

GRAPH THEORY AND DISCRETE MATHEMATICAL STRUCTURES

Max. Marks: 100

Time: 3 Hours

- Instructions:**
1. Answer any Five full Questions.
 2. Unit IV and V are compulsory.

UNIT - I

$$\left(\sim p \bigvee \sim q \right) \rightarrow \left(r \bigwedge_s \right)$$

$$\frac{\sim t}{\therefore p}$$

L CO PO M

- b. Define Universal and existential Quantifiers. Write down the following proposition in symbolic form and find its negation:
“All integers are rational numbers and some rational numbers are not integers”.

- c. Consider the following open statements with the set of all real numbers as the universe.

$$p(x): x \geq 0, \quad q(x): x^2 \geq 0, \quad r(x): x^2 - 3x - 4 = 0, \quad s(x): x^2 - 3 > 0$$

Determine the truth values of the following statements.

- 1) $\exists x, p(x) \wedge q(x)$
- 2) $\forall x, q(x) \rightarrow s(x)$
- 3) $\forall x, r(x) \vee s(x)$
- 4) $\exists x, p(x) \wedge q(x)$
- 5) $\forall x, r(x) \rightarrow p(x)$

OR

L CO PO M

2. a. Write Inverse and Domination laws of logic. Prove the $[(p \rightarrow q) \wedge (\sim q \wedge (r \vee q))] \Leftrightarrow \sim (q \vee p)$ without using truth table

(1) (1) (06)

- b. Define Tautology and Contradiction. Prove that, for any propositions p, q, r the compound proposition $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a Tautology.

(1) (1) (1) (07)

- c. 1) Define Inverse, converse, and Contrapositive for the conditional $p \rightarrow q$. State the Inverse, Converse, Contrapositive of the conditional.
“If Indayani wears a blue scarf, then she carries a white color hand bag”.

- 2) Find the truth values of p, q and r if compound proposition $p \rightarrow (q \vee r)$ is false.

(1) (1) (07)

UNIT - II

L CO PO M

3. a. Suppose $A, B, C \subseteq Z \times Z$ with $A = \{(x, y) | y = 5x - 1\}$, $B = \{(x, y) | y = 6x\}$, $C = \{(x, y) | 3x - y = -7\}$

Find 1) $A \cap B$, 2) $\overline{A} \cup \overline{C}$

- b. Define Zero-one matrix. Consider the sets $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$ and the relations $R \setminus \{(a, 1), (b, 1), (c, 2), (c, 3)\}$ and $S = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$ from A to B. Determine \overline{R} , $(R \cup S)$, S^c and their matrix representation.

(1) (2) (1) (06)
(2) (2) (1) (07)

- b. Define Planar Graph.

1) $G(V, E)$ be a connected planar graph with $|V| = v$ and $|E| = e$. Let r be the number of regions in the plane. Then $v - e + r = 2$

c. Define complement of graph and spanning of subgraph with example.

The sum of the degrees of all vertices in a graph is even and is equal to twice the number of edges in the graph.

(1) (06) (2) (4) (1) (07)
 (1), (4, 4), their matrix (1) (01) (07)

UNIT-V

27 pages.

-) (07)

- $$(06) \quad x^2 \text{ If } M$$

- (90)

(m)

- c. Define complement of graph and spanning of subgraph with example.
Prove that the sum of the degrees of all vertices in a graph is even and is equal to twice the number of edges in the graph.

UNIT -V

(I) L (4) CO (1) PO (07) M

- a. State and Prove Fermat's Theorem. (2) (5) (1) (06)
- b. Represent addition and multiplication tables for elements in $GF(2^3)$ (1) (5) (1) (07)
- c. Explain Elliptic curves over Z_p . (1) (5) (1) (07)

Fourth Semester B.E. Semester End Examination, May/June 2018/19

Time: 2 Hours

Max. Marks: 50

Instructions: 1. Answer all the questions from Section A.
2. Answer any 5 questions from Section B.

SECTION-A**UNIT I**

- A The term 'Ecology' has been derived from the French word --which means to encircle or surround
 a) Environ b) Oikos c) geo d) Aqua
- (1) (1) (1,8) (01)
- B Which of the following conceptual spheres of the environment is having the least storage capacity for matter
 a) Atmosphere b) Lithosphere c) Hydrosphere d) Biosphere
- (1) (1) (1,8) (01)
- C Atmosphere consists of 78 % Nitrogen and 21 % Oxygen by
 a) Volume b) Weight c) Density d) All the three
- (1) (1) (1,8) (01)
- D Which of the following is a key element of EIA?
 a) Scoping b) Screening c) Identifying and evaluating alternatives d) all
- (1) (1) (1,8) (01)
- E Plants usegas for photosynthesis
 a) Oxygen b) methane c) Nitrogen d) Carbon dioxide
- (1) (1) (1,8) (01)

UNIT II

- A Forests prevent soil erosion by binding soil particles in their
 a) stems b) roots c) leaves d) buds
- (1) (1) (1,8) (01)
- B Solar radiation consists of
 a) UV b) Visible light c) Infrared d) All of these
- (1) (2) (1,8) (01)

- C The most important fuel used by nuclear power plant is

- a) U - 235 b) U - 238 c) U - 245 d) U - 248

- (1) (2) (1,8) (01)

- D India's position in the Bio-gas plants globally

- a) 5th b) 2nd c) 4th d) 7th

- (1) (2) (1,8) (01)

- E Which resources are inexhaustible?

