Sensorless Speed and Position Estimation in a Stepper Motor

Azzeddine Ferrah, Jehad Al-Khalaf Bani-Younes, Mounir Bouzguenda, and Abdelkader Tami
Faculty of Engineering, Sohar University, Sohar, PO Box: 44, PC: 311,
Sohar, Sultanate of Oman, a.ferrah@soharuni.edu.om

***Abstract* - In this paper, the application of the extended Kalman filter (KF) to estimate the states of a stepper motor is described. A dynamic model, describing the operation of a two-phase stepper motor, was derived and used for sensorless speed and position measurement. The simulation results of full state estimation, using full order extended Kalman filter, is presented. The effects of noise and the sensitivity of the algorithm to motor parameters are thoroughly investigated.**

***Index Terms*– Kalman filter, sensorless, speed, position, stepper motor**

I. Introduction

Recently, there has been an increase in the case of AC motors in positioning applications. This increase is attributed to developments in power electronics and in computing technology. The faster computing capabilities have made it possible to perform more complex calculations in a shorter period of time. These advances have opened up new opportunities for advanced control methods such as nonlinear and optimal control. In addition, new opportunities exist for self-tuning and adaptive methods to determine unknown or slowly varying parameters. These methods are useful to automatically adjust the control system to maximize performance with minimal or no operator intervention.

Traditionally, DC motors have been chosen for industrial applications over stepper motors due to their linear input/output relationship, which in turn, allows for the use of standard control strategies. Over the past few years, the use of stepper motors has increased. The reasons for this include: better reliability due to the elimination of mechanical brushes, better heat dissipation as the windings are located on the stator and not on the rotor, higher torque-to-inertia ratio due to a lighter rotor, and lower prices.

Stepper motors were originally designed to be used in open loop control. Their inherent stepping ability allows for accurate positioning without feedback. An example of optimal open-loop control for a variable-reluctance stepper motor was demonstrated in [1]. Closed-loop control of stepper motors has been used increasingly in the last two decades to achieve faster response times and higher resolution capabilities. Adaptive control was successfully demonstrated in [2] to achieve higher precision by canceling torque-ripple effects. The stepper motor can also be operated at higher speeds, by taking nonlinear effects into consideration [3].

Kalman filter is certainly one of the greatest discoveries in the history of statistical estimation theory [4]. It has enabled engineers and scientists to do many things that could not have been done without it, and it has become as indispensable as silicon in the make up of many electronic systems. Its most immediate applications have been for the control of complex dynamic systems such as manufacturing processes, aircraft, ships, or spacecraft. The Kalman filter is also used for predicating the future courses of dynamic systems that people are not likely to control, such as the flow of rivers during flood, the trajectories of celestial bodies, or the prices of traded commodities.

In the present paper KF is applied to the estimation of speed and position of a two-phase stepper motor. The paper presents a brief review of the basics of stepper motors and provides a complete dynamic model of a two phase stepper motor. Simulation results of KF application to the problem in hand and their discussion is also provided.

II. STEPPER MOTOR DRIVE SYSTEM

A simple drive system for a stepping motor is represented by the block diagram in Fig. 1, the number of phases being four in this example. The block diagram is divided into two portions for convenience of explanation. Fig. 1a represents the portion from logic sequence to motor. When a step command pulse is applied to the logic sequencer, the states of the output terminal are changed to control the motor driver so as to rotate the motor a step angle in the commanded direction, the rotational direction is determined by the logic state at direction input, e.g. the H level for CW and L for the CCW direction input. In the same application, the logic sequence is unidirectional, having no direction signal terminal. If one increment of movement is performed by one step, the block diagram of Fig. 1a represents the whole system. But, when an increment is performed by two or more steps, another stage to produce a proper train of pulses is needed to put before the logic sequencer, and this is represented in Fig. 1b. This logic circuit is termed the “input controller”. In sophisticated applications the function of input controller is carried out pulse train to speed up, slew, and slow down the motor in the most efficient and reliable manner [5].

