DISCRETE Assignment
EE1205 Signals and Systems

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Question

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (S_1)? What is the sum of the first two terms? What is the second term? Similarly, find the 3rd, the 10th, and the nth terms.

Solution:

variable	value	description
y(n)	$(4n - n^2)u(n)$	Sum of first n-terms
x(n)	-	n^{th} term of the AP
d	-	common difference of AP

TABLE I
TABLE: INPUT PARAMETERS

$$y\left(n+1\right) = \left(4n - n^2\right) u\left(n\right) \tag{1}$$

refer equation(**??**) ,equation (**??**),equation (**??**) from appendix

$$z^{-1} Y\left(z\right) = 4\left(\frac{z^{-1}}{\left(1 - z^{-1}\right)^2}\right) - \frac{z^{-1}\left(1 + z^{-1}\right)}{\left(1 - z^{-1}\right)^3} \tag{2}$$

$$Y\left(z\right) = \frac{4}{\left(1 - z^{-1}\right)^2} - \frac{\left(1 + z^{-1}\right)}{\left(1 - z^{-1}\right)^3} \tag{3}$$

$$Y\left(z\right) = X\left(z\right) U\left(z\right) \tag{4}$$

$$X\left(z\right) = \frac{Y\left(z\right)}{U\left(z\right)} \tag{5}$$

$$X\left(z\right) = 4\left(\frac{1}{\left(1 - z^{-1}\right)}\right) - \frac{\left(1 + z^{-1}\right)}{\left(1 - z^{-1}\right)^2} \tag{6}$$

$$= \frac{\left(3 - 5z^{-1}\right)}{\left(1 - z^{-1}\right)^2} \tag{7}$$

$$= \frac{3z}{z-1} - \frac{2z}{\left(z-1\right)^2} \tag{8}$$

Using Contour Integration to find the inverse Z-transform,

$$x(n) = \frac{1}{2\pi j}\oint_C X(z)\, z^{n-1}\, dz \tag{9}$$

$$= \frac{1}{2\pi j}\oint_C \left(\frac{3}{z-1} - \frac{2}{(z-1)^2}\right) z^{n-1}\, dz \tag{10}$$

$$R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{11}$$

$$R = R_1 + R_2 \tag{12}$$

$$R_1 = \frac{1}{(0)!}\lim_{z\to 1}\frac{d^0}{dz^0}\left((z-1)\frac{3z^n}{z-1}\right) \tag{13}$$

$$= \lim_{z\to 1} 3z^n \tag{14}$$

$$= 3 \tag{15}$$

$$R_2 = frac1(1)!\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{-2z^n}{(z-1)^2}\right) \tag{16}$$

$$= -\lim_{z\to 1}\frac{d}{dz}\left(2z^n\right) \tag{17}$$

$$= -\lim_{z\to 1} 2nz^{n-1} \tag{18}$$

$$= -2n \tag{19}$$

$$R = 3 - 2n \tag{20}$$

$$x(n) = (3-2n)u(n) \tag{21}$$

$$\text{First term of AP } x(0) = 3 \tag{22}$$

$$\text{sum of first two terms is } y(2) = 4(2) - (2)^2 \tag{23}$$

$$= 4 \tag{24}$$

$$\text{second term of AP } x(1) = 1 \tag{25}$$

$$\text{third term of AP } x(2) = -1 \tag{26}$$

$$\text{tenth term of AP } x(9) = -15 \tag{27}$$

$$n^{th} \text{ term of AP } x(n) = (3-2n)u(n) \tag{28}$$