- a) Renewable b) fossil fuel c) nonrenewable d) mineral

- (1) (2) (1,8) (01)

UNIT III

- A Wind Farms are located in
 a) River basin b) Plain area c) Hilly area d) Valley area
- (1) (2) (1,8) (01)
- B Highest producer of Oil and petroleum is
 a) Middle East countries b) America c) China d) India
- (1) (3) (1,8) (01)
- C Which of the following is air pollutant?
 a) CO b) O₂ c) N₂ d) all
- (1) (3) (1,8) (01)
- D The liquid waste from baths and kitchens is called
 a) Sullage b) Domestic sewage c) Storm waste d) Run off
- (1) (3) (1,8) (01)

Fourth Semester B.E. Fast Track Semester End Examination, July/August 2019

DISCRETE MATHEMATICAL STRUCTURES AND GRAPH THEORY

Time: 3 Hours

Instructions: 1. Answer any Five full Questions.

UNIT - I

Max. Marks: 100

L CO PO M

1. a. Test the Validity of the argument

$$\frac{p \rightarrow q \\ r \rightarrow s \\ p \vee r}{\therefore (q \vee s)}$$

- b. Define the tautology and the Contradiction. Prove that, for any propositions p, q, r the compound proposition $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a Tautology.

(2) (1) (1) (07)

- c. Let $p(x): x^2 - 8x + 15 = 0$, $q(x): x$ is odd , $r(x): x > 0$ with the set of all integers as the universe. Determine the truth or falsity of each of the following statements.

- 1) $\forall x, [p(x) \rightarrow q(x)]$
- 2) $\exists x, [p(x) \rightarrow q(x)]$
- 3) $\exists x, [r(x) \rightarrow p(x)]$
- 4) $\exists x, [p(x) \rightarrow (q(x) \wedge r(x))]$
- 5) $\forall x, [(p(x) \vee q(x)) \rightarrow r(x)]$

OR

2. a. Define Inverse, converse, and Contrapositive for the conditional $p \rightarrow q$. State the Inverse, Converse, Contrapositive of the conditional “If Raghveer plays a Counterstrike game regularly, then he will participate in IIT Bombay video game competition”.

(1) (1) (1) (07)

- b. Construct the truth tables for the following compound propositions:

(2) (1) (1) (06)

- 1) $(p \vee q) \wedge \sim p$
- 2) $[(p \wedge q) \vee (\sim r)] \leftrightarrow p$

(1) (1) (1) (07)

- c. Find whether the following argument is valid:

No Engineering student of First or Second Semester Studies Logic.

Dev is an engineering student who studies Logic.

\therefore Dev is not in first Semester

(1) (1) (1) (07)

3. a. Define Cartesian Product. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(3) (1) (1) (07)
L CO PO M

- b. Define Zero-one relation matrix.

Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if “ a is a multiple of b ”. Write down the relation matrix $M(R)$ and draw its Digraph.

(2) (2) (1) (07)

- b. Define Zero-one matrix. Consider the sets $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$ and the relations $R = \{(a,1), (b,1), (c,2), (c,3)\}$ and $S = \{(a,1), (a,2), (b,1), (b,2)\}$ from A to B . Determine \overline{R} , $(R \cup S)_S^c$ and their matrix representation.

- c. On the set of Z of all integers, a relation R is defined by aRb if and only if $a^2 = b^2$. Verify that R is an equivalence relation. Determine the partition induced by this relation.

(1) (2) (1) (07)

OR

- 4 a. Define Cartesian Product, and Prove that $A \times (B - C) = (A \times B) - (A \times C)$.

(1) (2) (1) (06)

- b. Define Equivalence relations.

Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. On this set define the relation R by $(x, y) \in R$ if and only if $(x - y)$ is a multiple of 5. Verify that R is an equivalence relation.

(2) (2) (1) (07)

- c. Show that the D_{32} is a lattice.

(3) (2) (1) (07)

UNIT - III

- 5 a. Prove that the function $f: A \rightarrow B$ is invertible if and only if it is one-to-one and onto.

(3) (3) (1) (06)

- b. A bag contains 12 pairs of socks(each pair in different colors). If a person draws the socks one by one at random., determine at most how many draws are required to get at least one pair of matched socks.

(2) (3) (1) (07)

- c. Define Floor and ceiling Functions with example. Let $A = B = R$ Determine $\prod_A D$ and $\prod_B D$ for

i) $D = \{(x, y) | x = y^2, 0 \leq y \leq 2\}$ ii) $D = \{(x, y) | x^2 + y^2 = 1\}$

(1) (3) (1) (07)

OR

- 6 a. There are six programmers in the computer science department who can assist ten departments in the University. In how many ways can these departments be assisted by the six programmers so that each is working at least at one department?

