Gate Assignment CH 31

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1) **Question** : The position $x(t)$ of a particle, at constant ω, is described by the equation

$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

The initial conditions are $x(t = 0) = 1$ and $\left.\frac{dx}{dt}\right|_{t=0} = 0$.
The position of the particle at $t = \frac{3\pi}{\omega}$ is _____ (in integer). (GATE CH 2023)

Solution:

$x(t)$
$t = \frac{3\pi}{\omega}$.

$$x(t) = A\sin(\omega t) + B\cos(\omega t)$$

where A and B are constants to be determined based on the initial conditions.
Given:

$$x(0) = 1 \quad \text{and} \quad \left.\frac{dx}{dt}\right|_{t=0} = 0$$

We have:

$$x(0) = A\sin(0) + B\cos(0) = B = 1$$

$$\frac{dx}{dt} = A\omega\cos(\omega t) - B\omega\sin(\omega t)$$

$$\left.\frac{dx}{dt}\right|_{t=0} = A\omega = 0$$

From the second equation, $A = 0$.
Therefore, the solution to the differential equation is:

$$x(t) = \cos(\omega t)$$

Now, we need to find the position of the particle at $t = \frac{3\pi}{\omega}$. Substituting $t = \frac{3\pi}{\omega}$ into $x(t)$:

$$x\left(\frac{3\pi}{\omega}\right) = \cos\left(\omega \cdot \frac{3\pi}{\omega}\right) = \cos(3\pi) = -1$$

The position of the particle at $t = \frac{3\pi}{\omega}$ is -1 (in integer).