superconductor where magnetic field strength decreases by a factor of  $1/e$ .

Penetration depth depends on temperature and for  $0 < T < T_C$ ,

$$\lambda(T) = \frac{\lambda(0)}{\left[1 - \left(\frac{T}{T_C}\right)^4\right]^{\frac{1}{2}}}$$

where  $\lambda(0)$  is the penetration depth at 0 K.

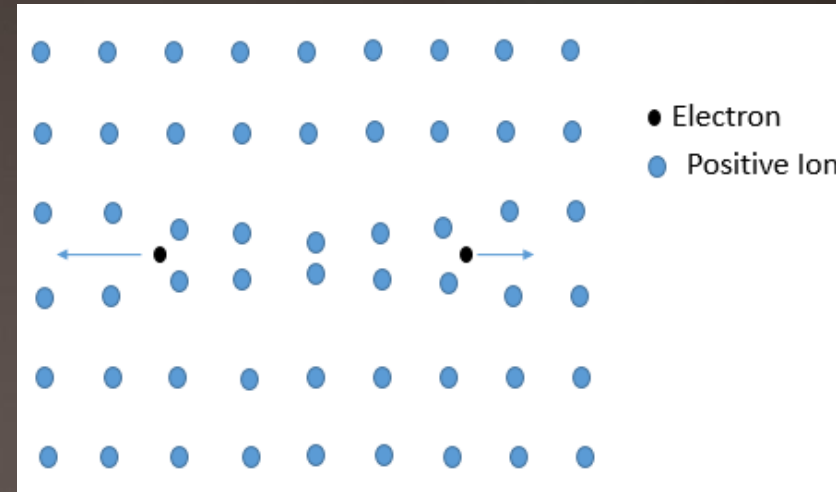
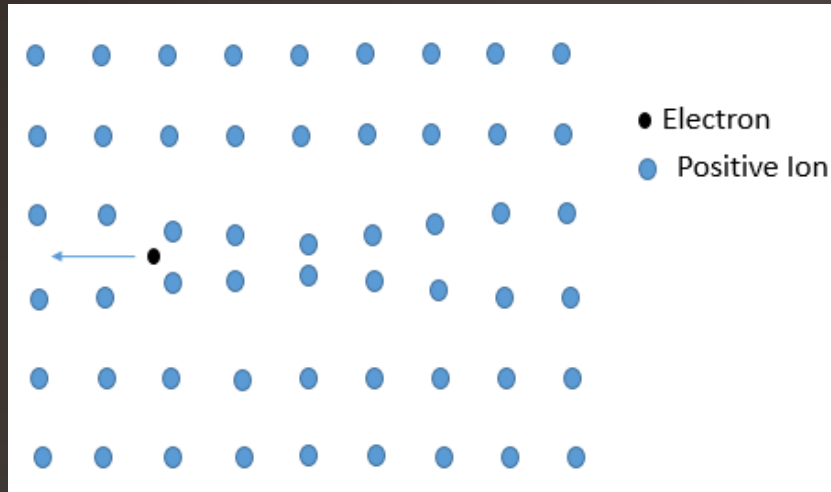
Isotope effect- The critical temperature of a superconductor depends on isotopic mass of the superconductor such that

$$M^{\frac{1}{2}} T_C = \text{constant}$$

This is known as isotope effect.

Persistent current is a steady current in superconductor that flows without any loss.

## BCS theory –



BCS theory predicts that Cooper pair, which is a bound pair of 2 electrons is responsible for superconductivity. In the absence of applied electric field, the formation of Cooper pair occurs as follows – An electron with spin up and momentum ' $p$ ' passes through lattice of positively charged ions towards left. As the electron moves through lattice, it creates a wake of positive charge. Consider another electron with spin down and momentum ' $p$ ' moving towards right. This electron gets attracted to the wake of positive charge.

