

19/08/2013

## A<sup>2</sup>d Intensity

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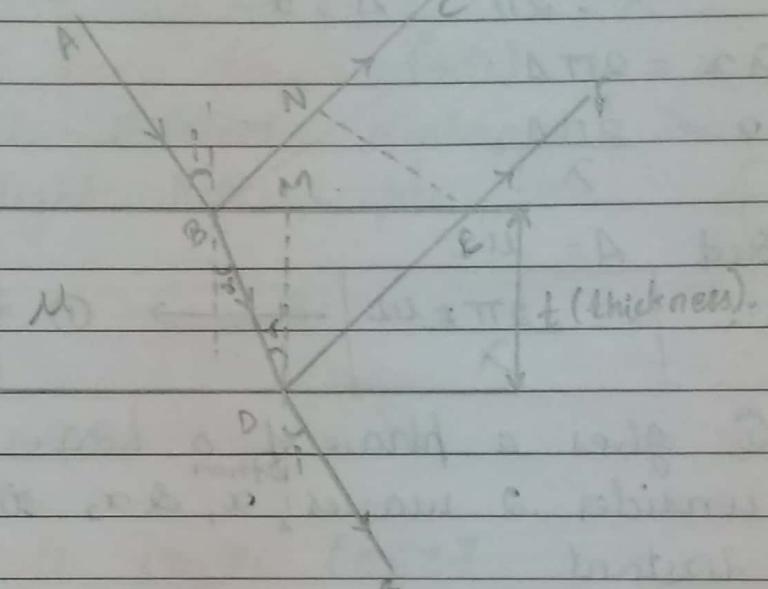
Date

- \* Technique of obtaining coherent sources for interference  
OR

• Expt  
Explain the two distinct methods of obtaining interference

method @ Amplitude division technique.

Ex: Interference due to thin film.



Explanation:

Consider a transparent thin film of thickness  $t$  kept in air medium. Let refractive index of material of the film be ' $n$ '. A plane wave  $AB$  from a monochromatic source of light is incident on the film with an angle of incidence ' $i$ '. Part of the wave gets reflected along  $BC$  & part of it gets refracted along  $BD$  which further gets reflected partially along  $BDE$ . The wave  $DF$  gets refracted along  $EF$ .

Since the two waves  $BC$  &  $BDEF$  are derived from the same source, they must be coherent waves & can produce interference.

Path difference b/w the 2 waves  
BC & BDEF

$$\text{i.e. } \delta = (BD + DF) - BN \rightarrow ①$$

From the  $\Delta^{\text{gle}}$  BDM,

$$\cos r = \frac{DM}{BD} \quad \text{i.e. } BD = \frac{DM}{\cos r} \rightarrow ②$$

From the right angled  $\Delta^{\text{gle}}$  MDP.

$$DE = \frac{DM}{\cos r} \rightarrow ③$$

Adding ② & ③

$$BD + DE = \frac{2DM}{\cos r}$$

But  $DM = t$  (thickness of the film)

$$\therefore BD + DF = \frac{2t}{\cos r} \rightarrow ④$$

From  $\Delta^{\text{gle}}$  BNE,  $\sin i = \frac{BN}{BE}$

$$BN = BE \sin i$$

$$BN = (BM + ME) \sin i \rightarrow ⑤$$

From  $\Delta^{\text{gle}}$  BMD,  $\tan r = \frac{BM}{DM} = \frac{BM}{t}$ .

$$\text{i.e. } BM = t \tan r \rightarrow ⑥$$

From figure,  $BM = MF$ .

: from eq ⑥

$$BM + ME = 2BM$$

$$\text{i.e. } BE = 2t \tan r \rightarrow ⑦$$

From eq ⑤ & ⑦,

$$BN = 2t \tan r \cdot \sin i \rightarrow ⑧$$

Sub eq ④ & ⑧ in eq ① Considering optical path  
we get,  $\delta = \frac{2tM}{\cos r} - 2t \tan r \sin i$

$$= \frac{2tM}{\cos r} - 2t \frac{\sin r \sin i}{\cos r}$$

$$\text{i.e. } \delta = \frac{2t}{\cos\alpha} [n - \sin\alpha \cdot \sin i] \rightarrow ⑨$$

Applying Snell's law for the reflection at the pt B

$$n_i \sin i = u \sin r$$

$$t \sin i = u \sin r$$

$$\text{i.e. } \sin i = u \sin r \rightarrow ⑩$$

Substituting eq<sup>n</sup> ⑩ in ⑨

We get

$$\delta = \frac{2t}{\cos\alpha} [n - u \sin^2 r]$$

$$= \frac{2ut}{\cos\alpha} [1 - \sin^2 r]$$

$$\delta = \frac{2ut}{\cos\alpha} \overline{\cos^2 r}$$

$$\delta = 2ut \cos\alpha \rightarrow ⑪$$

Since a wave of light, when undergoes reflection while moving from rarer to denser medium, it undergoes an additional path length of  $\frac{\lambda}{2}$ .  
 $\therefore$  eq<sup>n</sup> ⑪ can be written as

$$\boxed{\delta = 2ut \cos\alpha - \frac{\lambda}{2}} \rightarrow ⑫$$

which gives the expression for total path difference in case of interference due to reflected light at a thin film.

Case I: Condition for constructive interference.

