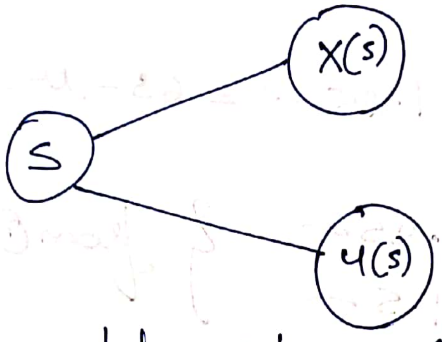
11/10/19

Unit - V

Joint probability distribution, stochastic process.



$X \times Y ; \; S \rightarrow X(s) \times Y(s)$

Let S be a sample space

Let $X = X(s)$ & $Y(s)$ be two random variable.

On the sample space S

$X(s) = \{ x_1, x_2, x_3, x_4 \ldots \ldots x_n \}$

$Y(s) = \{ y_1, y_2, y_3, y_4 \ldots \ldots y_m \}$

Examples.

(i) X : age Y : Blood pressure of a person.

(ii) X : crop yield Y : Rain fall in an area.

(iii) X : I.Q Y : Nutrition of an individual.

→ The J.P.D of $P(X = x_i, Y = y_j) = P(x_i, y_j)$

for the $P(x, y)$ satisfies the condition.

i) $P(x_i, y_j) \geq 0$

ii) $\sum_{j=1}^{m} \sum_{i=1}^{n} P(x_i, y_j) = 1$

x/y	y_1	y_2	y_3	.... y_m	Row Sum
x_1	P_{11}	P_{12}	P_{13}	.... P_{1m}	$f(x_1)$
x_2	P_{21}	P_{22}	P_{23}	.... P_{2m}	$f(x_2)$
x_3	P_{31}	P_{32}	P_{33}	.... P_{3m}	$f(x_3)$
⋮	⋮	⋮	⋮	....	⋮
x_n	P_{n1}	P_{n2}	P_{n3}	P_{nm}	$f(x_n)$
Column sum	$g(y_1)$	$g(y_2)$	$g(y_3)$	.... $g(y_m)$	1

Marginal distribution →

The marginal distribution of $f(x)$ of X is the probability distribution of only one variable 'X' & it is denoted by

$f(x) = P(X = n \quad y = \text{orbitary})$ ~~for~~

$= \sum_{j=1}^{m} P(x_i, y_j)$ is for fixed (i),

sum of ith row.

$|||^{ly}$ $g(y) = P(X = \text{orbitary}, y = m)$

$= \sum_{i=1}^{m} P(x_i, y_j)$ is for fixed (j),

sum of j^{th} row.

The marginal distribution of X & Y are given by

X	x_1	x_2	x_3	....	x_n
$f(x)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	....	$f(x_n)$

$\boxed{\sum f(x_n) = 1}$.

&

Y	y_1	y_2	y_3	....	y_n
$g(y)$	$g(y_1)$	$g(y_2)$	$g(y_3)$	....	$g(y_n)$

$\boxed{\sum g(y_n) = 1}$

Conditional probability of random variable.

Conditional probability of random variable Y_1, given that $X = x$ is

$$P(y/x) = \frac{P(x, y)}{f(x)}$$

|||ly for random Variable X, given that $X = y$ is

$$P(X|y) = \frac{P(X, y)}{g(y)}$$

Expectation →

$$E(X, Y) = \sum_{j=1}^{m} \sum_{i=1}^{n} P_{ij} \cdot x_i y_j$$

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Statistical independence

quiz

$$\boxed{P(x_i, y_j) = f(x_i) \cdot g(y_j)}$$

X|Y	y_1	y_2	y_3	.... y	Row sum
x_1					
x_2					
x_3					
⋮					
column row					1

The random X & Y are said to be statistically independent / independent if

$$\boxed{P(X_i, y_j) = f(x_i) \cdot g(y_j)}$$

Mean:

$$\mu_x = E(X) = \Sigma f(x_i) \cdot X_i$$

$$\mu_y = E(Y) = \Sigma g(y_i) \cdot y_i$$

Covariance → The covariance of X & Y is denoted by COV (X & Y).

$$COV(X, Y) = E(X, Y) - \mu(X) \cdot \mu(Y).$$

Correlation →

The correlation of X & Y is

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$

where,

$$\sigma_X^2 = E(X^2) - \mu_X^2$$

$$\sigma_Y^2 = E(Y^2) - \mu_Y^2$$

quiz / NOTE →

1) If X & Y are independent then
   a) $E(X,Y) = E(X).E(Y)$
   b) $Var(X+Y) = Var(X) + Var(Y)$
   c) $Cov(X,Y) = 0$.
2) X & Y are random variable then,
   $E(X+Y) = E(X) + E(Y)$.

