Acharya Narendra Dev College

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College Roll.No: AC-721

Course:-B.Sc.(Hons).Mathematics

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Subject:-Differential Equations (Lab Practical)

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SN Practical Name

1) Plotting of second and third order respective solution family of differential equation.

2) Growth and decay model

(exponential case only)
3)(i) Lake pollution model (with constant/seasonal flow and pollution concentration).
(ii) Case of single cold pill and a course of cold pills.
(iii) Limited growth of population (with and without harvesting).
4) (i) Predatory-prey model (basic Volterra model, with density dependence, effect of DDT, two prey one predator).
(ii) Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).

(iii) Battle model (basic battle model, jungle warfare, long range weapons).

5). Plotting of recursive sequences, and study of the convergence.

6)Find a value m ∊ N that will make the following inequality holds for all n>m :

7) Verify the Bolzano–Weierstrass theorem through plotting of sequences and hence identify convergent subsequences from the plot.

8) Study the convergence/divergence of infinite series of real numbers by plotting their sequences of partial

sum.

9) Cauchy’s root test by plotting nth roots.

10) D’Alembert’s ratio test by plotting the ratio of nth and (n+1)th term of the given series of Positive terms.

11)For the following sequence <a_n> ,ϵ=1/2^k,p=10^j,k=0,1,2,4..

find m ϵ N such that:-

For the following sequence <a_n> ,Given pϵN find mϵN Such that (i)$|a_{m+p}-a_m|<\epsilon$,(ii)$|a_{2m+p}-a_{2m}|<\epsilon$

12)For the following series $\sum_{\square=\square}^{\square} a_n$,

calculate i)$\left|\frac{a_{n+1}}{a_n}\right|$,

ii)$(| a_n |)^{\frac{1}{n}}$, for n=10^j,j=1,2,3,.......
and identify the convergent series,
where n a is given as:

1Q.)Plotting of second and third order respective solution family of differential equation.(Solve a differential for equation for the function y with independent variable x).

Plotting of second first differential equation

In[ ]:=
```
DSolve[{2 y'[t] + y[t] == 0}, y[t], t]
```

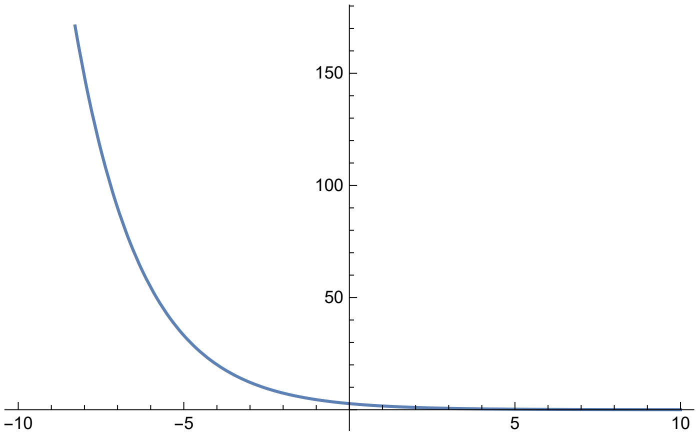
$\{\{y[t] \to e^{-t/2} c_1\}\}$

In[ ]:=
```
dpp = DSolve[{2 y'[t] + y[t] == 0, y[2] == 1}, y[t], t]
```

$\{\{y[t] \to e^{1-\frac{t}{2}}\}\}$

In[ ]:=
```
Plot[y[t] /. dpp, {t, -10, 10}]
```

Out[ ]=



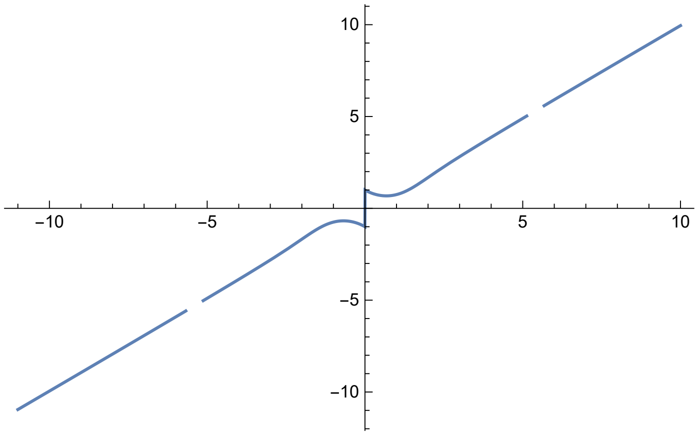
In[ ]:=
```
aas = DSolve[{ y'[x] == -y[x]^2 + x^2, y[0] == 1}, y[x], x]
```

Out[ ]=

$$\Big\{\Big\{y[x] \to \Big(\Big(\frac{1}{2} + \frac{i}{2}\Big)\Big((1+i)\, x^2\, \text{BesselJ}\Big[-\frac{3}{4}, \frac{i x^2}{2}\Big]\, \text{Gamma}\Big[\frac{1}{4}\Big] + i \sqrt{2}\, x^2\, \text{BesselJ}\Big[-\frac{5}{4}, \frac{i x^2}{2}\Big]\, \text{Gamma}\Big[\frac{3}{4}\Big] + \sqrt{2}\, \text{BesselJ}\Big[-\frac{1}{4}, \frac{i x^2}{2}\Big]\, \text{Gamma}\Big[\frac{3}{4}\Big] - i \sqrt{2}\, x^2\, \text{BesselJ}\Big[\frac{3}{4}, \frac{i x^2}{2}\Big]\, \text{Gamma}\Big[\frac{3}{4}\Big]\Big)\Big) \Big/ \Big(x \Big(\text{BesselJ}\Big[\frac{1}{4}, \frac{i x^2}{2}\Big]\, \text{Gamma}\Big[\frac{1}{4}\Big] + (1+i) \sqrt{2}\, \text{BesselJ}\Big[-\frac{1}{4}, \frac{i x^2}{2}\Big]\, \text{Gamma}\Big[\frac{3}{4}\Big]\Big)\Big)\Big\}\Big\}$$

In[∘]:=
```
Plot[y[x] /. aas, {x, -11, 10}]
```

Out[∘]=



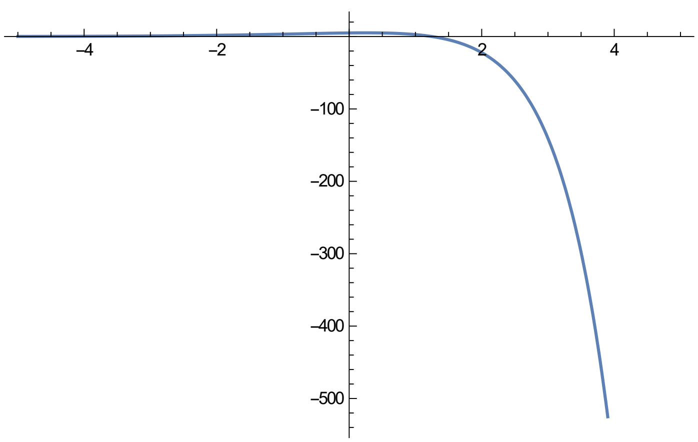
Second Differential equation

In[∘]:=
```
zs = DSolve[{y''[x] - 2 y'[x] + y[x] == 0, y[0] == 5, y'[0] == 1}, y[x], x]
```

Out[∘]=
```
{{y[x] → -ⅇ^x (-5 + 4 x)}}
```

In[∘]:=
```
Plot[y[x] /. zs, {x, -5, 5}]
```

Out[∘]=

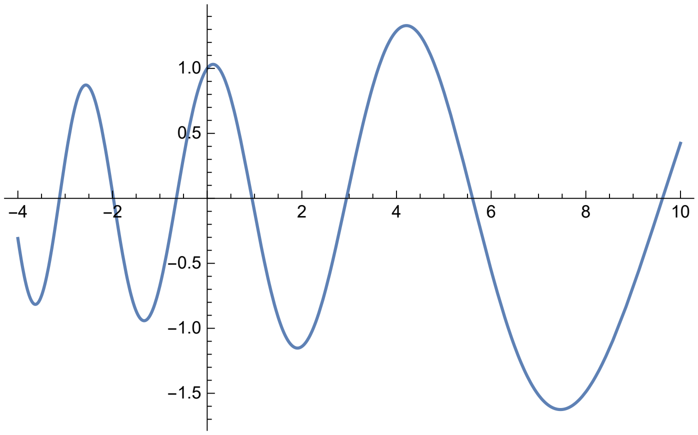


In[∘]:=
```
fg = DSolve[{f''[t] / f[t] == -4 Exp[-t/4], f[0] == 1, f'[0] == 1/2}, f[t], t]
```

Out[∘]=
```
{{f[t] → (BesselJ[0, 16 Sqrt[ⅇ^(-t/4)]] BesselY[0, 16] - BesselJ[0, 16] BesselY[0, 16 Sqrt[ⅇ^(-t/4)]] +
      4 BesselJ[1, 16] BesselY[0, 16 Sqrt[ⅇ^(-t/4)]] - 4 BesselJ[0, 16 Sqrt[ⅇ^(-t/4)]] BesselY[1, 16]) /
    (4 (BesselJ[1, 16] BesselY[0, 16] - BesselJ[0, 16] BesselY[1, 16]))}}
```

In[∘]:= `Plot[f[t] /. fg, {t, -4, 10}]`

Out[∘]=



In[∘]:=

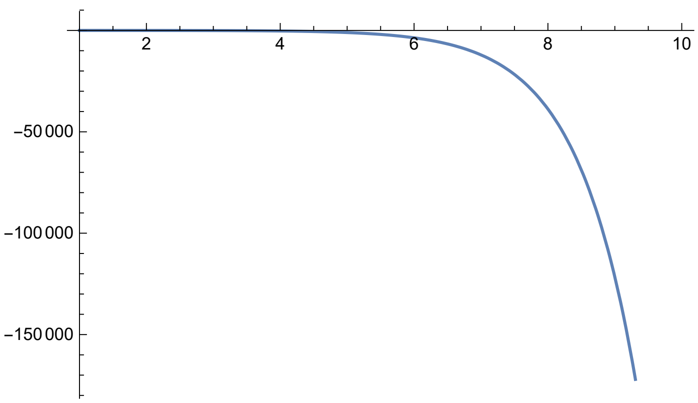
`sx = DSolve[{y''[x] - 2 y'[x] + y[x] == 0, y[0] == 3, y'[0] == 1}, y[x], x]`

In[∘]:= $\{\{y[x] \to -e^{x}(-3+2x)\}\}$

`Plot[y[x] /. sx, {x, 1, 10}]`

Out[∘]= $\{\{y[x] \to -e^{x}(-3+2x)\}\}$

Out[∘]=



Third Differential equation

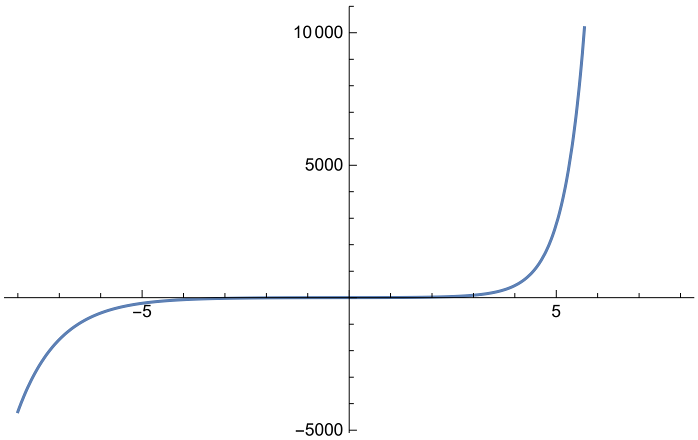
In[∘]:= `th = DSolve[{Y'''[s] + Y[s] == (e^s + 2)^2, Y[0] == 3, Y'[0] == 2, Y''[0] == 1}, Y[s], s]`

Out[∘]=

$$\left\{\left\{Y[s] \to \frac{1}{9} e^{-s}\left(-13+12 e^{s}+6 e^{2s}+e^{3s}-15 e^{3s/2}\,\mathrm{Cos}\left[\frac{\sqrt{3}\ s}{2}\right]+24 e^{s}\,\mathrm{Cos}\left[\frac{\sqrt{3}\ s}{2}\right]^2+12 e^{2s}\,\mathrm{Cos}\left[\frac{\sqrt{3}\ s}{2}\right]^2-5\sqrt{3}\ e^{3s/2}\,\mathrm{Sin}\left[\frac{\sqrt{3}\ s}{2}\right]+24 e^{s}\,\mathrm{Sin}\left[\frac{\sqrt{3}\ s}{2}\right]^2+12 e^{2s}\,\mathrm{Sin}\left[\frac{\sqrt{3}\ s}{2}\right]^2\right)\right\}\right\}$$

In[•]:=
```
Plot[Y[s] /. th, {s, -8, 8}]
```

Out[•]=

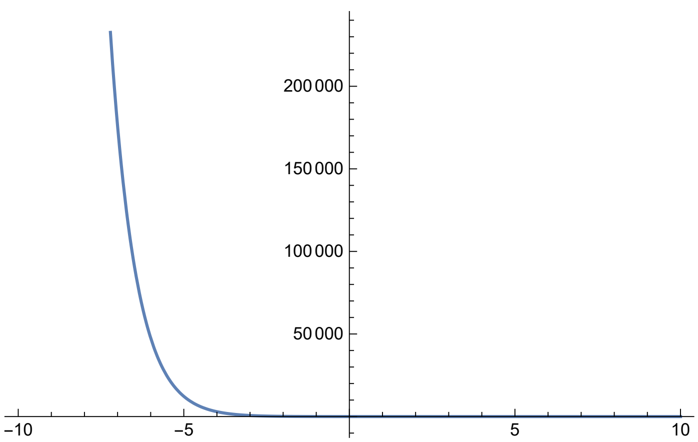


In[•]:=
```
tcs = DSolve[
   {g'''[x] + 3 g''[x] + 3 g'[x] + g[x] == 0, g[0] == 10, g'[0] == -7, g''[0] == 11}, g[x], x]
```

Out[•]= $\left\{\left\{g[x] \to \frac{1}{2} e^{-x} \left(20 + 6\,x + 7\,x^2\right)\right\}\right\}$

In[•]:=
```
Plot[g[x] /. tcs, {x, -10, 10}]
```

Out[•]=



In[•]:=
```
ClearAll
```

Out[•]= ClearAll

In[•]:=
```
ClearAll
```

Out[•]= ClearAll

2. Growth and decay model (exponential case only)

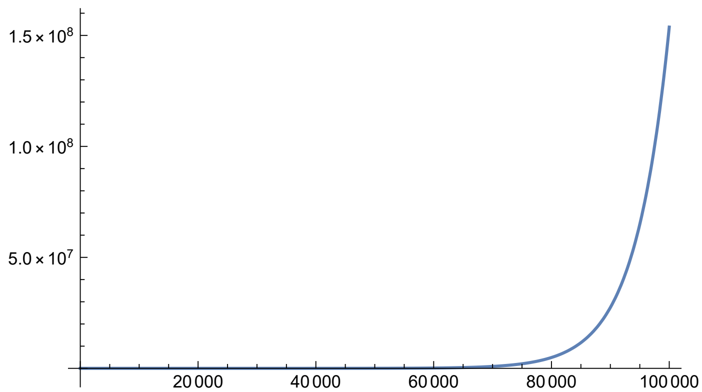
#Study the growth of modal

In[ ]:=
```
h12 = DSolve[{g'[t] == k * g[t], g[0] == r0}, g[t], t]
```

Out[ ]= $\{\{g[t] \to e^{k\,t}\, r0\}\}$

In[ ]:=
```
Plot[g[t] /. h12 /. {k → 1 / 5800, r0 → 5},
 {t, 0, 100000}, AxesOrigin → {0, 0}, PlotRange → All]
```

Out[ ]=



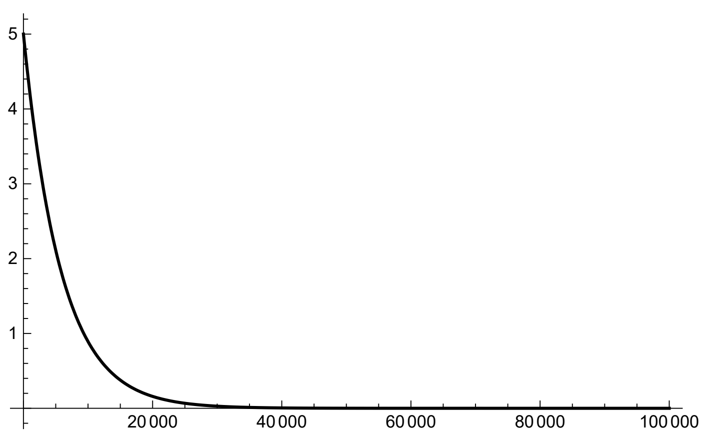
#Study of Decay modal

In[ ]:=
```
h35 = DSolve[{M'[t] == -k * M[t], M[0] == r0}, M[t], t]
```

Out[ ]= $\{\{M[t] \to e^{-k\,t}\, r0\}\}$

In[ ]:=
```
Plot[M[t] /. h35 /. {k → 1 / 5800, r0 → 5}, {t, 0, 100000},
 AxesOrigin → {0, 0}, PlotStyle → Black, PlotRange → All]
```

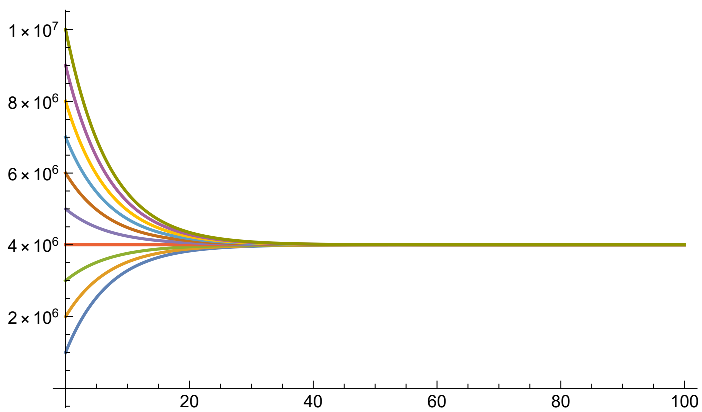
Out[ ]=



In[ ]:=
```
ClearAll
```
Out[ ]=
```
ClearAll
```

3. (i) Lake pollution model (with constant/seasonal flow and pollution concentration).

```
F = 4 * 10^6;
V = 28 * 10^6;
cin = 4 * 10^6;
re = DSolve[{P'[t] == (cin - P[t]) * F / V, P[0] == p}, P[t], t]
```
Out[ ]= $\left\{\left\{P[t] \to e^{-t/7}\left(-4\,000\,000 + 4\,000\,000\, e^{t/7} + p\right)\right\}\right\}$

In[ ]:=
```
Plot[Evaluate[Table[P[t] /. re /. p → i, {i, 10^6, 10^7, 10^6}]],
 {t, 0, 100}, AxesOrigin → {0, 0}, PlotRange → All]
```
Out[ ]=


