Rectangles in Rectangles

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NOTES

Rectangles in Rectangles

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Introduction When does one rectangle fit into another? In 1956, in the *Monthly*, L. Ford asked for a necessary and sufficient condition for a $p \times q$ rug to fit on an $a \times b$ floor. Necessary conditions are easy to find: If a $p \times q$ rectangle fits into an $a \times b$ rectangle (FIGURE 1), then

$pq \le ab$ (the area condition)
$p^2 + q^2 \le a^2 + b^2$ (the diameter condition)
$p + q \le a + b$ (the perimeter condition)
$\min\{p, q\} \le \min\{a, b\}$ (the thickness condition).

But none of these necessary conditions is sufficient. For example, a rectangle with sides 9 and 4 does not fit into a rectangle with sides 8 and 6, but all four necessary conditions are satisfied.

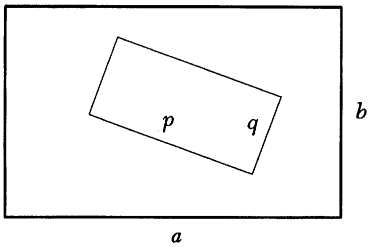

FIGURE 1

Sufficient conditions seem scarcer. One can show that if the thickness condition $\min\{p, q\} \le \min\{a, b\}$ is satisfied and if either of the conditions

$$\max\{p, q\} \le \min\{a, b\} \quad \text{and} \quad p + q \le \sqrt{a^2 + b^2}$$

holds, then a $p \times q$ rectangle fits into an $a \times b$ rectangle. But neither of these sufficient conditions is necessary; an 88×13 rectangle fits into an 81×59 rectangle (see below), but both of these two conditions are false.

In answer to Ford's question, W. Carver **[3]** gave the following mysterious-looking necessary and sufficient condition: a $p \times q$ rectangle $(p \ge q)$ fits into an $a \times b$ rectangle $(a \ge b)$ if and only if

$$p \le a \quad \text{and} \quad q \le b$$

or

$$p > a \quad \text{and} \quad b \ge \frac{2pqa + (p^2 - q^2)\sqrt{p^2 + q^2 - a^2}}{p^2 + q^2}.$$

Carver's ingenious elementary argument is entirely geometric. In this note we give a straightforward development of Carver's conditions that is quite different from the argument he suggested, and we prove:

THEOREM 1. (Carver [3]) *Suppose an $a \times b$ rectangle T is given, with the notation arranged so that $a \geq b$. Then a $p \times q$ rectangle R with $p \geq q$ fits into T if and only if*

(*a*) $p \leq a$ *and* $q \leq b$, *or*

(*b*) $p > a, q \leq b$, *and* $\left(\frac{a+b}{p+q}\right)^2 + \left(\frac{a-b}{p-q}\right)^2 \geq 2.$

In Section 4 we use this result to find the smallest rectangle of given shape that can accommodate first every rectangle of perimeter two, and then every rectangle of diameter one. In Section 5 we pose some related unsolved problems of undetermined difficulty.

2. Rectangles in rectangles In 1964 H. Steinhaus [**16**] asked for a necessary and sufficient condition on the six sides for one triangle to fit into another. Nearly thirty years later, K. A. Post [**12**] gave a set of 18 inequalities relating the sides of the triangles that are necessary and sufficient in the sense that if one of the inequalities is correct, the first triangle fits in the second; and if the first triangle fits in the second, then one (at least) of the inequalities is correct. Post's argument relies on a fitting lemma for triangles that is of interest in its own right: if one triangle fits in another, then it fits in such a way that two of its vertices lie on the same side of the containing triangle.

Our argument for rectangles also relies on a fitting lemma: The largest rectangle similar to a given rectangle that lies in a target rectangle T has its vertices on the sides of T.

