01/07/22

Fundamental Theorem on Equivalence relation:

Statement: If A is any non-empty set, then.

i) Any equivalence relation R on A induces partition of A.

ii) Any partition of A gives rise to an equivalence relation R on A.

Proof:

i) Given that R is an equivalence relation,

$\Rightarrow$ R is reflexive, symmetric and transitive.

Let us consider a set P of all distinct equivalence classes.

i.e $P = \{[a] : a \in A\}$.

where $[a] = \{x : xRa\}$.

We need to prove that, P is a partition of A, it means that P has to satisfy two condition of definition of partition.

1) Since R is reflexive relation

$\Rightarrow aRa \quad \forall a \in A$

$\Rightarrow a \in [a] \quad \forall a \in A$.

$\Rightarrow$ Union of all equivalence class gives A.

$\Rightarrow A = \bigcup_{a \in A} [a]$

2) We know that,

"Any two equivalence classes of A are either identical or disjoint".

Since P has distinct equivalence class, Intersection of 2 distinct equivalence classes is empty.

$\therefore$ P is partition of set A.

ii) Suppose $P = \{A_1, A_2, A_3, \cdots A_n\}$ is a partition of A.
Define a relation R on set A, such that aRb if and only if, (a,b) ∈ same block of P.

We need to prove that R is equivalence relation.

1) <u>Reflexive relation:</u>
Let us take element $a \in A$.
$\Rightarrow a \in A_i$, for only one $i = 1, 2, 3, \cdots n$
$\Rightarrow aRa$, as 'a' belongs to same block.
$\Rightarrow aRa \ \forall a \in A$.
$\therefore$ R is reflexive relation.

2) <u>Symmetric relation:</u>
If $aRb \Rightarrow a, b \in$ same block.
$\Rightarrow b, a \in$ same block.
$\Rightarrow bRa$
$\therefore$ R is symmetric relation

3) Transitive Relation:
If aRb and bRc
$\Rightarrow a, b \in$ same block & $b, c \in$ same block.
$\Rightarrow a, b, c \in$ same block
$\Rightarrow a, c \in$ same block
$\Rightarrow aRc$.
$\therefore$ R is transitive relation.
$\therefore$ R is an equivalence relation.

* Examples:
① For the equivalence relation $R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)\}$ defined on set $A = \{1,2,3,4\}$. Determine the partition induced by R.

Sol^n: Let $A = \{1,2,3,4\}$.
Equivalence relation on R is
$\{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)\}$.

Equivalence class of set A ~~is~~ are,
$[a] = \{x : xRa\} = \{x : (x,a) \in R\}$.
$[1] = \{x : (x,1) \in R\} = \{1,2\}$
$[2] = \{x : (x,2) \in R\} = \{1,2\}$
$[3] = \{x : (x,3) \in R\} = \{3,4\}$
$[4] = \{x : (x,4) \in R\} = \{3,4\}$.

Partition induced by R is
$P = \{[1], [3]\} = \{\{1,2\}, \{3,4\}\}$.

04/06/22

② For a set $A = \{1,2,3, \cdots\}$ consider the relation R on A defined as aRb ~~defi~~ if and only if $a-b$ is ~~defined~~ divisible by 5. Find the partition of A induced by R.

Sol^n: Let $A = \{1,2,3, \cdots\}$.
$R = \{(a,b) : (a-b) \text{ is divisible by } 5\}$.
$R = \{(a,b) : 5 \mid (a-b)\}$.

i) Reflexive Relation:
aRa as $5 \mid (a-a)$, $\forall a \in A$.
$(a,a) \in R$
$\therefore$ R is reflexive relation.

ii) Symmetric Relation:
If $aRb \Rightarrow 5 \mid a-b$
$\Rightarrow 5 \mid -(b-a)$

classmate
Date
Page

$\Rightarrow 5 | (b-a)$.

$\Rightarrow bRa$.

$\therefore$ R is symmetric Relation

iii) Transitive relation.