(2) (3) (1) (06)

- b. State pigeonhole Principle. Consider the functions f and g defined by $f(x) = x^3$ and $g(x) = x^2 + 1, \forall x \in R$ Find $g \circ f, f \circ g, f^2$ and g^2 .

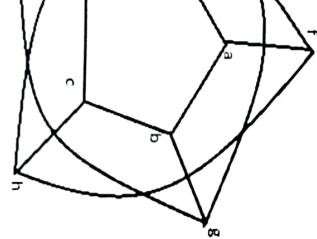
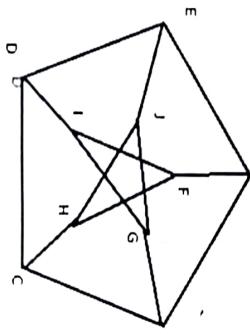
(1) (3) (1) (07)

- c. Define Generating function. Determine the coefficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$.

(1) (3) (1) (07)

UNIT - IV (Compulsory)

- 7 a. Define Isomorphism. Show that following two Graphs are isomorphic.



(1) (4) (1) (06)

c. Let $A = \{1, 2, 3, 4, 5\}$ Define a relation on $A \times A$ by $(a,b)R(c,d)$ iff $a+b=c+d$.

- 1) Verify that R is an equivalence relation on $A \times A$.
- 2) Determine the partition of $A \times A$ induced by R .

OR

4. a. Find the nature of the following relations on $A = \{p, q, r, s\}$ and draw its Digraphs represented by

$$1) \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad 2) \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad 3) \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

- b. Define Equivalence relations.

Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. On this set define the relation R by $(x, y) \in R$ if and only if $(x - y)$ is a multiple of 5. Verify that R is an equivalence relation.

- c. Show that D_{45} is a Lattice.

UNIT - III

5. a. Define one-one and onto function.

Determine if each function $f: A \rightarrow B$ is bijective

- 1) $f(x) = x^2$ $A = B = R$
- 2) $f(x) = \sqrt{x}$ $A = R^+, B = R$

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- b. ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle prove that at least two of these points are such that the distance between them is less than 0.5 cm

- c. Define 1) One-to-one function 2) Constant function 3) onto function.
Find the coefficient of x^{60} in $(x^8 + x^9 + x^{10} + \dots)^7$

- (1) (3) (1) (07)
- (2) (3) (1) (07)
- (3) (1) (06)

OR

6. a. In how many ways can a police captain distribute 24 rifles shells to four police officers so that each

officer gets at least three shells, but not more than eight?

- (2) (3) (1) (06)
- (2) (3) (1) (06)

- b. Let f, g, h be functions from Z to Z defined by $f(x) = x - 1, g(x) = 3x, h(x) = \{0, x \text{ is even}$

Determine $(f \circ (g \circ h))(x)$ and $((f \circ g) \circ h)(x)$ and verify that $f \circ (g \circ h) = (f \circ g) \circ h$

- (1) (3) (1) (07)
- (1) (3) (1) (07)

- c. If $f: A \rightarrow B$ and $g: B \rightarrow C$ are invertible functions, then $g \circ f: A \rightarrow C$ is an invertible function and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

- (1) (3) (1) (07)
- (1) (3) (1) (07)

• ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~13~~ ~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ ~~21~~ ~~22~~ ~~23~~ ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ ~~29~~ ~~30~~ ~~31~~ ~~32~~ ~~33~~ ~~34~~ ~~35~~ ~~36~~ ~~37~~ ~~38~~ ~~39~~ ~~40~~ ~~41~~ ~~42~~ ~~43~~ ~~44~~ ~~45~~ ~~46~~ ~~47~~ ~~48~~ ~~49~~ ~~50~~ ~~51~~ ~~52~~ ~~53~~ ~~54~~ ~~55~~ ~~56~~ ~~57~~ ~~58~~ ~~59~~ ~~60~~ ~~61~~ ~~62~~ ~~63~~ ~~64~~ ~~65~~ ~~66~~ ~~67~~ ~~68~~ ~~69~~ ~~70~~ ~~71~~ ~~72~~ ~~73~~ ~~74~~ ~~75~~ ~~76~~ ~~77~~ ~~78~~ ~~79~~ ~~80~~ ~~81~~ ~~82~~ ~~83~~ ~~84~~ ~~85~~ ~~86~~ ~~87~~ ~~88~~ ~~89~~ ~~90~~ ~~91~~ ~~92~~ ~~93~~ ~~94~~ ~~95~~ ~~96~~ ~~97~~ ~~98~~ ~~99~~ ~~100~~

GRAPH THEORY AND DISCRETE MATHEMATICAL STRUCTURES

Time: 3 Hours

Max. Marks: 100

Instructions:
 1. Answer any Five full Questions
 2. Unit II and V are compulsory.