Path difference,

$$\delta = n\lambda \text{ where } n = 0, 1, \dots$$

from eq<sup>n</sup> ⑫

$$\frac{2ut \cos\alpha - \frac{\lambda}{2}}{2} = n\lambda$$

$$\text{i.e. } 2nt \cos r = n\lambda + \frac{\lambda}{2}$$

$$2nt \cos r = \frac{2n\lambda + \lambda}{2}$$

$$\cancel{2nt \cos r} = \frac{(2n+1)\lambda}{2} \rightarrow ⑬$$

↑ which is the condition for constructive interference

case II Condit<sup>n</sup> for destructive interference

path difference,

$$\delta = (2n+1) \frac{\lambda}{2}$$

$$2nt \cos r = \frac{\lambda}{2} (2n+1) \frac{\lambda}{2}$$

$$2nt \cos r = (2n+1) \frac{\lambda}{2} + \frac{\lambda}{2}$$

$$= (2n+1+1) \frac{\lambda}{2}$$

$$= (2n+2) \frac{\lambda}{2}$$

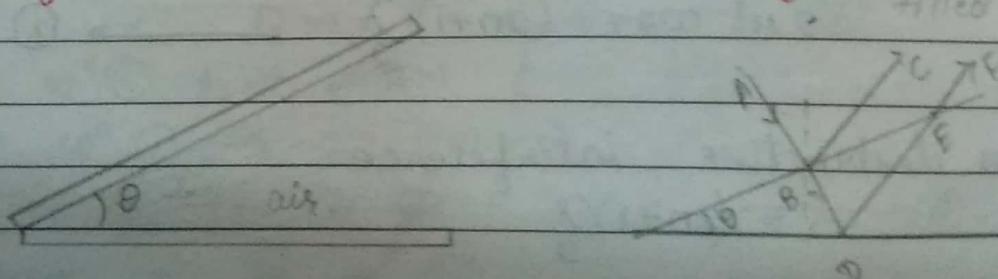
$$= 2(n+1) \frac{\lambda}{2}$$

$$= (n+1) \frac{\lambda}{2}$$

$$\cancel{2nt \cos r} = n\lambda \rightarrow ⑭$$

↑ which is the condit<sup>n</sup> for destructive interference

\* Interference due to an air wedge (triangular thin film filled with air)



- Air wedge is a thin film of varying thickness having a zero thickness at one end & progressively increasing thickness towards the other end.
- When a light wave AB is incident on the wedge it gets partially reflected along BC from the glass to air boundary at the top of the air film.
- Part of the light is transmitted through the air film & gets reflected partly at D along DF at the air to glass boundary.
- The wave DF further gets refracted along FF
- The two waves BC & FF are coherent as they are derived from the same wave AB, by means of amplitude division technique.
- Therefore the two waves can get interfere constructively or destructively depending of the path difference b/w them.
- W.R.t the optical path diff. b/w the two waves BC & FF is given by.

$$\delta = 2ut \cos r - \frac{\lambda}{2}$$

\* For constructive interference

$$\delta = n\lambda, \text{ where } n=1, 2, 3, \dots$$

$$\text{i.e. } 2ut \cos r - \frac{\lambda}{2} = n\lambda$$

$$\text{i.e. } 2ut \cos r = n\lambda + \frac{\lambda}{2}$$

$$2ut \cos r = (2n+1) \frac{\lambda}{2} \longrightarrow ①$$

\* For destructive interference

$$\delta = (2n+1) \frac{\lambda}{2}$$

$$\text{i.e. } 2ut \cos r - \frac{\lambda}{2} = (2n+1) \frac{\lambda}{2}$$

$$\text{i.e. } 2ut \cos\theta = (n+1)\lambda$$

or in general

$$2ut \cos\theta = n\lambda \longrightarrow \textcircled{2}$$

For normal incidence,

$$\cos\theta = 1 \quad (\because \theta = 0)$$

∴ eq<sup>n</sup> ② can be written as

$$2ut = n\lambda \longrightarrow \textcircled{3}$$

The interference pattern consist of rectangular fringes with equal distance apart. through out the interference pattern.

Let P' be the position of  $n^{\text{th}}$  dark fringe at which

thickness of the film is  $t = t_1$ .

Eq<sup>n</sup> ③ may be written as

$$2ut_1 = n\lambda \longrightarrow \textcircled{4}$$

Let the immediate next dark fringe

i.e.  $(n+1)^{\text{th}}$  dark fringe lying at Q.

where thickness of the film is  $t = t_2$

Eq<sup>n</sup> ③ may be written as

$$2ut_2 = (n+1)\lambda \longrightarrow \textcircled{5}$$

Subtracting ④ from ⑤

$$2u(t_2 - t_1) = (n+1)\lambda - n\lambda$$

$$2u(t_2 - t_1) < \lambda \longrightarrow \textcircled{6}$$

Consider  $\triangle MPQ$ ,

$$\text{we have } \tan\theta = \frac{QM}{PM}$$

$$\text{i.e., } QM = PM \tan\theta \longrightarrow \textcircled{7}$$

in eq<sup>n</sup> ⑥  $t_2 - t_1 = QM$  from figure.

$$\therefore \text{eq}^n ⑥ \Rightarrow 2uQM = \lambda$$

$$\text{i.e. } \cancel{QM} = \frac{\lambda}{2u} \longrightarrow \textcircled{8}$$

from eq<sup>n</sup> ⑦ & ⑧

$$\frac{\lambda}{2u} = PM \tan\theta \text{ or } PM = \frac{\lambda}{2u \tan\theta}$$

here  $P_M = \beta$ , the fringe width  
ie  $\boxed{\beta = \frac{\lambda}{2\mu \tan \theta}} \rightarrow ⑨$

For small angle of wedge  $\theta$

$$\tan \theta \approx \theta$$

$$\therefore \text{eq}^n ⑨ \rightarrow \boxed{\beta = \frac{\lambda}{2\mu \theta}} \rightarrow ⑩$$