If aRb and bRc

$\Rightarrow 5|(a-b)$ and $5|(b-c)$.

$\Rightarrow 5|(a-b)+(b-c)$

$\Rightarrow 5|(a-\cancel{b}+\cancel{b}-c)$

$\Rightarrow 5|(a-c)$

$\therefore aRc$

$\therefore$ R is transitive Relation

$\therefore$ R is an equivalence Relation.

Now, equivalences classes of set A are,

$[a] = \{x : xRa\} = \{x : (x,a) \in R\}$

$[a] = \{x : 5|(x-a)\}$

$[a] = \{x : (x-a) = 5n\}, \forall n \in Z$, provided $x \in A$.

$[a] = \{x : x = 5n+a\}, \forall n \in Z$, provided $x \in A$.

$[a] = \{5n+a\}, \forall n \in Z$, provided $5n+a \in A$.

$\therefore [1] = \{5n+1\}, \forall n \in Z$, provided $(5n+1) \in A$.

$[1] = \{1, 6, 11, 16, 21, 26, 31, \ldots\}$ $(\because \forall n = 0, 1, 2 \ldots)$

$[2] = \{5n+2\}, \forall n \in Z$, provided $(5n+2) \in A$.

$[2] = \{2, 7, 12, 17, 22, 27, \ldots\}$

$[3] = \{5n+3\}, \forall n \in Z$, provided $(5n+3) \in A$.

$[3] = \{3, 8, 13, 18, 23, 28, \ldots\}$

$[4] = \{5n+4\}, \forall n \in Z$, provided $(5n+4) \in A$

$[4] = \{4, 9, 14, 19, 24, 29, \ldots\}$

$[5] = \{5n+5\}, \forall n \in Z$, provided $(5n+5) \in A$.

$[5] = \{5, 10, 15, 20, 25, 30, \ldots\}$

$\therefore [1], [2], [3], [4]$ & $[5]$ are the only distinct equivalence classes of set A induced by the relation R.

$\therefore$ Partition of set A induced by ~~set~~ relation R is,
$R = \{ [1], [2], [3], [4], [5] \}$.

(3) On the set of all integers Z, the relation R is defined by aRb if and only if $a^2 - b^2$ is an even integer.
i) Show that R is an equivalence relation.
ii) Find the partition of A induced by R.

Sol:- Let $\cancel{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$.
or $I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
$R = \{(a, b) : (a^2 - b^2) \text{ is an even integer}\}$.
$R = \{(a, b) : (a^2 - b^2) \text{ is divisible by } 2\}$
$R = \{(a, b) : 2 | (a^2 - b^2)\}$.

i) Reflexive Relation:
aRa as $2 | (a^2 - b^2)$ $\forall a \in Z$.
R is reflexive relation.

ii) Symmetric Relation
If $aRb \Rightarrow 2 | (a^2 - b^2)$
$\Rightarrow 2 | -(b^2 - a^2)$
$\Rightarrow 2 | (b^2 - a^2)$
$\Rightarrow bRa$
$\therefore$ R is symmetric relation.

iii) Transitive relation
If aRb and bRc
$\Rightarrow 2 | (a^2 - b^2)$ and $2 | (b^2 - c^2)$
$\Rightarrow 2 | (a^2 - b^2) + (b^2 - c^2)$
$\Rightarrow 2 | (a^2 - \cancel{b^2} + \cancel{b^2} - c^2)$
$\Rightarrow 2 | (a^2 - c^2)$
$\Rightarrow aRc$
$\therefore$ R is transitive relation
$\therefore$ R is equivalence relation.

Now, equivalence class of set $\mathbb{Z}$ are,

$[a] = \{x : xRa\} = \{x : (x,a) \in R\}$.

$[a] = \{x : 2 \mid (x^2 - a^2)\}$.

$[a] = \{x : (x^2 - a^2) = 2n\}$, $\forall n \in \mathbb{Z}$, provided $x \in \mathbb{Z}$.

$[a] = \{x : x^2 = 2n + a^2\}$, $\forall n \in \mathbb{Z}$, provided $x \in \mathbb{Z}$.