UNIT - I

L	CO	PO	M
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1. a. Test the Validity of the argument by using laws of logic

$$\frac{p \rightarrow q \\ r \rightarrow s}{\therefore (q \vee s)}$$

- b. Define the tautology and the Contradiction. Prove that, for any propositions p, q, r the compound proposition $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a Tautology.

- c. Consider the following open statements with the set of all real numbers as the universe. Determine the truth values of the following statements.

- 1) $\exists x, p(x) \wedge q(x)$ (3) (1) (1) (06)
- 2) $\forall x, q(x) \rightarrow s(x)$
- 3) $\forall x, r(x) \vee s(x)$
- 4) $\exists x, p(x) \wedge q(x)$
- 5) $\forall x, r(x) \rightarrow p(x)$

(1) (1) (1) (07)

OR

2. a. Define Inverse, converse, and Contrapositive for the conditional $p \rightarrow q$. State the inverse, Converse, Contrapositive of the conditional.
 "If Indrayani wears a blue scarf, then she carries a white color hand bag".

- b. Construct the truth tables for the following compound propositions:

- 1) $(p \vee q) \wedge \sim p$ (2) (1) (1) (06)
- 2) $[(p \wedge q) \vee (\sim r)] \leftrightarrow p$

(1) (1) (1) (07)

- c. Find whether the following argument is valid:
 No engineering student of First or Second Semester studies Logic.
Dev is an engineering student who studies Logic.

\therefore Dev is not in first Semester

(3) (1) (1) (07)

UNIT - II

3. a. Suppose $A, B, C \subseteq Z \times Z$ with
 $A = \{(x,y)|y = 5x - 1\}, B = \{(x,y)|y = 6x\}, C = \{(x,y)|3x - y = -7\}$

Find 1) $A \cap B$, 2) $\overline{A} \cup \overline{C}$

(1) (2) (1) (06)

- b. Define Zero-one relation matrix.
 Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if " a is a multiple of b ". Write down the relation matrix $M(R)$ and draw its Digraph.

(2) (2) (1) (07)

Fourth semester B.E. Makeup Examination, May/June 2018-19
DISCRETE MATHEMATICAL STRUCTURES AND GRAPH THEORY

Time: 3 Hours

Max. Marks: 100

Instructions: 1. Answer any Five full Questions.
 2. Unit IV and V are compulsory.

UNIT - I

L CO PO M

- 1 a. Test the Validity of the argument by using laws of logic.

$$\begin{array}{c} (\sim p \vee \sim q) \rightarrow (r \wedge s) \\ r \rightarrow t \\ \hline \therefore p \end{array}$$

(3) (1) (1) (06)

- b. Define Tautology and Contradiction. Prove that, for any propositions p, q, r the compound proposition $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a Tautology.

(2) (1) (1) (07)

- c. Let $p(x): x^2 - 3x - 4 = 0$, $q(x): x \text{ is odd}$, $r(x): x > 0$ with the set of all integers as the universe. Determine the truth or falsity of each of the following statements.

- 1) $\forall x, [p(x) \rightarrow q(x)]$
- 2) $\exists x, [p(x) \rightarrow q(x)]$
- 3) $\exists x, [r(x) \rightarrow p(x)]$
- 4) $\exists x, [p(x) \rightarrow (q(x) \wedge r(x))]$
- 5) $\forall x, [(p(x) \vee q(x)) \rightarrow r(x)]$

(1) (1) (1) (07)

OR

- 2 a. Write down the following proposition in symbolic form and find its negation:
 "All integers are rational numbers and some rational numbers are not integers".
- (2) (1) (1) (06)
- b. Define Conjunction and Disjunction. Prove the logical equivalence by using Laws of Logic.
 $\{(p \rightarrow q) \wedge [\sim q \wedge (r \vee \sim q)]\} \Leftrightarrow \sim (q \vee p)$
- (1) (1) (1) (07)
- c. i) Define Inverse, converse, and Contrapositive for the conditional $p \rightarrow q$. State the Inverse, Converse, Contrapositive of the conditional.
 "If Indrayani wears a blue scarf, then she carries a white color hand bag".
- ii) Find the truth values of p, q and r if compound proposition $p \rightarrow (q \vee r)$ is false.

