Discrete Assignment

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1) **Question 11.9.4.9**: Find the sum to n terms of the series whose nth term is given by $n^2 + 2^n$?
Solution:

TABLE 1
INPUT PARAMETERS

Variable	Description	Value
$x(n)$	n-th term of sequence	$(n^2 + 2^n)u(n)$

$$S = \sum_{n=0}^{N} (n^2 + 2^n)u(n)$$

$$y(n) = n^2 + 2^n$$

a) Sum of n^2:

$$S_{n^2} = \sum_{n=0}^{N} n^2 u(n) = \frac{N(N+1)(2N+1)}{6}$$

b) Sum of 2^n:

$$S_{2^n} = \sum_{n=0}^{N} 2^n u(n) = 2(2^N - 1)$$

Therefore, the sum to N terms of the series $n^2 + 2^n$ is:

$$S = S_{n^2} + S_{2^n} = \frac{N(N+1)(2N+1)}{6} + 2(2^N - 1)$$

Now, we define $y(n)$ as the convolution of $x(n)$ and $u(n)$:

$$y(n) = (x * u)(n) = \sum_{k=-\infty}^{\infty} x(k)u(n-k)$$

Given $x(n) = n^2 + 2^n \cdot u(n)$, we have:

$$y(n) = \sum_{k=-\infty}^{\infty} (k^2 + 2^k)u(k)u(n-k)$$

Applying Z-transform of $y(n)$:

$$Y(z) = \sum_{n=0}^{\infty} y(n)z^{-n}$$

To find $Y(z)$, we need to use Z-transform pairs:

$$\mathcal{Z}\{n^2 \cdot u(n)\} = \frac{z(z+1)}{(z-1)^3}$$

$$\mathcal{Z}\{2^n \cdot u(n)\} = \frac{1}{1 - 2z^{-1}}$$

We need to express $y(n)$ in terms of known Z-transforms to find $Y(z)$.
Given:

$$Y(z) = \frac{z(z+1)}{(z-1)^3} + \frac{2}{1 - 2z^{-1}}$$

Applying inverse Z-transform:
1. For the term $\frac{z(z+1)}{(z-1)^3}$:

$$\mathcal{Z}^{-1}\left\{\frac{z(z+1)}{(z-1)^3}\right\} = n^2u(n) + nu(n)$$

2. For the term $\frac{2}{1-2z^{-1}}$:

$$\mathcal{Z}^{-1}\left\{\frac{2}{1 - 2z^{-1}}\right\} = 2 \cdot 2^n u(n)$$

Therefore, $y(n)$ is the sum of the above expressions:

$$y(n) = (n^2 + n)u(n) + 2 \cdot 2^n u(n)$$

This represents the complete expression for $y(n)$.