UNIT - IV

PROBABILITY

* A random variable X has the following probability function for various values of x

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

i) Find K ii) Evaluate $P(x<6)$, $P(x \geq 6)$ and $P(3 \leq x \leq 6)$

Sol^n : We have i) $\boxed{\sum P(x) = 1}$

$\therefore\ 0+K+2K+2K+3K+K^2+2K^2+7K^2+K=1$

$\Rightarrow\ 10K^2+9K-1=0 \quad \therefore K = \frac{1}{10}$

Hence

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$2/100$	$17/100$

ii) $P(x<6) = 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} = 0.81$

$P(x \geq 6) = P(6) + P(7) = \frac{2}{100} + \frac{17}{100} = 0.19$

$P(3 \leq x \leq 6) = P(3) + P(4) + P(5) + P(6)$

$= \frac{2}{10} + \frac{3}{10} + \frac{1}{100} + \frac{2}{100}$

$= 0.53$

* A Random variable x has the following probability function

X :	-2	-1	0	1	2	3
$P(X)$:	0.1	K	0.2	$2K$	0.3	K

Find K and find mean and variance.

Soln: We have $\Sigma P(x) = 1$

$$0.1 + K + 0.2 + 2K + 0.3 + K = 1$$
$$\Rightarrow K = 0.1$$

$$\text{Mean} = E(X) = \Sigma x\, P(x)$$
$$= -2(0.1) + (-1)(0.1) + 0(0.2) + (1)(0.2) + 2(0.3) + 3(0.1)$$
$$E(X) = 0.8$$

$$\text{Variance} = E(X^2) - [E(X)]^2$$

Now
$$E(X^2) = \Sigma x^2 \cdot P(x)$$
$$= (-2)^2(0.1) + (-1)^2(0.1) + 0(0.2) + (1)^2(0.2) + (2)^2(0.3) + (3)^2(0.1)$$
$$= 2.8$$

$$\therefore \text{Variance} = 2.8 - (0.8)^2$$
$$= 2.16$$

* From a sealed box containing a dozen apples it was found that 3 apples are perished. Obtain the probability distribution of the no. of perished apples when 2 apples are drawn at random. Also find mean and variance of this distribution.

Soln: Let X : No. of perished apples

2 apples out of 12 can be selected in $^{12}C_2$ ways.

Good apples are 9

$\therefore X = 0, 1, 2$

$P(X=0)$ = Probability of getting 0 perished apple $= \dfrac{3C_0 \cdot 9C_2}{12C_2} = \dfrac{6}{11}$

$$P(X=1) = \frac{3C_1 \cdot 9C_1}{12C_2} = \frac{9}{22}$$

$$P(X=2) = \frac{3C_2 \cdot 9C_0}{12C_2} = \frac{1}{22}$$

The prob. distr^ny is

x :	0	1	2
$P(x)$:	6/11	9/22	1/22

Mean $= \sum x \cdot P(x) = 1/2$ $\quad Var(x) = E(x^2) - [E(x)]^2 = 15/44$

* Binomial Distribution

m = Mean = np Var = npq

Probability fun of B.D is - $P(x) = {}^nC_x \, p^x q^{n-x}$

p - success, q - failure, n - trials.

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* 256 set of 12 tosses of a coin in how many cases one can expect 8 heads & 4 tails ?

soln: $p = 0.5$, $q = 0.5$, $n = 12$, 8

$P(x) = {}^nC_x \, p^x q^{n-x}$

$P(x=8) = {}^{12}C_8 (0.5)^8 (0.5)^4 = 0.30$

Expected no of such cases in 256 sets

is = 256×0.30

$= 76.80 \simeq 77$

* In sampling a large no of parts manufactured by a company, the mean no. of defectives in samples of 20 is 2. Out of 1000 such samples how many would be expected to contain atleast 3 defective parts.

Soln: Mean $= np = 2$ when $n = 20$

$\Rightarrow 20p = 2 \Rightarrow p = 1/10 = 0.1$

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{20}C_{x}\, (0.1)^{x} (0.9)^{20-x}$

Prob' of at least 3 defectives is $= P(3) + P(4) \dots + P(20)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \{ {}^{20}C_{0} (0.1)^{0} + (0.9)^{20-0} +$

${}^{20}C_{1} (0.1)^{1} (0.9)^{19} + {}^{20}C_{2} (0.1)^{2} (0.9)^{18} \}$

$= 0.323$

No of defectives in 1000 samples =

$1000 \times 0.323 = 323$

* Five dice were thrown 96 times and the number of times an odd nor acutually turned out in the experiment is given. Fit B.D to this data & calculate expected freqnencies.