$[a] = \{x : x = \pm\sqrt{2n+a^2}\}$, $\forall n \in \mathbb{Z}$, provide $x \in \mathbb{Z}$.

$[a] = \{-\sqrt{2n+a^2}, \sqrt{2n+a^2}\}$, $\forall n \in \mathbb{Z}$, provided $\sqrt{2n+a^2} \in \mathbb{Z}$.

$[1] = \{-\sqrt{2n+1}, \sqrt{2n+1}\}$, $\forall n \in \mathbb{Z}$, provided $\sqrt{2n+a^2} \in \mathbb{Z}$.

$[1] = \{-1, 1, -3, 3, -5, 5, -7, 7, \dots\}$.

$[1] = \{\dots\dots -7, -5, -3, -1, 1, 3, 5, 7, \dots\}$

$[2] = \{-\sqrt{2n+4}, \sqrt{2n+4}\}$, $\forall n \in \mathbb{Z}$, provided $\sqrt{2n+4} \in \mathbb{Z}$.

$[2] = \{0, -2, 2, -4, 4, -6, 6, \dots\}$

$[2] = \{\dots -6, -4, -2, 0, 2, 4, 6, \dots\}$.

$\therefore$ $[1], [2]$ are the only 2 distinct equivalence classes of set $\mathbb{Z}$ induced by the relation R.

$\therefore$ Partition of set $\mathbb{Z}$ induced by relation R is,

$R = \{[1], [2]\}$.

06/07/22

④ If $A = \{1, 2, 3, 4, 5\}$ and R is the equivalence relation on A that induces the partition $A = \{1,2\} \cup \{3,4\} \cup \{5\}$, then find R.

Sol^n: Let $A = \{1, 2, 3, 4, 5\}$.

Partition of A is

$P = \{\{1,2\}, \{3,4\}, \{5\}\}$.

$\therefore$ Equivalence relation R on A by the above partition P is,

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,4), (4,3), (5,5)\}$.

($\because$ aRb if & only if $(a,b) \in$ same block).

★ **Partial ordering Relation:**

A relation R, on set A is said to be partial ordering relation, if it satisfies reflexive, anti-symmetric and transitive relation.

★ **Partial ordered Set (OR) Poset:**

A set A, with partial ordering relation R, defined on it is called Partial ordered set of A (OR) Poset of A and is denoted by (A, R).

★ **Hasse Diagram:**

The Hasse diagram of a finite partial ordered set A (poset A) is a diagram whose vertices represent elements of A. If $(a, b) \in A$ and b is immediate successor of a $(a<b)$, then an edge (line) is drawn directed from a to b by placing element b at a higher level than element b.

To draw Hasse Diagram, of Poset (A, R), a digraph of the given partial ordering relation R is drawn and remove the following:

1) All the loops
2) All the edges implied by transitive relation.

★ **Examples:**

① Let $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and R be the relation on A defined by $x R y$ if and only if x divides y. Verify R is partial ordering relation and draw its Hasse diagram.

Sol: Let $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$.

$R = \{(x, y) : x \text{ divides } y\}$.

$R = \{(x, y) : x / y\}$.

i) Reflexive Relation

$x R x$ as x/x, $\forall x \in A$

∴ R is reflexive.

ii) Anti-Symmetric:

whenever $x \neq y$ and xRy, then $y \not R x$.

$\therefore$ R is anti-symmetric relation.

iii) Transitive relation:

If xRy and yRz.

$\Rightarrow$ $x|y$ and $y|z$.

$\Rightarrow$ $x|z$.

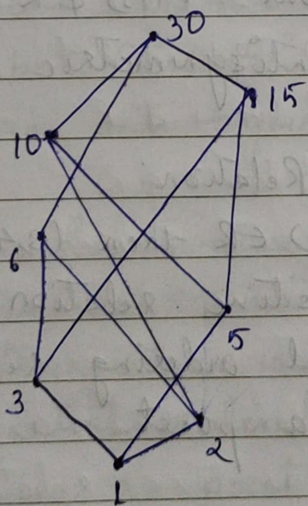
$\Rightarrow$ xRz.

