UNIT - IV

PROBABILITY

* A random variable X has the following probability function for various values of x

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

i) Find K ii) Evaluate $P(x<6)$, $P(x \geq 6)$ and $P(3 \leq x \leq 6)$

Sol^n: We have i) $\boxed{\sum P(x) = 1}$

$$\therefore 0+K+2K+2K+3K+K^2+2K^2+7K^2+K = 1$$

$$\Rightarrow 10K^2+9K-1 = 0 \quad \therefore K = \frac{1}{10}$$

Hence

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	1/10	2/10	2/10	3/10	1/100	2/100	17/100

ii) $P(x<6) = 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} = 0.81$

$P(x \geq 6) = P(6) + P(7) = \frac{2}{100} + \frac{17}{100} = 0.19$

$$P(3 \leq x \leq 6) = P(3) + P(4) + P(5) + P(6)$$

$$= \frac{2}{10} + \frac{3}{10} + \frac{1}{100} + \frac{2}{100}$$

$$= 0.53$$

* A Random variable x has the following probability function

X	:	-2	-1	0	1	2	3
$P(X)$	:	0.1	K	0.2	$2K$	0.3	K

Find K and find mean and variance.

Soln: We have $\sum P(x) = 1$

$$0.1 + K + 0.2 + 2K + 0.3 + K = 1$$

$$\Rightarrow K = 0.1$$

$$\text{Mean} = E(X) = \sum x\, P(x)$$

$$= -2(0.1) + (-1)(0.1) + 0(0.2) + (1)(0.2) + 2(0.3) + 3(0.1)$$

$$E(X) = 0.8$$

$$\text{Variance} = E(X^2) - [E(X)]^2$$

Now $E(X^2) = \sum x^2 \cdot P(x)$

$$= (-2)^2(0.1) + (-1)^2(0.1) + 0(0.2) + (1)^2(0.2) + (2)^2(0.3) + (3)^2(0.1)$$

$$= 2.8$$

$$\therefore \text{Variance} = 2.8 - (0.8)^2$$

$$= 2.16$$

* From a sealed box containing a dozen apples it was found that 3 apples are perished. Obtain the probability distribution of the no. of perished apples when 2 apples are drawn at random. Also find mean and variance of this distribution.

Soln: Let X : No. of perished apples

2 apples out of 12 can be selected in $^{12}C_2$ ways.

Good apples are 9

$\therefore X = 0, 1, 2$

$p(X=0)$ = Probability of getting 0 perished apple $= \dfrac{3C_0 \cdot 9C_2}{12C_2} = \dfrac{6}{11}$

$$P(X=1) = \frac{3C_1 \cdot 9C_1}{12C_2} = \frac{9}{22}$$

$$p(X=2) = \frac{3C_2 \cdot 9C_0}{12C_2} = \frac{1}{22}$$

The prob. distr^{ny} is

x :	0	1	2
$P(x)$:	$6/11$	$9/22$	$1/22$

Mean $= \sum x \cdot P(x) = \frac{1}{2}$ $Var(x) = E(x^2) - [E(x)]^2 = 15/44$

* Binomial Distribution

$m = \text{Mean} = np$ $\quad$ $\text{Var} = npq$

Probability func of B.D is - $P(x) = {}^nC_x \, p^x q^{n-x}$

p - success, q - failure, n - trials.

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* 256 set of 12 tosses of a coin in how many cases one can expect 8 heads & 4 tails ?

Soln: $p = 0.5$, $q = 0.5$, $n = 12$, 8

$P(x) = {}^nC_x \, p^x q^{n-x}$

$P(x=8) = {}^{12}C_8 \, (0.5)^8 \, (0.5)^4 = 0.1208$

Expected no of such cases in 256 sets

is $= 256 \times 0.1208$

$\simeq 30.92 \simeq 31$

* In sampling a large no of parts manufactured by a company, the mean no, of defectives in samples of 20 is 2. Out of 1000 such samples how many would be expected to contain atleast 3 defective parts.

Soln: Mean = $np = 2$ when $n = 20$

$\Rightarrow 20p = 2 \Rightarrow p = 1/10 = 0.1$

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{20}C_{x} (0.1)^{x} (0.9)^{20-x}$

Prob' of atleast 3 defectives is $= P(3) + P(4) \ldots + P(20)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \{ {}^{20}C_{0} (0.1)^{0} + (0.9)^{20-0} +$

${}^{20}C_{1} (0.1)^{1} (0.9)^{19} + {}^{20}C_{2} (0.1)^{2} (0.9)^{18} \}$

$= 0.323$

No of defectives in 1000 samples =

$1000 \times 0.323 = 323$

* Five dice were thrown 96 times and the number of times an odd no actually turned out in the experiment is given. Fit B.D to this data & calculate expected frequencies.

