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- The least squares method is based on the criterion of minimizing mean squared error.
- This is a good way to fit linear models to given data.
- As mentioned in the previous lecture, Fisher Linear Discriminant is another way of constructing linear classifiers.

Fisher Linear Discriminant

- A linear discriminant function based classifier is:

  Decide $X \in$ C-1 if $W^T X + w_0 > 0$

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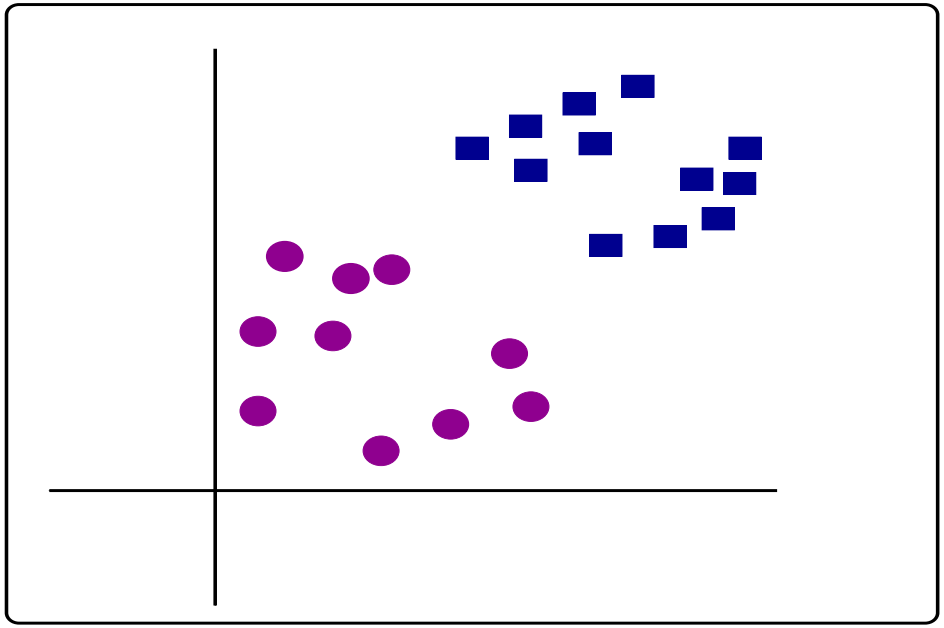
- We project the data along the direction W. Separation between points of different classes in the projected data is a good way to rate how good is W.
- Such a method is called Fisher Linear Discriminant.

- Consider the following 2-class example



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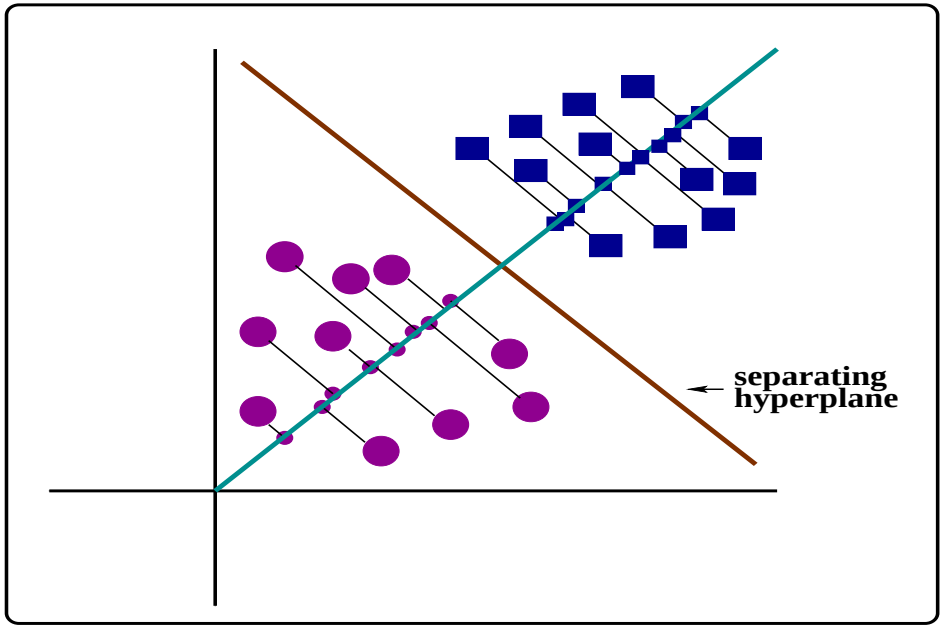