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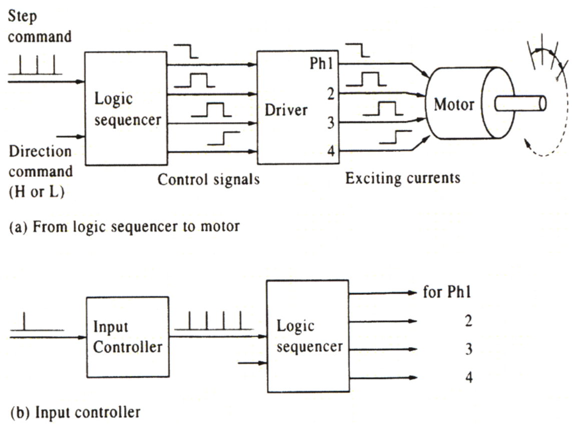

Fig. 1. Block diagram of a stepping motor drive system.

Here the step command pulses are given from an external source, and it is expected that the stepping motor is able to follow every pulse. This type of operation is referred to as the ‘open–loop’ drive. The open–loop drive is attractive and widely accepted in applications of speed and position controls. However, the performance of a stepping motor is limited under the open-loop mode. The drive may fail to follow a pulse command when the frequency of the pulse train is too high or inertial load is too heavy. Moreover the motor motion tends to be oscillatory in open–loop drives.

The performance of stepping motor can be improved to a great extent by employing position feedback and/or speed feedback to determine the proper phase(s) to be switched at proper timings. This type of control is termed ‘closed–loop’ drive. The closed–loop control is advantageous over the open-loop control not only in that step failure never occurs but also that the motion is much quicker and smoother [5].

In closed-loop control, a position sensor is needed for detecting the rotor position. As a typical sensor, nowadays, an optical encoder is used and it is usually coupled to the motor shaft. Position and speed sensors have some disadvantages:

- They are usually expensive.
- The sensor and the corresponding wires will take up space.
- In defective and aggressive environments, the sensor might be the weakest part of the system.

Especially, the last item degrades the system’s reliability and reduces the advantage of a closed-loop drive system. In the more advanced systems, instead of an additional mechanical sensor, rotor position and/or speed is sensed by observation of waveforms of the currents in the motor windings.

On the other hand, avoiding sensor means use of additional algorithms and added computational complexity that requires high-speed processors for real time application. As digital signal processors have became cheaper, and their performance greater, it has become possible to use them for controlling electrical drives as a cost effective solution. Some relatively new fully digitized methods, used for sensorless speed/position estimation, utilize this enhanced processing capacity [6].

Usually sensorless control is defined as a control scheme where no mechanical parameters like, speed and torque, are measured. In recent years, nonlinear observers are used to estimate parameters and states of electrical machines. Model-based Sensorless schemes, generally, rely on accurate system modeling and accurate model parameters values.

III. DYNAMIC MODELLING OF A STEPPER MOTOR

We consider a two-phase permanent magnet (PM) stepper motor. A simplified schematic of a motor with one pole-pair is shown in Fig. 1. Commanded voltages $(v_a \text{ and } v_b)$ control the two-phase currents (i_a and i_b). The magnetic field, due to the stator current, interacts with the permanent magnet on the rotor to create a torque, so that the rotor will tend to align itself with the magnetic field produced by the currents. Applying a sequence of voltage to each phase in succession will cause the rotor to step. The size of each step is 90° in the case of Fig. 1. In general, it is determined by n, the number of rotor teeth. A sinusoidal characteristic of the magnetic field in the air gap is assumed.

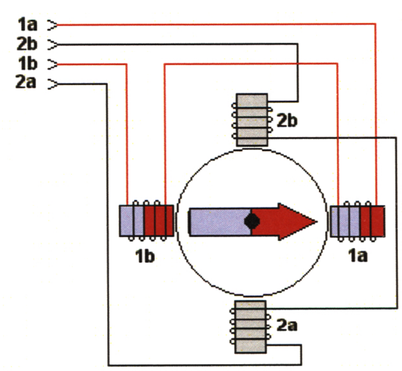

Fig. 2. Diagram of a two-phase stepper motor [7].