Xamples →

① A J.P.D is given by the following table. Find
① Covariance of (X,Y)
② Marginal distribution

X\Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

X|Y	-3	2	4	Row sum	
1	0.1	0.2	0.2	0.5	$f(x_1)$
3	0.3	0.1	0.1	0.5	$f(x_2)$
column sum	0.4	0.3	0.3	1	

ii) ∴ marginal distribution of X

X	1	3	
$f(x)$	0.5	0.5	$\Sigma f(x) = 1$

Y	-3	2	4	
$g(y)$	0.4	0.3	0.3	$\Sigma g(y) = 1$

i) $Cov(x, y) = E(X, Y) - \mu(x) \cdot \mu(y)$

Now

$$\mu_X = E(X) = \sum_{i=1}^{m} x_i f(x_i)$$

$$= x_1 f(x_1) + x_2 f(x_2)$$

$$= 1 \times 0.5 + 3 \times 0.5$$

$$= 0.5 + 1.5$$

$$= \underline{\underline{2}}.$$

$$\mu_Y = E(Y) = \sum_{j=1}^{n} y_j g(y_j)$$

$$= y_1 g(y_1) + y_2 g(y_2) + y_3 g(y_3)$$

$$= -3 \times 0.4 + 2 \times 0.3 + 4 \times 0.3$$

$$= -1.2 + 0.6 + 1.2 = 0.6 //$$

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$$E(X,Y) = \sum_{j=1}^{m} \sum_{i=1}^{n} P_{ij} x_i y_j$$

$$= \sum_{j=1}^{3} \sum_{i=1}^{2} P_{ij} x_i y_j$$

$$= \sum_{j=1}^{3} \left[P_{1j} \cdot x_1 y_j + P_{2j} \cdot x_2 \cdot y_j \right]$$

$$= P_{11} x_1 . y_1 + P_{21} . x_2 . y_1 +$$
$$P_{12} x_1 . y_2 + P_{22} . x_2 . y_2 +$$
$$P_{13} x_1 . y_3 + P_{23} . x_2 . y_3 .$$

$$= 0.1 \times 1 \times -3 + 0.3 . 3 \times -3 +$$
$$0.2 \times 1 \times 2 + 0.1 \times 3 \times 2 +$$
$$0.2 \times 1 \times 4 + 0.1 \times 3 \times 4$$

$$= -0.3 + (-2.7) + 0.4 + 0.6 + 0.8 + 1.2$$

$$= -3 + 1.8 + 1.2$$

$$= -3 + 3$$

$$\boxed{E(X,Y) = 0}$$

$$\therefore \operatorname{cov}(X,Y) = E(X,Y) - \mu_x \mu_y$$
$$= 0 - 2(0.6)$$
$$= -1.2$$

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12/10/19 (2) Find a) Marginal distribution $f(x)$ & $g(y)$.
b) $cov(X,Y)$

X|Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

X|Y	-2	-1	4	5	Row sum
1	0.1	0.2	0	0.3	0.6
2	0.2	0.1	0.1	0	0.4
Column sum	0.3	0.3	0.1	0.3	1

a) marginal distribution for X

X	1	2
f(X)	0.6	0.4

$\boxed{\sum f(x) = 1}$

b) marginal distribution for Y

Y	-2	-1	4	5
g(Y)	0.3	0.3	0.1	0.3

$\boxed{\sum g(y) = 1}$

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b) $Cov(X,Y) = E(X,Y) - \mu_X \cdot \mu_Y$