No of dice showing 1 or 3 or 5	0	1	2	3	4	5
observed freq	1	10	24	35	18	8.

soln: Prob' of getting 1 or 3 or 5 $= 3/6 = 1/2$

$$P(x) = {}^nC_x\, p^x q^{n-x} \quad ; \quad n=5$$

$$= {}^5C_x (1/2)^x (1/2)^{5-x} =$$

Now $f(x) = 96 \times P(x)$

$$f(0) = 96 \times {}^5C_0 (1/2)^0 (1/2)^{5-0} = 3$$

$$f(1) = 96 \times {}^5C_1 (1/2)^1 (1/2)^{5-1} = \cancel{30}\ 15$$

$$f(2) = 96 \times {}^5C_2 (1/2)^2 (1/2)^{5-2} = 30$$

$$f(3) = 96 \times {}^5C_3 (1/2)^3 (1/2)^{5-3} = 30$$

$$f(4) = 96 \times {}^5C_4 (1/2)^4 (1/2)^{5-4} = 15$$

$$f(5) = 96 \times {}^5C_5 (1/2)^5 (1/2)^{5-5} = 3$$

Expected frequencis are

3, 15, 30, 30, 15, 3.

Poisson Distribution

$$P(x) = \frac{e^{-m} m^{x}}{x!}$$

[Mean = m] [Var = m]

* The no of persons joining a cinema queue in a minute has poisson distribution with parameters 5.8. Find the probability i) no one joins the queue in a particular minute ii) 2 or more persons join the queue.

Soln: X : No of persons joining the queue

$$P(x) = \frac{e^{-m} m^{x}}{x!}$$ where $m = 5.8$ & $x = 0, 1, 2 \ldots$

$$P(x) = \frac{e^{-5.8} (5.8)^{x}}{x!}$$

i) $P(x=0) = \dfrac{e^{-5.8}(5.8)^{0}}{0!} = 0.003$

ii) $P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - [0.003 + 6.8]$

$= 0.9796.$

* For a poisson variable $3 \times P[X=2] = P[X=4]$ Find standard deviation.

Soln:
$$3 \times P[X=2] = P[X=4]$$
$$3 \times \frac{e^{-m} m^2}{2!} = \frac{e^{-m} m^4}{4!}$$
$$m^2 = 36 \Rightarrow m = 6$$
$$S.D = \sqrt{Var} = \sqrt{6} = 2.449 \quad [mean = Var = 6]$$

* Fit a Poisson distribution

x :	0	1	2	3	4
f :	122	60	15	2	1

soln : Mean = $\dfrac{\Sigma fx}{\Sigma f} = \dfrac{0+60+30+6+4}{200} = 0.5$

$$P(x) = \frac{e^{-m} m^x}{x!} \quad \& \quad f(x) = 200 \times P(x)$$

$$f(0) = 200 \times \frac{e^{-0.5}(0.5)^0}{0!} \simeq 121$$

$$f(1) = 200 \times \frac{e^{-0.5}(0.5)^1}{1!} \simeq 61$$

$$f(2) = 200 \times \frac{e^{-0.5}(0.5)^2}{2!} \simeq 15$$

$$f(3) = 200 \times \frac{e^{-0.5}(0.5)^3}{3!} \simeq 3$$

$$f(4) = 200 \times \frac{e^{-0.5}(0.5)^4}{4} \simeq 0.$$

$\therefore$ Expected freq' are 121, 61, 15, 03, 0.

Cumulative distribution funy

* The diameter of electric cable is assumed to be continuous with p.d.f $f(x) = \begin{cases} 6x(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

Verify that $f(x)$ is a p.d.f & find mean & var.

