

1. Prove that  $\{0^n 1^n 2^n : n \geq 1\}$  is not a regular language.

$\{0^n 1^n 2^n : n \geq 1\}$  not a regular language.

$xyz, xy^i z$ , where  $i \geq 1$

Some options possible for  $y$ :

$y = 11 \dots 1 \rightarrow$  would result in more 1's ( $xy^i yz$ )  
 $y = 000 \dots 0 \rightarrow$  would result in more 0's ( $xy^i yz$ )  
 $y = 22 \dots 2 \rightarrow$  similar to above would satisfy  $0^i 1^i 2^i$   
 $y = 00 \dots 1111 \dots 222 \rightarrow$  The pattern is lost here.  
 $(00001112220000)$

Impossible to have such a DFA,

$L = \{0^n 1^n 2^n : n \geq 1\}$  is non-regular

2. For arbitrary constant  $c$ , is  $\{0^n 1^n 2^n : n \geq c\}$  regular or not?

$\rightarrow$  For arbitrary constant  $c$ , is  $L = \{0^n 1^n 2^n : n \geq c\}$

Proving by Pumping lemma

If  $L$  is regular, then there exist  $n \geq 1$

— for any string  $w \in L$ ,  $|w| \geq n$ , we have

$w = xyz$  such that

—  $y \neq \epsilon$

—  $|xy| \leq n$

—  $xy^i z \in L$  for any  $i \geq 0$

Choose  $n = c$ ,  $y = 1 \dots 1_n$  which would imply

$x = 0^n$ ,  $z = 2^n$

This would fail for  $xy^2 z$ ,  $xy^3 z$  or  
 $xy^i z$ ,  $i \geq 1$ .

$\therefore L$  is not regular.

3. The decimal notation for a number is the number written in the usual way, as a string over the alphabet  $\{0, 1, \dots, 9\}$ . For example, the decimal notation for 13 is a string of length 2. In unary notation, only the symbol "1" is used; thus 5 would be represented as 11111 in unary

notation. Show that each of the following is or is not a regular language.

(For regular languages, write down its regular expression or describe the automata accepting it; for languages that are not regular, prove it using pumping lemma)

3.1  $\{w: w \text{ is the unary notation for a number that is a multiple of 7}\}$

$L = \{w: w \text{ is unary notation of no. multiple of 7}\}$   
 Consider  $L$  is regular, By Pumping lemma  
 For some  $n, |w| \geq n, w = xyz$   
 where  
 -  $y \neq \epsilon$   
 -  $|xy| \leq n$   
 -  $xy^iz \in L$  for  $i \geq 0$   
  
 Consider  $w = xy^iz \in L$   
 which means  $|w| = 7k$  where  $k \geq 1$   
  
 For  $xy^{i+1}z \in L$   
 $|y| = 7\ell$  where  $\ell \geq 1$   
  
 So  $L$  is regular.

3.2  $\{w: w \text{ is the unary notation for } 10^n, n \geq 1\}$

3.2  $L = \{w: w \text{ is unary notation for } 10^n, n \geq 1\}$   
 Consider  $L$  be regular, By Pumping lemma  
 For some  $n, |w| \geq n$ , then  $w = xyz$   
 where  
 -  $y \neq \epsilon$   
 -  $|xy| \leq n$   
 -  $xy^iz \in L$  for  $i \geq 0$   
  
 $|w| = 10^k$  where  $k \geq 1$   
  
 Let  $w = xy^iz \in L$  ( $|w| = n$ )  
 then  $xy^{i+1}z \in L$   
 But  $|y| = 10^m - 10^k$  ( $k \geq 1, n \geq k$ )  
 which means  $y$  is dynamic  
 not a fixed pattern.  
 So  $L$  is not regular.