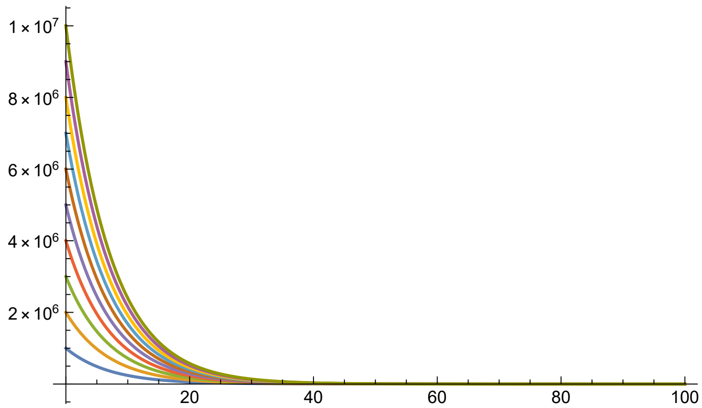
In[ ]:=
```
ClearAll
```
Out[ ]=
```
ClearAll
```

#Study the Lake Pollution modal with zero Pollution in lake

In[ ]:=
```
F = 4 * 10^6;
V = 28 * 10^6;
cin = 0;
rb = DSolve[{A'[t] == (cin - A[t]) * F / V, A[0] == a}, A[t], t]
```

Out[ ]= $\{\{A[t] \to a\, e^{-t/7}\}\}$

In[ ]:=
```
Plot[Evaluate[Table[A[t] /. rb /. a → j, {j, 10^6, 10^7, 10^6}]],
 {t, 0, 100}, AxesOrigin → {0, 0}, PlotRange → All]
```

Out[ ]=


#Lake Pollution Model with seasonal flow

In[ ]:=
```
cin = 3;
V = 28;
F = 50;
DU = D[J[t], t] + (F / V) * (1 + eps * Cos[2 Pi * t]) J[t] == 0
dc = DSolve[{DU, J[0] == j0}, J[t], t]
```

Out[ ]= $\frac{25}{14}\left(1 + eps\, Cos[2\pi t]\right) J[t] + J'[t] == 0$

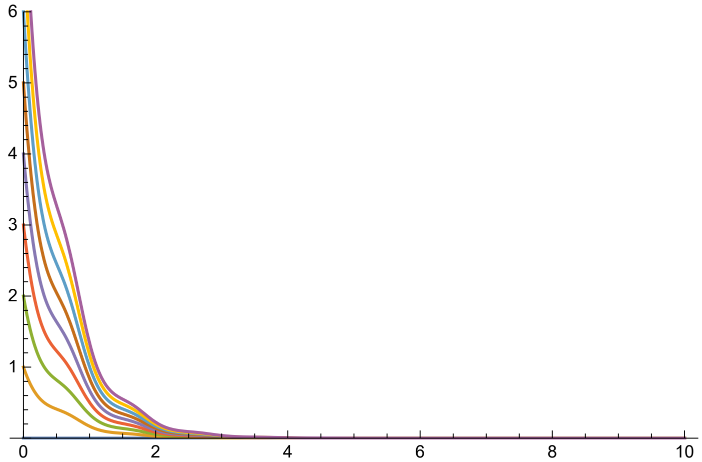
In[ ]:= $\{\{J[t] \to e^{-\frac{25 t}{14} - \frac{25\, eps\, Sin[2\pi t]}{28\pi}}\, j0\}\}$

```
eps = 0.5;
```

Out[ ]= $\{\{J[t] \to e^{-\frac{25 t}{14} - \frac{25\, eps\, Sin[2\pi t]}{28\pi}}\, j0\}\}$

In[∘]:=
```
Plot[Evaluate[J[t] /. dc /. j0 → Range[0, 8]], {t, 0, 10}, PlotRange → {0, 6}]
```

Out[∘]=



In[∘]:=
```
ClearAll
```

Out[∘]=
```
ClearAll
```

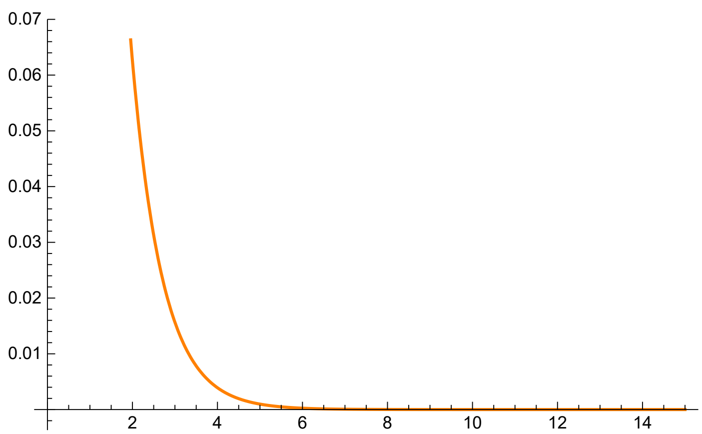
(ii) Case of single cold pill and a course of cold pills.

In[∘]:=
```
eq1 = x'[t] == -1.3860 x[t];
eq2 = y'[t] == 1.3860 x[t] - 0.1386 y[t];
sol = DSolve[{eq1, eq2, x[0] == 1, y[0] == 0}, {x[t], y[t]}, t]
```

Out[∘]= $\{\{x[t] \to 1.\, e^{-1.386\, t},\ y[t] \to -1.11111\, e^{-1.5246\, t}\, (1.\, e^{0.1386\, t} - 1.\, e^{1.386\, t})\}\}$

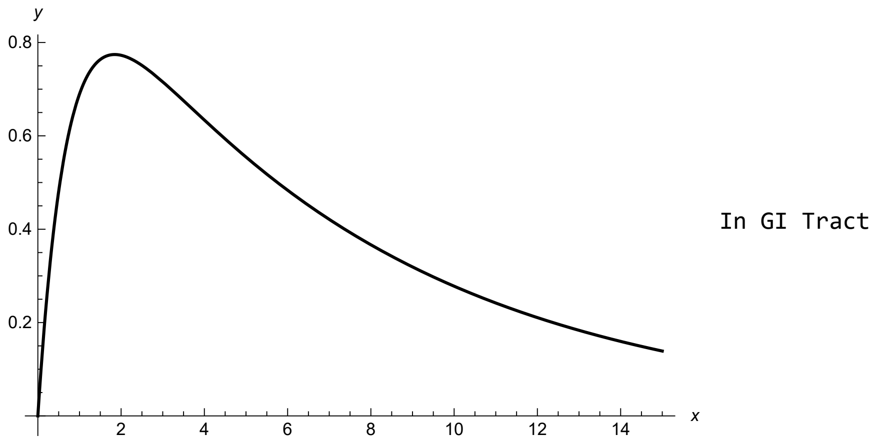
In[∘]:=
```
Plot[Evaluate[x[t] /. sol], {t, 0, 15}, PlotStyle → Orange]
```

Out[∘]=



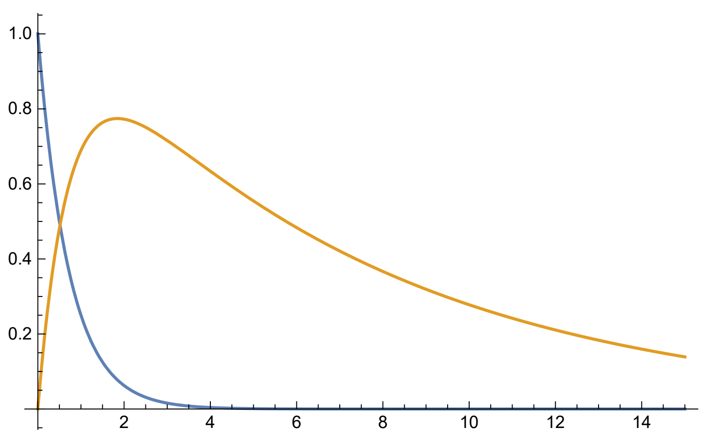
```
In[•]:= Plot[Evaluate[y[t] /. sol], {t, 0, 15}, PlotRange → All,
         PlotStyle → Black, PlotLegends → "In GI Tract", AxesLabel → {x, y}]
```

Out[•]=




```
In[•]:= Plot[Evaluate[{x[t], y[t]} /. sol], {t, 0, 15}]
```

Out[•]=



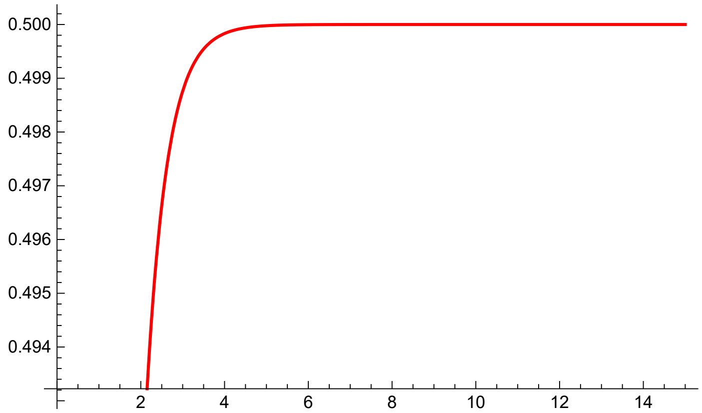
#(iii) Limited growth of population (with and without harvesting).

```
In[•]:= eq3 = x'[t] == 1 - 2 x[t];
        eq4 = y'[t] == 2 x[t] - 4 y[t];
        sol1 = DSolve[{eq3, eq4, x[0] == 0, y[0] == 0}, {x[t], y[t]}, t]
```

Out[•]= $\left\{\left\{x[t] \to \frac{1}{2} e^{-2t} \left(-1 + e^{2t}\right), y[t] \to \frac{1}{4} e^{-4t} \left(-1 + e^{2t}\right)^2\right\}\right\}$

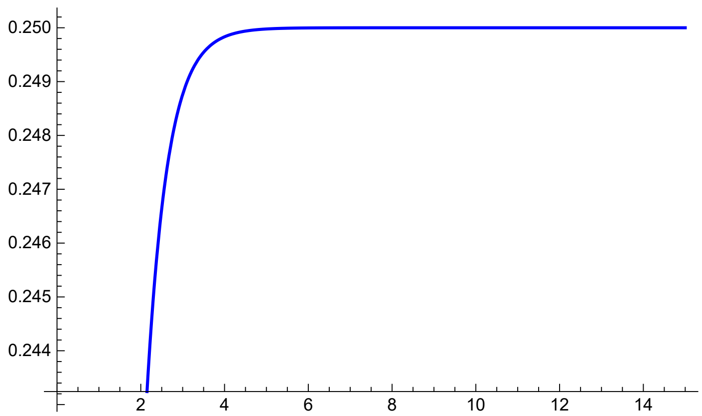
In[•]:=
```
Plot[Evaluate[x[t] /. sol1], {t, 0, 15}, PlotStyle → Red]
```

Out[•]=



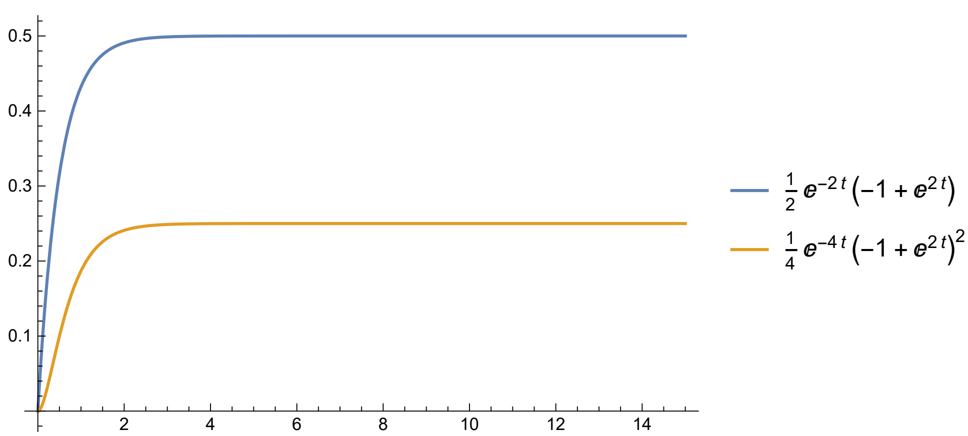
In[•]:=
```
Plot[Evaluate[y[t] /. sol1], {t, 0, 15}, PlotStyle → Blue]
```

Out[•]=



In[•]:=
```
Plot[Evaluate[{x[t], y[t]} /. sol1], {t, 0, 15}, PlotLegends → "Expressions"]
```

Out[•]=

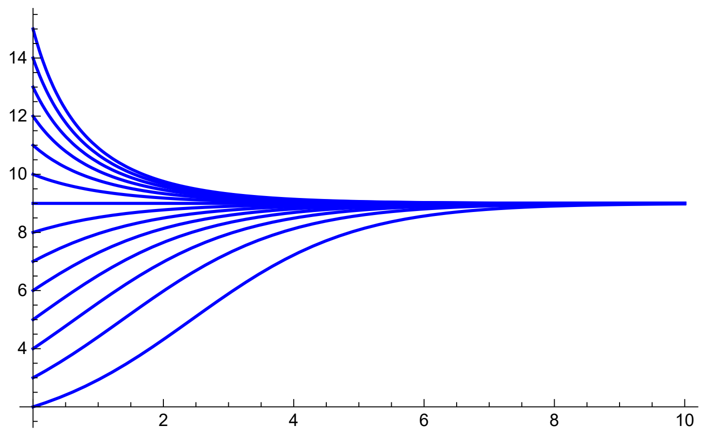


(iii) Limited growth of population (with and without harvesting)

In[ ]:=
```
r = 1;
k = 10;
h = 0.9;
sol6 = DSolve[{Z'[t] == (r * Z[t] * (1 - Z[t] / k)) - h, Z[0] == z0}, Z[t], t]
```

Out[ ]= $\left\{\left\{Z[t] \rightarrow \frac{9.\left(1. \times 2.71828^{0.8 t}-0.111111\left(-\frac{1.(-9.+1. z0)}{1.-1. z0}\right)^{1.}\right)}{2.71828^{0.8 t}-1.\left(-\frac{1.(-9.+1. z0)}{1.-1. z0}\right)^{1.}}\right\}\right\}$

In[ ]:=
```
Plot[Table[Z[t] /. sol6 /. z0 → j, {j, 2, 15, 1}],
 {t, 0, 10}, PlotRange → All, PlotStyle → Blue]
```

Out[ ]=


In[ ]:=
```
ClearAll
```

Out[ ]= ClearAll

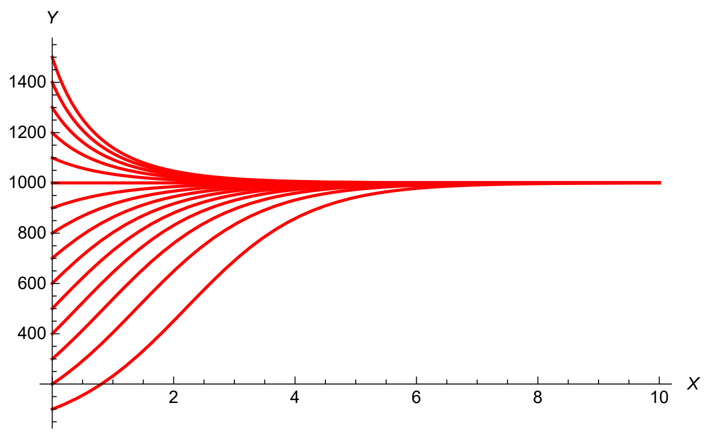
#Study the Logistic growth modal

In[ ]:=
```
s = 1;
c = 1000;
n = 0;
sol8 = DSolve[{F'[x] == (s * F[x] * (1 - F[x] / c)) - n, F[0] == f0}, F[x], x]
```

... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[ ]= $\left\{\left\{F[x] \rightarrow \frac{1000\, e^{x} f0}{1000 - f0 + e^{x} f0}\right\}\right\}$

In[ ]:=
```
Plot[Table[F[x] /. sol8 /. f0 → j, {j, 100, 1500, 100}],
  {x, 0, 10}, PlotRange → All, AxesLabel → {X, Y}, PlotStyle → Red]
```

Out[ ]=


In[ ]:=
```
ClearAll
```

Out[ ]=
```
ClearAll
```

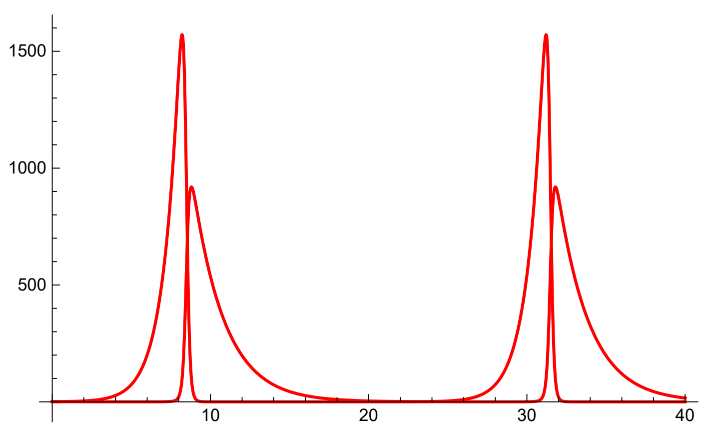
4. (i) Predatory-prey model (basic Volterra model, with density dependence, effect of DDT, two prey one predator).

