

Radial Basis Function (RBF) Networks

Introduction to Radial Basis Function Networks

- Radial basis function network is a type of artificial neural network used in soft computing.
- Mainly applied for function approximation, classification, and pattern recognition.

Note: It is a simple feedforward network with **faster training** compared to multilayer perceptrons.

Structure of Radial Basis Function Networks

An RBFN has 3 main layers:

1. **Input Layer** – passes input features to hidden layer.
2. **Hidden Layer** – contains neurons with **radial basis (Gaussian) activation**.
3. **Output Layer** – performs a **weighted linear combination** of these activations to produce the final network output.

Work of the Output Layer

- The output layer takes the **activations** from the hidden layer (the values of $\phi(x)$ from each radial basis neuron).
- It then performs a **weighted linear combination** of these activations to produce the final network output.

Mathematically:

$$y(x) = \sum_{i=1}^M w_i \phi_i(x) + b$$

Where:

- M = number of hidden neurons
- $\phi_i(x)$ = output of the i -th radial basis function
- w_i = weight connecting the i -th hidden neuron to the output
- b = bias term
- $y(x)$ = final output of the network

Radial Basis Function (Gaussian Function)

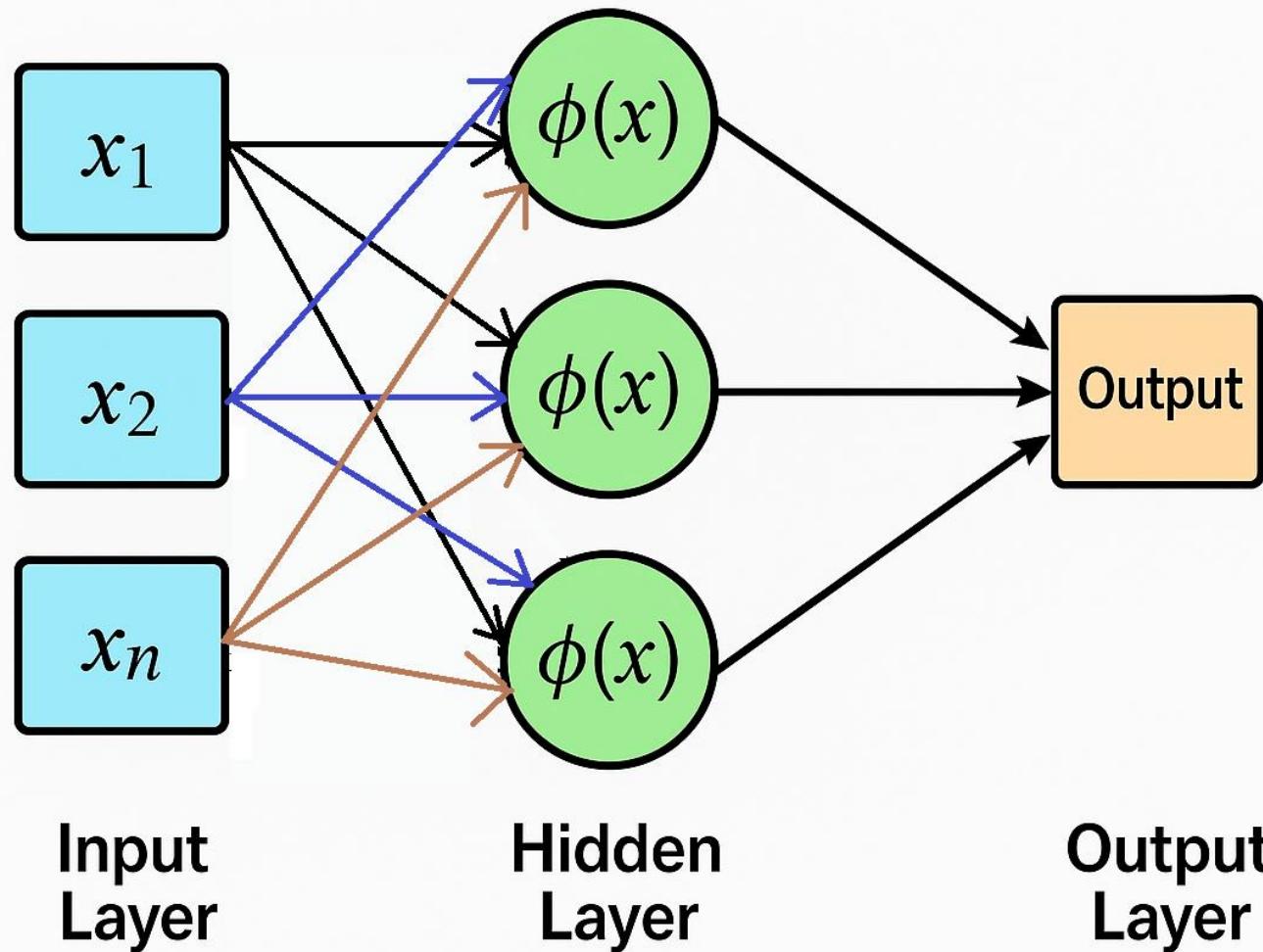
The hidden layer uses Gaussian radial basis functions. The **φ (phi)** symbol in refers to the **Radial Basis Function** itself — the **activation function** used in the hidden layer of an RBF Network. :

$$\phi(x) = \exp(-| |x - c| |^2 / (2\sigma^2))$$

Where:

- x = input vector
- $\exp()$ = exponential function
- c = n dimensional vector (center)
- σ = spread (width)

RADIAL BASIS FUNCTION NETWORKS



Breaking it down:

- x = input vector (e.g., $[x_1, x_2, \dots, x_n]$)
- c = center of the RBF (also an n-dimensional vector)
- $x - c$ = the difference vector (how far each component of x is from c)
- $\|x - c\|$ = the **Euclidean distance** between x and c :

$$\|x - c\| = \sqrt{(x_1 - c_1)^2 + (x_2 - c_2)^2 + \dots + (x_n - c_n)^2}$$

- Then, $\|x - c\|^2$ just means the **squared Euclidean distance** (no square root):

$$\|x - c\|^2 = (x_1 - c_1)^2 + (x_2 - c_2)^2 + \dots + (x_n - c_n)^2$$

Intuition

- If x is **very close** to c , then $\|x - c\|^2$ is small → the Gaussian value is near 1 (high activation).
- If x is **far away** from c , then $\|x - c\|^2$ is large → the Gaussian value is near 0 (low activation).

So the double bars are just a shorthand for "distance between vectors."

Key Features of RBF Networks

- **Localized Activation:** neurons respond only to nearby inputs.
- **Two-Stage Training:**
 - **Step 1:** Find centers & spreads (unsupervised).
 - **Step 2:** Learn weights (supervised).
- **Fast Training:** due to linear output layer.
- **Universal Approximator:** can approximate any continuous function.