LEMMA 2. (Mok-Kong Shen)[1] *Suppose rectangles T and R_0 are given. Then there is a largest rectangle R similar to R_0 that fits in T, and R fits in T with all four of its vertices on the sides of T.*

Proof. The existence of R is a routine consequence of compactness, but for completeness we include a short argument using sequences. Suppose that R_0 has sides u_0 and v_0. Let $\lambda = \sup\{t > 0 : \text{a } tu_0 \times tv_0 \text{ rectangle fits in } T\}$, and let R be the rectangle with sides $u = \lambda u_0$ and $v = \lambda v_0$. To show that R fits in T, take a sequence $\{\lambda_k\}$ of positive reals that increase to λ, and let R_k be the rectangle with sides $u_k = \lambda_k u_0$ and $v_k = \lambda_k v_0$. For each index k there are points K_k, L_k, M_k, N_k in T so that the rectangle $K_k L_k M_k N_k$ is congruent to R_k. Passing successively to convergent subsequences (using the Bolzano-Weierstrass theorem), we arrange for $\{K_k\}$, $\{L_k\}$, $\{M_k\}$, and $\{N_k\}$ all to converge, say to K, L, M, and N, respectively. Then the rectangle $R = KLMN$ is congruent to R_0 and lies in T.

How is such a maximal rectangle R situated in T? At least two vertices of R must lie on the sides of T, or a larger rectangle similar to R_0 could be found in T. We examine the possibilities.

If two vertices of R lie on the same side of T, then the sides of R must be parallel to those of T; and the maximality forces the other two vertices to lie on the sides of T as well.

[1] In response to my May 7, 1998 post to the newsgroup `sci.math.research` concerning this question, Mok-Kong Shen posted on May 11, 1998 a partial solution that includes a version of this fitting lemma. Mr. Shen declined my invitation to be coauthor of this note.

If two vertices of R lie on opposite sides of T and if at least one of the two other vertices of R is inside, a small translation of R moves both inside T; and then a suitable small rotation about its center moves R completely inside T, contrary to the maximality. So if two vertices of R lie on opposite sides of T, then all four vertices must lie on T.

If two vertices of R lie on adjacent sides of T and either of the other two vertices is on a side, we have two opposite vertices of R on the sides of T, and the previous argument applies. If both the other vertices are inside T, a small translation moves R completely inside T, contrary to the maximality. So in every case, all four vertices of R must lie on T. ∎

We say that a rectangle R is *inscribed* in a rectangle T if each (closed) side of T contains a vertex of R (see FIGURE 2). When a rectangle with sides p and q is inscribed in a rectangle with sides a and b, the lengths p, q, a, and b are related by the quartic equation

$$q^4 - (a^2 + b^2 + 2p^2)q^2 + 4abpq - p^2(a^2 + b^2 - p^2) = 0, \tag{1}$$

an equation that over the years has appeared frequently in the problem literature. The earliest references of which I am aware are in 1896, although (1) surely must have been known many years earlier. In a problem posed in the *Monthly*, M. Priest asked for the "... length of a piece of carpet that is a yard wide with square ends, that can be placed diagonally in a room 40 feet long and 30 feet wide, the corners of the carpet just touching the walls of the room." A solution **[14]** published the following year by C. D. Schmitt, et al., gives a trigonometric derivation of (1). Equation (1) also appears as an example in a discussion of quartic equations in Merriman and Woodward **[10**, p. 20], an 1896 mathematics reference book for engineers, with the curious footnote: "This example is known by civil engineers as the problem of finding the length of a strut in a panel of the Howe truss."[2]

The question was posed again in 1914 by C. E. Flanagan. The solution by Otto Dunkel **[4]**, published in 1920, includes a geometric derivation of (1) in the elegant symmetric form

$$2 - \left(\frac{a+b}{p+q}\right)^2 - \left(\frac{a-b}{p-q}\right)^2 = 0$$

and, regarding this as an equation in q for given a, b, and p, an elaborate investigation of the location of the roots.

3. The fitting region Suppose the given target rectangle T has sides of length a and b, where $a \geq b$. By the *fitting region* $F = F(T)$ we mean the set of points $\{p, q\}$ so that the rectangle with sides p and q fits in T. We find it convenient *not* to assume that $p \geq q$. It will simplify the language to regard a point as a 0×0 rectangle and a line segment of length $\ell > 0$ as an $\ell \times 0$ rectangle. With this agreement we see that F is a closed subset of the closed quarter-disk of radius $\sqrt{a^2 + b^2}$ that is symmetric in the 45° line.

The fitting region $F(T)$ is star-shaped with respect to the origin, that is to say, if $P \in F(T)$ and if Q lies on the segment OP (where O is the origin), then $Q \in F(T)$.

