A Feedback Linearization Controller for Trajectory Tracking of the Furuta Pendulum*

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Abstract— A Furuta pendulum is a two degrees-of-freedom mechanism consisting of an arm rotating in the horizontal plane and a pendulum rotating in the vertical plane. The pendulum is attached to the arm tip. The complexity in the control of this system relies on that only the arm is actuated. The problem addressed in this paper consists in the design of a controller such that the difference between the desired joint position and the actual one is uniformly ultimately bounded. In particular, the desired arm position is a time varying signal and the desired pendulum position is zero. Roughly speaking, this has the physical meaning that the arm should be moving while the pendulum should be kept at the upward position. To satisfy that control goal, in this document a new controller based on the feedback linearization technique is introduced. The new scheme has been experimentally compared with respect to a known algorithm.

I. INTRODUCTION

The rotary inverted pendulum, which is better known as Furuta pendulum named in inventor's honor [1], is a well-known underactuated mechanical system that is used extensively by many control researchers to test linear and non linear techniques [2], [3]. This mechanism consists in an arm rotating in the horizontal plane and pendulum rotating in the vertical plane. The system has only one actuator that provides torque $\tau \in \mathbb{R}$ at the arm.

Feedback linearization is a commonly control technique used in non linear systems; see for example [4] and [5]. A constructive methodology to control the Furuta pendulum by means of feedback linearization and Lyapunov design was presented in [6]. However, the methodology to calculate the zero-dynamics of the system disagrees the theory introduced in [4] and [5]. Moreover, in [7] an output tracking nonlinear controller was presented for a frictionless model of the Furuta pendulum; notwithstanding, again the zero-dynamics was not obtained according to the theory available. Finally, in [8] an adaptive fuzzy controller based on a feedback linearizing scheme was presented for the inverted-pendulum on a cart system. But again, the zero-dynamics was not determined according to the theory reported on the feedback linearization.

The control problem addressed in this paper is to keep the error trajectories uniformly ultimately bounded (UUB). The purpose of this document is to introduce a new controller, which is obtained by the feedback linearization technique,

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and to present a real-time evaluation of the performance with respect to the controller introduced in [7].

The Furuta pendulum dynamics and the control problem formulation are given in Section II. A new controller is proposed in Section III. The closed-loop system and zero dynamics are derived in Section IV. In Section V, to compare the performance of the new controller, the output tracking controller presented in [7] is revised and friction compensation is added with the aim to improve its performance. Subsequently in Section VI, by using an original experimental platform, which has been accurately identified, the real-time experimental tests are presented, as well as the performance comparative using the RMS error. Finally, some concluding remarks are provided in Section VII.

II. FURUTA PENDULUM DYNAMIC MODEL AND CONTROL PROBLEM FORMULATION

A. Model

As previously described, the Furuta pendulum is a mechanism consisting of an arm rotating in the horizontal plane and pendulum rotating in the vertical plane. See Fig. 1 for a description of the relative joint angle measurements and torque application.
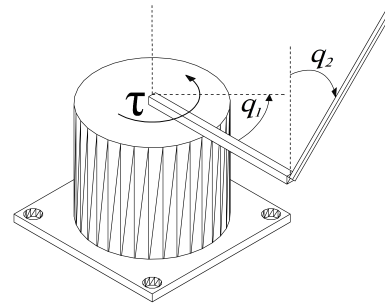

Fig. 1. Furuta pendulum.

The dynamic model of the Furuta pendulum in Euler-Langrange form is written as [2], [9], [10],

