Q2) (a) $u_e \to$ exhaust velocity w.r.t rocket $\to$ constant
$I_{sp} \to$ Specific impulse
$\dot{m} \to$ fuel rate $\to$ constant
$t_b \to$ fuel burnout time
$g_0 \to$ earth's grav pull

$$I_{sp} = \frac{F_{thrust}}{g_0\left(\frac{dm}{dt}\right)} = \frac{F_{thrust}}{\dot{m} g_0}$$

[Initial velocity of rocket is zero]

$$\Delta v = \cancel{u_0}\ u - 0 = u = I_{sp}\, g_0 \ln\left(\frac{M_0}{M_f}\right)$$

$M_0 \to$ Initial total mass of rocket

$$M_f = M_0 - \int_0^t \dot{m}\,dt = M_0 - \dot{m}\int_0^t dt$$

$$M_f = (M_0 - \dot{m}t)$$

$M_f \to$ final mass of rocket at time t.

$$\boxed{u(t) = I_{sp}\, g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right)} \quad \text{— (i)}$$

[For $0 \le t \le t_b$]

Also, $\Delta v = -v_{ex} \ln\left(\frac{M_f}{M_0}\right) \Rightarrow u = -u_e \ln\left(\frac{M_0 - \dot{m}t}{M_0}\right)$

$$\Rightarrow -\frac{u}{u_e} = \ln\left(1 - \frac{\dot{m}t}{M_0}\right) \Rightarrow \frac{\dot{m}t}{M_0} = 1 - e^{(-u/u_e)} \quad \text{(ii)}$$

$$\Rightarrow M_0 = \frac{\dot{m}t}{1 - e^{-u/u_e}}$$

Put (i) in (ii):

$$u = I_{sp}\, g_0 \ln\left(\frac{1}{1 - 1 + e^{-u/u_e}}\right) = -I_{sp}\, g_0 \ln(e^{-u/u_e})$$

$$= \frac{I_{sp}\, g_0\, u}{u_e}$$

$$\Rightarrow \boxed{u_e = I_{sp}\, g_0} \quad \text{(iii)}$$

Now the rocket will have constant upwards acc till time $t = t_b$

After that the rocket will have constant downward acc g_0

$$\boxed{u(t) = \begin{cases} u_e \ln\left(\dfrac{M_0}{M_0 - \dot{m}t}\right) & ; \ 0 \le t \le t_b \\ u_e \ln\left(\dfrac{M_0}{M_0 - \dot{m}t_b}\right) - gt & ; \ t > t_b \end{cases}}$$

(b) ~~At t_b~~ $u = \frac{dh}{dt} = u_e \ln\left(\frac{M_0}{M_0-\dot m t}\right)$

$$\Rightarrow \int_0^h dh = u_e \int_0^t \ln\left(\frac{M_0}{M_0-\dot m t}\right)dt = -u_e\int_0^t \ln\left(1-\frac{\dot m t}{M_0}\right)dt$$

$$= \cancel{-}\frac{u_e M_0}{\dot m}\int_1^{1-\dot m t/M_0} \ln(z)\,dz$$

$$= \frac{u_e M_0}{\dot m}\left[z\ln(z)\Big|_1^{1-\frac{\dot m t}{M_0}} - \int_1^{1-\dot m t/M_0} dz\right]$$

$$\boxed{\begin{array}{l} 1-\frac{\dot m t}{M_0} = z \\ \Rightarrow dt\left(\frac{-\dot m}{M_0}\right) = dz \\ \text{At } t=0,\ z=1 \\ t=t\ \ z = 1-\frac{\dot m t}{M_0} \end{array}}$$

$$\cancel{= \frac{u_e M_0}{\dot m}\left[\left(1-\frac{\dot m t}{M_0}\right)\ln\left(1-\frac{\dot m t}{M_0}\right) - \ln\left(1-\frac{\dot m t}{M_0}\right)\right]}$$

$$h = \frac{u_e M_0}{\dot m}\left[\left(1-\frac{\dot m t}{M_0}\right)\ln\left(1-\frac{\dot m t}{M_0}\right) + \frac{\dot m t}{M_0}\right]$$

$$\Rightarrow \boxed{h(t) = \frac{u_e M_0}{\dot m}\left[\left(1-\frac{\dot m t}{M_0}\right)\ln\left(1-\frac{\dot m t}{M_0}\right) + \frac{\dot m t}{M_0}\right]}$$

↳ Height attained till $0 \le t \le t_b$

At time $t_B = t_b$

$$\boxed{h(t_b) = \frac{u_e M_0}{\dot m}\left[\left(1-\frac{\dot m t_b}{M_0}\right)\ln\left(1-\frac{\dot m t_b}{M_0}\right) + \frac{\dot m t_b}{M_0}\right]}$$

