20 - Insem
20 - CCE
60 - Endsem
25 - Termwork

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Unit I Descriptive measures
Measures of central tendency (Mean, median, mod
measures of dispersion (variance, standard deviati
range), coefficients of variation, moments,
skewness and kurtosis.

Unit II Random variable and distribution function
Random variable, distribution functions (continious
and discrete), properties of distribution function
probability mass function (p.m.f), probability
density function (p.d.f) and commulative distributiv
function (continous. and discrete).

Unit III Mathematical Expectation and Generating
function.
Mathematical expectation, properties of expectation
, moment generating function.

Unit IV Probability distributions
Discrete distributions: Geometric, Binomial, poisson,
uniform distribution

Continous distribution: Normal distribution,
standard normal, uniform.

① Arithmetic mean ($\bar{x}$):

let $x_1, x_2, \ldots x_n$ be n observations.

then $\bar{x} = \dfrac{x_1 + x_2 + \ldots x_n}{n}$

$$\boxed{\bar{x} = \frac{\Sigma x_i}{n}}$$

if data is in form of frequency distribution then

x_i	x_1	x_2	....	x_n
f_i	f_1	f_2	...	f_n

$$\boxed{\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}}$$

To reduce the calculation:

$d_i = x_i - A$ (A is any value) Assume mean

$$f_i d_i = x_i f_i - A f_i$$

$$\Sigma f_i d_i = \Sigma x_i f_i - A \Sigma f_i$$

dividing by Σf_i

$$\frac{\Sigma f_i d_i}{\Sigma f_i} = \frac{\Sigma x_i f_i}{\Sigma f_i} - A \frac{\cancel{\Sigma f_i}}{\cancel{\Sigma f_i}}$$

$$\frac{\Sigma f_i d_i}{\Sigma f_i} = \bar{x} - A$$

$$\boxed{\bar{x} = \frac{\Sigma f_i d_i}{\Sigma f_i} + A}$$

further reduction of calculation,

$$u_i = \frac{x_i - A}{h}$$

h be class width (upper limit - lower lim

$h u_i = x_i - A$

multiplying by f_i

$h u_i f_i = x_i f_i - A f_i$

$h \Sigma u_i f_i = \Sigma x_i f_i - A \Sigma f_i$

dividing by Σf_i on b.s

$$\frac{h \Sigma u_i f_i}{\Sigma f_i} = \frac{\Sigma x_i f_i}{\Sigma f_i} - \frac{A \Sigma f_i}{\Sigma f_i}$$

$$\boxed{\bar{x} = A + \frac{h \Sigma u_i f_i}{\Sigma f_i}}$$

① calculate arithmetic mean for following frequency distribution.

class interval	0-8	8-16	16-24	24-32	32-40	40-48
frequency	8	7	16	24	15	7

→ ~~$\bar{x} = \Sigma x_i f_i$~~

Class interval	x_i	f_i	$x_i f_i$
0-8	4	8	32
8-16	12	7	84
16-24	20	16	320
24-32	28	24	672
32-40	36	15	540
40-48	44	7	308

$$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

$$\boxed{\bar{x} = 25.40}$$

② Find ~~basic~~ missing frequencies from the following data.

marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35
frequency	10	12	16	f_4	14	10	8

Average marks is 16.82

⇒ Given:

$\bar{x} = 16.82$ $A = 17.5$, $h = 5$

marks	x_i	f_i	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
0-5	2.5	10	-3	-30
5-10	7.5	12	-2	-24
10-15	12.5	16	-1	-16
15-20	17.5	f_4	0	0
20-25	22.5	14	1	14
25-30	27.5	10	2	20
30-35	32.5	8	3	24
		$70+f_4$		-12

$$\bar{x} = A + \frac{h\,\Sigma f_i u_i}{\Sigma f_i}$$ $h = 5$, $A = 17.5$

$$16.82 = 17.5 + \frac{5 \times -12}{70+f_4}$$

$$-0.68 = \frac{-60}{70+f_4}$$

$$-47.6 - 0.68 f_4 = -60$$

$$\boxed{f_4 = 18.23}$$

∴ Required missing frequency is 18.23.

③ Mean of following frequency table is 50. Find missing frequencies.

C.I	0-20	20-40	40-60	60-80	80-100	Total
frequency	17	f_1	32	f_2	19	120

$\Rightarrow$ mean $= \bar{x} = 50$

$\Sigma f_i = 120$

C.I	x_i	f_i	$f_i x_i$
0-20	10	17	170
20-40	30	f_1	$30f_1$
40-60	50	32	1600
60-80	70	f_2	$70f_2$
80-100	90	19	1710
Total		120	$3480 + 30f_1 + 70f_2$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$50 = \frac{3480 + 30f_1 + 70f_2}{120}$$

$$2520 = 30f_1 + 70f_2$$

$$3f_1 + 7f_2 = 252 \quad \text{—— (1)}$$

Total frequency given is 120

$$\Sigma f_i = 120$$

$$68 + f_1 + f_2 = 120$$

$$f_1 + f_2 = 52 \quad \text{—— (2)}$$

$$3f_1 + 7f_2 = 252 \times 1$$

$$f_1 + f_2 = 52 \times 3$$

$$3f_1 + 7f_2 = 252 \quad \text{—— (iii)}$$

$$-\;3f_1 + 3f_2 = 156 \quad \text{—— (iv)}$$

$$4f_2 = 96$$

$$f_2 = 1$$

$$\boxed{f_2 = 24}$$

$$f_1 + f_2 = 52$$

$$f_1 + 24 = 52$$

$$\boxed{f_1 = 28}$$

② Median :-

Median is a value of variate which divides given data into two equal parts.

For continious frequency distribution,

$$\boxed{\text{median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)}$$

l be lower limit of median class.

h be class width of median class.

f be frequency of median class.

N - total frequency

c - commulative frequency ~~before~~ / preceding the median class.

$\frac{N}{2}$ or just greater than $\frac{N}{2}$ will be median class.

③ Mode :

Mode is the value of variate which occurs most frequently in the data.

$$\boxed{\text{Mode} = l + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2}}$$

The modal class is the class which has maxm frequency.

l = lower limit of modal class

h = class width of modal class.

f_1 = frequency of modal class
f_0 = frequency preceding to modal class.
f_2 = frequency succeeding to modal class.

① Find median and mode for the following distribution.