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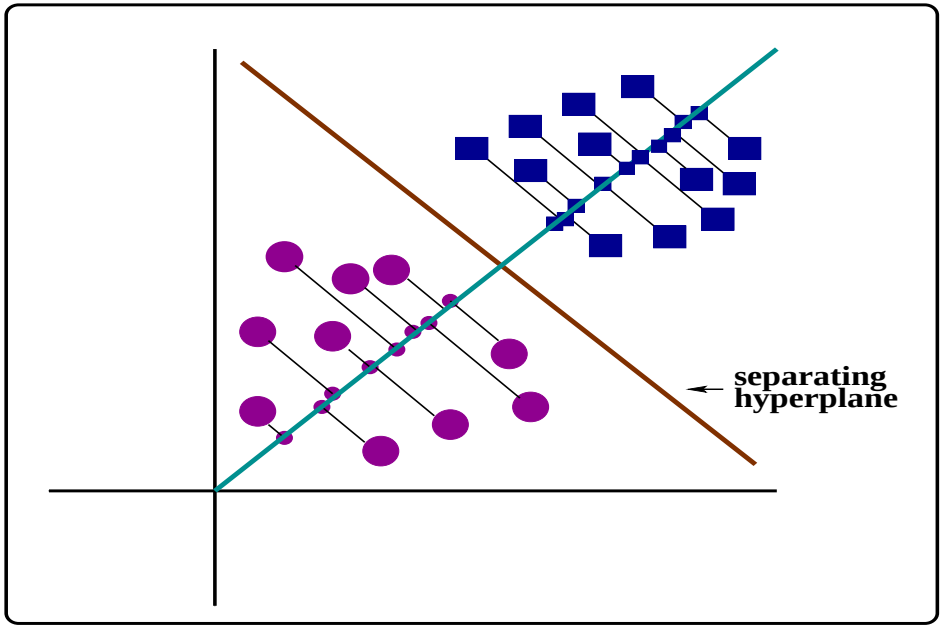

- Fisher Linear Discriminant is based on formalizing this notion

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- The idea is to find a direction W such that the training data of the two classes are well-separated if projected onto this direction.
- We need some figure of merit for each W to characterize how well the W results in such a separation.
- This is what we do now. We first consider the 2-class case.

- Let $\{(X_i,\ y_i),\ i = 1, \cdots, n\}$ be the data.

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- Let n_0 and n_1 denote the number of examples of each class. ($n = n_0 + n_1$)
- For any W, let $z_i = W^T X_i$.
- z_i are the one dimensional data that we get after projection.

- Let M_0 and M_1 be the means of data from the two classes:

$$M_0 = \frac{1}{n_0} \sum_{X_i \in C_0} X_i; \quad M_1 = \frac{1}{n_1} \sum_{X_i \in C_1} X_i$$

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The corresponding means of the projected data would be

$$m_0 = W^T M_0 \quad \text{and} \quad m_1 = W^T M_1$$

- The difference $(m_0 - m_1)$ gives us an idea of the separation between samples of the two classes after projecting the data onto the direction W.

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- Hence, we may want a W that maximizes $(m_0 - m_1)^2$.
- However, we have to make this scale independent.
- Also, the distance between means should be viewed relative to the variances.

- Define

$$s_0^2 = \sum_{X_i \in C_0} (W^T X_i - m_0)^2; \quad s_1^2 = \sum_{X_i \in C_1} (W^T X_i - m_1)^2$$

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  These give us the variances (upto a factor) of the two classes in the projected data.

- We want large separation between m_0 and m_1 relative to the variances.

- Hence we can take our objective to be to maximize

$$J(W) = \frac{(m_1 - m_0)^2}{s_0^2 + s_1^2}$$

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$$\begin{aligned}(m_1 - m_0)^2 &= (W^T M_1 - W^T M_0)^2 \\ &= W^T (M_1 - M_0)(M_1 - M_0)^T W\end{aligned}$$

- Thus we have $(m_1 - m_0)^2 = W^T S_B W$ where

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- It is called *between class* scatter matrix.
- We can similarly write s_0^2 and s_1^2 also as quadratic forms.

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- S_w is also $d \times d$ matrix and is called *within class* scatter matrix.

- Hence we can now write J as

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- Maximizing ratio of quadratic forms is a standard optimization problem.

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- Implies, $S_B W$ is in the same direction as $S_w W$.

- Thus, any maximizer of $J(W)$ has to satisfy

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- By solving the generalized eigen value problem we can find the best direction W.

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- This is a sum of large number of rank 1 matrices.
- Also each term in S_w is proportional to the sample-mean-estimate of the covariance matrix of one of the class conditional densities.

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- Now we get (since scale factor in W is not relevant)

$$W \;=\; S_w^{-1}(M_1 - M_0)$$

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- Even if S_w is not invertible, there are techniques to find the maximizer of $J(W)$ by solving the generalized eigen value problem.
- Thus we can find the best direction W.