[2]The Howe truss, a linear assemblage of cross-braced rectangular structural "panels," was invented by one William Howe (1803–1852) and patented in 1840 and 1842. It was the most popular bridge truss design in the United States in the latter half of the 19th century. See **[6]**.

More generally, if $(p, q) \in F(T)$ and if $0 \le r \le p$ and $0 \le s \le q$ (or if $0 \le r \le q$ and $0 \le s \le p$), then $(r, s) \in F(T)$.

An $x \times y$ rectangle R fits in T with its sides parallel to those of T precisely when $x \le a$ and $y \le b$, or $x \le b$ and $y \le a$. Writing $S_{uv} = \{(x, y): 0 \le x \le u,\ 0 \le y \le v\}$, we conclude that $S_{ab} \cup S_{ba} \subseteq F(T)$. We will find that $F(T)$ is formed from $S_{ab} \cup S_{ba}$ by adding two curvilinear triangular tabs T_1 and T_2 symmetric in the 45° line (see $T_1 = UVW$ in FIGURE 4).

It remains to analyze the situation in which the largest fitting rectangle $R = KLMN$ similar to a given rectangle R_0 fits in T with exactly one vertex on each side of $T = ABCD$. Suppose the vertices are labeled as in FIGURE 2, with $AB = CD = a$,

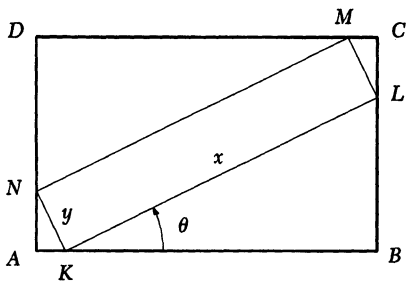

FIGURE 2

$BC = DA = b$, $K \in AB$, $L \in BC$, $M \in CD$, and $N \in DA$. Write $x = KL = NM$ and $y = KN = LM$. Taking the directed angle $\theta = \angle BKL$ as parameter, we see that

$$\begin{cases} x\cos\theta + y\sin\theta = a, \\ x\sin\theta + y\cos\theta = b \end{cases}$$

(cf. [14]). Solving for x and y gives parametric equations for part of the boundary curve of the fitting region:

$$\begin{cases} x = \dfrac{a\cos\theta - b\sin\theta}{\cos 2\theta}, \\[2ex] y = \dfrac{b\cos\theta - a\sin\theta}{\cos 2\theta}. \end{cases}$$

It follows that

$$\begin{cases} x + y = \dfrac{a+b}{\sqrt{2}} \dfrac{1}{\sin\left(\theta + \frac{\pi}{4}\right)}, \\[2ex] x - y = \dfrac{a-b}{\sqrt{2}} \dfrac{1}{\cos\left(\theta + \frac{\pi}{4}\right)}. \end{cases} \tag{2}$$

Rotating the coordinate axes through the angle $-\pi/4$ using the usual coordinate transformation $x = (\xi + \eta)/\sqrt{2}$, $y = (-\xi + \eta)/\sqrt{2}$, we arrive at the parametric equations

$$\begin{cases} \xi = \dfrac{a-b}{2} \dfrac{1}{\cos\left(\theta + \frac{\pi}{4}\right)}, \\[2ex] \eta = \dfrac{a+b}{2} \dfrac{1}{\sin\left(\theta + \frac{\pi}{4}\right)}. \end{cases}$$

To eliminate the parameter, we solve for $\sin(\theta + \frac{\pi}{4})$ and $\cos(\theta + \frac{\pi}{4})$ and square and add, obtaining the equation $\xi^2\eta^2 - c^2\xi^2 - d^2\eta^2 = 0$, i.e., (since $\eta = (x+y)/\sqrt{2} > 0$),

$$\eta = \frac{c|\xi|}{\sqrt{\xi^2 - d^2}},$$

where $c = \frac{1}{2}(a+b)$ and $d = \frac{1}{2}(a-b)$. Suppose $a \neq b$. Then $d \neq 0$, and the curve has two branches symmetric in the η-axis with asymptotes $\eta = c$ and $\xi = \pm d$ (see FIGURE 3). Write $\eta = \Gamma(\xi)$ for the branch with $\xi > 0$ and $\eta > 0$. Plainly Γ is a decreasing function whose graph is concave upwards.

