a) $\left(y_1 - \hat{\beta}_1 x_{11} - \hat{\beta}_2 x_{12}\right)^2 + \left(y_2 - \hat{\beta}_1 x_{21} - \hat{\beta}_2 x_{22}\right)^2 + \lambda\left(\beta_1^2 + \beta_2^2\right)$

$y_1 + y_2 = 0$

$x_{11} = x_{12} = x_1$

$x_{21} = x_{22} = x_2$

$x_{11} = -x_{21} \qquad x_1 = -x_2$

$x_{12} = -x_{22} \qquad x_1 = -x_2$

$$\left(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1\right)^2 + \left(y_2 - \hat{\beta}_1 x_2 - \hat{\beta}_2 x_2\right)^2 + \lambda\left(\beta_1^2 + \beta_2^2\right)$$

$$\left(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1\right)^2 + \left(y_2 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_1\right)^2 + \lambda\left(\beta_1^2 + \beta_2^2\right)$$

$$\left(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1\right)^2 + \left(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1\right)^2 + \lambda\left(\beta_1^2 + \beta_2^2\right)$$

$$\underline{\underline{2\left(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1\right)^2 + \lambda\left(\beta_1^2 + \beta_2^2\right)}}$$

b) $\dfrac{\partial}{\partial \beta_1} = 4\left(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1\right)(-x_1) + 2\lambda\beta_1$

$$-4y_1 x_1 + 4x_1^2 \hat{\beta}_1 + 4\hat{\beta}_2 x_1^2 + 2\lambda\beta_1 = 0$$

$$\frac{\partial}{\partial \beta_2} = 4(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1)(-x_1) + 2\lambda\beta_2$$

$$-4y_1x_1 + 4x_1^2\hat{\beta}_1 + 4\hat{\beta}_2 x_1^2 + 2\lambda\beta_2 = 0$$

$$\hat{\beta}_1(4x_1^2 + 2\lambda) + \hat{\beta}_2 4x_1^2 - 4x_1y_1 = 0 \rightarrow 1$$

$$\hat{\beta}_2(4x_1^2 + 2\lambda) + \hat{\beta}_1 4x_1^2 - 4x_1y_1 = 0 \rightarrow 2$$

From eq 1 & 2 $\boxed{\hat{\beta}_1 = \hat{\beta}_2}$

c) $(y_1 - \hat{\beta}_1 x_{11} - \hat{\beta}_2 x_{12})^2 + (y_2 - \hat{\beta}_1 x_{21} - \hat{\beta}_2 x_{22})^2 + \lambda(|\hat{\beta}_1| + |\hat{\beta}_2|)$

By substituting the conditions

$$2(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1)^2 + \lambda(|\hat{\beta}_1| + |\hat{\beta}_2|)$$

d) $$\frac{\partial}{\partial \beta_1} = 4(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1)(-x_1) \pm \lambda$$

$$-4x_1y_1 + 4x_1^2\hat{\beta}_1 + 4\hat{\beta}_2 x_1^2 \pm \lambda = 0$$

$$-4x_1y_1 + \hat{\beta}_1 4x_1^2 + \hat{\beta}_2 4x_1^2 + \lambda = 0 \rightarrow (1)$$

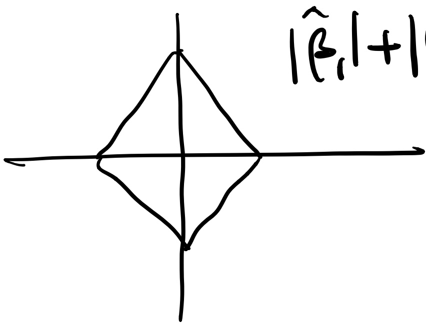
$$-4x_1y_1 + \hat{\beta}_1 4x_1^2 + \hat{\beta}_2 4x_1^2 - \lambda = 0 \rightarrow (2)$$

$$\frac{\partial}{\partial \beta_2} = 4(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1)(-x_1) \pm \lambda$$

$$-4x_1y_1 + \hat{\beta}_1 4x_1^2 + \hat{\beta}_2 4x_1^2 + \lambda = 0 \rightarrow (3)$$

$$-4x_1y_1 + \hat{\beta}_1 4x_1^2 + \hat{\beta}_2 4x_1^2 - \lambda = 0 \rightarrow (4)$$

Solving (1) & (2) gives many solution to $\beta_1$ & $\beta_2$



$|\hat{\beta}_1| + |\hat{\beta}_2| \leq s$

The solution lies on this plane.