No of dice showing 1 or 3 or 5 :	0	1	2	3	4	5
observed freq :	1	10	24	35	18	8.

Soln : Prob' of getting 1 or 3 or 5 $= 3/6 = 1/2$

$$P(x) = {}^nC_x \, p^x q^{n-x} \quad ; \quad n=5$$

$$= {}^5C_x (1/2)^x (1/2)^{5-x} =$$

Now $f(x) = 96 \times P(x)$

$$f(0) = 96 \times {}^5C_0 (1/2)^0 (1/2)^{5-0} = 3$$

$$f(1) = 96 \times {}^5C_1 (1/2)^1 (1/2)^{5-1} = \cancel{30}\ 15$$

$$f(2) = 96 \times {}^5C_2 (1/2)^2 (1/2)^{5-2} = 30$$

$$f(3) = 96 \times {}^5C_3 (1/2)^3 (1/2)^{5-3} = 30$$

$$f(4) = 96 \times {}^5C_4 (1/2)^4 (1/2)^{5-4} = 15$$

$$f(5) = 96 \times {}^5C_5 (1/2)^5 (1/2)^{5-5} = 3$$

Expected frequencis are

3, 15, 30, 30, 15, 3.

Poisson Distribution

$$P(x) = \frac{e^{-m} m^x}{x!}$$

[Mean = m] [Var = m]

* The no of persons joining a cinema queue in a minute has poisson distribution with parameters 5.8. Find the probability
i) no one joins the queue in a particular minute ii) 2 or more persons join the queue.

Soln: X : No of persons joining the queue

$$P(x) = \frac{e^{-m} m^x}{x!} \quad \text{where } m = 5.8 \ \& \ x = 0, 1, 2 \ldots$$

$$P(x) = \frac{e^{-5.8} (5.8)^x}{x!}$$

i) $P(x=0) = \frac{e^{-5.8} (5.8)^0}{0!} = 0.003$

ii) $P(x \geq 2) = 1 - P(x < 2)$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - [0.003 + 0.02]$$

$$= 0.9796.$$

* For a poisson variable ⑧

$3 \times P[X=2] = P[X=4]$ Find standard deviation.

soln: $3 \times P[X=2] = P[X=4]$

$$3 \times \frac{e^{-m} m^2}{2!} = \frac{e^{-m} m^4}{4!}$$

$$m^2 = 36 \Rightarrow m = 6$$

$$S.D = \sqrt{var} = \sqrt{6} = 2.449 \quad [mean = var = 6]$$

* Fit a Poisson distribution

x :	0	1	2	3	4
f :	122	60	15	2	1

soln : Mean $= \dfrac{\sum fx}{\sum f} = \dfrac{0+60+30+6+4}{200} = 0.5$

$$P(x) = \frac{e^{-m} m^x}{x!} \quad \& \quad f(x) = 200 \times P(x)$$

$$f(0) = 200 \times \frac{e^{-0.5}(0.5)^0}{0!} \simeq 121$$

$$f(1) = 200 \times \frac{e^{-0.5}(0.5)^1}{1!} \simeq 61$$

$$f(2) = 200 \times \frac{e^{-0.5}(0.5)^2}{2!} \simeq 15$$

$$f(3) = 200 \times \frac{e^{-0.5}(0.5)^3}{3!} \simeq 3$$

$$f(4) = 200 \times \frac{e^{-0.5}(0.5)^4}{4} \simeq 0.$$

$\therefore$ Expected freq' are 121, 61, 15, 03, 0.

Continuous ~~Cumulative~~ distribution fun

* The diameter of electric cable is assumed to be continuous with p.d.f $f(x) = \begin{cases} 6x(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

Verify that $f(x)$ is a p.d.f & find mean & var.

