11.15

EE23BTECH11030 - Shravya Kantayapalam

Question:
The transverse displacement of a string (clamped at its both ends) is given by

$$y(x,t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t) \tag{1}$$

where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg. Answer the following :

(a) Does the function represent a travelling wave or a stationary wave?
(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave ?
(c) Determine the tension in the string.

Solution:

Standing Wave	Traveling Wave
Formed by the interference of two waves traveling in opposite directions	Propagates through a medium without interference
Pattern appears stationary, with nodes and antinodes	Waveform moves through space
No net transport of energy	Transports energy from one place to another
Nodes do not move, so velocity is zero	Moves with a constant velocity

TABLE 0
TABLE-1:DIFFERENCE BETWEEN STANDING AND TRAVELLING WAVE

(a) The given function represents a stationary wave. In a stationary wave, the displacement pattern does not propagate through space, instead, it oscillates in a fixed position.The given wave can be interpreted as a superposition of two waves traveling in opposite directions as explained in next part.

(b) $y(x,t)$ can be represented as

$$y_1(x,t) = -A\sin(\omega t - kx) \tag{2}$$
$$y_2(x,t) = A\sin(\omega t + kx) \tag{3}$$
$$y(x,t) = y_1(x,t) + y_2(x,t) \tag{4}$$
$$= 2A\sin(kx)\cos(\omega t) \tag{5}$$

In the given function

$$y(x,t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t) \tag{6}$$

Camparing (5) and (6) then

$$y_1(x,t) = -0.03\sin\left(120\pi t - \frac{2\pi}{3}x\right) \tag{7}$$
$$y_2(x,t) = 0.03\sin\left(120\pi t + \frac{2\pi}{3}x\right) \tag{8}$$

we know

$$\lambda = \frac{2\pi}{k} \tag{9}$$
$$f = \frac{\omega}{2\pi} \tag{10}$$
$$v = \lambda f \tag{11}$$

Variable	Description	Value
λ	Wavelength	3m
f	Frequency	60Hz
v	Speed	180m/s

TABLE 0
TABLE-2:WAVELENGTH, FREQUENCY AND VELOCITY OF $y_1(x,t)$

Variable	Description	Value
λ	Wavelength	3m
f	Frequency	60Hz
v	Speed	180m/s

TABLE 0
TABLE-3:WAVELENGTH, FREQUENCY AND VELOCITY OF $y_2(x,t)$

(c) Velocity in terms of tension can be written as

$$v = \sqrt{\frac{T}{\mu}} \tag{12}$$
$$T = v^2\mu \tag{13}$$

μ is mass per unit length

$$\mu = \frac{\left(\frac{3}{100}\right)}{1.5} \tag{14}$$
$$= \frac{1}{50} \tag{15}$$
$$T = v^2\mu \tag{16}$$
$$= (180)^2\left(\frac{1}{50}\right) \tag{17}$$
$$= 648 \tag{18}$$

Tension in string = 648 N