Below critical temperature, random thermal motion of ions does not disturb the wake. Under such condition, the attraction between 2 electrons which is mediated by wake of positive charge exceeds the repulsion between them. Thus, 2 electrons form a bound pair.



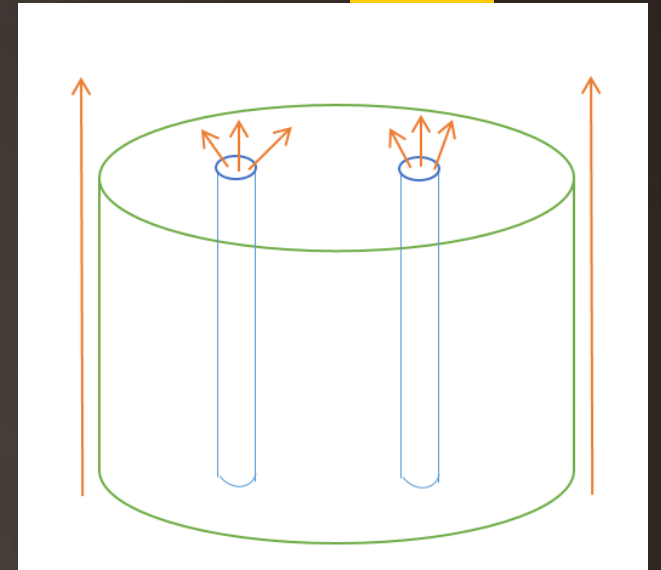
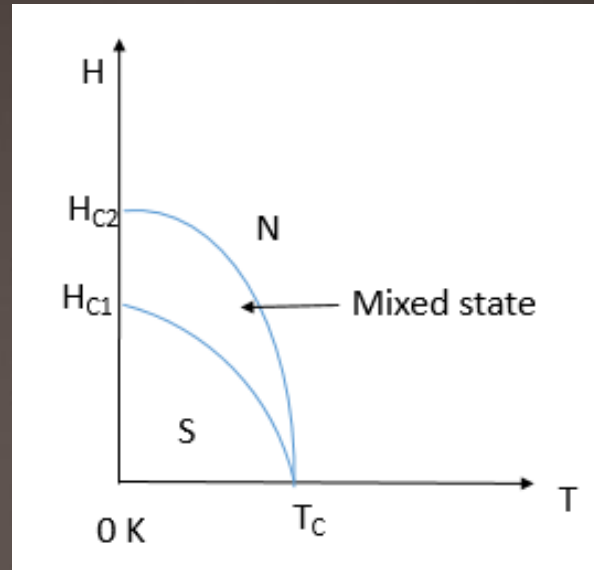
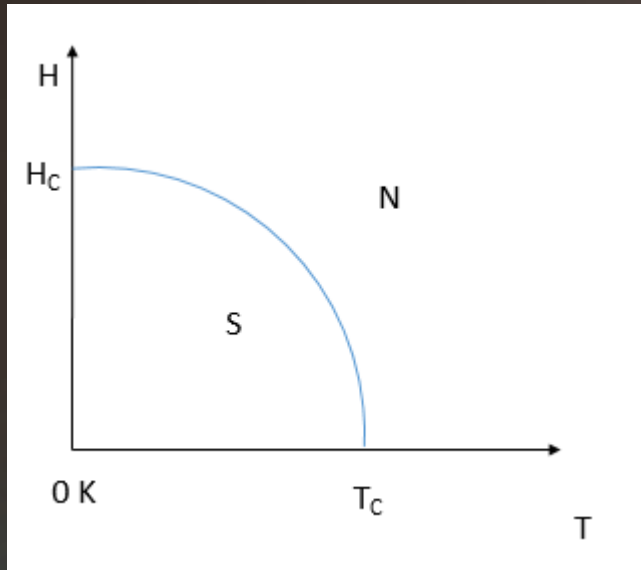
Above the critical temperature, random thermal motion of ions disturbs the wake. Thus, there is decrease in density of positive charge region. This in turn, reduces the attractive interaction between two electrons that has been mediated by the positive charge region and the bound pair breaks up.