Soln:
$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^{\infty} f(x)\,dx.$$

$$= 0 + \int_0^1 6x(1-x)\,dx + 0$$

$$= \int_0^1 (6x - 6x^2)\,dx = \left[3x^2 - 2x^3\right]_0^1 = 1$$

$\therefore$ $f(x)\,dx$ is pdf.

$$\text{Mean} = \int_{-\infty}^{\infty} x\, f(x)\,dx = \int_0^1 x \cdot 6x(1-x)\,dx$$

$$= \int_0^1 (6x^2 - 6x^3)\,dx = \left[2x^3 - \frac{3}{2}x^4\right]_0^1$$

$$= \frac{1}{2}$$

$$\text{Var} = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = \int_0^1 \left(x - \tfrac{1}{2}\right)^2 6x(1-x)\,dx.$$

$$= \int_0^1 \left(-6x^4 + \frac{9}{2}x^2 + \frac{3}{2}x\right)dx$$

$$= \frac{1}{20}$$

* Find the constant K s.t $f(x) = \begin{cases} Kx^2, & 0<x<3 \\ 0 & \text{otherwise} \end{cases}$

is a p.d.f. Also find i) $P(1<X<2)$
ii) $P(X \le 1)$ iii) $P(X>1)$ iv) Mean v) variance.

Sol^n: ~~To prove~~ $\int_{-\infty}^{\infty} f(x)\,dx = 1$

ie $\int_0^3 Kx^2\,dx = 1$

ie $\left[\frac{Kx^3}{3}\right]_0^3 = 1$ or $9K = 1$ $\therefore K = \frac{1}{9}$

i) $P(1<X<2) = \int_1^2 f(x)\,dx = \int_1^2 \frac{x^2}{9}\,dx$

$= \left[\frac{x^3}{27}\right]_1^3 = \frac{7}{27}$

ii) $P(X \le 1) = \int_0^1 f(x)\,dx = \int_0^1 \left[\frac{x^2}{9}\right] dx = \left[\frac{x^3}{27}\right]_0^1$

$= \frac{1}{27}$

iii) $P(X>1) = \int_1^3 f(x)\,dx = \int_1^3 \frac{x^2}{9}\,dx = \left(\frac{x^3}{27}\right)_1^3$

$= \frac{26}{27}$

iv) Mean $= \mu = \int_{-\infty}^{\infty} x f(x)\, dx$

$$= \int_0^3 x \cdot \frac{x^2}{9} dx = \left[\frac{x^4}{36}\right]_0^3 = \frac{81}{36} = \frac{9}{4}$$

v)

$$\text{Var} = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$

$$= \int_0^3 x^2 \cdot \frac{x^2}{9} dx - \left(\frac{9}{4}\right)^2$$

$$= \int_0^3 \frac{x^4}{9} dx - \left(\frac{9}{4}\right)^2$$

$$= \left[\frac{x^5}{45}\right]_0^3 - \frac{81}{16} = \frac{27}{80}.$$

* Find K st $f(x) = \begin{cases} kxe^{-x} & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$ is a pdf. Find the mean.

Soln: We know $\int_{-\infty}^{\infty} f(x)\,dx = 1$

ie $\int_0^1 kxe^{-x}\,dx = 1$

Applying Bernoulli's rule

$$K\left[x\cdot\left(\frac{e^{-x}}{-1}\right) - (1)\left(\frac{e^{-x}}{+1}\right)\right]_0^1 = 1$$

$$K\left\{\left[1\cdot\left(\frac{e^{-1}}{-1}\right) - \left(\frac{e^{-1}}{1}\right)\right] - \left[0 - \frac{e^0}{1}\right]\right\} = 1$$

$$K\left\{-\frac{2}{e} + 1\right\} = 1 \Rightarrow \left[\frac{-2+e}{e}\right]K = 1 \quad \therefore K = \frac{e}{e-2}$$

$$\text{Mean} = \mu = \int_{-\infty}^{\infty} x\cdot f(x)\,dx = \int_0^1 x\cdot\frac{e}{e-2}\cdot xe^{-x}\,dx$$

$$= \frac{e}{e-2}\int_0^1 x^2\cdot e^{-x}\,dx.$$

$$= \frac{e}{e-2}\left[x^2\left(\frac{e^{-x}}{-1}\right) - 2x\left(\frac{e^{-x}}{1}\right) + 2\left(\frac{e^{-x}}{-1}\right)\right]_0^1$$

$$= \frac{e}{e-2}\left\{\left[1\left(\frac{e^{-1}}{-1}\right) - 2\left(\frac{e^{-1}}{1}\right) + 2\left(\frac{e^{-1}}{-1}\right)\right] - \left[0 - 0 - 2e^{-0}\right]\right\}$$

$$= \frac{e}{e-2}\left\{-\frac{1}{e} - \frac{2}{e} - 2\left(\frac{1}{e}\right) + 2\right\} = \frac{2e-5}{e-2}$$

