COL765 : ILFP

Principal Typing Schemes

Solution to $\Gamma \vdash e : \tau \rhd_T C$ is

$\langle S, \sigma \rangle$

S : Substitution policy s.t. $S(\tau) = \sigma$

Once the type constraints are generated, solution to the constraints is obtained through an algorithm called Unification.

Unification

- Whenever two Types A & B are required to be "the same", the compiler must "unify" what it knows about A & B to produce a potentially more detailed description of their common type.

Eg: E1 : ('a * int)

E2 : (String * 'b) then

in the Exp. ITE <X, E_1, E_2>

the compiler can infer that

'a = String ^

'b = int

Unification's uses

- Type inference
- Pattern matching

Defn & Prop. of Unifiers

Unification is a process of finding a substitution S s.t. $aS = bS$

aS and bS: result of applying S to type exprs a & b.

Eg:

$E_1 = f \quad x \quad (g \; y)$

$E_2 = f (g \; z) \quad \omega$

then $S = \{ x \mapsto (g \; z), \quad \omega \mapsto (g \; y) \}$

$E_1 S = f \; (g \; z) \; (g \; y)$

$E_2 S = f \; (g \; z) \; (g \; y)$

Note:

- Unifiers donot necessary exist

  Eg: x & $f\,x$ can't be unified

- Neither is the case that unifiers are unique

  Eg: $T = \{ x \mapsto g\,(f\,a\,b),$
  $y \mapsto f\,b\,a,$
  $z \mapsto f\,a\,b,$
  $w \mapsto g\,(f\,b\,a) \}$

Applying T to E_1 & E_2 from the previous Example, we get

$E_1T = E_2T = f\,(g\,(f\,a\,b))\,(g\,(f\,b\,a))$

T is not the "most general unifier"

- Whenever a unifier exists ⇒ there is a mgu (most general unifier)
- Def^n of mgu: A unifier S for a & b is mgu if:
  - Any other unifier T for a & b can be obtained from S by doing further substitutions.

Eg: Consider: S, T, E_1, E_2 from previous examples

Consider $U = \{ z \longmapsto f\ a\ b,\ \ y \longmapsto f\ b\ a \}$

We find that

$T = SU$ s.t.