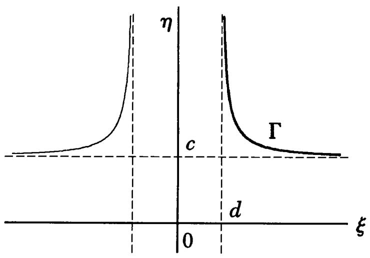

FIGURE 3

In terms of (x, y), Γ can be written (from (2)) in the form $f(x, y; a, b) = 2$, where

$$f(x, y; a, b) = \left(\frac{a+b}{x+y}\right)^2 + \left(\frac{a-b}{x-y}\right)^2.$$

The curve Γ is asymptotic to the lines $x + y = (a+b)/\sqrt{2}$ and $y - x = \pm(a-b)/\sqrt{2}$. For (x, y) in the quadrant $x > |y|$, we see that $f(x, y; a, b) < 2$ on the concave side of Γ and $f(x, y; a, b) > 2$ on the side of Γ that contains the asymptotes. The curve Γ meets the x-axis at $x = \sqrt{a^2 + b^2}$ with slope $-(a^2+b^2)/2ab < -1$, and it passes through the point (a, b) with slope a/b. Since Γ is concave, it meets the line segment $x = a$, $0 \leq y \leq b$ at (a, b) and again at a point U whose y-coordinate b_0 is the root of the cubic

$$y^3 + by^2 - 3a^2y + a^2b = 0$$

in the interval $0 < y < b$. Let U, V, W be the points with coordinates (a, b_0), $(a, 0)$, $(\sqrt{a^2+b^2}, 0)$, respectively, and let T_1 be the curvilinear triangular region bounded by the line segments UV and VW and the arc of Γ with endpoints U and W (see FIGURE 4). Let T_2 be the mirror image region of T_1 in the 45° line through the origin. We shall see that the fitting region $F(T)$ is precisely $S_{ab} \cup S_{ba} \cup T_1 \cup T_2$.

In the square case $a = b$ the graph Γ collapses to the line $\eta = c$, i.e., $x + y = a\sqrt{2}$; and T_1 and T_2 become ordinary triangles.

So finally we come to our result.

THEOREM 3. *A rectangle R with sides p and q fits in a rectangle T with sides a and b (with $a \geq b$) if and only if the point (p, q) lies in the set $S_{ab} \cup S_{ba} \cup T_1 \cup T_2$.*

Proof. If $(p, q) \in S_{ab} \cup S_{ba}$, then $p \leq a$ and $q \leq b$ or vice versa, and in either case the rectangle with sides p, q fits in T with its sides parallel to those of T. If

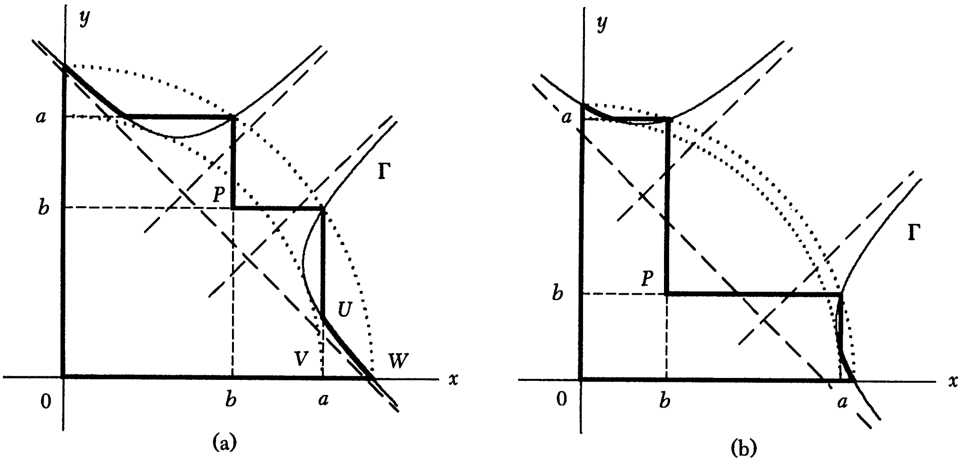

FIGURE 4

$(p, q) \in T_1$, the ray from the origin through (p, q) meets Γ at a point (x, y) whose coordinates are the sides of the largest rectangle R' in T similar to a $p \times q$ rectangle R; and R fits in R' and hence in T. If $(p, q) \in T_2$, then $(q, p) \in T_1$, and the above argument applies. So if (p, q) is any point in $S_{ab} \cup S_{ba} \cup T_1 \cup T_2$, the rectangle with sides p and q fits in T.