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Exponential Distribution

Probability density function is

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & \text{for } x > 0 \\ 0 & \text{otherwise, where } \alpha > 0. \end{cases}$$

Mean & S.D :

$$\text{Mean } (\mu) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{\infty} x\cdot\alpha e^{-\alpha x}\,dx.$$

Using Bernoulli's rule

$$\mu = \alpha\left[x\cdot\left(\frac{e^{-\alpha x}}{-\alpha}\right) - 1\left(\frac{e^{-\alpha x}}{\alpha^2}\right)\right]_0^\infty$$

$$= \alpha\left[0\cdot\frac{1}{\alpha^2}(0-1)\right] = \frac{1}{\alpha}$$

$$\therefore \mu = \frac{1}{\alpha}.$$

$$\text{Var} = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$$

$$= \int_0^{\infty}(x-\mu)^2\cdot\alpha e^{-\alpha x}\,dx$$

$$= \alpha\int_0^{\infty}(x-\mu)^2 e^{-\alpha x}\,dx$$

Applying Bernoulli's rule

$$= \alpha\left[(x-\mu)^2\left[\frac{e^{-\alpha x}}{-\alpha}\right] - 2(x-\mu)\cdot\left[\frac{e^{-\alpha x}}{\alpha^2}\right] + 2\left[\frac{e^{-\alpha x}}{-\alpha^3}\right]\right]_0^\infty$$

$$= \alpha\left[-\frac{1}{\alpha}(0-\mu)^2 - \frac{2}{\alpha^2}(0-(-\mu)) - \frac{2}{\alpha^3}(0-1)\right]$$

$$= \alpha\left\{\frac{\mu^2}{\alpha} - \frac{2\mu}{\alpha^2} + \frac{2}{\alpha^3}\right\} \quad \text{But } \mu = 1/\alpha$$

$$\sigma^2 = \alpha\left\{\frac{1}{\alpha^3} - \frac{2}{\alpha^3} + \frac{2}{\alpha^3}\right\} = \frac{1}{\alpha^2}$$

$$\sigma = \frac{1}{\alpha}$$

Mean $(\mu) = \frac{1}{\alpha}$; S.D $(\sigma) = \frac{1}{\alpha}$.

* If x is an exponential variate with mean 3 find i) $P(x>1)$ ii) $P(x<3)$ (15)

Sol^n: $\frac{1}{\alpha} = 3$, $\alpha = \frac{1}{3}$

$\because$ $f(x) = \begin{cases} \alpha e^{-\alpha x} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$

$f(x) = \begin{cases} \frac{1}{3} e^{-\frac{1}{3}x} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$

i) $P(x>1) = 1 - P(x \le 1)$

$= 1 - \int_0^1 f(x)\,dx.$

$= 1 - \int_0^1 \frac{1}{3} e^{-x/3} dx = 1 - \frac{1}{3}\left[\frac{e^{-x/3}}{-1/3}\right]_0^1$

$= 1 + [e^{-1/3} - 1] = e^{-1/3}$

ii) $P(x<3) = \int_0^3 f(x)\,dx$

$= \int_0^3 \frac{1}{3} e^{-x/3} dx$

$= \frac{1}{3}\left[\frac{e^{-x/3}}{-1/3}\right]_0^3$

$= -[e^{-1} - e^0] = 1 - \frac{1}{e}. \approx 0.6321$

* In a certain town duration of a shower is exponentially distributed withe mean 5 min. what is the probability that a shower will last for

i) 10 min or more ii) less than 10 min

iii) between 10 and 12 minutes.