$\therefore$ R is transitive relation.

$\therefore$ R is partial ordering relation

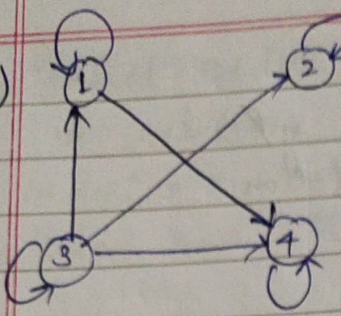
$\therefore$ (A, R) is poset.

Hasse diagram: of (A, R) is



08-04-22

(2) The diagraph of a relation defined on set $A = \{1, 2, 3, 4\}$ is as shown below, verify that (A, R) is a poset and draw corresponding Hasse diagram.

a) 

Sol: Relation R on set $A = \{1, 2, 3, 4\}$ defined by above diagraph is,

$R = \{(1,1), (1,4), (2,2), (3,1), (3,2), (3,3), (3,4), (4,4)\}$

i) Reflexive relation:

$(1,1), (2,2), (3,3), (4,4) \in R$

$\therefore$ R is reflexive

ii) Antisymmetric relation:

Since $(1,4) \in R$ but $(4,1) \notin R$

$(3,1) \in R$ but $(1,3) \notin R$

$(3,2) \in R$ but $(2,3) \notin R$

$(3,4) \in R$ but $(4,3) \notin R$.

$\therefore$ R is an antisymmetric relation

iii) ~~Re~~ Transitive Relation:

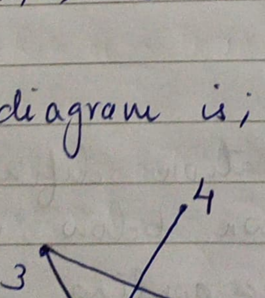
$(3,1) \in R$ & $(1,4) \in R$ then $(3,4) \in R$.

$\therefore$ R is transitive relation.

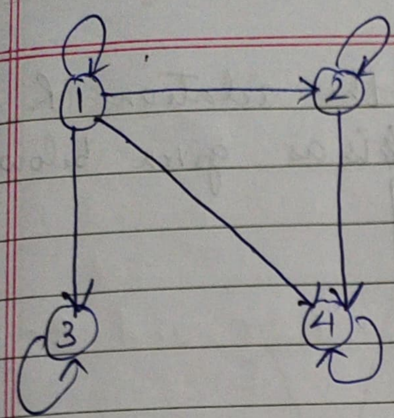
$\therefore$ ~~R~~ R is partial ordering relation.

(A, R) is a poset

Hasse diagram is;



b)



Sol: Relation R on set $A = \{1, 2, 3, 4\}$ defined by above diagraph is,

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

i) Reflexive Relation:

$(1,1), (2,2), (3,3), (4,4) \in R$

$\therefore$ R is Reflexive.

ii) Antisymmetric Relation:

$(1,2) \in R$ but $(2,1) \notin R$

$(1,3) \in R$ but $(3,1) \notin R$

$(1,4) \in R$ but $(4,1) \notin R$

$(2,4) \in R$ but $(4,2) \notin R$

$\therefore$ R is an anti-symmetric Relation.

iii) Transitive Relation:

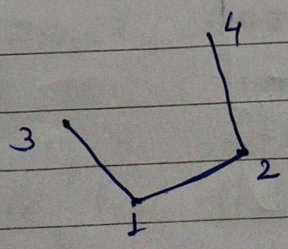
$(1,2) \in R$ & $(2,4) \in R$ then $(1,4) \in R$

$\therefore$ R is transitive relation.