Conversely, if a rectangle with sides p and q fits into T, the largest similar rectangle R', with sides $x \geq p$ and $y \geq q$, fits in T in one of only two possible ways. If two vertices of R' lie on the same side of T, then either $x \leq a$ and $y \leq b$ or vice versa, and $(x, y) \in S_{ab} \cup S_{ba}$; so $(p, q) \in S_{ab} \cup S_{ba}$. Otherwise, R' is inscribed in T, (x, y) lies on Γ, and (p, q) lies on the radius to (x, y) and so lies in $S_{ab} \cup S_{ba} \cup T_1 \cup T_2$. ■

It follows, as asserted, that $F(T) = S_{ab} \cup S_{ba} \cup T_1 \cup T_2$. Theorem 1 follows easily: if $p \geq q$, conditions (a) are equivalent to $(p, q) \in S_{ab}$; otherwise, $(p, q) \in T_1$ if and only if $p \geq a, q \leq b$, and $f(a, b; p, q) \geq 2$; and these are conditions (b).

The assertion in the Introduction that the conditions

$$\min\{p, q\} \leq \min\{a, b\} \quad \text{and} \quad p + q \leq \sqrt{a^2 + b^2}$$

are sufficient can be read off FIGURE 4, because if $p \geq q$ and $a \geq b$, then

$$\left\{(p, q) : q \leq b \text{ and } p + q \leq \sqrt{a^2 + b^2}\right\} \subseteq F(T).$$

It is worth noting that in the square case, a $p \times q$ rectangle fits into a square of side a if and only if $\max\{p, q\} \leq a$ or $p + q \leq a\sqrt{2}$ (cf. **[15]**).

4. Two rectangle covering problems Geometric covering problems have attracted considerable interest in recent years. We have recently described the smallest equilateral triangle that can cover any triangle of perimeter two (see Wetzel **[18]**), and in the same article we reported an easily verified claim by one J. Smith that the smallest equilateral triangle that can cover every triangle of diameter one (i.e., every triangle whose longest side has length one) has side $(2\cos 10°)/\sqrt{3}$. Here we use Theorem 3 to solve two analogous problems for rectangles.

First we determine the smallest rectangle R similar to a given rectangle R_0 that can accommodate every rectangle of perimeter two. Evidently the width of R must be at least $1/2$, and if R_0 is long and thin, taking the width of R to be $1/2$ suffices. However, if the sides of R_0 are more nearly equal, we need further to arrange for the diagonal of R to be at least one. More precisely, we prove the following:

THEOREM 4. *An $a \times b$ rectangle R_0 is given, with $a \geq b$. The smallest rectangle R similar to R_0 that can accommodate every rectangle of perimeter two is $a/2b \times 1/2$ if $a \geq b\sqrt{3}$ and $a/d \times b/d$ if $a \leq b\sqrt{3}$, where $d = \sqrt{a^2 + b^2}$ is the diagonal of R_0.*

Proof. The dual question is more natural in the present context: given a rectangle R with sides of lengths a and b, determine the largest possible ℓ so that R can accommodate every rectangle of perimeter ℓ. In geometric terms, we seek the largest ℓ so that the line segment $\sigma(\ell)$ with endpoints $(\ell/2, 0)$ and $(0, \ell/2)$ lies in the fitting region $F(R)$.

Since Γ meets the x-axis at the point $d = \sqrt{a^2 + b^2}$ with slope at most -1, as ℓ increases from 0, $\sigma(\ell)$ remains in $F(R)$ until it encounters either the point $P(b, b)$ or the intercept $(d, 0)$ (see FIGURE 4). The segment $\sigma(\ell)$ reaches the point P before the point $(d, 0)$ precisely when $2b \leq d$, i.e., precisely when $a \geq b\sqrt{3}$. So when $a \geq b\sqrt{3}$ the largest possible ℓ is $4b$, and when $a \leq b\sqrt{3}$ the largest possible ℓ is $2d$.