Soln: P.d.f of exponential distⁿ is

$$f(x) = \alpha e^{-\alpha x}, \quad x > 0$$

$$\frac{1}{\alpha} = 5 \quad \therefore \alpha = 1/5$$

$$\therefore f(x) = \frac{1}{5} e^{-x/5}$$

i) $P(x \geq 10) = \int_{10}^{\infty} \frac{1}{5} e^{-x/5} dx = \left[-e^{-x/5}\right]_{10}^{\infty}$

$$= -\left[e^{-\infty} - e^{-2}\right] = -\left[0 - e^{-2}\right] = e^{-2} = 0.1353$$

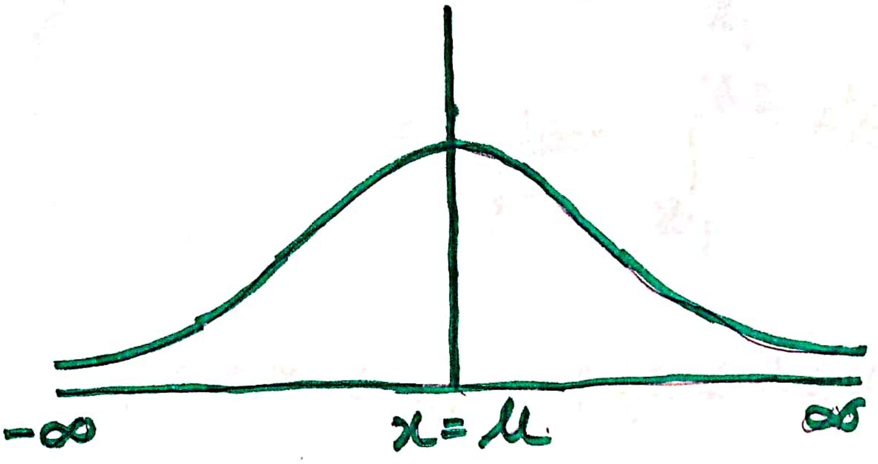
ii) $P(x < 10) = \int_{0}^{10} \frac{1}{5} e^{-x/5} dx = 1 - e^{-2} = 0.8647$

iii) $P(10 < x < 12) = \int_{10}^{12} \frac{1}{5} \cdot e^{-x/5} dx$

$$= -\left(e^{-12/5} - e^{-2}\right) = 0.0446$$

Normal Distribution

Normal Curve



The graph of the prob. fun $f(x)$ is bell shaped curve symmetrical about the line $x = \mu$ and is called normal probability curve.

Standard Normal Distribution

we have
$$P(a \leq x \leq b) = \int_a^b f(x)\,dx$$

For normal distr we have

$$P(a \leq x \leq b) = \frac{1}{\sigma\sqrt{2\pi}} \int_a^b e^{-(x-\mu)^2/2\sigma^2}\,dx \quad \text{— (1)}$$

Put $z = \frac{x-\mu}{\sigma}$ or $x = \mu + \sigma z$ then $dx = \sigma\,dz$

Let $z_1 = \frac{a-\mu}{\sigma}$ & $z_2 = \frac{b-\mu}{\sigma}$ be the values of z corresponding to $x = a$ & $x = b$ then

(1) becomes

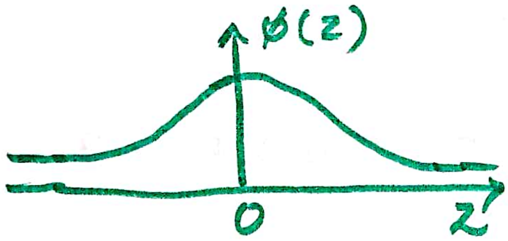
$$P(a \le x \le b) = \frac{1}{\sigma\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2} \cdot \sigma\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2} \cdot dz$$

$$\therefore P(a \le x \le b) = P(z_1 \le z \le z_2) = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2} dz.$$
$$= F(z)$$

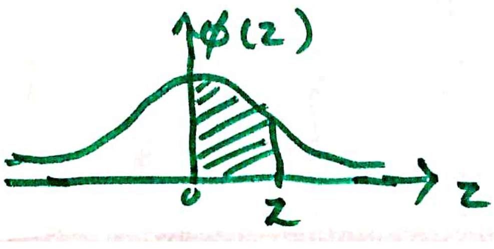
$\therefore$ $z = \frac{x-\mu}{\sigma}$ is known as the standard normal variate (SNV) with $\mu = 0, \sigma = 1$.