Study the Predatory-prey modal

In[ ]:=
```
r1 = 1;
r2 = 0.5;
c1 = 0.01;
c2 = 0.005;
sol10 = NDSolve[{X'[t] == r1 * X[t] - c1 * X[t] * S[t],
   S'[t] == c2 * X[t] * S[t] - r2 * S[t], X[0] == 200, X[0] == 80}, {X[t], S[t]}, {t, 0, 50}]
```

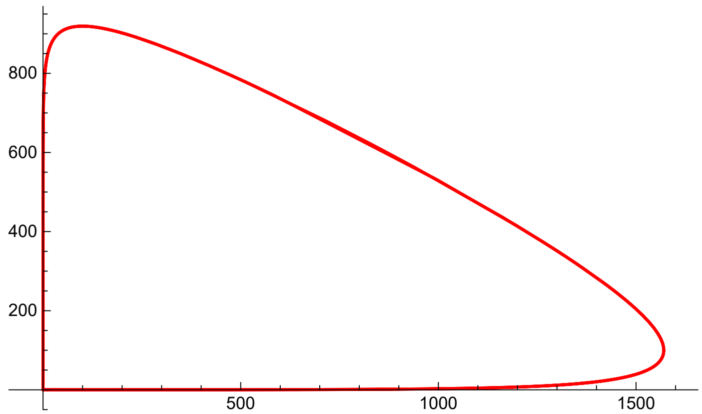
Out[ ]=
```
{{X[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t],
  S[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t]}}
```

In[ ]:=
```
Plot[{X[t], S[t]} /. sol10, {t, 0, 40}, AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Red]
```

Out[ ]=


In[ ]:=
```
ParametricPlot[{X[t], S[t]} /. sol10, {t, 0, 40},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Red]
```

Out[ ]=



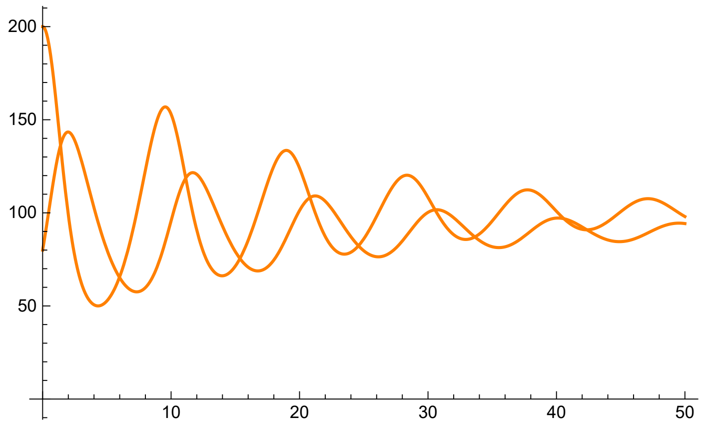
Density Dependent growth Modal

In[ ]:=
```
r1 = 1;
r2 = 0.5;
c1 = 0.01;
c2 = 0.005;
k = 1000;
```

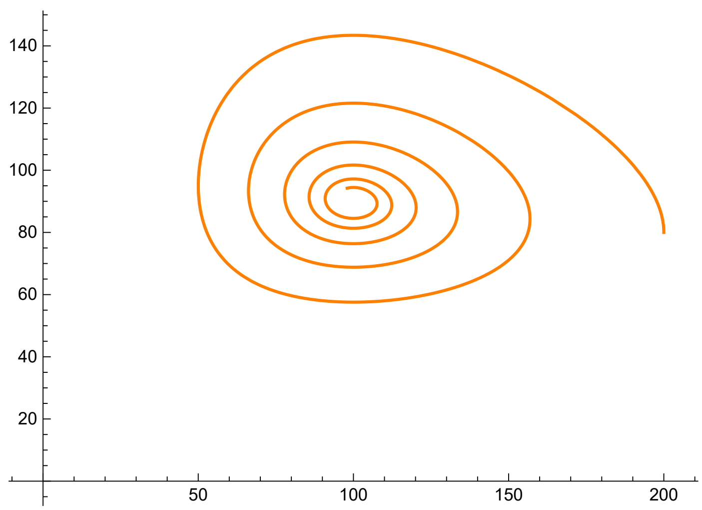
In[ ]:=
```
h7 = NDSolve[{V'[t] == r1 * V[t] (1 - V[t] / k) - c1 * V[t] * P[t],
   P'[t] == c2 * V[t] * P[t] - r2 * P[t], V[0] == 200, P[0] == 80}, {V[t], P[t]}, {t, 0, 50}]
```

Out[ ]=
```
{{V[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t],
  P[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t]}}
```

In[ ]:=
```
Plot[{V[t], P[t]} /. h7, {t, 0, 50},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Orange]
```

Out[ ]=


In[ ]:=
```
ParametricPlot[{V[t], P[t]} /. h7, {t, 0, 50},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Orange]
```

Out[ ]=


In[ ]:=
```
clearAll
```

Out[ ]=
```
clearAll
```

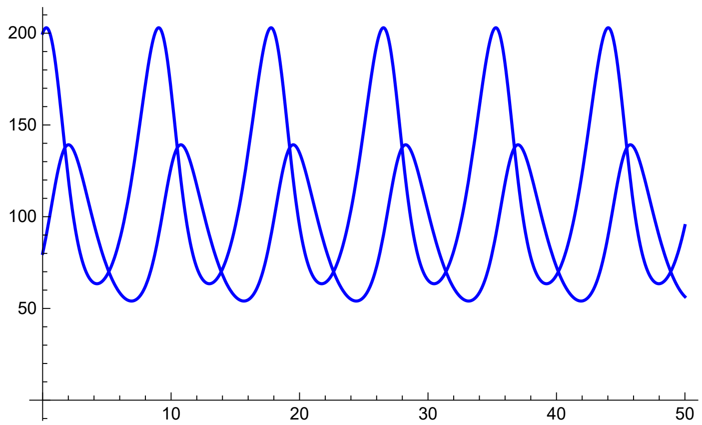
Predator Prey Model with DDT Effect

In[ ]:=
```
r5 = 1;
r6 = 0.5;
c5 = 0.01;
c6 = 0.005;
p5 = 0.1;
p6 = 0.1;
sol11 = NDSolve[{F'[t] == r5 * F[t] - c5 * F[t] * Y[t] - p5 * F[t], Y'[t] ==
    c6 * F[t] * Y[t] - r6 * Y[t] - p6 * Y[t], F[0] == 200, Y[0] == 80}, {F[t], Y[t]}, {t, 0, 50}]
```

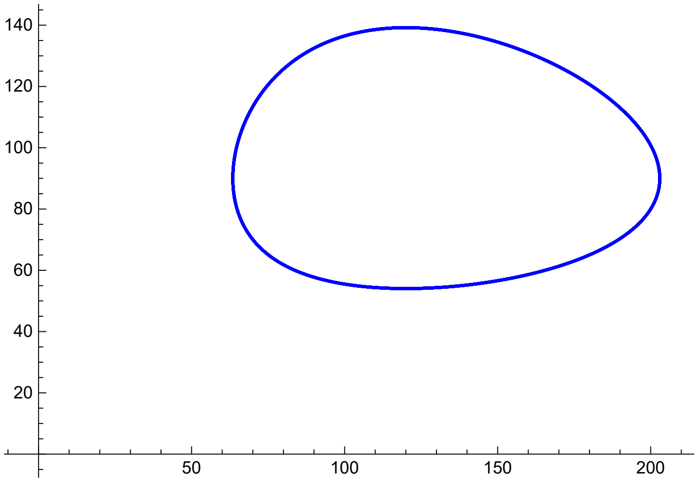
Out[ ]=
```
{{F[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t],
  Y[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t]}}
```

In[ ]:=
```
Plot[{F[t], Y[t]} /. sol11, {t, 0, 50},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Blue]
```

Out[ ]=



In[ ]:=
```
ParametricPlot[{F[t], Y[t]} /. sol11, {t, 0, 50},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Blue]
```

Out[ ]=


Battle Model

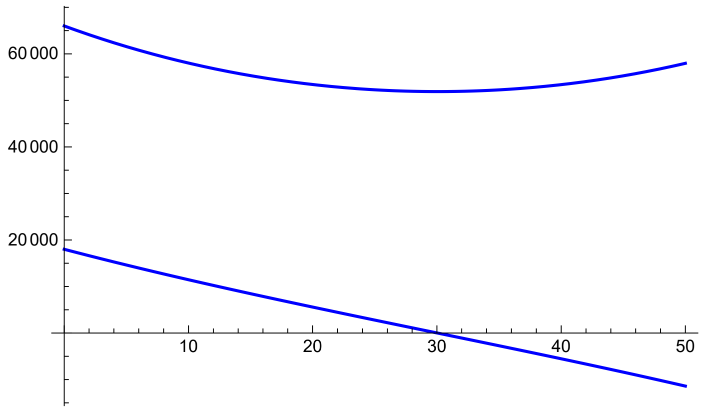
Normal

In[ ]:=
```
y = 0.0544;
r = 0.0106;
sol12 = NDSolve[{R'[t] == -y * Y[t], Y'[t] == -r * R[t], R[0] == 66000, Y[0] == 18000},
  {R[t], Y[t]}, {t, 0, 50}]
```

Out[ ]=
```
{{R[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t],
  Y[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t]}}
```

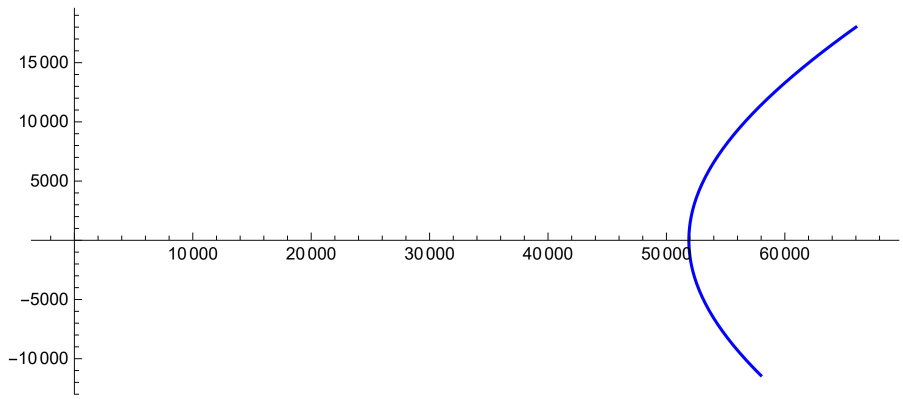
In[ ]:=
```
Plot[{R[t], Y[t]} /. sol12, {t, 0, 50},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Blue]
```

Out[ ]=



In[ ]:=
```
ParametricPlot[{R[t], Y[t]} /. sol12, {t, 0, 50},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Blue]
```

Out[ ]=



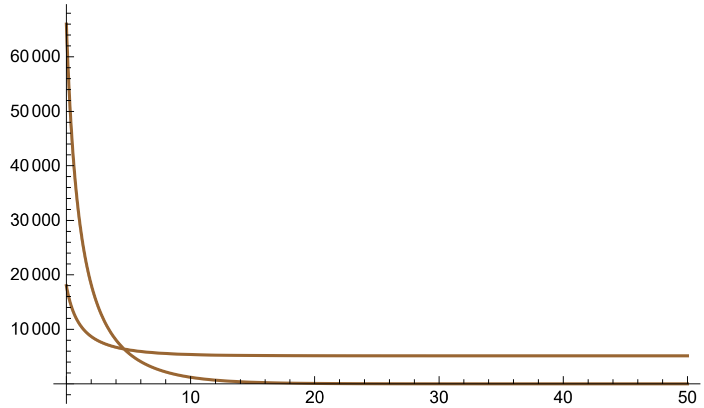
Long-Range

In[ ]:=
```
y = 0.0000544;
r = 0.0000106;
sol13 = NDSolve[{R'[t] == -y * R[t] * Y[t],
   Y'[t] == -r * R[t] * Y[t], R[0] == 66000, Y[0] == 18000}, {R[t], Y[t]}, {t, 0, 50}]
```

Out[ ]=
```
{{R[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t],
  Y[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t]}}
```

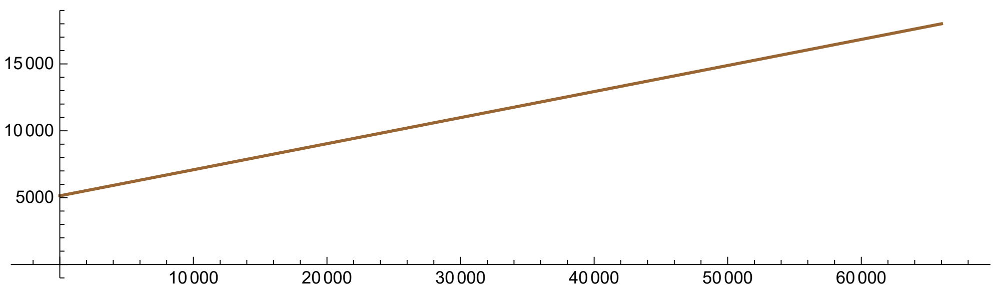
In[ ]:=
```
Plot[{R[t], Y[t]} /. sol13, {t, 0, 50},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Brown]
```

Out[ ]=



In[ ]:=
```
ParametricPlot[{R[t], Y[t]} /. sol13, {t, 0, 50},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Brown]
```

Out[ ]=



Jungle Warfare

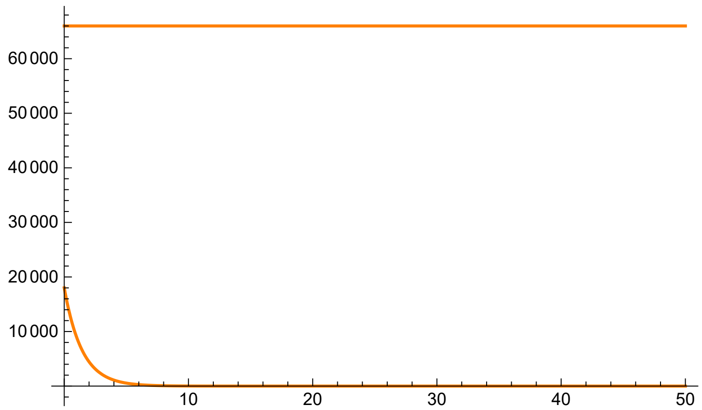
```
y = 0.0544;
r = 0.0106;
```

In[ ]:=
```
sol14 = NDSolve[{R'[t] == -y * Y[t], Y'[t] == -r * R[t] * Y[t], R[0] == 66 000, Y[0] == 18 000},
  {R[t], Y[t]}, {t, 0, 50}]
```

Out[ ]=
```
{{R[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t],
  Y[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t]}}
```

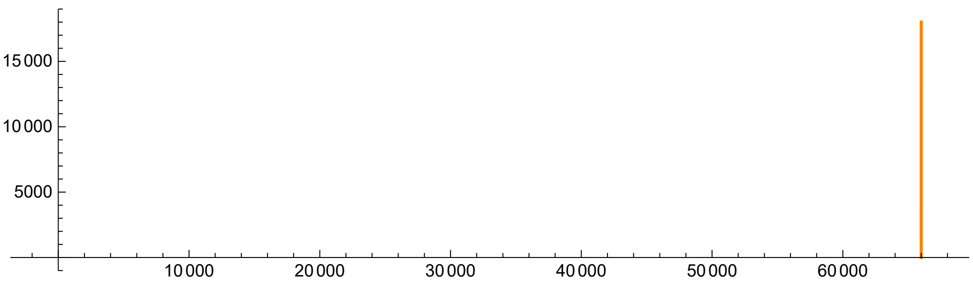
In[ ]:=
```
Plot[{R[t], Y[t]} /. sol14, {t, 0, 50},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Orange]
```

Out[ ]=



In[ ]:=
```
ParametricPlot[{R[t], Y[t]} /. sol14, {t, 0, 50},
 AxesOrigin → {0, 0}, PlotRange → All, PlotStyle → Orange]
```

Out[ ]=

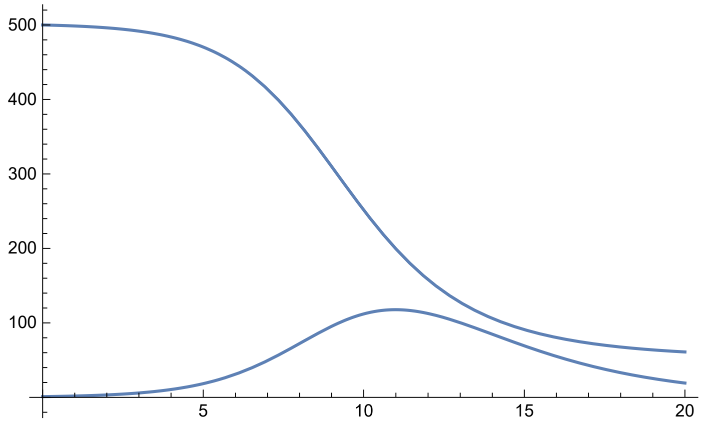


Epidemic Model on influenza

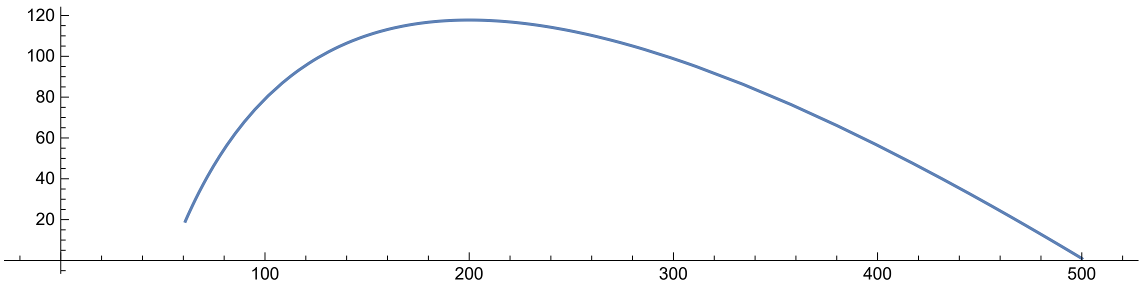
In[ ]:=
```
β = 0.002;
γ = 0.4;
sol15 = NDSolve[{S'[t] == -β * S[t] * H[t],
   H'[t] == β * S[t] * H[t] - γ * H[t], S[0] == 500, H[0] == 1}, {S[t], H[t]}, {t, 0, 20}]
```

Out[ ]=
```
{{S[t] → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ][t],
  H[t] → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ][t]}}
```

In[ ]:=
```
Plot[{S[t], H[t]} /. sol15, {t, 0, 20}, AxesOrigin → {0, 0}, PlotRange → All]
```

Out[ ]=


In[ ]:=
```
ParametricPlot[{S[t], H[t]} /. sol15, {t, 0, 20}, AxesOrigin → {0, 0}, PlotRange → All]
```

Out[ ]=


5. Plotting of recursive sequences, and study of the convergence

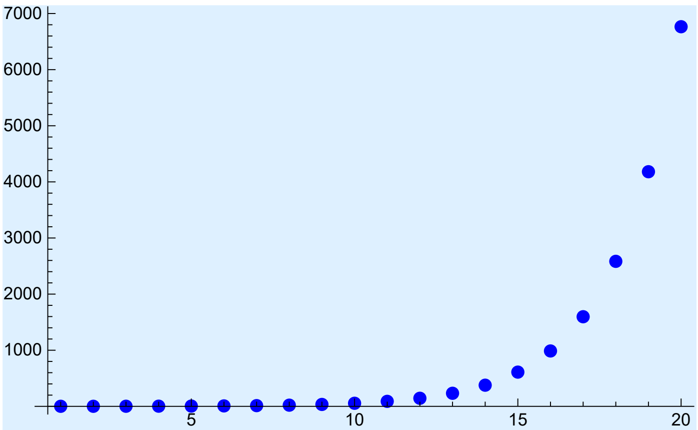
#Plotting of recursive sequences

In[ ]:=
```
a[1] = 1;
a[2] = 1;
a[n_] := a [n - 2] + a [n - 1];
aa = Table[a[n], {n, 1, 20}]
```

Out[ ]= {1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765}

In[∘]:=
```
ListPlot[aa, PlotStyle → {Blue, PointSize[0.02]}, PlotRange → All,
 PlotLegends → "grap[ for the sequence a[n]-a[n-2]-a[n-1]", Background → LightBlue]
```