$$\mu_X = E(X) = \sum_{i=1}^{n} x_i f(x_i)$$

$$= x_1 f(x_1) + x_2 f(x_2)$$

$$= 1 \times 0.6 + 2 \times 0.4$$

$$= 0.6 + 0.8$$

$$= \underline{\underline{1.4}}.$$

$$\mu_Y = E(Y) = \sum_{j=1}^{m} y_j g(y_j)$$

$$= y_1 g(y_1) + y_2 g(y_2) + y_3 g(y_3) + y_4 g(y_4)$$

$$= -2 \times 0.3 - 1 \times 0.3 + 4 \times 0.1 + 5 \times 0.3$$

$$= -0.6 - 0.3 + 0.4 + 1.5$$

$$= -0.9 + 1.9$$

$$= \underline{\underline{1}}$$

$$E(X,Y) = \sum_{j=1}^{m} \sum_{i=1}^{n} P_{ij} x_i y_j$$

$$= \sum_{j=1}^{m} \left[P_{1j} \cdot x_1 \cdot y_j + P_{2j} \cdot x_2 \cdot y_j \right]$$

$$= P_{11} \cdot x_1 \cdot y_1 + P_{21} \cdot x_2 \cdot y_1 +$$
$$P_{12} \cdot x_1 \cdot y_2 + P_{22} \cdot x_2 \cdot y_2 +$$
$$P_{13} \cdot x_1 \cdot y_3 + P_{23} \cdot x_2 \cdot y_3 +$$
$$P_{14} \cdot x_1 \cdot y_4 + P_{24} \cdot x_2 \cdot y_4.$$

$= 0.1 \times 1 \times -2 + 0.2 \times 2 \times -2 + 0.2 \times 1 \times -1 +$
$0.1 \times 2 \times -1 + 0 \times 1 \times 4 + 0.1 \times 2 \times 4 +$
$0.3 \times 1 \times 5 + 0 \times 2 \times 5$.

$= -0.2 - 0.8 - 0.2 - 0.2 + 0 + 0.8 + 1.5 + 0$

$= -1 - 0.4 + 2.3$

$= 1.3 - 0.4$

$E(X,Y) = \underline{\underline{0.9}}$

$\therefore$ COV $= E(X,Y) - \mu_x \cdot \mu_y$

$= 0.9 - 1.4 \times 1$

$= 0.9 - 1.4$

$= \underline{\underline{-0.5}}$.

(3) A fair coin is tossed three times. Let X denotes '0' or '1' accordingly as head or tail occurs on the first toss. Let Y denote the number of head which occur, find
a) distribution of X & Y
b) $E(X+Y)$.

$\rightarrow$ a) $S = 2^3$

$S = \{$ HHH TTT
HHT TTH
HTH THT
HTT THH $\}$

$X = \{1, 1, 1, 1, 0, 0, 0, 0\}$
$Y = \{3, 2, 2, 1, 0, 1, 1, 2\}$

X\Y	0	1	2	3	Row sum
0	0	1/8	2/8	1/8	4/8
1	1/8	2/8	1/8	0	4/8
Column sum	1/8	3/8	3/8	1/8	1

To checked X & Y are independent / not

$i=2 \quad j=3$

$P_{23} = f(x_2) \cdot g(y_3)$

$1/8 \neq 4/8 \cdot 3/8$

hence they are not independent.

b) $E(X+Y) = E(X) + E(Y)$

$= \mu_x + \mu_y$

$\mu_x = E(X) = \sum_{i=1}^{n} f(x_i) x_i$

$= x_1 f(x_1) + x_2 f(x_2)$

$= 0 \times 4/8 + 1 \times 4/8$

$= 0 + 4/8$

$= \underline{1/2}$

$\mu_y = E(Y) = \sum_{j=1}^{m} y_j \, g(y_j)$

$= y_1 g(y_1) + y_2 g(y_2) + y_3 g(y_3) + y_4 g(y_4)$.

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$= 0 + 3/8 + 6/8 + 3/8$

$= 12/8$

$= \underline{\underline{3/2}}$

$E(X+Y) = \frac{1}{2} + 3/2$

$= 4/2$

$= \underline{\underline{2}}$.

(4) Two cards are selected at a random from a box which contains five cards numbered 1, 1, 2, 2 & 3. find the joint distribution of X & Y where X denotes the sum & Y denotes the maximum of two number drawn.

$\rightarrow \quad 5C_2 = \frac{5 \times 4}{2} = 10$

not possible

$\therefore$ 10 ways

1, 1, 2, 2, 3

$S = \{ (1,1), (1,2), (1,2), (1,3)$
$(1,2), (1,2), (1,3)$
$(2,2), (2,3)$
$(2,3) \} = 10$ ways

$X = \{ 2, 3, 3, 4, 3, 3, 4, 4, 5, 5 \}$

$Y = \{ 1, 2, 2, 3, 2, 2, 3, 2, 3, 3 \}$

X|Y	1	2	3	Row sum
2	1/10	0	0	1/10
3	0	4/10	0	4/10
4	0	1/10	2/10	3/10
5	0	0	2/10	2/10
Column sum	1/10	5/10	4/10	1

(5) The J.P.D of two variables X & Y is given by

X|Y	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0
Column sum			

i) determine the marginal distribution & verify, that X & Y are not independent.

ii) find $P(X+Y>1)$.

