Gate Assignment CH 31

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1) **Question** : The position $x(t)$ of a particle, at constant ω, is described by the equation

$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

The initial conditions are $x(t=0) = 1$ and $\left.\frac{dx}{dt}\right|_{t=0} = 0$.
The position of the particle at $t = \frac{3\pi}{\omega}$ is __________ (in integer). (GATE CH 2023)

Solution:

TABLE I: Input Parameters

Parameter	Description
s	Complex frequency variable in Laplace domain
ω	Angular frequency
$X(s)$	Laplace transform of the function $x(t)$
$x(t)$	Time-domain function

The differential equation:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$x = c_1 \cos(\omega t) + c_2 \sin(\omega t)$$

$$x(0) = 1$$
$$\frac{dx}{dt}(0) = 0$$

The derivatives of $x(t)$:

$$\frac{dx}{dt} = -c_1 \sin(\omega t) \cdot \omega + c_2 \cos(\omega t) \cdot \omega$$

$$1 = c_1 + 0 \tag{1}$$
$$c_1 = 1 \tag{2}$$
$$0 = 0 + c_2 \cdot \omega \tag{3}$$
$$\omega \neq 0 \tag{4}$$
$$c_2 = 0 \tag{5}$$

Thus, the final solution is:

$$x(t) = \cos(\omega t)$$

And evaluating x at $t = \frac{3\pi}{\omega}$:

$$x\left(\frac{3\pi}{\omega}\right) = \cos\left(\omega \cdot \frac{3\pi}{\omega}\right) = \cos(3\pi) = -1$$