Sol:
$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx.$$

$$= 0 + \int_0^1 6x(1-x)\,dx + 0$$

$$= \int_0^1 (6x - 6x^2)\,dx = [3x^2 - 2x^3]_0^1 = 1$$

$\therefore$ $f(x)\,dx$ is pdf.

$$\text{Mean} = \int_{-\infty}^{\infty} x\, f(x)\,dx = \int_0^1 x \cdot 6x(1-x)\,dx$$

$$= \int_0^1 (6x^2 - 6x^3)\,dx = \left[2x^3 - \frac{3}{2}x^4\right]_0^1$$

$$= 1/2$$

$$\text{Var} = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = \int_0^1 (x - 1/2)^2\, 6x(1-x)\,dx.$$

$$= \int_0^1 \left(-6x^4 + \frac{9}{2}x^2 + \frac{3}{2}x\right)dx$$

$$= \frac{1}{20}$$

* Find the constant K s.t $f(x) = \begin{cases} Kx^2, & 0<x<3 \\ 0 & \text{otherwise} \end{cases}$

is a p.d.f. Also find i) $P(1<X<2)$
ii) $P(X\leq 1)$ iii) $P(X>1)$ iv) Mean v) Variance.

Sol^n: ~~To prove~~ $\int_{-\infty}^{\infty} f(x)\,dx = 1$

ie $\int_0^3 Kx^2\,dx = 1$

ie $\left[\frac{Kx^3}{3}\right]_0^3 = 1$ or $9K=1$ $\therefore K = \frac{1}{9}$

i) $P(1<X<2) = \int_1^2 f(x)\,dx = \int_1^2 \frac{x^2}{9}\,dx$

$= \left[\frac{x^3}{27}\right]_1^3 = \frac{7}{27}$

ii) $P(X\leq 1) = \int_0^1 f(x)\,dx = \int_0^1 \left[\frac{x^2}{9}\right] dx = \left[\frac{x^3}{27}\right]_0^1$

$= \frac{1}{27}$

iii) $P(X>1) = \int_1^3 f(x)\,dx = \int_1^3 \frac{x^2}{9}\,dx = \left(\frac{x^3}{27}\right)_1^3$

$= \frac{26}{27}$

iv)

iv) Mean $= \mu = \int_{-\infty}^{\infty} x f(x)\, dx$

$$= \int_0^3 x \cdot \frac{x^2}{9} dx = \left[\frac{x^4}{36}\right]_0^3 = \frac{81}{36} = \frac{9}{4}$$

v)

Var $= \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$

$$= \int_0^3 x^2 \cdot \frac{x^2}{9} dx - \left(\frac{9}{4}\right)^2$$

$$= \int_0^3 \frac{x^4}{9} dx - \left(\frac{9}{4}\right)^2$$

$$= \left[\frac{x^5}{45}\right]_0^3 - \frac{81}{16} = \frac{27}{80}.$$

(13)

Find K st $f(x) = \begin{cases} kxe^{-x} & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$ is a pdf. Find the mean.

Soln: We know $\int_{-\infty}^{\infty} f(x)\,dx = 1$

ie $\int_0^1 kxe^{-x}\,dx = 1$

Applying Bernoulli's rule

$K\left[x\cdot\left(\frac{e^{-x}}{-1}\right) - (1)\left(\frac{e^{-x}}{+1}\right)\right]_0^1 = 1$

$K\left\{\left[1\cdot\left(\frac{e^{-1}}{-1}\right) - \left(\frac{e^{-1}}{1}\right)\right] - \left[0 - \frac{e^0}{1}\right]\right\} = 1$

$K\left\{-\frac{2}{e} + 1\right\} = 1 \Rightarrow \left[\frac{-2+e}{e}\right]K = 1 \quad \therefore K = \frac{e}{e-2}$

$$\text{Mean} = \mu = \int_{-\infty}^{\infty} x\cdot f(x)\,dx = \int_0^1 x\cdot\frac{e}{e-2}\cdot xe^{-x}\,dx$$

$$= \frac{e}{e-2}\int_0^1 x^2\cdot e^{-x}\,dx.$$

$$= \frac{e}{e-2}\left[x^2\left(\frac{e^{-x}}{-1}\right) - 2x\left(\frac{e^{-x}}{1}\right) + 2\left(\frac{e^{-x}}{-1}\right)\right]_0^1$$

$$= \frac{e}{e-2}\left\{\left[1\left(\frac{e^{-1}}{-1}\right) - 2\left(\frac{e^{-1}}{1}\right) + 2\left(\frac{e^{-1}}{-1}\right)\right] - \left[0 - 0 - 2e^{-0}\right]\right\}$$

$$= \frac{e}{e-2}\left\{-\frac{1}{e} - \frac{2}{e} - 2\left(\frac{1}{e}\right) + 2\right\} = \frac{2e-5}{e-2}$$

