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## EXACT LINEARIZATION CONTROL OF A PM STEPPER MOTOR

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## ABSTRACT

This work considers the control of a PM stepper motor using the exact linearisation method. We show how this method leads naturally to the Blondel-Park transformation of Electric Machine Theory.

## I. INTRODUCTION

*Mathematical Model of the PM Stepper Motor*

The equations describing the stepper motor are given below [8], [9], [10], [11]. (See [8] for an especially clear and lucid derivation of these equations.)

$$\begin{cases} \frac{di_a}{dt} = \frac{1}{L} [v_a - Ri_a + K_m \omega \sin(N_r \theta)] \\ \frac{di_b}{dt} = \frac{1}{L} [v_b - Ri_b - K_m \omega \cos(N_r \theta)] \\ \frac{d\omega}{dt} = \frac{1}{J} [-K_m i_a \sin(N_r \theta) + K_m i_b \cos(N_r \theta) - B\omega] \\ \frac{d\theta}{dt} = \omega \end{cases} \quad (1.1)$$

$i_a, i_b$  and  $v_a, v_b$  are the currents and voltages in phases A and B respectively.  $L$  and  $R$  are the self-inductance and resistance of each phase winding,  $K_m$  is the motor torque constant,  $N_r$  is the number of rotor teeth,  $J$  is the rotor inertia,  $B$  is the viscous friction constant,  $\omega$  is the rotor speed,  $\theta$  is the motor position and  $\tau_L$  is the load torque.

Now assign the state variables  $x_1, x_2, x_3$  and  $x_4$  by

$$x^T = [i_a, i_b, \omega, \theta]^T. \quad (1.2)$$

For convenience let  $K_1 = R/L$ ,  $K_2 = K_m/L$ ,  $K_3 = K_m/J$ ,  $K_4 = B/J$ ,  $K_5 = N_r$ ,  $u_1 = v_a/L$ ,  $u_2 = v_b/L$ .

With  $\dot{x}_i = \frac{dx_i}{dt}$  ( $i = 1, 2, 3, 4$ ) the equations for the permanent-magnet stepper motor become:

$$\begin{cases} \dot{x}_1 = -K_1 x_1 + K_2 x_3 \sin(K_5 x_4) + u_1 \\ \dot{x}_2 = -K_1 x_2 - K_2 x_3 \cos(K_5 x_4) + u_2 \\ \dot{x}_3 = -K_3 x_1 \sin(K_5 x_4) + K_3 x_2 \cos(K_5 x_4) - K_4 x_3 \\ \dot{x}_4 = x_3 \end{cases} \quad (1.3)$$

## II. EXACT FEEDBACK LINEARIZATION CONTROLLER

Our goal here, following [5], [6], [14], [15], [16], is to find a nonlinear transformation  $T(x) : \mathbb{R}^4 \rightarrow \mathbb{R}^4$  that will transform the above nonlinear system into a *nonlinear controller canonical form*. In the new coordinates,  $x^* \triangleq T(x)$ , we can cancel out the nonlinearities with feedback and thus work with a linear system.

The transformation

$$\begin{aligned} x_1^* &= T_1(x) = x_1/K_3 \\ x_2^* &= T_2(x) = L_r(T_1) = x_2/K_3 \\ x_3^* &= T_3(x) = L_r(T_2) = -x_1 \sin(K_5 x_4) + x_2 \cos(K_5 x_4) - K_4 x_3/K_3 \\ x_4^* &= T_4(x) = x_1 \cos(K_5 x_4) + x_2 \sin(K_5 x_4) \end{aligned} \quad (2.5)$$

results in

$$\begin{aligned} \dot{x}^* &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\alpha_1 & -\alpha_2 & 0 \\ 0 & 0 & 0 & -K_1 \end{bmatrix} x^* + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\sin(K_5 x_4) & \cos(K_5 x_4) \\ \cos(K_5 x_4) & \sin(K_5 x_4) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ &+ K_2 x_3 \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\sin(K_5 x_4) & \cos(K_5 x_4) \\ \cos(K_5 x_4) & \sin(K_5 x_4) \end{bmatrix} \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix} \end{aligned} \quad (2.6)$$

where

$$\alpha_1 = (K_1 K_4 + K_5 K_3) \quad \text{and} \quad \alpha_2 = (K_1 + K_4)$$

## 2.2. The Control Law

Let

$$\begin{aligned} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} &= -K_5 x_3 \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix} - \begin{bmatrix} -\sin(K_5 x_4) & \cos(K_5 x_4) \\ \cos(K_5 x_4) & \sin(K_5 x_4) \end{bmatrix} M x^* \\ &+ \begin{bmatrix} -\sin(K_5 x_4) & \cos(K_5 x_4) \\ \cos(K_5 x_4) & \sin(K_5 x_4) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \end{aligned} \quad (2.7)$$

where

$$M = \begin{bmatrix} 0 & -\alpha_1 & -\alpha_2 & 0 \\ 0 & 0 & 0 & -K_1 \end{bmatrix}$$