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{f}_v(\dot{\boldsymbol{q}}) + \boldsymbol{f}_c(\dot{\boldsymbol{q}}) = \boldsymbol{u}, \quad (1)$$

where

$$\boldsymbol{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \text{ and } \boldsymbol{u} = \begin{bmatrix} \tau \\ 0 \end{bmatrix}$$

are the vector of joint position and the torque input vector, respectively, being $\tau \in \mathbb{R}$ the torque input of the arm,

$$M(\boldsymbol{q}) = \begin{bmatrix} \theta_1 + \theta_2 \sin^2(q_2) & \theta_3 \cos(q_2) \\ \theta_3 \cos(q_2) & \theta_4 \end{bmatrix},$$

$$C(\boldsymbol{q}, \dot{\boldsymbol{q}}) =$$
$$\left[\begin{array}{cc} \frac{1}{2}\theta_2 \dot{q}_2 \sin(2q_2) & -\theta_3 \dot{q}_2 \sin(q_2) + \frac{1}{2}\theta_2 \dot{q}_1 \sin(2q_2) \\ -\frac{1}{2}\theta_2 \dot{q}_1 \sin(2q_2) & 0 \end{array} \right],$$

$$\boldsymbol{g}(\boldsymbol{q}) = \left[\begin{array}{c} 0 \\ -\theta_5 \sin(q_2) \end{array} \right], \ \boldsymbol{f}_v(\dot{\boldsymbol{q}}) = \left[\begin{array}{c} \theta_6 \dot{q}_1 \\ \theta_7 \dot{q}_2 \end{array} \right],$$

$$\boldsymbol{f}_c(\dot{\boldsymbol{q}}) = \left[\begin{array}{c} \theta_8 \tanh(\beta \dot{q}_1) \\ \theta_9 \tanh(\beta \dot{q}_2) \end{array} \right],$$

where $M(\boldsymbol{q}) \in \mathbb{R}^{2 \times 2}$ is the positive definite inertia matrix and $C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \in \mathbb{R}^2$ is the centrifugal and Coriolis torque vector, $\boldsymbol{g}(\boldsymbol{q}) \in \mathbb{R}^2$ is known as the gravitational torque vector, $\boldsymbol{f}_v(\dot{\boldsymbol{q}}) \in \mathbb{R}^2$ is the vector containing the viscous friction terms of each joint, and $\boldsymbol{f}_c(\dot{\boldsymbol{q}}) \in \mathbb{R}^2$ is continuous and differentiable version of the Coulomb friction vector with $\beta > 0$ large enough. The constant parameters θ_i, with $i = \{1, 2, 3, ..., 9\}$ are related with the physical characteristics of the Furuta pendulum model.

The physical meaning of the input vector $\boldsymbol{u} \in \mathbb{R}^2$ is that the system is equipped with one actuator only, which delivers the torque input $\tau \in \mathbb{R}$.

B. Control Problem

First, let us define the following signals

$$\boldsymbol{e} = \left[\begin{array}{c} e_1 \\ e_2 \end{array} \right] = \left[\begin{array}{c} q_{d1} - q_1 \\ -q_2 \end{array} \right], \tag{2}$$

where $q_{d1}(t)$ is twice-differentiable signal that denotes the desired angular position of the arm.

The control problem consists in designing a controller $\tau \in \mathbb{R}$ such that the error trajectories $\boldsymbol{e}(t) \in \mathbb{R}^2$ satisfies the definition of a UUB signal. In other words, the controller should guarantee

$$\|\boldsymbol{e}(0)\| < a \ \Rightarrow \ \|\boldsymbol{e}(t)\| \leq b \ \forall \ t \geq t_0 + T. \tag{3}$$

III. PROPOSED SCHEME

A. Controller Derived from Feedback Linearization

In order to derive the feedback linearization controller, we express the open-loop dynamics (1) as

$$\frac{d}{dt}\dot{\boldsymbol{q}} = \boldsymbol{f}_z(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}_z(\boldsymbol{q})\tau, \tag{4}$$

where $\boldsymbol{f}_z = [f_{z1} \ f_{z2}]^T$ is the part of the Furuta pendulum dynamics that is not related whit the control input τ, given by

$$\boldsymbol{f}_z(\boldsymbol{q}, \dot{\boldsymbol{q}}) = M(\boldsymbol{q})^{-1}[-C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{f}_v(\dot{\boldsymbol{q}}) - \boldsymbol{f}_c(\dot{\boldsymbol{q}})], \tag{5}$$

and $\boldsymbol{g}_z = [g_{z1} \ g_{z2}]^T$ is the part of the Furuta pendulum dynamics that is directly related with the control input, such that

$$\boldsymbol{g}_z(\boldsymbol{q}) = \frac{1}{\det M} \left[\begin{array}{c} M_{22} \\ -M_{12} \end{array} \right]. \tag{6}$$

