Discrete Assignment

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EE23BTECH11030

Question 10.5.3.11

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (S_1)? What is the sum of the first two terms? What is the second term? Similarly, find the 3rd, the 10th, and the nth terms.

Answer

Given:

$$S_n = 4n - n^2$$

Where:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Symbol	Description
S_n	Sum of the first n terms
a	First term
d	Common difference

So, we have:

$$4n - n^2 = \frac{n}{2}[2a + (n-1)d]$$

To find a, the first term:

$$4n - n^2 = n(a + (n-1)d)$$
$$4 - n = a + (n-1)d$$

To find d, the common difference:

$$d = a_2 - a_1$$

Solving for a and d:

Using the equation $4 - n = a + (n-1)d$, we can derive a and d.

When $n = 1$:

$$4 - 1 = a + (1-1)d$$
$$3 = a$$

When $n = 2$:

$$4 - 2 = a + (2-1)d$$
$$2 = a + d$$

Substitute $a = 3$ into the equation $2 = a + d$:

$$2 = 3 + d$$
$$d = -1$$

Thus, the first term a is 3 and the common difference d is -1.

- First term (S_1): $a = 3$

- Sum of the first two terms (S_2):

$$\begin{aligned} S_2 &= 2\left(\frac{2a + (n-1)d}{2}\right) \\ &= 2\left(\frac{2(3) + (2-1)(-1)}{2}\right) \\ &= 2\left(\frac{6-1}{2}\right) \\ &= 2\left(\frac{5}{2}\right) \\ &= 5 \end{aligned}$$

- Second term: $S_2 - S_1 = 5 - 3 = 2$

In general, the nth term is $a + (n-1)d$, so:

- The 3rd term: $3 + (3-1)(-1) = 3 - 2 = 1$
- The 10th term: $3 + (10-1)(-1) = 3 - 9 = -6$
- The nth term: $3 + (n-1)(-1) = 3 - n + 1 = 4 - n$