CBCS SCHEME

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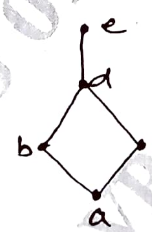
BCS405A

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define Tautology, show that $[(p \vee q) \wedge \{(p \to r) \wedge (q \to r)\}] \to r$	6	L1	CO1
	b.	Prove the following using the laws of logic : $\neg [\{(p \vee q) \wedge r\} \to \neg q] \Leftrightarrow \neg [\neg[(p \vee q) \wedge r] \vee \neg q] \Leftrightarrow q \wedge r$.	7	L2	CO1
	c.	Give i) a direct proof ii) an Indirect proof for the following statement "If n is an odd integer then n + 9 is an even integer".	7	L2	CO1
		OR			
Q.2	a.	Define i) an open statement ii) quantifiers.	6	L2	CO1
	b.	Test the validity of the following arguments. i) $\dfrac{\begin{array}{c} p \wedge q \\ p \to (q \to r) \end{array}}{\therefore r}$ ii) $\dfrac{\begin{array}{c} P \\ P \to \sim q \\ \sim q \to \sim r \end{array}}{\therefore \sim r}$	7	L2	CO1
	c.	For the following statements the universe comprises all non – zero integers. Determine the truth value of each statement. i) $\exists x, \exists y\ [xy = 1]$ ii) $\exists x, \forall y\ [xy = 1]$ iii) $\forall x, \exists y\ [xy = 1]$ iv) $\exists x, \exists y\ [(2x + y = 5) \wedge (x - 3y = -8)]$ v) $\exists x, \exists y\ [(3x - y = 17) \wedge (2x + 4y = 3)]$.	7	L2	CO1
		Module – 2			
Q.3	a.	Define the well ordering principle. By Mathematical induction, prove that $1 + 2 + 3 + \ldots.. + n = \frac{1}{2} n(n+1)$, $n \in z^+$.	6	L2	CO2
	b.	Prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. For F_0 , F_1, F_2 , …. are the Fibonacci numbers.	7	L2	CO2
	c.	Find the number of permutations of the letters of the word 'MASSASAUGA'. In how many of these all four A's are together? How many of them begin with S's?	7	L3	CO2
		OR			

Q.4	a.	Prove that $4n < n^2 - 7$ for all positive integers $n \geq 6$.	6	L2	CO3
	b.	Find the co-efficients of $x^9 y^3$ in the expansion of $(2x - 3y)^{12}$.	7	L3	CO3
	c.	Let $a_0 = 1$, $a_1 = 2$, $a_3 = 3$ and $a_n = a_{n-1} + a_{n-3}$ for $n \geq 3$, prove that $a_n \leq 3^n$ for all +ve integers n.	7	L2	CO3
		Module – 3			
Q.5	a.	State Pigeon hole principle. Prove that if 30 dictionaries in a library contains a total of 61,327 pages then atleast one of dictionaries must have atleast 2045 pages.	6	L2	CO3
	b.	Define power set. For any sets A, B, $C \subseteq U$, prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.	7	L2	CO3
	c.	Let f and g be functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$ if $(gof)(x) = 9x^2 - 9x + 3$, determine a & b.	7	L3	CO3
		OR			
Q.6	a.	Let $f : R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 5, \text{if } x > 0 \\ 1 - 3x, \text{if } x \leq 0 \end{cases}$ Find $f^{-1}(-5, 5)$ and $f^{-1}(-6, 5)$.	6	L2	CO3
	b.	Let N be the set of Natural numbers. Let a relation R be defined by $R = \{(a, b) / a \in N, b \in N, a - b \text{ is divisible by } 5\}$. Prove that R is an equivalence relation.	7	L2	CO3
	c.	For A = { a, b, c, d, e}, the Hasse diagram for the poset(A, R) is as shown below : i) Determine the relation matrix for R ii) Construct the diagraph for R. 	7	L3	CO3
		Module – 4			
Q.7	a.	Determine the number of integers between 1 and 250 that are divisible by 3 and not divisible by 5 and 7.	6	L3	CO4
	b.	Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$, where $n \geq 0$ and $F_0 = 0$, $F_1 = 1$.	7	L2	CO4
	c.	Define Derangement. Find the number of derangement of 1, 2, 3, and 4.	7	L3	CO4
		OR			

<table>
<tr><td>Q.8</td><td>a.</td><td>Find the Rook polynomial for the chess board contain 4 squares as shown in the Fig.Q8(a).
<table><tr><td>1</td><td>2</td></tr><tr><td>3</td><td>4</td></tr></table>Fig.Q8(a)</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Solve the recurrence relation $a_n = 5a_{n-1} + 6a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = 3$.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Find the distinct numbers which are multiples of at least one of 15, 40 and 35 not exceeding 1000.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">Module – 5</td></tr>
<tr><td>Q.9</td><td>a.</td><td>Define group and subgroup with example each.</td><td>6</td><td>L1</td><td>CO5</td></tr>
<tr><td></td><td>b.</td><td>State and prove Lagrange's theorem.</td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c.</td><td>Define Klein 4 group. Verify A = {e, a, b, c} is a Klein 4 group.</td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.10</td><td>a.</td><td>Prove that the intersection of two subgroup of a group is a subgroup of the group.</td><td>6</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b.</td><td>Prove that the cube roots of unity form a group under the multiplication.</td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c.</td><td>Let $G = S_4$, the symmetric group of order 4, for $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, find the subgroup $H = \langle a \rangle$, determine the number of left cosets of H in G.</td><td>7</td><td>L3</td><td>CO5</td></tr>
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