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- Or, we can take the one dimensional projected data and learn the best classifier by, e.g., modelling the class conditional densities as normal.
- This gives us the final Fisher Linear Discriminant (FLD) classifier.

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- Though the method looks quite different from that of linear least squares there are close connections between the two.
- We explain this connection next.

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- We now treat this as a data for a regression problem and learn a model $\hat{y} = W^T X + b$ using linear least squares.
- It can be shown that the least squares solution for W would be same as that of FLD.
- Thus FLD can be viewed as a special case of linear least squares.

- Consider a two class problem with class conditional densities to be normal with same covariance matrix.

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  Let $\hat{\Sigma}$ be the sample mean estimate for covariance matrix.

- Then the Bayes classifier, implemented with estimated class conditional densities, would be a linear classifier sign$(W^T X + b)$ where W is given by

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- It is easily seen that this would be essentially the same as the W given by FLD.

- Recall that the matrix S_w is defined by

$$S_w = \sum_{X_i \in C_0} (X_i - M_0)(X_i - M_0)^T + \sum_{X_i \in C_1} (X_i - M_1)(X_i - M_1)^T$$

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- Thus, S_w is proportional to sample mean estimator of covariance matrix.
- Thus, the FLD given by $W = S_w^{-1}(M_1 - M_0)$ would be essentially same as the Bayes optimal classifier.

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- In the case of regression, we have considered only estimating real-valued functions. We can generalize this to vector-valued functions.
- In classification we considered only 2-class problems. This can also be generalized to multi-class case.

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- We can obtain these by simply solving m number of linear least squares regression problems.

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- But neither of these approaches are really satisfactory for generalizing linear discriminant functions.

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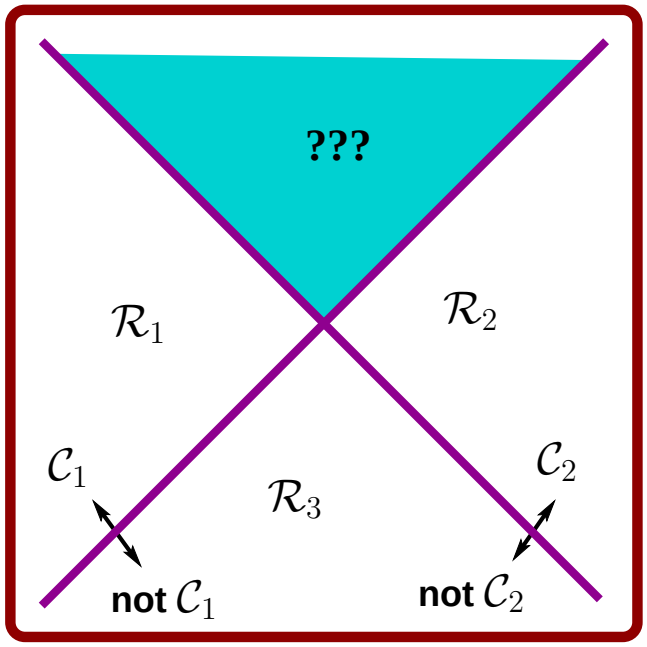

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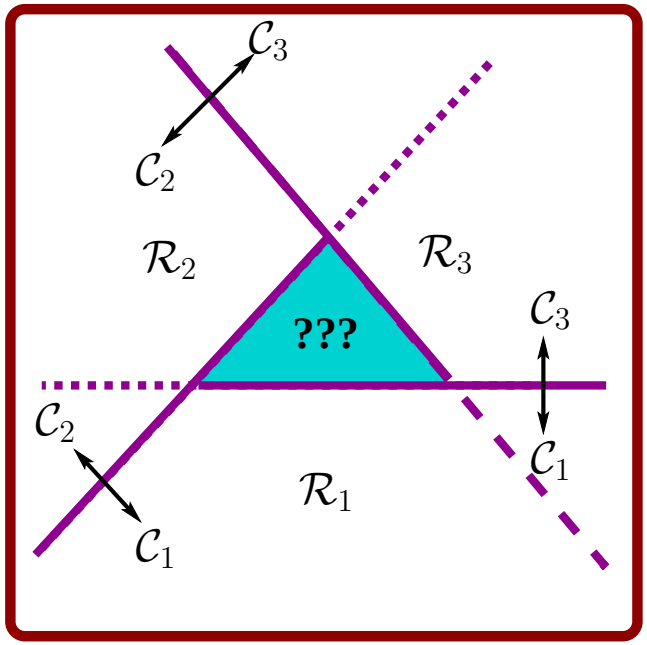

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- Recall that this is the way we generalized Bayes classifier to multi-class case.

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- We can similarly generalize logistic regression and FLD also for the K-class case.