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X\Y	0	1	2	Row Sum.
0	0.1	0.4	0.1	0.6
1	0.2	0.2	0	0.4
column sum	0.3	0.6	0.1	1

1) marginal distribution for X is

X	0	1
$f(x)$	0.6	0.4

$\therefore \sum f(x) = 1$

marginal distribution for Y is

Y	0	1	2
$g(y)$	0.3	0.6	0.1

$\therefore \sum g(y) = 1.$

Let $i = 1, 2 \quad j = 1, 2, 3$

if $i = 2 \quad j = 3$

$P_{23} = f(x_2) \cdot g(y_3)$

$0 = 0.4 \cdot 0.1$

$0 \neq 0.04$

Hence they are not independent.

11) $P(X+Y>1)$

$\rightarrow \quad P(0,2) + P(1,1) + P(1,2)$

$= 0.1 + 0.2 + 0$

$= \underline{\underline{0.3}}$.

(6) Evaluate the Conditional distribution $P(X|1)$ for the following joint distribution. Show that X & Y are not independent.

X|Y	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

$\rightarrow$

X|Y	1	2	3	row sum
1	1/12	1/6	0	1/4
2	0	1/9	1/5	14/45
3	1/18	1/4	2/15	79/180
column sum	5/36	19/36	7/3	1

$$P(x|y) = \frac{P(x,y)}{g(y)}$$

$$P(x|1) = \frac{P(x,1)}{g(y_1)}$$

$$P(1|1) = \frac{P(1,1)}{g(y_1)} = \frac{1/12}{5/36} = 3/5$$

$$P(2|1) = \frac{P(2,1)}{g(y_1)} = 0/5/36 = 0$$

$$P(3|1) = \frac{P(3,1)}{g(y_1)} = \frac{1/18}{5/36} = 2/5$$

X	1	2	3
P(x|1)	3/5	0	2/5

$\therefore \sum P(x|1) = 1.$

(7) Find the marginal distribution of X & Y & find P (Y=3 / X=2) if the JPD is

y\x	1	2	3
1	0.05	0.05	0.1
2	0.05	0.1	0.35
3	0	0.2	0.1

Y/X	1	2	3	Row sum
1	0.05	0.05	0.1	0.2
2	0.05	0.1	0.35	0.5
3	0	0.2	0.1	0.3
Column sum	0.1	0.35	0.55	1

marginal distribution of Y

Y	1	2	3
g(y)	0.2	0.5	0.3

marginal distribution of X

X	1	2	3
f(x)	0.1	0.35	0.55

$P(y=3 / x=2)$.

$\rightarrow$ $P(3,2) = \underline{\underline{0.2}}$.

—— x ——

Stocastic theory →

A family of random variable, indexed by a parameter (such as time) is known as stochastic process (i.e random process)

OR

The set of random variables $\{X_t \mid t \in T\}$

T = set of real numbers, is known as stochastic process.

where X_t is probability distribution

t is parameter

X is state.

Classification →

(1) Discrete → state.
Discrete → t (parameter).

(2) Discrete → state
Continuous → t (parameter).

(3) Continuous → state
discrete → t (parameter).

(4) Continuous → state
Continuous → t (parameter).

Markov Chain →

It's a process whose entire past history is summarilized in its current state.

A stochastic process such that generation of probability distribution depends only on the present state & this state is discret is called markov chain.

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Transition probability Matrix →

probability Vector →

It is a vector
$V = (V_1, V_2, \ldots V_n)$
if $V_i \geq 0$ for every I &

$$\sum_{i=1}^{n} V_i = 1$$

Stochastic Matrix →

It is a square matrix with each row ~~bing~~ being a probability vector, in other words all the entric of stochastic matrix are non negative & sum of entry of any row is 1.

Stochastic matrix is said to be regular if all the entries or sum power p^n are positive.

Key → **Fixed probability Vector →**

A probability vector is said to be fixed probability Vector, if

$$v = vp \quad \& \quad v \neq 0$$

Where p is transition probability Vector.