By using the definition of $\boldsymbol{e}$ in (2), the open-loop error dynamics can be written as

$$\frac{d}{dt}\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\tau, \tag{7}$$

where

$$\boldsymbol{f}(\boldsymbol{x}) = \left[\begin{array}{c} \dot{e}_1 \\ \dot{e}_2 \\ \ddot{q}_{d1} - f_{z1} \\ -f_{z2} \end{array} \right], \tag{8}$$

$$\boldsymbol{g}(\boldsymbol{x}) = \left[\begin{array}{cccc} 0 & 0 & -g_{z1} & -g_{z2} \end{array} \right]^T, \tag{9}$$

$$\boldsymbol{x} = \left[\begin{array}{cccc} e_1 & e_2 & \dot{e}_1 & \dot{e}_2 \end{array} \right]^T. \tag{10}$$

Feedback linearization is a control technique commonly used in non linear systems. This approach consists in the transformation of the non linear system into an equivalent system through a proper output signal.

Now, we propose the output function as

$$y = \Delta_1 e_1 + \Delta_2 e_2 + \dot{e}_1 + \dot{e}_2, \tag{11}$$

where Δ_1 and Δ_2 are positive constants.

According to Definition 6.7, page 244 in [5], for the output function defined as in (11) and the system in (7), the relative degree of the system is $r = 1$. Then, we have that the control input given by

$$\tau = \frac{-\Delta_1 \dot{e}_1 - \Delta_2 \dot{e}_2 - \ddot{q}_{d1} + f_{z1} + f_{z2} + v}{-[g_{z1} + g_{z2}]}, \tag{12}$$

results in a linear differential relation between the output y and a new function v to be defined, i.e.,

$$\frac{d}{dt}y = v. \tag{13}$$

The output in (11) can be turned into an exponentially convergent signal by defining

$$v = -K_p y, \tag{14}$$

where K_p is a positive constant.

Notice that the controller (12) is valid in a region of the state space where

$$g_{z1} + g_{z2} \neq 0.$$

However, based on the Furuta pendulum model and the numerical values of the parameters θ_3 and θ_4 to be presented later, we have

$$g_{z1} + g_{z2} < 0, \ \forall \ |q_2| < \arccos(\theta_4/\theta_3). \tag{15}$$

The dynamics of the output $y(t)$ is given by (13) and (14), for which we have that

$$\lim_{t \to \infty} y(t) = 0,$$

with exponential convergence rate.

IV. ZERO DYNAMICS

When a feedback linearization controller is applied to a nonlinear system, it is necessary to analyze the zero-dynamics of the system (a particular case of the internal dynamics). The zero-dynamics of the nonlinear system (7)-(11) is the dynamics of the system subjected to the constraint that the output be identically zero [5].

In agreement to the feedback linearization control technique, and for relative degree $r = 1$, we have for the

zero-dynamics analysis a coordinate transformation vector $\boldsymbol{z} = [\boldsymbol{\eta}^T \ \zeta_1]^T$, where $\zeta_1 = y(\boldsymbol{x})$ and the structure of the vector $\boldsymbol{\eta}$ is

$$\boldsymbol{\eta} = \left[\begin{array}{ccc} \eta_1 & \eta_2 & \eta_3 \end{array} \right]^T, \tag{16}$$

with each of the elements of the vector $\boldsymbol{\eta}$ linearly independent and satisfying the partial differential equation system

$$L_g \eta_i = \frac{\partial \eta_i}{\partial \boldsymbol{x}} \boldsymbol{g}(\boldsymbol{x}) = 0, \qquad i = 1, 2, 3, \tag{17}$$

where $\boldsymbol{g}(\boldsymbol{x})$ is defined in (9). Given the above, we propose a state transformation defined as

$$\boldsymbol{z} = H\boldsymbol{x}, \tag{18}$$

where

$$H = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{g_{z2}}{g_{z1}} & 1 \\ \Delta_1 & \Delta_2 & 1 & 1 \end{array} \right],$$

and $\boldsymbol{x} \in \mathbb{R}^4$ defined in (10).