The dynamic equations, derived in [8], are cast in state-space form as follows:

$$\frac{d}{dt}\begin{bmatrix} i_a \\ i_b \\ \omega \\ \theta \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & 0 & \frac{\lambda}{L}\sin\theta & 0 \\ 0 & \frac{-R}{L} & \frac{-\lambda}{L}\cos\theta & 0 \\ -\frac{\lambda}{J}\sin\theta & \frac{\lambda}{J}\cos\theta & \frac{-B}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} . \begin{bmatrix} i_a \\ i_b \\ \omega \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} . \begin{bmatrix} v_a & v_b & 0 & 0 \end{bmatrix}, \tag{1}$$

$$\begin{bmatrix} \omega \\ \theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ \omega \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} v_a & v_b \end{bmatrix} \tag{2}$$

where $\begin{bmatrix} \omega \\ \theta \end{bmatrix}$ is the output vector, and

R is the resistance of the coils.
L is the inductance of the coils.
λ is the motor constant depending on the design of the rotor.
J is the inertia of the rotor and the load.
$\theta(t)$ is the actual rotor position.
θ_{0a} is the location of the coil a in the stator.
ω is the rotational velocity of the rotor.
$i_a(t)$ is the current in the coil as function of time.

The model described above conforms to the general state-space model given by

$$\dot{x} = Ax + Bu \tag{3}$$

$$y = Cx + Du \tag{4}$$

x, **u** and **y** are called the state, input and output vectors, respectively.

where

A = $\begin{bmatrix} \frac{-R}{L} & 0 & \frac{\lambda}{L}\sin\theta & 0 \\ 0 & \frac{-R}{L} & \frac{-\lambda}{L}\cos\theta & 0 \\ -\frac{\lambda}{J}\sin\theta & \frac{\lambda}{J}\cos\theta & \frac{-B}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is called the state matrix,

B = $\begin{bmatrix} \frac{1}{L} \\ \frac{1}{L} \\ 0 \\ 0 \end{bmatrix}$ is called the input matrix,

C = $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is called output matrix, and

D = $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is the direct transmission matrix.

IV. STATES ESTIMATION USING KALMAN FILTER

A. Definition of Kalman Filter

Theoretically the Kalman filter is an estimator for what is called the linear quadratic problem, which is the problem of estimating the instantaneous "state" of a linear dynamic system perturbed by white noise by using measurement linearly related to the state but corrupted by white noise. The resulting estimator is statistically optimal with respect to any quadratic function of estimation error.

The most general nonlinear state-space model consists typically of two functions, f and h:

$$x_{k+1} = f(x_k, u_k, w_k) \tag{5}$$
$$z_k = h(x_k, v_k) \tag{6}$$

which govern state propagation and measurements respectively. Where u is process input, and w and v are state and measurement noise vector, respectively, k is the discrete time.

A linear state-space model is a model where the function f and h are linear. They can than be substituted with the matrices F, B and H, reducing state propagation calculations to linear algebra. This result in the following state-space model:

$$x_{k+1}=F_k x_k + B_k u_k + w_k \tag{7}$$
$$z_k=H_K x_k+v_k \tag{8}$$

B. BASICS OF KALMAN FILTER

Put in simple mathematical notation, KF imposes the following constraints:

$$f(x_k, u_k, w_k) =F_k x_k + B_k u_k + w_k \tag{9}$$
$$h(x_k, v_k) = H_K x_k + v_k \tag{10}$$

The covariance matrices of the noises are defined as

$$\text{cov}(w_k)=E\{w_i w_i^T\}=Q \tag{11}$$
$$\text{cov}(v_k)= E\{v_i v_i^T\}=R \tag{12}$$

where E{.} denotes the expected value . The constraints described a above reduce the model to:

$$x_{k+1}=F_k x_k + B_k u_k + w_k \tag{13}$$
$$z_k=H_k x_k+v_k \tag{14}$$

where F, B and H are matrices. This linear model is easier both to calculate and analyze, enabling easy investigation of properties such as observability and frequency response.

The state estimation technique can now be reduced to the Kalman filter, where f and h are replaced by the matrices F, B and H. The Kalman filter is decomposed into two steps: predict and update, as shown in Fig. 3.

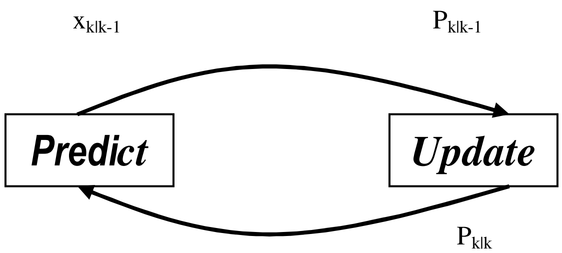

Fig. 3. Kalman filter loop.