$\therefore$ R is partial ordered relation

(A, R) is a poset

Hasse Diagram is,



③ Draw the Hasse diagram of the relations R on $A = \{1,2,3,4,5\}$, whose matrix is as given below:

$$M(R) = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Sol^n: Relation R on set $A = \{1,2,3,4,5\}$ defined by above matrix is;

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4),\ (1,5),\ (2,2),\ (3,3),\ (4,4),\ (5,5)\}$

i) Reflexive Relation:

$(1,1),\ (2,2),\ (3,3),\ (4,4),\ (5,5) \in R$

$\therefore$ R is reflexive relation.

ii) Anti-symmetric Relation:

Since,

$(1,2) \in R$ but $(2,1) \notin R$

$(1,3) \in R$ but $(3,1) \notin R$

$(1,4) \in R$ but $(4,1) \notin R$

$(1,5) \in R$ but $(5,1) \notin R$.

$\therefore$ R is an antisymmetric Relation.

iii) Transitive Relation:

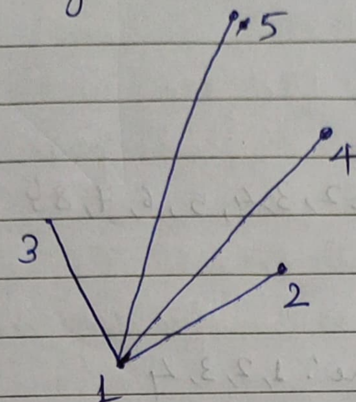
$(a,a) \in R$ & $(a,b) \in R$ then $(a,b) \in R$.

$\therefore$ R is transitive relation.

$\therefore$ R is partial ordering relation.

(A,R) is a poset

Hasse diagram is,



★ Lower Bound:
Let A be the poset and B is a subset of A. Then an element $a \in A$ is called lower bound of B if $a \leq b$, $\forall\, b \in B$.

★ Upper Bound:
Let A be the poset and B is a subset of A. Then an element $a \in A$ is called upper bound of B if $a \geq b$, $\forall\, b \in B$.

★ Greatest Lower Bound (GLB):
An element 'a' is called greatest lower bound of subset B of a poset A. If 'a' is a lower bound that is ~~les~~ greater than any other lower bounds of B.

★ Least Upper Bound (LUB):
An element 'a' is called least upper bound of subset B of a poset A. If 'a' is a upper bound that is less than any other upper

bounds of B.

★ Example:

(1) Consider a set $A = \{1,2,3,4,5,6,7,8\}$ and $B = \{4,5,6\}$ is the subset of A.

Sol: lower Bound of B are: 1, 2, 3, 4

Upper Bound of B are: 6, 7, 8.

Greatest lower Bound of B: 4

least Upper Bound of B: 6.

★ Lattice:

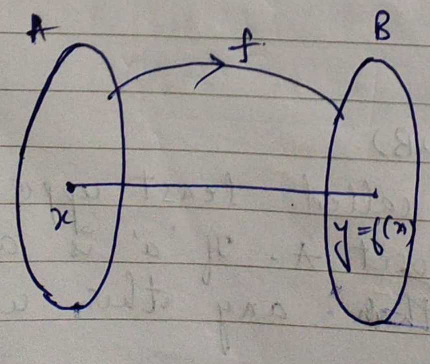
A partial ordering set (A, R) in which every pair of elements has both least upper bound and greatest lower bound. is called lattice.

★ Functions:

A relation R from non empty set A to non empty set B is said to be a function (Mapping) if from A to B, if each element of A has unique image (relation) in B.

And function is denoted by:

$f: A \to B$



01/07/22.

★ Types of Functions:

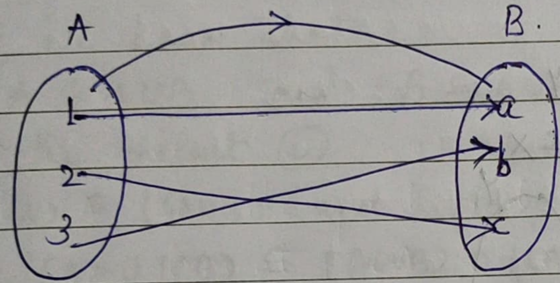
1) One-one function / One-to-one function / Injective

A function $f: A \to B$ is said to be one-one if distinct elements of A should have distinct ~~range~~ images in B.

i.e if $x_1 \neq x_2 \in A$, then $f(x_1) \neq f(x_2)$.

or: if $f(x_1) = f(x_2)$, then $x_1 = x_2$.

eg.