The internal dynamics of the system simply corresponds to the equations

$$\dot{\boldsymbol{\eta}} = \boldsymbol{w}(t, \zeta_1, \boldsymbol{\eta}) \tag{19}$$

of the normal form. The closed-loop system is defined by (13)-(14) and (19). The zero-dynamics is given by

$$\dot{\boldsymbol{\eta}} = \boldsymbol{w}(t, 0, \boldsymbol{\eta}). \tag{20}$$

From (18), we obtain the expression of the internal dynamics of the Furuta pendulum as

$$\dot{\boldsymbol{\eta}} = \left[\begin{array}{c} \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \end{array} \right] = \left[\begin{array}{c} \frac{d}{dt} e_1 \\ \frac{d}{dt} e_2 \\ \frac{d}{dt} \left[-\frac{g_{z2}}{g_{z1}} \dot{e}_1 + \dot{e}_2 \right] \end{array} \right], \tag{21}$$

by calculating the time derivative, substituting $\dot{e}_1$ and $\dot{e}_2$ from the inverse transformation of (18), substituting g_{z1} and g_{z2} obtained from (6), taking into account $\eta_1 = e_1$ and $\eta_2 = e_2$, and considering $\zeta_1 = 0$, the zero-dynamics is expressed as

$$\left[\begin{array}{c} \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \end{array} \right] =$$

$$\left[\begin{array}{c} -\Delta_1 G_1 \eta_1 - \Delta_2 G_1 \eta_2 - G_1 \eta_3 \\ -\Delta_1 G_2 \eta_1 - \Delta_2 G_2 \eta_2 + G_1 \eta_3 \\ \frac{\theta_3}{\theta_4} \cos(\eta_2) \ddot{q}_{d1} - \frac{\theta_3}{\theta_4} \cos(\eta_2) f_{z1} - \frac{\theta_3}{\theta_4} \sin(\eta_2) \dot{\eta}_1 \dot{\eta}_2 - f_{z2} \end{array} \right], \tag{22}$$

where

$$G_1 = \frac{g_{z1}}{g_{z1} + g_{z2}}, \ G_2 = \frac{g_{z2}}{g_{z1} + g_{z2}}.$$

It can be shown that a sufficient condition for the trajectories $\boldsymbol{\eta}(t)$ to be UUB is that

$$\Delta_2 > \kappa(\theta_i, \Delta_1), \tag{23}$$

where $\kappa(\theta_i, \Delta_1)$ is a function of the Furuta pendulum parameters θ_i, which are constants, and of the gain Δ_1.

However, the explicit proof of the derivation of (23) will be left out for shortening. In fact, the inequality (23) provides an explicit tuning guideline, which consists in selecting Δ_2 large enough. Also, it is possible to proof that, given the exponential convergence of the external dynamics trajectories $\zeta_1(t)$ and the uniformly ultimately boundedness of the internal dynamic trajectories $\boldsymbol{\eta}(t)$, the error state trajectories $\boldsymbol{x}(t)$ are UUB as well.

V. OUTPUT TRACKING NONLINEAR CONTROLLER

In this section we present the controller presented in [7]. We introduce terms corresponding to viscous friction and Coulomb friction in order to improve the controller performance and perform a comparison with respect to the proposed feedback linearization controller.

