Problem 9 tells us that Dawson's integral is a solution to the ODE $y' = 1 - 2ty = f(t,y)$ eq. (35)

To find the initial condition $y(0)$, we can take eq. (33) where $y(t) = \exp(-t^2)\int_0^t \exp(s^2)\,ds$ and set $t=0$.

This gives us. $y(0) = \exp(-0^2)\int_0^0 \exp(s^2)\,ds = e^0 \cdot 0 = 1\cdot 0 = 0$

So the initial condition is $\boxed{y(0) = 0}$ when solving the ODE.

To find an explicit expression for $y^{(k+1)}$ in terms of the value $y^{(k)}$, $t^{(k)}$, and Δt we use the trapezoidal rule:

$$y^{(k+1)} = y^{(k)} + \frac{\Delta t}{2}\left(f(t^{(k)}, y^{(k)}) + f(t^{(k+1)}, y^{(k+1)})\right). \qquad \text{eq. (36)}$$

First, we substitute $f(t,y) = 1-2ty$ into the trapezoidal rule:

$$y^{(k+1)} = y^{(k)} + \frac{\Delta t}{2}\left((1-2t^{(k)}y^{(k)}) + (1-2t^{(k+1)}y^{(k+1)})\right)$$

then we simplify

$$= y^{(k)} + \frac{\Delta t}{2}\left(2 - 2t^{(k)}y^{(k)} - 2t^{(k+1)}y^{(k+1)}\right)$$

$$= y^{(k)} + \frac{\Delta t}{2}(2) + \frac{\Delta t}{2}(-2t^{(k)}y^{(k)}) + \frac{\Delta t}{2}(-2t^{(k+1)}y^{(k+1)})$$

$$= y^{(k)} + \Delta t - t^{(k)}y^{(k)}\Delta t - t^{(k+1)}y^{(k+1)}\Delta t$$

Then, we move all the terms that have $y^{(k+1)}$ to one side:

$$y^{(k+1)} + t^{(k+1)}y^{(k+1)}\Delta t = y^{(k)} + \Delta t - t^{(k)}y^{(k)}\Delta t$$

Then we factor out $y^{(k+1)}$:

$$y^{(k+1)}(1+t^{(k+1)}\Delta t) = y^{(k)} + \Delta t - t^{(k)}y^{(k)}\Delta t$$

Then we isolate $y^{(k+1)}$ by dividing both sides by $(1+t^{(k+1)}\Delta t)$:

$$\boxed{y^{(k+1)} = \frac{y^{(k)} + \Delta t - t^{(k)}y^{(k)}\Delta t}{1+t^{(k+1)}\Delta t}}$$

This is the explicit expression for $y^{(k+1)}$.