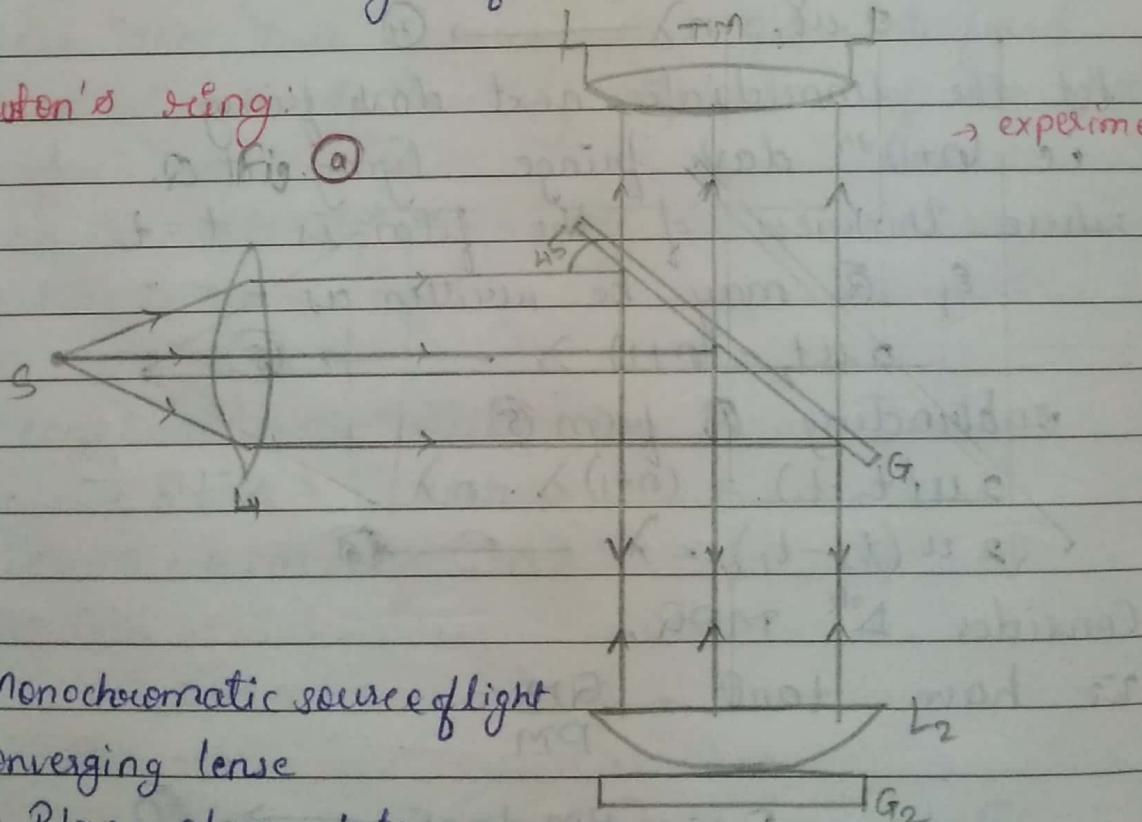
which gives the expression for fringe width.  
It indicates that as angle of the wedge increases  
the fringe width  $\beta$  decreases.

Thus the wedge shaped film can be used to  
determine refractive index of the material of the  
wedge shaped film or it can be used to  
determine wavelength of the incident light.

### \* Newton's ring:

Fig. ①

→ experimental setup



S - Monochromatic source of light

L<sub>1</sub> - Converging lens

G<sub>1</sub> - Plane glass plate

L<sub>2</sub> - Plano convex lens.

G<sub>2</sub> - Plane glass plate

TM - Travelling microscope

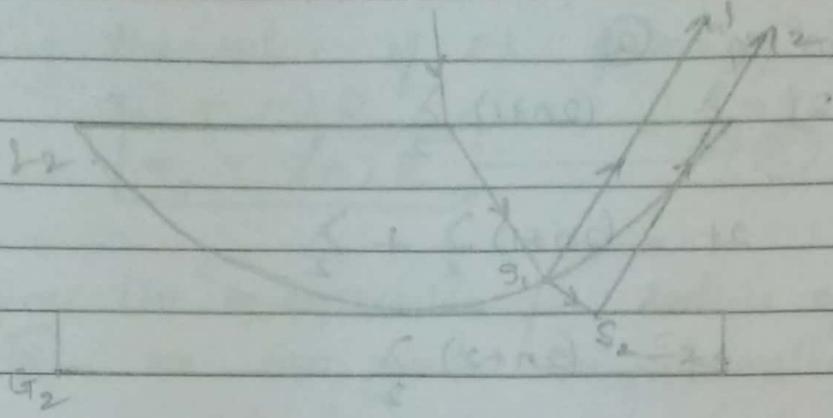


Fig (b) : Origin of Coherent waves

- \* The experimental setup is as shown in fig. a.
- \* A monochromatic beam of light from the source S is incident on a convex lens L<sub>1</sub> to obtain a 1<sup>st</sup> beam of light.
- \* The beam is allowed to incident on the glass plate G<sub>1</sub>, inclined at an angle of 45°.
- \* The beam after reflects propagates vertically downwards & is incident on the air film produced b/w plano convex lens L<sub>2</sub> & a plane glass plate G<sub>2</sub>.
- \* The coherent waves are originated from top surface & bottom surface of the air film as shown in the fig (b).
- \* These coherent waves propagate in upward direction, pass through the glass plate G<sub>2</sub> & enter into TM.
- \* Depending upon the path diff betw the coherent waves, they interfere constructively or destructively & a concentric rings pattern is obtained. These rings are called Newton's rings.

Path diff betw the coherent waves is given by

$$\delta = 2nt \cos r - \lambda/2 \rightarrow ①$$

For air film:  $n=1$  &

for normal incidence  $r=0$

$$\therefore ① \Rightarrow \delta = 2t - \frac{\lambda}{2} \rightarrow ②$$

But for dark ring

$$\delta = (2n+1) \frac{\lambda}{2}$$

From eq<sup>n</sup> ②

$$2t - \frac{\lambda}{2} = (2n+1) \frac{\lambda}{2}$$

$$2t = (2n+1) \frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2t = (2n+2) \frac{\lambda}{2}$$

$$2t = (n+1) \lambda$$

In general  $2t = n\lambda \longrightarrow ③$ .

\* Consider an  $n^{th}$  dark ring located at the point 'D', at which thickness of the film is  $DB = t$ .