Following the procedure of design presented in [7], this is, considering $\theta_2 = \theta_4$ for the constant parameters of the Furuta pendulum dynamic model in (1), and defining an output function $Z(t)$ as

$$Z = r^* q_1 + h \sin(q_2), \tag{24}$$

it is possible to obtain the output tracking controller with friction compensation

$$\begin{aligned} \tau = {} & (M(q_2)h - \theta_3)\sin(q_2)\dot{q}_2^2 + \theta_2 \sin(2q_2)\dot{q}_1 \dot{q}_2 \\ & + \theta_2 G(q_2)\sin(q_2)\cos^2(q_2)\dot{q}_1^2 + \theta_5 G(q_2)\sin(q_2)\cos(q_2) \\ & - G(q_2)\cos(q_2)\theta_7 \dot{q}_2 - G(q_2)\cos(q_2)\theta_9 \tanh(\beta \dot{q}_2) \\ & + \theta_6 \dot{q}_1 + \theta_8 \tanh(\beta \dot{q}_1) \\ & - K_1 M(q_2)\dot{Z} - K_2 M(q_2)Z + M(q_2)v, \end{aligned} \tag{25}$$

where

$$\begin{aligned} G(q_2) & = \frac{(\theta_1 + \theta_2 \sin^2(q_2))h - \theta_3 r^*}{\theta_3 h \cos^2(q_2) - \theta_2 r^*}, \\ M(q_2) & = \frac{1}{r^*}[(\theta_1 + \theta_2 \sin^2(q_2)) - G(q_2)\theta_3 \cos^2(q_2)], \\ v & = \ddot{Z}_d + K_1 \dot{Z}_d + K_2 Z_d - K_3 \dot{q}_2, \end{aligned}$$

with r^* and h being positive constants, and $Z_d(t)$ the desired output. Notice that, the controller in (25) corresponds to the controller in [7], but viscous and Coulomb friction are incorporated.

By following the steps in [7], it is possible to show that the trajectories $Z_e(t) = Z(t) - Z_d(t)$ and $q_2(t)$ are UUB, which also implies that $e_1(t) = q_{d1}(t) - q_1(t)$ is UUB. The proof of this fact will be left out for shortening.

VI. EXPERIMENTAL RESULTS

A. Experimental Platform

In this section, the real-time implementation of the proposed controller (12) and the output tracking controller with friction compensation in (25) are presented. The experimental tests have been conducted in a Furuta pendulum built at the *Instituto Politécnico Nacional-CITEDI* research center. See Fig. 2 for picture of the experimental system.

The constant parameters θ_i of the Furuta pendulum model in (1) have been identified. These parameters are shown in

Fig. 2. Furuta pendulum prototype built at *Instituto Politécnico Nacional-CITEDI* research center.

Table I, which were obtained by using the filtered dynamic model and the classical least squares identification; see for instance [11], [12] and [13], where identification procedures for mechanical systems are proposed. In the identification process, we considered $\beta = 100$, which is related to the vector of Coulomb friction $\boldsymbol{f}_c(\dot{\boldsymbol{q}}) \in \mathbb{R}^2$ in the Furuta pendulum model in (1).

TABLE I
NUMERICAL VALUES OF THE FURUTA PENDULUM PARAMETERS

Symbol	Value	Unit	Symbol	Value	Unit
θ_1	0.0619	$\mathrm{Kg\,m^2\,rad}$	θ_6	0.0083	$\mathrm{N\,m\,rad/s}$
θ_2	0.0149	$\mathrm{Kg\,m^2\,rad}$	θ_7	0.0007	$\mathrm{N\,m\,rad/s}$
θ_3	0.0185	$\mathrm{Kg\,m^2\,rad}$	θ_8	0.0188	$\mathrm{N\,m\,rad/s}$
θ_4	0.0131	$\mathrm{Kg\,m^2\,rad}$	θ_9	0.0087	$\mathrm{N\,m\,rad/s}$
θ_5	0.5076	$\mathrm{Kg\,m^2\,rad}$	$--$	$--$	$--$

The joint velocity is estimated through discrete differentiation from joint position measurements.

The initial condition of the experimental system was

$$\begin{bmatrix} q_1(0) \\ q_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ [rad], and } \begin{bmatrix} \dot{q}_1(0) \\ \dot{q}_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ [rad/s].}$$

In order to compare the tracking performance of the feedback linearization controller (FLC) in (12) with respect to the output tracking controller (OTC) in (25), the desired joint trajectory $q_{d1}(t)$ for the arm position was defined as

$$q_{d1}(t) = \sin(t).$$

B. Results for the Feedback Linearization Controller

Here, the results for the FLC in (12) are described. The control gains concerning the output function y in (11), we used $\Delta_1 = 6.0$, $\Delta_2 = 8.0$, and $K_p = 2.0$ in the FLC (12).

