Gate Assignment CH 31

Shravya Kantayapalam
EE23BTECH11030

1) **Question** : The position $x(t)$ of a particle, at constant ω, is described by the equation

$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

The initial conditions are $x(t = 0) = 1$ and $\left.\frac{dx}{dt}\right|_{t=0} = 0.$

The position of the particle at $t = \frac{3\pi}{\omega}$ is __________ (in integer). (GATE CH 2023)

Solution:

TABLE I: Input Parameters

Parameter	Description
s	Complex frequency variable in Laplace domain
ω	Angular frequency
$X(s)$	Laplace transform of the function $x(t)$
$x(t)$	Time-domain function

$$s^2X(s) - sx(0) - \frac{dx}{dt}(0) + \omega^2X(s) = 0$$

$$x(0) = 1 \quad \text{and} \quad \frac{dx}{dt}(0) = 0$$

$$s^2X(s) - s - \omega^2X(s) = 0$$

$$(s^2 + \omega^2)X(s) = s$$

$$X(s) = \frac{s}{s^2 + \omega^2}$$

$$X(s) = \frac{s}{s^2 + \omega^2} = \frac{A}{s} + \frac{Bs + C}{s^2 + \omega^2}$$

Multiplying both sides by $s(s^2 + \omega^2)$, we get:

$$s = A(s^2 + \omega^2) + (Bs + C)s$$

This implies $A = 0$, $B = 1$, and $C = 0$. Therefore,

$$X(s) = \frac{1}{s^2 + \omega^2}$$

$$x(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2 + \omega^2}\right\} = \cos(\omega t)$$

Finally, evaluating $x(t)$ at $t = \frac{3\pi}{\omega}$, we have:

$$x\left(\frac{3\pi}{\omega}\right) = \cos\left(\omega \cdot \frac{3\pi}{\omega}\right) = \cos(3\pi) = -1$$