Out[∘]=



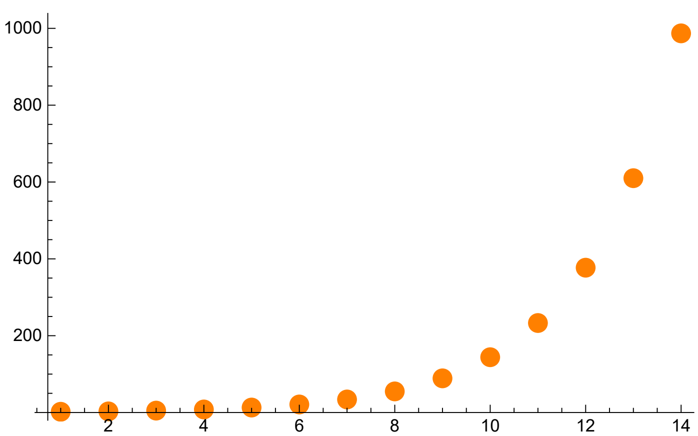
grap[for the sequence a[n]-a[n-2]-a[n

In[∘]:=
```
b[2] = 2;
b[3] = 3;
b[s_] := b[s - 1] + b[s - 2];
sn = Table[b[s], {s, 2, 15}]
```

Out[∘]= {2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987}

In[∘]:=
```
ListPlot[sn, PlotStyle → {Orange, PointSize[0.03]},
 PlotRange → All, PlotLegends → "Grap for the sequence b[s-1]+b[s-2]"]
```

Out[∘]=



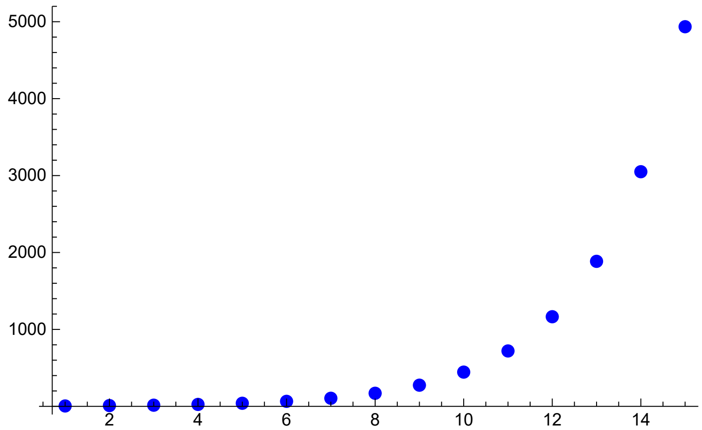
Grap for the sequence b[s-1]+b[s-2]

In[∘]:=
```
w[1] = 5;
w[2] = 10;
w[x_] := w[x - 2] + w[x - 1];
cd = Table[w[x], {x, 1, 15}]
```

Out[∘]= {5, 10, 15, 25, 40, 65, 105, 170, 275, 445, 720, 1165, 1885, 3050, 4935}

In[ ]:=
```
ListPlot[cd, PlotStyle → {Blue, PointSize[0.02]},
 PlotRange → All, PlotLegends → "Grap for the sequence  w[x-2]+w[x-1]"]
```

Out[ ]=



Grap for the sequence w[x-2]+w[x-1]

In[ ]:=
```
ClearAll
```

Out[ ]=
```
ClearAll
```

```
"Ques:Define Recurrence Sequence and Find its first 20 terms  and Plot"
```

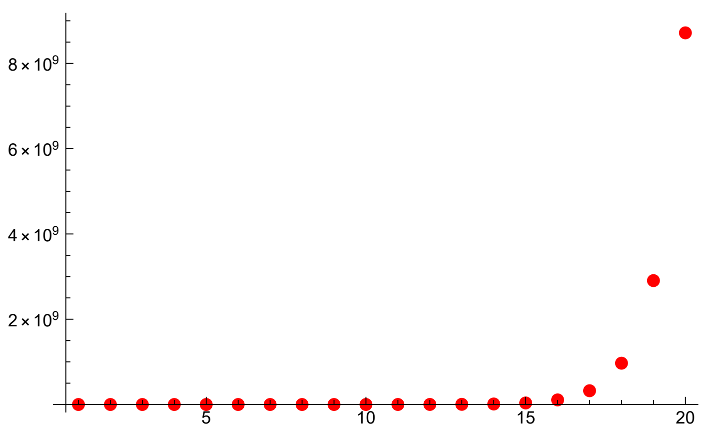
In[ ]:=
```
f[1] = 2;
```

In[ ]:=
```
f[n_] := 3 * f[n - 1] + 11;
ff = Table[f[n], {n, 1, 20}]
```

Out[ ]=
```
{2, 17, 62, 197, 602, 1817, 5462, 16 397, 49 202, 147 617, 442 862, 1 328 597, 3 985 802,
 11 957 417, 35 872 262, 107 616 797, 322 850 402, 968 551 217, 2 905 653 662, 8 716 960 997}
```

In[ ]:=
```
ListPlot[ff, PlotStyle → {Red, PointSize[0.02]}, PlotRange → All,
 PlotLegends → "Grap for the sequence  f[n_]:=3*f[n-1]+11"]
```

Out[ ]=



Grap for the sequence f[n_]:=3*f[n-1]

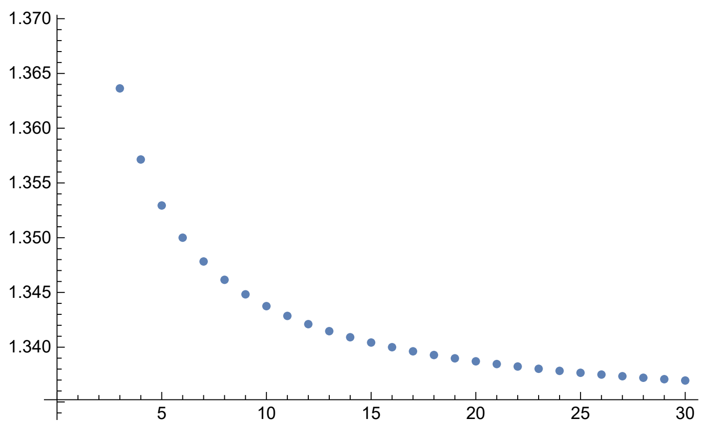
Q1.study the convergence of sequence a[n_]=(4*n+3)/ (3*n+2) through Plotting

In[∘]:=
$$a[n\_] = \frac{(4*n+3)}{(3*n+2)};$$

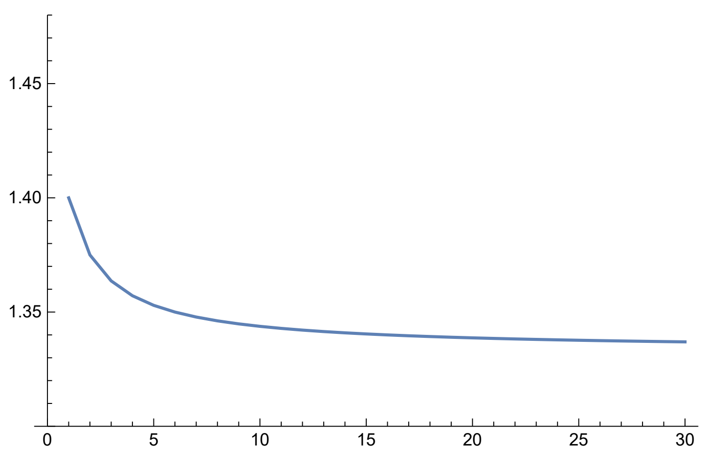
```
hemant = Table[{n, a[n]}, {n, 30}]
```

Out[∘]=
$$\left\{\left\{1, \frac{7}{5}\right\}, \left\{2, \frac{11}{8}\right\}, \left\{3, \frac{15}{11}\right\}, \left\{4, \frac{19}{14}\right\}, \left\{5, \frac{23}{17}\right\}, \left\{6, \frac{27}{20}\right\}, \left\{7, \frac{31}{23}\right\}, \left\{8, \frac{35}{26}\right\}, \left\{9, \frac{39}{29}\right\}, \left\{10, \frac{43}{32}\right\}, \left\{11, \frac{47}{35}\right\}, \left\{12, \frac{51}{38}\right\}, \left\{13, \frac{55}{41}\right\}, \left\{14, \frac{59}{44}\right\}, \left\{15, \frac{63}{47}\right\}, \left\{16, \frac{67}{50}\right\}, \left\{17, \frac{71}{53}\right\}, \left\{18, \frac{75}{56}\right\}, \left\{19, \frac{79}{59}\right\}, \left\{20, \frac{83}{62}\right\}, \left\{21, \frac{87}{65}\right\}, \left\{22, \frac{91}{68}\right\}, \left\{23, \frac{95}{71}\right\}, \left\{24, \frac{99}{74}\right\}, \left\{25, \frac{103}{77}\right\}, \left\{26, \frac{107}{80}\right\}, \left\{27, \frac{111}{83}\right\}, \left\{28, \frac{115}{86}\right\}, \left\{29, \frac{119}{89}\right\}, \left\{30, \frac{123}{92}\right\}\right\}$$

In[∘]:=
```
ListPlot[hemant]
```

Out[∘]=


In[∘]:=
```
ListLinePlot[hemant, PlotRange → {1.3, 1.48}]
```

Out[∘]=


In[ ]:=
```
TableForm[Table[{n, a[n]}, {n, 30}]]
```

Out[ ]//TableForm=

1	$\frac{7}{5}$
2	$\frac{11}{8}$
3	$\frac{15}{11}$
4	$\frac{19}{14}$
5	$\frac{23}{17}$
6	$\frac{27}{20}$
7	$\frac{31}{23}$
8	$\frac{35}{26}$
9	$\frac{39}{29}$
10	$\frac{43}{32}$
11	$\frac{47}{35}$
12	$\frac{51}{38}$
13	$\frac{55}{41}$
14	$\frac{59}{44}$
15	$\frac{63}{47}$
16	$\frac{67}{50}$
17	$\frac{71}{53}$
18	$\frac{75}{56}$
19	$\frac{79}{59}$
20	$\frac{83}{62}$
21	$\frac{87}{65}$
22	$\frac{91}{68}$
23	$\frac{95}{71}$
24	$\frac{99}{74}$
25	$\frac{103}{77}$
26	$\frac{107}{80}$
27	$\frac{111}{83}$
28	$\frac{115}{86}$
29	$\frac{119}{89}$
30	$\frac{123}{92}$

In[ ]:=
```
Limit[a[n], n → Infinity]
```

Out[ ]= $\frac{4}{3}$

Q2 Study the convergence of Sequence a[n_]=(4*x^3+5x)/(3*x^2+8)

through Plotting

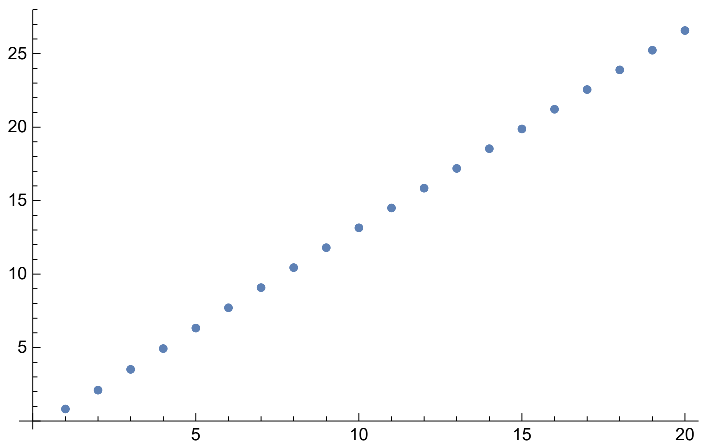
In[ ]:=

```
b[n_] = (5 n + 4 n^3)/(8 + 3 n^2);
raja = Table[{n, b[ n]}, {n, 20}]
ListPlot[b[n]]
```

Out[ ]= $\{\{1, \frac{9}{11}\}, \{2, \frac{21}{10}\}, \{3, \frac{123}{35}\}, \{4, \frac{69}{14}\}, \{5, \frac{525}{83}\}, \{6, \frac{447}{58}\}, \{7, \frac{1407}{155}\}, \{8, \frac{261}{25}\}, \{9, \frac{2961}{251}\}, \{10, \frac{2025}{154}\}, \{11, \frac{5379}{371}\}, \{12, \frac{1743}{110}\}, \{13, \frac{8853}{515}\}, \{14, \frac{5523}{298}\}, \{15, \frac{13575}{683}\}, \{16, \frac{2058}{97}\}, \{17, \frac{19737}{875}\}, \{18, \frac{11709}{490}\}, \{19, \frac{27531}{1091}\}, \{20, \frac{8025}{302}\}\}$

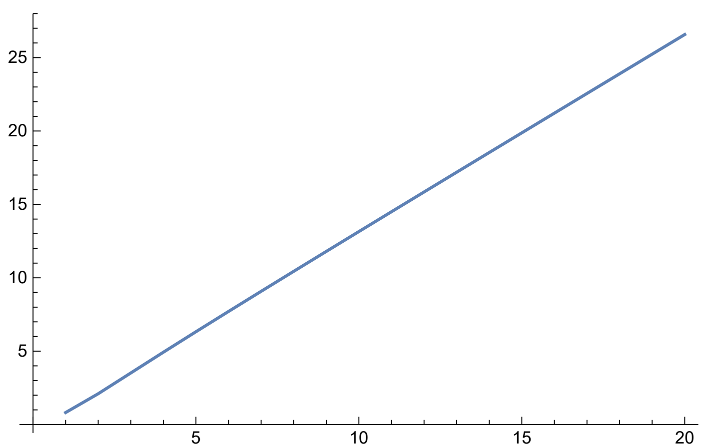
In[ ]:= `ListPlot[raja]`

Out[ ]=



In[ ]:= `ListLinePlot[raja]`

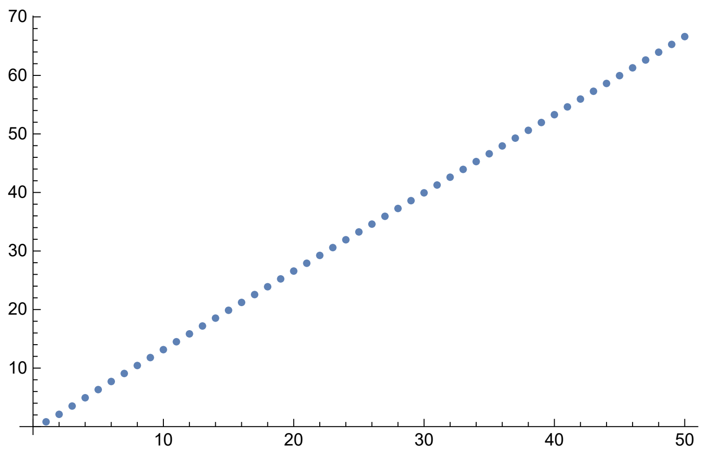
Out[ ]=



In[ ]:= ss = Table[{n, N[b[n]]}, {n, 50}]

Out[ ]= {{1, 0.818182}, {2, 2.1}, {3, 3.51429}, {4, 4.92857}, {5, 6.3253}, {6, 7.7069}, {7, 9.07742}, {8, 10.44}, {9, 11.7968}, {10, 13.1494}, {11, 14.4987}, {12, 15.8455}, {13, 17.1903}, {14, 18.5336}, {15, 19.8755}, {16, 21.2165}, {17, 22.5566}, {18, 23.8959}, {19, 25.2346}, {20, 26.5728}, {21, 27.9106}, {22, 29.2479}, {23, 30.585}, {24, 31.9217}, {25, 33.2581}, {26, 34.5943}, {27, 35.9303}, {28, 37.2661}, {29, 38.6017}, {30, 39.9372}, {31, 41.2726}, {32, 42.6078}, {33, 43.9429}, {34, 45.2779}, {35, 46.6128}, {36, 47.9476}, {37, 49.2824}, {38, 50.6171}, {39, 51.9517}, {40, 53.2862}, {41, 54.6207}, {42, 55.9551}, {43, 57.2895}, {44, 58.6238}, {45, 59.9581}, {46, 61.2923}, {47, 62.6265}, {48, 63.9607}, {49, 65.2948}, {50, 66.6289}}

In[ ]:= ListPlot[ss]

Out[ ]=


#observation : sequence is not convergent

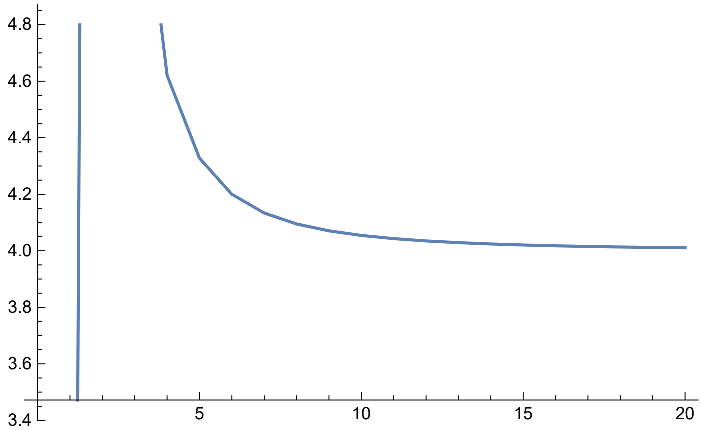
3Q.study the convergence of sequence a[n_]= $\frac{4*n^3+3*n}{n^3-6}$ through Plotting

In[ ]:= $d[n\_] = \frac{4*n^3+3*n}{n^3-6};$

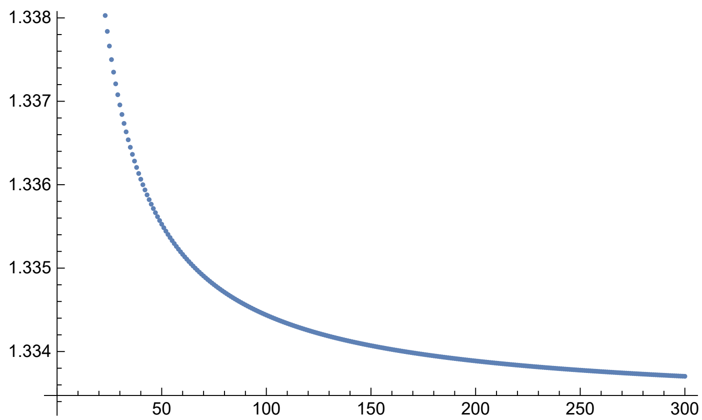
fb = Table[{n, d[n]}, {n, 20}]

Out[ ]= $\{\{1, -\frac{7}{5}\}, \{2, 19\}, \{3, \frac{39}{7}\}, \{4, \frac{134}{29}\}, \{5, \frac{515}{119}\}, \{6, \frac{21}{5}\}, \{7, \frac{1393}{337}\}, \{8, \frac{1036}{253}\}, \{9, \frac{981}{241}\}, \{10, \frac{2015}{497}\}, \{11, \frac{5357}{1325}\}, \{12, \frac{1158}{287}\}, \{13, \frac{1261}{313}\}, \{14, \frac{5509}{1369}\}, \{15, \frac{4515}{1123}\}, \{16, \frac{8216}{2045}\}, \{17, \frac{19703}{4907}\}, \{18, \frac{3897}{971}\}, \{19, \frac{27493}{6853}\}, \{20, \frac{2290}{571}\}\}$

```
In[•]:= ListLinePlot[fb]
```

Out[•]=



```
In[•]:= cd = Table[{n, N[a[n]]}, {n, 300}];
ListPlot[cd]
```

Out[•]=


6.Find a value m ∊ N that will make the following inequality holds for all n>m :

Ques1:Inequality holds for all n> m:

```
In[•]:= For[n = 1, Abs[(0.5)^(1/n) - 1] ≥ (10)^(-3), n = n + 1]
Print[n]
```

693

```
In[ ]:= If[Abs[(0.5)^(1/n) - 1] < (10)^(-3), Print["The inequality does hold for n"],
         Print["The Inequality  does not hold for n"]]
```

The inequality does hold for n

Obervation : "The inequality does hold for n"

Ques2:Inequality holds for all n> m:

```
In[ ]:= For[n = 1, Abs[n^(1/n) - 1] < 10^(-3), n = n + 1]

In[ ]:= Print[n]
```

2

```
In[ ]:= If[Abs[n^(1/n) - 1] < (10)^(-3), Print["The inequality does hold for n"],
         Print["The Inequality  does not hold for n"]]
```

The Inequality does not hold for n

Obervation : "The Inequality does not hold for n"

Ques3:Inequality holds for all n> m:

```
In[ ]:= For[n = 1, Abs[(0.9)^n] < 10^(-3), n = n + 1]
        Print[n]

Out[ ]= Null
```

1

```
In[ ]:= If[Abs[(0.9)^n] < (10)^(-3), Print["The inequality does hold for n"],
         Print["The Inequality  does not hold for n"]]
```

The Inequality does not hold for n

Obervation : "The Inequality does not hold for n"

Ques4:Inequality holds for all n> m:

```
In[ ]:= For[n = 1, Abs[2^n/n!] < 10^(-7), n = n + 1]

In[ ]:= Print[n]
```

1

```
In[ ]:= If[Abs[2^n/n!] < (10)^(-7), Print["The inequality does hold for n"],
         Print["The Inequality  does not hold for n"]]
```

The Inequality does not hold for n

Obervation : "The Inequality does not hold for n"

7. Verify the Bolzano–Weierstrass

theorem through plotting of sequences and hence identify convergent subsequences from the plot.