(3) (1) (1) (07)

UNIT - II

L CO PO M

- 3 a. Define Cartesian Product. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
- (1) (2) (1) (06)
- b. For $A = \{1, 2, 3, 4\}$ Let $R = \{(1,1), (1,2), (2,3), (3,3), (3,4)\}$ be a relation on A. Find R^2 , R^3 and R^4 and Draw their Digraphs. Also, Determine their matrix representations.
- (2) (2) (1) (07)
- c. In the set Z^+ . A relation R is defined by aRb if and only if "a Divides b" Prove that R is reflexive, Transitive and antisymmetric, but not symmetric.
- (1) (2) (1) (07)

OR

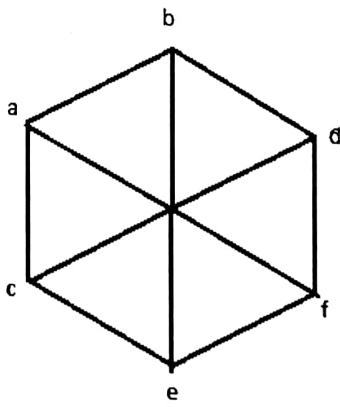
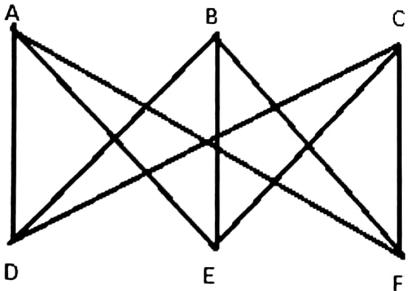
- 4 a. Explain with an example Symmetric, Antisymmetric and Transitive relations.
- (1) (2) (1) (06)

- b. 1) Define Hamiltonian Path, Hamiltonian cycle with examples.
 2) Prove that a Graph G is Eulerian if and only if every vertex of G has even degree.
- c. Define Connected Graph and Bipartite Graph.
 i) Show that K_5 is non-planar
 ii) Give example of Euler path but no Euler Circuit.
- | | UNIT –V (Compulsory) | | | | |
|----|---|------------|------------|------------|--------------|
| a. | State and Prove Fermat's Theorem. | (1)
L | (4)
CO | (1)
PO | (07)
M |
| b. | Find the GCD of 2040 and 1098 by Euclid Algorithm. Express it as a linear combination of the two. | (2)
(1) | (5)
(5) | (1)
(1) | (06)
(07) |
| c. | Explain Elliptic curves over Z_p . | (1)
(1) | (5)
(5) | (1)
(1) | (07)
(07) |

- b. Prove that the relation "congruent modulo 7" is an equivalence relation on the set of all integers \mathbb{Z} .
Find partition of \mathbb{Z} .
- c. If R is a relation on the set $A = \{1, 2, 3, 4, 6, 12\}$ defined by xRy if and only if x divides y . Prove that (A, R) is a poset. Draw its Hasse Diagram. (2) (2) (1) (07)
- 5 a. **UNIT - III** (3) (2) (1) (07)
 Let $f: R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 5, & x > 0 \\ -3x + 1, & x \leq 0 \end{cases}$
 1) Determine $f\left(\frac{5}{3}\right), f(-\frac{5}{3})$.
 2) Find $f^{-1}(3), f^{-1}(-6)$.
 3) What are $f^{-1}([-5, 5])$ and $f^{-1}([-6, 5])$.
- b. State Pigeonhole principle. Prove that if 60 dictionaries in a library contain a total of 95371 pages, then at least one of the dictionaries must have at least 1561 pages. (3) (3) (1) (06)
- c. Prove that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are invertible functions, then $g \circ f: A \rightarrow C$ is an invertible function and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. (2) (3) (1) (07)

- 6 a. Define
 1) One-to-one function
 2) Constant function
 3) onto function.
 Find the coefficient of x^{60} in $(x^8 + x^9 + x^{10} + \dots)^7$. (1) (3) (1) (07)
- b. Define Composition of functions. (2) (3) (1) (06)
 Let f and g be functions from \mathbb{R} to \mathbb{R} defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$. If $(gof)(x) = 9x^2 - 9x + 3$ determine a, b .
- c. Define Floor and ceiling Functions with example. (1) (3) (1) (07)
 1) Let $A = B = \mathbb{R}$. Determine $\prod_A D$ and $\prod_B D$ for each of the following sets $D \subseteq A \times B$
 a) $D = \{(x, y) | x = y^2, 0 \leq y \leq 2\}$
 b) $D = \{(x, y) | x^2 + y^2 = 1\}$

- 7 a. Define Isomorphism. Show that following two Graphs are isomorphic. (1) (3) (1) (07)
UNIT - IV L CO PO M



- b. Define Planar Graph. (1) (4) (1) (06)
 1) $G = (V, E)$ be a connected planar graph with $|V| = v$ and $|E| = e$. Let r be the number of regions in the plane. Then $v - e + r = 2$ (2) (4) (1) (07)

- c. Prove that the relation "congruent modulo 7" is an equivalence relation on the set of all integers Z.
Find partition of Z.

OR

4 a. Explain with an example Symmetric, Antisymmetric and Transitive relations.

- b. If A = {1, 2, 3, 4} and R, S are relations on A defined by $R = \{(1,2), (1,3), (2,4), (4,4)\}$, $S = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4)\}$ find $R \circ S, S \circ R, R^2$ and S^2 . And determine their matrix representations.

- c. Show that D_{32} is a Lattice.

UNIT - III

- 5 a. If $f: A \rightarrow B$ and $g: B \rightarrow C$ are invertible functions, then $g \circ f: A \rightarrow C$ is an invertible function and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

- b. State pigeonhole principle. Prove that if 30 dictionaries in a library contain a total of 61,327 pages, then at least one of the dictionaries must have at least 2045 pages.