Dually, the smallest rectangle similar to a given $a \times b$ rectangle (with $a \geq b$) that can accommodate every rectangle of perimeter ℓ is $\ell a/4b \times \ell/4$ when $a \geq b\sqrt{3}$ and $\ell a/2d \times \ell b/2d$ when $a \leq b\sqrt{3}$. The assertion follows when $\ell = 2$. ■

COROLLARY 5. *The rectangle of least area that can accommodate every rectangle of perimeter two has sides $1/2$ and $\sqrt{3}/2$.*

As a second example, we determine the smallest rectangle similar to a given rectangle that can accommodate every rectangle of diameter one.

THEOREM 6. *An $a \times b$ rectangle R_0 is given, with $a \geq b$. The smallest rectangle R similar to R_0 that can accommodate every rectangle of diameter one is $a/(b\sqrt{2}) \times 1/\sqrt{2}$ if $a \geq b\sqrt{2}$ and $1 \times b/a$ if $a \leq b\sqrt{2}$.*

Proof. Direct arguments are easy to give, but a dual argument that parallels the proof of Theorem 4 is straightforward. For the given rectangle R_0 with fitting region $F(R_0)$, we seek the largest d so that every rectangle with diagonal d fits in R_0. Write $\Phi(\rho)$ for the quarter circular arc $x^2 + y^2 = d^2$, $0 \leq x \leq d$. In geometric terms, we seek the largest d so that $\Phi(d) \subseteq F(R_0)$.

It is a calculus exercise to show that Γ (as a function of x and y) has slope $-x/y$ at precisely one point, the point (x_1, y_1) where

$$\begin{cases} x_1 = \sqrt{\frac{a}{2}\left(a + \sqrt{a^2 - b^2}\right)}, \\ y_1 = \sqrt{\frac{a}{2}\left(a - \sqrt{a^2 - b^2}\right)}. \end{cases} \tag{3}$$

Since $x_1^2 + y_1^2 = a^2$, it follows that Γ and the quarter circle $\Phi(a)$ are tangent at the point (x_1, y_1) given by (3) (see FIGURE 4). Consequently, the largest radius d so that $\Phi(d) \subseteq F(R_0)$ is just $\min\{a, b\sqrt{2}\}$, depending on whether for increasing d, $\Phi(d)$ meets P or Γ first.

Dually, the smallest rectangle similar to a given $a \times b$ rectangle (with $a \geq b$) that can accommodate every rectangle of diameter d is $ad/b\sqrt{2} \times d/\sqrt{2}$ when $a \geq b\sqrt{2}$ and $d \times bd/a$ when $a \leq b\sqrt{2}$. The assertion follows when $d = 1$. ■

COROLLARY 7. *The rectangle of least area that can accommodate every rectangle of diameter one has sides 1 and $1/\sqrt{2}$.*

5. Some further questions Here are a few interesting related problems, of undetermined difficulty.

Find necessary and sufficient conditions on a, b, c, s for a triangle with sides a, b, c to fit in a square with side s; for a square with side s to fit in a triangle with sides a, b, c.

More generally, find necessary and sufficient conditions on a, b, c, p, q for a triangle with sides a, b, c to fit in a rectangle with sides p, q; for a rectangle with sides p, q to fit in a triangle with sides a, b, c.

Each of these fitting problems has associated covering problems. How small a square (or rectangle of prescribed shape) can accommodate every triangle of perimeter two? Of diameter one? How small a triangle of prescribed shape can accommodate every rectangle of perimeter two? Of diameter one?

Find necessary and sufficient conditions on the lengths a, b, c and p, q, r for a box (i.e., a rectangular parallelepiped) with edges p, q, r to fit in a box with edges a, b, c. (F.M. Garnett asked this question in 1923, and W.B. Carver **[2]** supplied a fragmentary answer in 1925.) What are the analogous results for orthotopes in $\mathbb{E}^d$? Again, there are related covering problems.

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