\* From  $\Delta ACB$ , we can write

$$AB^2 = AC^2 + BC^2$$

$$\text{i.e. } AB^2 = (AO - OC)^2 + BC^2 \longrightarrow ④$$

\* If centre of curvature is located at pt 'A' then  $AO = AB = R$  i.e. radius of curvature of the plane convex lens

\* Hence eq<sup>n</sup> ④ can be written as

$$R^2 = (R - t)^2 + BC^2$$

$$R^2 = R^2 + t^2 - 2Rt + BC^2$$

$$2Rt = t^2 + BC^2 \longrightarrow ⑤$$

From the fig. it is clear that  $OD = BC = r_n$  (i.e. radius of  $n^{th}$  dark ring)

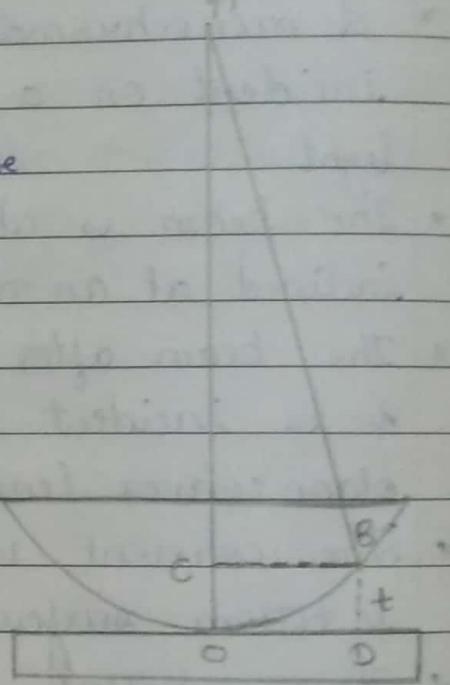
$$\therefore ⑤ \Rightarrow 2Rt = t^2 + r_n^2 \longrightarrow ⑥$$

Since  $2Rt \gg t^2$ .

as thickness of the film  $t \ll R$ . we can neglect the term  $t^2$

$$\therefore \text{eq}^n ⑥ \Rightarrow 2Rt = r_n^2$$

$$\text{i.e. } r_n^2 = 2Rt \longrightarrow ⑦$$



substituting the value of  $\sigma$  from eq<sup>n</sup> ⑧  
 we get  $\sigma_n^2 = n\lambda R$   
 i.e.  $\sigma_n = \sqrt{n\lambda R}$   $\rightarrow$  ⑧

which gives the expression for radius of  $n^{th}$  darkening.  
 From eq<sup>n</sup> ⑧, we can write the diameter of  $n^{th}$  ring as

$$D_n = 2\sqrt{n\lambda R} \rightarrow ⑨$$

since  $\sigma_n \propto \sqrt{n}$ ,  $\therefore D_n \propto \sqrt{n}$ .

the diameter of the Newton's ring doesn't increase in the same proportion as the order of the ring. Therefore the rings gets closer as  $n$  increases.  
 $\therefore$  The rings are not equally spaced.

### \* Application of Newton's ring

① Determinants of wavelength of light.

Consider an ' $n^{th}$ ' dark ring, its diameter is given by,  $D_n = 2\sqrt{2n\lambda R}$

By squaring

$$D_n^2 = 4n\lambda R \rightarrow ⑩$$

Consider another  $(n+m)^{th}$  ring, whose diameter is given by,

$$D_{(n+m)}^2 = 4(n+m)\lambda R \rightarrow ⑪$$

Subtracting eq<sup>n</sup> ⑩ from ⑪ we get.

$$\begin{aligned} D_{(n+m)}^2 - D_n^2 &= 4(n+m)\lambda R + 4m\lambda R - 4n\lambda R \\ &= 4m\lambda R \end{aligned}$$

$$\text{i.e. } \lambda = \frac{D_{(n+m)}^2 - D_n^2}{4m\lambda R} \rightarrow ⑫$$

Thus by knowing the radius of curvature of the plano convex lens & diameters of the 2 given rings we can evaluate wavelength of the incident light.

② Determination of refractive index of a liquid:  
 The liquid whose refractive index is to be determined is introduced to fill the gap bet" plano convex lens & plane glass plate. Therefore interference pattern obtained will be due to liquid film.

Let  $n$  be the R.I. of the liquid, then the condition for dark ring can be written as,

$$2ut \cos\theta = n\lambda$$

For normal incident  $\theta=0$  &  $\cos\theta=1$   
 $; 2ut = n\lambda$ .

i.e. diameter of  $n^{\text{th}}$  film ring is given by.

$$D_{nL}^2 = \frac{4n\lambda R}{\mu} \rightarrow (13)$$

Note, Consider another  $(n+m)^{\text{th}}$  ring for which diameter of the dark ring is given by,

$$D_{(n+m)L}^2 = \frac{4(n+m)\lambda R}{\mu} \rightarrow (14)$$

Subtracting eq<sup>n</sup> (13) from (14)  
 we get,

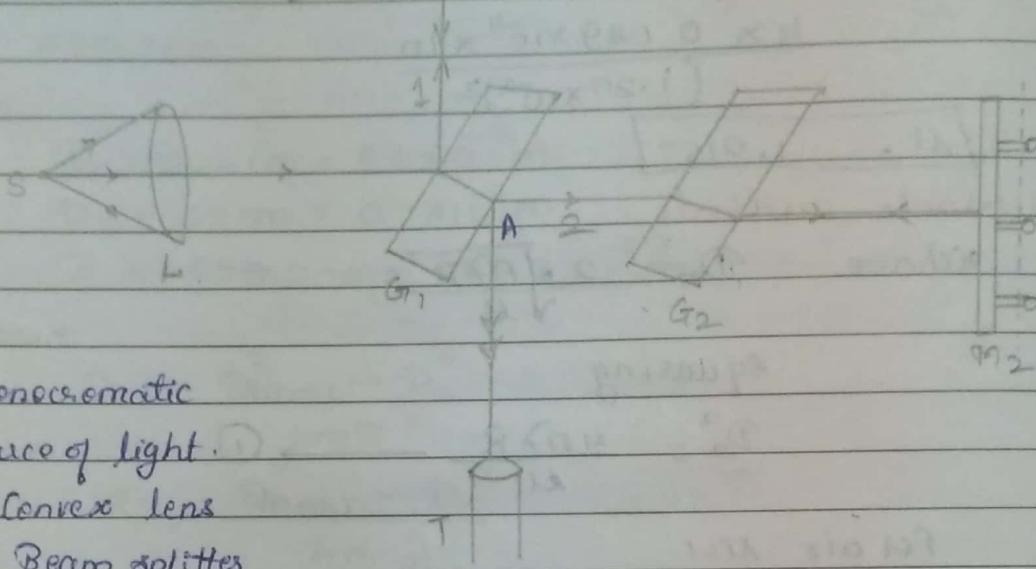
$$D_{(n+m)L}^2 - D_{nL}^2 = \frac{4m\lambda R}{\mu}$$

i.e.  $R = \frac{4m\lambda R}{(D_{(n+m)L}^2 - D_{nL}^2)} \rightarrow (15)$

By knowing radius of curvature of plane convex lens & diameters of two rings we can calculate R.I. of the liquid.