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Exponential Distribution

Probability density function is

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & \text{for } x > 0 \\ 0 & \text{otherwise, where } \alpha > 0. \end{cases}$$

Mean & S.D :

$$\text{Mean } (\mu) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{\infty} x \cdot \alpha e^{-\alpha x}\,dx.$$

Using Bernoulli's rule

$$\mu = \alpha \left[x \cdot \left(\frac{e^{-\alpha x}}{-\alpha} \right) - 1 \left(\frac{e^{-\alpha x}}{\alpha^2} \right) \right]_0^{\infty}$$

$$= \alpha \left[0 \cdot \frac{1}{\alpha^2}(0-1) \right] = \frac{1}{\alpha}$$

$$\therefore \mu = \frac{1}{\alpha}.$$

$$\text{Var} = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 x \cdot \alpha e^{-\alpha x}\,dx$$

On simplifying

$$\text{Var} = \frac{1}{\alpha^2}$$

* If x is an exponential variate with mean 3 find i) $P(x>1)$ ii) $P(x<3)$ (15)

Sol^n: $\frac{1}{\alpha} = 3$. $\alpha = 1/3$

$\because f(x) = \begin{cases} \alpha e^{-\alpha x} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$

$f(x) = \begin{cases} \frac{1}{3} e^{-1/3 x} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$

i) $P(x>1) = 1 - P(x \le 1)$

$= 1 - \int_0^1 f(x)\,dx.$

$= 1 - \int_0^1 \frac{1}{3} e^{-x/3} dx = 1 - \frac{1}{\cancel{3}}\left[\frac{e^{-x/3}}{-1/\cancel{3}}\right]_0^1$

$= 1 + \left[e^{-1/3} - 1\right] = e^{-1/3}$

ii) $P(x<3) = \int_0^3 f(x)\,dx$

$= \int_0^3 \frac{1}{3} e^{-x/3} dx$

$= \frac{1}{\cancel{3}}\left[\frac{e^{-x/3}}{-1/\cancel{3}}\right]_0^3$

$= -\left[e^{-1} - e^{0}\right] = 1 - \frac{1}{e}. = 0.6321$

* In a certain town duration of a shower is exponentially distributed withe mean 5 min. what is the probability that a shower will last for
i) 10 min or more ii) less than 10 min
iii) between 10 and 12 minutes.

∴ **Soln**: P.d.f of exponential distⁿ is

$f(x) = \alpha e^{-\alpha x}, \quad x > 0$

$\frac{1}{\alpha} = 5 \quad \therefore \alpha = 1/5$

$\therefore f(x) = \frac{1}{5} e^{-x/5}$

i) $P(x \geq 10) = \int_{10}^{\infty} \frac{1}{5} e^{-x/5} dx = \left[-e^{-x/5}\right]_{10}^{\infty}$

$= -\left[e^{-\infty} - e^{-2}\right] = -\left[0 - e^{-2}\right] = e^{-2} = 0.1353$

ii) $P(x < 10) = \int_{0}^{10} \frac{1}{5} e^{-x/5} dx = 1 - e^{-2} = 0.8647$

iii) $P(10 < x < 12) = \int_{10}^{12} \frac{1}{5} \cdot e^{-x/5} dx$

$= -\left(e^{-12/5} - e^{-2}\right) = 0.0446$

Normal Distribution

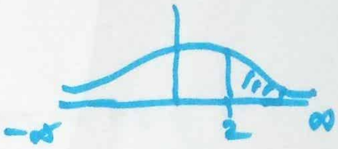
* If x is normally distributed with mean 12 and S.D 4 find the following
i) $P(x \geq 20)$ ii) $P(x \leq 20)$

Soln: Let $z = \frac{x-\mu}{\sigma} = \frac{x-12}{4}$

$$P(x \geq 20) = P\left[\frac{x-12}{4} \geq \frac{20-12}{4}\right]$$

$= P[z \geq 2]$

= are from 2 to ∞



= [are frm 0 to ∞] - [area from 0 to 2]

