

GATE 2021 Assignment

EE1205 Signals and Systems

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Question

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (S_1)? What is the sum of the first two terms? What is the second term? Similarly, find the 3rd, the 10th, and the n th terms.

Solution:

variable	value	description
$y(n)$	$(4n - n^2)u(n)$	Sum of first n-terms
$x(n)$	-	n^{th} term of the AP
d	-	common difference of AP

TABLE I

TABLE: INPUT PARAMETERS

$$y(n+1) = (4n - n^2)u(n) \quad (1)$$

refer equation(??) ,equation (??),equation (??) from appendix

$$z^{-1}Y(z) = 4 \left(\frac{z^{-1}}{(1 - z^{-1})^2} \right) - \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} \quad (2)$$

$$Y(z) = \frac{4}{(1 - z^{-1})^2} - \frac{(1 + z^{-1})}{(1 - z^{-1})^3} \quad (3)$$

$$Y(z) = X(z) U(z) \quad (4)$$

$$X(z) = \frac{Y(z)}{U(z)} \quad (5)$$

$$X(z) = 4 \left(\frac{1}{(1 - z^{-1})} \right) - \frac{(1 + z^{-1})}{(1 - z^{-1})^2} \quad (6)$$

$$= \frac{(3 - 5z^{-1})}{(1 - z^{-1})^2} \quad (7)$$

$$= \frac{3z}{z - 1} - \frac{2z}{(z - 1)^2} \quad (8)$$

Using Contour Integration to find the inverse Z-transform,

$$x(n) = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \quad (9)$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{3}{z-1} - \frac{2}{(z-1)^2} \right) z^{n-1} dz \quad (10)$$

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (11)$$

$$R = R_1 + R_2 \quad (12)$$

$$R_1 = \frac{1}{(0)!} \lim_{z \rightarrow 1} \frac{d^0}{dz^0} \left((z-1) \frac{3z^n}{z-1} \right) \quad (13)$$

$$= \lim_{z \rightarrow 1} 3z^n \quad (14)$$

$$= 3 \quad (15)$$

$$R_2 = \frac{1}{1(1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{-2z^n}{(z-1)^2} \right) \quad (16)$$

$$= - \lim_{z \rightarrow 1} \frac{d}{dz} (2z^n) \quad (17)$$

$$= - \lim_{z \rightarrow 1} 2nz^{n-1} \quad (18)$$

$$= -2n \quad (19)$$

$$R = 3 - 2n \quad (20)$$

$$x(n) = (3 - 2n)u(n) \quad (21)$$

$$\text{First term of AP } x(0) = 3 \quad (22)$$

$$\text{sum of first two terms is } y(2) = 4(2) - (2)^2 \quad (23)$$

$$= 4 \quad (24)$$

$$\text{second term of AP } x(1) = 1 \quad (25)$$

$$\text{third term of AP } x(2) = -1 \quad (26)$$

$$\text{tenth term of AP } x(9) = -15 \quad (27)$$

$$n^{\text{th}} \text{ term of AP } x(n) = (3 - 2n)u(n) \quad (28)$$