## Micelielton's Interferometer

Construction:



S - Monochromatic

source of light.

L - Convex lens

G<sub>1</sub> - Beam splitter

G<sub>2</sub> - Compensator

M<sub>1</sub> & M<sub>2</sub> → Plane mirror

T - Telescope

M<sub>2</sub>' - Virtual image of mirror M<sub>2</sub>

d - distance bet<sup>n</sup> the mirror M<sub>1</sub> & virtual image M<sub>2</sub>'

\* It consists of a monochromatic source 'S', converging lens 'L', a beam splitter G<sub>1</sub>, a compensating plate G<sub>2</sub> & two plane mirror M<sub>1</sub> & M<sub>2</sub>. The beam splitter is partially silvered plane glass plate, which can transmit the light partially & reflects the rest.

\* G<sub>1</sub> being a plane glass plate has the thickness same as that of G<sub>2</sub>.

\* G<sub>1</sub> & G<sub>2</sub> are held 11° to each other but inclined at an angle of 45° with respect to the mirror M<sub>2</sub>.

\* The mirrors M<sub>1</sub> & M<sub>2</sub> can be set 1° to each other with the help of the screw provided at their back. The interference pattern as observed in Telescope

### Working

- \* A monochromatic beam of light is allowed to incident on the beam splitter G.
- \* It is partially reflected at the back surface of G, along  $AM_1$ , & partially transmitted along  $AM_2$ .
- \* The beam  $AM_1$  gets reflected back along the same path & enters the telescope T.
- \* The beam  $AM_2$  gets reflected back along the same path along  $M_2$  to A & undergoes reflection once again by G, & enters the telescope T.
- \* Since these two beams are produced from a single source through division of amplitude & hence are coherent.
- \* It is clear that the beam 1 undergoes reflection at G, passes the glass plate G, two times.
- \* Therefore ~~to~~ to compensate the path of the beam ~~to~~ the glass plate  $G_2$  is used.
- \* Now the optical path of the wave 1 is given by

$$P_1 = 2AM_1 + \frac{\lambda}{2} + \frac{\lambda}{2} \longrightarrow ①$$

"optical path of the wave 2 is given by,

$$P_2 = 2AM_2 + \frac{\lambda}{2} + \frac{\lambda}{2} \longrightarrow ②$$

subtracting eq<sup>n</sup> ② from ① we get

we get the path diff betn the waves 1 & 2. as

$$\delta = P_1 - P_2$$

$$\delta = 2(AM_1 - AM_2) \longrightarrow ③$$

Since  $M_2'$  the virtual image of mirror  $M_2$  at the same distance as  $M_2$  from the ft 'A'. Therefore

$$AM_2 = AM_2'$$

$$eq^{\prime} ③ \rightarrow \delta = 2(AM_1 - AM_2') \longrightarrow ④$$

From the fig. it is clear that

$$AM_1 - AM_2' = d \longrightarrow ⑤$$

$$\therefore \delta = 2d \rightarrow ①$$

\* The effect is same as the interference due to thin film of thickness  $d$ .

\* If we observe the system at an angle  $\theta$  the path diff betn the 2 waves will be.

$$\delta = 2d \cos\theta \rightarrow ②$$

\* Since the waves 2 undergoes reflect from denser medium at G, it covers an additional path of  $\frac{\lambda}{2}$

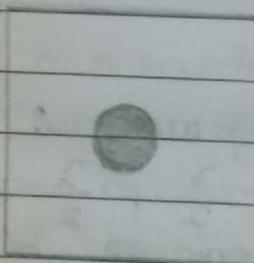
: The total path diff betn the two waves ① & ②.

$$\delta = 2d \cos\theta + \frac{\lambda}{2} \rightarrow ③$$

For the given values of  $d$ ,  $\lambda$  &  $\theta$ , the interference fringes are obtained to be in circular shape.

case(i) : If  $M_2$  coincides with  $M_1$ , then  $d = 0$ .

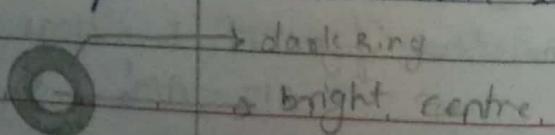
This leads to the additional path diff of zero. In other words total path diff  $\delta = \frac{\lambda}{2}$ . This leads to the destructive interference & therefore centre of the field of view will be dark as shown below:



case(ii) : If one of the mirrors is moved through a distance of  $\frac{\lambda}{4}$ , the path changes by  $\frac{\lambda}{2}$  & leads to the total path diff of,

$$\delta = \frac{\lambda}{2} + \frac{\lambda}{2} - \lambda \rightarrow \text{constructive}$$

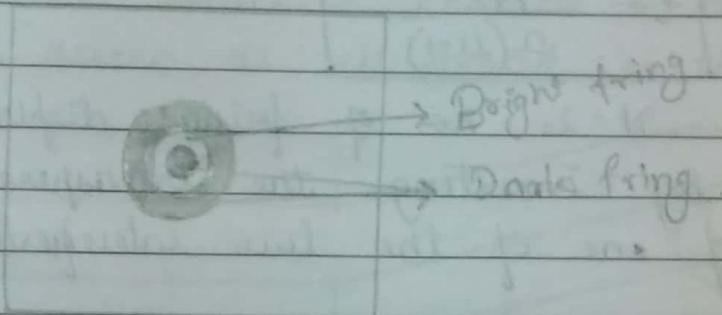
This leads to the constructive interference at the centre of the interference pattern as shown in the fig



case(3) : By moving mirror  $a$  through by another dist  $\frac{\lambda}{4}$  dist, the total path diff is caused to be

$$\delta = \lambda + \frac{\lambda}{2} = \frac{3\lambda}{2}, \text{ which corresponds to destructive}$$

interference & again dark spot is obtained at the centre by displacing the bright fring outwards as shown in fig.