We then have

$$\dot{x}^* = Ax^* + Bv$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (2.8)$$

The pair (A,B) is in control canonical form so that by state feedback

$$v = -Lx^* = -LT(x), L \in \mathbb{R}^{2 \times 4} \quad (2.9)$$

the closed loop poles may be arbitrarily located (e.g.,  $L = \begin{bmatrix} a_{30} & a_{31} & a_{32} & 0 \\ 0 & 0 & 0 & a_{40} \end{bmatrix}$ )

Let  $\theta_d(t)$ ,  $\omega_d(t)$ ,  $\alpha_d(t)$  be desired position speed, and acceleration profiles. A possible control law is then

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = - \begin{bmatrix} a_{30} & a_{31} & a_{32} & 0 \\ 0 & 0 & 0 & a_{40} \end{bmatrix} \begin{bmatrix} x_1^* - x_{1d} \\ x_2^* - x_{2d} \\ x_3^* - x_{3d} \\ x_4^* - x_{4d} \end{bmatrix} - \begin{bmatrix} \int_{t_0}^t (\theta - \theta_d) dt \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\alpha}_d(t) \\ 0 \end{bmatrix} \quad (2.10)$$

where by (2.8) we see that  $[x_1^*, x_2^*, x_3^*] = [\theta, \omega, \alpha]/K_s$ . The equilibrium value of  $x_4^*$  is  $i_0$ .  $r(t) = \dot{\alpha}_d(t)$  is the reference input.

The integral control term  $\int_{t_0}^t (\theta - \theta_d) dt$  helps eliminate position error due to disturbances, unmodeled dynamics and imperfect cancellation of the nonlinear terms.

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### III. DQ TRANSFORMATION

The transformation (2.5) is really just the well known Blondel-Park or DQ transformation. In fact the DQ transformation is usually given as [20]:

$$\begin{bmatrix} i_a^* \\ i_b^* \\ \omega \\ \theta \end{bmatrix} = \begin{bmatrix} \cos(K_s x_4) & \sin(K_s x_4) & 0 & 0 \\ -\sin(K_s x_4) & \cos(K_s x_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

where  $[x_1, x_2, x_3, x_4] = [i_a, i_b, \omega, \theta]$  as in (1.2).

In these coordinates the system becomes

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} i_a^* \\ i_b^* \\ \omega \\ \theta \end{bmatrix} &= \begin{bmatrix} -K_1 & 0 & 0 & 0 \\ 0 & -K_1 & -K_2 & 0 \\ 0 & K_3 & -K_4 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a^* \\ i_b^* \\ \omega \\ \theta \end{bmatrix} \\ &+ \begin{bmatrix} \cos(K_s \theta) & \sin(K_s \theta) \\ -\sin(K_s \theta) & \cos(K_s \theta) \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + K_s \omega \begin{bmatrix} i_b^* \\ -i_a^* \\ 0 \\ 0 \end{bmatrix} \end{aligned} \quad (3.1)$$

Choose

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -K_s \omega \begin{bmatrix} i_b^* \\ -i_a^* \end{bmatrix} + \begin{bmatrix} \cos(K_s \theta) & -\sin(K_s \theta) \\ \sin(K_s \theta) & \cos(K_s \theta) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (3.2)$$

to get a linear system (recall  $x_1 = i_a$  and  $x_2 = i_b$ ).

The invertible transformation Q given by

$$x^* \triangleq Q \begin{bmatrix} i_a^* \\ i_b^* \\ \omega \\ \theta \end{bmatrix} \triangleq \begin{bmatrix} 0 & 0 & 0 & 1/K_3 \\ 0 & 0 & 1/K_3 & 0 \\ 0 & 1 & -K_4/K_3 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_a^* \\ i_b^* \\ \omega \\ \theta \end{bmatrix} \quad (3.3)$$

takes the system (3.1) and puts it into control canonical form. Furthermore by adding the feedback term  $-Mx^*$  to  $[v_1, v_2]^T$  in (3.2) results in the system (2.8).

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