FFT and Sound processing

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FFT (DFT)

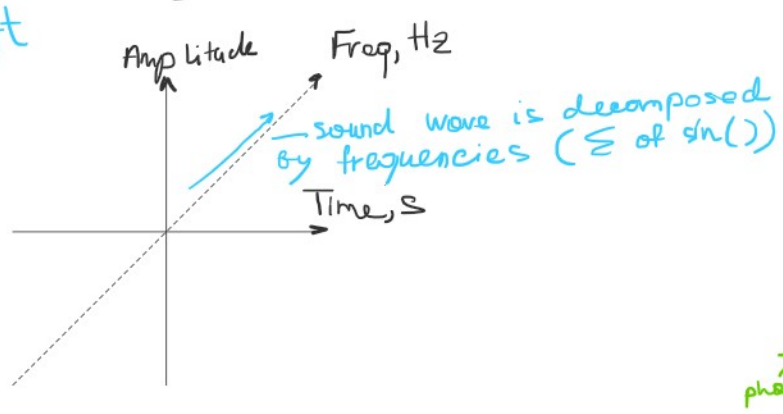
Signal → Wave (discrete)

Wave → (fft) → spectrum

Wave → (stft) → spectrogram

Two domains

1) Time domains
2) Frequency domain

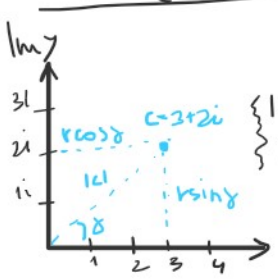


$\sin(2\pi(\underset{1}{f}\cdot t - \underset{2}{\varphi}))$ - formula of sin-wave

1 - frequency ($2\pi \cdot f = w$)

2 - phase

FT (with complex numbers and math)



$\{|c| = \sqrt{a^2+b^2} = r,\ \gamma = \operatorname{arctg}\frac{a}{b}\} \Rightarrow c = r(\cos\gamma + i\sin\gamma) = r\cdot e^{i\varphi}$ (3)

(dist from origin = magnitude); polar coordinates representation; direction

$e^{i\pi} + 1 = 0$ - Euler Identity

complex numbers recap.-

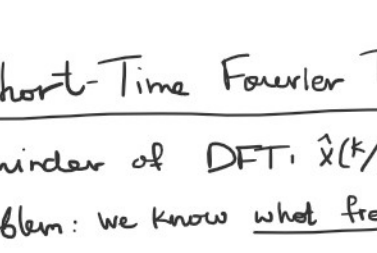
now for FFT.

how to go from (1),(2) to (3)?

Intuition: use magnitude and phase as polar coordinates.

$c_f = \frac{d_f}{\sqrt{2}} \cdot e^{-i2\pi\varphi_f}$ - complex FFT coefs.

($\frac{d_f}{\sqrt{2}} = |c|$; $\gamma = 2\pi\varphi_f$)



* - added for 2) rotation

Idea of FFT:

- compare signal with different sinusoids
- FFT(freq) → magnitude + phase
- high magnitude => sin ≈ signal

Choose freq.
⇓
Optimize phase
⇓
Calculate magnitude

$\varphi_f = \underset{p\in[0,1)}{\arg\max} \int s(t)\cdot\sin(2\pi(ft-\varphi))dt$ (1)

(φ_f - phase; $s(t)$ - signal; $\sin(2\pi(ft-\varphi))$ - chosen sinusoid)

$d_f = \max_{\varphi\in[0,1)} \int s(t)\cdot\sin(2\pi(ft-\varphi))dt$ (2)

(d_f - magnitude)

Fourier representation (ift)

$g(t) = \int c_f \cdot e^{i2\pi ft} df$

add up all weighted sinusoids and get original sound.