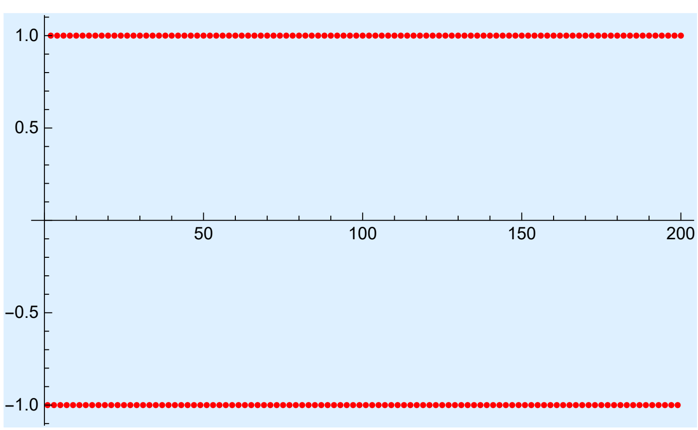
ques1):Verify the Bolzano–Weierstrass theorem through plotting of sequences a[n_]:=(-1)^n and identify the convergence

In[ ]:=
```
a[n_] := (-1)^n;
ra = Table[a[n], {n, 1, 2000}];
lb = Min[ra];
ub = Max[ra];
If[(-Infinity < lb) && (ub < Infinity), Print[" Bolzano weirstrass thm is applicable "],
 Print[" Bolzano weirstress thm is not applicable"]]
```

```
Bolzano weirstrass thm is applicable
```

In[ ]:=
```
bs = Table[{n, a[n]}, {n, 1, 200}];
ListPlot[bs, PlotLegends → "Grap of sequence -1^n",
 PlotStyle → Red, PlotRange → All, Background → LightBlue]
```

Out[ ]=

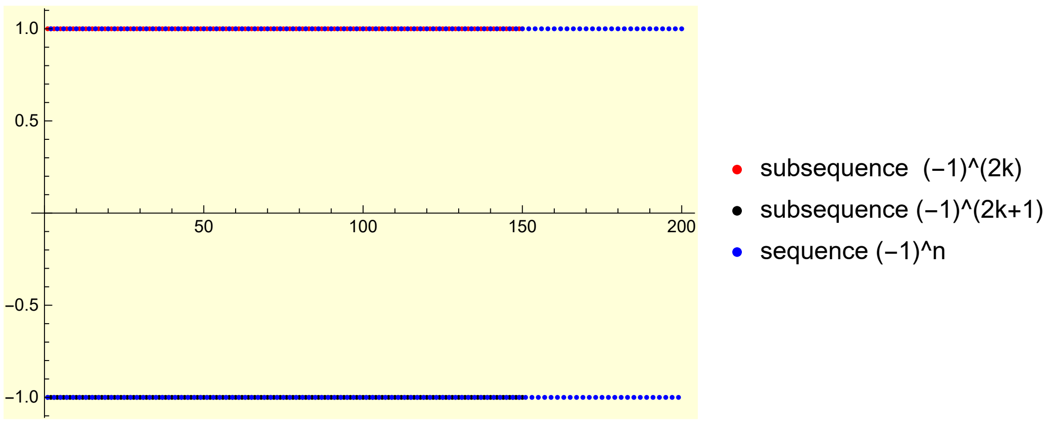


Grap of sequence -1^n

```
In[◦]:= b[k_] := (-1) ^ (2 k);
c[k_] := (-1) ^ (2 k + 1);
bd = Table[{k, b[k]}, {k, 1, 150}];
cd = Table[{k, c[k]}, {k, 1, 150}];
```

```
In[◦]:= ListPlot[{bd, cd, bs},
 PlotLegends → {"subsequence  (-1)^(2k)", "subsequence (-1)^(2k+1) ", "sequence (-1)^n" },
 PlotStyle → {Red, Black, Blue}, PlotRange → All, Background → LightYellow, PlotRange → All]
```

Out[◦]=



Observation : " Bolzano weirstrass theorem is applicable " and "sequence is not convergent"

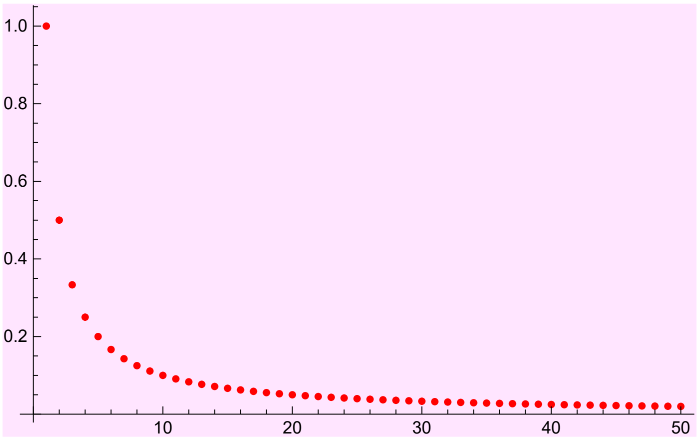
ques2:Verify Bolzano Weirstrass theorem through plotting of the sequence a[n_]:=(1/n) and identify the convergence

```
In[◦]:= a[n_] := 1 / n;
ra = Table[a[n], {n, 1, 2000}];
lb = Min[ra];
ub = Max[ra];
If[(-Infinity < lb) && (ub < Infinity), Print[" Bolzano weirstrass thm is applicable "],
 Print[" Bolzano weirstress thm is not applicable"]]
```

```
Bolzano weirstrass thm is applicable
```

In[∘]:=
```
aa = Table[{n, a[n]}, {n, 1, 50}];
ListPlot[aa, PlotLegends → "Grap of sequence -1^n",
 PlotStyle → Red, PlotRange → All, Background → LightMagenta]
```

Out[∘]=

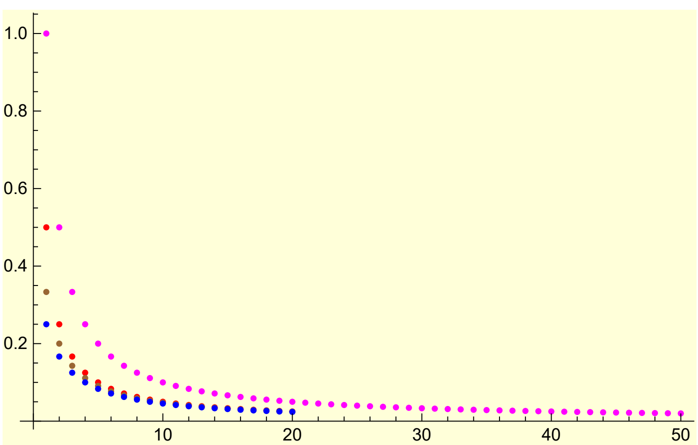


Grap of sequence -1^n

In[∘]:=
```
b[j_] := 1/(2*j);
c[j_] := 1/(2*j+1);
d[j_] := 1/(2*j+2);
bb = Table[{j, b[j]}, {j, 1, 20}];
cc = Table[{j, c[j]}, {j, 1, 20}];
dd = Table[{j, d[j]}, {j, 1, 20}];
```

In[∘]:=
```
ListPlot[{bb, cc, dd, aa},
 PlotLegends → {"subsequence 1/2j", "subsequence 1/(2j+1)", "sequence 1/n" },
 PlotStyle → {Red, Brown, Blue, Magenta}, PlotRange → All,
 Background → LightYellow, PlotRange → { { }, {0, 0.6}}]
```

Out[∘]=



- subsequence 1/2j
- subsequence 1/(2j+1)
- sequence 1/n

Observation : "Bolzano weirstrass thm is applicable" and "sequence is convergent"

8. Study the convergence/divergence

of infinite series of real numbers by plotting their sequences of partial sum.

In[ ]:=
```
ClearAll
```

Out[ ]=
```
ClearAll
```

1Q.Study the convergence or divergence of the infinite series a[n_]=$\sum_{i=1}^{n} \frac{1}{(4*i^2)-1}$ by Plotting the sequence of Partial sum

In[ ]:=
```
b[n_] := Sum[1/((4*i^2) - 1), {i, 1, n}];
```

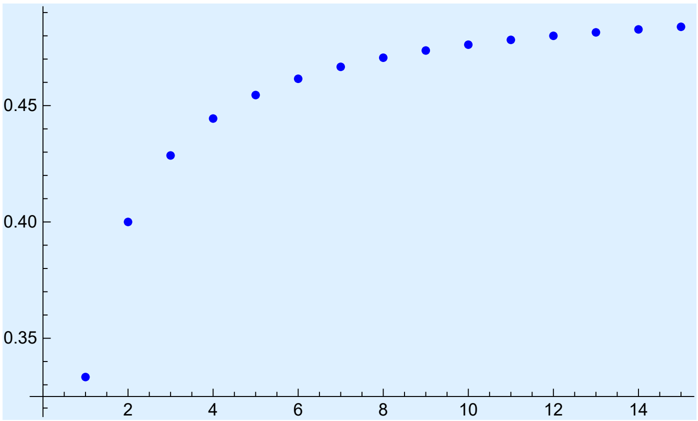
In[ ]:=
```
mk = Table[{n, N[b[n]]}, {n, 15}];
```

In[ ]:=
```
TableForm[mk, TableHeadings → {None , {"n", "nth Partial sum"}}]
```

Out[ ]//TableForm=

n	nth Partial sum
1	0.333333
2	0.4
3	0.428571
4	0.444444
5	0.454545
6	0.461538
7	0.466667
8	0.470588
9	0.473684
10	0.47619
11	0.478261
12	0.48
13	0.481481
14	0.482759
15	0.483871

In[ ]:= ListPlot[mk, PlotStyle → Blue, Background → LightBlue]

Out[ ]= 

In[ ]:= Limit[b[n], n → Infinity]

Out[ ]= $\frac{1}{2}$

#Obervation : Series is convergent

2Q-Study the convergence or divergence of the infinite series a[n_]:= $1/\left(\sqrt{i\,(1+i)}\right)$ by Plotting the sequence of Partial sum

In[ ]:= a[n_] := $\sum_{i=1}^{n}\left(1/\left(\sqrt{i\,(1+i)}\right)\right)$;

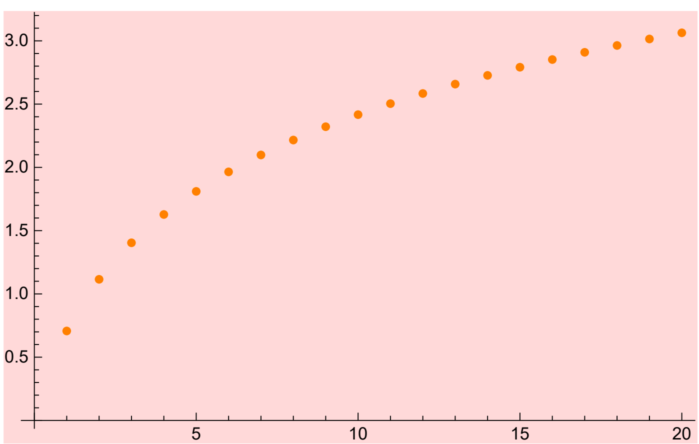
In[∘]:=
```
abc = Table[{n, N[a[n]]}, {n, 20}];
TableForm[abc, TableHeadings → {None, {"n", "nth Partial sum"}}]
```

Out[∘]//TableForm=

n	nth Partial sum
1	0.707107
2	1.11536
3	1.40403
4	1.62764
5	1.81021
6	1.96451
7	2.09815
8	2.216
9	2.32141
10	2.41675
11	2.50379
12	2.58385
13	2.65798
14	2.72699
15	2.79154
16	2.85217
17	2.90934
18	2.96341
19	3.01471
20	3.0635

In[∘]:=
```
ListPlot[abc, PlotStyle → Orange, Background → LightRed]
```

Out[∘]=



In[∘]:=
```
Limit[a[n], n → ∞]
```

Out[∘]=

$$\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{1}{\sqrt{i\,(1+i)}}\right)$$

Observation : Series is Divergent

Q3 : Study the convergence or divergence of the infinite series

$a[n\_] = \sum_{i=1}^{n} \left(\frac{1}{\sqrt{i}} - \frac{1}{\sqrt{i+1}}\right)$ by plotting the sequence of Partial sum

In[ ]:= $a[n\_] = \sum_{i=1}^{n} \left(\frac{1}{\sqrt{i}} - \frac{1}{\sqrt{i+1}}\right);$

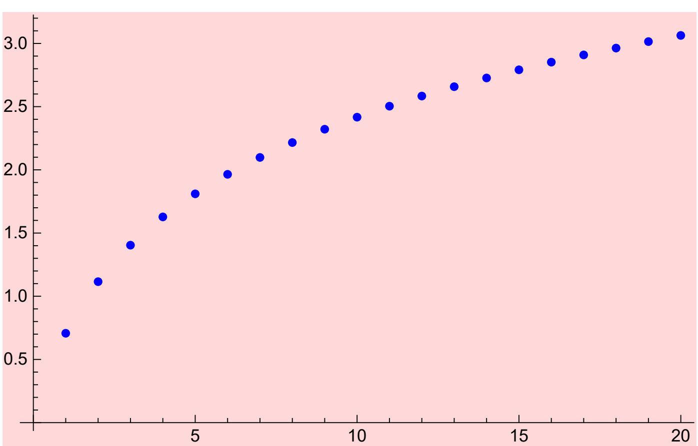
In[ ]:=
```
sol16 = Table[{n, N[a[n]]}, {n, 14}];
TableForm[sol16, TableHeadings → {None, {"n", "nth Partial sum"}}]
```

Out[ ]//TableForm=

n	nth Partial sum
1	0.292893
2	0.42265
3	0.5
4	0.552786
5	0.591752
6	0.622036
7	0.646447
8	0.666667
9	0.683772
10	0.698489
11	0.711325
12	0.72265
13	0.732739
14	0.741801

TLiListPlot[abc, PlotStyle → Orange, Background → LightRed] ×
ListPlot[abc, PlotStyle → Orange, Background → LightRed] ×
stPlot[abc, PlotStyle → Orange, Background → LightRed] ×
TableForm[abc, TableHeadings → {None, {"n", "nth Partial sum"}}] ×
ableForm[abc, TableHeadings → {None, {"n", "nth Partial sum"}}]

In[ ]:=
```
ListPlot[abc, PlotStyle → Blue, Background → LightRed]
```

Out[ ]=


In[•]:= `Limit[a[n], n → Infinity]`

Out[•]= 1

Observation : Series is Convergent

Q4 : Study the convergence or divergence of the infinite series $a[n\_] = \sum_{i=1}^{n}$ (Log[i+1]-Log[i]) by plotting the sequence of Partial sum

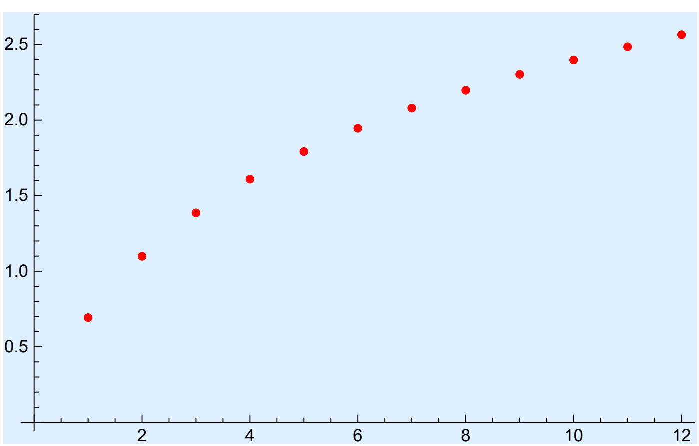
In[•]:= $a[n\_] = \sum_{i=1}^{n} (\text{Log}[i+1] - \text{Log}[i]);$

In[•]:=
```
sol16 = Table[{n, N[a[n]]}, {n, 12}];
TableForm[sol16, TableHeadings → {None, {"n", "nth Partial sum"}}]
```

Out[•]//TableForm=

n	nth Partial sum
1	0.693147
2	1.09861
3	1.38629
4	1.60944
5	1.79176
6	1.94591
7	2.07944
8	2.19722
9	2.30259
10	2.3979
11	2.48491
12	2.56495