C.I	0-10	10-20	20-30	30-40	40-50	50-60
frequency	12	18	27	20	17	6

⇒ ①

C.I	frequency	Commulative frequency
0-10	12	12
10-20	18	30
20-30	27	57
30-40	20	77
40-50	17	94
50-60	6	100
	100	

$\frac{N}{2} = \frac{100}{2} = 50$ ~~N=6 observations~~

20-30 be median class.

$$\text{median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

$l = 20,\ h = 10,\ f = 27,\ N = 100,\ c = 30$

$$= 20 + \frac{10}{27}\left(\frac{100}{2} - 30\right)$$

$$\boxed{\text{median} = 27.40}$$

② To find mode :-

modal class will be 20-30 because 27 is maximum frequency.

$$mode = l + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2}$$

$l = 20$, $h = 10$, $f_1 = 27$, $f_0 = 18$, $f_2 = 20$

$$mode = 20 + \frac{10(27-18)}{2(27)-18-20}$$

$$= \frac{410}{16}$$

$$\boxed{mode = 25.62}$$

② An incomplete frequency distribution table is given find missing frequencies.

C.I	10-20	20-30	30-40	40-50	50-60	60-70	70
frequency	12	30	f_1	65	f_2	25	

Total 22

Given median value is 46.

→ Given:

Total frequency = 229

$12 + 30 + f_1 + 65 + f_2 + 25 + 18 = 229$

$150 + f_1 + f_2 = 229$

$f_1 + f_2 = 79$ —①

median = 46

C.I	frequency	c.f
10-20	12	12
20-30	30	42
30-40	f_1	$42 + f_1$

40-50	65	$107+f_1$
50-60	f_2	$107+f_1+f_2$
60-70	25	$132+f_1+f_2$
70-80	18	$150+f_1+f_2$
	229	

$N = 229$

$\frac{N}{2} = \frac{229}{2} = 114.5$

As median is 46 lies betn 40-50

median class is 40-50

$h = 10$ $\qquad l = 40$

$\text{median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$ $\qquad c = 42+f_1$

$46 = 40 + \frac{10}{65}\left(\frac{229}{2} - \cancel{107}^{42} + f_1\right)$

$39 = 114.5 - \cancel{107}^{42} + f_1$

$\cancel{39 =}$ $\boxed{f_1 = 33.5}$

put f_1 in ①

$f_1 + f_2 = 79$

$31.5 + f_2 = 79$

$\boxed{f_2 = 45.5}$

∴ The missing frequencies are $f_1 = 33.5$ and $f_2 = 45.5$

③ The median and mode of the distance are known to be Rs. 3350 and Rs. 3400 respectively. find values of f_1, f_2, f_3.

→

wages in RS.	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000	6000-7000
No. of employees	4	16	f_1	f_2	f_3	6	4
				Total 230.			

→ Given: median = 3350

mode = 3400

Total frequency = 230

$4+16+f_1+f_2+f_3+10=230$

$\therefore f_1+f_2+f_3=200$ —— ①

C.I	frequency	C.f
0-1000	4	4
1000-2000	16	20
2000-3000	f_1	$20+f_1$
3000-4000	f_2	$20+f_1+f_2$
4000-5000	f_3	$20+f_1+f_2+f_3$
5000-6000	6	$26+f_1+f_2+f_3$
6000-7000	4	$30+f_1+f_2+f_3$

median class is 3000-4000

$$\text{median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

$l = 3000$

$h = 1000$

$f = f_2$

$N = 230$

$c = 20+f_1$

$$3350 = 3000 + \frac{1000}{f_2}\left(\frac{230}{2} - 20 - f_1\right)$$

$$\frac{350 f_2}{1000} = \frac{230}{2} - 20 - f_1$$

$$0.350 f_2 = 115 - 20 - f_1$$

$$f_1 + 0.350 f_2 = +95 \quad —— ②$$

$$f_1 = -95 + 0.350 f_2$$

$$100 f_1 + 35 f_2 = 9500 \quad —— ②$$

modal class is 3000 - 4000

$$\text{mode} = l + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2}$$

$$3400 = 3000 + \frac{1000(f_2 - f_1)}{2f_2 - f_1 - f_3}$$

$$\frac{400}{1000} = \frac{f_2 - f_1}{2f_2 - f_1 - f_3}$$

$$0.4 = \frac{f_2 - f_1}{2f_2 - f_1 - f_3}$$

$$0.8f_2 - 0.4f_1 - 0.4f_3 = f_2 - f_1$$

$$-0.4f_1 + f_1 + 0.2f_2 + 0.4f_3 = 0$$

$$0.6f_1 - 0.2f_2 - 0.4f_3 = 0$$

$$6f_1 + 2f_2 - 4f_3 = 0 \text{ ——— (2)}$$

$$3f_1 - f_2 - 2f_3 = 0$$

$f_1 = 60$
$f_2 = 100$
$f_3 = 40$

(4) The following table shows the distance of 100 students according to the marks secured by them in M-I median of the distance is 30 marks. find missing frequencies.

marks	0-10	10-20	20-30	30-40	40-50	50-60
no. of Students	10	f_1	25	30	f_2	10

→ Given:-

Median = 30 marks.

Total = N = 100

Marks	no of students	c.f
0-10	10	10
10-20	f_1	$10+f_1$
20-30	25	$35+f_1$
* 30-40	30	$65+f_1$
40-50	f_2	$65+2f_1+f_2$
50-60	10	$75+2f_1+f_2$
	100	

median class is 30-40.

$$\text{median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

$f = 30$, $c = 35 + f_1$, $l = 30$, $h = 10$, $N = 100$

median = 30

$$30 = 30 + \frac{10}{30}\left(\frac{100}{2} - 35 - f_1\right)$$

$$50 - 35 - f_1 = 0$$

$$-f_1 = 15$$

$$\boxed{f_1 = 15}$$

from given,

$$75 + f_1 + f_2 = 100$$

$$75 + 15 + f_2 = 100$$

$$f_2 = 100 - 90$$

$$\boxed{f_2 = 10}$$

* Combined arithmetic mean :-

Let $\bar{x_1}$ be the mean of series one whose no. of observation is n_1.

Let $\bar{x_2}$ is mean of series 2 whose no. of obs. be n_2

The combined arithmetic mean =

$$\boxed{\bar{z} = \frac{n_1\bar{x_1} + n_2\bar{x_2}}{n_1+n_2}} \quad \ldots \text{(for two series)}$$

① Average salary of male employes in form class 2500 and that of female was 2000.
The main salary of all employees was 2200.
Find percentage of male and female employees.

→

$\bar{x_1}$ = Average salary of male employes = 2500

$\bar{x_2}$ = Average salary of male employes = 2000

$\bar{z}$ = average salary of all employes = 2200

let assume that,

n_1 = be no. of male employees.

n_2 = be no. of female employees.