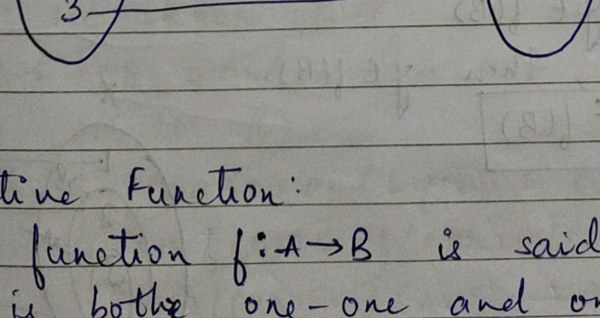


2) Onto Function / Surjective:

A function $f: A \to B$. is said to be onto if every element $y \in B$ should be the image of atleast one element $x \in A$.

<u>Note:</u> If a function f is onto, then co-domain and range are equal.

eg.



3) Bijective Function:

A function $f: A \to B$ is said to be bijective if f is both one-one and onto.

★ Properties of Function:

Theorem - I:

Let $f: X \to Y$ be a function and A and B are non-empty subsets of X. Then prove that,

i) If $A \subseteq B$, then $f(A) \subseteq f(B)$.

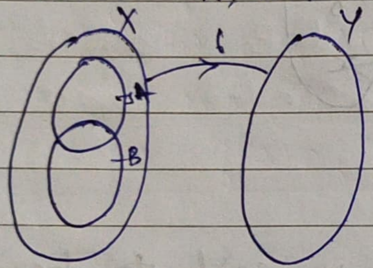
ii) $f(A \cup B) = f(A) \cup f(B)$.

iii) $f(A \cap B) = f(A) \cap f(B)$, and equality holds if f is one-one function.

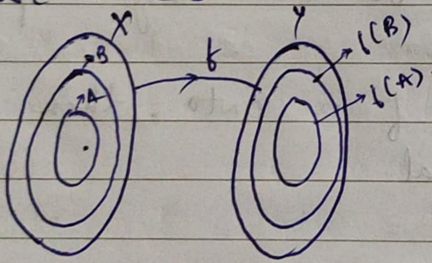
<u>Proof</u>:

Let $f: X \to Y$

Given that $A, B \subseteq X$.



i) Suppose $A \subseteq B$.



Let us consider an arbitary $y \in Y$, and if $y \in f(A)$,

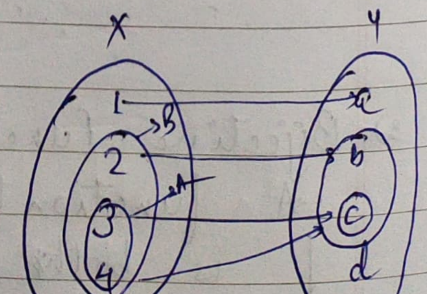
$y \in f(A) \Rightarrow y = f(x)$, for some $x \in A$.

$\Rightarrow y = f(x)$, for some $x \in B$ ($\because A \subseteq B$)

$\Rightarrow y \in f(B)$

$\therefore$ if $y \in f(A)$, then $y \in f(B)$.

$\therefore \boxed{f(A) \subseteq f(B)}$



$c \in f(A) \Rightarrow c = f(x)$, for some $x \in A$ ↓ 3, 4.

ii) If $y \in f(A \cup B) \Rightarrow y = f(x)$ for some $x \in A \cup B$.

$\Rightarrow y = f(x)$ for some $x \in A$ or $x \in B$.