The $F(z)$ is SNV which is symmetrical about the line $z=0$



The integral in RHS represent the area bounded by $z = z_1$ and $z = z_2$. Further if $z_1 = 0$ we have

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2} dz$$ represent the area from $z=0$ to z



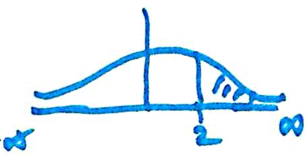
Normal Distribution

* If x is normally distributed with mean 12 and S.D 4 find the following
i) $P(x \geq 20)$ ii) $P(x \leq 20)$

Soln: let $z = \dfrac{x-\mu}{\sigma} = \dfrac{x-12}{4}$

$P(x \geq 20) = P\left[\dfrac{x-12}{4} \geq \dfrac{20-12}{4}\right]$

$= P[z \geq 2]$



= are from 2 to ∞

= [are frm 0 to ∞] − [area from 0 to 2]

$= 0.5 - 0.52$

$= 0.0228$

$P(x \leq 20) = P\left[\dfrac{x-12}{4} \leq \dfrac{20-12}{4}\right]$

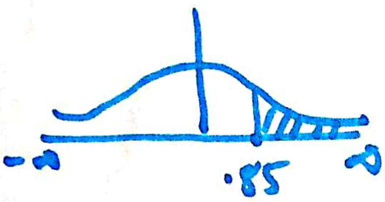
$= 0.9772$

* Evaluate the following probabilities with

i) $P(z \geq 0.85)$ ii) $P(-1.64 \leq z \leq -0.88)$

iii) $P(z \leq -2.43)$ iv) $P(|z| \leq 1.94)$

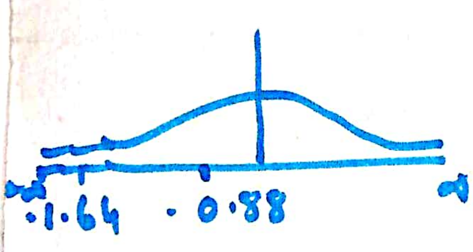
Soln: i) $P(z \geq 0.85) = P(\text{area from } 0 \text{ to } \infty) - P(\text{area from } 0 \text{ to } 0.85)$



$= 0.5 - \phi(0.85)$

$= 0.5 - 0.3023 = 0.1977$

ii) $P(-1.64 \leq z \leq -0.88) = P(\text{area from } -1.64 \text{ to } -0.88)$

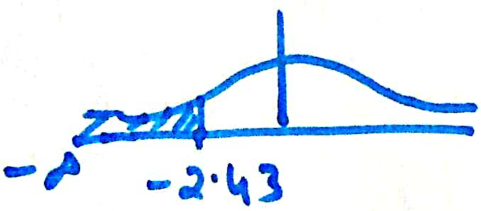


$= P(\text{area from } 0 \text{ to } 1.64) - P(\text{area from } 0 \text{ to } 0.88)$

$= \phi(1.64) - \phi(0.88)$

$= 0.4495 - 0.3106 = 0.1389$

iii) $P(z \leq -2.43) = P(\text{area from } -\infty \text{ to } -2.43)$



$= P(\text{area from } -\infty \text{ to } 0) - P(\text{area from } -2.43 \text{ to } 0)$

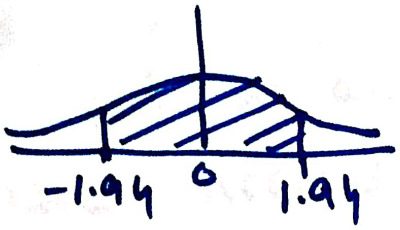
$= 0.5 - \phi(+2.43)$

$= 0.5 - 0.4925$

$= 0.0075.$

iv) $P(|z| \leq 1.94) = P(-1.94 \leq z \leq 1.94)$

$= P(\text{area from } -1.94 \text{ to } 1.94)$



$= P(\text{area from } -1.94 \text{ to } 0) +$
$P(\text{area from } 0 \text{ to } 1.94)$

$= \phi(1.94) + \phi(1.94)$

$= 0.4738 + 0.4738$

$= .9476$

* In normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find μ and σ. [$\phi(0.5) = 0.19$ & $\phi(1.4) = 0.42$]

Soln:
Let μ and σ be the mean and S.D of the normal distribution

By data we have

$$P(x < 45) = 0.31 \quad \& \quad P(x < 64) = 0.08$$

we have SNV $z = \dfrac{x - \mu}{\sigma}$

when $x = 45$, $\quad z = \dfrac{45 - \mu}{\sigma} = z_1$

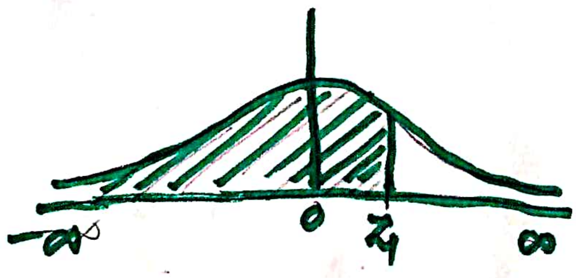
$x = 64$, $\quad z = \dfrac{64 - \mu}{\sigma} = z_2$

