

# CS 5381 Analysis of Algorithms

## Homework 1

Fall 2022

**Total 180 points**

### **Growth of Functions**

1. (20 points) Let  $f(n)$  and  $g(n)$  be asymptotically nonnegative functions. Show that

$$\max(f(n), g(n)) = \Theta(f(n) + g(n)).$$

2. (20 points) Show that for any real constants  $a$  and  $b$  with  $b > 0$ , we have

$$(n + a)^b = \Theta(n^b).$$

### **Divide-and-Conquer**

3. (20 points) Use the substitution method to show that the running time of the recurrence

$$T(n) = T(n - 1) + n$$

is  $O(n^2)$ .

4. (20 points) Use the recursion-tree method to find the running time of the recurrence

$$T(n) = T(n/2) + T(n/4) + T(n/8) + n.$$

5. (20 points) Use the master theorem to find the running time of the recurrence

$$T(n) = 8T(n/2) + \Theta(n^2).$$

6. (20 points) Find the running time of the recurrence

$$T(n) = 2T(n/2) + n^3.$$

7. (20 points) Find the running time of the recurrence

$$T(n) = 3T(n/2) + n \lg n.$$

### **Probabilistic Analysis**

8. (20 points) In HIRE-ASSISTANT algorithm, assuming that the candidates are presented in a random order, what is the probability that you hire exactly one time? What is the probability that you hire exactly  $n$  times?

9. (20 points) Use indicator random variables to solve the following problem, which is known as the hat-check problem. Each of  $n$  customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers who get back their own hat?