Using combined arithmetic mean formula,

$$\bar{z} = \frac{n_1\bar{x_1} + n_2\bar{x_2}}{n_1+n_2}$$

$$2200 = \frac{2500n_1 + 2000n_2}{n_1+n_2}$$

$$2200n_1 + 2200n_2 = 2500n_1 + 2000n_2$$

$$300n_1 - 200n_2 = 0$$

$$3n_1 - 2n_2 = 0$$

$$\frac{n_1}{n_2} = \frac{2}{3}$$

Percentage of male employees $= \left(\frac{2}{2+3}\right) \times 100$

$$= \frac{200}{5} = 40\%$$

% female = 100 - 40 = 60%.

Percentage of female employees = $\left(\frac{3}{2+3}\right) \times 100$

$$= \frac{300}{5} = 60\%$$

* Measures of dispersion :-

① Range - The difference betn highest value & least value is called range.

ex. 21, 25, 28, 31, 35, 38, 48

$$\text{Range} = 48 - 21 = 27$$

② Standard deviation (σ) :

$$\boxed{\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}}}$$

③ Variance - σ^2

$$\sigma = \sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2}$$

$$\sigma = \sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\bar{X})^2}$$

To reduced the calculation,

$$d_i = x_i - A$$

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2}$$

$$u_i = \frac{x_i - A}{h}$$

$$\boxed{\sigma = h\sqrt{\frac{\Sigma f_i u_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)^2}}$$

① find Standard deviation of following data.

①

x_i	102	106	110	114	118	122	126
f_i	3	9	25	35	17	10	1

→

x_i	f_i	$d_i = x_i - A$	d_i^2	$f_i d_i$	$f_i d_i^2$
102	3	-12	144	-36	432
106	9	-8	64	-72	576
110	25	-4	16	-100	400
114	35	0	0	0	0
118	17	4	16	68	272
122	10	8	64	80	640
126	1	12	144	12	144
Total	100			-48	2464

$$\text{deviation} = \sigma = \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2}$$

$$= \sqrt{\frac{2464}{100} - \left(\frac{-48}{100}\right)^2}$$

$$\boxed{\sigma = 4.94}$$

②

x_i	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f_i	1	5	12	22	17	9	4

→

x_i	f_i	$u_i = \frac{x_i - A}{h}$ $(h=10)$	$f_i u_i$	$f_i u_i^2$	u_i^2
4.5	1	-3	-3	9	9
14.5	5	-2	-10	20	4
24.5	12	-1	-12	12	1
* 34.5	22	0	0	0	0
44.5	17	1	17	17	1
54.5	9	2	18	36	4
64.5	4	3	12	36	9
	70		22	130	

$n = 9$,

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{360}{9} = 40$

$\bar{y} = \frac{\Sigma y_i}{n} = \frac{227}{9} = 30.7$

$\sigma x = \sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2}$

$= \sqrt{\frac{22700}{9} - (40)^2}$

$\boxed{\sigma x = 30.37}$

$\sigma y = \sqrt{\frac{\Sigma y_i^2}{n} - (\bar{y})^2}$

$= \sqrt{\left(\frac{11877}{9}\right) - \left(\frac{30.7}{8}\right)^2}$

$\boxed{\sigma y = 19.42}$

coefficient of variation of 1st series $= \frac{\sigma x}{\bar{x}} \times 100$

$= \frac{30.37}{4\cancel{0}} \times 10\cancel{0}$

$= \frac{303.7}{4} = 75.9$

coefficient of variation of 2nd series $= \frac{\sigma y}{\bar{y}} \times 100$

$= \frac{19.42}{30.77} \times 100$

$= 62.72$

c.v of 1st series > c.v of 2nd series

player A player B

So the player B is more consistent.

* <u>Coefficient of variation:-</u>

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

① The prices of share X and Y are given below. State which share is more stable.

x	55	54	52	53	56	58	52	50	51	49
y	108	107	105	105	106	107	104	103	104	101

→ $A = 56$, B

x	y	$u_i = x_i - A$	$v_i = y_i - 105$ ($y_i - B$)	u_i^2	v_i^2
55	108	-1	3	1	9
54	107	-2	2	4	4
52	105	-4	0	16	0
53	105	-3	0	9	0
56	106	0	1	0	1
58	107	2	2	4	4
52	104	-4	-1	16	1
50	103	-6	-2	36	4
51	104	-5	-1	25	1
49	101	-7	-4	49	16
530	1050	-30	0	160	40

$$\bar{x} = A + \frac{\Sigma u_i}{n}$$

$$= 56 + \frac{(-30)}{10}$$

$$\boxed{\bar{x} = 53}$$

$\bar{y} = B + \frac{\Sigma vi}{n}$ $\quad B = 105$

$= \cancel{56 - \frac{0}{10}}$ $\quad = 105 + \frac{0}{10}$

$\boxed{\bar{y} = 56}$ $\quad y = 105$

$\sigma x = \sqrt{\frac{\Sigma ui^2}{n} - \left(\frac{\Sigma ui}{n}\right)^2}$

$= \sqrt{\frac{160}{10} - \left(\frac{-30}{10}\right)^2}$

$\sigma x = 2.64$

$\sigma y = \sqrt{\frac{\Sigma vi^2}{n} - \left(\frac{\Sigma vi}{n}\right)^2}$

$= \sqrt{\frac{40}{10} - \left(\frac{0}{10}\right)^2}$

$= \sqrt{4}$

$\sigma y = 2$

coefficient of variation of first series $= X$

$= \frac{\sigma x}{\bar{X}} \times 100$

$= \frac{2.64}{53} \times 100$

$= 4.98$

coefficient of variation of second series y:

$= \frac{\sigma y}{\bar{y}} \times 100 = \frac{2}{56} \times 100$

$= \cancel{3.57}\ 1.50$

Here C.V of share y < C.V of share x
so share y is more consistent.

② The following are the scores of two batsmen A & B 10 consecutive matches:

A	12	115	6	73	7	19	119	36	84	29
B	47	12	16	42	4	51	37	48	13	0

compared Avg (mean) — more — cmp C.V

who is better scorer and best consistent?

⟹ $A = 73$ $B = 42$

A	B	$u_i = x_i - A$	$v_i = y_i - B$	u_i^2	v_i^2
12	47	−61			
115	12	42			
6	16	−67			
73	42	0			
7	4	−66			
19	51	−54			
119	37	46			
36	48	−37			
84	13	11			
29	0	−44			

* combined standard deviation :-

let $\bar{x}_1, \sigma_1$ be the arithmetic mean and standar deviation of sample size n_1.

& $\bar{x}_2, \sigma_2$ be the A.M and S.D of sample size n_2.

$\bar{x}$ = combined arithmetic mean

σ = combined standard deviation.

$$\boxed{(n_1+n_2)\sigma^2 = n_1\sigma_1^2 + n_2\sigma_2^2 + n_1 D_1^2 + n_2 D_2^2}$$

$\therefore \sigma^2$ = variance

$D_i = \bar{x}_i - \bar{x}$

$D_1 = \bar{x}_1 - \bar{x}$ ($\bar{x}_1$ - A.M of 1st series, $\bar{x}$ - combined A.M.)