In[•]:= `ListPlot[sol16, PlotStyle → Red, Background → LightBlue]`

Out[•]=


In[•]:= `Limit[a[n], n → Infinity]`

Out[•]= ∞

Observation : Series is divergent

In[ ]:=
```
ClearAll
```
Out[ ]=
```
ClearAll
```

9. Cauchy’s root test by plotting nth roots

QCheck the convergence for the series a[n] by plotting the nth term

In[ ]:= $a[n\_] := \frac{n}{n^n};$

In[ ]:= $caRoot[n\_] := \sqrt[n]{Abs[a[n]]};$

In[ ]:=
```
ans = Table[{n, caRoot[n]}, {n, 1, 30}];
```

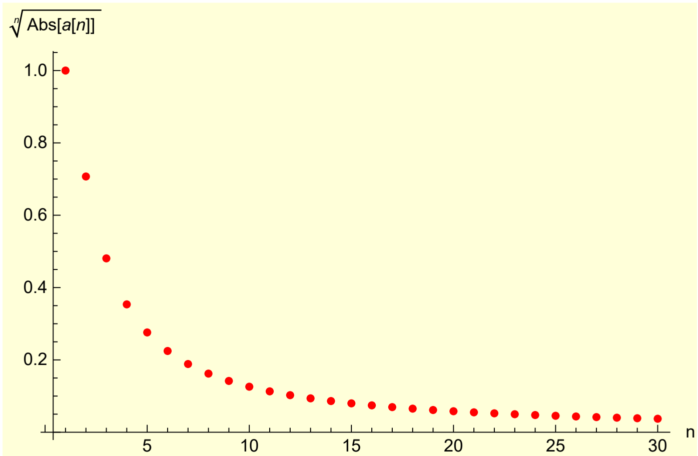
In[ ]:= TableForm[Table[{n, N[caRoot[n]]}, {n, 1, 20}],
 TableHeadings → {{}, {"n", "$\sqrt[n]{Abs[a[n]]}$"}}]

Out[ ]//TableForm=

n	$\sqrt[n]{Abs[a[n]]}$
1	1.
2	0.707107
3	0.48075
4	0.353553
5	0.275946
6	0.224668
7	0.188638
8	0.162105
9	0.141835
10	0.125893
11	0.113052
12	0.102506
13	0.0937011
14	0.0862459
15	0.0798573
16	0.0743254
17	0.0694913
18	0.0652326
19	0.0614539
20	0.0580793

```
In[•]:= ListPlot[ans, PlotStyle → Red, Background → LightYellow, PlotRange → All,
  PlotLegends → "Graph of ⁿ√Abs[a[n]] ", AxesLabel → {"n", " ⁿ√Abs[a[n]] "}]
```

Out[•]=



Graph of $\sqrt[n]{\text{Abs}[a[n]]}$

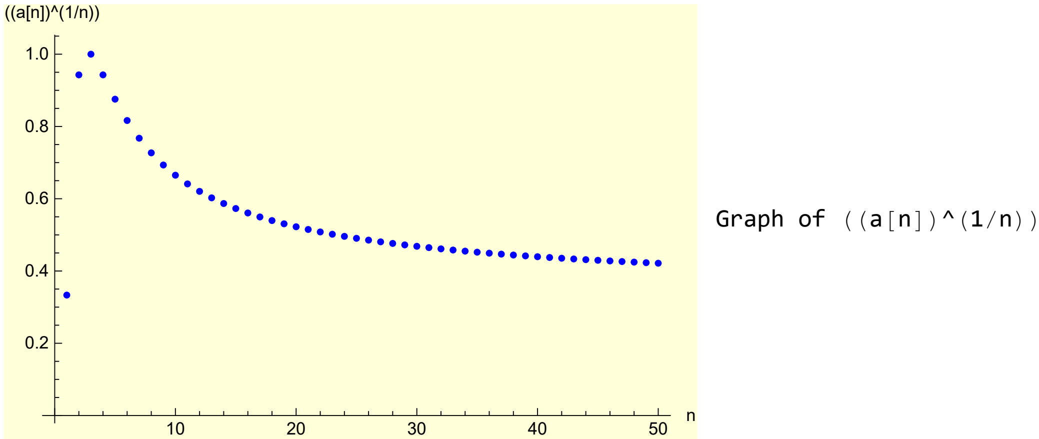
```
In[•]:= Limit[caRoot[n], n → Infinity]

Out[•]= 0
```

```
In[•]:= a[n_] := (n^3/3^n);

In[•]:= cauch[n_] := ((a[n])^(1/n));

In[•]:= sol18 = Table[{n, cauch[n]}, {n, 1, 50}];

In[•]:= TableForm[Table[{n, N[cauch[n]]}, {n, 1, 20}],
  TableHeadings → {{}, {"n", "((a[n])^(1/n))"}}]
```

Out[•]//TableForm=

n	((a[n])^(1/n))
1	0.333333
2	0.942809
3	1.
4	0.942809
5	0.875509
6	0.816497
7	0.767474
8	0.727005
9	0.693361
10	0.665087
11	0.641054
12	0.620403
13	0.60248
14	0.586783
15	0.572924
16	0.560598
17	0.549562
18	0.539623
19	0.530624
20	0.522436

In[ ]:=
```
ListPlot[sol18, PlotStyle → Blue, Background → LightYellow, PlotRange → All,
 PlotLegends → "Graph of ((a[n])^(1/n))", AxesLabel → {"n", "((a[n])^(1/n)) "}]
```

Out[ ]=


In[ ]:=
```
Limit[cauch[n], n → Infinity]
```

Out[ ]= $\frac{1}{3}$

In[ ]:= `a[n_] := ` $\left(1+\frac{1}{\sqrt{n}}\right)^{\left(-n^{(3/2)}\right)}$`;`

In[ ]:= `caroot[n_] := ((a[n])^(1/n));`

In[ ]:= `sol19 = Table[{n, caroot[n]}, {n, 1, 50}];`

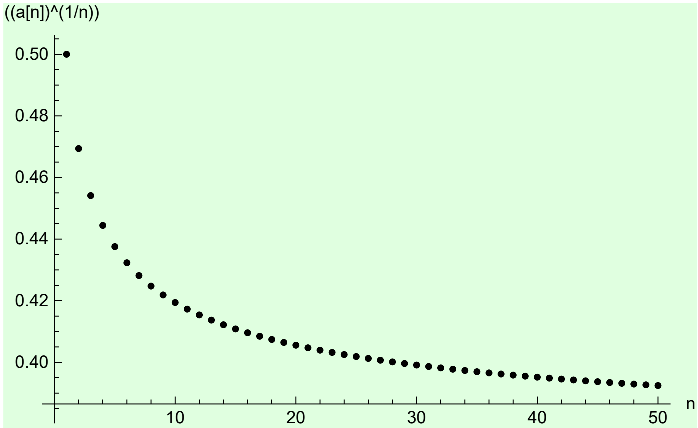
In[ ]:=
```
TableForm[Table[{n, N[caroot[n]]}, {n, 1, 20}],
 TableHeadings → {{}, {"n", "((a[n])^(1/n))"}}]
```

Out[ ]//TableForm=

n	((a[n])^(1/n))
1	0.5
2	0.46939
3	0.454128
4	0.444444
5	0.437561
6	0.432326
7	0.428165
8	0.424748
9	0.421875
10	0.419413
11	0.417272
12	0.415386
13	0.413708
14	0.412202
15	0.41084
16	0.4096
17	0.408465
18	0.40742
19	0.406455
20	0.405559

In[∘]:=
```
ListPlot[sol19, PlotStyle → Black, Background → LightGreen, PlotRange → All,
 PlotLegends → "Graph of ((a[n])^(1/n))", AxesLabel → {"n", "((a[n])^(1/n)) "}]
```

Out[∘]=



Graph of ((a[n])^(1/n))

In[∘]:=
```
Limit[caroot[n], n → Infinity]
```

Out[∘]= $\frac{1}{e}$

In[∘]:= $a[n\_] := \left(\frac{e^n}{n^n}\right);$

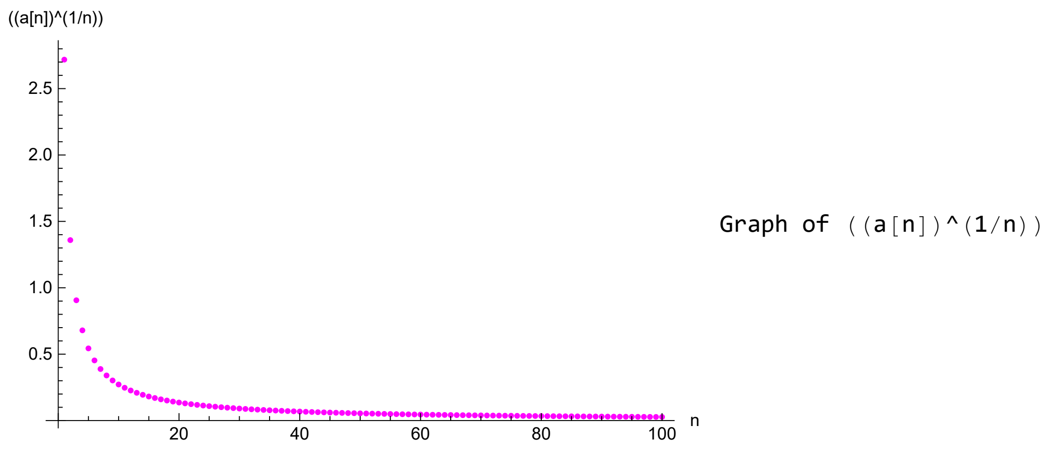
In[∘]:=
```
cac[n_] := ((a[n])^(1/n));
sol20 = Table[{n, cac[n]}, {n, 1, 100}];
TableForm[Table[{n, N[cac[n]]}, {n, 1, 20}],
 TableHeadings → {{}, {"n", "((a[n])^(1/n))"}}]
```

Out[∘]//TableForm=

n	((a[n])^(1/n))
1	2.71828
2	1.35914
3	0.906094
4	0.67957
5	0.543656
6	0.453047
7	0.388326
8	0.339785
9	0.302031
10	0.271828
11	0.247117
12	0.226523
13	0.209099
14	0.194163
15	0.181219
16	0.169893
17	0.159899
18	0.151016
19	0.143067
20	0.135914

```
In[]:= ListPlot[sol20, PlotStyle → Magenta, PlotRange → All,
        PlotLegends → "Graph of ((a[n])^(1/n))", AxesLabel → {"n", "((a[n])^(1/n)) "}]
```

Out[]=



10. D’Alembert’s ratio test by plotting the ratio of nth and (n+1)th term of the given series of positive terms

In[]:= $a[n\_] := \frac{n!}{n^n};$

$DalRatio[n\_] := \frac{a[n+1]}{a[n]};$

In[◦]:=
```
hemant = Table[{n, DalRatio[n]}, {n, 1, 30}];
TableForm[Table[{n, N[DalRatio[n]]}, {n, 1, 20}], TableHeadings → {{}, {"n", "a[n + 1]/a[n]"}}]
```

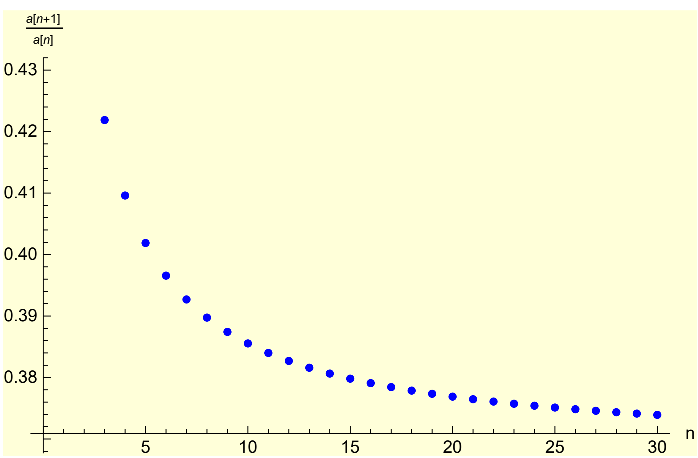
Out[◦]//TableForm=

n	$\frac{a[n+1]}{a[n]}$
1	0.5
2	0.444444
3	0.421875
4	0.4096
5	0.401878
6	0.396569
7	0.392696
8	0.389744
9	0.38742
10	0.385543
11	0.383995
12	0.382697
13	0.381592
14	0.38064
15	0.379812
16	0.379085
17	0.378442
18	0.377868
19	0.377354
20	0.376889

In[◦]:=
```
ListPlot[hemant, PlotRange → {{}, {0.5, 1}}, Background → LightYellow,
 PlotStyle → Blue, PlotLegends → "Graph of a[n + 1]/a[n]", AxesLabel → {"n", "a[n + 1]/a[n]"}]
```

ListPlot: Value of option PlotRange -> {{}, {0.5, 1}} is not All, Full, Automatic, a positive machine number, or an appropriate list of range specifications.

Out[◦]=



Graph of $\frac{a[n+1]}{a[n]}$

In[◦]:=
```
Limit[DalRatio[n], n → Infinity]
```

Out[◦]=

$$\frac{1}{e}$$

#Observation := From the above plot it is observed that the ration of nth and $(n+1)$ th term of given series is convergent to a limit i.e $\frac{1}{e}$, hence applying Dal embert ration test the series is Convergent since Limit[DalRatio[n], n → Infinity] is $\frac{1}{e}$ which is less then 1.

Ques2

In[ ]:=
```
b[n_] := (2^n)/(n^2 + 1);
DalRatio1[n_] := b[n + 1]/b[n] ;
```

In[ ]:=
```
hemant2 = Table[{n, DalRatio1[n]}, {n, 1, 300}];
TableForm[Table[{n, N[DalRatio1[n]]}, {n, 1, 20}],
 TableHeadings → {{}, {"n", "a[n + 1]/a[n]"}}]
```

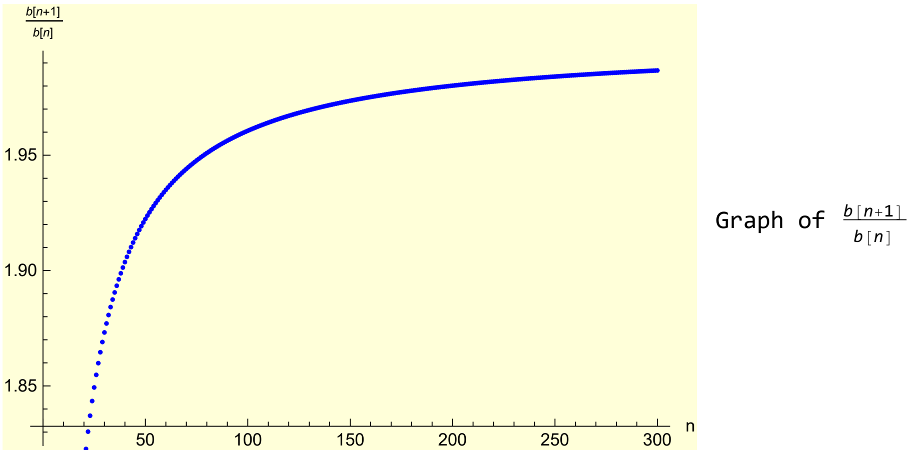
Out[ ]//TableForm=

n	$\frac{a[n+1]}{a[n]}$
1	0.8
2	1.
3	1.17647
4	1.30769
5	1.40541
6	1.48
7	1.53846
8	1.58537
9	1.62376
10	1.65574
11	1.68276
12	1.70588
13	1.72589
14	1.74336
15	1.75875
16	1.77241
17	1.78462
18	1.79558
19	1.80549
20	1.81448

In[ ]:=
```
ListPlot[hemant2, PlotRange → {{}, {0.5, 1}}, Background → LightYellow,
 PlotStyle → Blue, PlotLegends → "Graph of b[n + 1]/b[n]", AxesLabel → {"n", "b[n + 1]/b[n]"}]
```

ListPlot: Value of option PlotRange –> {{}, {0.5, 1}} is not All, Full, Automatic, a positive machine number, or an appropriate list of range specifications.

Out[ ]=



In[ ]:=
```
Limit[DalRatio1[n], n → Infinity]
```

Out[ ]= 2

#Observation := From the above plot it is observed that the ration of nth and (n + 1) th term of given series is convergent to a limit i.e 2, hence applying Dal embert ration test the series is divergent since Limit[DalRatio[n], n → Infinity] is 2 which is less then 1

Ques3

In[ ]:=
$$c[n\_] := \frac{2^n + n!}{n^n};$$

In[ ]:=
$$DalRatio12[n\_] := \frac{c[n+1]}{c[n]};$$

In[◦]:=
```
hemant3 = Table[{n, DalRatio12[n]}, {n, 1, 300}];
TableForm[Table[{n, N[DalRatio12[n]]}, {n, 1, 20}],
 TableHeadings → {{}, {"n", "c[n + 1]/c[n]"}}]
```

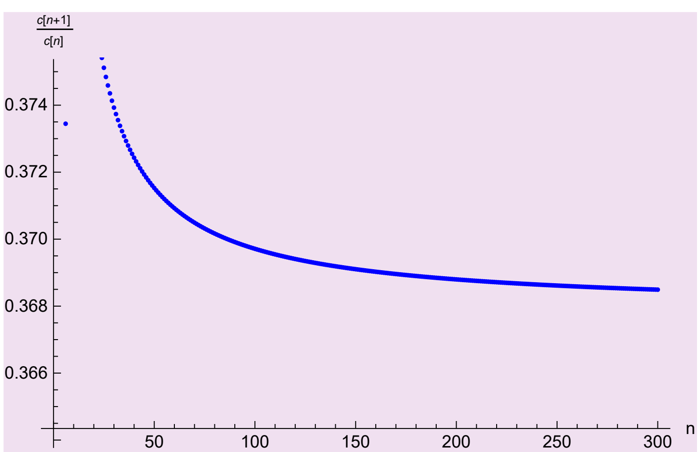
Out[◦]//TableForm=

n	$\frac{c[n+1]}{c[n]}$
1	0.5
2	0.345679
3	0.301339
4	0.311296
5	0.345474
6	0.373446
7	0.385401
8	0.387832
9	0.386984
10	0.385454
11	0.383979
12	0.382694
13	0.381591
14	0.38064
15	0.379812
16	0.379085
17	0.378442
18	0.377868
19	0.377354
20	0.376889

In[◦]:=
```
ListPlot[hemant3, PlotRange → {{}, {0.5, 1}}, Background → LightPurple,
 PlotStyle → Blue, PlotLegends → "Graph of c[n + 1]/c[n]", AxesLabel → {"n", "c[n + 1]/c[n]"}]
```

ListPlot: Value of option PlotRange -> {{}, {0.5, 1}} is not All, Full, Automatic, a positive machine number, or an appropriate list of range specifications.

Out[◦]=



Graph of $\frac{c[n+1]}{c[n]}$

In[◦]:=
```
Limit[DalRatio12[n], n → Infinity]
```

Out[◦]= $\frac{1}{e}$

#Observation := From the above plot it is observed that the ration of nth and $(n+1)$ th term of given series is convergent to a limit i.e $\frac{1}{e}$, hence applying Dal embert ration test the series is Convergent since Limit[DalRatio[n], n → Infinity] is $\frac{1}{e}$ which is less then 1.

