Homework – 5

1. Consider the grammar $G = (V, \Sigma, R, S)$, where

$$V = \{a, b, S, A\},$$
$$\Sigma = \{a, b\},$$
$$R = \{S \to AA,$$
$$A \to AAA,.$$
$$A \to a,$$
$$A \to bA,$$
$$A \to Ab\}.$$

a). Give a string of $L(G)$ that can be produced by applying the rules at most 4 times

$S \Rightarrow AA \Rightarrow bAA \Rightarrow baA \Rightarrow \boldsymbol{baa}$

b). Same string can be derived in different ways, e.g., $S \Rightarrow AA \Rightarrow aA \Rightarrow aa$, $S \Rightarrow AA \Rightarrow Aa \Rightarrow aa$. Give at least 2 distinct derivations for the string $babbab$

Ans.

1. $S \Rightarrow AA \Rightarrow bAA \Rightarrow bAAb \Rightarrow bAbAb \Rightarrow bAbbAb \Rightarrow babbab$
2. $S \Rightarrow AA \Rightarrow AAb \Rightarrow bAbAb \Rightarrow bAbbAb \Rightarrow babbab$

c). For any $m, n, p > 0$, describe a derivation in G of the string $b^m ab^n ab^p$

$S \Rightarrow AA \Rightarrow bAA \Rightarrow bAbAb$

After the above string, we apply only the below 2 rules in random fashion

- $A \longrightarrow bA$
- $A \longrightarrow Ab$

In any case we end up with $b^*Ab^*Ab^*$

At the end we just apply the below rule for 2 times

$A \longrightarrow a$

2. Consider the grammar $G = (V, \Sigma, R, S)$, where

$$V = \{a, b, S, A\},$$
$$\Sigma = \{a, b\},$$
$$R = \{S \to aAa,$$
$$S \to bAb,$$
$$S \to e,$$
$$A \to SS\}.$$

Give a derivation of the string $\boldsymbol{baabbb}$ in $\boldsymbol{G}$.

$S \Rightarrow bAb \Rightarrow bSSb \Rightarrow baAaSb \Rightarrow baAabAbb \Rightarrow baSSabSSbb \Rightarrow baabbb$

3. Show that the following languages are context-free by exhibiting context free grammars generating each.

a). $\{a^m b^n : m \geq n \geq 0\}$

V = {a,b,S,A}

$\sum = \{a, b\}$

$R = \{ S \longrightarrow aAb$

$A \longrightarrow aA$

$A \longrightarrow aAb$

$A \longrightarrow e$

b). $\{a^m b^n c^{2m+n} : m, n \geq 0\}$

V = {a,b,c,S,A}

$\sum = \{a, b, c\}$

$R = \{ S \longrightarrow aAcc$

$A \longrightarrow aAcc$

$A \longrightarrow aBcc$

$B \longrightarrow bBc$

$B \longrightarrow e$

$A \longrightarrow e$