The experimental results are illustrated in Fig. 3-6. In particular, the Fig. 3 shows the time evolution of $q_1(t)$ and $q_{d1}(t)$, and the Fig. 4 depicts the time evolution of $q_2(t)$. The applied torque $\tau(t)$ and the output signal $y(t)$ are observed in Fig. 5 and 6, respectively.

The obtained control action $\tau(t)$ has high frequency components. This is attributed to the PWM switching of the servo amplifier and to the discrete velocity estimation.
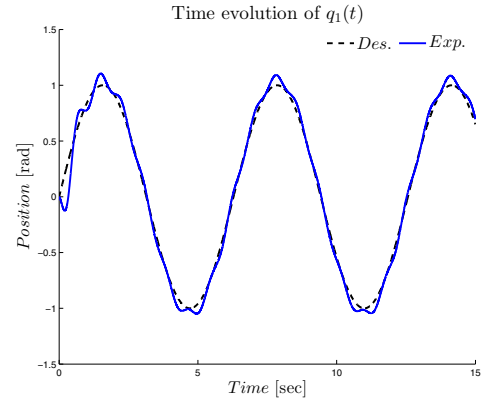

Fig. 3. **FLC**: Time evolution of $q_1(t)$ and $q_{d1}(t)$ obtained by experiment.
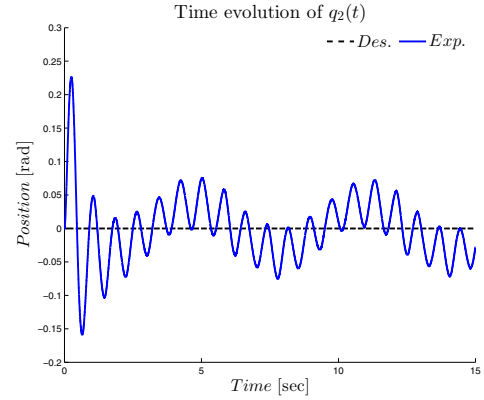

Fig. 4. **FLC**: Time evolution of $q_2(t)$ obtained by experiment.

Particularly, in Fig. 6 the response of the output function $y(t)$ in (11) is appreciated. Although theory predicts exponential convergence of $y(t)$, an oscillatory behavior with high frequency components is presented. It is noteworthy that the output function $y(t)$ in (11) depends of the joint velocity $\dot{\boldsymbol{q}}(t) \in \mathbb{R}^2$. The main reason for the oscillatory behavior of $y(t)$ is that the joint velocity $\dot{\boldsymbol{q}}(t)$ is estimated via the "dirty" derivative algorithm and the relative high value of the gains Δ_1 and Δ_2.

By means of numerical simulation assuming a continuous time implementation of the controller and non quantized position and velocity measurements, the exponential convergence of $y(t)$ was corroborated.

C. Results for the Output Tracking Controller

For the experimental implementation of the scheme OTC in (25), for which the desired trajectory $q_{d1}(t)$ is defined in relation with the desired output $Z_d(t)$, we have