11Q.For the following sequence <a_n> ,ϵ=1/2^k,p=10^j,k=0,1,2,4.. find m ϵ N such that:-

Ques1):For the following sequence <a_n> ,Given pϵN find mϵN Such that (i)$|a_{m+p}-a_m|<\epsilon$,(ii)$|a_{2m+p}-a_{2m}|<\epsilon$

```
In[*]:= a[n_] := (n + 1) / n;
ϵ = Table[1 / 2^k, {k, {0, 1, 2, 5}}];
p = Table[10^j, {j, {1, 2, 3, 4, 5}}];
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[m + p[[j]]] - a[m]] ≥ ϵ[[k]], m++] ×
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a2m+p-a2m|<ϵ", "when m=", m]]]
```

For p=10 and $\in 1 | a_{2m+p} - a_{2m} | < \in$when m=1

For p=100 and $\in 1 | a_{2m+p} - a_{2m} | < \in$when m=1

For p=1000 and $\in 1 | a_{2m+p} - a_{2m} | < \in$when m=1

For p=10 000 and $\in 1 | a_{2m+p} - a_{2m} | < \in$when m=1

For p=10 and $\in \frac{1}{2} | a_{2m+p} - a_{2m} | < \in$when m=2

For p=100 and $\in \frac{1}{2} | a_{2m+p} - a_{2m} | < \in$when m=2

For p=1000 and $\in \frac{1}{2} | a_{2m+p} - a_{2m} | < \in$when m=2

For p=10 000 and $\in \frac{1}{2} | a_{2m+p} - a_{2m} | < \in$when m=2

For p=10 and $\in \frac{1}{4} | a_{2m+p} - a_{2m} | < \in$when m=4

For p=100 and $\in \frac{1}{4} | a_{2m+p} - a_{2m} | < \in$when m=4

For p=1000 and $\in \frac{1}{4} | a_{2m+p} - a_{2m} | < \in$when m=4

For p=10 000 and $\in \frac{1}{4} | a_{2m+p} - a_{2m} | < \in$when m=4

In[∘]:=
```
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[2 m + p[[j]]] - a[2 m]] ≥ ϵ[[k]], m++] ×
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a2m+p-a2m|<ϵ", "when m=", m]]]
```

For p=10 and $\in 1 | a_{2m+p} - a_{2m} | < \in$when m=1

For p=100 and $\in 1 | a_{2m+p} - a_{2m} | < \in$when m=1

For p=1000 and $\in 1 | a_{2m+p} - a_{2m} | < \in$when m=1

For p=10 000 and $\in 1 | a_{2m+p} - a_{2m} | < \in$when m=1

For p=10 and $\in \frac{1}{2} | a_{2m+p} - a_{2m} | < \in$when m=1

For p=100 and $\in \frac{1}{2} | a_{2m+p} - a_{2m} | < \in$when m=1

For p=1000 and $\in \frac{1}{2} | a_{2m+p} - a_{2m} | < \in$when m=1

For p=10 000 and $\in \frac{1}{2} | a_{2m+p} - a_{2m} | < \in$when m=1

For p=10 and $\in \frac{1}{4} | a_{2m+p} - a_{2m} | < \in$when m=2

For p=100 and $\in \frac{1}{4} | a_{2m+p} - a_{2m} | < \in$when m=2

For p=1000 and $\in \frac{1}{4} | a_{2m+p} - a_{2m} | < \in$when m=2

For p=10 000 and $\in \frac{1}{4} | a_{2m+p} - a_{2m} | < \in$when m=2

In[ ]:= ClearAll

Out[ ]= ClearAll

Ques2:For the following sequence $<a_n>$,given $\epsilon>0$ and pϵN find me N such that (i) $a_{m+p} - a_m \; |< \epsilon$,(ii) $| \; a_{2m+p} - a_{2m} \; |< \epsilon$.

In[ ]:=
```
a[n_] := 1/n;
ϵ = Table[1/2^k, {k, {0, 1, 2, 5}}];
p = Table[10^j, {j, {1, 2, 3, 4, 5}}];
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[m + p[[j]]] - a[m]] ≥ ϵ[[k]], m++]
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a_{m+p}-a_m|<ϵ", "when m=", m]]]
```

For p=10 and $\epsilon 1 \, | a_{m+p} - a_m | < \epsilon$when m=1

For p=100 and $\epsilon 1 \, | a_{m+p} - a_m | < \epsilon$when m=1

For p=1000 and $\epsilon 1 \, | a_{m+p} - a_m | < \epsilon$when m=1

For p=10 000 and $\epsilon 1 \, | a_{m+p} - a_m | < \epsilon$when m=1

For p=10 and $\epsilon \frac{1}{2} | a_{m+p} - a_m | < \epsilon$when m=2

For p=100 and $\epsilon \frac{1}{2} | a_{m+p} - a_m | < \epsilon$when m=2

For p=1000 and $\epsilon \frac{1}{2} | a_{m+p} - a_m | < \epsilon$when m=2

For p=10 000 and $\epsilon \frac{1}{2} | a_{m+p} - a_m | < \epsilon$when m=2

For p=10 and $\epsilon \frac{1}{4} | a_{m+p} - a_m | < \epsilon$when m=4

For p=100 and $\epsilon \frac{1}{4} | a_{m+p} - a_m | < \epsilon$when m=4

For p=1000 and $\epsilon \frac{1}{4} | a_{m+p} - a_m | < \epsilon$when m=4

For p=10 000 and $\epsilon \frac{1}{4} | a_{m+p} - a_m | < \epsilon$when m=4

In[ ]:=
```
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[2 m + p[[j]]] - a[2 m]] ≥ ϵ[[k]], m++]
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a_{2m+p}-a_{2m}|<ϵ", "when m=", m]]]
```

For p=10 and ∈1 | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=100 and ∈1 | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=1000 and ∈1 | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=10 000 and ∈1 | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=10 and ∈ $\frac{1}{2}$ | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=100 and ∈ $\frac{1}{2}$ | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=1000 and ∈ $\frac{1}{2}$ | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=10 000 and ∈ $\frac{1}{2}$ | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=10 and ∈ $\frac{1}{4}$ | $a_{2m+p}-a_{2m}$ | <∈when m=2

For p=100 and ∈ $\frac{1}{4}$ | $a_{2m+p}-a_{2m}$ | <∈when m=2

For p=1000 and ∈ $\frac{1}{4}$ | $a_{2m+p}-a_{2m}$ | <∈when m=2

For p=10 000 and ∈ $\frac{1}{4}$ | $a_{2m+p}-a_{2m}$ | <∈when m=2

In[∘]:= **ClearAll**

Out[∘]= ClearAll

Ques3:For the following sequence <a_n> ,given ϵ>0 and pϵN find me N such that (i) $a_{m+p} - a_m$ | < ϵ,(ii) | $a_{2m+p} - a_{2m}$ | < ϵ.

In[∘]:= $a[n\_] := \sum_{i=1}^{n} \frac{1}{i!}$;

In[∘]:= ϵ = Table[$\frac{1}{2^k}$, {k, {0, 1, 2}}];

```
p = Table[10^j, {j, {1, 2, 3}}];
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[m + p[[j]]] - a[m]] ≥ ϵ[[k]], m++] ×
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a_{m+p}-a_m|<ϵ", "when m=", m]]]
```

For p=10 and ∈1 | $a_{m+p}-a_m$ | <∈when m=1

For p=100 and ∈1 | $a_{m+p}-a_m$ | <∈when m=1

For p=10 and ∈ $\frac{1}{2}$ | $a_{m+p}-a_m$ | <∈when m=2

For p=100 and ∈ $\frac{1}{2}$ | $a_{m+p}-a_m$ | <∈when m=2

In[∘]:=
```
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[2 m + p[[j]]] - a[2 m]] ≥ ϵ[[k]], m++]
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a2 m+p-a2 m|<ϵ", "when m=", m]]]
```

For p=10 and ϵ1 $|a_{2m+p} - a_{2m}|$ <ϵwhen m=1

For p=100 and ϵ1 $|a_{2m+p} - a_{2m}|$ <ϵwhen m=1

For p=10 and ϵ $\frac{1}{2}$ $|a_{2m+p} - a_{2m}|$ <ϵwhen m=1

For p=100 and ϵ $\frac{1}{2}$ $|a_{2m+p} - a_{2m}|$ <ϵwhen m=1

In[∘]:= ClearAll

Out[∘]= ClearAll

Ques4:For the following sequence <a_n> ,given ϵ>0 and p∈N find me N such that (i) $a_{m+p} - a_m$ $|<\epsilon$,(ii) $|$ $a_{2m+p} - a_{2m}$ $|<\epsilon$.

In[∘]:=
```
a[n_] := (-1)^n / n!;
ϵ = Table[1/2^k, {k, {0, 1, 2, 5}}];
p = Table[10^j, {j, {1, 2, 3, 4}}];
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[m + p[[j]]] - a[m]] ≥ ϵ[[k]], m++]
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|am+p-a m|<ϵ", "when m=", m]]]
```

For p=10 and ϵ1 $|a_{m+p} - a_m|$ <ϵwhen m=1

For p=100 and ϵ1 $|a_{m+p} - a_m|$ <ϵwhen m=1

For p=1000 and ϵ1 $|a_{m+p} - a_m|$ <ϵwhen m=1

For p=10 and ϵ $\frac{1}{2}$ $|a_{m+p} - a_m|$ <ϵwhen m=2

For p=100 and ϵ $\frac{1}{2}$ $|a_{m+p} - a_m|$ <ϵwhen m=2

For p=1000 and ϵ $\frac{1}{2}$ $|a_{m+p} - a_m|$ <ϵwhen m=2

For p=10 and ϵ $\frac{1}{4}$ $|a_{m+p} - a_m|$ <ϵwhen m=3

For p=100 and ϵ $\frac{1}{4}$ $|a_{m+p} - a_m|$ <ϵwhen m=3

For p=1000 and ϵ $\frac{1}{4}$ $|a_{m+p} - a_m|$ <ϵwhen m=3

In[ ]:=
```
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[2 m + p[[j]]] - a[2 m]] ≥ ϵ[[k]], m++] ×
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a2m+p-a2m|<ϵ", "when m=", m]]]
```

For p=10 and $\epsilon 1|a_{2m+p}-a_{2m}|<\epsilon$when m=1

For p=100 and $\epsilon 1|a_{2m+p}-a_{2m}|<\epsilon$when m=1

For p=1000 and $\epsilon 1|a_{2m+p}-a_{2m}|<\epsilon$when m=1

For p=10 and $\epsilon \frac{1}{2}|a_{2m+p}-a_{2m}|<\epsilon$when m=1

For p=100 and $\epsilon \frac{1}{2}|a_{2m+p}-a_{2m}|<\epsilon$when m=1

For p=1000 and $\epsilon \frac{1}{2}|a_{2m+p}-a_{2m}|<\epsilon$when m=1

For p=10 and $\epsilon \frac{1}{4}|a_{2m+p}-a_{2m}|<\epsilon$when m=2

For p=100 and $\epsilon \frac{1}{4}|a_{2m+p}-a_{2m}|<\epsilon$when m=2

For p=1000 and $\epsilon \frac{1}{4}|a_{2m+p}-a_{2m}|<\epsilon$when m=2

Ques5:For the following sequence $<a_n>$,given $\epsilon>0$ and p∈N find me N such that (i) $a_{m+p} - a_m \; | < \epsilon$,(ii) $| \; a_{2m+p} - a_{2m} \; | < \epsilon$.

In[ ]:=
$$a[n\_] := \sum_{i=1}^{n} \frac{(-1)\text{^}i}{i!};$$

$$\epsilon = \text{Table}\left[\frac{1}{2^k}, \{k, \{0, 1, 2, 6, 7\}\}\right];$$

$$p = \text{Table}\left[10^j, \{j, \{1, 2, 3, 4\}\}\right];$$

```
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[m + p[[j]]] - a[m]] ≥ ϵ[[k]], m++] ×
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|am+p-am|<ϵ", "when m=", m]]]
```

For p=10 and $\in 1 \,|\, a_{m+p} - a_m \,| < \in$ when m=1

For p=100 and $\in 1 \,|\, a_{m+p} - a_m \,| < \in$ when m=1

For p=1000 and $\in 1 \,|\, a_{m+p} - a_m \,| < \in$ when m=1

For p=10 and $\in \frac{1}{2} \,|\, a_{m+p} - a_m \,| < \in$ when m=1

For p=100 and $\in \frac{1}{2} \,|\, a_{m+p} - a_m \,| < \in$ when m=1

For p=1000 and $\in \frac{1}{2} \,|\, a_{m+p} - a_m \,| < \in$ when m=1

For p=10 and $\in \frac{1}{4} \,|\, a_{m+p} - a_m \,| < \in$ when m=2

For p=100 and $\in \frac{1}{4} \,|\, a_{m+p} - a_m \,| < \in$ when m=2

For p=1000 and $\in \frac{1}{4} \,|\, a_{m+p} - a_m \,| < \in$ when m=2

For p=10 and $\in \frac{1}{64} \,|\, a_{m+p} - a_m \,| < \in$ when m=4

For p=100 and $\in \frac{1}{64} \,|\, a_{m+p} - a_m \,| < \in$ when m=4

For p=1000 and $\in \frac{1}{64} \,|\, a_{m+p} - a_m \,| < \in$ when m=4

In[ ]:=
```
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[2 m + p[[j]]] - a[2 m]] ≥ ϵ[[k]], m++]
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a_{2 m+p}-a_{2 m}|<ϵ", "when m=", m]]]
```

For p=10 and ∈1 | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=100 and ∈1 | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=1000 and ∈1 | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=10 and ∈$\frac{1}{2}$ | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=100 and ∈$\frac{1}{2}$ | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=1000 and ∈$\frac{1}{2}$ | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=10 and ∈$\frac{1}{4}$ | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=100 and ∈$\frac{1}{4}$ | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=1000 and ∈$\frac{1}{4}$ | $a_{2m+p}-a_{2m}$ | <∈when m=1

For p=10 and ∈$\frac{1}{64}$ | $a_{2m+p}-a_{2m}$ | <∈when m=2

For p=100 and ∈$\frac{1}{64}$ | $a_{2m+p}-a_{2m}$ | <∈when m=2

For p=1000 and ∈$\frac{1}{64}$ | $a_{2m+p}-a_{2m}$ | <∈when m=2

In[ ]:= **ClearAll**

Out[ ]= ClearAll

Ques6: For the following sequence <a_n> ,given ∈>0 and p∈N find me N such that (i) $a_{m+p} - a_m$ | < ∈,(ii) | $a_{2m+p} - a_{2m}$ | < ∈.

In[ ]:=
```
a[n_] := Sum[n^2/2^n, {i, 1, n}];
ϵ = Table[1/2^k, {k, {0, 1, 2, 4}}];
p = Table[10^j, {j, {1, 2, 3, 4}}];
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[m + p[[j]]] - a[m]] ≥ ϵ[[k]], m++] ×
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a_{m+p}-a_m|<ϵ", "when m=", m]]]
```

$$a[n\_] := \sum_{i=1}^{n} \frac{n^2}{2^n};$$

For p=10 and $\in 1\,|a_{m+p}-a_m|<\in$when m=1

For p=100 and $\in 1\,|a_{m+p}-a_m|<\in$when m=1

For p=1000 and $\in 1\,|a_{m+p}-a_m|<\in$when m=1

For p=10 and $\in \frac{1}{2}\,|a_{m+p}-a_m|<\in$when m=1

For p=100 and $\in \frac{1}{2}\,|a_{m+p}-a_m|<\in$when m=1

For p=1000 and $\in \frac{1}{2}\,|a_{m+p}-a_m|<\in$when m=1

For p=10 and $\in \frac{1}{4}\,|a_{m+p}-a_m|<\in$when m=1

For p=100 and $\in \frac{1}{4}\,|a_{m+p}-a_m|<\in$when m=14

For p=1000 and $\in \frac{1}{4}\,|a_{m+p}-a_m|<\in$when m=14

In[∘]:=
```
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[2 m + p[[j]]] - a[2 m]] ≥ ϵ[[k]], m++] ×
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a_{2 m+p}-a_{2 m}|<ϵ", "when m=", m]]]
```

For p=10 and $\in 1\,|a_{2m+p}-a_{2m}|<\in$when m=5

For p=100 and $\in 1\,|a_{2m+p}-a_{2m}|<\in$when m=5

For p=1000 and $\in 1\,|a_{2m+p}-a_{2m}|<\in$when m=5

For p=10 and $\in \frac{1}{2}\,|a_{2m+p}-a_{2m}|<\in$when m=6

For p=100 and $\in \frac{1}{2}\,|a_{2m+p}-a_{2m}|<\in$when m=6

For p=1000 and $\in \frac{1}{2}\,|a_{2m+p}-a_{2m}|<\in$when m=6

For p=10 and $\in \frac{1}{4}\,|a_{2m+p}-a_{2m}|<\in$when m=7

For p=100 and $\in \frac{1}{4}\,|a_{2m+p}-a_{2m}|<\in$when m=7

For p=1000 and $\in \frac{1}{4}\,|a_{2m+p}-a_{2m}|<\in$when m=7

Ques7:For the following sequence $<a_n>$,given ϵ>0 and p$\in$N find me N such that (i) $a_{m+p} - a_m \ |< \epsilon$,(ii) $|\ a_{2\,m+p} - a_{2\,m}\ |< \epsilon$.