For nonlinear processes the extended Kalman filter (EKF) is used, where the Jacobians of f and h are calculated around the estimated state as follows:

$$F_k = \left. \frac{\partial f(x,u,w)}{\partial x} \right|_{\hat{x}_{k|k}, u_k, 0} \tag{15}$$

$$H_k = \left. \frac{\partial h(x,v)}{\partial x} \right|_{\hat{x}_{k|k-1}, 0} \tag{16}$$

The EKF actual calculations required are:
Predict next state, before measurement taken:

$$\begin{aligned} \hat{x}_{k|k-1} &= f(\hat{x}_{k-1|k-1}, u_k, 0) \\ \hat{P}_{k|k-1} &= F_k \hat{P}_{k-1|k-1} F_k^{\,T} + Q_k \end{aligned} \tag{17-18}$$

Update state, after measurement taken:

$$\begin{aligned} K_k &= \hat{P}_{k|k-1} H_k^{\,T} (H_k \hat{P}_{k|k-1} H_k^{\,T} + R_k)^{-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k (z_k - h(\hat{x}_{k|k-1}, 0)) \\ \hat{P}_{k|k} &= (I - K_k H_k) \hat{P}_{k|k-1} \end{aligned} \tag{19-20}$$

where K is the Kalman gain matrix, used in the update observer, and $\hat{P}$ is the covariance matrix for the state estimation, containing information about the accuracy of the estimate.

V. SPEED AND POSITION ESTIMATION USING THE EKF

In the present work, the EKF is applied to estimate the speed and position, of a two phase stepper motor drive, using current measurements. The Jacobians of the dynamic stepper model described are calculated prior to the application of the EKF. The sensorless speed and position estimator was implemented, in Matlab, using the EKF algorithm given by (17) through (20).

For our simulation we selected three G series 34 Frame stepper motors manufactured by SmartDrive [9]. For the three motors, selected, the following parameters are used [10]:

Table I
Stepper motor parameters

	SMR341	SMR342	SMR343
R [Ω]	**0.4**	**0.62**	**0.84**
L [mH]	**1.75**	**3.1**	**4.7**
J [kg.m^2]	**1.6*10^{-4}**	**3.2*10^{-4}**	**4.8*10^{-4}**
λ [Nm/A]		**0.1**	
B [Nms/rad]		**0.001**	

Simulation is used to estimate motor speed and position using the **EKF** algorithm described in the previous section. The simulation provides the computed (true) and the estimated winding currents, i_a and i_b, together with speed, ω, and position, θ. The state estimation covariance, P, is also computed. This is an indicator of the goodness of the estimate. Effects of measurement noise and sensitivity to changes in motor parameters, R and L, are also investigated. The noise covariances R and Q used throughout the simulation are given below:

Measurement noise covariance:

$$R = \begin{bmatrix} 0.8^2 & 0 \\ 0 & 0.8^2 \end{bmatrix} \tag{21}$$

Process noise covariance:

$$Q = \begin{bmatrix} \frac{0.01^2}{L} & 0 & 0 & 0 \\ 0 & \frac{0.01^2}{L} & 0 & 0 \\ 0 & 0 & (0.5)^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

The values used in both matrices are obtained using trial and error approach. Exact values are extremely difficult to find. This requires a lengthy statistical analysis that is beyond the scope of this paper. Due to page number limitation, for this paper, only results relevant to the motor SMR 341 are presented. This motor has the following parameters: R=0.4Ω, L=1.75 mH, J=1.6*10^{-4}mH, λ=0.1 Nm/A, B=0.001 Nms/rad.

Figure 4 shows that the estimated value of rotor speed converges to the true value within 0.27 seconds. The tracking of rotor position is shown in Fig. 5. It can be seen that there is a sharp increase in the estimated value before the algorithm is able to lock on the true value. The overall estimation error is shown in Fig. 6. It is clear that good estimation is possible in less than 0.3 seconds.