Q) Find which matrices are stochastic —

① $\begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 1/4 & 1/4 \end{bmatrix} \longrightarrow$ ✗ [not a square matrix]

② $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow$ ✓

③ $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/4 \end{bmatrix} \longrightarrow$ ✗ [sum isn't 1]

④ $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \longrightarrow$ ✓

⑤ $\begin{bmatrix} 0 & 1 \\ -1/2 & 3/2 \end{bmatrix} \longrightarrow$ ✗ [non positive number is present].

Xamples

① Find the unique fixed probability vector of

$$p = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$$

→ wkt $v = vp$.

$$[x, y, z] = [x, y, z]\, p.$$

$$[x, y, z] = [x, y, z] \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$$

$$[x, y, z] = \left[\frac{y}{2} + \frac{z}{2},\ x + \frac{z}{4},\ \frac{y}{2} + \frac{z}{4}\right]$$

On Equating,

$$x = \frac{y}{2} + \frac{z}{2}, \quad y = x + \frac{z}{4}, \quad z = \frac{y}{2} + \frac{z}{4}$$

Now, $z = \frac{y}{2} + \frac{z}{4} \rightarrow (1)$

We also know that,

$$\sum_{i=1}^{n} v_i = 1 \rightarrow \text{i.e } x + y + z = 1$$

$$x = 1 - y - z, \quad y = 1 - x - z, \quad z = 1 - x - y.$$

from (1),

$$z - \frac{y}{2} = \frac{z}{4}$$

$$z - \frac{z}{4} = \frac{y}{2}$$

$$\frac{4z - z}{4} = \frac{y}{2}$$

$$\frac{3z}{4} = \frac{y}{2}$$

$$\Rightarrow 6z = 4y$$

$$\boxed{z = \frac{2}{3}y}$$

$\therefore$ $y = 1 - x - z$

$$= 1 - \left[\frac{y}{2} + \frac{y}{3}\right] - \frac{2y}{3}$$

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$$y = 1 - \frac{y}{2} - \frac{y}{3} + \frac{2}{3}y$$

$$y = \frac{6 - 3y - 2y + 4y}{6}$$

$$6y = 6 - 5y + 4y$$

$$6y = 6 - 9y$$

$$15y = 6$$

$$\boxed{y = \frac{6}{15}} \Rightarrow \boxed{y = 2/5}$$

$$z = \frac{2}{3}y$$

$$= \frac{2}{3} \times \frac{6}{15}$$

$$= \frac{2}{3} \times \frac{2}{5}$$

$$\boxed{z = \frac{4}{15}}$$

$$x = \frac{2}{10} + \frac{4}{30}$$

$$x = \frac{1}{5} + \frac{2}{15}$$

$$= \frac{3+2}{15} \rightarrow \frac{5}{15} = 1/3$$

$$\boxed{x = 1/3}$$

The unique fixed probability

$[x, y, z] = [1/3, 2/5, 4/15]$.

(2) $P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/3 & 2/3 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$$\rightarrow [x, y, z] = [x, y, z] \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/3 & 1/2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= [y/3, \; x/2 + y/2 + z, \; x/2]$$

On Equating,

$x = y/3, \quad y = x/2 + y/2 + z, \quad z = x/2.$

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$$\sum_{i=1}^{n} v_i = 1$$

i.e $x + y + z = 1$

wkt, $x = 1 - y - z$ $\quad y = 1 - x - z$ $\quad z = 1 - x - y$

wkt $\frac{y}{3} = x$, $\boxed{2z = x}$

$\Rightarrow \boxed{3x = y}$ $\quad \rightarrow z = x/2$

$x = 1 - y - z$

$x = 1 - 3x - \frac{x}{2}$

$2x = 2 - 6x - x$

$2x + 1x + 6x = 2$

$\boxed{x = 2/9}$

$3x = y$

$\Rightarrow \frac{3 \times 2}{9} = y$

$\boxed{y = 6/9}$

$2z = \frac{2}{9}$

$z = \frac{\cancel{2}}{9 \times \cancel{2}}$

$\boxed{z = 1/9}$

$\therefore$ Unique fixed probability $[x, y, z] = [2/9, 6/9, 1/9]$.

③ Verify that $A = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1 & 0 \end{bmatrix}$ is regular matrix.

