COL765
Quiz 3 solutions

Problem 1:

```haskell
module Queue where

-- Auxiliary functions
qempty = []

qlen_aux []     o = o
qlen_aux (x:xs) o = qlen_aux xs (o+1)
qlen :: [a] -> Integer
qlen q = qlen_aux q 0

-- Required functions
enq :: [a] -> Integer -> a -> [a]
enq q n e = if qlen q > n then error "Overflow"
                          else q++[e]

deq :: [a] -> [a]
deq [] = error "Underflow"
deq (x:xs) = xs

qhd :: [a] -> a
qhd [] = error "Underflow"
qhd (x:xs) = x

isempty :: [a] -> Bool
isempty [] = True
isempty (x:xs) = False
```

Problem 2:

BNF:

$$e, e' := n \in \mathbb{N} | (e + e') | (e * e') | (e - e')$$

To prove: For e generated by above BNF, property $P(e)$: $L(e) = R(e)$, where $L(e)$ is the number of (in e and $R(e)$ is the number of) in e.

Proof:

Basis: Basis set $n \in \mathbb{N}$ contains no (or) so $P(e)$ trivially holds.

Induction Hypothesis: For any e of the form $(f \odot g)$, where $\odot \in \{+, *, -\}$ and f,g are generated from BNF, $P(f)$ and $P(g)$ hold.

Induction Step: Case analysis for each constructor.

- Case $e = (f + g)$: $L(e) = 1 + L(f) + L(g)$, $R(e) = 1 + R(f) + R(g)$. By IH, $L(f) = R(f)$ and $L(g) = R(g)$. Therefore, $L(e) = R(e)$ and $P(e)$.
- Case $e = (f * g)$: Same as above.
- Case $e = (f - g)$: Same as above.