$= 0.5 - 0.52$

$= 0.0228$

$$P(x \leq 20) = P\left[\frac{x-12}{4} \leq \frac{20-12}{4}\right]$$

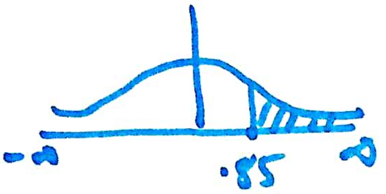
$= 0.9772$

* Evaluate the following probabilities with

i) $P(Z \geq 0.85)$ ii) $P(-1.64 \leq Z \leq -0.88)$

iii) $P(Z \leq -2.43)$ iv) $P(|Z| \leq 1.94)$

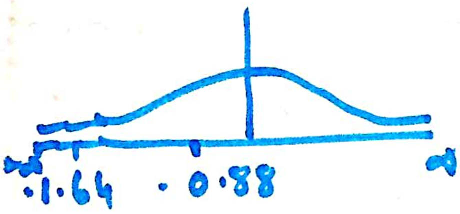
Soln: i) $P(Z \geq 0.85) = P(\text{area from } 0 \text{ to } \infty) - P(\text{area from } 0 \text{ to } 0.85)$



$= 0.5 - \phi(0.85)$

$= 0.5 - 0.3023 = 0.1977$

ii) $P(-1.64 \leq Z \leq -0.88) = P(\text{area from } -1.64 \text{ to } -0.88)$

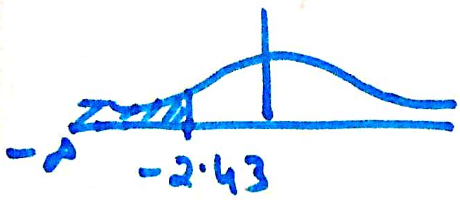


$= P(\text{area from } 0 \text{ to } 1.64) - P(\text{area from } 0 \text{ to } 0.88)$

$= \phi(1.64) - \phi(0.88)$

$= 0.4495 - 0.3106 = 0.1389$

iii) $P(Z \leq -2.43) = P(\text{area from } -\infty \text{ to } -2.43)$



$= P(\text{area from } -\infty \text{ to } 0) - P(\text{area from } -2.43 \text{ to } 0)$

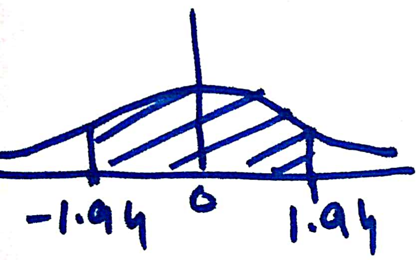
$= 0.5 - \phi(+2.43)$

$= 0.5 - 0.4925$

$= 0.0075.$

iv) $P(|z| \leq 1.94) = P(-1.94 \leq z \leq 1.94)$

$= P(\text{area from } -1.94 \text{ to } 1.94)$



$= P(\text{area from } -1.94 \text{ to } 0) +$
$P(\text{area from } 0 \text{ to } 1.94)$

$= \phi(1.94) + \phi(1.94)$

$= 0.4738 + 0.4738$

$= .9476$

Mean, Variance & S.D of Binomial Distribution

* Mean (μ):

$$\mu = \sum_{x=0}^{n} x\, p(x)$$

$$= \sum_{x=0}^{n} x \, {}^nC_x \, p^x q^{n-x}$$

$$= 0 + 1 \cdot {}^nC_1 p^1 q^{n-1} + 2 \cdot {}^nC_2 p^2 q^{n-2} + 3 \cdot {}^nC_3 p^3 q^{n-3} + \cdots + n \cdot {}^nC_n p^n q^{n-n}$$

$$= npq^{n-1} + 2 \cdot \frac{n!}{(n-2)!\,2!} p^2 q^{n-2} + 3 \cdot \frac{n!}{(n-3)!\,3!} p^3 q^{n-3} + \cdots + np^n$$

$$= npq^{n-1} + 2\left[\frac{n(n-1)(n-2)!}{(n-2)! \cdot 2} p^2 q^{n-2}\right] + 3\left[\frac{n(n-1)(n-2)(n-3)!}{(n-3)! \cdot 3 \times 2 \times 1}\right] p^3 q^{n-3} + \cdots + np^n$$

$$= npq^{n-1} + n(n-1)p^2 q^{n-2} + \frac{n(n-1)(n-2)}{2!} p^3 q^{n-3} + \cdots + np^n$$

$$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2!} p^2 q^{n-3} + \cdots + p^{n-1}\right]$$

$$= np\,[q+p]^{n-1}$$

$$\mu = np\,(1)^{n-1}$$

$$\therefore \boxed{\mu = np}$$

Variance (V)