At absolute zero, the binding energy of Cooper pair is about  $3kT_C$ . As temperature increases from absolute zero to critical temperature, the binding energy reduces and is zero at critical temperature.

The Cooper pair are weakly bound and hence the constant process of breaking of pair and formation of pair (with different partners) goes on. This implies, all pairs have same constant total momentum which is zero and thus we have highly ordered state. When electric field is applied, Cooper pair move through lattice under applied electric field, During such motion, the pairs maintain the order as no. of pairs is to be maintained maximum. So, the motion of each pair is locked into the motion of all the rest.

Due to this, random scattering of electrons is avoided by lattice imperfections at low temperature giving rise to superconductivity.

## Classification of superconductors – Type I and Type II superconductors

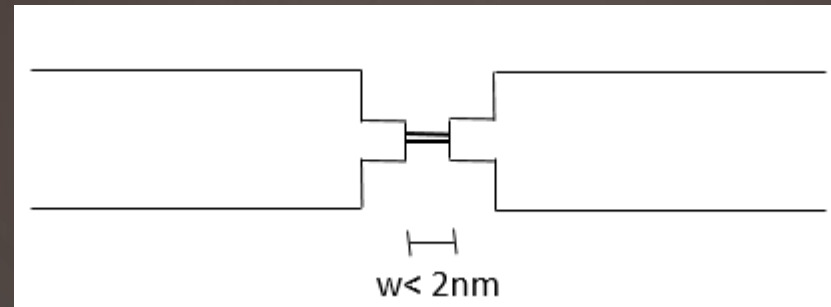


For type I superconductors, the critical magnetic field that destroys the superconducting state is unique at absolute zero. Above this field, superconductor goes into normal state. In case of type II superconductors, two critical fields are observed at absolute zero.

Below  $H_{c1}$ , the material is in the superconducting state. In between and , the material develops set of channels called as vortex cores. In these channels, the material is in the normal state while in the rest of the region, it is in the superconducting state.

So, for type II superconductors, magnetic flux can penetrate vortex core regions. As the external field approaches  $H_{c2}$ , the size of the vortex core increases and above  $H_{c2}$ , the material goes into normal state. The values of  $H_{c2}$ , is much greater than  $H_{c1}$ , This leads to the use of type II superconductor as magnets in particle accelerators.

Josephson junction-



In superconductors, tunnelling of Cooper pair is observed when two superconductors are connected by a thin insulating layer of thickness not more than 2 nm. The tunnelling of Cooper pair is known as Josephson effect.

In DC Josephson effect, where no voltage is applied across the junction, the current through junction is given by  $I = I_{max} \sin \phi$ , where  $\phi$  is the phase difference between wave functions of Cooper pair on either side of junction and  $I_{max}$  is the maximum current which depends on thickness of insulating region.

In AC Josephson effect, where voltage  $V$  is applied across the junction, the phase difference between wave functions on either side of the junction starts changing with time as

$$\nu = \frac{d\phi}{dt} = \frac{2Ve}{h}$$

The variation of current with time is known as AC Josephson effect. Thus, oscillating current emits radiation of frequency  $\nu$ , which is proportional to  $V$ .

Thus, AC Josephson junction gives precise measurement of voltage.

Problem- Determine the critical current and critical current density of a superconducting ring of diameter 1mm at 4.2 K. ( Given :  $T_C = 7.18 \text{ K}$ ,  $H_C = 6.5 \times 10^4 \text{ A/m}$  at 0K.)

$$I_C = 2\pi r H_C$$

$$H'_C = H_C \left( 1 - \left( \frac{T}{T_C} \right)^2 \right) = 6.5 \times 10^4 \left( 1 - \left( \frac{4.2}{7.18} \right)^2 \right) = 4.28 \times 10^4 \text{ A/m}$$

$$I_C = 2\pi \left( \frac{10^{-3}}{2} \right) \times 4.28 \times 10^4 = 134.3 \text{ A}$$