$$q_{d1}(t) = Z_d/r^*,$$
$$Z_d = r^* \sin(t),$$

Concerning to the output function $Z(t)$ in (24), we used $r^* = 0.55$ and $h = 0.60$. For the OTC in (25), the control gains that showed better experimental performance were $K_1 = 6.0$, $K_2 = 8.0$ and $K_3 = 1.0$.
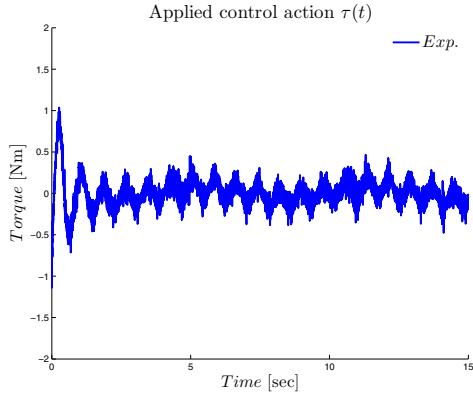
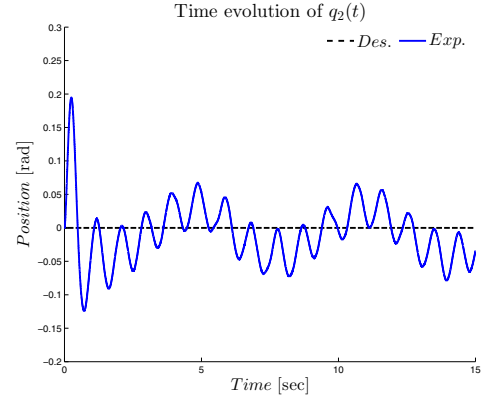Fig. 5. **FLC**: Time evolution of control input $\tau(t)$ obtained by experiment.

Fig. 6. **FLC**: Time evolution of the output signal $y(t)$ obtained by experiment.

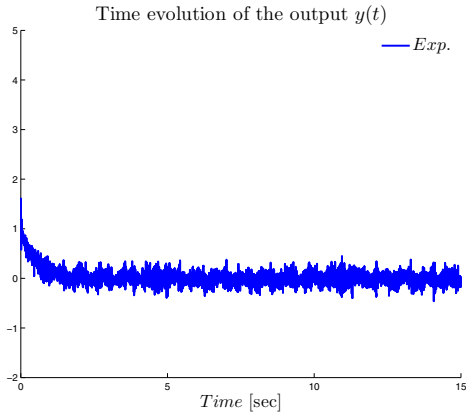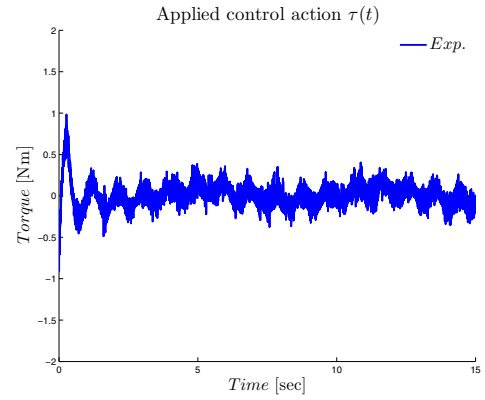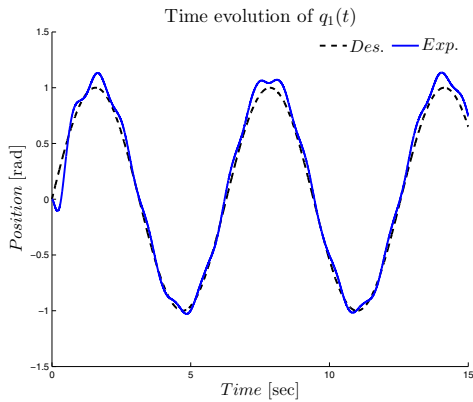Fig. 7. **OTC**: Time evolution of $q_1(t)$ and $q_{d1}(t)$ obtained by experiment.

Fig. 8. **OTC**: Time evolution of $q_2(t)$ obtained by experiment.

Fig. 9. **OTC**: Time evolution of control input $\tau(t)$ obtained by experiment.
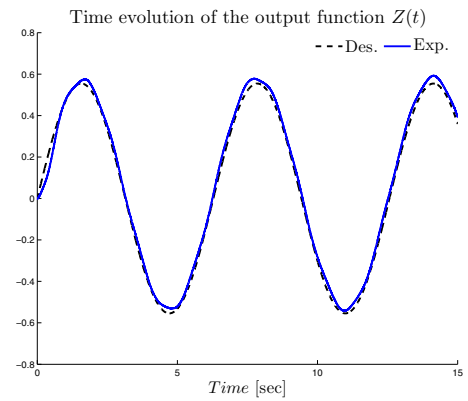