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$$V = \sum_{x=0}^{n} x^2 p(x) - \mu^2$$

Consider, $\sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} [x(x-1) + x] p(x)$

$$= \sum_{x=0}^{n} x(x-1) p(x) + \sum_{x=0}^{n} x p(x)$$

$$= \sum_{x=0}^{n} x(x-1)\, nC_x p^x q^{n-x} + np$$

$$= 0 + 0 + 2\, nC_2 p^2 q^{n-2} + 3\cdot 2\, nC_3 p^3 q^{n-3} + 4\cdot 3\cdot nC_4 p^4 q^{n-4} + \dots +$$

$$n(n-1)\, nC_n p^n q^0 + np$$

$$= 2\left[\frac{n!}{(n-2)!\,2!} p^2 q^{n-2}\right] + 3\cdot 2\left[\frac{n!}{(n-3)!\,3!} p^3 q^{n-3}\right] + 4\cdot 3\left[\frac{n!}{(n-4)!\,4!} p^4 q^{n-4}\right]$$

$$+ \dots + n(n-1) p^n + np$$

$$= \frac{n!}{(n-2)!} p^2 q^{n-2} + \frac{n!}{(n-3)!} p^3 q^{n-3} + \frac{n!}{(n-4)!\,2!} p^4 q^{n-4} + \dots + n(n-1) p^n$$

$$+ np$$

$$= \frac{n(n-1)(n-2)!}{(n-2)!} p^2 q^{n-2} + \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} p^3 q^{n-3} +$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!\,2!} p^4 q^{n-4} + \dots + n(n-1) p^n + np$$

$$= n(n-1)p^2\left[\cancel{p}\, q^{n-2} + (n-2) p q^{n-3} + \frac{(n-2)(n-3)}{2!} p^2 q^{n-4} + \dots + p^{n-2}\right]$$

$$+ np$$

$$= n(n-1) p^2 [q+p]^{n-2} + np$$

$$= n(n-1) p^2 (1) + np$$

$= n^2p^2 - np^2 + np$

Now, $V = \sum_{x=0}^{n} x^2 p(x) - \mu^2$

$= n^2p^2 - np^2 + np - (np)^2$

$= n^2p^2 - np^2 + np - n^2p^2$

$= np(1-p)$

$$\boxed{V = npq}$$

$\therefore$ $S.D = \sqrt{V} = \sqrt{npq}$

Mean, Variance and S.D of ~~Exponential~~ Poisson Distribution

* Mean (μ)

$$\mu = \sum_{x=0}^{n} x\, p(x)$$

$$= \sum_{x=0}^{n} x \frac{e^{-m} m^x}{x!}$$

$$= 0 + 1.\frac{me^{-m}}{1!} + 2.\frac{m^2 e^{-m}}{2!} + 3.\frac{m^3 e^{-m}}{3!} + 4.\frac{m^4 e^{-m}}{4!} + \ldots$$

$$= me^{-m}\left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \ldots\right]$$

$$= me^{-m}.e^{m}$$

$$\boxed{\mu = m}$$

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* Variance (v)

$$V = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$$

Consider, $\sum_{x=0}^{\infty} x^2 p(x) = \sum_{x=0}^{\infty} \left(x[x-1]+x\right) p(x)$

$$= \sum_{x=0}^{\infty} x(x-1) p(x) + \sum_{x=0}^{\infty} x p(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-m} m^x}{x!} + m$$

$$= \left[0 + 0 + 2.1 \frac{e^{-m} m^2}{2!} + 3(2)\frac{e^{-m} m^3}{3!} + 4(3)\frac{e^{-m} m^4}{4!} + \dots\right] + m$$

$$= m^2 e^{-m} \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \dots\right] + m$$

$$= m^2 e^{-m} . e^{m} + m$$

$$\boxed{\sum_{x=0}^{\infty} x^2 p(x) = m^2 + m}$$

$$V = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$$

$$= m^2 + m - m^2$$

$$\boxed{V = m}$$

$$\therefore \quad S.D = \sqrt{V}$$

$$\boxed{S.D = \sqrt{m}}$$

The mean & variance are equal in Poission Distribution