Thus, critical current density  $J_C = \frac{I_C}{\pi r^2} = \frac{134.3}{\pi \times (0.5 \times 10^{-3})^2} = 1.71 \times 10^8 \text{ A/m}^2$

Problem – The critical temperature for mercury with isotopic mass 199.5 is 4.185 K. What is the critical temperature when isotopic mass is 203.4?

$$\sqrt{M_1}T_{C1} = \sqrt{M_2}T_{C2}$$

$$T_{C1} = \frac{\sqrt{M_2}}{\sqrt{M_1}} T_{C2} = \frac{\sqrt{199.5}}{\sqrt{203.4}} 4.185 = 4.144 \text{ K}$$

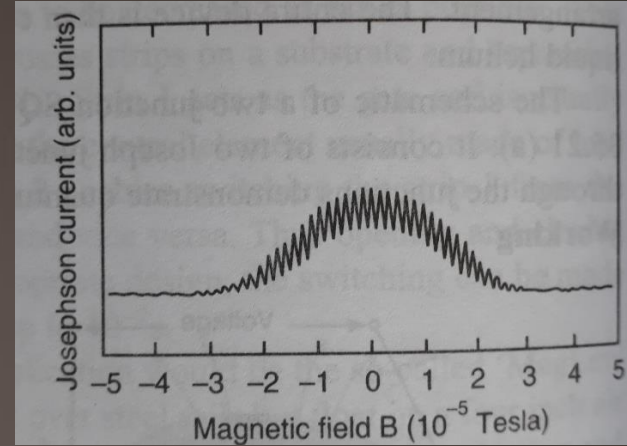
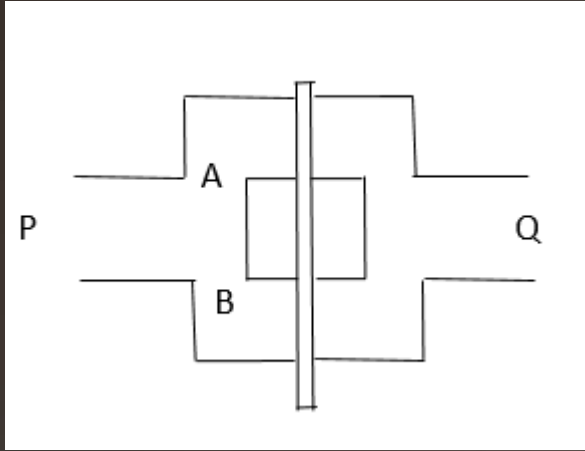
Problem- Determine penetration depth in mercury at 0K if critical temperature of mercury is 4.2 K and penetration depth is 57 nm at 2.9 K.

$$\text{As } \lambda(T) = \frac{\lambda(0)}{\left[1 - \left(\frac{T}{T_C}\right)^4\right]^{\frac{1}{2}}},$$

$$57 \times 10^{-9} = \frac{\lambda(0)}{\sqrt{1 - \left(\frac{2.9}{4.2}\right)^4}} \text{ gives } \lambda(0) = 57 \times 10^{-9} \times \sqrt{1 - 0.2273} = 50.1 \text{ nm}$$