- c. Let $f: R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 5, & x > 0 \\ -3x + 1, & x \leq 0 \end{cases}$

- 1) Determine $f(5/3)$, $f(-5/3)$.
- 2) Find $f^{-1}(3)$, $f^{-1}(-6)$.
- 3) What are $f^{-1}([-5, 5])$ and $f^{-1}([-6, 5])$.

OR

- 6 a. There are six programmers in the computer science department who can assist ten departments in the University. In how many ways can these departments be assisted by the six programmers so that each is working at least at one department?

- b. Define Floor and ceiling Functions with example.

- 1) Let $A = B = R$ Determine $\prod_A D$ and $\prod_B D$ for each of the following sets $D \subseteq A \times B$

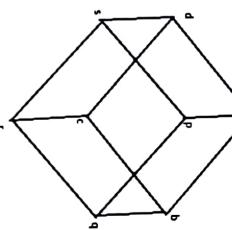
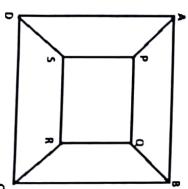
- a) $D = \{(x,y) | x = y^2, 0 \leq y \leq 2\}$
- b) $D = \{(x,y) | x^2 + y^2 = 1\}$

- c. Define Composition of functions.

- Le f and g be functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$. If $(gof)(x) = 9x^2 - 9x + 3$ determine a, b.

UNIT - IV

- 7 a. Define Isomorphism. Verify that the two graphs shown below are isomorphic



- (1) (2) (1) (07)

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- reg

- (1) (2) (1) (06)

- Define Plan

- The sum o

- the graph.

- a. State and I

- Find the C

- b. Explain E

- c. Define

- c. Let $A = \{1, 2, 3, 4, 5\}$. Define a relation on $A \times A$ by (a,b)R(c,d) iff $a + b = c + d$.
- Verify that R is an equivalence relation on $A \times A$.
 - Determine the partition of $A \times A$ induced by R.

OR

- 4 a. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and R be the equivalence relation on A that induces the partition $A = \{1,2\} \cup \{3\} \cup \{4,5,6\} \cup \{7\}$ find R.
- | | | | |
|-----|-----|-----|------|
| (1) | (2) | (1) | (07) |
|-----|-----|-----|------|

- b. a) Define 1) Reflexive 2) Transitive 3) Irreflexive relations.
 b) Let $R = \{(1,2), (1,3), (2,4), (3,2)\}$ be a relation on $A = \{1, 2, 3, 4\}$. Write the relation matrix $M(R)$ of R. Compute $[M(R)]^2$ and hence obtain R^2 .

- c. Define Hasse Diagram. Draw the Hasse diagram representing the positive divisors of 36.
- | | | | |
|-----|-----|-----|------|
| (2) | (2) | (1) | (07) |
|-----|-----|-----|------|

UNIT - III

(3)	(2)	(1)	(07)
L	CO	PO	M

- 5 a. Define one-one, onto and on-one correspondence functions.
 Consider the function $f: R \rightarrow R$ defined by $f(x) = x^2, \forall x \in R$ Is f invertible? Give Reason.
 b. State pigeonhole principle. Prove that in any set of 29 persons at least five must have been born on the same day of the week.
- Let $A = B = R$ Determine $\prod_A D$ and $\prod_B D$ for each of the following sets $D \subseteq A \times B$
 - $D = \{(x,y)|x = y^2, 0 \leq y \leq 2\}$
 - $D = \{(x,y)|x^2 + y^2 = 1\}$
- | | | | |
|-----|-----|-----|------|
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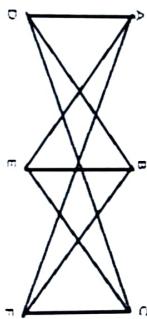
OR

- 6 a. In how many ways can a police captain distribute 24 rifles shells to four police officers so that each officer gets at least three shells, but not more than eight?
 b. Consider two functions $f: A \rightarrow B$ and $g: B \rightarrow C$. Prove that if $g \circ f: A \rightarrow C$ is one-one then f is one-one and if $g \circ f: A \rightarrow C$ is onto, then g is onto.

- c. Prove that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are invertible functions, then $g \circ f: A \rightarrow C$ is an invertible function and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

UNIT - IV (Compulsory)

- 7 a. Define Isomorphism. Verify that the two graphs shown below are isomorphic



- b. 1) Define Hamiltonian Path, Hamiltonian cycle with examples.

- 2) Prove that a Graph G is Eulerian if and only if every vertex of G has even degree.

Note: L (Level), CO (Course Outcome), PO (Programme Outcome), M (Marks)

- c. Define CC
 1) If
 2) G

b.

- If $A = \{1, 2, 3, 4\}$. Let $R = \{(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)\}$ be the relation on A . Determine whether the relation R is reflexive, irreflexive, symmetric, anti-symmetric or transitive.

c.