Thus as 'd' increase the new ring appears at the centre & the field becomes more & more crowded with thinner rings.

### \* Application of Michelson's Interferometer

① To determine the wavelength of incident light

The monochromatic source of light whose wavelength is to be determine is kept at the position  $s$ .

\* The wavelength of the light can be calculated by using the eq<sup>n</sup>

$$\lambda = \frac{2(d_2 - d_1)}{N} \quad \text{where } N \text{ is the no. of frings disappearing at the}$$

centre of the field of views, when one of the mirror is moved to change the dist. appear from  $d_1$  to  $d_2$ .

② To determine the difference b/w the wavelength of two waves.

It can be calculated by using the relation  $\Delta\lambda = \frac{\lambda^2}{2d}$  where  $\lambda$  is mean wavelength.

③ Measurement of thickness of a thin transparent sheet.

It can be calculated by using the rel?

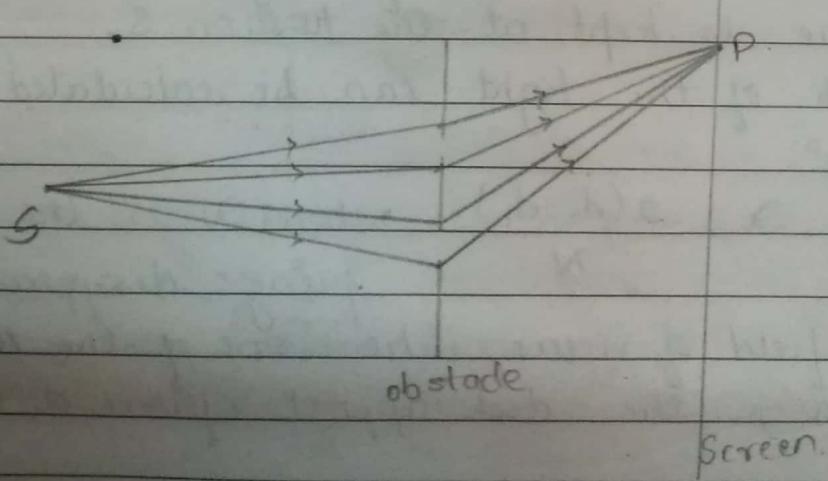
$$t = \frac{N\lambda}{2(u-1)}$$

where -  $N$  - is no of fringes displaced in the field of view by inserting the transparent sheet in the path of one of the two interfering beams.

\* Diffract: There are two types of diffract.

\* Fresnel diffract: In this type of diffract source of light & the screen are effectively at finite distances from the obstacle.

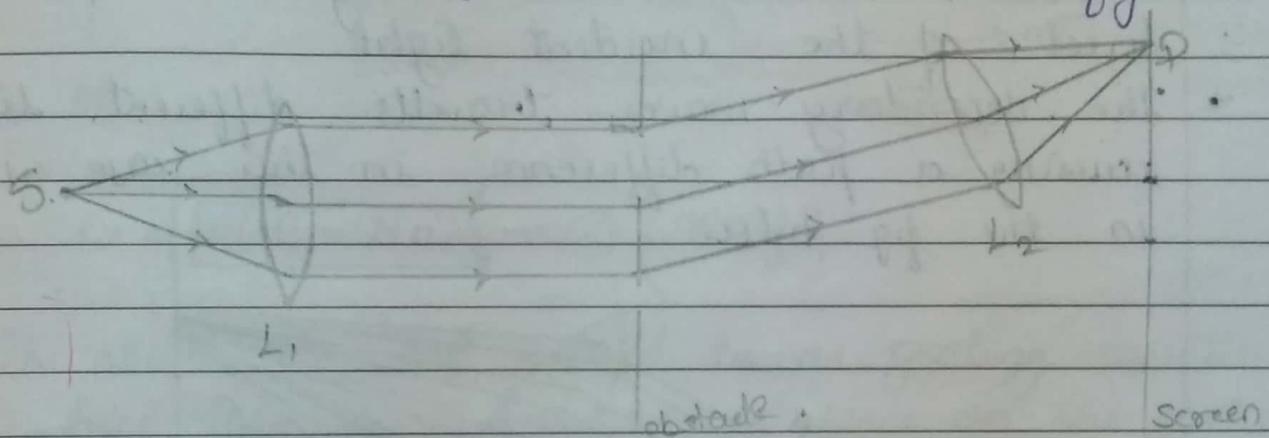
\* Spherical wavefronts or cylindrical wavefronts are employed to obtain the diffract pattern in this case. The fig. A shown below illustrates the Fresnel diffract.



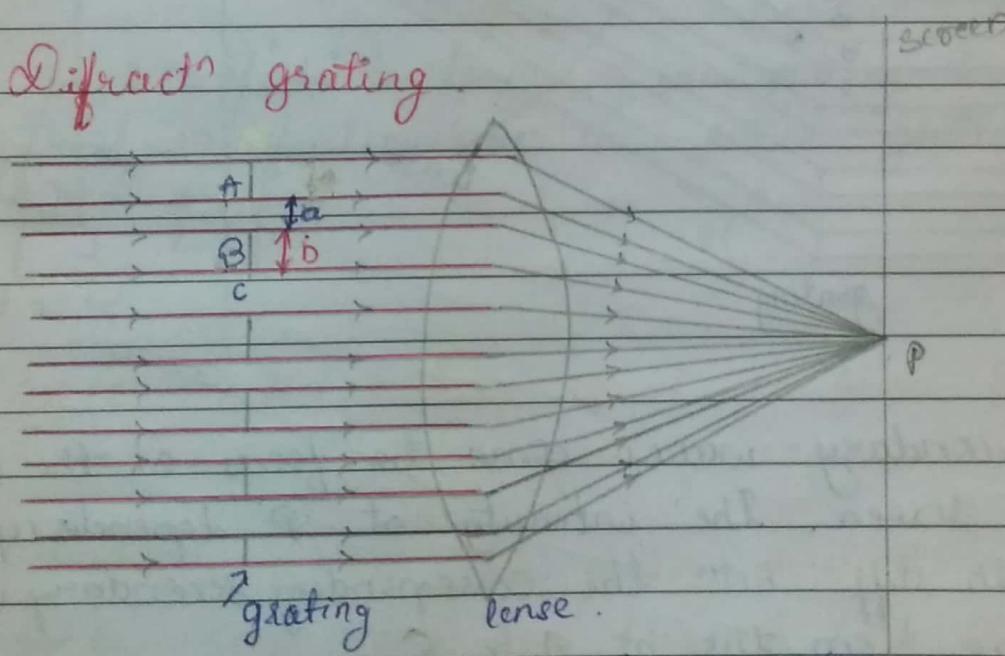
## ② Fraunhofer diffraction

In this type of diffraction source of light & the screen are effectively kept at infinite distances from the obstacle.