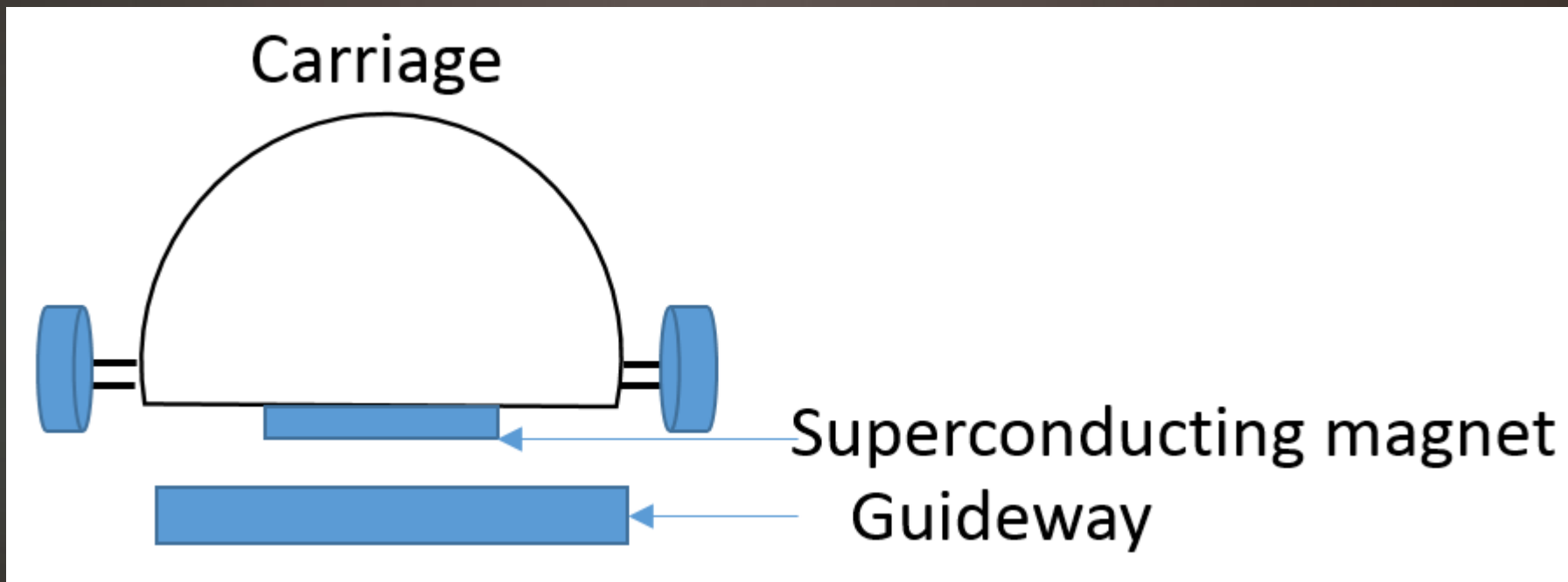
# SQUID- Superconducting Quantum Interference Device



The DC SQUID consists of two Josephson junctions. The current through superconductor at point P splits into two branches A and B and passes through Josephson junctions. The current through these branches meet at point Q.

The current at point Q is the result of interference of De-Broglie waves associated with Cooper pairs that tunnel through the junction. In absence of magnetic field, the phase difference between wavefunction of Cooper pairs passing through branches A and B is zero. In presence of magnetic field, phase difference becomes non-zero. The current through the junction is  $I = 2(I_{max} \sin \phi) \cos \frac{e\Phi}{\hbar c}$  where  $\Phi$  is magnetic field. These variations in the magnetic field results in cosine variations in the oscillating current in the graph. Using SQUID measurement of magnetic field ( $10^{-13}$  to  $10^{-21}$  T) can be carried out.

Maglev trains – Perfect diamagnetic properties of superconductors is used in Maglev trains. The carriages of train are fitted with superconducting magnets. Aluminium guideway is laid down on the track and it carries current. The train undergoes levitation because of repulsion between magnetic field produced by superconductor and magnetic field produced by guideway. The levitation which is about 10-15 cm eliminates friction and enables train to move at higher speed with much efficiency.



## High temperature superconductors –

High temperature superconductors that exhibit superconductivity at boiling temperature of liquid nitrogen. One example is oxide of yttrium-barium-copper. For high temperature superconductors, superconductivity cannot be explained by BCS theory. The Hall effect studies of these materials indicate that holes are causing the current. The material has anisotropic properties and isotope effect is not observed in these materials.