$D_2 = \bar{x}_2 - \bar{x}$

① The mean and standard deviation in each of examination by three medical examinatio are given below. find mean & S.D for the entire data when grouped together.

medical exam	No. of examined	mean	standard deviation
A	$50 = n_1$	$113 = \bar{x}_1$	$6 = \sigma_1$
B	$60 = n_2$	$120 = \bar{x}_2$	$7 = \sigma_2$
C	$90 = n_3$	$115 = \bar{x}_3$	$8 = \sigma_3$

$\Rightarrow$ $\bar{x}$ is combined arithmetic mean

σ is combined standard deviation.

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1+n_2+n_3}$$

$$\bar{x} = \frac{50\times113 + 60\times120 + 90\times115}{50+60+90}$$

$\bar{x} = 116$

combined standard deviation

$$= \frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_3\sigma_3^2 + n_1 D_1^2 + n_2 D_2^2 + n_3 D_3^2}{n_1 + n_2 + n_3}$$

$D_1 = \bar{x}_1 - \bar{x} = 113 - 116 = -3$

$D_2 = \bar{x}_2 - \bar{x} = 120 - 116 = 4$

$D_3 = \bar{x}_3 - \bar{x} = 115 - 116 = -1$

$$\sigma^2 = \frac{50 \times 36 + 60 \times 49 + 90 \times 64 + 50 \times 9 + 60 \times 16 + 90 \times 64}{50 + 60 + 90}$$

$\sigma^2 = 60$

$\boxed{\sigma = 7.745}$

* Moments, Skewness & Kurtosis :-

① Raw moments (moment about any value A).

The rth moment about any value A is denoted as μ_r'.

$$\boxed{\mu_r' = \frac{\Sigma f_i (x_i - A)^r}{\Sigma f_i}}$$

A is any value here we have taken A=0.

A=0

$$\therefore \mu_r' = \frac{\Sigma f_i (x_i)^r}{\Sigma f_i}$$

r=0

$$\mu_0' = \frac{\Sigma f_i}{\Sigma f_i} \quad \therefore \boxed{\mu_0' = 1}$$

$r=1, \quad \mu_1' = \frac{\Sigma f_i (x_i)}{\Sigma f_i}$

$\boxed{\mu_1' = \bar{x}}$

$r=2 \quad \mu_2' = \frac{\Sigma f_i (x_i)^2}{\Sigma f_i}$

$r=3 \quad \mu_3' = \frac{\Sigma f_i (x_i)^3}{\Sigma f_i}$

$r=4 \quad \mu_4' = \frac{\Sigma f_i (x_i)^4}{\Sigma f_i}$

② Central moments: (moments about arithmetic mean)
- rth moment about mean is denoted as μ_r

$$\boxed{\mu_r = \frac{\Sigma f_i (x_i - \bar{x})^r}{\Sigma f_i}}$$

if $r=0$:

$\mu_0 = \frac{\Sigma f_i (x_i - \bar{x})^0}{\Sigma f_i} = \frac{\Sigma f_i}{\Sigma f_i} = 1$

$\boxed{\mu_0 = 1}$

$r=1$

$\mu_1 = \frac{\Sigma f_i (x_i - \bar{x})}{\Sigma f_i} = \frac{\Sigma f_i x_i}{\Sigma f_i} - \frac{\Sigma f_i \bar{x}}{\Sigma f_i}$
$= \bar{x} - \bar{x} = 0$

$r=2$

$\mu_2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i} = \sigma^2 = V(x)$

$\boxed{\mu_2 = \sigma^2}$

$\sigma = \sqrt{\mu_2}$

$r=3 \quad \mu_3 = \frac{\Sigma f_i (x_i - \bar{x})^3}{\Sigma f_i}$

$r=4 \quad \mu_4 = \frac{\Sigma f_i (x_i - \bar{x})^4}{\Sigma f_i}$

* Relation between μ_r' and μ_r:

$\mu_0 = 1$, $\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

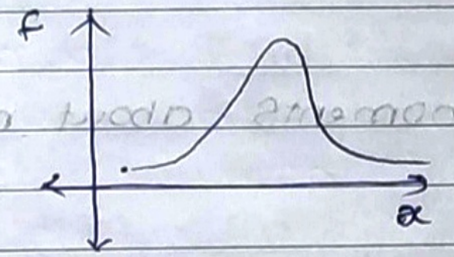
* ~~Skewness:~~

Note: i) $\mu_1' = \frac{\Sigma f_i(x_i - A)}{\Sigma f_i}$

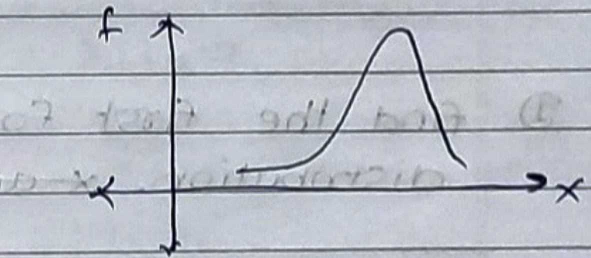
$\mu_1' = \bar{x} - A$ $\quad\therefore \boxed{\bar{x} = \mu_1' + A}$

ii) $V(x) = \mu_2 = \sigma = \sqrt{\mu_2}$

* Skewness: It signifies departure from symmetry.



+vely skewed



-vely skewed

coefficient of skewness $= \beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3}$

If β_1 is +ve $\rightarrow$

If β_1 is -ve $\rightarrow$

* Kurtosis:- To get the complete idea of distribution

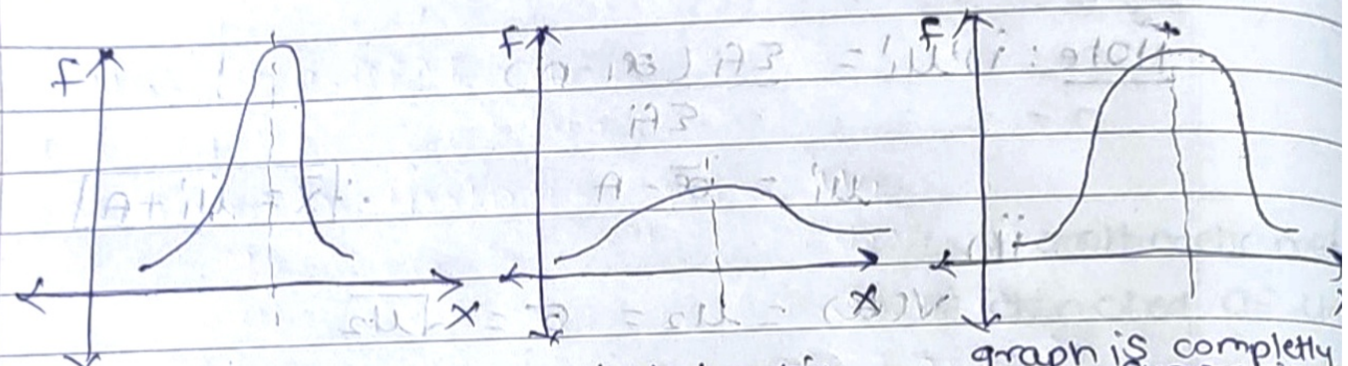
- To get the graph nature either it is symmetrical

Coefficient of kurtosis $= \beta_2 = \dfrac{\mu_4}{(\mu_2)^2}$

If $\beta_2 > 3$ then the distribution is called leptokurtic

If $\beta_2 < 3$ then the distribution is called platykurtic.

