Open with

TRANSFORMS OF PERIODIC FUNCTIONS

If $f(t)$ is a periodic function with period T, i.e., $f(t+T) = f(t)$, then

$$L\{f(t)\} = \frac{\int_0^T e^{-st} f(t)dt}{1-e^{-sT}}$$

We have $$L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt = \int_0^T e^{-st} f(t)dt + \int_T^{2T} e^{-st} f(t)dt + \int_{2T}^{3T} e^{-st} f(t)dt + \ldots$$

In the second integral put $t = u + T$, in the third integral put $t = u + 2T$, and so on. Then

$$L\{f(t)\} = \int_0^T e^{-st} f(t)dt + \underline{\int_0^T e^{-s(u+T)} f(u+T)du} + \int_0^T e^{-s(u+2T)} f(u+2T)du + \ldots$$

$$= \int_0^T e^{-st} f(t)dt + e^{-sT}\int_0^T e^{-su} f(u)du + e^{-2sT}\int_0^T e^{-su} f(u)du + \ldots$$

$[\because \ f(u) = f(u+T) = f(u+2T) \text{ etc.}]$

$$= \int_0^T e^{-st} f(t)dt + e^{-sT}\int_0^T e^{-st} f(t)dt + e^{-2sT}\int_0^T e^{-st} f(t)dt + \ldots$$

$$= (1 + e^{-sT} + e^{-2sT} + \ldots \infty)\int_0^T e^{-st} f(t)dt$$

$$L[f(t)] = \ = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)dt.$$

$1+D+D^2+ \ldots$

LAPLACE TRANSFORMS

Date

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt$$

$$L[e^{at} f(t)] = \bar{f}(s-a)$$

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}[f(s)]$$

$$L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(s)\,ds$$

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^{T} e^{-st} f(t)\,dt$$

$$L\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$$

LAPLACE TRANSFORMS

$$L(1) = \frac{1}{s}$$

$$L(t^n) = \frac{n!}{s^{n+1}}$$

$$L(e^{at}) = \frac{1}{s-a}$$

$$L(\sin at) = \frac{a}{s^2+a^2}$$

$$L(\cos at) = \frac{s}{s^2+a^2}$$

$$L(\sinh at) = \frac{a}{s^2-a^2}$$

$$L(\cosh at) = \frac{s}{s^2-a^2}$$

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}[f(s)]$$

$$L(e^{at}) = \frac{1}{s-a}$$

$$L(e^{at} t^n) = \frac{n!}{(s-a)^{n+1}}$$

$$L(e^{at} \sin bt) = \frac{b}{(s-a)^2+b^2}$$

$$L(e^{at} \cos bt) = \frac{s-a}{(s-a)^2+b^2}$$

$$L(e^{at} \sinh bt) = \frac{b}{(s-a)^2-b^2}$$

$$L(e^{at} \cosh bt) = \frac{s-a}{(s-a)^2-b^2}$$

$$\cosh t = \frac{e^{at}+e^{-at}}{2}$$

$$\sinh t = \frac{e^{t}-e^{-t}}{2}$$