DFT case (what is actually going on)

$g(t) \mapsto x(n),\ n\in\mathbb{N}$

$t = n\cdot T$ (T - sampling rate)

$\hat{x}(f) = \sum_{n=0}^{N-1} x(n)\cdot e^{-i2\pi fn}$ - discrete version of $\int_{-\infty}^{+\infty}$

- consider n where freq ≠ 0
- compute for finite set of $\{f_1,\dots,f_n\}$, M = N - number of samples (allows ift and efficient)

$f = \frac{k}{N},\ k\in[0,M-1]=[0,N-1]$

$F(k) = \frac{k}{NT} = \frac{k\cdot sr}{N}$ - actual freq. value

BUT: redundancy - plot will be symmetrical around Nyquist fr. = $\frac{N}{2}$ > $F(N/2) = sr/2$. So we can throw everything after it.

Note on FT to FFT (very fast computing)
- $O(N^2) \to O(N\log N)$
- $N = 2^x$ ($x\in\mathbb{N}$)

$g(t): \mathbb{R}\to\mathbb{R}$ - original audio signal (time domain)

$\hat{g}(f): \mathbb{R}\to\mathbb{C}$ - FT, outputs a complex number (c_f)

$\hat{g}(f) = \int_{-\infty}^{+\infty} g(t)\cdot e^{-i2\pi ft} dt$

($g(t)$ - sound signal (amplitude at time t); $e^{-i2\pi ft}$ - phase term; $2\pi f$ ~ speed of unit circle rotation)

Interpretation of f - center of mass. The greater $|c_m|$ - the greater the alignment with true sinusoid.

* Since sound is discrete, instead of $\int$ we do Σ over all timestamps.

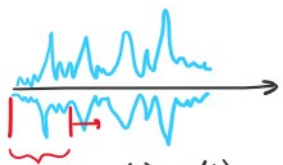
$d_f = \sqrt{2}\cdot|\hat{g}(f)|,\ \varphi_f = -\frac{\gamma_f}{2\pi}$

Short-Time Fourier Transform (signal → spectrogram)

reminder of DFT: $\hat{x}(k/N) = \sum_{n=0}^{N-1} x(n)\cdot e^{-i2\pi n\cdot\frac{k}{N}}$

Problem: we know what freq. are present, but don't know when. (cause they are averaged over time)

Solution: use windowing



window size - number of samples W() is applied to

frame size - number of samples in chunk

ws can be < then fs (then samples outside of ws will be 0-padded)

1) $x_w(t) = x(k)\cdot W(k)$ (W(k) - window function)

2) apply fft on window

1) windows are overlapping. Each window position is defined by hop size parameter.

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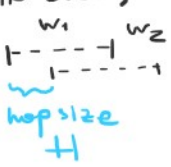

$$S(m,k) = \sum_{n=0}^{N-1} x(n+mH)\cdot w(n)\cdot e^{-i2\pi n\frac{k}{N}}$$

- phase thingy
- proxy time value, frame number
- N = frame size we are Σ over frame
- all samples from m frame.
- window function

DFT → 1-d vector, N complex FT

STFT → 2-d matrix, freq-bins x frames

$\#\text{freq-bins} = \frac{\text{framesize}}{2} + 1$ (related to Nyq. freq.)

$\#\text{frames} = \frac{\text{samples} - \text{framesize}}{\text{hopsize}} + 1$

Example:

signal – 10k samples

fs = ws = 1000

hs = 500

number of fr. bins = $\frac{1000}{2} + 1 = 501 \rightarrow (0, \frac{fs}{2})$

num. of frames = (10k − 1k)/500 + 1 = 19

STFT → (501, 19)

Parameters study

- fs – (512, 1024, .., 2^n)
  - fs↓ ⇒ freq.res↑ time res.↓ (time/freq trade off)
  - fs↑ ⇒ freq.res.↓ time res.↑
- hs (256, 512, .., 2^m), $m = (n-1, n-2, \dots) \sim hs = \frac{fs}{4}$
- w(n) – - butter, like Hann window

How to get to spectrogram?

$$Y(m,k) = |S(m,k)|^2$$

↖ real-valued matrix