$\Rightarrow (y = f(x)$, for some $x \in A)$ (or) $(y = f(x)$, for some $x \in B)$

$\Rightarrow y \in f(A)$ (or) $y \in f(B)$

$\Rightarrow y \in f(A) \cup f(B)$

$\Rightarrow f(A \cup B) \subseteq f(A) \cup f(B)$ — ①

Next, we know that,

$A \subseteq A \cup B$ and $B \subseteq A \cup B$.

By the result (i)

$f(A) \subseteq f(A \cup B)$ and $f(B) \subseteq f(A \cup B)$

$\therefore f(A) \cup f(B) \subseteq f(A \cup B) \cup f(A \cup B)$

$f(A) \cup f(B) \subseteq f(A \cup B)$ — ②

from ① and ②, we get

$\boxed{f(A \cup B) = f(A) \cup f(B).}$

iii) If $y \in (A \cap B) \Rightarrow y = f(x)$ for some $x \in A \cap B$

$\Rightarrow y = f(x)$ for some $x \in A$ and $x \in B$.

$\Rightarrow (y = f(x)$, for some $x \in A)$ (and) $(y = f(x)$, for some $x \in B)$

$\Rightarrow y \in f(A)$ (and) $y \in f(B)$

$\Rightarrow y \in f(A) \cap f(B)$

$\Rightarrow f(A \cap B) \subseteq f(A) \cap f(B)$ — ③

Next, we know that,

$A \subseteq A \cap B$ and $B \subseteq A \cap B$.

By result (i)

$f(A) \subseteq f(A \cap B)$ and $f(B) \subseteq f(A \cap B)$.

$\therefore f(A) \cap f(B) \subseteq f(A \cap B) \cap f(A \cap B)$

$f(A) \cap f(B) \subseteq f(A \cap B)$. — ④

$\therefore$ ~~By (3) & (4)~~

~~$\boxed{f(A\cap B) = f(A)\cap f(B)}$~~

Given that f is one-one function.

If $y \in f(A)\cap f(B) \Rightarrow y \in f(A)$ and $y \in f(B)$

$\Rightarrow (y = f(x_1)$, for some $x_1 \in A)$ and $(y = f(x_2)$, for some $x_2 \in B)$

$\Rightarrow y = f(x_1) = f(x_2)$, for ~~some~~ $x_1 \in A$ & $x_2 \in B$.

$\Rightarrow x_1 = x_2$ as f is one-one function.

$\Rightarrow y = f(x_1)$, for $x_1 \in A$ and $x_1 \in B$.

$\Rightarrow y = f(x_1)$, for $x_1 \in A\cap B$.

$\Rightarrow y \in f(A\cap B)$.

$\therefore f(A)\cap f(B) \subseteq f(A\cap B)$ — (4)

from (3) and (4), we get

$\boxed{f(A\cap B) = f(A)\cap f(B)}$

11-07-22:

Theorem-2:

Let A and B be the finite set, and $f: A \to B$ be a function. Then prove that:

i) If f is one-one, then $n(A) \le n(B)$.

ii) If f is onto, then $n(B) \le n(A)$.

iii) If f is bijective, then $n(A) = n(B)$.

iv) If $n(A) > n(B)$ then atleast two different elements of A have the same image under f.

Proof:

Let $f: A \to B$.

Let us consider $A = \{a_1, a_2, a_3, \dots a_n\}$.

and $B = \{b_1, b_2, b_3, \dots b_m\}$

$\therefore n(A) = n$ $\quad n(B) = m$.

i) Suppose f is one-one.
$\Rightarrow$ Each distinct elements of A must have distinct images in B.
$\Rightarrow$ B must have atleast 'n' elements.
$\Rightarrow a_1, a_2, \dots a_n \in A$ must have images $f(a_1), f(a_2), \dots f(a_n) \in B$.
$\Rightarrow n(B) \geq n$.
$\Rightarrow n(B) \geq n(A)$ ($\because n(A) = n$)
$\Rightarrow$ $\boxed{n(A) \leq n(B)}$ — ⓐ.