$E_1 SU = E_2 SU = T$

$E_1 SU = f\ x\ (g\ y)\ SU = f\ (g\ z)\ (g\ y)\ U$

$= f\ (g\ (f\ a\ b))\ (g\ (f\ b\ a))$

$= E_1 T$

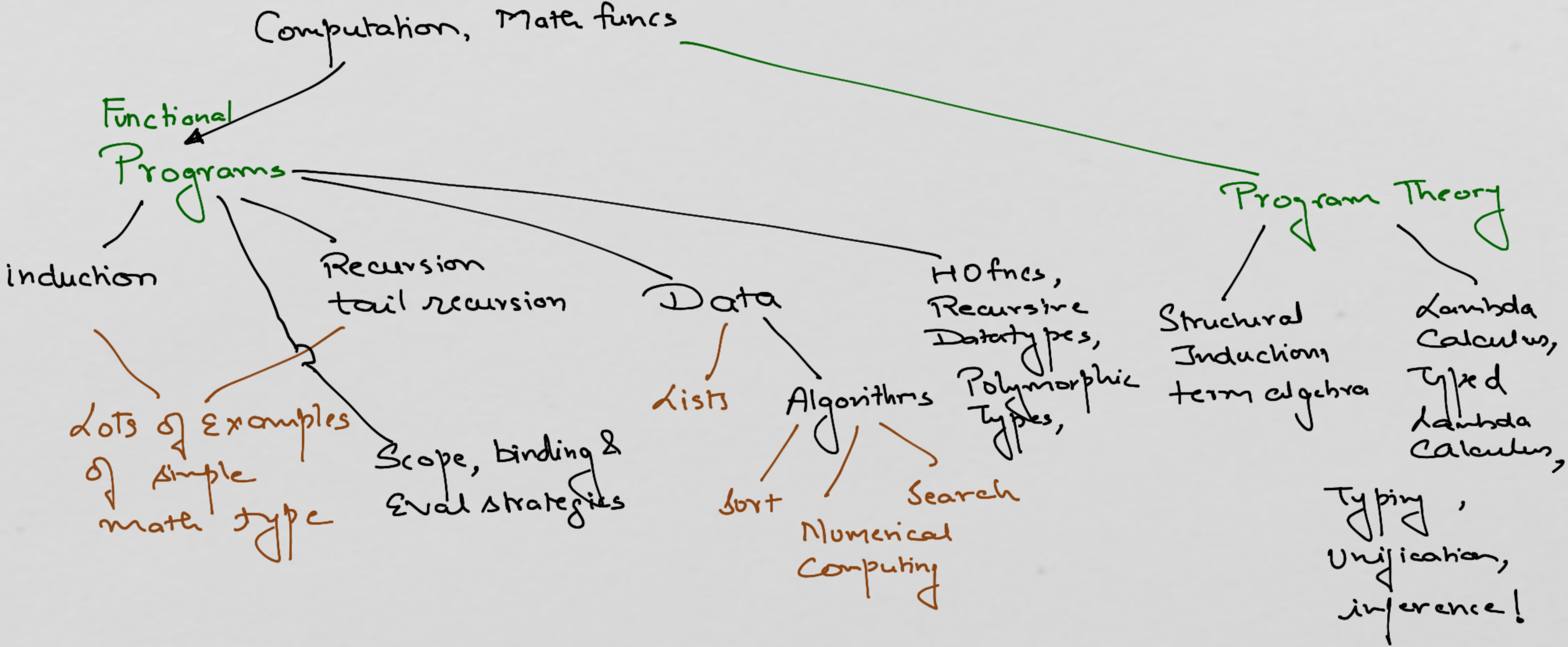
Unification Alg for a not just a pair of terms
but also for a set of pair of terms

```
type  id = String
data  term = Var id | Term (id * Term list)

type  Substitution = (id * term) list

(* check if var x occurs in term t *)
fun occurs (x::id)(t::term):: bool =
    case t of
      Var y  ⟶  (x = y)
    | Term(_, s) ⟶ (occurs x s)
```

```
(* apply a substitution right to left *)
fun apply (s:: substitution) (t::term) :: term =
    foldr (\x u ⇒ subst u x) s t

fun subst (s::term) (t::term) :: term =
  (* subst. term s for all occurrences of x in t*)
    case t of
      Var y → if x=y then s else t
    | Term (i,u) → Term (i, map (subst s x) u)
```

```
fun unify_one (s::term) (t::term):: substitution =
    case (s, t) of
      (Var x, Var y) ⟶ if x=y then [] else [(x, t)]

      Term (f, sl), Term (g, tl)
          ⟶ if f==g && length sl == length tl
            then unify (combine sl tl)
(* Combine :: [a] ⟶ [b] ⟶ [(a,b)] *)
            else error "head symbol conflict"

      Var x, (Term (_,_) as t),
      Term (_,_) as t, Var x ⟶
            if occurs x t
            then error "not unifiable"
            else [(x, t)]
```

```
fun unify (s :: [(term * term)]) :: substitution =
    case s of
      [] ⟶ []
    | (x, y) :: sl ⟶ let t2 = unify sl in
                     let t1 = unify-one
                         (apply t2 x)
                         (apply t2 y) in
                     t1 ++ t2
```


Computation, Math funcs
Functional
Programs
Program Theory
Induction
Recursion
tail recursion
Data
HOfncs,
Recursive
Datatypes,
Polymorphic
Types,
Structural
Induction,
term algebra
Lambda
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lambda
Calculus,
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of simple
math type
Scope, binding &
Eval strategies
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Why f.P. & PLT

- Great features : Elegance, Crisp & well defined semantics
  HO fncs, Lazy evaluation, Conciseness, forces you
  to think on Essentials of computation
- Big data → Conceptually all functional
- Native concurrency
- PLT – Math + S.E. + Linguistics + CogSc.
  [design, implm., analysis & characterisation
  of formal languages]