Model Question Paper Set - 1 with effect from 2022(CBCS Scheme)

USN

Fourth Semester B.E Degree Examination

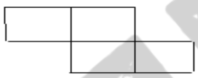
DISCRETE MATHEMATICAL STRUCTURES (BCS405A)

TIME: 03Hours **Max.Marks:100**

Note:

1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**
2. M: Marks, L: RBT levels, C: Course outcomes.

		Module – 1	M	L	C
Q.1	a	Show that the compound proposition $\left[(p \leftrightarrow q)\wedge(q \leftrightarrow r)\wedge(r \leftrightarrow p)\right] \Leftrightarrow \left[(p \to q)\wedge(q \to r)\wedge(r \to p)\right]$ for primitive statements p, q, r is logically equivalence.	6	L2	CO1
	b	Establish the validity of the following argument using the Rules of Inference: $\{p \wedge (p \to q) \wedge (s \vee r) \wedge (r \to \sim q)\} \longrightarrow (s \vee t)$.	7	L2	CO1
	c	For the universe of all integers, let p(x), q(x), r(x), s(x) and t(x) denote the following open statements: p(x): x>0, q(x): x is even, r(x): x is a perfect square, s(x): x is divisible by 3, t(x): x is divisible by 7. Write the following statements in symbolic form: i) At least one integer is even. ii) There exists a positive integer that is even. iii) If x is even, then x is not divisible by 3. iv) No even integer is divisible by 7. v) There exists even integer divisible by 3.	7	L1	CO1
		OR			
Q.2	a	Define a tautology. Prove that, for any propositions p, q, r the compound propositions, $\{(p\to q)\wedge(q\to r)\}\to\{(p\to r)\}$ is tautology.	6	L2	CO1
	b	Prove the following using laws of logic: $p\to(q\to r)\Leftrightarrow(p\wedge q)\to r$.	7	L2	CO1
	c	Give i) direct proof ii) indirect proof iii) proof by contradiction for the following statement: "if n is an odd integer then n+9 is an even integer".	7	L3	CO1
		Module – 2			
Q.3	a	Prove that $1^2+3^2+5^2+\cdots.+(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}$ by Mathematical Induction.	6	L2	CO2
	b	Let $a_0=1, a_1=2, a_2=3$ and $a_n=a_{n-1}+a_{n-2}+a_{n-3}$ for $n\geq 3$. Prove that $a_n \leq 3^n\ \forall\ n \in z^+$.	7	L2	CO2
	c	Find the number of ways of arrangement of the letters of the word 'TALLAHASSEE' which have no adjacent A's.	7	L2	CO2
		OR			
Q.4	a	Determine the coefficient of xyz^2 in the expansion of $(2x-y-z)^4$.	6	L2	CO2
	b	In how many ways one can distribute 8 identical marbles in 4 distinct containers so that i) no container is empty ii) the fourth container has an odd number of marbles in it.	7	L2	CO2
	c	How many positive integers n can we form using the digits 3,4,4,5,5,6,7 if we want n to exceed 5,000,000?	7	L2	CO2
		Module – 3			
Q.5	a	Let f: R → R be defined by, $f(x)=\begin{cases}3x-5 \text{ if } x>0\\ 1-3x \text{ if } x\leq 0\end{cases}$. Find $f^{-1}(0), f^{-1}(1), f^{-1}(-1), f^{-1}(3), f^{-1}(6),\ f^{-1}([-6,5])$ and $f^{-1}([-5,5])$	6	L2	CO3

	b	State Pigeon hole principle. Prove that in any set of 29 persons; at least 5 persons have been born on the same day of the week.	7	L2	CO3
	c	Let A={1,2,3,4,6} and 'R' be a relation on 'A' defined by aRb if and only if "a is multiple of b" represent the relation 'R' as a matrix, draw the diagraph and relation R.	7	L2	CO3
		OR			
Q.6	a	If $f{:}A\to B$, $g{:}B\to C$, $h{:}C\to D$ are three functions, then Prove that h∘(g∘f)=(h∘g)∘f.	6	L2	CO3
	b	Show that if n+1 numbers are chosen from 1 to 2n then at least one pair add to 2n+1.	7	L2	CO3
	c	Draw the Hasse diagram representing the positive divisors of 72.	7	L2	CO3
		Module – 4			
Q.7	a	In how many ways the 26 letters of English alphabet are permuted so that none of the pattern's CAR, DOG, PUN or BYTE occurs?	6	L2	CO4
	b	Define Derangement. In how many ways can each of 10 people select a left glove and a right glove out of a total of 10 pairs of gloves so that no person selects a matching pair of gloves?	7	L2	CO4
	c	Solve the recurrence relation: $C_n = 3C_{n-1} - 2C_{n-2}, for\ n \geq 2$, given $C_1 = 5, C_2 = 3$.	7	L3	CO4
		OR			
Q.8	a	In how many ways one can arrange the letters of the word **CORRESPONDENTS** so that there are i) exactly 2 pairs of consecutive identical letters? ii) at least 3 pairs of consecutive identical letters? iii) no pair of consecutive identical letters?	6	L2	CO4
	b	Find the rook polynomial for the chess board as shown in the figure 	7	L2	CO4
	c	Solve the recurrence relation $a_{n+2} - 3a_{n+1} + 2a_n = 0, a_0 = 1, a_1 = 6$.	7	L3	CO4
		Module – 5			
Q.9	a	If H, K are subgroups of a group G, prove that $H \cap K$ is also a subgroup of G. Is $H \cup K$ a subgroup of G.	6	L2	CO5
	b	Define Klein 4 group. Verify $A = \{1, 3, 5, 7\}$ is a klein 4 group.	7	L2	CO5
	c	State and prove Lagrange's Theorem.	7	L2	CO5
		OR			
Q.10	a	Show that i) the identity of G is unique. ii) the inverse of each element of G is Unique.	6	L3	CO5
	b	Show that $(A, \cdot)$ is an abelian group where $A = \{a \in Q \mid a \neq -1\}$ and for any $a, b \in A, a \,.\, b = a + b + ab$.	7	L3	CO5
	c	Let $G = S_4$, for $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, find the subgroup $H = \langle \alpha \rangle$. Determine the left cosets of H in G.	7	L3	CO5