If $\beta_2 = 3$ then the distribution is called mesokurtic.



leptokurtic $\beta_2 > 3$ | platykurtic $\beta_2 < 3$ | graph is completely symmetric, mesokurtic $\beta_2 = 3$

① find the first four moments about mean for the distribution.

x_i	1	2	3	4	5	6	7	8	9
f_i	1	6	13	25	30	22	15	10	5

Also find coefficient of skewness and kurtosis.

$\Rightarrow$ $\mu_0 = 1$,

$$\mu_1' = \frac{\sum f_i(x_i - A)}{\sum f_i} = \frac{\sum f_i d_i}{\sum f_i}$$

$$\mu_2' = \frac{\sum f_i d_i^2}{\sum f_i}$$

$$\mu_3' = \frac{\sum f_i d_i^3}{\sum f_i} \qquad \mu_4' = \frac{\sum f_i d_i^4}{\sum f_i}$$

$A = 5$

x	f_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$	$f_i d_i^3$	$f_i d_i^4$
1	1	-4	-4	16	-64	+256
2	6	-3	-18	54	-162	+486
3	13	-2	-26	52	-104	208
4	25	-1	-25	25	-25	25
5	30	0	0	0	0	0
6	22	1	22	22	22	22
7	15	2	30	60	120	240
8	10	3	30	90	270	810
9	5	4	20	80	320	1280
45	127		29	399	377	2047 3327

i) All four central moments:

$\mu_0' = 1$

$$\mu_1' = \frac{\Sigma f_i d_i}{\Sigma f_i} = \frac{29}{127} = 0.2283$$

$$\mu_2' = \frac{\Sigma f_i d_i^2}{\Sigma f_i} = \frac{399}{127} = 3.1417$$

$$\mu_3' = \frac{\Sigma f_i d_i^3}{\Sigma f_i} = \frac{377}{127} = 2.9685$$

$$\mu_4' = \frac{\Sigma f_i d_i^4}{\Sigma f_i} = \frac{3327}{127} = 26.1968$$

Using relation between μ_r and μ_r'

$\mu_0 = 1$

$\mu_1 = 0$

$$\mu_2 = \mu_2' - (\mu_1')^2$$
$$= 3.1417 - (0.2283)^2$$
$$\mu_2 = 3.08957$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$= 2.9685 - 3(3.1417 \times 0.2283) + 2(0.2283)^3$$

$$\mu_3 = 0.8432$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

$$\mu_4 = 26.1968 - 4(2.9685 \times 0.2283) + 6(3.1417) \times (0.2283)^2 - 3(0.2283)^4$$

$$\mu_4 = 24.4613$$

coefficient of kurtosis $= \beta_2 = \dfrac{\mu_4}{(\mu_2)^2}$

$$= \frac{24.4613}{(3.0895)^2}$$

$$\beta_2 = 25623$$

coefficient of skewness

$$= \beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3}$$

$$= \frac{(0.8405)^2}{(3.0897)^3} = 0.0239$$

② find 1st four central moments.

x_i	0	1	2	3	4	5	6	7	8
f_i	1	8	28	156	170	56	28	8	1

Also find β_1 & β_2.

→

$A = 4$

x_i	f_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$	$f_i d_i^3$	$f_i d_i^4$
0	1	-4	-4	16	-64	256
1	8	-3	-24	72	-216	648
2	28	-2	-56	112	-224	448
3	156	-1	-156	156	-156	156
A = 4	170	0	0	0	0	0
5	56	1	56	56	56	56
6	28	2	56	112	224	448
7	8	3	24	72	216	648
8		4	4	16	64	256
Total	456		-100	612	-100	2916

$\mu_0' = 1$, $\mu_1' = \frac{\Sigma f_i d_i}{\Sigma f_i} = \frac{-100}{456}$

$= -0.2192$

$\mu_2' = \frac{\Sigma f_i d_i^2}{\Sigma f_i} = \frac{612}{456} = 1.34210$

$\mu_3' = \frac{\Sigma f_i d_i^3}{\Sigma f_i} = \frac{-100}{456} = -0.2192$

$\mu_4' = \frac{\Sigma f_i d_i^4}{\Sigma f_i} = \frac{2916}{456} = 6.3947$

$\boxed{\mu_0 = 1}$, $\boxed{\mu_1 = 0}$

$\mu_2 = \mu_2' - (\mu_1')^2 = 1.3421 - (-0.2192)^2$

$\boxed{\mu_2 = 1.2940}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^2$

$= -0.2192 - 3(1.34210 \times -0.2192) + 2(-0.2192)^2$

$\boxed{\mu_3 = 0.7594}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 6.3947 - 4(0.2192)^2 + 6(1.3421 \times (0.2192)^2)$
$- 3(0.2192)^4$

$\boxed{\mu_4 = 6.5824}$

$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = \frac{(0.7594)^2}{(1.2940)^3} = 0.2661$

$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{(6.5824)}{(1.2940)^2} = 3.93114$

③ find first four central moments.
If moment about value (μ_r') five are -
7, 70, 140, 175. Also find coefficient of skewness and kurtosis.

→ $A = 5$

$\mu_1' = 7,\ \mu_2' = 70,\ \mu_3' = 140,\ \mu_4' = 175$

$\mu_0 = 1,\ \mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2$
$= 70 - (7)^2$
$= 70 - 49$
$\mu_2 = 21$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1)^3$
$= 140 - 3(140 \times 70) + 2(7)^3$
$= 175 - (3(140 \times 70) + 2(7)^3)$

$\mu_3 = -644$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 175 - 4(140 \times 7) + 6(170 \times (7)^2 - 3(7)^4$

$\mu_4 = 9632$

coefficient of kurtosis $= \beta_2 = \frac{\mu_4}{(\mu_2)^2}$

$= \frac{9632}{(21)^2}$

$\beta_2 > 3$ leptokurtic. $\beta_2 = 21.8412$

coefficient of skewness $= \beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3}$

$= \frac{(644)^2}{(21)^3}$

$\beta_1 = 44.78$

Mean $= \bar{x} = \mu_1' + A$

$= 5 + 7$

$\bar{x} = 12$

Standard deviation $= \sigma = \sqrt{\mu_2}$

$= \sqrt{21}$

$\sigma = 4.58$

variance $= \sigma^2 = \mu_2 = 21$

(4) The first four moment about origin are 1, 4, 10, 46 respectively. find 1st four moments about mean and also find β_1 and β_2, $\bar{x}$ and S.D.