- \* Plane wavefronts are employed to obtain the diffraction pattern in this case.
- \* Two convex lenses are used to meet the above condition, one b/w source & obstacle & the other b/w obstacle & the screen as illustrated in the fig. below.

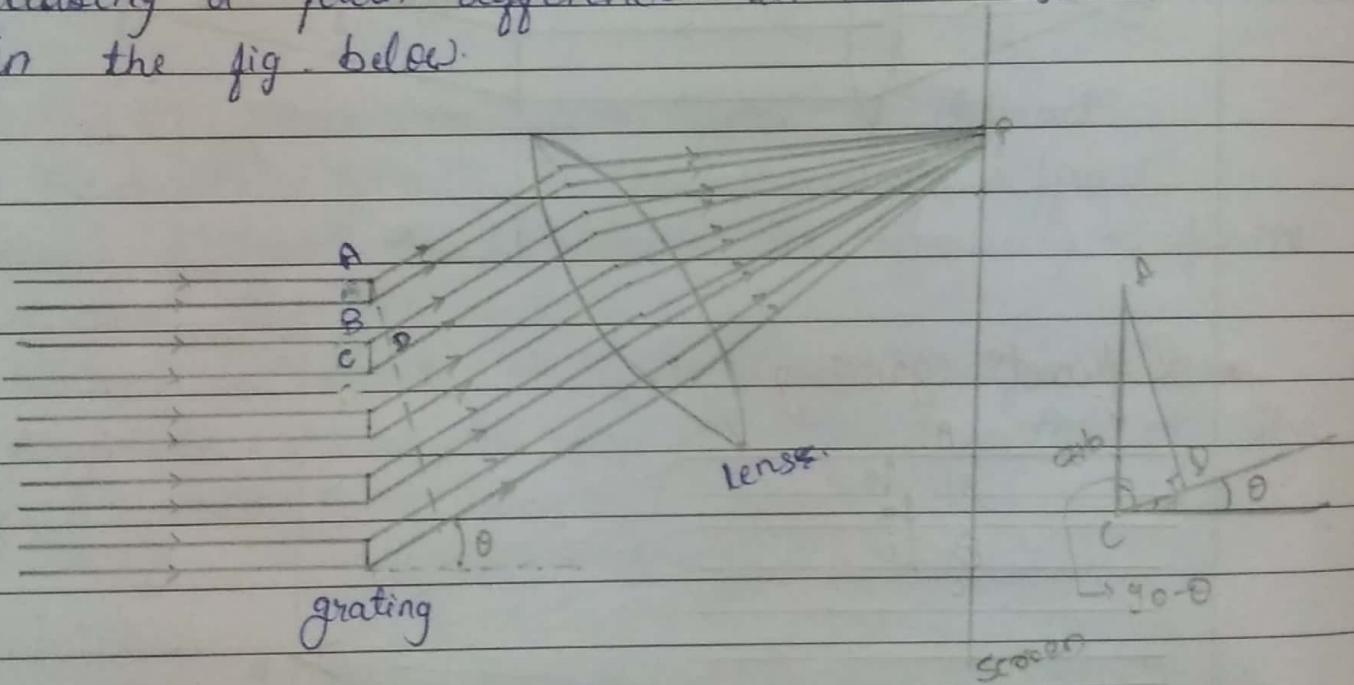


## \* Diffraction grating



- \* Consider a grating transmission <sup>grating</sup> in which width of a transparent sheet AB = a & width of an opacity B.
- \* The distance a+b+d is called grating constant or grating element. It is also called grating period.

- \* When a parallel beam of monochromatic light is incident normally on the grating surface, the secondary waves travelling in the same direction come to focus at the pt 'P' on the screen. The pt P corresponds to the position of central maximum.
- \* Now consider the secondary waves travelling in a direction inclined at an angle  $\theta$  with the direction of the incident light.
- \* The secondary waves travel different distances causing a path difference in this case as shown in the fig. below.



- \* The secondary waves come to focus at the pt 'P' on the screen. The intensity at 'P' depends upon the path diff betw the corresponding secondary waves originating from the pt A & C.
- \* It is clear that path diff betw the corresponding waves  $\Delta \delta = CD \rightarrow ①$

From  $\triangle ACD$

$$\sin \theta = \frac{CD}{AC} \quad \text{But } AC = ab = d$$

i.e.  $a+b=d$  (the grating const.)  
&  $CD=\delta$  (path diff).

$$\therefore \sin \theta = \frac{\delta}{(a+b)}$$

$$\text{i.e. } [\delta = (a+b) \sin \theta] \longrightarrow \textcircled{2}$$

The fit 'P' corresponds to the max. intensity if the path diff.  $\delta = n\lambda$   $\xrightarrow{\text{constructive interference}}$   $\textcircled{3}$

$$\therefore \text{from eq}^n \textcircled{2} \text{ & } \textcircled{3}$$

$$a+b = n\lambda$$

$$\text{in general, } (a+b) \sin \theta_n = n\lambda \longrightarrow \textcircled{4}$$

which can also be written as

$$[\delta \sin \theta_n = n\lambda] \longrightarrow \textcircled{5} \text{ d-grating const.}$$

Q2

$$\sin \theta_n = \frac{1}{d} n\lambda$$

$$\text{i.e. } [\delta \sin \theta_n = N n\lambda] \xrightarrow{d} \textcircled{6}, \left( N = \frac{1}{d} \right)$$

The above eq<sup>n</sup>'s are referred to as grating equations.  
here  $N = \frac{1}{d}$  refers to no of lines on the grating per unit length.