In[ ]:=
```
a[n_] := Sum[(-1)^(n-1)/n, {i, 1, n}];
ϵ = Table[1/2^k, {k, {0, 1, 2, 4, 5}}];
p = Table[10^j, {j, {1, 2, 3, 4}}];
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[m + p[[j]]] - a[m]] ≥ ϵ[[k]], m++]
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a_(m+p)-a_m|<ϵ", "when m=", m]]]
```

For p=10 and ϵ1 $|a_{m+p}-a_m|<\epsilon$when m=1

For p=100 and ϵ1 $|a_{m+p}-a_m|<\epsilon$when m=1

For p=1000 and ϵ1 $|a_{m+p}-a_m|<\epsilon$when m=1

For p=10 and $\epsilon\frac{1}{2}|a_{m+p}-a_m|<\epsilon$when m=1

For p=100 and $\epsilon\frac{1}{2}|a_{m+p}-a_m|<\epsilon$when m=1

For p=1000 and $\epsilon\frac{1}{2}|a_{m+p}-a_m|<\epsilon$when m=1

For p=10 and $\epsilon\frac{1}{4}|a_{m+p}-a_m|<\epsilon$when m=1

For p=100 and $\epsilon\frac{1}{4}|a_{m+p}-a_m|<\epsilon$when m=1

For p=1000 and $\epsilon\frac{1}{4}|a_{m+p}-a_m|<\epsilon$when m=1

For p=10 and $\epsilon\frac{1}{16}|a_{m+p}-a_m|<\epsilon$when m=1

For p=100 and $\epsilon\frac{1}{16}|a_{m+p}-a_m|<\epsilon$when m=1

For p=1000 and $\epsilon\frac{1}{16}|a_{m+p}-a_m|<\epsilon$when m=1

In[ ]:=
```
For[k = 1, k < Length[ϵ], k++,
 For[j = 1, j < Length[p], j++,
  For[m = 1, Abs[a[2 m + p[[j]]] - a[2 m]] ≥ ϵ[[k]], m++]
   Print["For p=", p[[j]], " and ϵ", ϵ[[k]], "|a_(2m+p)-a_(2m)|<ϵ", "when m=", m]]]
```

For p=10 and $\epsilon 1 \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=100 and $\epsilon 1 \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=1000 and $\epsilon 1 \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=10 and $\epsilon \frac{1}{2} \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=100 and $\epsilon \frac{1}{2} \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=1000 and $\epsilon \frac{1}{2} \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=10 and $\epsilon \frac{1}{4} \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=100 and $\epsilon \frac{1}{4} \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=1000 and $\epsilon \frac{1}{4} \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=10 and $\epsilon \frac{1}{16} \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=100 and $\epsilon \frac{1}{16} \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

For p=1000 and $\epsilon \frac{1}{16} \, |a_{2m+p} - a_{2m}| < \epsilon$ when m=1

12. For the following series $\sum_{\square=\square}^{\square} a_n$, calculate i) $\left|\frac{a_{n+1}}{a_n}\right|$, ii) $(\,|\,a_n\,|\,)^{\frac{1}{n}}$, for n=$10^j$,j=1,2,3,....... and identify the convergent series, where n a is given as:

Ques1 $\left(\frac{1}{n}\right)^{\frac{1}{n}}$

```
In[•]:= a[n_] := (1/n)^(1/n);

In[•]:= cauch[n_] := (a[n])^(1/n);

In[•]:= dalembert[n_] := a[n + 1]/a[n];

In[•]:= l1 = Limit[cauch[n], n → Infinity];

In[•]:= l2 = Limit[dalembert[n], n → Infinity];

In[•]:= If[l1 < 1, Print["The seriers is convergent according to cauchy nth root test"],
         If[l1 > 1, Print["The seriers is divergent according to cauchy nth root test"],
          Print["The cauchy test fails "]]]

        The cauchy test fails

In[•]:= If[l2 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
         If[l2 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
          Print["The dalembert test fails "]]]

        The dalembert test fails

In[•]:= For[j = 1, j < 7, j++, n = 10^j;
         Print["For n=", n, "cauchy=", N[cauch[n]]]]

        For n=10cauchy=0.977237

        For n=100cauchy=0.99954

        For n=1000cauchy=0.999993

        For n=10 000cauchy=1.

        For n=100 000cauchy=1.

        For n=1 000 000cauchy=1.

In[•]:= For[j = 1, j < 7, j++, n = 10^j;
         Print["For n=", n, "Dalembert=", N[dalembert[n]]]]

        For n=10Dalembert=1.01234

        For n=100Dalembert=1.00036

        For n=1000Dalembert=1.00001

        For n=10 000Dalembert=1.

        For n=100 000Dalembert=1.

        For n=1 000 000Dalembert=1.

In[•]:= ClearAll

Out[•]= ClearAll
```

Ques2 . $\frac{1}{n}$

In[ ]:=
```
a[n_] := 1/n;
```

In[ ]:=
```
cauch1[n_] := (a[n])^(1/n);
dalembert1[n_] := a[n + 1]/a[n];
l3 = Limit[cauch1[n], n → Infinity];
l4 = Limit[dalembert1[n], n → Infinity];
```

In[ ]:=
```
If[l3 < 1, Print["The seriers is convergent according to cauchy nth root test"],
 If[l4 > 1, Print["The seriers is divergent according to cauchy nth root test"],
  Print["The cauchy test fails "]]]
```

```
The cauchy test fails
```

In[ ]:=
```
If[l3 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
 If[l4 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
  Print["The dalembert test fails "]]]
```

```
The dalembert test fails
```

In[ ]:=
```
For[j = 1, j < 7, j++, n = 10^j;
 Print["For n=", n, "cauchy1=", N[cauch1[n]]]]
```

```
For n=10cauchy1=0.794328
For n=100cauchy1=0.954993
For n=1000cauchy1=0.993116
For n=10 000cauchy1=0.999079
For n=100 000cauchy1=0.999885
For n=1 000 000cauchy1=0.999986
```

In[ ]:=
```
For[j = 1, j < 7, j++, n = 10^j;
 Print["For n=", n, "Dalembert=", N[dalembert1[n]]]]
```

```
For n=10Dalembert=0.909091
For n=100Dalembert=0.990099
For n=1000Dalembert=0.999001
For n=10 000Dalembert=0.9999
For n=100 000Dalembert=0.99999
For n=1 000 000Dalembert=0.999999
```

In[ ]:=
```
ClearAll
```

Out[ ]=
```
ClearAll
```

Ques3 . $\frac{1}{n^2}$;

```
In[]:= a[n_] := 1/n^2;

In[]:= cauch2[n_] := (a[n])^(1/n);
       dalembert2[n_] := a[n + 1]/a[n];
       l5 = Limit[cauch2[n], n → Infinity];
       l6 = Limit[dalembert2[n], n → Infinity];

In[]:= If[l5 < 1, Print["The seriers is convergent according to cauchy nth root test"],
        If[l5 > 1, Print["The seriers is divergent according to cauchy nth root test"],
         Print["The cauchy test fails "]]]

       The cauchy test fails

In[]:= If[l6 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
        If[l6 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
         Print["The dalembert test fails "]]]

       The dalembert test fails

In[]:= For[j = 1, j < 7, j++, n = 10^j;
        Print["For n=", n, "cauchy2=", N[cauch2[n]]]]

       For n=10cauchy2=0.630957
       For n=100cauchy2=0.912011
       For n=1000cauchy2=0.986279
       For n=10 000cauchy2=0.99816
       For n=100 000cauchy2=0.99977
       For n=1 000 000cauchy2=0.999972

In[]:= For[j = 1, j < 7, j++, n = 10^j;
        Print["For n=", n, "Dalembert2=", N[dalembert2[n]]]]

       For n=10Dalembert2=0.826446
       For n=100Dalembert2=0.980296
       For n=1000Dalembert2=0.998003
       For n=10 000Dalembert2=0.9998
       For n=100 000Dalembert2=0.99998
       For n=1 000 000Dalembert2=0.999998

In[]:= ClearAll
Out[]= ClearAll
```

Ques4 . $\left(1 + \frac{1}{n^{1/2}}\right)^{-n^{\frac{3}{2}}}$;

```
In[•]:= a[n_] := (1 + 1/√n)^(-n^(3/2));

In[•]:= cauch3[n_] := (a[n])^(1/n);
        dalembert3[n_] := a[n + 1]/a[n];

In[•]:= l7 = Limit[cauch3[n], n → Infinity];
        l8 = Limit[dalembert3[n], n → Infinity];
        If[l7 < 1, Print["The seriers is convergent according to cauchy nth root test"],
         If[l7 > 1, Print["The seriers is divergent according to cauchy nth root test"],
          Print["The cauchy test fails "]]]

        The seriers is convergent according to cauchy nth root test

In[•]:= If[l8 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
         If[l8 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
          Print["The dalembert test fails "]]]

        The seriers is convergent according to dalembert nth ratio test

In[•]:= For[j = 1, j < 7, j++, n = 10^j;
         Print["For n=", n, "cauchy3=", N[cauch3[n]]]]

        For n=10cauchy3=0.419413

        For n=100cauchy3=0.385543

        For n=1000cauchy3=0.373621

        For n=10 000cauchy3=0.369711

        For n=100 000cauchy3=0.36846

In[•]:= For[j = 1, j < 3, j++, n = 10^j;
         Print["For n=", n, "Dalembert3=", N[dalembert3[n]]]]

        For n=10Dalembert3=dalembert3[10.]

        For n=100Dalembert3=dalembert3[100.]

In[•]:= ClearAll

Out[•]= ClearAll
```

Ques:5 a[n_] := $\frac{n!}{n^n}$;

```
In[•]:= a[n_] := n!/n^n;
```

```
In[•]:= cauch4[n_] := (a[n])^(1/n);
        dalembert4[n_] := a[n + 1] / a[n];

In[•]:= l9 = Limit[cauch4[n], n → Infinity];
        l10 = Limit[dalembert4[n], n → Infinity];

In[•]:= If[l9 < 1, Print["The seriers is convergent according to cauchy nth root test"],
         If[l9 > 1, Print["The seriers is divergent according to cauchy nth root test"],
          Print["The cauchy test fails "]]]

        The seriers is convergent according to cauchy nth root test

In[•]:= If[l10 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
         If[l10 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
          Print["The dalembert test fails "]]]

        The seriers is convergent according to dalembert nth ratio test

In[•]:= For[j = 1, j < 7, j++, n = 10^j;
         Print["For n=", n, "cauchy4=", N[cauch4[n]]]]

        For n=10cauchy4=0.452873
        For n=100cauchy4=0.379927
        For n=1000cauchy4=0.369492
        For n=10 000cauchy4=0.368083
        For n=100 000cauchy4=0.367904
        For n=1 000 000cauchy4=0.367882

        For[j = 1, j < 7, j++, n = 10^j;
         Print["For n=", n, "Dalembert4=", N[dalembert4[n]]]]

        For n=10Dalembert4=0.385543
        For n=100Dalembert4=0.369711
        For n=1000Dalembert4=0.368063
        For n=10 000Dalembert4=0.367898
        For n=100 000Dalembert4=0.367881
        For n=1 000 000Dalembert4=0.36788
```

Ques6 : $\frac{n^3 \_+5}{3^n+2}$

```
In[•]:= a[n_] := (n^3 + 5) / (3^n + 2);

In[•]:= cauch5[n_] := (a[n])^(1/n);
        dalembert5[n_] := a[n + 1] / a[n];
```

```
In[•]:= l11 = Limit[cauch5[n], n → Infinity];
        l12 = Limit[dalembert5[n], n → Infinity];

In[•]:= If[l11 < 1, Print["The seriers is convergent according to cauchy nth root test"],
         If[l11 > 1, Print["The seriers is divergent according to cauchy nth root test"],
          Print["The cauchy test fails "]]]

        The seriers is convergent according to cauchy nth root test

In[•]:= If[l12 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
         If[l12 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
          Print["The dalembert test fails "]]]

        The seriers is convergent according to dalembert nth ratio test

In[•]:= For[j = 1, j < 3, j++, n = 10^j;
         Print["For n=", n, "cauchy5=", N[cauch5[n]]]]

        For n=10cauchy5=0.665417

        For n=100cauchy5=0.382718

In[•]:= For[j = 1, j < 7, j++, n = 10^j;
         Print["For n=", n, "Dalembert5=", N[dalembert5[n]]]]

        For n=10Dalembert5=0.443128

        For n=100Dalembert5=0.343434

        For n=1000Dalembert5=0.334334

        For n=10 000Dalembert5=0.333433

        For n=100 000Dalembert5=0.333343

        For n=1 000 000Dalembert5=0.333334

In[•]:= ClearAll

Out[•]= ClearAll
```

Ques7 : $\frac{1}{n^2+n}$;

```
In[•]:= a[n_] := 1/(n^2 + n);

In[•]:= cauch6[n_] := (a[n])^(1/n);
        dalembert6[n_] := a[n + 1]/a[n];

In[•]:= l13 = Limit[cauch6[n], n → Infinity];
        l14 = Limit[dalembert6[n], n → Infinity];

In[•]:= If[l13 < 1, Print["The seriers is convergent according to cauchy nth root test"],
         If[l13 > 1, Print["The seriers is divergent according to cauchy nth root test"],
          Print["The cauchy test fails "]]]

        The cauchy test fails
```

```
In[]:= If[l14 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
        If[l14 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
         Print["The dalembert test fails "]]]
```

The dalembert test fails

```
In[]:= For[j = 1, j < 7, j++, n = 10^j;
        Print["For n=", n, "cauchy6=", N[cauch6[n]]]]
```

For n=10cauchy6=$Null^{1/10}$

For n=100cauchy6=$Null^{1/100}$

For n=1000cauchy6=$Null^{1/1000}$

For n=10 000cauchy6=$Null^{1/10\,000}$

For n=100 000cauchy6=$Null^{1/100\,000}$

For n=1 000 000cauchy6=$Null^{1/1\,000\,000}$

```
In[]:= For[j = 1, j < 7, j++, n = 10^j;
        Print["For n=", n, "Dalembert6=", N[dalembert6[n]]]]
```

For n=10Dalembert6=1.

For n=100Dalembert6=1.

For n=1000Dalembert6=1.

For n=10 000Dalembert6=1.

For n=100 000Dalembert6=1.

For n=1 000 000Dalembert6=1.

Ques8 : $\frac{1}{\sqrt{n+1}}$

```
In[]:= a[n_] := 1/Sqrt[n + 1];
```

```
In[]:= cauch7[n_] := (a[n])^(1/n);
       dalembert7[n_] := a[n + 1]/a[n];
```

```
In[]:= l15 = Limit[cauch7[n], n → Infinity];
       l16 = Limit[dalembert7[n], n → Infinity];
```

```
In[]:= If[l15 < 1, Print["The seriers is convergent according to cauchy nth root test"],
        If[l15 > 1, Print["The seriers is divergent according to cauchy nth root test"],
         Print["The cauchy test fails "]]]
```

The cauchy test fails

```
In[]:= If[l16 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
        If[l16 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
         Print["The dalembert test fails "]]]
```

The dalembert test fails

In[ ]:=
```
For[j = 1, j < 7, j++, n = 10^j;
 Print["For n=", n, "cauchy7=", N[cauch7[n]]]]
```

```
For n=10cauchy7=0.887014
For n=100cauchy7=0.977189
For n=1000cauchy7=0.996552
For n=10 000cauchy7=0.99954
For n=100 000cauchy7=0.999942
For n=1 000 000cauchy7=0.999993
```

In[ ]:=
```
For[j = 1, j < 7, j++, n = 10^j;
 Print["For n=", n, "Dalembert7=", N[dalembert7[n]]]]
```

```
For n=10Dalembert7=0.957427
For n=100Dalembert7=0.995086
For n=1000Dalembert7=0.999501
For n=10 000Dalembert7=0.99995
For n=100 000Dalembert7=0.999995
For n=1 000 000Dalembert7=1.
```

Ques 9: Cos[n]

In[ ]:=
```
a[n_] := Cos[n];
```

In[ ]:=
```
cauch8[n_] := (a[n])^(1/n);
dalembert8[n_] := a[n + 1]/a[n];

l17 = Limit[cauch8[n], n -> Infinity];
l18 = Limit[dalembert8[n], n -> Infinity];
```

In[ ]:=
```
If[l17 < 1, Print["The seriers is convergent according to cauchy nth root test"],
 If[l17 > 1, Print["The seriers is divergent according to cauchy nth root test"],
  Print["The cauchy test fails "]]]
```

Out[ ]=
```
If[Interval[{0, 1}] < 1,
 Print[The seriers is convergent according to cauchy nth root test],
 If[l17 > 1, Print[The seriers is divergent according to cauchy nth root test],
  Print[The cauchy test fails ]]]
```

In[ ]:=
```
If[l18 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
 If[l18 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
  Print["The dalembert test fails "]]]
```

Out[ ]=
```
If[Indeterminate < 1,
 Print[The seriers is convergent according to dalembert nth ratio test],
 If[l18 > 1, Print[The seriers is divergent according to dalembert nth ratio test],
  Print[The dalembert test fails ]]]
```

In[ ]:=
```
For[j = 1, j < 7, j++, n = 10^j;
 Print["For n=", n, "cauchy8=", N[cauch8[n]]]]
```

```
For n=10cauchy8=0.934515 + 0.303642 i
For n=100cauchy8=0.99852
For n=1000cauchy8=0.999425
For n=10 000cauchy8=0.999995 + 0.000314158 i
For n=100 000cauchy8=1. + 0.0000314159 i
For n=1 000 000cauchy8=1.
```

In[ ]:=
```
For[j = 1, j < 7, j++, n = 10^j;
 Print["For n=", n, "Dalembert8=", N[dalembert8[n]]]]
```

```
For n=10Dalembert8=-0.00527452
For n=100Dalembert8=1.03443
For n=1000Dalembert8=-0.696933
For n=10 000Dalembert8=0.270214
For n=100 000Dalembert8=0.570403
For n=1 000 000Dalembert8=0.854696
```

Ques10. $\frac{1}{n\,\mathrm{Log}[n]}$;

In[ ]:=
```
a[n_] := 1/(n * Log[n]);
```

In[ ]:=
```
cauch9[n_] := (a[n])^(1/n);
dalembert9[n_] := a[n + 1]/a[n];
```

In[ ]:=
```
l19 = Limit[cauch9[n], n → Infinity];
l20 = Limit[dalembert9[n], n → Infinity];
```

In[ ]:=
```
If[l19 < 1, Print["The seriers is convergent according to cauchy nth root test"],
 If[l19 > 1, Print["The seriers is divergent according to cauchy nth root test"],
  Print["The cauchy test fails "]]]
```

```
The cauchy test fails
```

In[•]:=
```
If[l20 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
 If[l20 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
  Print["The dalembert test fails "]]]
```

The dalembert test fails

In[•]:=
```
For[j = 1, j < 7, j++, n = 10^j;
 Print["For n=", n, "cauchy9=", N[cauch9[n]]]]
```

For n=10cauchy9=0.730766

For n=100cauchy9=0.940519

For n=1000cauchy9=0.991199

For n=10 000cauchy9=0.998858

For n=100 000cauchy9=0.99986

For n=1 000 000cauchy9=0.999984

In[•]:=
```
For[j = 1, j < 7, j++, n = 10^j;
 Print["For n=", n, "Dalembert9=", N[dalembert9[n]]]]
```

For n=10Dalembert9=0.872957

For n=100Dalembert9=0.987964

For n=1000Dalembert9=0.998856

For n=10 000Dalembert9=0.999889

For n=100 000Dalembert9=0.999989

For n=1 000 000Dalembert9=0.999999

Ques11.

In[•]:= a[n_] := $\frac{1}{n*(\text{Log}[n])\text{^}2}$;

In[•]:= cauch10[n_] := $(a[n])^{\frac{1}{n}}$;

dalembert10[n_] := $\frac{a[n+1]}{a[n]}$;

In[•]:=
```
l21 = Limit[cauch10[n], n → Infinity];
l22 = Limit[dalembert10[n], n → Infinity];
```

In[•]:=
```
If[l21 < 1, Print["The seriers is convergent according to cauchy nth root test"],
 If[l21 > 1, Print["The seriers is divergent according to cauchy nth root test"],
  Print["The cauchy test fails "]]]
```

The cauchy test fails

In[•]:=
```
If[l22 < 1, Print["The seriers is convergent according to dalembert nth ratio test"],
 If[l22 > 1, Print["The seriers is divergent according to dalembert nth ratio test"],
  Print["The dalembert test fails "]]]
```

The dalembert test fails

```
In[ ]:= For[j = 1, j < 7, j++, n = 10^j;
         Print["For n=", n, "cauchy10=", N[cauch10[n]]]]
```

```
For n=10cauchy10=0.67229
For n=100cauchy10=0.926265
For n=1000cauchy10=0.989285
For n=10 000cauchy10=0.998636
For n=100 000cauchy10=0.999836
For n=1 000 000cauchy10=0.999981
```

```
In[ ]:= For[j = 1, j < 7, j++, n = 10^j;
         Print["For n=", n, "Dalembert10=", N[dalembert10[n]]]]
```

```
For n=10Dalembert10=0.838259
For n=100Dalembert10=0.985834
For n=1000Dalembert10=0.998712
For n=10 000Dalembert10=0.999878
For n=100 000Dalembert10=0.999988
For n=1 000 000Dalembert10=0.999999
```