→ first four moment about origin are 1, 4, 10, 46

$\therefore A = 0, \; \mu_1' = 1, \; \mu_2' = 4, \; \mu_3' = 10, \; \mu_4' = 46$

$\mu_0 = 1, \; \mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 4 - 1 = 3$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= 10 - 3(4 \times 1) + 2(1)^3$

$= 10 - 12 + 2$

$= -2 + 2$

$\mu_3 = 0$

$\mu_4 = \mu_4' - 4\mu_3\mu_1 + 6\mu_2\mu_1^2 - 3(\mu_1)^4$

$= 46 - 4(10 \times 1) + 6(4)(1)^2 - 3(1)^4$

$= 46 - 40 + 24 - 3$

$= 6 + 24 - 3$

$\mu_4 = 27$

Coefficient of skewness $= \beta_1 = \dfrac{(\mu_3)^2}{(\mu_2)^3}$

$= \dfrac{(0)^2}{(3)^3}$

$\therefore$ ~~$\beta_1 = 0$ so its mesokurtic.~~ $\beta_1 = 0$ its symmetric distribution $\beta_1 = 0$

Coefficient of kurtosis $= \beta_2 = \dfrac{\mu_4}{(\mu_2)^2}$

$= \dfrac{27}{(3)^2}$

$= \dfrac{27}{9}$

$\beta_2 = 3$

$\beta_2 = 3$ so its mesokurtic.

mean $= \bar{x} = \mu_1' + A$

$= 1 + 0$

$= 1$

S.D $= \sigma = \sqrt{\mu_2} = \sqrt{3}$

$\sigma = 1.732$

(4) The first four moment about value 2 are 1, 2.5, 5.5, & 16 resp. find first four moment about mean (about value - A) and also find $\beta_1, \beta_2, \bar{x}$, and S.D

→ $A = 2$, $\mu_1' = 1$, $\mu_2' = 2.5$, $\mu_3' = 5.5$, $\mu_4' = 16$

$\mu_0 = 1$, $\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 2.5 - (1)^2 = 2.5 - 1 = 1.5$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= 5.5 - 3(2.5 \times 1) + 2(1)^3$

$= 5.5 - 7.5 + 2$

$= 0$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 16 - 4(5.5 \times 1) + 6(2.5 \times (1)^2) - 3(1)^4$

$= 16 - 22.0 + 15.0 - 3$

$\mu_4 = 6$

coefficient of skewness $= \beta_1 = \dfrac{(\mu_3)^2}{(\mu_2)^3}$

$\beta_1 = \dfrac{(0)^2}{(1.5)^3} = 0$

coefficient of curtosis $= \beta_2 = \dfrac{\mu_4}{(\mu_2)^2} = \dfrac{6}{(1.5)^2} = 2.666$

$\beta_2 < 3$. so it is platykurtic.

mean $= \bar{x} = \mu_1' + A = 1 + 2 = 3$

standard deviation $= \sigma = \sqrt{\mu_2}$

$= \sqrt{1.5}$

$\boxed{\sigma = 1.2247}$

Random Variable & distribution function (1)

* Random variable :-
- It is real valued function defined on sample space.

$X : S \rightarrow R$

X from sample space to real.

Note: A random variable may be finite or countably infinite or uncountably infinite.

ex:-
X - No. of heads

* Two types of random variable:-

(1) Discrete Random Variable:-
A random variable X takes finite or countably infinite values then it is called discrete random variable.

ex:- 1) No. of children's in family.
2) No. of stars in sky.
~~3) c~~

(2) continious Random variable
If a random variable X takes uncountably infinite values it is called continious random variable.
ex:- The weight of person.

* Probability mass function (PMF):-
- If discrete random variable X takes value $x_1, x_2, \ldots, x_n$
and corresponding probabilities are $P_1, P_2 \ldots$

$x_1, x_2, \ldots x_n$

$p_1, p_2, \ldots p_n$

is called probobility mass function, if it satisfies the two condition:

i) $0 \leq p_i \leq 1$ $\quad i = 1, 2 \ldots n$

ii) $\sum_{i=1}^{n} p_i = 1$

Properties of pmf :-

① Mean or expected value (μ) or (Ex)

$$\mu = \frac{\sum x_i p_i}{\sum p_i}$$

if it is pmf then $\sum p_i = 1$

$$\boxed{E(x) = \mu = \sum_{i=1}^{n} x_i p_i}$$

② variance = $\boxed{V(x) = E(x^2) - [E(x)]^2}$

$$\boxed{E(x^2) = \sum x_i^2 p_i}$$

③ commulative distribution function :- (cdf)

- denoted by $F(x)$

$$\boxed{F(x) = p(x \leq x_i)}$$

* Probability density function :-

If X is continious random variable and it takes all values in some interval (a, b) and it satisfies . .

i) $f(x) \geq 0$ for all $x \in (a,b)$

ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

<u>Note</u>: ① $P(a < x < b) = \int_a^b f(x)\, dx$

② Mean or expected value

$$\mu = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

① Obtain distribution function of total number of heads occuring in three tosses of and unbaised coin. & find expected value.

→ X - No. of heads.

X = 0, 1, 2, 3

X	0	1	2	3
p(x)	1/8	3/8	3/8	1/8

S = { HHH, HHT, HTH, HTT, THH, THT
TTH, TTT }

$$E(x) = \sum_{i=0}^{3} p_i x_i \quad \left(\sum_{i=0}^{3} x_i\, P(x = x_i)\right)$$

$$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$$

$$\frac{6}{8} + \frac{6}{8}$$

$$E(x) = \frac{12}{8} = 1.5$$

$= 0 \times P(x=0) + 1 \times P(x=1) + 2 \times P(x=2) + 3 \times P(x=3)$

(2) obtain the distribution function of sum on the uppermost face when two dice are rolled.

→ X = sum on uppermost face.