- If $A = \{1, 2, 3, 4, 6, 12\}$; On A , define the relation R by aRb if and only if "a divides b". Prove that R is a partial order on A . Draw Hasse diagram for this relation.

UNIT - III

- 5 a. Define one-one and onto function.

Determine if each function $f: A \rightarrow B$ is bijective

1) $f(x) = x^2 A = B = R$

2) $f(x) = \sqrt{x} A = R^+, B = R$

- b. ABC is an equilateral triangle whose sides are of length 1 cm each. If we select 5 points inside the triangle prove that at least two of these points are such that the distance between them is less than 0.5 cm

- c. Prove that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are invertible functions, then $g \circ f: A \rightarrow C$ is an invertible function and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

(1) (3) (1) (7)

(1) (3) (1) (7)

(1) (3) (1) (7)

6

- a. Define Stirling Numbers of the second kind. Find how many functions are there from A to B . How many of these are one-to-one? How many are onto?

Where $A = \{1, 2, 3, 4\}$. $B = \{1, 2, 3, 4, 5, 6\}$.

- b. Define Floor and ceiling Functions with example.

1) Let $A = B = R$ Determine $\prod_A D$ and $\prod_B D$ for each of the following sets $D \subseteq A \times B$

a) $D = \{(x, y) | x = y^2, 0 \leq y \leq 2\}$

b) $D = \{(x, y) | x^2 + y^2 = 1\}$

(1) (3) (1) (7)

(2) (3) (1) (6)

(1) (3) (1) (6)

- c. Let f, g, h be functions from Z to Z defined by $f(x) = x - 1, g(x) = 3x, h(x) = \begin{cases} 0, & x \text{ is even} \\ 1, & x \text{ is odd} \end{cases}$

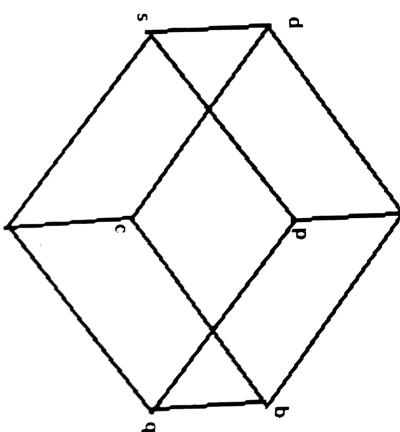
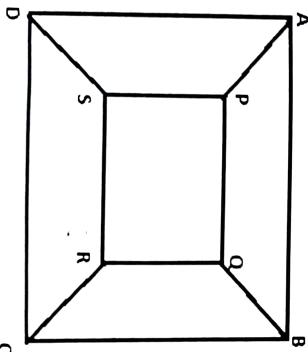
Determine $(f \circ (g \circ h))(x)$ and $((f \circ g) \circ h)(x)$ and verify that $f \circ (g \circ h) = (f \circ g) \circ h$

(1) (3) (1) (7)

L CO PO M

UNIT - IV (Compulsory)

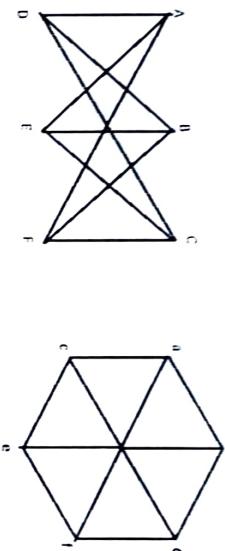
- 7 a. Define Isomorphism. Verify that the two graphs shown below are isomorphic



(2) (4) (1) (6)

UNIT - IV

- (1) (2) (1) (06) a. Define Isomorphism. Verify that the two graphs shown below are isomorphic.



L CO PO M

- (1) (2) (1) (06) b. Define complement of graph and spanning of subgraph with example.

- (2) (2) (1) (07) c. The sum of the degrees of all vertices in a graph is even and is equal to twice the number of edges in the graph.

- (3) (2) (1) (07) L CO PO M d. Define Connected Graph and Bipartite Graph.

- 1) Show that K_5 is non-planar
2) Give example of Euler path but no Euler Circuit.

UNIT - V

- (1) (4) (1) (07)

- (2) (5) (1) (06)

- (1) (5) (1) (07)

- (1) (5) (1) (07)

- (1) (5) (1) (07)

- (3) (3) (1) (06) e. a. State and Prove Euler's Theorem.
b. Represent addition and multiplication tables for elements in $GF(2^3)$
c. State Chinese Remainder Theorem Solve $x \equiv 2 \pmod{3}$, $x \equiv 3 \pmod{5}$, $x \equiv 2 \pmod{7}$.

