Discrete Assignment

Shravya Kantayapalam
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Question 10.5.3.11

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (S_1)? What is the sum of the first two terms? What is the second term? Similarly, find the 3rd, the 10th, and the nth terms.

Answer

If the sum of the first n terms of an AP is $4n - n^2$.

Let S_n represent the sum of the first n terms of the arithmetic progression (AP).

$$S_n = 4n - n^2$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where:

$$\begin{aligned} S_n &\text{ is the sum of the first } n \text{ terms,} \\ a &\text{ is the first term,} \\ d &\text{ is the common difference.} \end{aligned}$$

$$4n - n^2 = \frac{n}{2}[2a + (n-1)d]$$

To find the first term a:

$$\begin{aligned} 4 - n &= a + (n-1)d \\ 4 - n &= a + (n-1)d \\ 4 - n &= a + nd - d \\ 4 - n &= a - d + nd - d \\ 4 - n &= a - d(1-n) \end{aligned}$$

Comparing coefficients:

$$a = 4, \quad d = -1$$

Thus, the first term S_1 is $a = 4$.

To find the sum of the first two terms (S_2):

$$\begin{aligned} S_2 &= 2\left(\frac{2a + (n-1)d}{2}\right) \\ &= 2\left(\frac{2(4) + (2-1)(-1)}{2}\right) \\ &= 2\left(\frac{8-1}{2}\right) \\ &= 2\left(\frac{7}{2}\right) \\ &= 7 \end{aligned}$$

To find the second term ($S_2 - S_1$):

$$S_2 - S_1 = 7 - 4$$
$$= 3$$

To find the values of a and d

$$S_n = 4n - n^2$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where:

S_n is the sum of the first n terms,
a is the first term, and
d is the common difference.

$$4n - n^2 = \frac{n}{2}[2a + (n-1)d]$$

To find a, the first term:

$$4n - n^2 = n(a + (n-1)d)$$
$$4 - n = a + (n-1)d$$

To find d, the common difference:

$$d = a_2 - a_1$$

Solving for a and d: Using the equation $4 - n = a + (n-1)d$, we can derive a and d.
When $n = 1$:

$$4 - 1 = a + (1-1)d$$
$$3 = a$$

When $n = 2$:

$$4 - 2 = a + (2-1)d$$
$$2 = a + d$$

Substitute $a = 3$ into the equation $2 = a + d$:

$$2 = 3 + d$$
$$d = -1$$

Thus, the first term a is 3 and the common difference d is -1.

- First term (S_1): $a = 3$
- Sum of the first two terms (S_2):

$$S_2 = 2\left(\frac{2a + (n-1)d}{2}\right)$$
$$S_2 = 2\left(\frac{2(3) + (2-1)(-1)}{2}\right) = 2(3) = 6$$

- Second term: $S_2 - S_1 = 6 - 3 = 3$

In general, the nth term is $a + (n-1)d$, so:

- The 3rd term: $3 + (3-1)(-1) = 3 - 2 = 1$
- The 10th term: $3 + (10-1)(-1) = 3 - 9 = -6$
- The nth term: $3 + (n-1)(-1) = 3 - n + 1 = 4 - n$