Fig. 10. **OTC**: Time evolution of the output signal $Z(t)$ obtained by experiment.
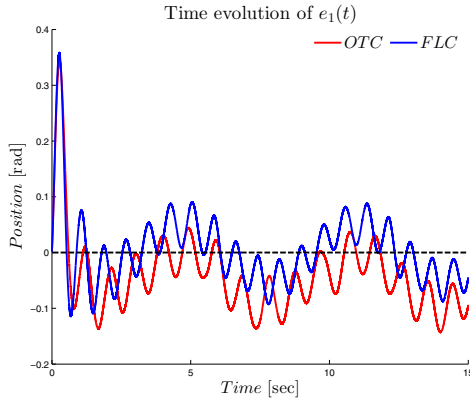
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Fig. 11. **Experimental comparative**: Time evolution of error $e_1(t)$ for both controllers.
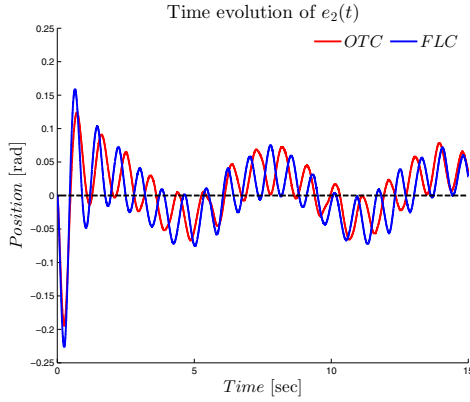

Fig. 12. **Experimental comparative**: Time evolution of error $e_2(t)$ for both controllers.

The experimental results are illustrated in Fig. 7-10. In particular, the Fig. 7 shows the time evolution of $q_1(t)$ and $q_{d1}(t)$, and the Fig. 8 depicts the time evolution of $q_2(t)$. The applied torque $\tau(t)$ and the output signal $Z(t)$ are observed in Fig. 9 and 10, respectively.

D. Performance Comparative

The discussion of the experimental results for the controllers is now presented. With this aim, in Fig. 11 the tracking error $e_1(t)$ for both implementations is given. Likewise, in Fig. 12 the regulation error of the pendulum $e_2(t)$ for both implementations is presented.

To obtain a better interpretation of the information presented in Fig. 11-12, in Table II the RMS values of $e_1(t)$, $e_2(t)$ and $\tau(t)$ for both controllers are shown. Let us notice that the RMS value of a signal is less affected than other norms by large but infrequent values of the signal [14]. To calculate the RMS values shown in Table II, the time interval considered is $10 \leq t \leq 15$, with the purpose of eliminating the transitory response.

As can be seen in Table II, the FLC in (12) performs better than the OTC in (25). The FLC reduces the RMS value of $e_1(t)$ by 34.25% with respect to the OTC algorithm, while the RMS value of $e_2(t)$ is reduced by 0.82%. This at the

cost of a slightly greater control action for the FLC, with the RMS value of $\tau(t)$ increased by 3.86%.

TABLE II
COMPARISON OF THE RMS VALUE FOR $10 \leq t \leq 15$

Index	OTC	FLC	*%
$\|e_1(t)\|_{rms}$ [rad]	0.0721	0.0474	34.25%
$\|e_2(t)\|_{rms}$ [rad]	0.0381	0.0378	0.84%
$\|\tau(t)\|_{rms}$ [Nm]	0.1155	0.1200	-3.86%

*Percentage of improvement of FLC with respect to OTC

VII. CONCLUSIONS

A new controller based on the feedback linearization technique has been introduced in this paper. The output function $y(t)$ was selected as a linear combination of the position and velocity tracking errors. In fact, the output function $y(t)$ is inspired from the filtered tracking error used in passivity-based controllers for fully actuated mechanical systems.

The main theoretical result consisted in proving that the output function $y(t)$ converges to zero in an exponential form while the trajectories of the zero-dynamics are UUB. Real-time experiments confirmed the validity of the main result.

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