(1) (3) (1) (07),

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- E** Physical pollution of water is due to
 a) Dissolved oxygen b) Turbidity c) pH d) none
4. **A** UNIT IV
 Alternative eco-friendly fuel for automobiles is
 a) Petrol b) Diesel c) CNG
B Ozone layers absorbs
 a) UV rays b) infra-red rays c) Cosmic rays
C Environmental (Protection) Act was enacted in the year
 a) 1986 b) 1992 c) 1984
D The Wild Life Protection Act was enacted in the year
 a) 1986 b) 1974 c) 1994
E Environmental protection is the responsibility of
 a) Govt. of India b) NGOs
5. **A** 'Earth Day' is observed on:
 a) 1st December b) 5th June c) April 22nd d) 1st January.
B The leader of Chipko movement is :
 a) Sundarlal Bahuguna b) MedhaPatkar c)Vandana Shiva
C ISO 14000 standards deal with :
 a) Pollution Management b) Risk management c) Environmental Management d) None of the above.
D An important NGO involved in Global environmental protection is
 a) UNICEF b) Green Peace c) WHO d) CPCB
E Which of the following animals is endangered species of India:
 b) Black buck b) Elephant c) Fox
SECTION-B
6. Explain with neat sketch CNS Cycle.
7. Explain briefly the Structure of Ecosystem
8. Explain the energy crisis in India
9. Explain the application of using solar energy
10. Write a short note on Chernobyl disaster
11. Write a short note on Tsunami disasters warning and mitigation measures
12. List different Environmental Protection Acts
13. Explain the various sources and types of solid waste
- Note: L (level), CO (Course Outcome), PO (Programme Outcome), M (Marks)

- 8
- b.
- Define Planar Graph.
 - $G(V, E)$ be a connected planar graph with $|V| = v$ and $|E| = e$. Let r be the number of regions in the plane, then prove that $v - e + r = 2$
 - Define Hamiltonian Path, Hamiltonian cycle with examples.

UNIT - V (Compulsory)

- | | L | CO | PO | M |
|---|-----|-----|-----|------|
| a. State and Prove Euler's Theorem. | (1) | (4) | (1) | (07) |
| b. State Chinese Remainder Theorem. Solve $x \equiv 0 \pmod{3}$, $x \equiv 2 \pmod{5}$, $x \equiv 69 \pmod{11}$. | (2) | (5) | (1) | (06) |
| c. Explain Testing of Primality. | (1) | (5) | (1) | (07) |

Fourth semester B.E. Semester End Examination, May/June 2018-19

GRAPH THEORY AND DISCRETE MATHEMATICAL STRUCTURES

Time: 3 Hours

Instructions: 1. Answer any Five full Questions.

Max. Marks: 100

- UNIT - I**
- | L | CO | PO | M |
|-----|-----|-----|------|
| (1) | (1) | (1) | (06) |
| (2) | (1) | (1) | (06) |
| (2) | (1) | (1) | (07) |
| (1) | (1) | (1) | (07) |

- a. Define Inverse, converse, and Contrapositive for the conditional $p \rightarrow q$. State the Inverse, Converse, Contrapositive of the conditional. “If Raghveer plays a Counterstrike game regularly, then he will participate in IIT Bombay video game competition”.

- b. Prove that, for any propositions p, q, r the compound proposition $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a Tautology.

- c. Find whether the following argument is valid:

No engineering student of First or Second Semester studies Logic.
Dev is an engineering student who studies Logic.
 \therefore Dev is not in first Semester

(1) (1) (1) (07)

OR

- 2** a. For any propositions p, q, r, prove the logical equivalence $[p \rightarrow (q \rightarrow r)] \Leftrightarrow [(p \rightarrow r) \vee (q \rightarrow r)]$
- | (2) | (1) | (1) | (06) |
|-----|-----|-----|------|
| (1) | (1) | (1) | (06) |

- b. Test the Validity of the argument by using laws of logic

$$\begin{array}{c} (p \wedge q) \\ p \rightarrow (r \wedge q) \\ r \rightarrow (s \vee t) \\ \hline \sim s \\ \therefore t \end{array}$$

(3) (1) (1) (07)

- c. a) Define 1) Exclusive Disjunction 2) Disjunction 3) Conditional.

- b) Let x be a specified number. Write down the negation of the following conditionals:

- 1) “If x is an integer, then x is a rational number”.

- 2) “If x is not real number, then it is not a rational number and not an irrational number”.

UNIT - II

- 3** a. Define a set, proper subset and power set, with an example each.
- | (1) | (2) | (1) | (06) |
|-----|-----|-----|------|
| (1) | (2) | (1) | (06) |

- b. Using venn diagrams, investigate the truth or falsity of:

- (i) $A\Delta(B \cap C) = (A\Delta B) \cap (A\Delta C)$ (ii) $A - (B \cup C) = (A - B) \cap (A - C)$, For any sets A, B and C.

- c. Let $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{7, 8, 9\}$. Find (i) $(A \cup B) \times C, A \cup (B \times C), (A \times B) \cup C$ and $A \times (B \cup C)$.

(1) (2) (1) (07)

OR

- 4** a. Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if “a is a multiple of b”. Represent the relation R as a matrix and draw its digraph.
- | (1) | (2) | (1) | (07) |
|-----|-----|-----|------|
| (1) | (2) | (1) | (07) |