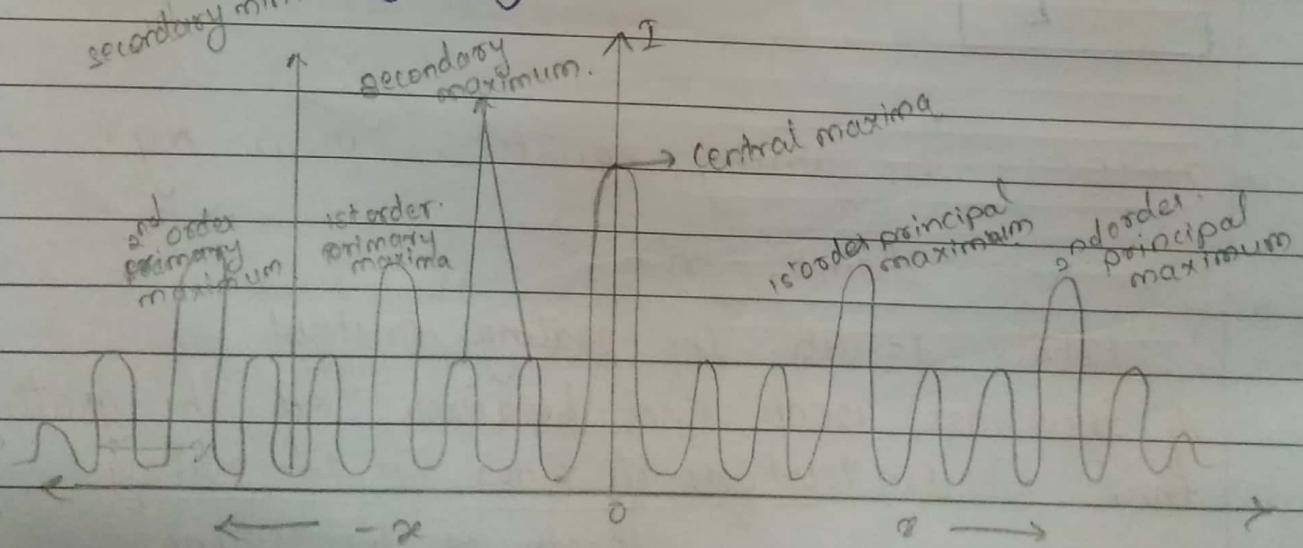
The intensity distribution of the diffraction pattern by a grating in ~~no~~ no of slits is given by

$$I = I_0 \left( \frac{\sin^2 \alpha}{\alpha^2} \right) \cdot \frac{\sin^2 N\alpha}{\sin^2 \alpha}$$

where  $\alpha = \pi d \sin \theta$

$\lambda$

- the expression for the intensity consists of 2 terms  
 $I_0 \left( \frac{\sin^2 \alpha}{\alpha^2} \right)$  → which represents the intensity distribution due to a single slit diffraction.
- And the 2nd term,  
 $\left( \frac{\sin^2 N\alpha}{\sin^2 \alpha} \right)$  represents the intensity distribution due to the diffraction by  $N$  no of equally spaced slits.
- The resultant intensity distribution is as shown in the following figure.



## \* Resolving Power

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An ability of an optical instrument to produce distinctly separate images of two objects located very close to each other is called its resolving power.

- It is given by the reciprocal of the smallest angle subtended at the objective by two pt objects which can just be distinguish separate.

### ~~Grating~~ Expression for resolving power of a grating

The spectral resolving power of a grating is given by

$$RP = \frac{\lambda}{d\theta} \rightarrow ①$$

where  $\lambda$  is average wavelengths of two spectral lines

$d\lambda$  is the difference betn the wavelengths ~~as~~ spectral wavelengths interval

eq<sup>n</sup> ① can be modified as follows

$$RP = \frac{\lambda}{d\theta} \frac{d\theta}{d\lambda} \rightarrow ②$$

the grating eq<sup>n</sup> is given by

$$(a+b) \sin\theta = n\lambda \rightarrow ③$$

diff eq<sup>n</sup> ③ ~~isnt~~ totally

$$(a+b) \cos\theta, d\theta = n \cdot d\lambda$$

$$\text{i.e. } \frac{d\theta}{d\lambda} = \frac{n}{(a+b) \cos\theta} \rightarrow ④$$

Consider a telescope objective capturing the beam refracted by a grating at an angle of  $\theta'$  with the direct<sup>n</sup> of the incident beam as shown in fig below.

Incident  
light  
beam

grating

object  
lens of telescope

The angular limit of resolution of a telescope objective is given by.

$$\frac{\text{Imp formula}}{\theta} = \frac{\lambda}{d} \rightarrow ⑤$$

where  $d \rightarrow$  diameter of the objective lens.

From the geometry of the figure, it's clear from the  $\triangle ACB$  that

$$\cos\theta = \frac{AC}{AB} = \frac{d}{AB}$$

$$d = AB \cos\theta$$

Let 'L' be the length of the grating.  
i.e.  $AB = L$ .

$$\therefore d = L \cos\theta \rightarrow ⑥$$

Eq" ⑤ may be written as.

$$\frac{\theta}{d} = \frac{\lambda}{L \cos\theta} \rightarrow ⑦ \text{ } ⑧$$

$$\frac{\theta}{d} = \frac{\lambda}{L \cos\theta} = L \cos\theta \rightarrow ⑦b$$

Substituting eq ④ & ⑦b in eq" ②  
we get

$$R.P. = \frac{L \cos\theta \cdot n}{(a+b) \cos\theta}$$

$$R.P. = \frac{Ln}{(a+b)} \rightarrow ⑧a$$

since  $\frac{L}{(a+b)} = N$ , the total no of lines on the grating

eq<sup>n</sup> ⑧a may be written as

$$RP = nN \longrightarrow \textcircled{8b}$$

which gives the expression for resolving power of  
the grating.