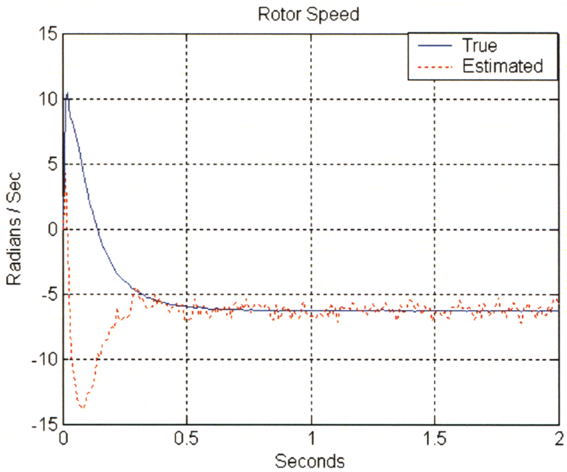

Fig. 4. Rotor speed for SMR341.

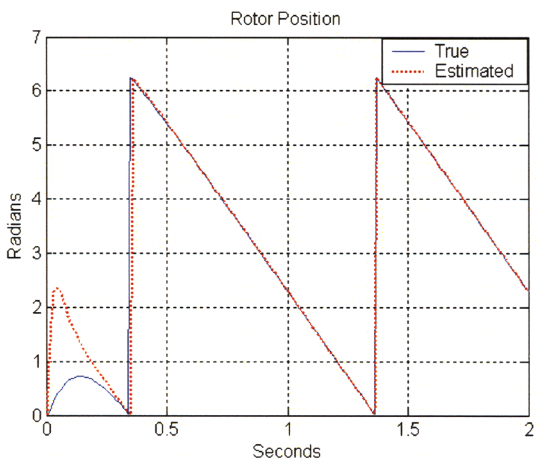

Fig. 5. Rotor position for SMR341.

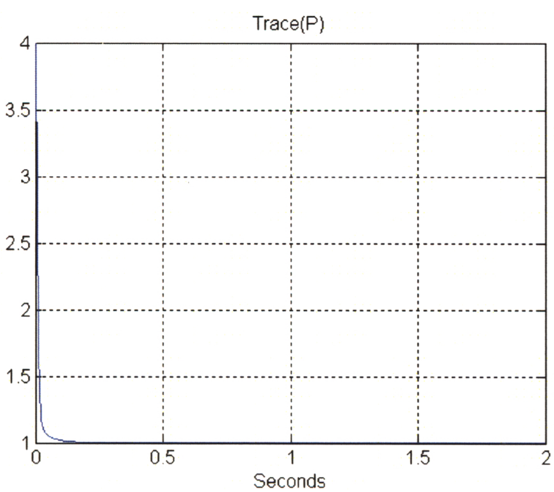

Fig. 6. Trace (P) for SMR341.

VI. EFFECTS OF MEASUREMENT NOISE

Considering the SM343, and using the process noise covariance Q given by (23), leads to deterioration of the algorithm capability in tracking the system states. This yields meaningless results as shown in Fig. 7 and 8. These figures show clearly the strong dependency of the algorithm on the process noise. Fig. 9 shows that the estimation covariance is high. This is an evidence of poor estimation. Measurement noise, R, was found to be less significant in estimating the states. Therefore, it is advisable to take a great care in choosing the elements of the matrix Q.

$$Q = \begin{bmatrix} \frac{0.09^2}{L} & 0 & 0 & 0 \\ 0 & \frac{0.09^2}{L} & 0 & 0 \\ 0 & 0 & (0.5)^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{23}$$

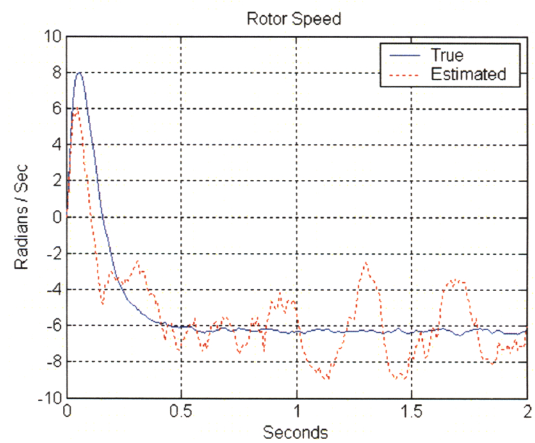

Fig. 7. Rotor speed estimation, for SMR343, showing effect of process noise.