S = {(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

X = 2, 3, 4 ... 12

X	2	3	4	5	6	7	8	9	10	11	12
p(x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Expected value = $\mu = E(x) = \sum_{i=2}^{12} x_i P(x = x_i)$

$= 2\times\frac{1}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$

$= \frac{252}{36}$

$\boxed{E(x) = 7}$

(2) Variance :- $E(x^2) - [E(x)]^2$

$=$

$= \left[\frac{4}{36} + \frac{18}{36} + \frac{48}{36} + \frac{100}{36} + \frac{180}{36} + \frac{343}{36} + \frac{320}{36} + \frac{324}{36} + \frac{300}{36} + \frac{242}{36} + \frac{144}{36}\right] - [7]^2$

$$\text{Variance} = 5\cancel{6.8972}4 - 49$$

$$\cancel{V(x) = 7.1944}$$

$$V(x) =$$

$$\therefore \boxed{V(x) = 5.8333}$$

$\therefore$ variance is 5.833

③ The probability distribution of x is as follows.

X	0	1	2	3	4
$P(x)$	0.1	k	$2k$	$2k$	k

is a probability mass function then find:

i) The value of k

ii) $P(x < 2)$

iii) $P(x \geq 3)$

iv) $P(1 \leq x \leq 3)$

$\Rightarrow$ X is probability distribution function.

$\Sigma pmf = 1$

i) $\therefore$ According to condition,

$$\sum_{i=0}^{n} P(x = x_i) = 1$$

$$0.1 + k + 2k + 2k + k = 1$$

$$0.1 + 6k = 1$$

$$6k = 1 - 0.1$$

$$6k = 0.9$$

$$k = \frac{0.9}{6}$$

$$\boxed{k = 0.15}$$

ii)

X	0	1	2	3	4
$p(x)$	0.1	0.15	0.3	0.3	0.15

$p(x<2) = p(x=0) + p(x=1)$

$= 0.1 + 0.15$

$p(x<2) = 0.25$

iii) $p(x \geq 3) = p(x=3) + p(x=4)$

$= 0.3 + 0.15$

$= 0.45$

iv) $p(1 \leq x \leq 3) = p(x=1) + p(x=2) + p(x=3)$

$= 0.15 + 0.3 + 0.3$

$= 0.15 + 0.6$

$= 0.75$

(4) Let the random variable X takes values $-2, -1, 0, 1, 2$ and $p(x=-2) = p(x=-1) = p(x=1) = p(x=2)$;

$p(x<0) = p(x=0) = p(x>0)$. Determine pmf of X and also find variance.

→ Let X be random variable. It takes values $-2, -1, 0, 1, 2$.

$p(x=-2) = p(x=-1) = p(x=1) = p(x=2) = k$

$\sum p(x=x_i) = 1$

$p(x=-2) + p(x=-1) + p(x=0) + p(x=1) + p(x=2) = 1$

$p(x=0) + 4k = 1$

$p(x=0) = 1-4k$ —— (1)

Also, $p(x<0) = p(x=0)$

$p(x=0) = p(x=-1) + p(x=-2)$

$p(x=0) = 2k$ —— (2)

from (I) and (II)

$2k = 1-4k$

$6k = 1$

$k = \frac{1}{6}$

$p(x=0) = 1-4k$

$= 1-4\left(\frac{1}{6}\right)$

$= 2/3$

pmf:

x_i	-2	-1	0	1	2
$p(x=x_i)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

variance $= E(x^2) - [E(x)]^2$

$= \frac{4}{6} + \frac{1}{6} + \frac{1}{6} + \frac{4}{6} - \left[\frac{-\cancel{2}}{\cancel{6}} - \frac{\cancel{1}}{\cancel{6}} + \frac{\cancel{1}}{\cancel{6}} + \frac{\cancel{2}}{\cancel{6}}\right]$

$= \frac{8}{6} + \frac{2}{6} = \frac{10}{6}$

$\boxed{V(x) = 5/3}$

$\therefore$ variance is 1.66

⑤ The probability density function

$f(x) = \lambda x e^{-x}$, $x > 0$. find i) The value of λ

ii) $p(2 < x < 5)$

iii) $E(x)$

→ $f(x) = \lambda x e^{-x}$, $x > 0$

Using properties of probability distribution function.

$\int_{-\infty}^{\infty} f(x) = 1$

$$\int_0^\infty \lambda x \cdot e^{-x}\, dx = 1$$

$$\lim_{x\to 0}\left[\frac{x}{e^x}\right]$$

$$\lim_{x\to 0}\left[\frac{1}{e^x}\right] = \frac{1}{\infty} = 0$$

$$\int u \cdot v = u \cdot \int v\,dv - \int\left[\frac{du}{dx}\int v\,dv\right] dx$$

$$1 = \lambda\left[x \cdot \frac{e^{x}}{-1} + e^{x}\right]_0^\infty$$

$$1 = \lambda\left[(0 \mp 0) - (0 + -1)\right]$$

$$1 = \lambda(1)$$

$$\boxed{\lambda = 1}$$

ii) $$p(2 < x < 5) = \int_2^5 f(x)\,.dx$$

$$= \int_2^5 x \cdot e^{-x}\, dx$$

$$= \int_2^5 \left[x \cdot \frac{e^{-x}}{-1} + e^{-x}\right]_2^5$$

$$= \left[5 \cdot \frac{e^{-5}}{-1} + e^{-5}\right] - \left[2 \cdot \frac{e^{2}}{-1} + e^{-2}\right]$$

$$= \left[-5 \times e^{-5} + e^{-5}\right] + \cancel{2}e^{-2} - 3e^{-2}$$

$$= -6e^{-5} - 3e^{-2}$$

$$= 0.3655$$

iii) $E(x) = \int_0^\infty x \, f(x) \, dx$

$= \int_0^\infty x \cdot x \cdot e^{-x} \, dx$

$= \int_0^\infty x^2 e^{-x} \, dx$

$= \left[x^2 \frac{e^{-x}}{-1} - \int \left[2x \cdot \frac{e^{-x}}{-1} \right] dx \right]_0^\infty$

$= \left[-x^2 e^{-x} + 2 \left[x \cdot \frac{e^{-x}}{-1} - \frac{e^{-x}}{-1 \times -1} \right] \right]_0^\infty$

$= \left[-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} \right]_0^\infty$

$= \left[-\infty^2 e^{-\infty} - 2\infty e^{-\infty} - 2 e^{-\infty} \right] - [-2]$

$E(x) = +2$

$\int x^2 \cdot e^{-x} \, dx$

$\int u.v. = u v_1 + u' v_2 + u'' v_3 .$

$= x^2 \frac{e^{-x}}{-1} + 2x \cdot -1 \times \frac{e^{-x}}{-1} + 2 \times -e^{-x}$

$= \left[-x^2 e^{-x} + 2x \, e^{-x} + -2e^{-x} \right]_0^\infty$

$= [0] - [-2]$

$= 2$

① The diameter x of a electric cable is assume to be continious random variable with pdf is given by such that,

$f(x) = 6x(1-x) \quad 0 \le x \le 1$

i) check that above is pdf

ii) obtain the expression for distributive function $F(x)$.

iii) Determine the number of k such that

$p(x < k) = p(x > k)$,

→ Given : x be diameter of electric cable.

i) $f(x) = 6x(1-x)$

$0 \le x \le 1$

$$\int_0^1 f(x)\,dx = \int_0^1 6x(1-x)\,dx$$

$$= \int_0^1 (6x - 6x^2)\,dx$$

$$= \left[\frac{6x^2}{2} - \frac{6x^3}{3}\right]_0^1$$

$$= \left[\frac{6}{2} - \frac{6}{3}\right] - \left[0 - 0\right]$$

$$= 3 - 2$$

$$\int_0^1 f(x)\,dx = 1$$

∴ The f(x) is an p.d.f

As. $f(x) \ge 0$ for all $0 \le x \le 1$

so f(x) is an p.d.f.

ii) distributive function = $F(x)$ = c.d.f.

