Gate Assignment CH 31

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1) **Question** : The position $x(t)$ of a particle, at constant ω, is described by the equation

$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

The initial conditions are $x(t = 0) = 1$ and $\left.\frac{dx}{dt}\right|_{t=0} = 0$.
The position of the particle at $t = \frac{3\pi}{\omega}$ is __________ (in integer). (GATE CH 2023)

Solution:

TABLE I: Input Parameters

Parameter	Description
s	Complex frequency variable in Laplace domain
ω	Angular frequency
$X(s)$	Laplace transform of the function $x(t)$
$x(t)$	Time-domain function

Given:

$$X(s) = \frac{s}{s^2 + \omega^2} = \frac{A}{s} + \frac{Bs + C}{s^2 + \omega^2}$$

Multiplying both sides by $s^2 + \omega^2$ yields:

$$s = A(s^2 + \omega^2) + (Bs + C) \cdot s$$
$$s = As^2 + A\omega^2 + Bs^2 + Cs$$
$$(A + B)s^2 + Cs + A\omega^2 = s$$

This equation must hold for all values of s. Therefore, the coefficients must match term by term.

$$A\omega^2 = 0 \implies A = 0$$
$$\text{For the } s \text{ terms:} \quad C = 1$$
$$\text{For the } s^2 \text{ terms:} \quad A + B = 0 \implies B = -A = 0$$

Thus, $A = 0$, $B = 0$, and $C = 1$. Therefore:

$$X(s) = \frac{s}{s^2 + \omega^2} = \frac{1}{s^2 + \omega^2}$$

The inverse Laplace transform of $X(s)$ is:

$$x(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2 + \omega^2}\right\} = \sin(\omega t)$$