~~Max~~ ~~Max altitude when $u(t)=0$ → max alt at $t=t_m$~~

~~i.e.~~ $\cancel{u_e \ln\left(\frac{M_0}{M_0-\dot m t_b}\right) - g_0 t_m = 0}$

$$\cancel{\Rightarrow t_m = \frac{u_e}{g_0}\ln\left(\frac{M_0}{M_0-\dot m t_b}\right)}$$

$$\cancel{\Rightarrow h_{max} =}$$

After $t=t_b$ $\quad \frac{dh}{dt} = u_e \ln\left(\frac{M_0}{M_0-\dot m t_b}\right) - gt$

$$\Rightarrow \int_{h(t_b)}^{h} dh = u_e \ln\left(\frac{M_0}{M_0-\dot m t_b}\right)\int_{t_b}^{t} dt - g\int_{t_b}^{t} t\,dt$$

$$h - h(t_b) = \left[u_e \ln\left(\frac{M_0}{M_0 - \dot m t_b}\right)\right]\left[t - t_b\right] - \frac{g}{2}\left[t^2 - t_b^2\right]$$

$$\boxed{h(t) = \begin{cases} \dfrac{u_e M_0}{\dot m} \ln\left[\left(1 - \dfrac{\dot m t}{M_0}\right)\ln\left(1 - \dfrac{\dot m t}{M_0}\right) + \dfrac{\dot m t}{M_0}\right] \\[2ex] h(t_b) + \left[u_e \ln\left(\dfrac{M_0}{M_0 - \dot m t_b}\right)\right](t - t_b) - \dfrac{g}{2}\left[t^2 - t_b^2\right] \end{cases}}$$

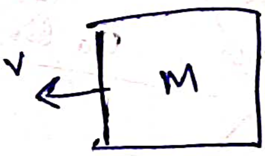
Max altitude at $t = t_m$ when $u(t_m) = 0$

$$u_e \ln\left[\frac{M_0}{M_0 - \dot m t_b}\right] = g t_m \Rightarrow t_m = \frac{u_e}{g} \ln\left(\frac{M_0}{M_0 - \dot m t_b}\right)$$

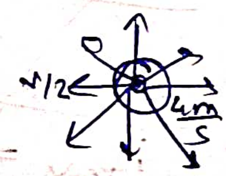
$$\Rightarrow \boxed{h_{max} = h(t_b) + \left[u_e \ln\left(\frac{M_0}{M_0 - \dot m t_b}\right)\right]\left[\frac{u_e}{g} \ln\left(\frac{M_0}{M_0 - \dot m t_b}\right) - t_b\right] - \frac{g}{2}\left[\left(\frac{u_e}{g} \ln\left(\frac{M_0}{M_0 - \dot m t_b}\right)\right)^2 - t_b^2\right]}$$

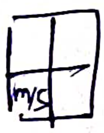
Q.3) Suppose n-boosters are used

$M_0 = m/5$ Suppose $t_b \rightarrow$ burnout time.



At min $\dot m$, speed of quadrant after fuel is exhausted $= v/2$



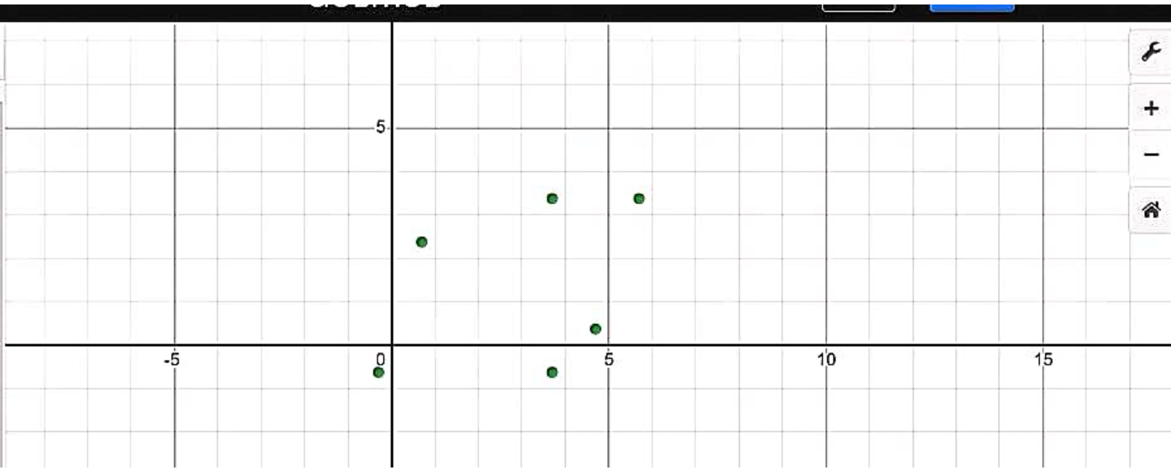


i.e. $\cancel{}$ $u_e \ln\left(\dfrac{m/5}{m/5 - \dot m t_b}\right) = \dfrac{v}{2}$

$$\Rightarrow \frac{m}{m - 5\dot m t_b} = e^{\left(\frac{v}{2u_e}\right)} \Rightarrow 1 - \frac{5\dot m t_b}{m} = e^{(v/2u_e)}$$

$$\Rightarrow \boxed{(\dot{m})_{min} = \frac{m}{5t_b}\left[1 - e^{(V/2u_e)}\right]}$$

x_1	y_1
−0.30738	−0.62208
3.69262	3.37792
4.69262	0.37792
3.69262	−0.62208
5.69262	3.37792
0.69262	2.37792



$$\frac{\left(x_1-a\right)^2}{c^2}+\frac{\left(y_1-b\right)^2}{d^2}\sim 1$$

STATISTICS

RESIDUALS

$RMSE = 5.53 \times 10^{-10}$,

PARAMETERS

$a = 203.613$

$c = -2.004 \times 10^{6}$

$b = -5.8462 \times 10^{9}$

$d = 5.8462 \times 10^{9}$