$F(x) = p(x \le x_i)$

$$= \int_0^x f(x)\,dx$$

$$= \int_0^x (6x - 6x^2)\,dx$$

$$= \left[\frac{6x^2}{2} - \frac{6x^3}{3}\right]_0^x$$

$$F(x) = 3x^2 - 2x^3$$

$\therefore$ Distribution function $= F(x) = 3x^2 - 2x^3$

iii) $p(x < k) = p(x > k)$

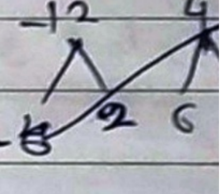
$$\int_0^k f(x)\,dx = \int_k^1 f(x)\,dx$$

$$\int_0^k (6x - 6x^2)\,dx = \int_k^1 (6x - 6x^2)\,dx$$

$$[3x^2 - 2x^3]_0^k = [3x^2 - 2x^3]_k^1$$

$$3k^2 - 2k^3 = [3 - 1] - [3k^2 - 2k^3]$$

$$3k^2 - 2k3 = 2 - 3k^2 + 2k3$$



$$- 6k3 - 4k3 - 2 = 0$$

$$6k3 - 6k + 2k - 2 = 0$$

~~6k~~

$$6k(k+1) - 2(k+k) = 0$$

$\therefore \boxed{k = -1}$ and $6k = 2$

$$k = \frac{2}{6}$$

~~6k(k-1)~~

$$4k^3 - 6k^2 + 1 = 0 \qquad k = \frac{1}{3}$$

$$4k^3 - 6k^2 + 1 = 0 \qquad \boxed{\cancel{k = 0.33}}$$

$$k = \frac{1 \pm \sqrt{3}}{2} \text{ and } k = \frac{1}{2}$$

② find the value k if p.d.f. $f(x) = \frac{k}{1+x^2}$; $-\infty < x < \infty$

Also find, $P(0 \le x \le 1)$,

$P(\frac{1}{\sqrt{3}} \le x \le \sqrt{3})$.

$\Rightarrow$

$f(x) = \frac{k}{1+x^2}$, $-\infty < x < \infty$

$\int_{-\infty}^{\infty} f(x) . dx = 1$

$\int_{-\infty}^{\infty} \frac{k}{1+x^2} dx = 1$

$\int_{-a}^{a} f(x) . dx = 2 \int_{0}^{a} f(x) dx$

when $f(x)$ is even function

$2k \int_{0}^{\infty} \frac{1}{1+x^2} = 1$

$f(-x) = f(x) =$ even

$f(-x) = -f(x) =$ odd

for odd

$\int_{-a}^{a} f(x) . dx = 0$ when $f(x)$ is odd function

$2k \left[\tan^{-1} x\right]_{0}^{\infty} = 1$

$2k \left[\tan^{-1}\infty - \tan^{-1}(0)\right] = 1$

$2k \left[\frac{\pi}{2} - 0\right] = 1$

$\pi k = 1$

$k = \frac{1}{\pi}$

i) $P(0 \le x \le 1) = \int_{0}^{1} f(x). dx$

$= \int_{0}^{1} \frac{k}{1+x^2} . dx$

$= \frac{1}{\pi} \left[\tan^{-1} x\right]_{0}^{1}$

$= \frac{1}{\pi} \left[\frac{\pi}{4} - 0\right]$

$$\boxed{P(0 \le x \le 1) = \frac{1}{4}}$$

ii) $P\left(\frac{1}{\sqrt{3}} \le x \le \sqrt{3}\right) = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} f(x).\,dx$

$$= \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \left[\frac{k}{1+x^2}\right] dx$$

$$= \frac{1}{\pi}\left[\tan^{-1} x\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$$

$$= \frac{1}{\pi}\left[\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right]$$

$$= \frac{1}{\pi}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \frac{1}{\pi} \cdot \frac{3\pi}{18}$$

$$\boxed{P\left(\frac{1}{\sqrt{3}} \le x \le \sqrt{3}\right) = \frac{1}{6}}$$

③ X is continious random variable with p.d.f

$$f(x) = \begin{cases} kx & 0 \le x \le 2 \\ 2k & 2 < x \le 4 \\ -kx + 6k & 4 < x \le 6 \end{cases}$$

find the value of k & mean.

→ $f(x) = kx$

$$\int_0^{6} f(x) = 1$$

$$1 = \int_0^2 kx.\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx+6k)\,dx$$

$$\left[\frac{kx^2}{2}\right]_0^2 + \left[2kx\right]_2^4 + \left[-\frac{kx^2}{2} + 6k\right]_4^6 = 1$$

$$\frac{4k}{2} + 2k - \frac{36k}{2} + 6k + \frac{16k}{2} - 6k = 1$$

$$4k - \frac{20k}{2} = 1$$

$$4k - 10k = 1$$

$$\cancel{4k = 1 - 10k}$$

$$-6k = 1$$

$$k = -1/6$$

$$\frac{4k^2}{2} + 2k(2) + \left[-\frac{36k}{2} + 36k + \frac{16k}{2} - 24k\right] = 1$$

$$2k^2 + 4k + 2k = 1$$

$$8k = 1$$

$$\boxed{k = 1/8}$$

$$\text{Mean} = E(x) = \int_0^6 x \cdot f(x)\, dx = 1$$

$$= \int_0^2 kx^2\, dx + \int_2^4 2kx\, dx + \int_4^6 (-kx^2 + 6kx)\, dx$$

$$= \left[\frac{kx^3}{3}\right]_0^2 + \left[2k\frac{x^2}{2}\right]_2^4 + \left[-\frac{kx^3}{3} + 6\frac{kx^2}{2}\right]_4^6$$

$$= \left[\frac{1}{8} \times \frac{x^3}{3}\right]_0^2 + \frac{1}{8}\left[12\right] + \frac{1}{8}\left[-\frac{36}{3} +\right.$$