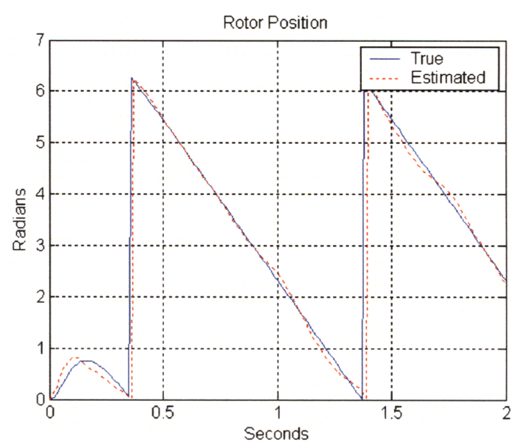

Fig. 8. Rotor position estimation, for SMR343, showing effect of process noise.

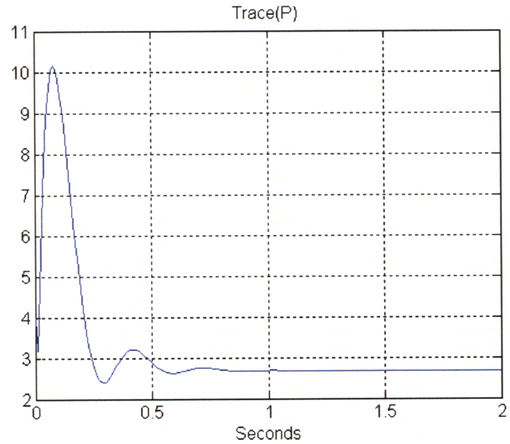

Fig. 9. Estimation covariance, for SMR343, showing effect of process noise.

VII. SENSITIVITY TO MOTOR PARAMETERS

Four cases were considered by changing R_a and L by ±10%. It has been found that increasing R_a 10% leads to a large overshoot at the onset of the tracking, as shown in Fig. 10. Then, shifted and noisy position estimation follows. Reducing R_a by the same amount will not result in overshoot, but leads to noisy estimation. However, increasing or reducing L will lead to large overshoot and distorted estimation. This is due to the fact that L is directly linked to the process noise covariance. So any change in L will lead to a change in Q. To prevent this from happening Q has to be adjusted according to any change incurred in L.

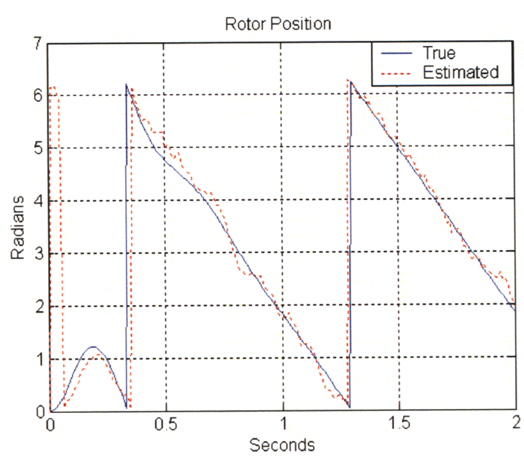

Fig. 10. Rotor position for SMR341 when increasing R_a by 10%.

VIII. CONCLUSIONS

The focus of this work was the sensorless speed and position measurement of a two-phase stepper motor. A dynamic mathematical model of the stepper motor was derived and formulated into state space model for easy analysis and manipulation. The EKF was, then, applied to the developed motor model. The model, which is inherently nonlinear, was linearized prior to the EKF application. Three motors of different sizes, whose parameters were provided in the manufacturer's data sheet, were used to test the effectiveness of the developed EKF-based sensorless algorithm. Simulation results have shown good agreement between true and estimated states. Effects of process noise and sensitivity to motor parameters were also investigated. This clearly indicated that the application of the EKF requires good evaluation of measurement noise linked with thorough understanding of the system under investigation.

The simulations carried out, in this work, show the great potential of the investigated technique. However, due to hardware and equipment limitations this technique was not tested experimentally. Therefore, it will be very beneficial and useful if the developed technique is implemented in real-time using microcontrollers. There is a variety of such chips widely available in the market.

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