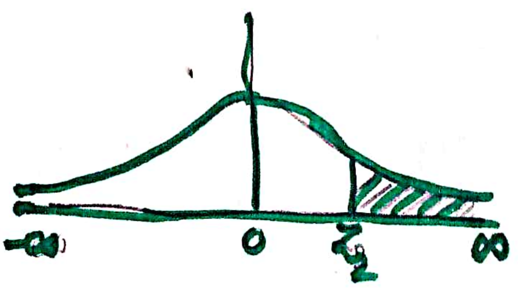
we have

$$P(z < z_1) = 0.31 \quad \& \quad P(z > z_2) = 0.08$$

ie $0.5 + \phi(z_1) = 0.31 \quad \& \quad 0.5 - \phi(z_2) = 0.08$

$$\phi(z_1) = -0.19 \qquad \phi(z_2) = 0.42$$





From the data we have

$0.19 = \phi(0.5)$ & $0.42 = \phi(1.4)$

$\therefore \phi(z_1) = -\phi(0.5)$ and $\phi(z_2) = \phi(1.4)$

$\therefore z_1 = -0.5$ & $z_2 = 1.4$

ie $\frac{45-\mu}{\sigma} = -0.5$ & $\frac{64-\mu}{\sigma} = 1.4$

$\mu - 0.5\sigma = 45$ & $\mu + 1.4\sigma = 64$

On solving $\mu = 50$, $\sigma = 10$

Mean $= 50$ & S.D $= 10$

★ The marks of 1000 students in an examination follows a normal distribution with mean 70 and S.D 5. Find the no. of students whose marks will be i) less than 65 ii) more than 75 iii) betⁿ 65 & 75.

Solⁿ: x – marks of students.

$\mu = 70,\ \sigma = 5.$ SNV $z = \dfrac{x-\mu}{\sigma} = \dfrac{x-70}{5}$

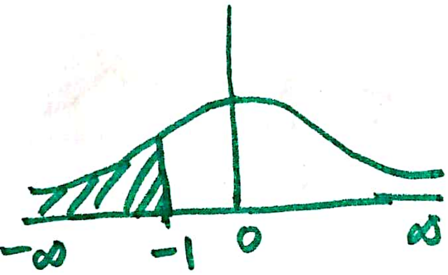
i) $P(x<65) = P\left(\dfrac{x-\mu}{\sigma} < \dfrac{65-70}{5}\right)$

$= P(z < -1)$

= area from $-\infty$ to -1

= area from $(1$ to $\infty)$

= (area from 0 to ∞) – (area from 0 to 1)



$= 0.5 - \phi(1)$

$= 0.5 - 0.3413 = 0.1587$

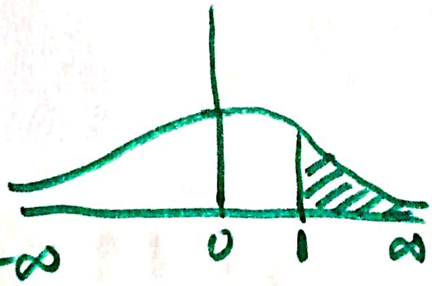
For 1000 students $= 1000 \times 0.1587 \simeq 159$

ii) $P(x>75) = P\left[\dfrac{x-70}{5} > \dfrac{75-70}{5}\right]$

$= P[z>1]$

= [area from 1 to ∞]

= [area from 0 to ∞] – [area from 0 to 1]



$= 0.5 - 0.3413$

$= 0.1587$

For 1000 students no of students scoring more than 75 marks $= 75 \times 1000 = 158.7$

$\simeq 159.$

iii) $P(65 \le x \le 75) = P\left(\frac{65-70}{5} \le \frac{x-70}{5} \le \frac{75-70}{5}\right)$

$= P(-1 \le Z \le 1)$

= (area from -1 to 1)

= (area from -1 to 0) +
(area from 0 to 1)

$= 2(0.3413) = 0.6826$

out of 1000 students $= 1000 \times 0.6826$

$= 682.6$

$\simeq 683$