$\rightarrow$ Said to be regular, if all the entries are positive or sum power p^n are positive.

$$A \cdot A = A^2 \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1/8 & 5/16 & 9/16 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$$

④ find the unique fixed probability of the matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$

$\rightarrow$ $v = vp$

$$[x, y, z] = [x, y, z] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

$$[x, y, z] = \left[\frac{z}{2}, \; x + \frac{z}{2}, \; y\right]$$

$$x = \frac{z}{2}, \quad y = x + \frac{z}{2}, \quad z = y.$$

$\sum_{i=1}^{n} v_i = 1$, ie $x + y + z = 1$

$x = 1 - y - z$, $\quad y = 1 - x - z$, $\quad z = 1 - x - y$.

$$z = 1 - \frac{z}{2} - z$$

$$z = \frac{2 - z - 2z}{2}$$

$$2z = 2 - 3z$$

$$5z = 2$$

$$\boxed{z = 2/5}$$

$z = y$

$z = 2/5 = y$

$$\boxed{y = 2/5}$$

$$x = \frac{z}{2}$$

$$x = \frac{\cancel{2}}{5 \times \cancel{2}}$$

$$\boxed{x = 1/5}$$

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The Unique fixed probability is

$[x, y, z] = [1/5, 2/5, 2/5]$.

(5) Prove that markov chain with transition matrix

$$P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

$\longrightarrow$

(1) A man smoking habits are as follows, if he smokes filter cigarettes one week, he switches to non filter cigarette the next week with the probability 0.2 on the other hand if he smokes non filter cigarette one week, there is a probability of 0.7 that he will smoke non filter cigarette for another week as well. In a long run how often does he smokes filter cigarette.

$\rightarrow$ $V = vp$

$$P = \begin{array}{c|cc} & F & NF \\ \hline F & 0.8 & 0.2 \\ NF & 0.3 & 0.7 \end{array}$$

To know that he opts filter cigarette,
$\therefore$ we have to find $F = PV$,

$$[x, y] = [x, y] \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

$$= [x0.8 + 0.3y, \quad 0.2x + 0.7y]$$

$x = x0.8 + 0.3y \qquad y = 0.2x + 0.7y.$

$\sum_{i=1}^{n} V_i = 1$, i.e $x + y = 1$

$\Rightarrow x = 1 - y.$

$x = 0.8x + 0.3(1-x)$

$= 0.8x + 0.3 - 0.3x$

$x = 0.5x + 0.3$

$x - 0.5x = 0.3$

$0.5x = 0.3 \Rightarrow x = 0.3/0.5 \rightarrow \boxed{x = 3/5}$

$x = 0.6$

Scanned by CamScanner

$$0.6 = x(0.8) + y(0.3)$$

$$0.6 = 0.6(0.8) + y(0.3)$$

$$0.6 - .48 = y(0.3)$$

$$0.12 = y(0.3)$$

$$\frac{0.12}{0.3} = y$$

$$\Rightarrow \boxed{y = 0.4}$$

$$\boxed{y = 2/5}$$

$$[x, y] = [3/5, 2/5].$$

man smoking filter cigaratte is 60%

man smoking non filter cigaratte is 40%.

15/10/19

(7). A students study habit are as follows. If the student studies one night he is 60% sure not to study the next night. If he does not study one night he is 80% sure to study the next day. In the long run how often does he study.

$\rightarrow$

$$\begin{array}{ccc} & S & NS \\ S & 0.4 & 0.6 \\ NS & 0.8 & 0.2 \end{array}$$

$\therefore$ we have $v = vp$

$$[x, y, \cancel{z}] = [x, y, \cancel{z}] \begin{bmatrix} 0.4 & 0.6 \\ 0.8 & 0.2 \end{bmatrix}$$

$$[x, y, \cancel{z}] = [0.4x + 0.8y, \ 0.6x + 0.2y]$$

$[x, y] = [0.4x + 0.8y, \ 0.6x + 0.2y]$

On Equating,

$x = 0.4x + 0.8y \qquad y = 0.6x + 0.2y.$

$\sum_{i=1}^{n} v_i = 1 \quad$ i.e $\quad x + y + \cancel{z} = 1$

$x = 1 - y - \cancel{z} \qquad y = 1 - x - \cancel{z} \qquad \cancel{z = 1 - x - y}$