ii) Suppose f is onto.
$\Rightarrow$ Each element of B is the image of atleast one element of A.
$\Rightarrow$ ~~B~~ A must have atleast m elements.
$\Rightarrow n(A) \geq m$.
$\Rightarrow n(A) \geq n(B)$ ($\because n(B) = m$).
$\Rightarrow$ $\boxed{n(A) \geq n(B)}$ — ⓑ

iii) Suppose f is bijective.
From the result (i) & (ii), we get
$\boxed{n(A) = n(B)}$

iv) Suppose $n(A) > n(B)$.
By the contrapositive of the result (i), we get
If $n(A) > n(B)$, then f is not one-one.
$\Rightarrow$ There are atleast 2 elements $x_1 \neq x_2 \in A$, have the same image y in B.
$\Rightarrow$ $y = f(x_1)$ & $y = f(x_2)$ for $x_1 \neq x_2 \in A$.
It means that atleast two elements of A have the same image under f.

Theorem-3:

Suppose A and B are finite sets having same number of elements and $f: A \to B$ is a function. Then f is one-one if and only if f is onto.

Proof:

Let $f: A \to B$

Given that, number of elements in A and B are same

$\therefore \boxed{n(A) = n(B) = n}$

Let $A = \{a_1, a_2, \cdots a_n\}$ & $n(A) = n$.

Suppose f is one-one;

$\Rightarrow$ Each element $a_1, a_2, \cdots a_n \in A$ must have distinct images $f(a_1), f(a_2), \cdots f(a_n)$.

$\Rightarrow$ The set of these images constitute a range of 'f' and it is denoted by $f(A)$.

$\Rightarrow f(A) = \{f(a_1), f(a_2), f(a_3), \cdots f(a_n)\}$.

$\Rightarrow n(f(A)) = n$.

$\Rightarrow \boxed{n(f(A)) = n(B)}$ $(\because n(B) = n)$

$\therefore$ f is onto.

Conversely,

Suppose f is onto,

$\Rightarrow$ co-domain (B) and range set $(f(A))$ have same number of elements.

$\Rightarrow B = f(A) = \{f(a_1), f(a_2), \cdots, f(a_n)\}$. & $n(B) = n$.

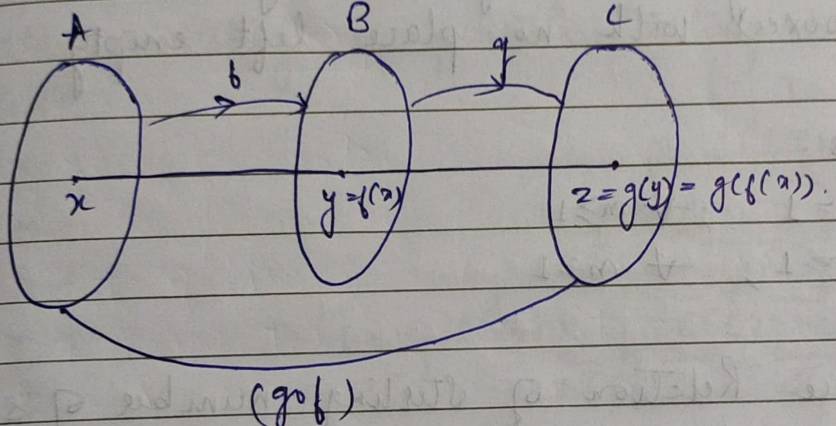
$\Rightarrow f(a_1), f(a_2), \cdots f(a_n)$ are distinct elements.

$\therefore$ f is one-one

★ Composition of Functions:

Let $f: A \to B$ and $g: B \to C$ be two functions. Then the compositions of two functions f and g is denoted by gof. is defined by the function $gof: A \to C$ such that, ~~gof~~

$$gof(x) = g(f(x)), \quad \forall\, x \in A.$$



★ Invertible Function:

A function $f: A \to B$ is said to be invertible if there exists a function $g: B \to A$ such that $(gof)(x) = x$ & $(fog)(y) = y$. Then the function g is called inverse of function f and it is denoted by f^{-1}.