CBCS SCHEME

USN 4 V P 2

BMATS201

Second Semester B.E./B.Tech. Degree Examination, June/July 2024
Mathematics – II for CSE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. VTU Hand book is permitted.

		Module – 1	M	L	C
Q.1	a.	Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{(x+y+z)}dzdydx$.	7	L2	CO1
	b.	By changing the order of integration evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xydydx$.	7	L3	CO1
	c.	With usual notation, prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.	6	L2	CO1
		OR			
Q.2	a.	Evaluate $\int_0^a\int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\,dxdy$ by changing into polar coordinates.	7	L3	CO1
	b.	Find the area bounded between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ by double integration.	7	L2	CO1
	c.	Using Mathematical tool, write the code to find the volume bounded by the sphere $x^2+y^2+z^2=a^2$ by double integration.	6	L3	CO5
		Module – 2			
Q.3	a.	Find the directional derivative of $\phi = xy^3 + yz^3$ at the point (2, -1, 1) in the direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$.	7	L2	CO2
	b.	Verify whether the vector $\vec{F}=\frac{x\hat{i}+y\hat{j}}{x^2+y^2}$ is both solenoidal and irrotational.	7	L2	CO2
	c.	Prove that the cylindrical coordinate system is orthogonal.	6	L2	CO2
		OR			
Q.4	a.	If $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$ find div $\vec{F}$ and curl $\vec{F}$.	7	L2	CO2
	b.	Find the angle between the normal's to the surface $x^2yz = 1$ at the points (-1, 1, 1) and (1, -1, -1).	7	L3	CO2
	c.	Using mathematical tool write the code to find divergence and curl of the vector $\vec{F}=(4xy-z^3)\hat{i}+2x^2\hat{j}-3xz^2\hat{k}$.	6	L3	CO5

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<tr><td colspan="6"></td></tr>
<tr><td colspan="6">Module – 3</td></tr>
<tr><td>Q.5</td><td>a.</td><td>Let W be a subset of $V_3(R)$ consisting of vectors of the form (a, a^2, b) where the second component is the square of the first. Is W a subspace of $V_3(R)$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Let P_n be the vector space of real polynomial functions of degree $\leq n$. Verify that the transformation $T : P_2 \rightarrow P_1$ defined by $T(ax^2 + bx + c) = (a + b)x + c$ is linear.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Find the Kernel and range of the linear transformation $T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (x + y, z)$.</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.6</td><td>a.</td><td>Determine whether or not each of the following $x_1 = (2, 2, 1)$, $x_2 = (1, 3, 7)$, $x_3 = (1, 2, 3)$ forms a basis in R^3.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Verify Rank-nullity theorem for the transformation $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>The inner product of the polynomials $f(t) = t + 2$, $g(t) = 3t - 2$ in $p(t)$ is given by $\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt$. Find i) $\langle f, g\rangle$ ii) $\|f\|$ iii) $\|g\|$</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">Module – 4</td></tr>
<tr><td>Q.7</td><td>a.</td><td>Find an approximate root of the equation $\cos x = 3x - 1$ correct to four decimal places using Regula Falsi method between 0.5 and 0.7.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>The area 'A' of a circle of diameter 'd' is given by the following table:
<table>
<tr><td>d:</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr>
<tr><td>A:</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr>
</table>
Using appropriate Newton's interpolation formula for equispaced values of x, find area of the circle corresponding to the diameter 105.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Evaluate $I = \int_0^5 \frac{1}{4x+5}\,dx$ by Simpson's $1/3^{rd}$ rule by considering 10 sub intervals. Hence find an approximate value of log5.</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.8</td><td>a.</td><td>Find the real root of $x \log_{10} x = 1.2$ correct to four decimals that lies near 2.5 using Newton Raphson method.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Fit a polynomial for the following data using Newton's divided difference formula:
<table>
<tr><td>x:</td><td>-4</td><td>-1</td><td>0</td><td>2</td><td>5</td></tr>
<tr><td>y:</td><td>1245</td><td>33</td><td>5</td><td>9</td><td>1335</td></tr>
</table></td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Use trapezoidal rule to find $\int_0^{0.6} e^{-x^2}\,dx$ by taking seven ordinates.</td><td>6</td><td>L3</td><td>CO4</td></tr>
</table>

		Module – 5			
Q.9	a.	Employ Taylor's series method to obtain approximate solution at $x = 0.1$ and $x = 0.2$ for the initial value problem $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$.	7	L2	CO4
	b.	Apply Runge-Kutta method of fourth order to find an approximate solution at $x = 0.1$ given $\frac{dy}{dx} = 3x + y/2$, $y(0) = 1$.	7	L2	CO4
	c.	Apply Milne's predictor – corrector method to solve the equation $(y^2 + 1)dy - x^2dx = 0$ at $x = 1$ given $y(0) = 1$, $y(0.25) = 1.0026$, $y(0.5) = 1.0206$, $y(0.75) = 1.0679$.	6	L2	CO4
		OR			
Q.10	a.	Apply modified Euler's method to find solution at $x = 0.1$ by taking $h = 0.1$ given $y' = x^2 + y^2$, $y(0) = 0$.	7	L2	CO4
	b.	Find an approximate solution of $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ at $x = 0.2$ using Runge-Kutta method of order four.	7	L2	CO4
	c.	Write the mathematical tool code to solve $\frac{dy}{dx} = x^2 + y$, $y(0) = 10$ using Taylor's series method at $x = 0.1(0.1)0.3$. Consider the terms upto fourth degree.	6	L3	CO5

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