Now, $\quad x = 0.4x + 0.8(1 - x)$

$x = 0.4x + 0.8 - 0.8x$

$x = -0.4x + 0.8$

$x + 0.4x = 0.8$

$\cancel{1.4}x = 0.8$

$x = \dfrac{0.8}{\cancel{1.4}}$

$x = \dfrac{0.8}{1.4} \Rightarrow 8/14 = 4/7$

$= 0.5714$

$\boxed{x = 0.5714}$

$x = 1 - y$

$\Rightarrow y = 1 - x$

$= 1 - \dfrac{4}{7}$

$= \dfrac{7 - 4}{7}$

$= 3/7$

$\boxed{y = 0.4286}$

$[x, y] = [0.5714, \ 0.4286]$

Therefore in a long run studying is 57% & non studying is 43%.

(8) A salesman territory consists of 3 cities A, B, C. He never sells in the same city on successive days. If he sells in the city A, then the next day he sells in city B. However if he sells in city B or C then the next day he is twice likely to sell in city A in other city. In the long run how often does he sells in each of the city.

$\rightarrow$

$$P = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & 2/3 & 0 & 1/3 \\ C & 2/3 & 1/3 & 0 \end{array}$$

we know that,

$$V = PV$$

$$[x, y, z] = [x, y, z] \begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix}$$

$$[x, y, z] = \left[\frac{2y}{3} + \frac{2z}{3}, \; x + \frac{z}{3}, \; \frac{y}{3} \right]$$

On Equating,

$$x = \frac{2y}{3} + \frac{2z}{3} \qquad y = x + \frac{z}{3} \qquad z = y/3$$

wkt

$$\sum_{i=1}^{n} V_i = 1 \longrightarrow x + y + z = 1$$

$$x = 1 - y - z \qquad y = 1 - x - z \qquad z = 1 - x - y$$

Now, $\cancel{x = 1 - y - z}$

$$\cancel{x = 1 - \left[x + \frac{1}{3}\left(\frac{y}{3}\right)\right] - y/3}$$

$$\cancel{x = 1 - x}$$

$$y = 1 - x - z$$

$$y = 1 - \left[\frac{2}{3}y + \frac{2}{3}\left(\frac{y}{3}\right)\right] - y/3$$

$$y = 1 - \frac{2}{3}y - \frac{2}{9}y - y/3$$

$$y = 1 - y - \frac{2}{9}y$$

$$9y = 9 - 9y - 2y$$

$$9y = 9 - 11y$$

$$20y = 9 \Rightarrow \boxed{y = 9/20}$$

Now, $z = y/3$

$$z = \frac{9}{20} \times \frac{1}{3}$$

$$z = \frac{9}{60} \rightarrow \boxed{z = 3/20}$$

$$x = 1 - y - z$$

$$x = 1 - \frac{9}{20} - \frac{3}{20}$$

$$20x = 20 - 12$$

$$20x = 8$$

$$x = 8/20 \Rightarrow \boxed{x = 2/5}$$

$$[x, y, z] = [2/5, 9/20, 3/20]$$

Theorem $\rightarrow$ The probability distribution of the system n-steps later is given by

$$\boxed{p^n = p^0 \cdot p^n}$$

where n is steps, p^0 = initial state.

Q) Every year a man trades his car for a new car. If he has maruti, he trades it for an ambassador. If he has an ambassador he trades it for santro, however if he has santro he is just as likely to trade it for a new maruti or ambassador. In 2000 he brought his first car which was santro.

① find the probability that he has

a) 2002 santro

b) 2002 maruti

c) 2003 ambassodor

d) 2003 santro

② In the long run how often will he sell on santro.

→
$$P = \begin{array}{c|ccc} & M & A & S \\ \hline M & 0 & 1 & 0 \\ A & 0 & 0 & 1 \\ S & 1/3 & 1/3 & 1/3 \end{array}$$

$$p^0 = \begin{array}{ccc} M & A & S \\ \end{array}$$
$$p^0 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \rightarrow \text{initial}$$

i) a) 2000 : initial state $p^0 = [0\ 0\ 1]$
since he has santro car

To reach 2002 years (2 years later)
2000 → 2002 i.e 2 steps

$$p^n = p^0 \cdot p^n$$

$$n = 2$$

$$p^2 = p^0 \cdot p^2$$

$$\phi^2 = [0\ 0\ 1] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$\phi^2 = \left[1/9\ ,\ 4/9\ ,\ \underline{\underline{4/9}} \right].$$